UNIT-3

Pushdown Automata:-

① Pushdown Automaton (PDA):-

A pushdown automaton or PDA is represented by 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F, Z_0)$ where

$Q \rightarrow$ set of states

$\Sigma \rightarrow$ set of ~~output~~ input alphabets

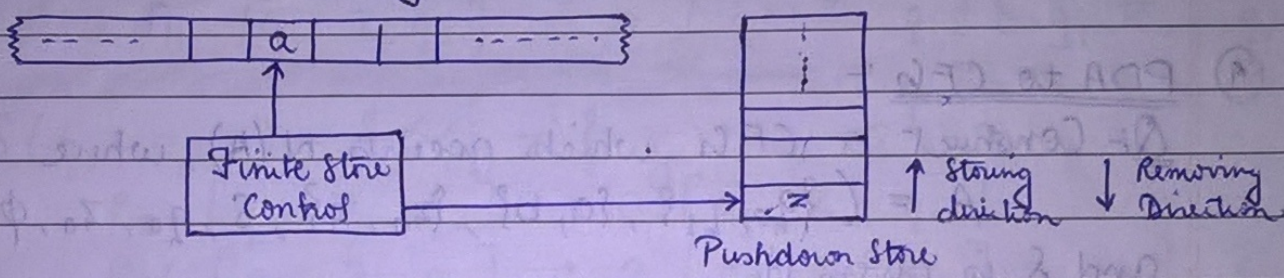
$\Gamma \rightarrow$ set of stack symbols

$\delta \rightarrow$ Transition function $(Q \times (\Sigma \cup \{\Lambda\}) \times \Gamma \rightarrow Q \times \Gamma^*)$

$q_0 \rightarrow$ Initial State

$F \rightarrow$ ~~Final State(s)~~ set of final states

Z_0 ~~B~~ $\rightarrow$ ~~Blank~~ stack symbol (~~b~~)



Model of a pushdown automaton

② Deterministic Pushdown Automata:-

A PDA, $A = (Q, \Sigma, \Gamma, \delta, q_0, F, Z_0)$ is deterministic if

(i) $\delta(q, a, Z)$ is either empty or a singleton, and

(ii) $\delta(q, \Lambda, Z) \neq \phi$ implies $\delta(q, a, Z) = \phi$ for each $a \in \Sigma$.

③ CFG to PDA:-

Q. Construct a PDA to the following CFG: $S \rightarrow 0BB$, $B \rightarrow 0S \mid 1S \mid 0$. Test whether 010^4 is in $N(A)$.

A:- $A = (\{q\}, \{0, 1\}, \{S, B, 0, 1\}, \delta, q, S, \phi)$

δ is defined by the following rules:

R_1: $\delta(q, \Lambda, S) = \{(q, 0BB)\}$

R_2: $\delta(q, \Lambda, B) = \{(q, 0S), (q, 1S), (q, 0)\}$

R_3: $\delta(q, 0, 0) = \{(q, \Lambda)\}$

R_4: $\delta(q, 1, 1) = \{(q, \Lambda)\}$

NEXT VARIABLE   STACK TOP

$(q, 010^4, S)$

$\vdash (q, 010^4, 0BB)$

$\vdash (q, 10^4, BB)$

$\vdash (q, 10^4, 1SB)$

$\vdash (q, 0^4, SB)$

$\vdash (q, 0^4, 0BBB)$

$\vdash (q, 0^3, BBB)$

$\vdash^* (q, 0^3, 000)$

$\vdash^* (q, \Lambda, \Lambda)$

Thus, $010^4 \subseteq N(A)$

④ **PDA to CFG :-**

Q:- Construct a CFG which accepts $N(A)$, where

$$A = (\{q_0, q_1\}, \{a, b\}, \{Z_0, Z\}, \delta, q_0, Z_0, \phi)$$

and δ is given by

$$\delta(q_0, b, Z_0) = \{(q_0, ZZ_0)\}$$
$$\delta(q_0, \Lambda, Z_0) = \{(q_0, \Lambda)\}$$
$$\delta(q_0, b, Z) = \{(q_0, ZZ)\}$$
$$\delta(q_0, a, Z) = \{(q_1, Z)\}$$
$$\delta(q_1, b, Z) = \{(q_1, \Lambda)\}$$
$$\delta(q_1, a, Z_0) = \{(q_0, Z_0)\}$$

