RGPV SOLUTION BE-3001 (EC) MATHEMATICS-3 JUN 2018

1. a) Find the Fourier series to represent the function $f(x)=x^2$ in $(-\pi,\pi)$

Solution : Given : $f(x)=x^2,\ -\pi<x<\pi$ …(1)

Here, $2L=\pi-(-\pi) i.e. 2L=2\pi \Rightarrow L=\pi$

Suppose the Fourier series of $f(x)$ with period $2L$ is,

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{L}\right)++\sum_{n=1}^{\infty}b_n\sin\left(\frac{n\pi x}{L}\right)$$

$$\Rightarrow \quad f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx \qquad \text{[Since } L=\pi\text{]} \qquad \ldots(2)$$

Now, $a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=\frac{1}{\pi}\int_{-\pi}^{\pi}x^2dx$

$$\Rightarrow \quad =2\int_0^{\pi}x^2dx \qquad \text{[Since } x^2=\text{Even]}$$

$$\Rightarrow \quad a_0=2\left[\frac{x^3}{3}\right]_0^{\pi}=\frac{2}{3}\left[\pi^3-0\right]=\frac{2\pi^2}{3}$$

and $a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx dx=\frac{1}{\pi}\int_{-\pi}^{\pi}x^2\cos nx dx$ [$x\cos nx=\text{odd}$]

$$\Rightarrow \quad =2\int_0^{\pi}x^2\cos nx dx$$

$$\Rightarrow \quad =\frac{2}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right)-2x\left(-\frac{\cos nx}{n^2}\right)+2\left(-\frac{\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$\Rightarrow \quad a_n=\frac{2}{\pi}\left[\left\{0+\frac{2\pi(-1)^n}{n^2}\right\}-\{0-0-0\}\right]=\frac{4(-1)^n}{n^2}$$

and $b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx dx=\frac{1}{\pi}\int_{-\pi}^{\pi}x^2\sin nx dx$

$$\Rightarrow \quad =0 \qquad [x^2\sin nx=odd]$$

Putting in equation (1), we get

$$f(x)=\frac{\pi^2}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos nx$$

$$\Rightarrow \quad \boxed{f(x)=\frac{\pi^2}{3}-4\left[\frac{\cos x}{1^2}-\frac{\cos 2x}{2^2}+\frac{\cos 3x}{3^2}-....\right]} \qquad \ldots(3)$$

Proved

b) Expand $\pi x - x^2$; 0<x< π in a half-range sine series.

Solution : Given : $f(x) = \pi x - x^2$

Here $L = \pi$, then the half range sine series of given function is,

$$f(x) = \sum_{n=1}^{n} b_n \sin\left(\frac{n\pi x}{L}\right) = \sum_{n=1}^{n} b_n \sin(nx)$$

$$\Rightarrow \quad f(x) = b_n \sin x + b_2 \sin 2x + b_3 \sin 3x \qquad \ldots(1)$$

Since, $b_n = \dfrac{2}{\pi}\int_0^\pi f(x)\sin nx dx$

$$\Rightarrow \quad = \frac{2}{\pi}\int_0^\pi \left[\pi x - x^2\right]\sin nx\, dx$$

$$\Rightarrow \quad = \frac{2}{\pi}\left[\left(\pi x - x^2\right)\left(-\frac{\cos nx}{n}\right) - (\pi - 2x)\left(-\frac{\sin nx}{n^2}\right) + (-2)\left(\frac{\cos nx}{n^3}\right)\right]_0^\pi$$

$$\Rightarrow \quad b_n = \frac{2}{\pi}\left[\left\{0 + 0 - 2\frac{(-1)^n}{n^3}\right\} - \left\{0 + 0 - \frac{2}{n^3}\right\}\right] = \frac{4}{n^3\pi}\left[1 - (-1)^n\right]$$

If n is odd, then $\left[1 - (-1)^n\right] = 2$

$$\therefore \quad b_1 = \frac{8}{\pi}, b_3 = \frac{8}{3^3\pi} \text{ and } b_5 = \frac{8}{5^3\pi}$$

and if n is an even, then $\left[1 - (-1)^n\right] = 0$

$$\therefore \quad b_2 = b_4 = \ldots = 0$$

Putting in equation (1), we get

$$f(x) = \sum_{n=1}^{n} \frac{4}{n^3\pi}\left[1 - (-1)^n\right]\sin(nx)$$

$$\Rightarrow \quad \boxed{f(x) = \frac{8}{\pi}\left[\frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \ldots\right]} \qquad \textbf{Answer}$$

2. a) Find the Fourier sine transform of $f(x) = \dfrac{e^{-ax}}{x}$.

