Subject Name: Microwave Engineering **Subject Code**: EC7001

Subject Notes

UNIT-2

Microwave Networks and Component

Transmission line ports of microwave network, Scattering matrix, Properties of scattering matrix of reciprocal, Non reciprocal, loss less, Passive networks, Examples of two, three and four port networks, wave guide components like attenuator, Phase shifters and couplers, Flanges, Bends, Irises, Posts, Loads, Principle of operation and properties of E-plane, H-plane Tee junctions of wave guides, Hybrid T, Multi-hole directional coupler, Directional couplers, Microwave resonators- rectangular. Excitation of wave guide and resonators by couplers. Principles of operation of non reciprocal devices, properties of ferrites, Isolators and phase shifters.

Microwave Devices

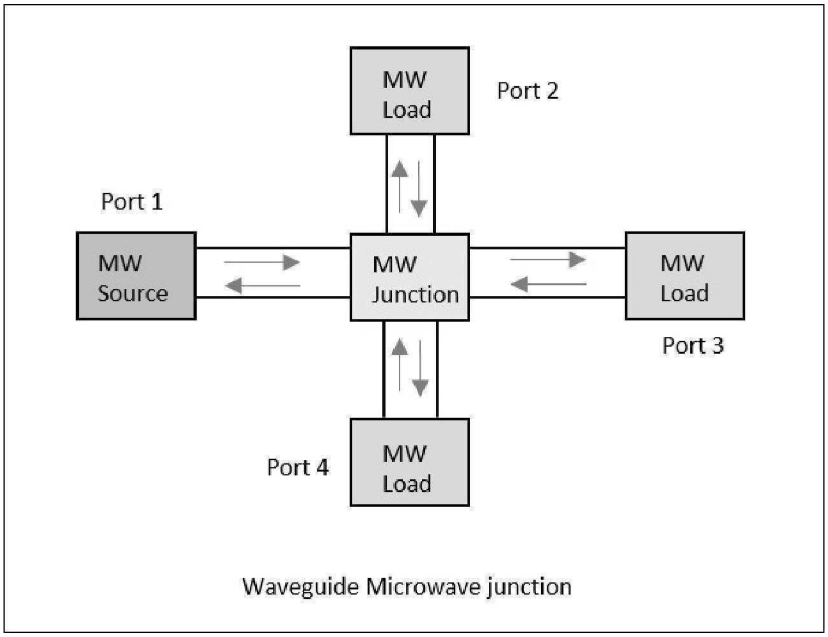
Just like other systems, the Microwave systems consists of many Microwave components, mainly with source at one end and load at the other, which are all connected with waveguides or coaxial cable or transmission line systems.

Following are the properties of waveguides.

- High SNR
- Low attenuation
- Lower insertion loss

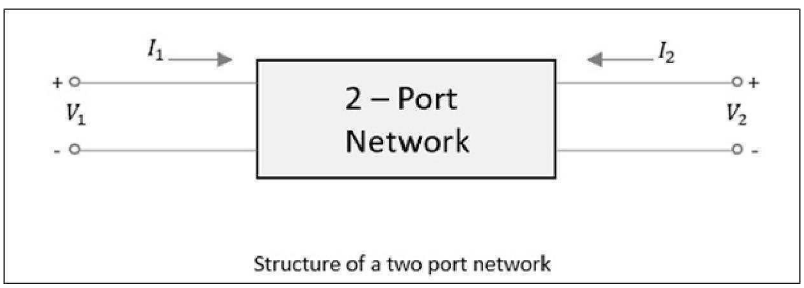
Waveguide Microwave Functions

Consider a waveguide having 4 ports. If the power is applied to one port, it goes through all the 3 ports in some proportions where some of it might reflect back from the same port. This concept is clearly depicted in the following figure.



Scattering Parameters

For a two-port network, as shown in the following figure, if the power is applied at one port, as we just discussed, most of the power escapes from the other port, while some of it reflects back to the same port. In the following figure, if $\mathbf{V_1}$ or $\mathbf{V_2}$ is applied, then $\mathbf{I_1}$ or $\mathbf{I_2}$ current flows respectively.



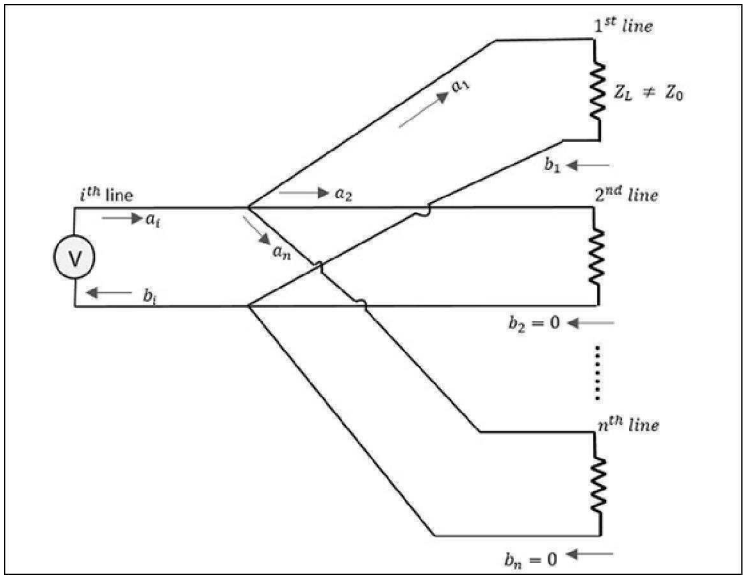
If the source is applied to the opposite port, another two combinations are to be considered. So, for a two-port network, $2 \times 2 = 4$ combinations are likely to occur.

The travelling waves with associated powers when scatter out through the ports, the Microwave junction can be defined by S-Parameters or **Scattering Parameters**, which are represented in a matrix form, called as "**Scattering Matrix**".

Scattering Matrix

It is a square matrix which gives all the combinations of power relationships between the various input and output ports of a Microwave junction. The elements of this matrix are called **"Scattering Coefficients"** or **"Scattering (S) Parameters"**.

Consider the following figure.



Here, the source is connected through i_{th} i_{th} line while $a_1 a_1$ is the incident wave and $b_1 b_1$ is the reflected wave.

If a relation is given between b_1 and a_1,

b_1=(reflectioncoefficient)$a_1 = S_{1i} a_1$

Where

- S_{1i}= Reflection coefficient of 1st1st line (where ii is the input port and 11is the output port)
- 1 = Reflection from 1st line
- i = Source connected at ith line

If the impedance matches, then the power gets transferred to the load. Unlikely, if the load impedance doesn't match with the characteristic impedance. Then, the reflection occurs. That means, reflection occurs if

$$Z_l \neq Z_o$$

However, if this mismatch is there for more than one port, example ′n′ port, then i=1to n (since i can be any line from 1 to nn).

