Total No. of Questions : 8] [Total No. of Printed Pages : 4

IT/CS–4507

B. E. (Fourth Semester) EXAMINATION, June, 2003

(Information Technology & Computer Science Engg.)

DISCRETE STRUCTURES

(IT/CS – 4507)

Time : Three Hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : Attempt any *five* questions. All questions carry equal marks.

1. (a) Let R be a binary relation on the set of all integers such that $R = \{(a, b) \mid (a - b)$ is an even integer $\}$. Is R an equivalence relation ? 5

   (b) Explain the principle of inclusion and exclusion. 4

   (c) By principle of mathematical induction prove that :

   $$2 \cdot 7^n + 3 \cdot 5^n - 5$$

   is divisible by 24 for all natural numbers n. 7

2. (a) Let $A = \{a, b, c, d\}$ and P (A) its power set. Draw Hasse diagram of $(P(A), \subseteq)$. 7

   (b) Explain any *three* of the following : 3 each

   (i) Partitions

   (ii) Chains and Antichains

   (iii) Pigeon hole principle

   (iv) Lattice

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3. (a) Using truth table, determine whether each of the following is a tautology, a contingency or an absurdity ? 5 each

   (i) $(p \rightarrow q) \wedge (p \vee q)$

   (ii) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

   (b) State whether the argument given below is valid or not valid. If it is valid, identify the Tautology or Tautologies used : 6

   I will become famous or I will be a writer

   I will not be a writer

   $\therefore$ I will become famous

4. (a) For the finite state machine shown below, find all equivalent states and obtain an equivalent finite state machine with the smallest number of states : 10

State		INPUT		Output
		0	1	
⇒	A	F	B	0
	B	D	C	0
	C	G	B	0
	D	E	A	1
	E	D	A	0
	F	A	G	1
	G	C	H	1
	H	A	H	1

   (b) Show that the language :

   $$L = \{a^k \mid k = i^2,\ i \geq 1\}$$

   is not a finite state language. 6

5. (a) Explain any *two* of the following : 6

   (i) Eulerian Path and Circuit

   (ii) Rooted tree

   (iii) Binary Search Tree

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(b) Write an algorithm for shortest path in weighted graph and use it to find shortest path for a to z in the graph shown in fig. 1 where numbers associated with the edges are the weights. 4, 6

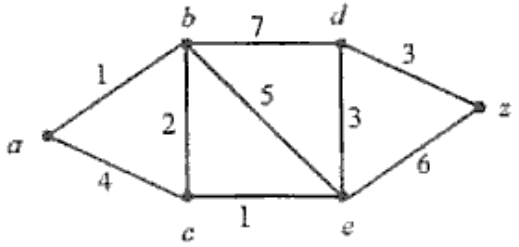

Fig. 1

6. (a) Write an algorithm to find minimum spanning tree and use it to find a minimum spanning tree for the graph shown in fig. 2. 4, 6

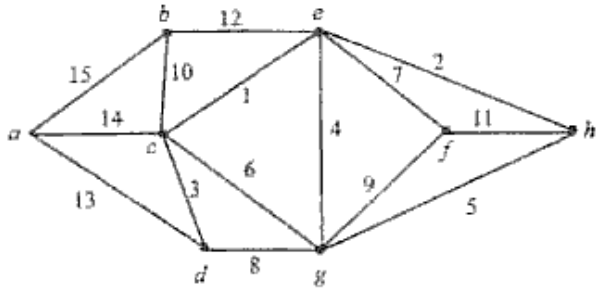

Fig. 2

(b) Let a, b and c be numeric functions such that $a * b = c$. Given that : 6

$$a_r = \begin{cases} 1, & r = 0 \\ 2, & r = 1 \\ 0, & r \geq 2 \end{cases}$$

$$c_r = \begin{cases} 1, & r = 0 \\ 0, & r \geq 1 \end{cases}$$

find b. 6

7. (a) Solve the recurrence relation : 8

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)^2, r \geq 2$$

(b) Write short notes on any *two* of the following : 4 each

(i) Isomorphism and Automorphism

(ii) Cosets

(iii) Groups and semigroups

8. (a) Let G be the set of the non-zero real numbers and let $a * b = \frac{ab}{2}$ then show that (G, *) is an abelian group. 8

(b) If R is a ring, then for all $a, b, c \in R$ show that : 4 each

(i) $ao = oa = 0$

(ii) $a(-b) = -(ab) = (-a)b$

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