Solution : Given, $f(x) = \dfrac{e^{-ax}}{x}$

By Fourier sine Transform,

$$F\{f(x)\} = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\, dx$$

$$\Rightarrow \quad F\{f(x)\} = \sqrt{\frac{2}{\pi}}\int_0^\infty \left(\frac{e^{-ax}}{x}\right)\sin sx\, dx = I \qquad \ldots(1)$$

Differentiate w.r.t. s, on both sides, we get

$$\frac{dI}{ds} = \sqrt{\frac{2}{\pi}} \frac{d}{dx}\left[\int_0^\infty \left(\frac{d^{-ax}}{x}\right) \sin sx dx\right]$$

$$\Rightarrow \quad \frac{dI}{ds} = \sqrt{\frac{2}{\pi}} \int_0^\infty \left(\frac{e^{-ax}}{x}\right) \frac{\partial}{\partial s}(\sin sx) dx = \sqrt{\frac{2}{\pi}} \int_0^\infty \left(\frac{e^{-ax}}{x}\right)(x \cos sx) dx$$

$$\Rightarrow \quad \frac{dI}{ds} = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \cos sx\, dx = \sqrt{\frac{2}{\pi}} \left[\frac{e^{-ax}}{(-a)^2 + s^2}\{-a \cos sx + s \sin sx\}\right]_0^\infty$$

$$\Rightarrow \quad \frac{dI}{ds} = \sqrt{\frac{2}{\pi}} \left(\frac{1}{s^2 + a^2}\right)[\{0\} - \{-a + 0\}] = \sqrt{\frac{2}{\pi}} \left(\frac{a}{s^2 + a^2}\right)$$

Integrating both sides, w.r.t s, we get

$$I = \sqrt{\frac{2}{\pi}} \left[\tan^{-1}\left(\frac{s}{a}\right)\right] + c \qquad \ldots(2)$$

For the initial condition, putting s=0, then c = 0

$\therefore$ From (2), we have

$$I = \sqrt{\frac{2}{\pi}} \left[\tan^{-1}\left(\frac{x}{a}\right)\right] \Rightarrow F\{f(x)\} = \sqrt{\frac{2}{\pi}} \left[\tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$\Rightarrow \quad \boxed{F(s) = \sqrt{\frac{2}{\pi}} \left[\tan^{-1}\left(\frac{x}{a}\right)\right]}$$ **Answer**

b) Find the cosine transform of $\frac{1}{x^2 + a^2}$

Solution : Suppose $F(x) = \frac{1}{x^2 + a^2}$

The Fourier cosine transform of $F(x)$ is,

$$f_C(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty F(x) \cos px\, dx$$

$$\therefore \quad f_C(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{1}{x^2 + a^2} \cos px\, dx = I \qquad \text{[Say]} \qquad \ldots (1)$$

Differentiating w.r.t., p, we get

$$\frac{d}{dp} I = \frac{d}{dp} \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{1}{x^2 + a^2} \cos px\, dx$$

$$\Rightarrow \quad = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{1}{x^2 + a^2} \frac{\partial}{\partial p}(\cos px) dx = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{-x}{x^2 + a^2} \sin px\, dx$$

$$\Rightarrow \quad \frac{dI}{dp} = -\sqrt{\frac{2}{\pi}} \int_0^\infty \frac{x^2}{x(x^2 + a^2)} \sin px dx$$

$$\Rightarrow \quad = -\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\left(a^2+x^2-a^2\right)}{x\left(x^2+a^2\right)}\sin\ px\,dx \qquad \text{[Adding and subtract 1]}$$

$$\Rightarrow \quad \frac{dI}{dp} = -\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin\ px}{x}dx + a^2\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin\ px}{x\left(x^2+a^2\right)}dx$$

$$\Rightarrow \quad \frac{dI}{dp} = -\sqrt{\frac{2}{\pi}}\left(\frac{\pi}{2}\right) + a^2\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin\ px}{x\left(x^2+a^2\right)}dx$$

$$\Rightarrow \quad \frac{dI}{dp} = -\sqrt{\frac{2}{\pi}} + a^2\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin\ px}{x\left(x^2+a^2\right)}dx \quad \ldots.(2) \qquad \left[\Theta \int_0^\infty \frac{\sin ax}{x}dx = \frac{\pi}{2}\right]$$

Again differentiating w.r.t., p, we get

$$\frac{d^2 I}{dp^2} = 0 + a^2\frac{d}{dp}\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin\ px}{x\left(x^2+a^2\right)}dx$$

$$\Rightarrow \quad = a^2\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{x\cos px}{x\left(x^2+a^2\right)}dx = a^2\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\cos px}{x^2+a^2}dx = a^2 I \qquad \text{From (1)}$$

$$\Rightarrow \quad \frac{d^2 I}{dp^2} - a^2 I = 0 \qquad \ldots.(3)$$

This is Linear differential equation of higher order.