Therefore, we have

$$b_1 = S_{11}a1 + S_{12}a_2 + S_{13}a_3 + ... + S_{1n}a_n$$

$$b_2 = S_{21}a1 + S_{22}a_2 + S_{23}a_3 + ... + S_{2n}a_n$$

..

..

..

..

$$b_n=S_{n1}a1+S_{n2}a_2+S_{n3}a_3+..............+S_{nn}a_n$$

When this whole thing is kept in a matrix form,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ . \\ b_n \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & . & . & s_{1n} \\ s_{21} & s_{22} & s_{23} & . & . & s_{2n} \\ s_{31} & s_{32} & s_{33} & . & . & s_{3n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ s_{n1} & s_{n1} & s_{n2} & . & . & s_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ . \\ . \\ a_n \end{bmatrix}$$

Column matrix [b] **Scattering matrix** [S] Input **Matrix** [a]

The column matrix [b][b] corresponds to the reflected waves or the output, while the matrix [a] corresponds to the incident waves or the input. The scattering column matrix [s] which is of the order of n×nn×n contains the reflection coefficients and transmission coefficients. Therefore,

$$[b]=[S][a]$$

Properties of [S] Matrix

The scattering matrix is indicated as [S] matrix. There are few standard properties for [S] matrix. They are −

- [S] is always a square matrix of order (nxn) $[S]_{n\times n}$
- [S][S] is a symmetric matrix

i.e., Sij=SjiSij=Sji

- [S][S] is a unitary matrix

i.e., [S][S] ∗=I[S][S] ∗=I

- The sum of the products of each term of any row or column multiplied by the complex conjugate of the corresponding terms of any other row or column is zero. i.e.,

$$\sum_{i=j}^{n} s_{ik} s_{ik}^{\ *} = 0 \ \text{ for } k \neq j.$$

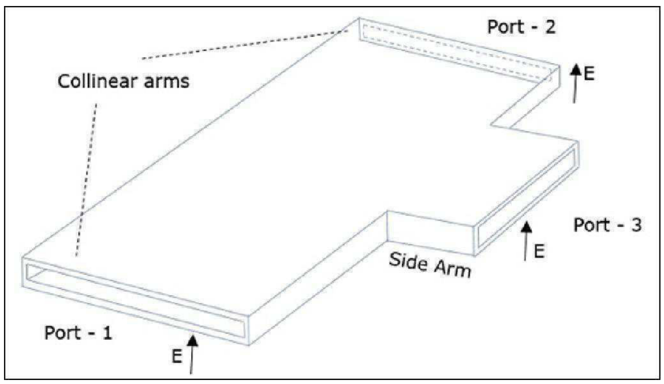
(k=1,2,3,...n)and(j=1,2,3,...n)

If the electrical distance between some k^{th} port and the junction is $\beta_k I_k$, then the coefficients of S_{ij} involving k, will be multiplied by the factor $e^{-j\beta\ kIk} s_{ik}$

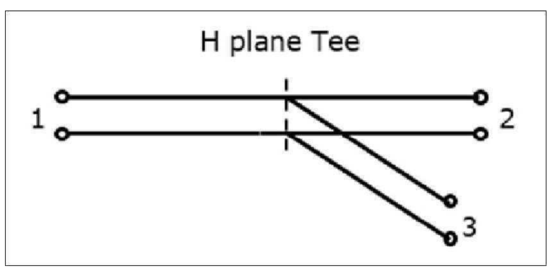
H plane T

An H-Plane Tee junction is formed by attaching a simple waveguide to a rectangular waveguide which already has two ports. The arms of rectangular waveguides make two ports called collinear ports i.e., Port1 and Port2, while the new one, Port3 is called as Side arm or H-arm. This H-plane Tee is also called as Shunt Tee.

As the axis of the side arm is parallel to the magnetic field, this junction is called H-Plane Tee junction. This is also called as Current junction, as the magnetic field divides itself into arms. The cross-sectional details of H-plane tee can be understood by the following figure.



The following figure shows the connection made by the sidearm to the bi-directional waveguide to form the serial port.



Properties of H-Plane Tee

The properties of H-Plane Tee can be defined by its $[S]_{3\times3}$ matrix.

It is a 3×3 matrix as there are 3 possible inputs and 3 possible outputs.

$$[s]=\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \qquad (1)$$

From the symmetric property,

$$S_{ij}=S_{ji}$$

So

$$S_{12}= S_{21}, S_{23}= S_{32}= S_{13}, S_{13}= S_{31}\ S_{12}= S_{21}$$

For perfectly matched junctions:

$$S_{33}=0$$

Now, the [S] matrix can be written as,

$$[s]=\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{13} \\ s_{13} & s_{13} & 0 \end{bmatrix}$$

We can say that we have four unknowns, considering the symmetry property.

From the Unitary matrix property

$$[S][S]^{*}=[I]$$

$$[s]=\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{13} \\ s_{13} & s_{13} & 0 \end{bmatrix}\begin{bmatrix} s_{11}^{*} & s_{12}^{*} & s_{13}^{*} \\ s_{12}^{*} & s_{22}^{*} & s_{13}^{*} \\ s_{13}^{*} & s_{13}^{*} & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying we get,

Performing row and column operations:

$$R_1C_1: |s_{11}|^2+|s_{12}|^2+|s_{13}|^2=1 \qquad (2)$$

$$R_2C_2: |s_{12}|^2+|s_{22}|^2+|s_{13}|^2=1 \qquad (3)$$

$$R_3C_3: |s_{13}|^2+|s_{13}|^2=1 \qquad (4)$$

$$|s_{13}|=1/\sqrt{2} \qquad (5)$$

$$R_3C_1: s_{13}s_{11}^{*}-s_{13}s_{12}^{*}=0 \qquad (6)$$

On solving above :

$$s_{11}=s_{22} \text{ and } s_{12}=-s_{12}$$

Put values in (2) equation we get $s_{11}=1/2$ and $s_{22}=1/2$

Now we get scattering matrix:

$$[s] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

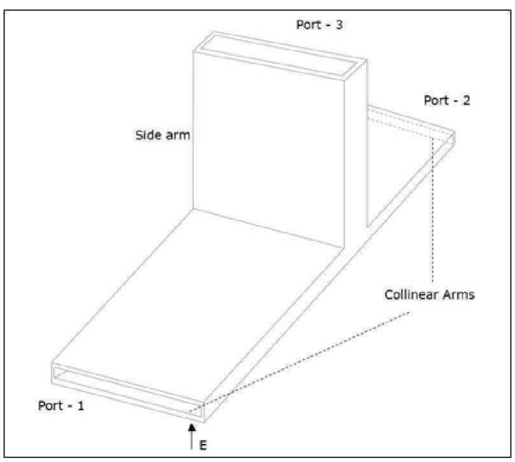
We know that [b]=[S][a]

