BE–301(GS)

B. E. (Third Semester) EXAMINATION, Dec., 2011

(Grading System)

(Common for all Branches)

ENGINEERING MATHEMATICS – II

[BE – 301(GS)]

Time : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 22 (Grade-D)

Note : Attempt *one* question from each Unit. All questions carry equal marks.

Unit – I

1. (a) Find the Fourier transform for the function :

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

(b) Find the Fourier transform of :

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

Or

2. (a) Express $f(x) = x$ as a half range cosine series in $0 < x < 2$.

(b) Find the Fourier sine transform of $e^{-|x|}$. Hence, show that :

$$\int_0^\infty \frac{x \sin mx}{1+x^2}\, dx = \frac{\pi e^{-m}}{2}, m > 0$$

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Unit – II

3. (a) If $f(t)$ is a periodic function with period T, then prove that :

$$L\{f(t)\} = \frac{\int_0^T e^{-sT} f(t)\, dt}{1 - e^{-sT}}$$

(b) Apply Convolution theorem to evaluate :

$$L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$$

Or

4. (a) Find the Laplace transforms of :

(i) $\frac{1-e^t}{t}$

(ii) $t^2 \sin at$

(b) Solve the equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4e^{2t}$, $y(0) = -3$, $y'(0) = 5$ using Laplace transform.

Unit – III

5. (a) Solve the equation :

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

given that $y = x$ is a solution.

(b) Solve the Bessel's equation :

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$$

Or

6. (a) Solve the equation :

$$\frac{d^2y}{dx^2} + \cot x\frac{dy}{dx} + 4y\,\mathrm{cosec}^2 x = 0$$

(b) Find the series solution of the equation :

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$$

Unit – IV

7. (a) Solve the equations :

(i) $\frac{y^2z}{x}p+xz\,q=y^2$

(ii) $x^2p^2+y^2q^2=z^2$

(b) Solve the equation :

$$\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x\,\partial y}+4\frac{\partial^2 z}{\partial y^2}=e^{2x+y}$$

Or

8. Solve the wave equation :

$$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial \pi^2}$$

subject to the conditions :

$y(0,t)=0,\quad y(l,t)=0;\quad y(x,0)=a\sin\left(\frac{\pi x}{\lambda}\right)$ and $\left(\frac{\partial y}{\partial t}\right)_{t=0}=0.$

Unit – V

9. (a) Prove that :

(i) $\nabla r^n=n\,r^{n-2}\vec{r}$, where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$.

(ii) Curl $\vec{F}=0$, where $\vec{F}=\text{grad}\,(x^3+y^3+z^3-3xyz)$.

(b) (i) Find a unit vector normal to the surface $xy^3z^2=4$ at the point $(-1,-1,2)$.

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(ii) Show that the vector :
$\vec{A}=(-x^2+yz)\hat{i}+(4y+z^2x)\hat{j}+(2xz-4z)\hat{k}$
is solenoidal.

Or

10. (a) Use Stokes' theorem to evaluate :

$$\int_C [(x+y)\,dx+(2x-z)\,dy+(y+z)\,dz]$$

where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6).

(b) Evaluate :

$$\int_S \vec{F}\cdot dS$$

where $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ and S is the surface bounding the region $x^2+y^2=4$, $z=0$ and $z=3$.