$\therefore$ The solution of (3) is,

$$I = c_1 e^{ap} + c_2 e^{-ap} \qquad \ldots.(4)$$

Differentiating w.r.t, p, we get

$$\frac{dI}{dp} = ac_1 e^{ap} - ac_2 e^{-ap} \qquad \ldots.(5)$$

Putting $p = 0,$ in equation (1) and (4) we get

$$I = \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{1}{x^2+x^2}dx = \left(\frac{1}{a}\right)\sqrt{\frac{2}{\pi}}\left[\tan^{-1}\left(\frac{x}{a}\right)\right]_0^\infty = \left(\frac{1}{a}\right)\sqrt{\frac{2}{\pi}}\left(\frac{\pi}{2}\right) = \left(\frac{1}{a}\right)\sqrt{\frac{\pi}{2}}$$

and $\quad c_1 + c_2 = I \Rightarrow c_1 + c_2 = \left(\frac{1}{a}\right)\sqrt{\frac{\pi}{2}} \qquad \ldots.(6)$

Again Putting p=0, in equation (2) and (5) we get

$$\frac{dI}{dp} = -\sqrt{\frac{\pi}{2}} + 0 \Rightarrow \frac{dI}{dp} = -\sqrt{\frac{\pi}{2}} \text{ and } c_1 - c_2 = -\frac{1}{a}\sqrt{\frac{\pi}{2}} \qquad \ldots(7)$$

Solve (6) and (7), we get

$$c_1 = 0 \text{ and } c_2 = \frac{1}{a}\sqrt{\frac{\pi}{2}}$$

$\therefore$ From (4), we get

$$I = \frac{1}{a}\sqrt{\frac{\pi}{2}}e^{-ap}$$

$\Rightarrow$ $\boxed{\text{i.e., } F_C\left\{\frac{1}{x^2+a^2}\right\}=\frac{1}{a}\sqrt{\frac{\pi}{2}}e^{-ap}}$ **Answer**

3. a) Find the Laplace transform of the following:

(i). $2\sin t\cos t$ **(ii).** $(t^2+1)^2$

Solution : (i). $L(2\sin t\cos t\}=L\{\sin 2t\}=\frac{2}{p^2+4}$ **Answer**

(ii). $L\left\{(t^2+1)^2\right\}=L\left\{t^4+1+2t^2\right\}$

$\Rightarrow$ $=L\left\{t^4\right\}+L\{1\}+2t\left\{t^2\right\}$

$\Rightarrow$ $=\frac{\lfloor 4}{p^5}+\frac{1}{p}+2\frac{\lfloor 2}{p^3}$

$\Rightarrow$ $=\frac{24}{p^5}+\frac{1}{p}+\frac{4}{p^3}$

Thus, $\boxed{L\left\{(t^2+1)^2\right\}=\frac{24}{p^5}+\frac{1}{p}+\frac{4}{p^3}}$ **Answer**

b) Find the Laplace transform of the following :

(i). $t\sin at$ **(ii).** $t^n e^{at}$

Solution : (i). Since $L\{\sin at\}=\frac{a}{p^2+a^2}=f(p)$

By Multiplication property, we have

$$L\{t\sin at\}=(-1)\frac{d}{dp}f(p)$$

$\Rightarrow$ $=(-1)\frac{d}{dp}\left[\frac{a}{p^2+a^2}\right]$

$\Rightarrow$ $=(-1)\left[-\frac{2ap}{(p^2+a^2)}\right]=\frac{2ap}{(p^2+a^2)^2}$

Thus, $\boxed{L\{t\sin at\}=\frac{2ap}{(p^2+a^2)^2}}$ **Answer**

(iii). Since $L\left\{t^n\right\}=\frac{\Gamma n+1}{p^{n+1}}=f(p)$

By First shifting theorem we get

$$L\left\{e^{at}t^n\right\}=f(p-a)$$

$\Rightarrow$ $=\frac{\Gamma n+1}{(p-a)^{n+1}}$ **Answer**

4. a) Evaluate the following

(i). $L^{-1}\left\{\dfrac{3s+7}{s^2-2s-3}\right\}$ **(ii).** $L^{-1}\left\{\dfrac{3s-2}{s^2-4s+20}\right\}$

