

Note : Attempt any five questions. All questions carry equal marks.

1. (a) Show that if $f(n) = O(n^2)$ and $g(n) = O(n^3)$, then $f(n) + g(n) = O(n^3)$ and $f(n)g(n) = O(n^6)$. 5
- (b) Show that every positive integer can be expressed as the product of prime numbers. 5
- (c) Construct an NFA with three states that accepts the language $\{ab, abc\}^*$. Comment it into DFA. 10
2. (a) Let G_1 and G_2 be two regular grammars. Show how one can derive regular grammars for the language :

$$L(G_1) \cup L(G_2)$$

- (b) Find context-free grammars for the following languages (with $n \geq 0, m \geq 0, k \geq 0$):

$$L = \{a^n b^m c^k : n = m \text{ or } m \leq k\}$$

3. (a) Consider the grammar with productions :

$$S \rightarrow aaB$$

$$A \rightarrow bBb/\lambda$$

$$B \rightarrow Aa$$

Show that the string $aabbabba$ is not in the language generated by this grammar.

- (b) Construct an NPDA corresponding to the grammar :

$$S \rightarrow aABB/aAA$$

$$A \rightarrow aBB/a$$

$$B \rightarrow bBB/A$$

4. (a) Eliminate all useless productions from the grammar :

$$S \rightarrow aS/AB, A \rightarrow bA, B \rightarrow AA$$

What language does this grammar generate ?

- (b) Transform the grammar with productions :

$$S \rightarrow abAB, A \rightarrow bAB/\lambda, B \rightarrow BAa/A/\lambda$$

into Chomsky normal form.

5. (a) Find an DPDA with no more than two internal states that accepts the language $L(aa^*ba^*)$.
- (b) Show that a deterministic context-free language is never inherently ambiguous.
6. (a) Construct Turing machines that will accept the language :
 $L = L(aba^*b)$ on $\{a, b\}$
- (b) Give a formal definition of a Turing machine with a semi-infinite tape. Then prove that the class of Turing machines with semi-infinite tape is equivalent to the class of standard Turing machine.
7. (a) Show that the family of recursively enumerable languages is closed under intersection.
- (b) What difficulties would arise if we allowed the empty string as the left side of a production in an unrestricted grammar ? Explain with the help of an example. RGPVONLINE.COM
8. Write short notes on any four of the following :
 - (a) Post correspondence problem
 - (b) Ackerman's function
 - (c) Computability and Decidability
 - (d) Equivalence of CFGs for PDA
 - (e) Regular languages
 - (f) Markov Algorithm