UNIT-5

Tractable and Untractable Problems -

① **Tractable Problems :-**

The set of all the problems that can be solved within polynomial amount of time using deterministic machine.

Untractable problems :-

The set of all the problems that can't be solved within polynomial amount of time using deterministic machine.

② **P class problem :-**

A language L is in class P if there exists some polynomial $T(n)$ such that $L = T(M)$ for some deterministic TM M of time complexity $T(n)$.

NP class problem :-

A language L is in class NP if there is a non-deterministic TM M and a polynomial time complexity $T(n)$ such that $L = T(M)$ and M executes at most $T(n)$ moves for every input w of length n.

③ **Polynomial time reduction :-**

Let P_1 and P_2 be two problems. A reduction from P_1 to P_2 is an algorithm which converts an instance of P_1 to an instance of P_2. If the time taken by the algorithm is a polynomial $p(n)$, n being the length of the input of P_1, then the reduction is called a polynomial reduction P_1 to P_2.

→ If there is a polynomial time reduction from P_1 to P_2 and if P_2 is in P then P_1 is in P.

④ **NP-complete problem :-**

Let L be a language or problem in NP. The L is NP-complete if

(i) L is in NP

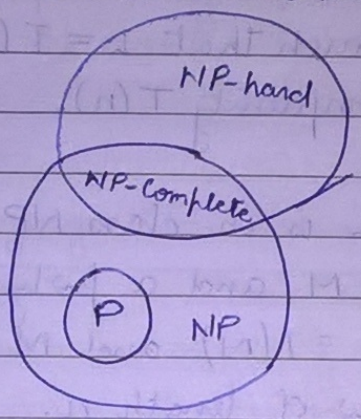
(ii) For every language L' in NP there exists a polynomial time reduction of L' to L.

→ If P_1 is NP-complete, and there is a polynomial-time reduction of P_1 to P_2, then P_2 is NP-complete.
→ If some NP-complete problem is in P, then P = NP.

(5) NP-hard problems :-

These problems are at least as hard as the hardest problem in NP but not necessarily in NP.

The problem to which all NP-class problems are reducible in polynomial time are known as NP-hard problems



(6) NP-complete problems :-

(i) SAT problem (satisfiability problem for boolean expression)
(ii) Hamiltonian Path Problem (HPP)
(iii) Traveling Salesman Problem (TSP)
(iv) Vertex Cover Problem (VCP)
(v) Partition problem. (PP)

Cook's theorem → SAT is NP-complete.