Total No. of Questions : 8] *[Total No. of Printed Pages : 2*

Roll No ....

BE-3001(AU/CM/ME/MI) (CBGS)

B.E. IV Semester

Examination, November 2018

Choice Based Grading System (CBGS)

Mathematics - III

Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions.

ii) All questions carry equal marks.

1. a) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Also deduce $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$ Answer

   b) Expand $f(x) = x \sin x$ as a Fourier series in the range $0 < x < 2\pi$. Answer

2. a) Find the Fourier transform of $F(x) = \begin{cases} 1 - x^2, & |x| \leq \\ 0 \quad , & |x| > \end{cases}$ Answer

   Answer b) Find the Fourier cosine transform of $f(x) = e^{-3x} + e^{-4x}$

3. a) Find $L\left\{\frac{1-\cos 2t}{t}\right\}$ Answer

   b) Evaluate $L^{-1}\left\{\frac{6s^2 + 22s - 18}{s^3 + 6s^2 + 11s + 6}\right\}$

4. a) Solve $(D^3 - 3D^2 + 3D - 1)y = t^2 e^t$ given that $y(0) = 1$, $y'(0) = 0$, $y''(0) = -2$, using Laplace transform.

   b) By convolution theorem, evaluate

   $$L^{-1}\left\{\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right\}$$

5. a) If $u(x, y)$ and $v(x, y)$ are harmonic functions in a region R, prove that the function

   $$\left[\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right]$$

   is an analytic function of $z = x + iy$.

   b) Evaluate $\oint_c \frac{e^z}{(z+1)^2} dz$, where C is the circle $|z - 1| = 3$.

6. a) Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2 (z+2)}$ and the residue at each pole.

   b) Apply the calculus of Residue to show that

   $$\int_0^{\pi} \frac{(1 + 2\cos\theta)}{(5 + 4\cos\theta)} d\theta = 0$$

7. a) Find by Taylor's series method the value of y at $x = 0.1$ and $x = 0.2$ to five places of decimals, from

   $$\frac{dy}{dx} = x^2 y - 1, \; y(0) = 1$$

   b) Use Runge-Kutta method to solve the equation $\frac{dy}{dx} = 1 + y^2$ for $x = 0.2$ to $x = 0.4$ with $h = 0.2$. Given that initially at $x = 0$, $y = 0$.

8. a) Using Euler's modified method, find a solution of the equation $\frac{dy}{dx} = \log(x + y)$, $y(0) = 2$ at $x = 1.2$ and 1.4 with $h = 0.2$.

   b) Use Picard's method to approximate the value of y when $x = 0.1$, given that $y = 1$ when $x = 0$ and $\frac{dy}{dx} = 3x + y^2$.

******