Solution: (i). $L^{-1}\left\{\dfrac{3s+7}{s^2-2s-3}\right\}=L^{-1}\left\{\dfrac{3s+7}{(s-1)^2-4}\right\}$

$\Rightarrow \quad =L^{-1}\left\{\dfrac{3(s-1)+10}{(s-1)^2-4}\right\}=e^t L^{-1}\left\{\dfrac{3s+10}{s^2-4}\right\}$ [By FST]

$\Rightarrow \quad e^t[3\cosh 2t+\sinh 2t]$ **Answer**

(ii). $L^{-1}\left\{\dfrac{3s-2}{s^2-4s+20}\right\}=L^{-1}\left\{\dfrac{3s-2}{(s-2)^2+16}\right\}$

$\Rightarrow \quad =L^{-1}\left\{\dfrac{3(s-2)-4}{(s-2)^2+16}\right\}=e^{2t}L^{-1}\left\{\dfrac{3s-4}{s^2+16}\right\}$

$\Rightarrow \quad =e^{2t}[3\cos 4t-\sin 4t]$ **Answer**

b) Using convolution theorem evaluate

$$L^{-1}\left\{\frac{1}{(s-1)(s-2)}\right\}$$

Solution : Suppose

$f(s)=\dfrac{1}{s-1}$ and $g(s)=\dfrac{1}{s-2}$

Taking inverse Laplace transform on both sides, we get

$L^{-1}\{f(s)\}=L^{-1}\left\{\dfrac{1}{s-1}\right\}$ and $L^{-1}\{g(s)\}=L^{-1}\left\{\dfrac{1}{s-2}\right\}$

$\Rightarrow \quad =e^t=F(t)$

By Convolution theorem, we have

$$L^{-1}\{f(s)g(s)\}=\int_0^t F(x)G(t-x)dx$$

$\therefore \quad =\int_0^t e^x e^{2(t-x)}dx$

$\Rightarrow \quad =e^{3t}\int_0^t e^{-x}dx=-e^{3t}\left[e^{-x}\right]_0^t$

$\Rightarrow \quad =-e^{3t}\left[e^{-t}-1\right]=-e^{2t}+e^{3t}$

Thus, $\boxed{L^{-1}\left\{\dfrac{1}{(s-1)(s-2)}\right\}=-e^{2t}+e^{3t}}$ **Answer**

5. a) Find the value of k for which the function the p.d.f. :

$$f(x)=\begin{cases} k\,x^2, & 0\le x\le 3 \\ 0, & otherwise \end{cases}$$ **is a probability density function. Also compute** $P(1\le x\ge 2)$

Solution : Given : $f(x)=\begin{cases} k\,x^2, & 0\le x\le 3 \\ 0, & otherwise \end{cases}$

By the definition of Probability density function of $f(x)$ is

$$\int_{-\infty}^{\infty} f(x)\,dx=1 \qquad \ldots(1)$$

$$\Rightarrow \quad k\int_0^3 x^2 dx=1$$

$$\Rightarrow \quad k\left[\frac{x^3}{3}\right]_0^3=1 \;\Rightarrow\; k(9-0)=1$$

Thus $\boxed{k=\frac{1}{9}}$ **Answer**

(i). To find P(1 $\le x \le$ 2)

We know that $P(a\le x\le b)=\int_a^b f(x)dx$

$$\therefore \quad P(1\le x\le 2)=\int^2 k\,x^2 dx=\frac{1}{9}\left[\frac{x^3}{3}\right]_1^2 \qquad \left[\Theta\ k=\frac{1}{9}\right]$$

$$=\frac{1}{27}[8-1]=\frac{7}{27}$$

Thus $\boxed{P(1\le x\le 2)=\frac{7}{27}}$ **Answer**

b) A coin is tossed 4 times. What is the probability of getting

(i) two heads (ii). atleast two heads

Solution : The probability of head $p=\frac{1}{2}$, so that $q=\frac{1}{2}$ and $n=4$

(i). Probability of two heads $=P(X=2)$

$$={}^4C_2 p^2 q^2$$

$$=6\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2=\frac{3}{8}$$ **Answer**

(ii). Probability of atleast two heads $P(X\ge 2)$

$$=P(X=2)+P(X=3)+P(X=4)$$

$$=1-\left[P(X=0)+P(X=1)\right]$$

$$=1-\left[{}^4C_0 p^0 q^4+{}^4C_1 p^1 q^3\right]$$

$$=1-\left[1\times1\times\left(\frac{1}{2}\right)^4+4\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)^3\right]$$

$$=1-\frac{1}{16}[1+4]=\frac{16-5}{16}=\frac{11}{16}$$ **Answer**

6. a) Use Poisson distribution to find the probability of at most 5 defective fuses in a box of 200 fuses. Experience shows that 2 percent of such fuses are defective.

