Roll No ....

BE - 102

B.E. I & II Semester Examination, December 2014

Engineering Mathematics-I

Time : Three Hours

Maximum Marks : 70

Note: i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.

ii) All parts of each questions are to be attempted at one place.

iii) All questions carry equal marks, out of which part A and B (Max.50 words) carry 2 marks, part C (Max.100 words) carry 3 marks, part D (Max.400 words) carry 7 marks.

iv) Except numericals, Derivation, Design and Drawing etc.

1. a) Define curvature of a curve at a point and find the radius of curvature at any point (s, ψ) of the curve $s = 4a \sin\psi$.

b) If $u = f\left(\frac{y}{x}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

c) Discuss the maxima and minima of the function $x^3 + y^3 - 3axy$.

d) Compute the approximate value of $\sqrt{11}$ to four decimal place by taking the first five terms of an approximate Taylor's expansion.

Or

If $x^x y^y z^z = c$, then show that $\frac{\partial^2 z}{\partial x \partial y} = -\left[x \log(ex)\right]^{-1}$.

2. a) Using Gamma function, evaluate $\int_0^\infty \sqrt{x}\, e^{-3\sqrt{x}}\, dx$.

b) Evaluate $\int_0^2 \int_0^1 \left(x^2 + y^2\right) dx\, dy$

c) Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dx\, dy\, dz$

d) Evaluate $\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\ldots\left(1+\frac{n^2}{n^2}\right)\right]$

Or

Prove the Legendre's duplication formula $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$

3. a) State whether the differential equation $\left(e^y + 1\right)\cos x\, dx + e^y \sin x\, dy = 0$ is exact differential equation or not.

b) Solve the differential equation $p = \sin(y - xp)$

c) Solve the differential equation $\frac{dy}{dx}-\frac{dx}{dy}=\frac{x}{y}-\frac{y}{x}$

d) Solve $x^2\frac{dy}{dx}-3x\frac{dy}{dx}+4y=(1+x)^2$

Or

Solve the simultaneous equations: $\frac{dx}{dt}+5x+y=e^t$;

$$\frac{dy}{dt}-x+3y=e^{2t}$$

4. a) Find one non zero minor of highest order of the matrix $A=\begin{pmatrix}-1 & -2 & 3\\ -2 & 4 & -1\\ -1 & 2 & 7\end{pmatrix}$ and hence find the rank of the matrix A.

b) Find the sum and product of eigen values of the matrix A = $\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & 1\\ 2 & -1 & 3\end{pmatrix}$ without actually computing them.

c) Find the characteristic equation of the matrix $A=\begin{pmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{pmatrix}$.

d) Find the normal form of the matrix $A=\begin{pmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\\ 6 & 3 & 0 & -7\end{pmatrix}$ and hence find its rank.

Or

For what values of λ, the equations

$x+y+z=1,\ x+2y+4z=\lambda,\ x+4y+10z=\lambda^2$

have a solution and solve completely in each case.

5. a) Let $p\equiv$ *Raju is tall,* $q\equiv$ *Raju is handsome and* $r\equiv$ *People like Raju* then write the following statements in language.

i) $(p\Rightarrow q)\vee(p\Rightarrow r)$ ii) $p\Rightarrow(q\vee r)$ iii) $\sim p\vee\sim q$ iv) $\sim(\sim p\vee\sim q)$

b) In a Boolean algebra B, prove that $a+b=b\Rightarrow a.b=a, \forall a,b\in B$.

c) Draw the switching circuit for the following function and replace it by simpler one:

$F(x,y,z)=x.y.z+(x+y).(x+z)$

d) Prove that a tree with n vertices has $(n-1)$ edges.

Or

If p, q, r are three statements then show that $(p\Leftrightarrow q)\wedge(q\Leftrightarrow r)\Rightarrow(p\Leftrightarrow r)$ is a tautology.

******