Total No. of Questions :8] *[Total No. of Printed Pages :3*

Roll No .....

MVSE-101

M.E/M.Tech. I Semester

Examination, June 2017

Advance Mathematics and Numerical Analysis

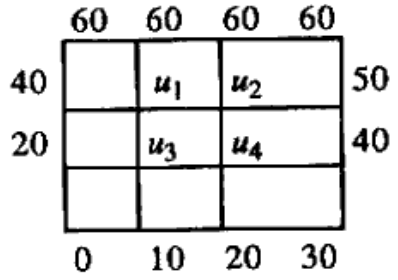
Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions.
ii) All questions carry equal marks.

1. a) Solve the boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0, t) = u(1, t) = 0$ and $u(x,0) = \sin \pi x$, $0 \le x \le 1$ taking $h = 0.2$ and $k = 0.02$.

   b) Solve the elliptic equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ for the square mesh with the boundary values shown in figure



2. a) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x^2}$ with the conditions $u(0,t) = u(1,t) = 0, u(x,0) = \frac{1}{2}x(1-x)$ and $u_t(x,0) = 0$ taking $h = k = 0.1$ for $0 \le t \le 0.4$.

   b) Solve the partial differential equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y, x = 3 = y$ with $u = 0$ on the boundary and mesh length 1.

3. a) Use Fourier transform to solve the boundary value problem $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t) = 0$, $u(2,t) = 0$, $u(x,0) = 0.05x(2-x)$ and $u_t(x,0) = 0$ where $0 < x < 2, t > 0$.

   b) Find the Fourier transform of e^{-ax^2}, where $a > 0$.

4. a) Show that the function $u(x) = xe^x$ is a solution of the Volterra integral equation $u(x) = \sin x + 2\int_0^x \cos(x - \xi)u(\xi)d\xi$.

   b) Form an integral equation corresponding to the differential equation $y'' + xy' + 2y = 0$, $y(0) = 1$, $y'(0) = 0$ into an integral equation.

5. a) Solve, by using method of successive approximations, the integral equation $y(x) = 1 + \lambda \int_0^1 xt\, y(t)dt$.

   b) Solve the integral equation

   $$y(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)\, y(t)dt.$$

6. a) Solve the Euler's equation for the functional

   $$\int_{x_1}^{x_2} (1 + x^2 y')\, y'\, dx.$$

   b) Using Rayleigh-Ritz method, solve the boundary value problem $y'' - y + x = 0;\ (0 \le x \le 1),\ y(0) = 0, y(1) = 0$.

7. a) Show that the shortest distance between two points in a plane is a straight line.

   b) Solve the integral equation $\int_0^x y(t)\, y(x-t)dt = 4 \sin 9x$

8. Write short note on each of the followings:
   a) Green's function
   b) Finite difference method
   c) Abel's integral equations

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