8. Define each of the following :
   i) Null Hypothesis
   ii) Test of significance
   iii) Markov Chain
   iv) Traffic intensity
   v) Hermite Polynomial

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Roll No ....

MMPD/MMCM/MMMD/MMTP/MMIE - 101

M.E./M.Tech., I Semester

Examination, December 2013

Advance Mathematics

Time : Three Hours

Maximum Marks : 70

Note: *Attempt any five questions.*
All questions carry equal marks.

1. a) Show that the Set V of all real valued continuous function of x defined on $[0,1]$ is a vector space over the R of real numbers with respect to point wise vector addition and scalar multiplication defined by :

$$(f_1+f_2)x=f_1(x)+f_2(x) \quad \forall \quad f_1,f_2\in V$$

$$(a\,f_1)x=a\,f_1(x) \quad \forall \quad a\in R, f_1\in V$$

   b) Define the linear transformation. Show that the mapping $f:V_3(R)\rightarrow V_2(R)$ defined by : $f(x,y,z)=(x-y,x-z)$ is a linear transformation.

2. a) Solve by method of Separation of variables :

$$\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$$

b) Find the numerical solution of Poisson's equation :

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x^2}=f(x,y) \text{ using finite difference method.}$$

3. a) Prove that the Poisson's distribution is a limiting form of Binomial distribution when p (or q) is very small and n is very large so that the average number of successes np is a finite constant m (say).

b) A coin was tossed 400 times and the head turned up 316 times. Test the hypothesis that the coin is unbiased.

4. a) Define Stochastic process and Markov process with example.

b) In a railway marshalling yard, goods arrive at a rate of 30 trains per day. Assuming that the inter arrival time follows a exponential distribution and service time distribution is also exponential with the average 36 minutes. Then calculate:

i) The mean queue size.

ii) The probability that the queue size exceeds 10.

If the input of trains increases to average 33 per day what will be change in I and II.

5. a) Explain discretization in finite element methods.

b) Use Galerkin's method to solve the equation:

$$\frac{d^2 y}{dx^2}-y+x=0 \quad , \quad y(0)=1, y(1)=0$$

6. a) Find the Fourier Transform of

$$f(x)=\begin{cases}1-x^2, & for\,|x|<1\\ 0, & For\,|x|>1\end{cases}$$

Hence evaluate $\int_0^{\infty}\frac{x\cos x-\sin x}{x^3}\cos\frac{x}{2}dx$.

b) Prove that for Hermite polynomial, if m < n then

$$\frac{d^m}{dx^m}\{H_n(x)\}=\frac{2^m.n!}{(n-m)!}H_{(n-m)}(x)$$

7. a) The mean and variance of Binomial distribution are 4 and $\frac{4}{3}$ respectively. Find

i) the probability of 2 successes

ii) the probability of more than two successes.

b) The number of units of an item that are withdrawn from inventory on a day basis is a Markov chain process in which requirements for tomorrow depend on today's requirements. A one day transition matrix is given below:

Number of units withdrawn from inventory

$$\begin{array}{c c} & \begin{array}{ccc} 5 & 10 & 12 \end{array} \\ \begin{array}{c} 5 \\ 10 \\ 12 \end{array} & \begin{bmatrix} 0.6 & 0.4 & 0.0 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.3 & 0.6 \end{bmatrix} \end{array}$$