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

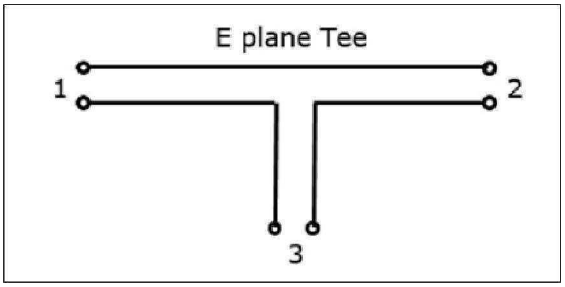
<u>Expression for E-plane T junctions.</u>

Ans- An E-Plane Tee junction is formed by attaching a simple waveguide to the broader dimension of a rectangular waveguide, which already has two ports. The arms of rectangular waveguides make two ports called collinear ports i.e., Port1 and Port2, while the new one, Port3 is called as Side arm or E-arm. T his E-plane Tee is also called as Series Tee.

As the axis of the side arm is parallel to the electric field, this junction is called E-Plane Tee junction. This is also called as Voltage or Series junction. The ports 1 and 2 are 180° out of phase with each other. The cross-sectional details of E-plane tee can be understood by the following figure.



The following figure shows the connection made by the sidearm to the bi-directional waveguide to form the parallel port.



Properties of E-Plane Tee

The properties of E-Plane Tee can be defined by its $[S]_{3\times3}$ matrix.

It is a 3×3 matrix as there are 3 possible inputs and 3 possible outputs.

$$[s]=\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \qquad (1)$$

From property of E-plane T: $S_{23}= -S_{13}$

From the symmetric property,

$$S_{ij}=S_{ji}$$

So

$$S_{12}= S_{21},\ S_{23}= S_{32}= S_{13},\ S_{13}= S_{31}$$

For perfectly matched junctions:

$$S_{33}=0$$

Now, the [S] matrix can be written as,

$$[s]=\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & -s_{13} \\ s_{13} & -s_{13} & 0 \end{bmatrix}$$

We can say that we have four unknowns, considering the symmetry property.

From the Unitary matrix property

$$[S][S]^*=[I]$$

$$[s]=\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & -s_{13} \\ s_{13} & -s_{13} & 0 \end{bmatrix}\begin{bmatrix} s_{11}^* & s_{12}^* & s_{13}^* \\ s_{12}^* & s_{22}^* & -s_{13}^* \\ s_{13}^* & -s_{13}^* & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying we get,

Performing row and column operations:

$$R_1C_1: |s_{11}|^2 + |s_{12}|^2 + |s_{13}|^2 = 1 \quad (2)$$

$$R_2C_2: |s_{12}|^2 + |s_{22}|^2 + |s_{13}|^2 = 1 \quad (3)$$

$$R_3C_3: |s_{13}|^2 + |s_{13}|^2 = 1 \quad (4)$$

$$|s_{13}| = 1/\sqrt{2} \quad (5)$$

$$R_3C_1: s_{13}s_{11}^{*} - s_{13}s_{12}^{*} = 0 \quad (6)$$

On solving above :

$$s_{11} = s_{22} \text{ and } s_{12} = -s_{12}$$

Put values in (2) equation we get $s_{11} = 1/2$ and $s_{22} = 1/2$

Now we get scattering matrix:

$$[s] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

We know that [b]=[S][a]

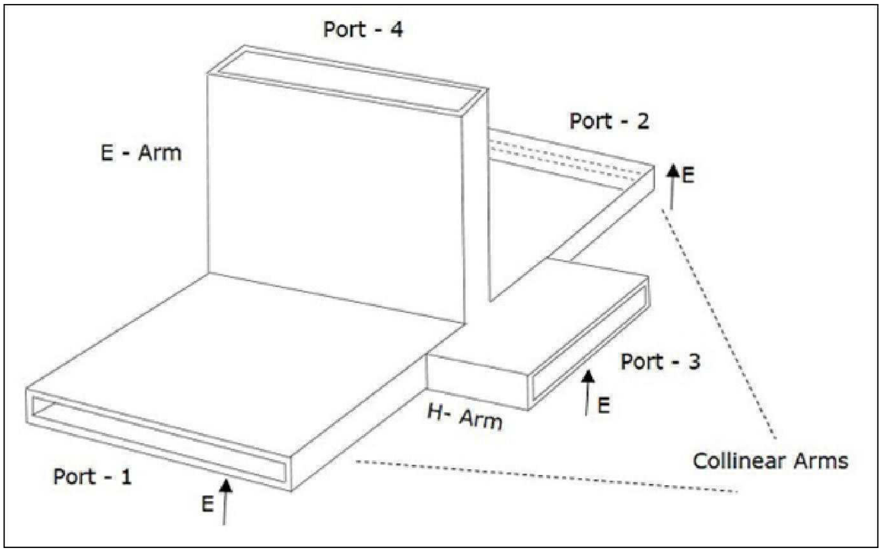
$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Expression for E-H-plane T junctions/ Magic T Junctions.

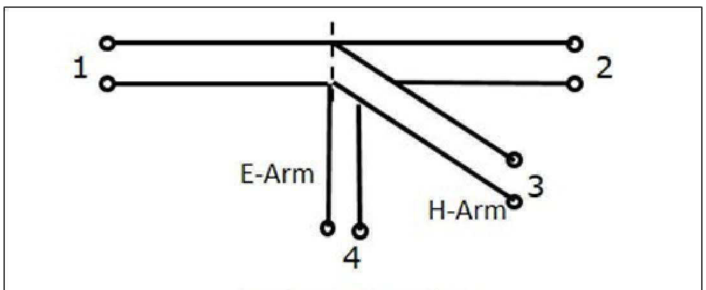
Ans-15- An E-H Plane Tee junction is formed by attaching two simple waveguides one parallel and the other series, to a rectangular waveguide which already has two ports. This is also called as Magic Tee, or Hybrid or 3dB coupler.

The arms of rectangular waveguides make two ports called collinear ports i.e., Port 1 and Port 2, while the Port 3 is called as H-Arm or Sum port or Parallel port. Port 4 is called as E-Arm or Difference port or Series port.

The cross-sectional details of Magic Tee can be understood by the following figure.



The following figure shows the connection made by the side arms to the bi-directional waveguide to form both parallel and serial ports.



