实用优化算法作业(1)

1 【习题1.6】设$f(x) = e^{-x} + x^2$, 试用黄金分割法求其极小点. 取初始区间为$[-1, 1]$, $\epsilon = 0.1$

【解：】$x_2 = -1 + 0.618(1 - (-1)) = 0.236$. $f_2 = f(x_2) = 0.845$, $x_1 = -1 + 0.382(1 - (-1)) = -0.236$. $f_1 = f(x_1) = 1.322$

由于$f_1 > f_2$, 则$a = -0.236$, $x_1 = 0.236$, $f_1 = 0.845$. 令$x_2 = -0.236 + 0.618(1 - (-0.236)) = 0.528$, $f_2 = f(x_2) = 0.869$.

由于这时$f_1 < f_2$, 故有$b = x_2 = 0.528$, $x_2 = x_1 = 0.236$, $f_2 = f_1 = 0.845$. 令$x_1 = -0.236 + 0.382(0.528 - (-0.236)) = 0.0558$. $f_1 = f(x_1) = 0.949$.

由于这时$f_1 > f_2$, 故取$a = 0.0558$, $x_1 = 0.236$, $f_1 = 0.845$. $x_2 = 0.348$, $f_2 = 0.827$.

由于$f_1 > f_2$, 取$a = 0.236$, $x_1 = x_2 = 0.348$, $f_1 = 0.827$, $x_2 = 0.416$, $f_2 = 0.833$.

由于$f_1 < f_2$, 取$b = x_2 = 0.416$, $x_2 = x_1 = 0.348$, $f_2 = f_1 = 0.827$, $x_1 = 0.305$, $f_1 = 0.830$.

由于$f_1 > f_2$, 故取$a = x_1 = 0.305$, $x_1 = x_2 = 0.348$, $f_1 = f_2 = 0.827$, $x_2 = 0.374$, $f_2 = 0.827$;

由于$f_1 < f_2$, 故取$b = x_2 = 0.374$, $x_2 = x_1 = 0.348$, $f_2 = f_1 = 0.827$, $x_1 = 0.331$, $f_1 = 0.828$.

这时, $b - a = 0.069 < 0.1$, 中止算法，得近似解$x^* = 0.339$.

2 【习题3.5】试用最速下降法求解

$$\min \ x_1^2 + 2x_2^2.$$

设初始点为$x_0=(4,4)$, 迭代三次, 并验证相邻两次迭代的搜索方向是正交的.

【解：】记$f(x)=x_1^2+2x_2^2$, 则

$$g(x)=\nabla f(x)=\begin{pmatrix}2x_1\\4x_2\end{pmatrix}.$$

在初始点处$f(x^0)=48$, $g(x^0)=(8,16)^T$, 搜索方向为$p^0=-g(x^0)=(-8,-16)$. 考虑

$$\min_{a\geq 0} f(x^0+\alpha p^0)=16(3-20\alpha+36\alpha^2).$$

其极小点为$\alpha^0=5/18$. 则

$$x^1=x^0+\alpha^0 p^0=\begin{pmatrix}16/9\\-4/9\end{pmatrix}.$$

此时

$$g^1=g(x^1)=\begin{pmatrix}32/9\\-16/9\end{pmatrix}, p^1=-g^1=\begin{pmatrix}-32/9\\16/9\end{pmatrix}.$$

考虑

$$\min_{a\geq 0} f(x^1+\alpha p^1)=\frac{32}{81}(9-40\alpha+48\alpha^2).$$

其极小点为$\alpha^1=5/12$.则

$$x^2=x^1+\alpha^1 p^1=\begin{pmatrix}8/27\\8/27\end{pmatrix}.$$

此时

$$g^2=g(x^2)=\begin{pmatrix}16/27\\32/27\end{pmatrix}, p^2=-g^2=\begin{pmatrix}-16/27\\-32/27\end{pmatrix}.$$

考虑

$$\min_{a\geq 0} f(x^2+\alpha p^2)=\left(\frac{8}{27}\right)(3-20\alpha+36\alpha^2).$$

其极小点为$\alpha^2=5/18$. 则

$$x^3=x^2+\alpha^2 p^2=\begin{pmatrix}32/243\\-8/243\end{pmatrix}.$$