Solution : Give the no. of fuses in a box n = 200

Probability of defective fuses p = 2%=0.02

So that mean of Poisson distribution m=np=200×0.02=4

The probability of at most 5 defective fuses = $P(X\le 5)$

$$=P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$$

$$\Rightarrow \quad =\frac{e^{-4}(4)^0}{\lfloor 0}+\frac{e^{-4}(4)^1}{\lfloor 1}+\frac{e^{-4}(4)^2}{\lfloor 2}+\frac{e^{-4}(4)^3}{\lfloor 3}+\frac{e^{-4}(4)^4}{\lfloor 4}+\frac{e^{-4}(4)^5}{\lfloor 5}$$

$$\Rightarrow \quad =e^{-4}\left[1+4+8+\frac{32}{3}+\frac{32}{3}+\frac{128}{15}\right]=0.7851$$ **Answer**

b) Find the mean and variance for Binomial distribution.

Solution : (i) *Mean of Binomial Distribution:*

We know that by binomial distribution

$$P(X=r)={}^nC_r p^r q^{n-r}$$

Formula for mean of B.D. is,

$$m=\sum_{r=0}^{n} r.P(X=r)$$

$$\therefore \quad =\sum_{r=0}^{n} r.{}^nC_r p^r q^{n-r}=\sum_{r=1}^{n} r.{}^nC_r p^r q^{n-r} \qquad [\because \text{ first term is zero}]$$

$$\Rightarrow \quad \sum_{r=1}^{n} n.{}^{n-1}C_{r-1}\, p^r q^{n-r} \qquad [\because r.{}^nC_r=n.{}^{n-1}C_{r-1}]$$

$$\Rightarrow \quad =n\,p\sum_{r=1}^{n} {}^{n-1}C_{r-1}p^{r-1}q^{(n-1)-(r-1)}$$

$$\Rightarrow \quad =np(q+p)^{n-1}=np \qquad [\because q+p=1]$$

Hence, $\boxed{m=np}$

(ii). *Variance of Binomial Distribution:*

We know that by binomial distribution

$$P(X=r)={}^nC_r p^r q^{n-r}$$

Formula for variance of B.D. is,

$$V = \sum_{r=0}^{n} r^2 . P(X = r) - (mean)^2$$

$$\therefore \quad = \sum_{r=0}^{n} [r + r(r-1)] . {}^nC_r p^r q^{n-r} - m^2$$

$$\Rightarrow \quad = \sum_{r=0}^{n} r . {}^nC_r p^r q^{n-r} + \sum_{r=0}^{n} r.(r-1) {}^nC_r \ p^r \ q^{n-r} - n^2 p^2$$

$$\Rightarrow \quad = n\,p + \sum_{r=2}^{n} n.(n-1)\, {}^{n-2}C_{r-2} p^r q^{n-r} - n^2 p^2 \qquad [\because r.(r-1)\ {}^nC_r = n.(n-1)\, {}^{n-2}C_{r-2}]$$

$$\Rightarrow \quad = np + n.(n-1)\, p^2 \sum_{r=2}^{n} {}^{n-2}C_{r-2} r^{r-2} q^{(n-2)-(r-2)} - n^2 p^2$$

$$\Rightarrow \quad = n\,p + (n^2 p^2 - np^2)(q + p)^{n-2} - n^2 p^2$$

$$\Rightarrow \quad = n\,p + \cancel{n^2 p^2} - np^2 - \cancel{n^2 p^2} \qquad [\because \ q + p = 1\,]$$

$$\Rightarrow \quad = np(1 - p) = npq$$

Hence, $\boxed{V = n\,p\,q}$

7. a) Use least square method to fit a straight line to the data

x	**:**	**1**	**2**	**3**	**4**
y	**:**	**3**	**7**	**13**	**21**

Solution: Suppose straight line y as dependent and x as a independent variable is

$$y = a + bx \qquad \ldots(1)$$

Here two unknown constants, then the two normal equations are,

$$\sum y = ma + b\sum x \qquad \ldots(2)$$

and $$\sum xy = a\sum x + b\sum x^2 \qquad \ldots(3)$$

Table :

x	y	x.y	x^2
1	3	3	1
2	7	14	4
3	13	39	9
4	21	84	16
$\sum x = 10$	$\sum y = 44$	$\sum x.y = 140$	$\sum x^2 = 30$