Magic Tee junction

Characteristics of E-H Plane Tee

- If a signal of equal phase and magnitude is sent to port 1 and port 2, then the output at port 4 is zero and the output at port 3 will be the additive of both the ports 1 and 2.
- If a signal is sent to port 4, (E-arm) then the power is divided between port 1 and 2 equally but in opposite phase, while there would be no output at port 3. Hence, $S_{34} = 0$.
- If a signal is fed at port 3, then the power is divided between port 1 and 2 equally, while there would be no output at port 4. Hence, $S_{43} = 0$.
- If a signal is fed at one of the collinear ports, then there appears no output at the other collinear port, as the E-arm produces a phase delay and the H-arm produces a phase advance. So, $S_{12} = S_{21} = 0$.

Properties of E-H Plane Tee

The properties of E-H Plane Tee can be defined by its [S]4×4[S]4×4 matrix.

It is a 4×4 matrix as there are 4 possible inputs and 4 possible outputs.

$$[s]=\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix} \qquad (1)$$

From property of E-plane T: $S_{24}= -S_{14}$

From property of H-plane T: $S_{23}= S_{13}$

From the symmetric property,

$$S_{ij}=S_{ji}$$

So

$$S_{12}= S_{21}, S_{23}= S_{32}, S_{13}= S_{31}, S_{14}= S_{41}, S_{24}= S_{42}, S_{34}= S_{43}$$

For perfectly matched junctions:

$S_{33}=0$ and $S_{44}=0$

Due to isolated port $S_{43}= S_{34}=0$

Now, the [S] matrix can be written as,

$$[s]=\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{12} & s_{22} & s_{13} & -s_{14} \\ s_{13} & s_{13} & 0 & 0 \\ s_{14} & -s_{14} & 0 & 0 \end{bmatrix}$$

We can say that we have four unknowns, considering the symmetry property.

From the Unitary matrix property

$$[S][S]^{*}=[I]$$

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{12} & s_{22} & s_{13} & -s_{14} \\ s_{13} & s_{13} & 0 & 0 \\ s_{14} & -s_{14} & 0 & 0 \end{bmatrix}\begin{bmatrix} s_{11}^{*} & s_{12}^{*} & s_{13}^{*} & s_{14}^{*} \\ s_{12}^{*} & s_{22}^{*} & s_{13}^{*} & -s_{14}^{*} \\ s_{13}^{*} & s_{13}^{*} & 0 & 0 \\ s_{14}^{*} & -s_{14}^{*} & 0 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Multiplying we get,

Performing row and column operations:

$$R_1C_1 : |s_{11}|^2 + |s_{12}|^2 + |s_{13}|^2 + |s_{14}|^2 = 1 \quad (2)$$

$$R_2C_2 : |s_{12}|^2 + |s_{22}|^2 + |s_{13}|^2 + |s_{14}|^2 = 1 \quad (3)$$

$$R_3C_3 : |s_{13}|^2 + |s_{13}|^2 = 1 \quad (4)$$

$$R_4C_4 : |s_{14}|^2 + |s_{14}|^2 = 1 \quad (5)$$

$$|s_{13}| = 1/\sqrt{2} \quad (6)$$

$$|s_{14}| = 1/\sqrt{2} \quad (7)$$

On solving above :

$$s_{11} = s_{22} \text{ and } s_{12} = -s_{12}$$

Put values in (2) equation we get $s_{11} = 0$ and $s_{22} = 0$

Now we understand that ports 1 and 2 are perfectly matched to the junction. As this is a 4 port junction, whenever two ports are perfectly matched, the other two ports are also perfectly matched to the junction.

The junction where all the four ports are perfectly matched is called as Magic Tee Junction.

By substituting the equations from 12 to 16, in the [S] matrix of equation, we obtain the scattering matrix of Magic Tee as

Now we get scattering matrix:

$$[s] = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

We know that [b]=[S][a]

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

Applications of E-H Plane Tee

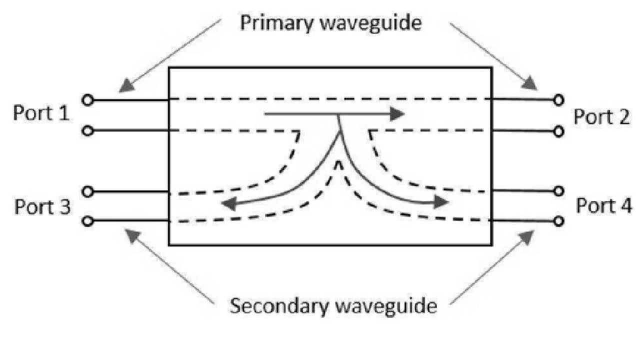
Some of the most common applications of E-H Plane Tee are as follows –

- E-H Plane junction is used to measure the impedance – A null detector is connected to E-Arm port while the Microwave source is connected to H-Arm port. The collinear ports together with these ports make a bridge and the impedance measurement is done by balancing the bridge.
- E-H Plane Tee is used as a duplexer – A duplexer is a circuit which works as both the transmitter and the receiver, using a single antenna for both purposes. Port 1 and 2 are used as receiver and transmitter where they are isolated and hence will not interfere. Antenna is connected to E-Arm port. A matched load is connected to H-Arm port, which provides no reflections. Now, there exists transmission or reception without any problem.
- E-H Plane Tee is used as a mixer – E-Arm port is connected with antenna and the H-Arm port is connected with local oscillator. Port 2 has a matched load which has no reflections and port 1 has the mixer circuit, which gets half of the signal power and half of the oscillator power to produce IF frequency.
- In addition to the above applications, an E-H Plane Tee junction is also used as Microwave bridge, Microwave discriminator, etc.

Directional Coupler

A **Directional coupler** is a device that samples a small amount of Microwave power for measurement purposes. The power measurements include incident power, reflected power, VSWR values, etc.

Directional Coupler is a 4-port waveguide junction consisting of a primary main waveguide and a secondary auxiliary waveguide. The following figure shows the image of a directional coupler.



Directional Coupler

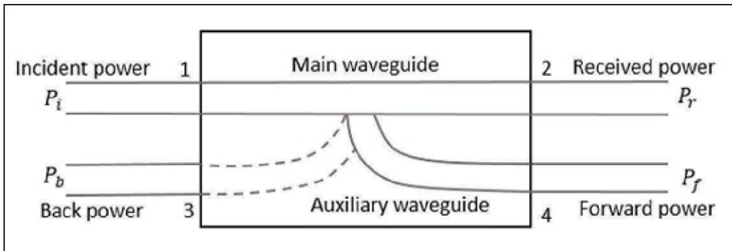
Directional coupler is used to couple the Microwave power which may be unidirectional or bi-directional.

Properties of Directional Couplers

The properties of an ideal directional coupler are as follows.

