Total No. of Questions : 10] [Total No. of Printed Pages : 4

Roll No. ....

CS/EI/IT–405

B. E. (Fourth Semester) EXAMINATION, June, 2009

(Old Scheme)

(Common for CS, EI & IT Engg.)

DISCRETE STRUCTURE

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : Answer all questions. All questions carry equal marks.

Unit–I

1. (a) Let I be the set of all integers, and let m be a fixed positive integer. Two integers a and b are said to be congruent modulo m-symbolised by $a \equiv b \pmod{m}$ if $a - b$ is exactly divisible by m, i. e., $a - b$ is an integer multiple of m. Show that this is an equivalence relation, describe the equivalence sets. 10

(b) If n pigeons are assigned to m pigeonholes $(n > m)$ then some pigeonhole must contain at least $\left[\left(\frac{n-1}{m}\right)\right] + 1$ pigeons, where $[k]$ denotes the largest integer not greater than k. 10

Or

2. (a) (i) If A, B, C are three sets, prove that : 10

$$(A - C) \cap (B - C) = (A \cap B) - C$$

P. T. O.

(ii) Prove that $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$ is divisible by 25 for all $n \in N$. 10

(b) In a Boolean algebra (B, ∧, ∨, ′) a relation "≤" is defined by $a \leq b$ if $a \vee b = b$ or $a \wedge b = a$. Prove that the relation "≤" is a partial order in B and (B, ≤) is a lattice.

Unit – II

3. (a) (i) What do you mean by contingency and prove that the statement. 10

$$(p \Rightarrow q) \Rightarrow (p \wedge q)$$

is a contingency.

(ii) Express the formula as given by :

$$(\sim p \Rightarrow r) \wedge (q \Leftrightarrow p)$$

in its principal conjunctive normal form.

(b) Show that the language : 10

$$L = \{a^k \mid k = i^2, i \geq 1\}$$

is not a finite state language.

Or

4. (a) (i) Prove that : 10

$$(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$$

is a tautology.

(ii) If $4x - 2 = 10$ then $x = 3$. Find converse, inverse and contrapositive.

(b) Let M be the finite state machine with state table : 10

F	A	B
S0	S2, x	S2, y
S1	S3, y	S1, z
S2	S1, z	S0, x
S3	S0, z	S2, x

Find the input set A, the state set S, the output set z and the initial state.

Unit – III

5. (a) Define graph, plannar graph and multigraph and prove that the number of edges in a simple graph is $\frac{n(n-1)}{2}$. 10

(b) Determine the minimum weight spanning tree for the following graph using Kruskal's algorithm. 10

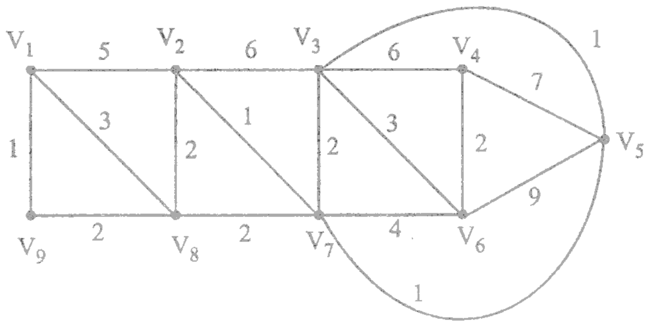

Fig. 1

Or

6. (a) Use Dijkstra's algorithm to find the shortest path from a to z for the following graph. 10

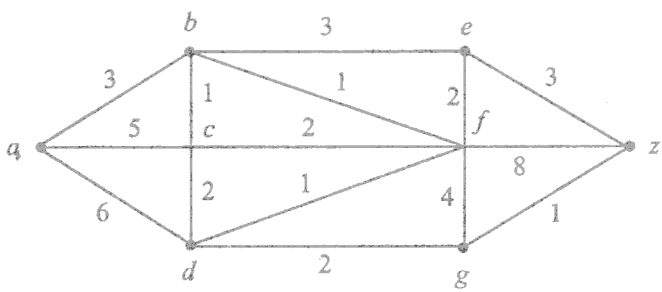

Fig. 2

(b) Define any *two* of the following : 5 each

(i) Eulerian path

(ii) Hamiltonian circuit

(iii) Cut-set

Unit – IV

7. (a) Solve the following recurrence relation : $2\,a_r - 5\,a_{r-1} + 2\,a_{r-2} = 0$, given $a_0 = 0$, $a_1 = 1$. 10

(b) Solve : 10

$$y_{x+2} + y_{x+1} + y_x = x^2 + x + 1$$

Or

8. (a) Solve the recurrence relation : 10

$$a_r - 7\,a_{r-1} + 10\,a_{r-2} = 0$$

given $a_0 = 0, a_1 = 3$.

(b) Solve : 10

$$y_{x+1} - 3\,y_x = 3^x \cdot x^2$$

Unit – V

9. (a) Let G be the set of the non-zero real numbers and let $a * b = \dfrac{ab}{2}$, then show that (G, *) is an abelian graph. 10

(b) Prove that every field is an integral domain. 10

Or

10. (a) Find the orders of each element of the group $G = \{0, 1, 2, 3, 4, 5\}$, the composition in G is 'addition modulo σ'. 10

(b) Prove that a subgroup H of a group G is normal if and only if : 10

$$x\,H\,x^{-1} = H \;\forall\, x \in G$$