Total No. of Questions :5] [Total No. of Printed Pages :3

Roll No ....

CS/IT - 302

B.E. III Semester Examination, December 2014

Discrete Structure

Time : Three Hours

Maximum Marks : 70

Note: i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.

ii) All parts of each question are to be attempted at one place.

iii) All questions carry equal marks, out of which part A and B (Max.50 words) carry 2 marks, part C (Max.100 words) carry 3 marks, part D (Max.400 words) carry 7 marks.

iv) Except numericals, Derivation, Design and Drawing etc.

Unit - I

1. a) Prove that intersection of sets is distributive w.r.t. union of sets i.e $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

b) Define subset of a set with an example. Also define proper subset and improper subset.

c) Prove by mathematical induction-

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + ... + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

d) Let $X = \{a, b, c\}$. Define $f : X \to X$ such that $f = \{(a,b), (b,a), (c,c)\}$

Find (i) f^{-1} (ii) f^2 (iii) f^3 (iv) f^4

OR

Among the first 500 positive integers:

i) Determine the integers which are not divisible by 2, nor by 3, nor by 5.

ii) Determine the integers which are exactly divisible by one of them.

Unit - II

2. a) Define monoid with an example.

b) Determine whether a semigroup with more than one idempotent element can be group.

c) Explain Homomorphism and Isomorphism of groups with an example.

d) Consider an algebraic system $(\theta, *)$ where θ is the set of rational numbers and $*$ is a binary operation defined by

$$a * b = a + b - ab \ \forall \ a, b \in \theta$$

Determine whether $(\theta, +)$ is a group.

OR

Let $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and multiplication modulo 8, that is $x \oplus y = (xy)$ mod 8.

i) Prove that $(\{0,1\}, \oplus)$ is not a group.

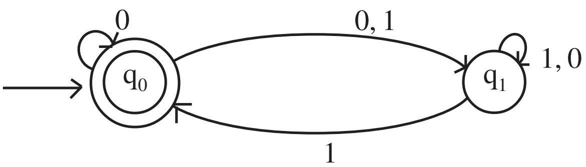
ii) Write three distinct groups $(G, \otimes)$ where GCS and Ghrs 2 elements.

Unit - III

3. a) Write down the principle of Duality.

b) Show that $(p \oplus q) \vee (p \downarrow q)$ is equivalent to $p \uparrow q$.

c) Convert the following NFA to DFA



d) Determine whether the following are equivalent using biconditional statement.

i) $p \leftrightarrow q \cong (p \wedge q) \vee (\sim p \wedge \sim q)$

ii) $(p \to q) \to t \cong (p \wedge \sim q) \to t$

OR

Show that the rule of hypothetical syllogism is valid

$p \to q$

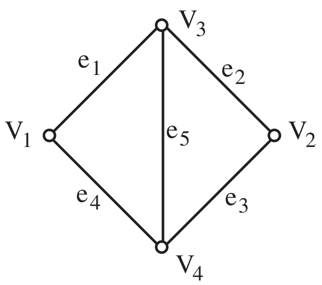
$q \to r$

$p \to r$

Unit - IV

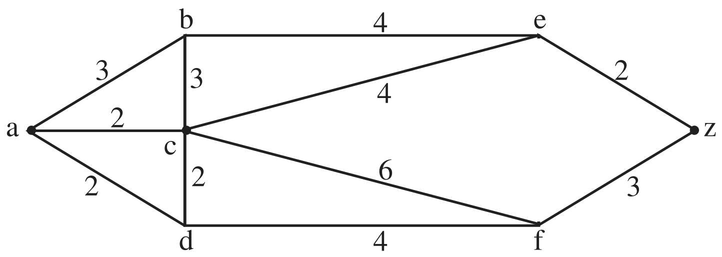
4. a) Prove that in any graph, there are an even number of vertices of odd degree.

b) Consider the undirected graph G as show in fig below. Find in incidence matrix M_I.



c) Prove that every Planar graph has at least one vertex of degree 5 or less than 5.

d) Find the shortest path between a and z in the graph shown below.



OR

Explain briefly:

i) Euler Path ii) Hamiltonian Path

iii) Graph Coloring iv) Chromatic Number

Unit - V

5. a) Prove that if L be a lattice then $a \wedge b = a$ if and only if $a \vee b = b$.

b) There are n objects out of which r objects are to be arranged. Find total number of permutations when

i) Four particular objects always occur

ii) Four particular objects never occur.

c) Solve the difference equation $Y_K - Y_{K-1} - Y_{K-2} = 0$.

d) Solve the recurrence relation $a_{r+2} - 2a_{r+1} + a_r = r^2 . 2^r$.

OR

Write short notes : (any three)

i) Bounded Lattice

ii) Isomorphic Lattice

iii) Hasse Diagram

iv) Binomial Theorem

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