Total No. of Questions : 10] [Total No. of Printed Pages : 4

Roll No. .....

BE–102(GS)

B. E. (First/Second Semester) EXAMINATION, Dec., 2011

(Grading System)

(Common for all Branches)

ENGINEERING MATHEMATICS–I

[BE–102(GS)]

Time : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 22 (D Grade)

Note : Attempt *one* question from each Unit. All questions carry equal marks.

Unit–I

1. (a) Expand $e^{a \sin^{-1} x}$ in ascending powers of x.

   (b) If $p = x \cos \alpha + y \sin \alpha$, touches the curve $\left(\frac{x}{a}\right)^{\frac{n}{n-1}} + \left(\frac{y}{b}\right)^{\frac{n}{n-1}} = 1$, prove that :

   $$p^n = (a \cos \alpha)^n + (b \sin \alpha)^n$$

   Or

2. (a) Show that the radius of curvature at any point of the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ is $4a \cos\left(\frac{\theta}{2}\right)$

   (b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that :

   (i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$

   (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4 \cos^3 u}$

Unit–II

3. (a) Find the limit as $n \to \infty$ of the series :

   $$\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots\ldots + \frac{1}{2n}$$

   (b) Find the volume common to the cylinders $x^2 + y^2 = a^2$, $x^2 + z^2 = a^2$.

   Or

4. (a) Evaluate :

   $$\int_0^\infty \int_0^x x e^{-x^2/y} \, dy \, dx$$

   by chaning the order of integration.

   (b) Prove that :

   (i) $\frac{\beta(m+1, n)}{m} = \frac{\beta(m, n+1)}{n} = \frac{\beta(m, n)}{m+n}$

   (ii) $\boxed{m}\left|\left(m - \frac{1}{2}\right)\right. = \frac{\sqrt{\pi}}{2^{2m-1}} \boxed{2m}$

Unit–III

5. (a) Solve the equation :

   $$(y - x) \frac{dy}{dx} = a^2$$

   (b) Solve the equation :

   $$\frac{d^2 y}{dx^2} + 4y = \sec 2x$$

   by the method of variation of parameters.

Or

6. (a) Solve the equation :

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + \log x$$

(b) Solve the simultaneous equations :

$$\frac{dx}{dt} + y = \sin t$$

$$\frac{dy}{dt} + x = \cos t$$

Unit–IV

7. (a) Reduce the matrix :

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

to normal form and find its range.

(b) Find the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Or

8. (a) Test for consistency and solve :

$$5x + 3y + 7z = 4$$

$$3x + 26y + 2z = 9$$

$$7x + 2y + 10z = 5$$

(b) Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

and find its inverse.

P. T. O.

Unit–V

9. (a) Define the following terms with examples :

(i) Simple graph

(ii) Degree of a vertex

(iii) Isomorphic graphs

(iv) Spanning tree

(b) Express the following function into disjunctive normal form :

$$f(x, y, z) = (x + y + z)(x'y + x'z)'$$

Or

10. (a) Let $X = \{a, b, c, d\}$ be a universe of discourse and A, B be the fuzzy sets on X defined by :

$$A = \left\{\frac{0.3}{a}, \frac{0.5}{b}, \frac{0.6}{c}, \frac{0.4}{d}\right\}$$

$$B = \left\{\frac{0.2}{a}, \frac{0.6}{b}, \frac{0.3}{c}, \frac{0.7}{d}\right\}$$

Find :

(i) Height of $A \cup B$

(ii) α-cut of $A \cap B$ for $\alpha = 0.4$

(iii) $(A \cup B)'$

(iv) $A' \cap B'$

(b) Prove that the number of vertices of odd degree in a graph is always even.