- All the terminations are matched to the ports.
- When the power travels from Port 1 to Port 2, some portion of it gets coupled to Port 4 but not to Port 3.
- As it is also a bi-directional coupler, when the power travels from Port 2 to Port 1, some portion of it gets coupled to Port 3 but not to Port 4.
- If the power is incident through Port 3, a portion of it is coupled to Port 2, but not to Port 1.
- If the power is incident through Port 4, a portion of it is coupled to Port 1, but not to Port 2.
- Port 1 and 3 are decoupled as are Port 2 and Port 4.

Ideally, the output of Port 3 should be zero. However, practically, a small amount of power called **back power** is observed at Port 3. The following figure indicates the power flow in a directional coupler.



Directional Coupler indicating powers

Where

- P_i = Incident power at Port 1
- P_r = Received power at Port 2
- P_f = Forward coupled power at Port 4
- P_b = Back power at Port 3

Following are the parameters used to define the performance of a directional coupler.

Coupling Factor (C)

The Coupling factor of a directional coupler is the ratio of incident power to the forward power, measured in dB.

$$C=10\ \log_{10}P_i/P_f \quad dB$$

Directivity (D)

The Directivity of a directional coupler is the ratio of forward power to the back power, measured in dB.

$$D=10\log_{10}P_f/P_b dB$$

Isolation

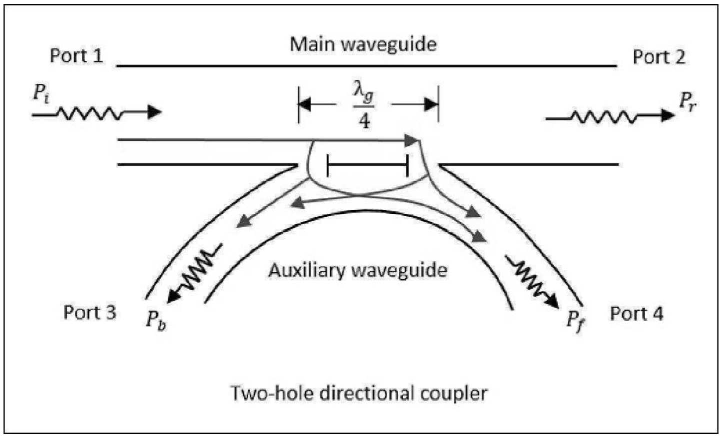
It defines the directive properties of a directional coupler. It is the ratio of incident power to the back power, measured in dB.

$$I=10\log_{10}P_i/P_b dB$$

Isolation in dB = Coupling factor + Directivity

Two-Hole Directional Coupler

This is a directional coupler with same main and auxiliary waveguides, but with two small holes that are common between them. These holes are λ g/4distance apart where λ g is the guide wavelength. The following figure shows the image of a two-hole directional coupler.



A two-hole directional coupler is designed to meet the ideal requirement of directional coupler, which is to avoid back power. Some of the power while travelling between Port 1 and Port 2, escapes through the holes 1 and 2.

The magnitude of the power depends upon the dimensions of the holes. This leakage power at both the holes are in phase at hole 2, adding up the power contributing to the forward power $\mathbf{P_f}$. However, it is out of phase at hole 1, cancelling each other and preventing the back power to occur.

Hence, the directivity of a directional coupler improves.

Waveguide Joints

As a waveguide system cannot be built in a single piece always, sometimes it is necessary to join different waveguides. This joining must be carefully done to prevent problems such as – Reflection effects, creation of standing waves, and increasing the attenuation, etc.

The waveguide joints besides avoiding irregularities, should also take care of E and H field patterns by not affecting them. There are many types of waveguide joints such as bolted flange, flange joint, choke joint, etc.

Derivation for 4 port directional couplers:

Scattering matrix for Directional coupler:

$$[s]=\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix}$$

All 4 ports are perfectly matched ports:

S11=s22=s33=s44

From symmetric property:

S12= S21, S13= S31, S14= S41, S23= S32, S24= S42, S34= S43,

Ideally back power is 0 so: S13= S31=0 and S24= S42=0.

Scattering matrix for directional coupler:

$$[s]=\begin{bmatrix} 0 & s_{12} & 0 & s_{14} \\ s_{21} & 0 & s_{23} & 0 \\ 0 & s_{32} & 0 & s_{34} \\ s_{41} & 0 & s_{43} & 0 \end{bmatrix}$$

Unitary matrix property:[S][S*]=[I]

$$\begin{bmatrix} 0 & s_{12} & 0 & s_{14} \\ s_{21} & 0 & s_{23} & 0 \\ 0 & s_{32} & 0 & s_{34} \\ s_{41} & 0 & s_{43} & 0 \end{bmatrix}\begin{bmatrix} 0 & s_{12}^* & 0 & s_{14}^* \\ s_{21}^* & 0 & s_{23}^* & 0 \\ 0 & s_{32}^* & 0 & s_{34}^* \\ s_{41}^* & 0 & s_{43}^* & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Performing row and coloumn operations:

R1C1: $|s_{12}|^2+|s_{14}|^2=1$ (1)

R2C2: $|s_{12}|^2+|s_{23}|^2=1$ (2)

R3C3: $|s_{23}|^2+|s_{34}|^2=1$ (3)

R1C3: $s_{12}\, s_{23}^*+ s_{14}\, s_{34}^*$ (4)

Compare 1 and 2: $s_{14}=s_{23}$

Compare 2 and 3: $s_{12}=s_{34}$

Let $s_{12}=s_{34}=P= s_{34}^*$

Equation 4 will be $s_{34}(s_{23}^* +s_{14})=0$

$P(s_{23}^* +s_{14})=0$

Let $s_{23}=jq$ and $s_{23}^* =-jq$

So Scattering matrix will be:

$$[s]=\begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$

Waveguide Joints

Since an entire waveguide system cannot possibly be molded into one piece, the waveguide must be constructed in sections and the sections connected with joints. The three basic types of waveguide joints are:

Permanent joints
The permanent joint is a factory-welded joint that requires no maintenance.

Semipermanent joints
Sections of waveguide must be taken apart for maintenance and repair. A semipermanent joint, called a "choke joint", is most commonly used for this purpose. The choke joint provides good electromagnetic continuity between sections of waveguide with very little power loss.