A:- Let $G = (V_N, \{a, b\}, P, S)$

$V_N \rightarrow \{S, [q_0, Z_0, q_0], [q_0, Z_0, q_1], [q_0, Z, q_0], [q_1, Z, q_1], [q_0, Z, q_1], [q_1, Z_0, q_0], [q_1, Z_0, q_1], [q_1, Z, q_0]$

$P \rightarrow$ $P_1: S \rightarrow [q_0, Z_0, q_0]$

$P_2: S \rightarrow [q_0, Z_0, q_1]$

$\delta(q_0, b, Z_0) = \{(q_0, ZZ_0)\}$ yields $2^2 = 4$

$P_3: [q_0, Z_0, q_0] \longrightarrow b \cdot [q_0\ Z\ q_0]\ [q_0\ Z_0\ q_0]$

$P_4: [q_0, Z_0, q_0] \longrightarrow b \cdot [q_0\ Z\ q_1]\ [q_1\ Z_0\ q_0]$

P_5: $[q_0, Z_0, q_1] \longrightarrow b.\ [q_0\ Z_0\ q_0]\ [q_0\ Z_0\ q_1]$
P_6: $[q_0, Z_0, q_1] \longrightarrow b.\ [q_0\ Z\ q_1]\ [q_1\ Z_0\ q_1]$

$\delta(q_0, \Lambda, Z_0) = \{(q_0, \Lambda)\}$ yields $2^0 = 1$
P_7: $[q_0, Z_0, q_0] \longrightarrow \Lambda$

$\delta(q_0, b, Z) = \{(q_0, ZZ)\}$ yields $2^2 = 4$
P_8: $[q_0\ Z\ q_0] \longrightarrow b.\ [q_0\ Z\ q_0]\ [q_0\ Z\ q_0]$
P_9: $[q_0\ Z\ q_0] \longrightarrow b.\ [q_0\ Z\ q_1]\ [q_1\ Z\ q_0]$
P_{10}: $[q_0\ Z\ q_1] \longrightarrow b.\ [q_0\ Z\ q_0]\ [q_0\ Z\ q_1]$
P_{11}: $[q_0\ Z\ q_1] \longrightarrow b.\ [q_0\ Z\ q_1]\ [q_1\ Z\ q_1]$

$\delta(q_0, a, Z) = \{(q_1, Z)\}$ yields $2^1 = 2$
P_{12}: $[q_0\ Z\ q_0] \longrightarrow a.\ [q_1\ Z\ q_0]$
P_{13}: $[q_0\ Z\ q_1] \longrightarrow a.\ [q_1\ Z\ q_1]$

$\delta(q_1, b, Z) = \{(q_1, \Lambda)\}$ yields $2^0 = 1$
P_{14}: $[q_1\ Z\ q_1] \longrightarrow$ ~~ab $[q_1$~~ b. Λ

$\delta(q_1, a, Z_0) = \{(q_0, Z_0)\}$ yields $2^1 = 2$
P_{15}: $[q_1\ Z_0\ q_0] \longrightarrow a.\ [q_0\ Z_0\ q_0]$
P_{16}: $[q_1\ Z_0\ q_1] \longrightarrow a.\ [q_0\ Z_0\ q_1]$

(5) **Intantaneous Description of PDA :-**

Intantaneous description describe the configuration of a PDA at a given instant. ID is a triple such as (q, w, γ) where q is a state, w is a string of input symbols and γ is a string of stack symbols.

(6) Input string is accepted by the PDA if
- → The final state is reached.
- → The stack is empty.

Content Free Languages -(CFL) -

① Pumping Lemma for CFL :-

Let L be a content free language. Then we can find a natural number n such that:

(i) Every $z \in L$ with $|z| \geq n$ can be written as $uvwxy$ for some strings u, v, w, x, y. $(z = uvwxy)$

(ii) $|vx| \geq 1$

(iii) $|vwx| \leq n$

(iv) $uv^k w x^k y \in L$ for all $k \geq 0$

② Closure properties of CFL :-

(1) Union (2) Concatenation (3) Intersection

(4) Complementation (5) set difference (6) Intersection with Regular languages (7) Substitution

③ Decision problems involving CFL :-

→ Algorithm for deciding whether a content free language L

(i) is empty ⇒ L is nonempty if and only if $S \in W_K$ (vertices/states)

(ii) is finite ⇒ L is finite if and only if the directed graph has no cycles.