MCSE/MSE - 101

M.E/M.Tech., I Semester

Examination, December 2014

Advanced Computational Mathematics

Time : Three Hours

Maximum Marks : 70

Note: i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.

ii) All parts of each questions are to be attempted at one place.

iii) All questions carry equal marks, out of which part A and B (Max.50 words) carry 2 marks, part C (Max.100 words) carry 3 marks, part D (Max.400 words) carry 7 marks.

iv) Except numericals, Derivation, Design and Drawing etc.

1. a) Let V be a vector space of dimension n. Then show that any set of n linearly independent elements of V is a basis of V.

b) Show that $erf(x) + erf_c(x) = 1$

c) If V be a finite dimensional vector space over a field F and S∈F is a characteristic root of the linear transformation T∈ A(V), where A(V) is an algebra of linear transformations on V. Then show that for any polynomial $q(x) \in F[x]$, q(λ) is a characteristic root of q(T).

d) If V and V' are vector spaces of dimensions m and n respectively over the field F. Then prove that dimension L(V, V') = mn, where L(V, V') is the space of all linear transformations of V to V'.

d) Let u_w be the Yager class of t-conorms defined by

$u_w(a,b) = \min\left\{1, \left(a^w + b^w\right)^{\frac{1}{w}}\right\}, w > 0$ then show that for all

$a, b \in [0, 1]$

$\max(a, b) \le u_w(a, b) \le u_{max}(a, b)$.

Where $u_{max}(a,b) = \begin{cases} a & when\ b = 0 \\ b & when\ a = 0 \\ 1 & otherwise \end{cases}$

OR

Define transitive closure of a crisp fuzzy relation and write the 3 steps algorithm to find it. Find the maximum minimum transitive closure of the following fuzzy relation

$$R(X, X) = \begin{bmatrix} .7 & .5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & .4 & 0 & 0 \\ 0 & 0 & .8 & 0 \end{bmatrix}.$$

OR

Let $H_n(x)$ be the Hermite polynomial of degree n. Then show that $H_1(x) = x$, $H_2(x) = x^2 - 1$, $H_3(x) = x^3 - 3x$ and $H_4(x) = x^4 - 6x^2 + 3$.

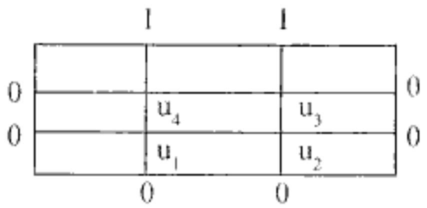
2. a) Write the Jacobi's and Gauss-seidel iterations schemes for solving the Laplace equation $u_{xx} + u_{yy} = 0$ using finite difference method.

b+c) Discuss the numerical solution of the Laplace equation to get the standard five-point and diagonal five point formulae.

d) Using the method of separation of variables, solve the equation $\frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t}$, given that $v = 0$ when $t \to \infty$, also $v = 0$ at $x = 0$ and $x = l$.

OR

Solve the equation $u_{xx} + u_{yy} = 0$ by Gauss-Seidel method in the following domain:



3. a) A can hit a target 3 times out of 5 shots, B can hit 2 times out of 5 shots and C can hit 3 times out of 4 shots. All of them fire one shot each simultaneously at the target. Find the probability that 2 shots hit the target.

b) If X is a normal variate with parameters μ and σ, then show that mean $E(X) = \mu$.

c) If the mean of a Binomial distribution is 3 and the variance is 3/2, find the probability of obtaining atmost 3 successes.

d) A discrete random variable X has the following probability mass function:

Values of X:x	0	1	2	3	4	5	6	7
$P(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) Find k

ii) Evaluate $P(X < 6)$ and $P(0 < X < 5)$

iii) Determine the distribution function of X.

OR

What do you mean by standard error? For the following sampling distribution find the standard error:

$\bar{X}$:	Probability:
1	$1/16$
2	$2/16$
3	$1/16$
4	$2/16$
5	$4/16$
6	$2/16$
7	$1/16$
8	$2/16$
9	$1/16$

4. a) Two brands of tooth paste, A and B, are available, A Customer C_1 who uses brand A, there is 80% chance that he would buy the same brand in the next purchase, where as a customer C_2, using brand B now, has 90% chance to buy the same brand in his next purchase. Express the initial transition probability matrix. Find the probability of buying the same brands by C_1 and C_2 three periods from now.

b) The arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between two successive arrivals. The call time is assumed to be distributed exponentially with mean of 3 minutes.

i) What is the probability that a person arriving at the booth with have to wait?

ii) The telephone department plans to install a 2nd booth if it is convinced that an arrival would expect waiting for at least 3 minutes for a call. By how much should the flow of arrivals increase in order to justify a 2nd booth.

c) Recently in a market survey by a firm it was found that three brands A, B, C of talcum powders are used by the customers in a city with 20% choice to A, 50% to brand B and 30% to brand C. Firm further analysed the data and found the following brand switching matrix:

$$
\begin{array}{lc}
 & \text{Next choice of brand} \\
 & \begin{array}{ccc} \quad A & \quad\quad B & \quad\quad C \end{array} \\
\begin{array}{ll} \text{Present} & A \\ \text{Brand} & B \\ \text{Choice} & C \end{array} & \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.1 & 0.7 \end{bmatrix}
\end{array}
$$

Find the distribution of customers choice three periods later from now.

d) Find the steady state probabilities for a Markov process with the following transition matrix:

$$
\begin{array}{cc}
 & \text{States} \\
 & \begin{array}{cccc} 1 & \quad 2 & \quad 3 & \quad 4 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & .75 & .25 & 0 \\ 0 & .5 & .5 & 0 \\ 0 & 0 & .5 & .5 \\ 1 & 0 & 0 & 0 \end{bmatrix}
\end{array}
$$

OR

At the sales counter of a super market there are two sales girls. If the service time for each customer is exponential with a mean at 4 minutes and customers arrive in Poisson fashion at the rate of 10 an hour. Then determine

i) Probability of waiting for a service.

ii) Expected percentage of idle time for each girl

iii) For a waiting customer, the expected length of his waiting time.

5. a) Let $C:[0,1] \to [0,1]$ be a continuous fuzzy complement function, then show that C has a unique equilibrium.

b) Describe two different functions with their application.

c) Let $i : [0,1] \times [0,1] \to [0,1]$ be a fuzzy intersection function, then prove that $i_{\min}(a,b) \le i(a,b) \le \min(a,b)$

where $i_{\min}(a,b) = \begin{cases} a & when\, b = 1 \\ b & when\, a = 1 \\ 0 & otherwise \end{cases}$