Note : Attempt any five questions.

1. (a) Prove by Induction : 5

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$$

(b) Find transitive closure and reflexive and transitive closure for the following relation set : 5

$$R = \{(a, b), (a, c), (c, d), (a, a), (b, a)\}$$
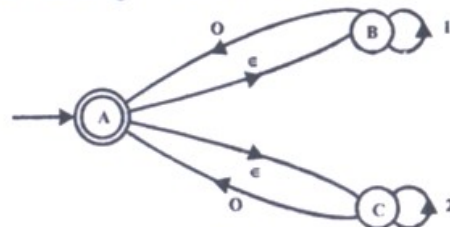
(c) Convert the following into NFA without ϵ move. 5

Fig. 1

(iii)

(d) Construct DFA for the following Language : 5

$L(M) = \{\omega \in (0, 1)^* | \omega$ has neither 'aa' nor 'bb' as a substring$\}$

2. (a) Convert the following regular expression to NFA and then it to DFA : 6

$(a + b)^*$ ab
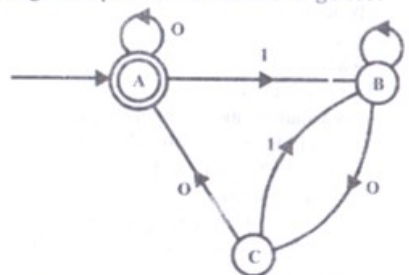
(b) Write regular expression for the following DFA : 4

Fig. 2

(c) Construct Moore machine equivalent to the given Mealey machine : 10

State	Input			
	0		1	
	State	Output	State	Output
→ q_1	q_1	1	q_2	0
q_2	q_4	1	q_4	1
q_3	q_2	1	q_3	1
q_4	q_3	0	q_1	1

3. (a) Prove that regular sets are closed under intersection. 5

(b) Apply Pumping Lemma to prove that $L = \{a^n b^{2n} | n \geq 1\}$ is not regular set. 5

(c) Construct CFG for the following : 10

(i) The set of all strings over $\{a, b\}$ with exactly twice as many a's as b's.

(ii) $0^* + 1^*$

4. (a) Eliminate ϵ productions from CFG : 5

S → ABCD
A → CDa
B → Cb
C → a|ϵ
D → bD|ϵ

(b) Convert the CFG into CNF : 5

S → aS |b Sc Sd|p|q

(c) Convert CFG into GNF : 10

A → BC
B → CA/b
C → AB|a

5. (a) Construct PDA for the following set : 10

$L = \{a^{2n} b^n | n \geq 1\}$

(b) Convert the CFG into PDA : 10

S → aB|bA
A → a | aS | bAA
B → b | bS | aBB

6. (a) Convert the following PDA into CFG : 10

$M = (\{q_0, q_1\}, \{0, 1\}, \{z_0, x\}, \delta, q_0, z_0, \phi)$

$\delta(q_0, 1, z_0) = (q_0, xz_0), \delta(q_0, \epsilon, z_0) = (q_0, \epsilon)$
$\delta(q_0, 1, x) = (q_0, xx), \delta(q_1, 0, z_0) = (q_0, z_0)$
$\delta(q_0, 0, x) = (q_1, x), \delta(q_1, 1, x) = (q_1, \epsilon)$

(b) (i) Prove CFLS are not closed under intersection. 10

(ii) Prove CFLS are closed under kleen closure. 10

7. (a) Design turing machine to accept : 10

$L = \{a^n b^b c^n | n \geq 1\}$

(b) Discuss how turing machine can be designed to compute : 10

(i) n! (ii) $\log_2 n$

8. (a) Prove that union of two recursive language is recursive and if a language L and its compliment $\overline{L}$ are both recursively enurable then L is recursive. 10

(b) For the regular expression $0(10)^*$ write right linear grammar and convert it into NFA. 10