Roll No ....

MMIE/MMPD/MMIP/ MMTP/MMMD-101

M.E/M.Tech. I Semester

Examination, June 2017

Advanced Mathematics

Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions.
ii) All questions carry equal marks.

1. a) Show that the set $W = \{(a_1, a_2, a_3) / a_1, a_2, a_3 \in F$ is a subspace of $V_3(F)$.

   b) Express $p(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Hermite polynomials.

2. a) Show that the mapping $T : V_3(R) \to V_3(R)$ defined by $T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$ is a linear transformation. Also, find matrix of T with respect to the standard basis of R^3.

   b) Prove that $H_n(x) = (-1)^n . e^{x^2} . \frac{d^n}{dx^n}(e^{-x^2})$.

3. a) Solve $\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}$ taking $h = 1$ upto $t = 1.25$ under the conditions $u(0, t) = u(5, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x^2(5 - x)$

   b) Write short note on each of the following:
      i) Discrete Fourier transform
      ii) Wavelet transform

4. a) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ by method of separation of variables, given that $u(x, 0) = 6e^{-3x}$

   b) Explain Markov process and Stochastic process with examples.

5. a) Urn A contains 2 white and 5 black balls and urn B contains 3 white and 6 black balls. A ball is taken out at random from urn A and transferred to urn B. Then a ball is drawn at random from urn B. What is the probability that it is a black ball.

   b) What do you mean by parameter estimation. Explain in detail point estimate and interval estimate of a parameter. A sample of size 9 is taken from a normal population. For this sample calculation yielded $\overline{x} = 15.8$ and $s^2 = 10.3$. Find a 99% interval estimate for the population mean. Given that the value of t-statistic for 8 degrees of freedom and 99% confidence level is 3.36.

6. a) A continuous RV x has the density function

$$f(x) = \begin{cases} k(1-x^2), & \text{for};0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Then, find (i) value of k, (ii) P(0.1<x<0.2) and (iii) the distribution function. Using the distribution function, determine the probability P(X < 0.3)

b) Find mean and variance of Poisson distribution.

7. a) Explain the queuing model $(M / M / S : \infty / \infty / F\bar{C}FS)$. Write down the system of differential difference equations. Also, obtain system of steady-state equations.

b) If the number of arrival in time t follows the Poisson distribution. Then the interatrial time follows negative exponential distribution.

8. a) Solve the boundary value problem

$y'' - y + x = 0,\ 0 \le x \le 1,\ y(0) = 0, y(1) = 0$

by Rayleigh-Ritz method.

b) Prove that the shortest distance between two points in a plane is straight line.

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