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Roll No. .....

301

B. E. (Third Semester) EXAMINATION, June, 2009

(New Scheme)

(Common for all Branches)

MATHEMATICS – III

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : Attempt all questions. All questions carry equal marks. *One* complete question solve at one place.

1. (a) Show that the Polar form of Cauchy-Reimann equations are :

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}; \frac{dv}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

and deduce that :

$$\frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

(b) If :

$$F(t) = \int_C \frac{4z^2 + z + 5}{z - t} dz$$

where C is the Ellipse :

$$(x/2)^2 + (y/3)^2 = 1$$

Find the value of :

(i) $F(3 \cdot 5)$

(ii) $F(i)$, $F'(-1)$ and $F''(-i)$

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Or

(a) Expand :

$$f(z) = \frac{1}{(z-1)(z-2)}$$

in the region $|z| > 2$.

(b) Show that :

$$\int_0^{2\pi} \frac{\cos 2\theta \, d\theta}{1 - 2a\cos\theta + a^2} = \frac{2\pi a^2}{1 - a^2} \; (a^2 < 1)$$

2. (a) Show that the nth difference of a polynomial of degree n will be constant and all $(n+1)$th and higher order difference are low.

(b) Find the Ist and IInd derivative of the function tabulated below at $x = 1{\cdot}1$:

x	$f(x)$
1·0	0
1·2	0·128
1·4	0·544
1·6	1·296
1·8	2·482
2·0	4·0

Or

(a) Find :

$$\int_0^{0.6} e^{-x^2} \, dx$$

by taking seven ordinates using Simpson's 1/3 rule.

(b) Define forward, backward, central difference operator and shift average operator also.

3. (a) Solve the following by Euler modified method :

$$\frac{dy}{dx} = \log (x + y) \quad y(0) = 2$$

at $x = 1 \cdot 4$ with $h = 0 \cdot 2$.

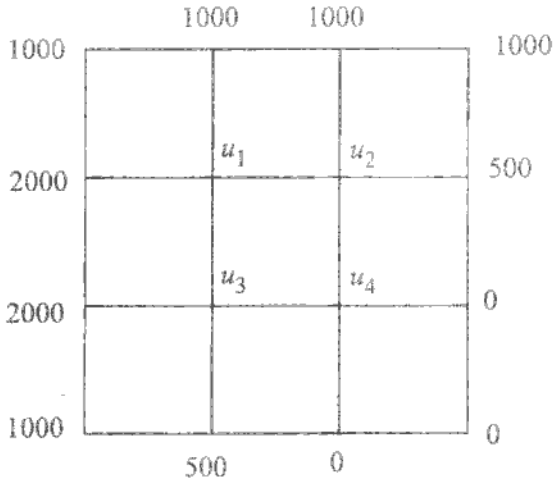
(b) Find the roots of equation $x e^x = \cos x$ using Regula-Falsi method, correct to four decimal places.

Or

(a) Solve by Gauss-Seidel method :

$$10 x_1 - 2 x_2 - x_3 - x_4 = 3$$
$$- 2 x_1 + 10 x_2 - x_3 - x_4 = 15$$
$$- x_1 - x_2 + 10 x_3 - 2 x_4 = 27$$
$$- x_1 - x_2 - 2 x_3 + 10 x_4 = - 9$$

(b) Given the values of $u(x, y)$ on the boundary of the square in below. Evaluate the function $u(x, y)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the Pivotal points of this figure.



4. (a) A company makes two kinds of leather belts. Belt A is a high quality belt and belt B the lower quality. The

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respective profit are Rs. 4·00 and Rs. 3·00 per belt. Each belt of type A requires twice as much time as a belt of type B and if all belts were of type B, the company could make 1000 per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). The belt A requires a fancy buckle and only 400 per day are available. There are 700 buckles a day available for belt B. Determine the optimal product mix by graphical method.

(b) Solve the following transportation problem :

		Destination				Supply
		D_1	D_2	D_3	D_4	
Source	S_1	19	30	50	10	7
	S_2	70	30	40	60	9
	S_3	40	8	70	20	18
Demand		5	8	7	14	

Or

(a) Four jobs are to be done on four different machines. The cost (in rupees) of producing *i*th job on the Jth machine is given below :

		Machine			
		M_1	M_2	M_3	M_4
Job	J_1	15	11	13	15
	J_2	17	12	12	13
	J_3	14	15	10	14
	J_4	16	13	11	17

Assign the jobs to different machines so as to minimize the total cost.

(b) Solve by Simplex method :

Minimize :

$$z = x_1 - 3x_2 + 2x_3$$

Subject to :

$$3x_1 - x_2 + 3x_3 \leq 7$$

$$-2x_1 + 4x_2 \leq 12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

5. (a) In a railway marshalling yard goods train arrive at a rate of 30 train per day. If the distribution of arrivals is the Poisson and that of service time is exponential with an average 36 minutes, then find :

(i) Mean queue size.

(ii) The probability that queue size exceeds.

If the input of trains increases to an average 33 per day what will be the change in (i) and (ii) ? 10

(b) The state transition matrix for retentions gains and losses of firms A, B and C is given below. Using this matrix determine the steady state equilibrium condition :

From	To		
	A	B	C
A	0·700	0·100	0·200
B	0·100	0·800	0·100
C	0·200	0·100	0·700

Or

(a) Write the advantage and disadvantage of Robust design method.

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(b) The number of units of an item that are withdrawn from inventory on a day to day basis is a Markov Chain Process in which requirement for tomorrow depend on today's requirement. A one day transition matrix is given below.

Number of units withdrawn from inventory :

$$\begin{array}{cc} & \text{Tomorrow} \\ & \begin{array}{ccc} 5 & 10 & 12 \end{array} \\ \text{Today} \begin{array}{c} 5 \\ 10 \\ 12 \end{array} & \begin{bmatrix} 0{\cdot}6 & 0{\cdot}4 & 0{\cdot}0 \\ 0{\cdot}3 & 0{\cdot}3 & 0{\cdot}4 \\ 0{\cdot}1 & 0{\cdot}3 & 0{\cdot}6 \end{bmatrix} \end{array}$$

(i) Construct a tree diagram showing inventory requirements on two consecutive days.

(ii) Develop a two day transition matrix.