BE - 102

B.E. I & II Semester

Examination, December 2012

Engineering Mathematics-I

(Grading System)

Time : Three Hours

Maximum Marks : 70

Note : 1. *Attempt all questions.*

2. *All questions carry equal marks.*

3. *Internal choices are also given.*

Unit - I

1. a) Expand $\sin x$ in powers of $(x - \pi/2)$. Hence. Find the value of $\sin 91°$ correct to 4 decimal places.

   b) Prove that if the perimeter of a triangle is constant its area is maximum when the triangle is equilateral.

OR

2. a) If $u = x\phi(y/x) + \psi(y/x)$. prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$

   b) Show that the radius of curvature at any point on the cardioid.

$$r = a(1 - \cos\theta) \text{ is } \frac{2}{3}\sqrt{2ar}$$

Unit - II

3. a) Evaluate $\underset{n \to \infty}{Lim} \left\{ \frac{n!}{n^n} \right\} yn$

   b) Find the whole area of astroid $x^{u3} + y^{u3} = a^{u3}$

OR

4. a) Find. by triple integration. the volume of the sphere

$x^2 + y^2 + z^2 = a^2$.

   b) Prove that $\beta(m,n) = \frac{\overline{|m|n}}{|m+n}$

Unit - III

5. a) Solve the differential equation.

$$\frac{d^2 y}{dx^3} - 3\frac{d^2 x}{dx^2} + 4\frac{dy}{dx} - 2y = e^x - \cos x$$

   b) Solve the following differential equation by method of variation of parameters

$(D^2 + a^2)y = \sec ax$.

OR

6. a) Solve the differential equation.

$$x^2 \frac{d^2 y}{dx^2} + 2x\frac{dy}{dx} - 12y = x^3 \log x$$

   b) Solve

$$\frac{dx}{dt} - 7x + y = 0$$

$$\frac{dy}{dt} - 2x - 5y = 0,$$

Unit - IV

7. a) Find the normal form of the matrix A and hence find its rank, where

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

   b) For the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$. Find non-singular matrices P and Q such that PAQ is in the normal form. Also find rank of A.

OR

8. a) Determine the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

b) Test the consistency of the following system of equations and solve using matrix methods.

$5x + 3y + 7z = 4$

$3x + 26y + 2z = 9$

$7x + 2y + 10z = 5$

Unit - V

9. a) Prove that the proposition

$P \rightarrow (q \rightarrow r) \leftrightarrow (p \wedge q) \rightarrow r$ is a fautology.

b) Define a tree and prove that a tree T with n vertices has exactly $(n-1)$ edges.

OR

10. a) Let $(B, +, \cdot, ')$ be a Boolean algebra and a, b be any two elements of B. Then prove that

i) $(a+b)' = a' \cdot b'$

ii) $(a \cdot b)' = a' + b'$

b) Define the following terms:
   i) Support of a fuzzy set.
   ii) Complement of a fuzzy set.
   iii) Union of two fuzzy set.
   iv) Intersection of two fuzzy set.