6. a) Define probability density function of a random variable. A continuous random variable X has the density function
   $f(x) = cx^2, 0 < x < 1$
   Then find the value of c and determine the probability that $1/3 < x < 1/2$.

   b) There are three similar coins, one of which is ideal and other two are biased. The chances of heads on the biased coins are respectively 1/3 and 2/3. A coin is selected at random and tossed twice. If head occurs both the times, find the probability that the ideal coin was selected.

7. a) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses, if experience shows that 2 percent of such fuses are defective.

   b) Write a short note on small sample test. also, find the student's t for following variable values in a sample of eight.
   $-4, -2, -2, 0, 2, 2, 3, 3.$

8. a) What are the basic elements of reliability? State and explain the factors to be considered in designing reliability.

   b) Write short note on the following:
   i) Theory of testing hypothesis
   ii) Sampling distribution
   iii) System reliability
   iv) Maintenance and reliability.

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Total No. of Questions :8] *[Total No. of Printed Pages :4*

Roll No ....

MVCT/MBCT/MVCP - 101

M.E./M.Tech., I Semester

Examination, December 2015

Advance Mathematics

Time : Three Hours

Maximum Marks : 70

Note : i) Attempt any five out of eight questions.
ii) All questions carry equal marks.

1. a) Using graphical method, find the maximum value of
   $Z = 7x_1+3x_2$
   Subject to constraints
   $$x_1 + 2x_2 \geq 3$$
   $$x_1 + x_2 \leq 4$$
   and $0 \leq x_1 \leq 3, 0 \leq x_2 \leq 5$

   b) Four jobs are to be done on four different machines. The cost (in rupees) of producing i^{th} job on j^{th} machine is given below.

Job ↓ / Machine →	M_1	M_2	M_3	M_4
J_1	15	11	13	15
J_2	17	12	12	13
J_3	14	15	10	14
J_4	16	13	11	17

   Assign the jobs to different machines so as to minimize the total cost.

2. a) Using Simplex method, solve the following LPP:

Maximize $Z = 3x_1 + 2x_2$

Subject to constraints

$$x_1 + x_2 \le 4$$
$$x_1 - x_2 \le 2$$
$$x_1, x_2 \ge 0$$

b) Explain the concept of duality in linear programming problem. Write the dual of the following linear programming problem.

Minimize $Z = 7x_1 + 3x_2 + 8x_3$

Subject to constraints

$$8x_1 + 2x_2 + x_3 \ge 3$$
$$3x_1 + 6x_2 + 4x_3 \ge 4$$
$$4x_1 + x_2 + 5x_3 \ge 1$$
$$x_1 + 5x_2 + 2x_3 \le 7$$
$$x_1, x_2, x_3 \ge 0$$

3. a) A project consists of a series of tasks labelled A, B,....H, I with the following constraints A < D, A < E, B < F, D < F, C < G, C < H, F < I, G < I.

Where the notation X < Y means that the task X must be completed before Y is started. Draw a graph to represent the sequence of tasks and find the minimum time of completion of the project. When the time (in days) of completion of each task is as follows.

Task	A	B	C	D	E	F	G	H	I
Time	8	10	8	10	16	17	18	14	9

b) Write the difference between PERT and CPM. Also, explain their importance in management of projects.

4. a) Discuss the queueing model [(M/M/I) : (∞/∞/FCFS)]. Also, obtain system of differential difference equation.

b) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following:

i) The mean queue size (line length) and

ii) The probability that the queue size exceeds.

5. a) Patients arrive of a clinic according to a Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate 20 per hour:

i) Find the effective rate at the clinic

ii) What is the expected waiting time until a patient is discharged from the clinic?

b) Solve the game whose pay-off matrix is given by

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \text{Player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix} \end{array}$$