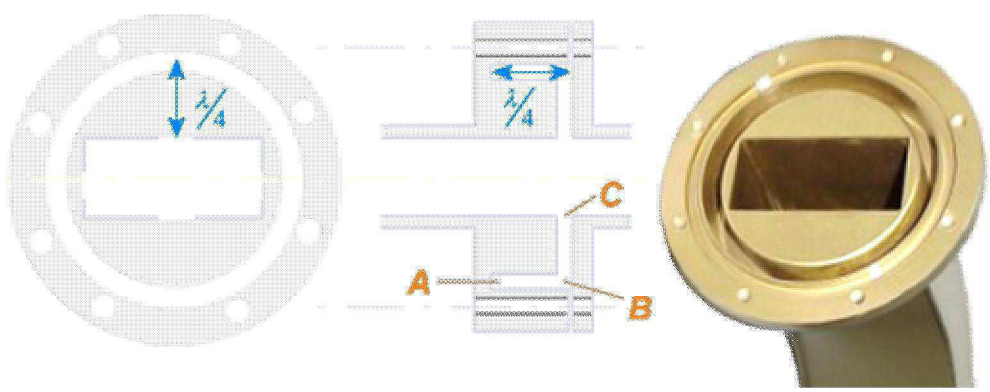
Rotating joints
Whenever a stationary rectangular waveguide is to be connected to a rotating antenna, a rotating joint must be used. A circular waveguide is normally used in a rotating joint.
The rotating section of the joint also uses a choke joint to complete the electrical connection with the stationary section.

Choke Joint

Sections of waveguide must be taken apart for maintenance and repair. A semipermanent joint, called a „choke joint", is most commonly used for this purpose. The choke joint provides good electromagnetic continuity between sections of waveguide with very little power loss.

A cross-sectional view of a choke joint is shown in the figure. The pressure gasket shown between the two metal surfaces forms an airtight seal.

Notice that the slot is exactly ¼λ from the "a" wall of the waveguide.



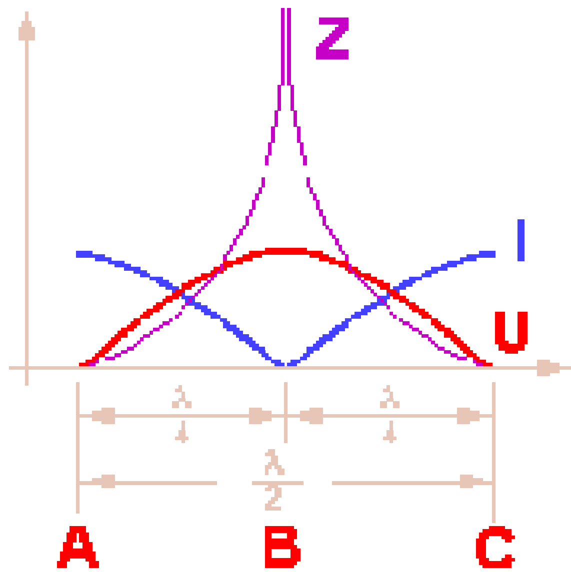


Figure 2: Transformation of the impedance

The slot is also ¼λ deep, as shown, and because it is shorted at the end, a high impedance results after ¼λ . The high impedance after ¼λ results in a low impedance, or short, after ½λ .

This effect creates a good electrical connection between the two sections that permits energy to pass with very little reflection or loss. There is therefore a (HF) electrically conducting connection between the two waveguide sections. A galvanic connection of the two wave guide sections or the flanges isn't necessary at this. Therefore it is possible to mount a seal between the two flanges to lock the inside of the waveguide air tightly.

The expression “choke joint” for the described flange connection was chosen because of its effect like a HF choke. As when a HF choke the HF energy shall be prevented from the migration by the connecting piece to the outside.

Waveguide Bends

Waveguide is normally rigid, except for flexible waveguide, and therefore it is often necessary to direct the waveguide in a particular direction. Using waveguide bends and twists it is possible to arrange the waveguide into the positions required.

When using waveguide bends and waveguide twists, it is necessary to ensure the bending and twisting is accomplished in the correct manner otherwise the electric and magnetic fields will be unduly distorted and the signal will not propagate in the manner required causing loss and reflections. Accordingly waveguide bend and waveguide twist sections are manufactured specifically to allow the waveguide direction to be altered without unduly destroying the field patterns and introducing loss.

Types of waveguide bend

There are several ways in which waveguide bends can be accomplished. They may be used according to the applications and the requirements.

- Waveguide E bend
- Waveguide H bend
- Waveguide sharp E bend
- Waveguide sharp H bend

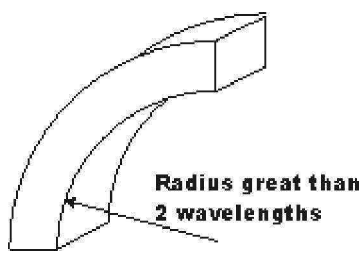
Each type of bend is achieved in a way that enables the signal to propagate correctly and with the minimum of disruption to the fields and hence to the overall signal.

Ideally the waveguide should be bent very gradually, but this is normally not viable and therefore specific waveguide bends are used.

Most proprietary waveguide bends are common angles - 90° waveguide bends are the most common by far.

Waveguide E bend

This form of waveguide bend is called an E bend because it distorts or changes the electric field to enable the waveguide to be bent in the required direction.

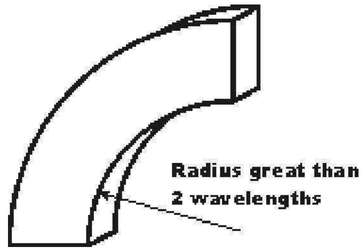


Waveguide E bend

To prevent reflections this waveguide bend must have a radius greater than two wavelengths.

Waveguide H bend

This form of waveguide bend is very similar to the E bend, except that it distorts the H or magnetic field. It creates the bend around the thinner side of the waveguide.

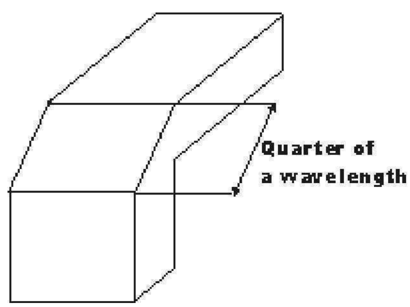


Waveguide H bend

As with the E bend, this form of waveguide bend must also have a radius greater than 2 wavelengths to prevent undue reflections and disturbance of the field.

Waveguide sharp E bend

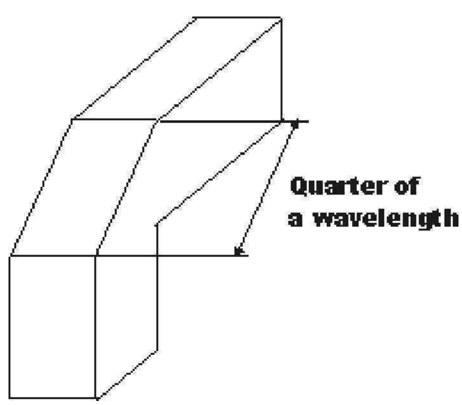
In some circumstances a much shorter or sharper bend may be required. This can be accomplished in a slightly different manner. The techniques is to use a 45° bend in the waveguide. Effectively the signal is reflected, and using a 45° surface the reflections occur in such a way that the fields are left undisturbed, although the phase is inverted and in some applications this may need accounting for or correcting.



