5. a) Show that a linearly ordered poset is a distributive lattice.

b) Let A = {a, b, c, d} and R be relation on A whose matrix is

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Draw the Hasse diagram of R.

c) Define the following:

i) Binomial theorem

ii) Multinomial coefficient

d) Solve the recurrence relation:

$a_r - 7a_{r-1} + 10a_{r-2} = 0$ given $a_0 = 0$ and $a_1 = 6$

Or

Write short notes:

i) Posets

ii) Lattices

iii) Permutation and Combination

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Total No. of Questions : 5] *[Total No. of Printed Pages : 4*

Roll No ....

CS/IT-302

B.E. III Semester

Examination, December 2016

Discrete Structure

Time : Three Hours

Maximum Marks : 70

Note: i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.

ii) All parts of each question are to be attempted at one place.

iii) All questions carry equal marks, out of which part A and B (Max. 50 words) carry 2 marks, part C (Max. 100 words) carry 3 marks, part D (Max. 400 words) carry 7 marks.

iv) Except numericals, Derivation, Design and Drawing etc.

1. a) Show that $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$

b) Give A = the set of real numbers; prove that aRb if and only if $a^2 + b^2 = 4$

c) Define a relation on the set {a, b, c, d} that is

i) Reflexive and symmetric, but not transitive

ii) Reflexive and transitive, but not symmetric

d) What is Mathematical Induction? Prove by induction that $3 + 11 + \ldots\ldots\ldots + (8n - 5) = 4n^2 - n$ for $n \in$ N and $n \geq 1$.

Or

Let A = B = C = R, and Let f : A → B, g : B → C be defined by $f(a) = a + 1$ and $g(b) = b^2 + 2$, find

i) (gof) (–2)

ii) (fog) (x)

iii) (gog) (y)

iv) (gof) (x)

2\. a) Let A be a set with n elements. How many binary operations and commutative binary operations can be defined on A.

b) Prove or disprove that the intersection of two subsemigroup of a semigroup $(S, *)$ is a subsemigroup of $(S, *)$.

c) Define homomorphism and isomorphism of groups with an example.

d) Let G be the set of all nonzero real numbers and let

$$a * b = \frac{ab}{2}$$

Show that $(G, *)$ in an Abelian group.

Or

Let H and K be subgroups of a group G.

i) Prove that $H \cap K$ is a subgroup of G,

ii) Show that $H \cup K$ need not be a subgroup of G.

3\. a) Write an English sentence corresponding to each of the following:

i) $\forall_x \exists y R(x, y)$ ii) $\forall_x (\sim \theta(x))$

b) Prove that $(\sim q \wedge (p \Rightarrow q)) \Rightarrow \sim p$ in a tautology.

c) Why is it important to recognize the converse and the contrapositive of a conditional statement.

d) Let p : An internet business is cheaper to start.

q : I will start an internet business and

r : An internet business makes less money.

For each of the following write the argument in English sentence and also determine the validity of argument.

i) $$\begin{array}{l} r \Rightarrow (q \Rightarrow p) \\ \sim p \\ \hline \therefore (\sim r) \vee (\sim q) \end{array}$$

ii) $$\begin{array}{l} p \Rightarrow q \\ q \Rightarrow r \\ p \\ \hline \therefore r \end{array}$$

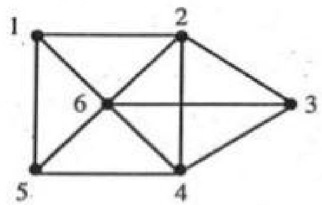
Or

Let I = {0, 1} construct a Moore machine that accepts those input sequences, w that contain the string 01 or the string 10 anywhere within them in other words we are to accept exactly those strings that do not consist entirely of 0's or entirely of 1's.

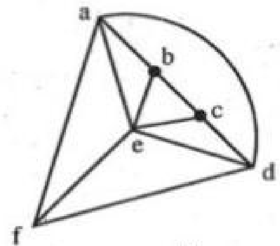
4\. a) Give an example of regular graph connected graph on six vertices that is not complete.

b) Draw a graph that has an Euler circuit and Hamiltonian circuit that are not the same.

c) Prove that G and H are isomorphic



G



H

d) i) Prove that a finite connected graph G is Eulerian if and only if each vertex has even degree.

ii) Define planar graph, regular and bipartite graph. Draw the graph $K_{2,5}$

Or

Find the shortest path between a to f in the following weighted graph.