Total No. of Questions : 8] [Total No. of Printed Pages : 3

CS/EI/IT–405 EI

B. E. (Fourth Semester) EXAMINATION, Dec., 2004

(Common for CS, EI & IT Engg.)

DISCRETE STRUCTURE

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : Attempt any *five* questions. All questions carry equal marks.

1. (a) If R and S are equivalence relation in a set X, then prove that $R \cap S$ is also an equivalence relation in the set X.

   (b) Show that :

$$1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6};\ n \geq 1$$

   by Mathematical induction.

2. (a) Show that the relation defined by "≤" on the set of all natural numbers is reflexive and transitive but not symmetric.

   (b) Show that the following propositions are tautologies :

   (i) $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

   (ii) $\{(p \vee \sim q) \wedge (\sim p \vee \sim p)\} \vee q$

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3. (a) For each of the sets described below, find a deterministic finite state machine that recognizes the set :

$$L = \{(01)^i 1^{2j} \mid i \geq 1, j \geq 1\}$$

   (b) For the finite state machine shown in the figure :

State	Input 0	Input 1	Output
A	B	F	0
B	B	C	1
C	D	C	1
D	A	E	1
E	A	D	0
F	F	D	0

   (i) List all 0-equivalent states.

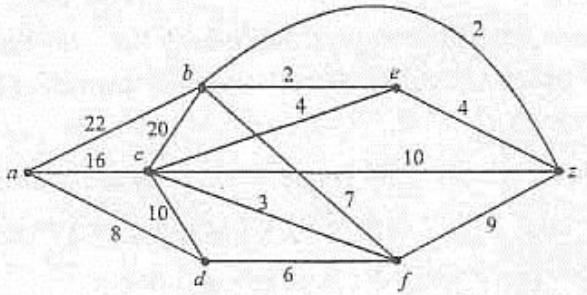
   (ii) Find all equivalent states and obtain an equivalent finite state machine with the smallest number of states.

4. (a) Prove that the following statement is logically equivalent :

$$(p \Rightarrow q) \vee r \equiv (p \vee r) \Rightarrow (q \vee r)$$

   (b) Define walk, path and circuit in a directed graph. What is the metric in a graph ?

5. (a) Determine shortest path between '*a*' and '*z*' in the graph shown below, where the numbers associated with the edges are the distance between vertices :



(b) A graph is given by the following adjacency matrix :

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

Check whether it is connected or not.

6. (a) Solve the recurrence relation :

$$a_r + 6\,a_{r-1} + 9\,a_{r-2} = 3$$

given that $a_0 = 0, a_1 = 1$.

(b) Determine the generating function of the numeric function a_r where

$$ar = \begin{cases} 2^r, & \text{if } r \text{ is even} \\ -2^r, & \text{if } r \text{ is odd} \end{cases}$$

7. (a) Find total solution for the difference equation :

$$a_r - 4\,a_{r-1} + 4\,a_{r-2} = (r+1)\,2^r$$

Given $a_0 = 1, a_1 = 2, a_2 = 3$

(b) Write generating function for the following discrete numeric functions :

(i) 2, 3, 5, 9, 17, 33, ...

(ii) 2, 5, 13, 35, ...

8. (a) Let (A, *) be a semigroup. Further more for every a and b in A, if $a \neq b$ then :

$$a * b \neq b * a;$$

Show that for every a, b in A, $a * b * a = a$.

(b) Show that kernel of homomorphism of ring R to R' is an ideal.