Here, m=5

Putting in equation (2) and (3), we get

$$5a + 10b = 44 \qquad \ldots(4)$$

and $$10a + 30b = 140 \qquad \ldots(5)$$

Solving equation (4) and (5), we get

$$a = -1.6 \text{ and } b = 5.2$$

Putting in equation (1), we get

$$\boxed{y = 1.6 + 5.2x} \qquad \textbf{Answer}$$

b) Fit a Poisson's distribution to the set of observations :

x	**:**	**0**	**1**	**2**	**3**	**4**
y	**:**	**122**	**60**	**15**	**2**	**1**

Solution : Given, n=4 and $N = \sum f = 200$

The expected frequency of Poisson distribution

$$f_e = N\,P(X = r) = 200\left[\frac{e^{-m} m^r}{\lfloor r}\right] \qquad \ldots(1)$$

Mean $m = \dfrac{\sum f\, r}{N} = \dfrac{100}{200} = 0.5$

Expected frequency distribution table

r	f	$f.r$	$f_e = 122 \times \left[\frac{(0.5)^r}{\lfloor r}\right]$
0	122	0	122
1	60	60	61
2	15	30	15.25~15
3	2	6	2.541~3
4	1	4	0.3177~0
Total	**200**	$\sum f.r = 100$	

Putting in equation (1), we get

$$f_e = 200\left[\frac{e^{-0.5}(0.5)^r}{\lfloor r}\right] = 200 \times 0.61\left[\frac{(0.5)^r}{\lfloor r}\right]$$

$$\Rightarrow \qquad f_e = 122 \times \left[\frac{(0.5)^r}{\lfloor r}\right]$$

Putting r=0,1,2,3,4, we get the expected frequency are 122, 61, 15, 3 and 0 respectively.

8. a) If there are 3 misprint in a book of 1000 pages, find probability that a given page will contain

(i). No Misprint **(ii). More than 2 misprint**

Solution: Given the number of pages $n = 1000$

$\therefore$ The probability of misprints in a page $p = \dfrac{3}{1000} = 0.003$

and Mean of Poisson distribution $m = np = 1000\left(\dfrac{3}{1000}\right) = 3$

(i). The probability no misprint = $P(X = 0)$

$$= \frac{e^{-3}(3)^0}{\lfloor 0} = e^{-3} = 0.04978$$ **Answer**

(ii). The Probability of more than 2 misprint $= P(X > 2)$

$$= P(X = 3) + P(X = 4) + ... + P(X = 1000)$$

$$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$

$$= 1 - \left[\frac{e^{-3}(3)^0}{\lfloor 0} + \frac{e^{-3}(3)^1}{\lfloor 1} + \frac{e^{-3}(3)^2}{\lfloor 2}\right]$$

$$= 1 - e^{-3}\left[1 + 3 + \frac{9}{4}\right] = 1 - \frac{25}{4}e^{-3}$$

= 0.68883 **Answer**

b) Find $L\left\{\dfrac{1-e^t}{t}\right\}$

Solution : Suppose $F(t) = 1 - e^t$

Taking laplace transform on both sides, we get

$$L\{F(t)\} = L\{1\} - L\{e^t\}$$

$\Rightarrow$ $$= \frac{1}{p} - \frac{1}{p-1} = f(p)$$

By Division property of Laplace transform, we have

$$L\left\{\frac{F(t)}{t}\right\} = \int_p^\infty f(p)dp$$

$\therefore$ $$L\left\{\frac{1-e^t}{t}\right\} = \int_p^\infty \left[\frac{1}{p} - \frac{1}{p-1}\right]dp$$

$\Rightarrow$ $$= [\log p - \log(p-1)]_p^\infty = \left[\log\left(\frac{p}{p-1}\right)\right]_p^\infty$$

$$\Rightarrow \qquad = 0 - \log\left(\frac{p}{p-1}\right) = -\log\left(\frac{p}{p-1}\right)$$

Thus, $\boxed{L\left\{\frac{1-e^{t}}{t}\right\} = -\log\left(\frac{p}{p-1}\right)}$ **Answer**