Waveguide sharp E bend

Waveguide sharp H bend

This for of waveguide bend is the same as the sharp E bend, except that the waveguide bend affects the H field rather than the E field.



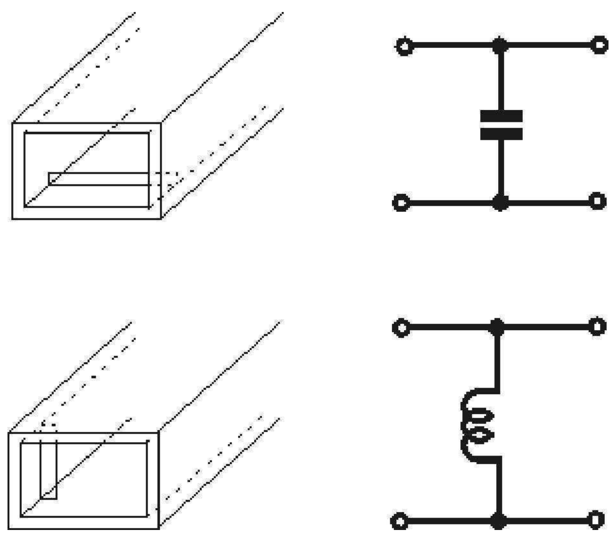
Waveguide sharp H bend

Impedance matching using a waveguide iris

Irises are effectively obstructions within the waveguide that provide a capacitive or inductive element within the waveguide to provide the impedance matching.

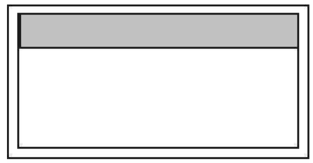
The obstruction or waveguide iris is located in either the transverse plane of the magnetic or electric field. A waveguide iris places a shunt capacitance or inductance across the waveguide and it is directly proportional to the size of the waveguide iris.

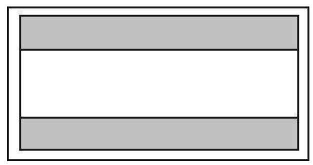
An inductive waveguide iris is placed within the magnetic field, and a capacitive waveguide iris is placed within the electric field. These can be susceptible to breakdown under high power conditions - particularly the electric plane irises as they concentrate the electric field. Accordingly the use of a waveguide iris or screw / post can limit the power handling capacity.



Impedance matching using a waveguide iris

The waveguide iris may either be on only one side of the waveguide, or there may be a waveguide iris on both sides to balance the system. A single waveguide iris is often referred to as an asymmetric waveguide iris or diaphragm and one where there are two, one either side is known as a symmetrical waveguide iris.





Symmetrical and asymmetrical waveguide iris implementations

A combination of both E and H plane waveguide irises can be used to provide both inductive and capacitive reactance. This forms a tuned circuit. At resonance, the iris acts as a high impedance shunt. Above or below resonance, the iris acts as a capacitive or inductive reactance.

Impedance matching using a waveguide post or screw

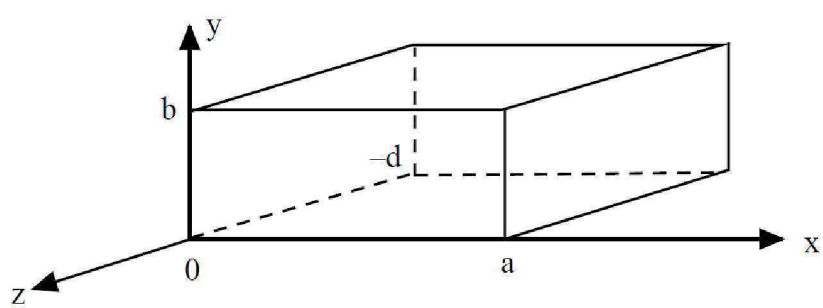
In addition to using a waveguide iris, post or screw can also be used to give a similar effect and thereby provide waveguide impedance matching.

The waveguide post or screw is made from a conductive material. To make the post or screw inductive, it should extend through the waveguide completely making contact with both top and bottom walls. For a capacitive reactance the post or screw should only extend part of the way through.

When a screw is used, the level can be varied to adjust the waveguide to the right conditions.

Microwave Resonator or Cavity Resonator

A cavity resonator is a useful microwave device. If we close off two ends of a rectangular waveguide with metallic walls, we have a rectangular cavity resonator. In this case, the wave propagating in the z direction will bounce off the two walls resulting in a standing wave in the z direction.



When a shorting plate is placed at distance multiple of $\lambda g/2$ then the hollw space so formed can support a signal which bounces back and forth between 2 shorting plates. The result is resonance and hence the hollow space is called cavity and resonator as the cavity resonator.

It is a tuned circuit at low frequency having frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

For d=3/2

A,b,m and n are constants

$$\lambda_0 = \frac{c}{f_0}$$

Expression for cut off frequency in rectangular cavity resonator

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \gamma^2$$

For propagation of waves $\gamma = j\beta$

$$\gamma^2 = -B^2$$

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + B^2$$

If wave exist in cavity resonator there is phase change to given guide wavelength

$$\beta = \frac{2\pi}{\lambda_g} = \frac{p\pi}{d}$$ condition for resonance

P= constant 1,2,. . . . . ∞ indicates half wave either dielectric or magnetic field variation on z-direction.

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

On solving this:

$$f_0 = \frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2\right]^{1/2}$$

Applications of cavity resonators

1. Tuned circuits

2. UHF Tubes, Klystron Amplifier or Oscillators

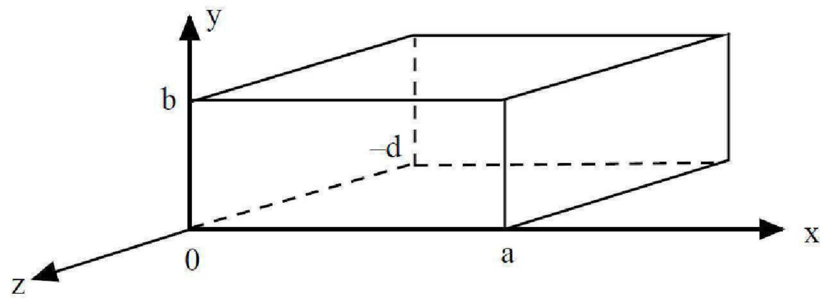
3. Cavity magnetrons

4. Duplexers of Radars

Field expressions for TM_{mnp} and TE_{mnp} modes in rectangular cavity resonator

