

Roll No

MEEM-101**M.E/M.Tech. I Semester**

Examination, June 2017

Applied Mathematics**Time : Three Hours****Maximum Marks : 70****Note:** i) Attempt any five questions.

ii) All questions carry equal marks.

1. a) Using Laplace transform solve the following differential equation:

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t, x(0) = 0, x'(0) = 1.$$

- b) Solve the Simultaneous equations:

$$(D^2 - 3)x - 4y = 0 \text{ and } x + (D^2 + 1)y = 0 \text{ for } t > 0.$$

given that $x = y = \frac{dy}{dt} = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$.

33 445

[2]

2. a) Find the Fourier transform of $F(x)$ defined by

$$F(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases} \text{ and hence evaluate :}$$

i) $\int_{-\infty}^{\infty} \frac{\sin Sa \cos Sx}{S} ds$ and

ii) $\int_0^{\infty} \frac{\sin S}{S} ds$

- b) Find the Z-transform of the unit step function

$$U[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

3. a) Using Euler's method, solve for y at $x = 0.1$ from

$$\frac{dy}{dx} = x + y + xy, y(0) = 1 \text{ taking step size } h = 0.025.$$

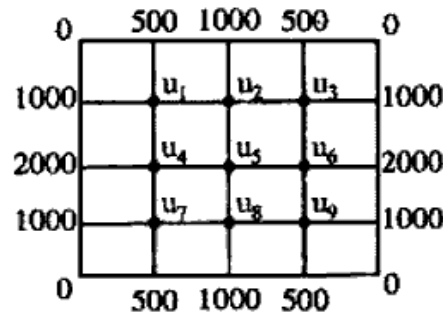
- b) Given $\frac{dy}{dx} = x^2(1 + y)$ and $y(1) = 1, y(1.1) = 1.233,$

$y(1.2) = 1.548, y(1.3) = 1.979$, evaluate $y(1.4)$ by Adams-Bashgonth method.

33

445

4. Solve the Elliptic equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, for the following square mesh with boundary values as shown:



5. a) The following results were obtained when 100 batches of seeds were allowed to germinate on damp filter paper in a laboratory.

$$\beta_1 = \frac{1}{15}, \beta_2 = \frac{89}{30}$$

Determine the Binomial distribution. Calculate the expected frequency for $x = 8$, assuming $p > q$.

- b) Find the mean and variance of Poisson's Distribution.
6. a) A coin is tossed 400 times and it turns up head 216 times. Discuss whether the coin may be unbiased one.
- b) In a sample of 600 men from a certain large city 400 are found to be smokers. In one of 900 from another large city, 450 are smokers. Do the data indicate that cities are significantly different with respect to prevalence of smoking among men?

- a) Write a note on Mathematical modelling of an ordinary differential equation.
- b) Define Non-Linear Growth and decay population model.

- a) Compute the integral $\int_5^{12} \frac{dx}{x}$, using Gauss's quadrature formula for the given numerical values of u 's and w 's:

$$u_{-1} = 0.4372, u_0 = 0.3874, u_1 = 0.3873$$

$$w_{-1} = \frac{4}{9}, w_0 = \frac{5}{18}, w_1 = \frac{5}{18}$$

- b) Find the values of $u(x, t)$, satisfy the parabola equations

$$\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2} \text{ and the boundary conditions } u(0, t) = 0 = u(8, t)$$

$$\text{and } u = (x, 0) = 4x - \frac{x^2}{2}, \text{ at the points } x = i, i = 0, 1, 2, \dots, 7$$

$$\text{and } t = \frac{1}{8} j, j = 0, 1, 2, \dots, 5$$

443