Roll No ....

MVCT/MVSE-101

M.E./M.Tech., I Semester

Examination, June 2013

Advance Mathematics & Numerical Analysis

Time : Three Hours

Maximum Marks : 70

Note: Attempt any five questions.
All questions carry equal marks.

1. a) Solve the partial differential equation

$$\Delta^2 u = -10(x^2 + y^2 + 10)$$

over the square with sides $x = 0 = y$, $x = 3 = y$ with $u = 0$ on the boundary and mesh length = 1

b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

subject to condition

$u(x,0) = \sin \pi x \;\; 0 \le x \le 1; u(0,t) = u(1,t) = 0$. Carry out

computations for two levels, taking $h = 1/3, K = \frac{1}{36}$

2. a) Find the Mellin transforms of

i) $\operatorname{Sin} x$ ii) $(1+x)^{-1}$

b) Define Fourier sine transform, also state and prove change of scale property in Fourier transform.

3. a) Find the integral equation corresponding to the boundary value problem.

$$y''(x) + \lambda\, y(x) = 0, \quad y(0) = y(1) = 0$$

b) Define Green's Function and find the Green function for the boundary value problem

$$\frac{d^2 y}{dx^2} + \mu^2 x = 0$$
$$y(0) = 0 = y(1)$$

4. a) Using the method of successive approximations, solve the volterra integral equation.

$$y(x) = 1 + x + \int_0^x (x - t)\, y(t)\, dt$$

b) Show that y(x) = 1 is a solution of the Fredholm integral equation

$$y(x) + \int_0^1 x(e^{tx} - 1)\, y(t)\, dt = e^x - x$$

5. a) Test for an extreme the functional

$$\pm\left[y(x)\right] = \int_0^1 \left(xy + y^2 - 2y^2 y^1\right) dx$$
$$y(0) = 1, \quad y(1) = 2$$

b) Find the solid of maximum volume formed by the revolution of a given surface area.

6. a) Find the externals of the functional and extreme value of the following.

$$\mathrm{I}\left[y(x)\right] = \int_{x0}^{x1} \frac{1 + y^2}{(y^1)^2}\, dx$$

b) Find the surface with the smallest area which enclose a given volume.

7. a) Use Galerkin's method to solve the equation.

$$\frac{d^2 y}{dx^2} - y + x = 0$$
$$y(0) = 1, \; y(1) = 0$$

b) State and prove convolution theorem for the Fourier transform.

8. a) Explain discretization in finite elements method

b) Use Rayleigh-Ritz method to solve the equation :

$$\frac{d^2 y}{dx^2} + y = x$$
$$y(0) = 0, \; y(1) = 1$$

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