b) Evaluate by using Laplace transform

i) $\int_0^\infty t\, e^{-4t} \sin t\, dt$

ii) $\int_0^\infty e^{-t} \frac{\sin t}{t} dt$

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Roll No .....

MA-220 (CS/IT/EC) (CBCS)

B.E., III Semester

Examination, December 2017

Choice Based Credit System (CBCS)

Mathematics - III

Time : Three Hours

Maximum Marks : 60

Note: i) Attempt any five questions.
ii) All questions carry equal marks.

1. a) Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}, \quad -\pi < x < \pi$$

Hence show that $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$

b) Obtain half-range sine series for e^x in $0 < x < 1$

2. a) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$

Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2} dx$

b) Using Laplace transform to solve the diff. equation

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$$

When $x = 2$, $\frac{dx}{dt} = -1$ at $t = 0$

3. a) Find the Laplace transform of $\frac{1-\cos t}{t^2}$

b) Using the convolution theorem, find

$$L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$$

4. a) Define:

i) Probability density function for continuous random variables.

ii) Mean and variance of random variables.

b) Find the mean and variance for Binomial distribution.

5. a) Fit Poisson's distribution to the following and calculate theoretical frequencies ($e^{-0.5} = 0.61$)

Deaths:	0	1	2	3	4
Frequency:	122	60	15	2	1

b) Show that the mean deviation from the mean of the normal distribution is $\frac{4}{5}$ times its standard deviation.

6. a) By the method of least squares. Find the straight line that best fits the following data:

x	1	2	3	4	5
y	14	27	40	55	68

b) The profit of certain company in the x^{th} year of its life are given by:

x	1	2	3	4	5
y	1250	1400	1650	1950	2300

Taking $u = x - 3$ and $50v = y - 1650$, show that the parabola of second degree of y on x is $y = 1140.05 + 72.1x + 32.15x^2$.

7. a) Find the Fourier series for $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$

b) Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$ and hence find Fourier sine transform of $F(x) = \frac{x}{1+x^2}$

8. a) For a Poisson distribution with mean m, show that

$$\mu_{r+1} = mr.\mu_{r-1} + m\frac{d\mu r}{dm}$$

Where $\mu_r = \sum_{x=0}^{\infty}(x-m)^r e^{-m}\frac{m^x}{\lfloor x}$