

Roll No .....

**MVSE-101****M.E./M.Tech., I Semester**

Examination, December 2016

**Advance Mathematics and Num. Analysis***Time : Three Hours**Maximum Marks : 70*

- Note :* i) Attempt any five questions.  
ii) All questions carry equal marks.

1. Solve the hyperbolic equation  $\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}$  taking  $h = 1$  upto  $t = 1.25$  under the conditions  $u(0, t) = u(5, t) = 0$ ,  $u_t(x, 0) = 0$  and  $u(x, 0) = x^2(5 - x)$ .

2. Solve the boundary value problem  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  under the conditions  $u(0, t) = u(1, t) = 0$  and  $u(x, 0) = \sin \pi x$ ,  $0 \leq x \leq 1$ , taking  $h = 0.2$  and  $k = 0.02$ .

3. a) Find the Fourier sine transform of  $\frac{e^{-ax}}{x}$ . Hence, find

Fourier sine transform of  $\frac{1}{x}$ .

- b) Find  $f(x)$  if  
i) Its sine transform is  $e^{-ax}$ ,  
ii) Its cosine transform is  $e^{-as}$ .

4. a) Define Mellin transform. Find the Mellin transform of  
i)  $e^{-x}$  and  
ii)  $\sin x$   
b) Find Hankel transform of  $x^{-2}e^{-x}$ , taking  ${}_xJ_1(px)$  as the kernel.
5. a) Verify that the function  $u(x) = 1-x$  is solution of the integral equation  $x = \int_0^x e^{x-\xi} u(\xi) d\xi$ .  
b) Convert the differential equation  $y'' - 2xy' - 3y = 0$  with the initial conditions  $y(0) = 1$  and  $y'(0) = 0$  to integral equation.
6. a) Solve the Fredholm integral equation  
 $u(x) = \cos x + \lambda \int_0^\pi \sin xu(t) dt$   
b) Using the method of successive approximations, solve the integral equation  $y(x) = 1 + \int_0^x y(t) dt$ .
7. a) Prove that the shortest distance between two points in a plane is a straight line.  
b) On which curve the functional  $\int_0^{\frac{\pi}{2}} (y'^2 - y^2 + 2xy) dx$  with  $y(0) = 0$  and  $y\left(\frac{\pi}{2}\right) = 0$  can be extremised.
8. Using Galerkin's method, solve the boundary value problem  $y'' = 3x + 4y$ ;  $y(0) = 0$ ,  $y(1) = 1$ .

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