Total No. of Questions : 8] [Total No. of Printed Pages : 4

Roll No

MMIE/MMPD/MMIP/ MMTP/MMMD-101

M.E./M.Tech. I Semester

Examination, December 2017

Advanced Mathematics

Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions.

ii) All questions carry equal marks.

1. a) Let V be the set of all pairs (x, y) of real numbers and let R be the field of real numbers. In each of the following, examine whether V is a vector space over the field of real numbers or not?

i) $(x, y)+(x_1, y_1)=(x+x_1, y+y_1)$; $c(x, y)=(|c|x, |c|y)$

ii) $(x, y)+(x_1, y_1)=(x+x_1, y+y_1)$; $c(x, y)=(0, cy)$

iii) $(x, y)+(x_1, y_1)=(x+x_1, y+y_1)$; $c(x, y)=(c^2x, c^2y)$

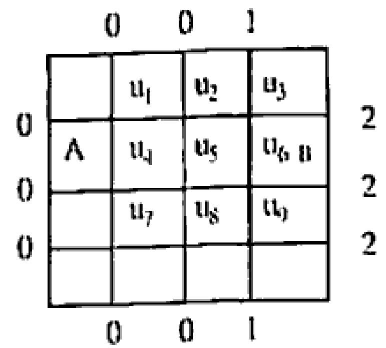
b) Show that $e^{2tx-t^2}=\sum_{n=0}^{\infty}\frac{t^n}{\underline{|n}}\cdot H_n(x)$, where $H_n(x)$ is the Hermite polynomial.

2. a) i) Find the Laplace transform of error function defined by $erf(\sqrt{t})=\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}du$.

ii) Prepare the tables of addition and multiplication in the modular system of numbers with modulo 5.

b) Solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$ by the method of separation of variables, where $u(x,0)=6e^{-3x}$.

3. a) Solve the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, for the square mesh with boundary values as shown in figure



b) Solve the Poisson's equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-10(x^2+y^2+10)$ over the square mesh with sides $x=0=y$, $x=3=y$ with $u(x, y)=0$ on the boundary and mesh length $=1$.

4. a) Find the mean and variance for Poisson distribution.

b) The mean and variance of a Binomial distribution $P(x, n, p)$ are 4 and $4/3$ respectively. Find $P(x \geq 2)$ and the probability of two successes.

5. a) Solve the recurrence relation $a_r = a_{r-1} + a_{r-2}$ given $a_0 = 1$, $a_1 = 1$.

b) Prove that a stochastic process with independent increments has the Markov property.

6. a) Find the system of differential difference equations for model $(M/M/1 : N/\infty/FCFS)$.

b) A TV repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairer sets in the order in which they come in and if the arrival of sets is approx. Poisson with an average rate of 10 per 8 hr day what is repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

7. a) Find the extremals of the following functionals :

i) $\int_1^2 \frac{\sqrt{1+y'^2}}{x} dx$; $y(1)=0$, $y(2)=1$

ii) $\int_{x_0}^{x_1} \frac{1+y^2}{y'^2} dx$

b) Solve the Boundary value problem $y'' + y + x = 0 (0 \leq x \leq 1)$, $y(0) = y(1) = 0$ by Ritz method.

8. a) State and prove Euler Lagrange's equation.

b) Define the following:

i) Basis of vector space.

ii) Wavelet transform.

iii) Sample event and compound event.

iv) Level of significance.

v) Markov chain.

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