Total No. of Questions : 10] [Total No. of Printed Pages : 4

Roll No. ....

BE–102

B. E. (First/Second Semester)
EXAMINATION, June, 2010
(Common for all Branches)
ENGINEERING MATHEMATICS – I
(BE – 102)

Time : Three Hours
Maximum Marks : 100
Minimum Pass Marks : 35

Note : Attempt *one* question from each Unit. All questions carry equal marks.

Unit – I

1. (a) If $f(x) = \log(1+x), x > 0$, using Maclaurin's theorem, show that for $0 < \theta < 1$:

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3(1+\theta x)^3}$$

Deduce that $\log(1+x) < x - \frac{x^2}{2} + \frac{x^3}{3}$ for $x > 0$. 10

(b) Find the radius of curvature at any point of the cycloid : 10

$$x = a(\theta + \sin\theta)$$
$$y = a(1 - \cos\theta)$$

Or

2. (a) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, show that : 10

$$\frac{\partial^2 u}{\partial x \, \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$$

P. T. O.

(b) Discuss the maxima and minima of : 10

$$F(x, y) = x^3 y^2 (1 - x - y)$$

Unit – II

3. (a) Change the order of integration in :

$$I = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$$

and hence evaluate. 10

(b) Show that : 10

$$\beta(p, q) = \int_0^{\infty} \frac{y^{q-1}}{(1+y)^{p+q}}\, dy$$

Or

4. (a) Find by triple integration, the volume of the sphere : 10

$$x^2 + y^2 + z^2 = a^2$$

(b) Give the definition of triple integrals. Evaluate : 10

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dx\, dy\, dz$$

Unit – III

5. (a) Solve : 10

$$(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$$

(b) Solve : 10

$$y = 2px + p^n \text{ where } p = \frac{dy}{dx}.$$

Or

6. (a) Solve : 10

$$(D^2 - 4D + 4)\, y = 8x^2 e^{2x} \sin 2x$$

(b) Solve the simultaneous equations : 10

$$\frac{dx}{dt} + 2y + \sin t = 0$$

$$\frac{dy}{dt} - 2x - \cos t = 0$$

given $x = 0$ and $y = 1$ when $t = 0$.

Unit–IV

7. (a) Define rank of the matrix. Determine the rank of the following matrix : 10

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(b) Investigate the values of λ and μ so that the equations : 10

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + \lambda z = \mu$$

have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

Or

8. (a) Find the eigen values and eigen vectors of the matrix : 10

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(b) Find the characteristic equation of the matrix :

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

and hence, find A^{-1}. 10

Unit – V

9. (a) Prove that the complement of an element is unique in Boolean algebra. 10

(b) Write the function :

$$F(x, y, z) = x \cdot y' + x \cdot z + x \cdot y$$

into conjunctive normal forms in three variables x, y and z. 10

Or

10. (a) Prove that the sum of degree of all the vertices in a graph G is even. 10

(b) If p, q and r are three statements, then prove that : 10

$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$