Case-1: TM waves

Expression similar to TM waves in rectangular waveguide:



$$E_z = C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y e^{(j\omega t+\gamma z)}$$ for –ve direction and general equation

$$E_z = C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y e^{(j\omega t-\gamma z)}$$ for +ve direction

$$E_z = C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y e^{(j\omega t\pm\gamma z)}$$

Let the amplitude in +ve direction be A+

Let the amplitude in -ve direction be A-

$$E_z = \left[A^+ e^{-j\beta z} + A^- e^{j\beta z}\right] C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y e^{(j\omega t)}$$

To make E_z vanish at z=0 and for z=d $A^+=A^-$

$$E_z = A\left[e^{-j\beta d} + e^{j\beta d}\right] C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y e^{(j\omega t)}$$

$$E_z = 2AC_2 C_4 \cos\beta d \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y e^{(j\omega t)}$$

E_z at the surface of conductor so

$$0 = 2AC_2 C_4 \cos\beta d \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y e^{(j\omega t)}$$

$$0 = \cos \beta d$$

So

$$\beta d = \frac{(2n+1)\pi}{2}$$

Let p=(2n+1)/2

So

$$\beta = \frac{p\pi}{d}$$

So

$$E_z = C\cos\left(\frac{p\pi}{d}\right)z\sin\left(\frac{m\pi}{a}\right)x\sin\left(\frac{n\pi}{b}\right)ye^{(j\omega t-\gamma z)}$$

For TE waves:

$$H_z = C\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)ye^{(j\omega t\pm\beta z)}$$

Let the amplitude in +ve direction be A+

Let the amplitude in -ve direction be A-

$$H_z = \left[A^+e^{-j\beta z} + A^-e^{j\beta z}\right]C\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)ye^{(j\omega t)}$$

To make E_y vanish at z=0 and z=d $A^+=A^-$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial}{\partial y}E_z + \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}H_z$$

For TE waves E_z=0

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}H_z$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}\left[\left[A^+e^{-j\beta z} + A^-e^{j\beta z}\right]C\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)ye^{(j\omega t)}\right]$$

E_y vanish at z=0 and z=d

$$0 = -\frac{j\omega\mu}{h^2}\left[A^+e^{-j\beta z} + A^-e^{j\beta z}\right]C\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)ye^{(j\omega t)}$$

$$-2j\sin\beta d = 0$$

$$\beta = \frac{p\pi}{d}$$

$$H_z = -2jAC\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y\sin\left(\frac{p\pi}{d}\right)ze^{(j\omega t-\gamma z)}$$

$$H_z = C\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y\sin\left(\frac{p\pi}{d}\right)ze^{(j\omega t-\gamma z)}$$

Ferrite Devices

Ferrites are non-metallic material with resistivity nearly 10^{14} times greater than metals with dielectric constants around 10-15 and relative permeability of order 1000.

They have magnetic properties similar to ferrous metals. They are oxide based compounds having general composition of the form metal oxides and ferric oxides like MeO, Me_2O_3 that represents divalent metallic oxides such as MnO, ZnO, CdO, NiO, or mixtures of these.

Production: They are obtained by firing powdered oxides of materials at 1100^0C or more and pressing them into different shapes. This processing gives them added characteristics of ceramic insulators so that they can be used at microwave frequency.

Basic properties of ferrites:

Ferrites have atoms with large number of spinning electrons resulting in strong magnetic properties. These magnetic properties are due to magnetic dipole moment associated with the electron spin.

This property used to reduce reflected power for modulation purpose and in switching circuit.

Due to high resistivity they can used upto 100GHz.

At microwave frequency ferrites are non-reciprocal property.

Isolator with Faraday Rotation

Faraday rotation in ferrites: When an electromagnetic wave passes through ferrites, plane of polarization continuous to rotate at angle θ in one particular direction (either clockwise or anticlockwise). This plane of polarization changes in the same direction whatever may be the direction of propagation of wave. This is called as Faraday rotation. Isolator comprises of four components

i) rectangular waveguide with planar resistive card,

ii)Mechanical bend of 45^0 in anticlockwise direction. It is reciprocal device

iii) circular waveguide with ferrite rod to give a polarization rotation of 45^0 in clockwise direction. It is nonreciprocal device,

iv) rectangular waveguide with resistive card.

Working of Isolator:

Case1) A TE_{10} mode is applied to the rectangular waveguide so the field is oriented in vertical direction as shown in figure. When the wave passes through mechanical twist (bend) the field polarization rotates by 45^0 in anticlockwise direction. This field now passed through the Ferrite material and the field polarization now is rotated in clockwise direction by 45^0. Due to anticlockwise and clockwise rotation of 45^0 the overall rotation in polarization is nullified and the field becomes vertical as it was at the input. This vertical polarized field when passes through the last component of rectangular waveguide, it does not interact with the resistive card since card is horizontally placed.

Case2) When a vertically polarized wave is reflected from the second port, it travels with same orientation of the field till the ferrite rod. When it passes through ferrite material it under goes a rotation of 45^0 in clockwise direction and this rotated filed when passes through the mechanical twist, field is further rotated by the 45^0 in clockwise as the mechanical twist is reciprocal in nature. Now the total filed rotation is of 90^0 in clockwise and due to this field becomes parallel with the resistive card at the input of isolator and is absorbed by the card. Thus the isolator gives different response in different direction of propagation.

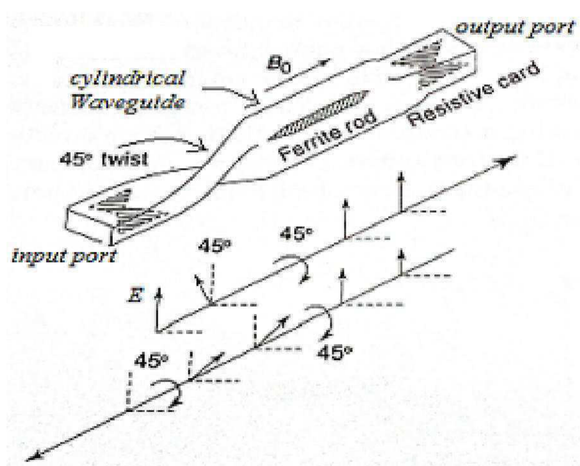


Figure: Propagation through Isolator

Non-Reciprocal Phase Shifter

A nonreciprocal phase shifter consists of a thin slab of ferrite placed in a rectangular waveguide at a point where the de magnetic field of the incident wave mode is circularly polarized. Ferrite is a family of MeO · Fez 03, where Me is a divalent iron metal. When a piece of ferrite is affected by a de magnetic field, the ferrite exhibits Faraday rotation.