401(GS)

B. E. (Fourth Semester) EXAMINATION, June, 2012

(Grading System)
(Common for all Branches)

MATHEMATICS—III

Time : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 22 (D Grade)

Note :   All questions are compulsory. All questions carry equal marks. Internal choices are also given.

1. (a) Show that the function $f(z) = e^{-z^{-4}}, z \neq 0$ and $f(0) = 0$ is not analytic at $z = 0$, although Cauchy-Riemann equations are satisfied at this point.

   (b) Use Cauchy's integral formula to evaluate

   $\oint \dfrac{e^{2z}}{(z + 1)^4} dz$ where C is the circle $|z| = 2$.

<div align="center">Or</div>

   (a) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Find the conjugate function v and express $u + iv$ as an analytic function of z.

   (b) Define Residue and evaluate :

   $$\int_0^{} \frac{d\theta}{1 - 2a\sin\theta + a^2}, 0 < a < 1$$

   by using Residue theorem.

2. (a) By using Newton-Raphson method, find the real roots of the equation $x^3 - x - 1 = 0$.

(b) Apply Gauss-Jordan method to solve the equations :

$$10x + y + z = 9$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$

Or

(a) Using Regula-Falsi method, compute the real root of equation $x\,e^x = 2$, correct to four decimal places.

(b) Apply Gauss elimination method to solve the equations :

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

3. (a) The area 'A' of a circle of diameter 'd' is given for the following values :

d	A
80	5026
85	5674
90	6362
95	7088
100	7854

By using appropriate interpolating formulae, find approximate values for the areas of circles of diameter 82 and 91 respectively.

(b) A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given ahead for various values of the time t-seconds. Find the velocity of the slider and its acceleration when $t = 0.3$ second :

t	x
0·0	30·13
0·1	31·62
0·2	32·87
0·3	33·64
0·4	33·95
0·5	33·81
0·6	33·24

Or

(a) Using Newton's divided difference formula, evaluate $f(8)$ and $f(15)$, for given values :

x	$f(x)$
4	48
5	100
7	294
10	900
11	1210
13	2028

(b) Calculate by Simpson's $\frac{1}{3}$rd rule an approximate value of $\int_0^1 \frac{dx}{1+x}$ by taking seven ordinates. Compare it with exact value and deduce the value of $\log_e 2$.

4. (a) Using Picard's process of successive approximation, obtain a solution upto the fifth approximation of the equation $\frac{dy}{dx} = y + x$, such that $y = 1$ when $x = 0$.

(b) Find the Karl Pearson's coefficient of correlation for the following data :

x	y
10	18
14	12
18	24
22	6
26	30
30	36

Or

(a) Apply Runge-Kutta method of fourth order to find an approximate value of y when $x = 0 \cdot 2$, given that $\frac{dy}{dx} = x + y^2$ and $y = 1$ when $x = 0$.

(b) Fit a second degree parabola, by using the concept of regression lines :

x	y
1	2
2	6
3	7
4	8
5	10
6	11
7	11
8	10
9	9

5. (a) Define the following terms :

Probability function, Probability mass function and Probability density function.

(b) Derive the expression for mean and variance of Binomial distribution.

Or

(a) Define Normal distribution. Give the basic properties and the standard form of Normal distribution.

(b) Write short notes on any *two* of the following :

(i) Discrete and Continuous random variable

(ii) Student's t-test

(iii) Properties of Chi-square distribution

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