CS/IT–402(N)

B. E. (Fourth Semester) EXAMINATION, June, 2010

(New Scheme)

(Common for CS & IT Engg. Branch)

DISCRETE STRUCTURE

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : Attempt *one* question from each Unit. All questions carry equal marks.

Unit – I

1. (a) Write the principle of mathematical induction and by using this prove that $n(n^2 + 5)$ is an integer multiple of 6 for all positive integers n.

   (b) Define relations P and Q on $\{1, 2, 3, 4, \}$ by

   $$P = \{(a, b) : | a - b | = 1\}$$

   and $Q = \{(a, b) : a - b$ is even$\}$.

   Also represent P and Q as both graphs and matrices.

Or

2. (a) If A and B are two sets then prove that :

   $$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$

   (b) If $f : A \to B$ and $g : B \to C$ be one to one onto functions, then prove that $g \circ f$ is also one to one onto and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

Unit – II

3. (a) Prove that a non-empty subset S of a group $(G, *)$ is a subgroup of G if and only if for every pair of elements $a, b \in S$, $a * b^{-1} \in S$.

   (b) Define field and prove that the set of complex numbers is a field w. r. t. ordinary addition and multiplication.

Or

4. (a) Define field and show that the set of real numbers of the form $a + b\sqrt{3}$, where a and b are rational numbers is a field with respect to addition and multiplication.

   (b) Define the following :

   (i) Monoid (ii) Cyclic group

   (iii) Cosets (iv) Factor group

   (v) Normal subgroup

Unit – III

5. (a) Prove by the truth table that the following are tautologies or contradiction :

   (i) $(p \to (q \wedge r)) \to (\sim r \to \sim q)$

   (ii) $(p \vee q) \wedge \{p \vee (\sim q)\} \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

   (b) Make a finite state system to multiply a given binary integer by 3.

Or

6. (a) Write short notes on the following :

   (i) Universal and existential quantifiers

   (ii) Predicates

(b) Let $A = \{0, 1\}$, $S = \{S_0, S_1, S_2\}$, $O = \{0, 1\}$ and the functions f and g be given by the following state table :

State	Input		Output
	0	1	
S_0	S_1	S_0	0
S_1	S_2	S_1	1
S_2	S_2	S_0	1

Draw the state graph of the finite state machine $M = \{S, A, O, f, g, S_0\}$.

Unit—IV

(a) Prove that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges.

(b) Define the following terms giving examples :

(i) Incidence matrix

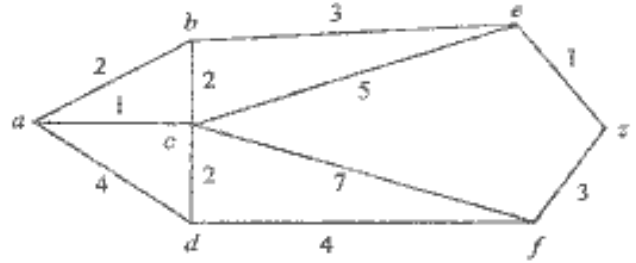
(ii) Eulerian graph

(iii) Graph coloring

(iv) Connected graph

Or

(a) Use Dijkstra's algorithm to find the shortest path between a to z in the graph :



(b) Every tree has either one or two centers.

Unit—V

9. (a) What is Hasse diagram ? Draw the Hasse diagram of the set D_{30} of positive integral divisors of 30 with the relation '|'.

(b) Solve the recurrence relation $a_r + 6a_{r-1} + 9a_{r-2} = 3$, given $a_0 = 0$, $a_1 = 1$.

Or

10. (a) Prove that in a distributive lattice, if an element has a complement then this complement is unique.

(b) Write short notes on the following :

(i) Binomial theorem

(ii) Multimonial coefficients

(iii) Permutation and combination

(iv) Generating function