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Roll No. ....

BE–102

B. E. (First Semester) EXAMINATION, April, 2009

(Common to all Branches)

ENGINEERING MATHEMATICS – I

(BE – 102)

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : Attempt *five* questions in all selecting *one* question from each Unit. All questions carry equal marks.

Unit – I

1. (a) Expand $e^{a \sin^{-1} x}$ in ascending power of x.

(b) In the catenary $y = c \cosh \left(\frac{x}{c}\right)$, prove that the length of the subtangent is $c \cosh \left(\frac{x}{c}\right)$ and that of subnormal is $c \sinh \left(\frac{3x}{c}\right)$.

Or

2. (a) Show that the function $\sin 3x - 3 \sin x$ is minimum when $x = \frac{\pi}{2}$ and maximum when $x = \frac{3\pi}{2}$.

P. T. O.

(b) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, prove that :

(i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial y^2} + y^2 \frac{\partial^2 u}{\partial x^2} = 2 \cos 3u \sin u$

Unit – II

3. (a) Find the limit, when $n \to \infty$ of the series :

$$\frac{n}{n^2} + \frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \ldots\ldots \frac{n}{n^2 + (n-1)^2}$$

(b) Compute the value of $\int\int_R dx\, dy$ when R is the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

of positive quadrant.

Or

4. (a) Prove that :

$$\int_0^\infty x^n e^{-k^2 x^2} dx = \frac{1}{2k^{n+1}} \Gamma\left(\frac{n+1}{2}\right), n > -1$$

(b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and $y + z = 4$ and $x = 0$.

Unit – III

5. (a) Solve the differential equation :

$$(1 + xy^2) \frac{dy}{dx} = 1$$

(b) Solve the differential equation :

$$\frac{d^2 y}{dx^2} + a^2 y = \tan ax$$

Or

6. (a) Solve the differential equation :

$$y = 2px + p^n$$

(b) Solve the differential equation :

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

Unit – IV

7. (a) Reduce the matrix A to the normal form and find its rank :

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

(b) Test for consistency and solve :

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

Or

8. (a) Find the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(b) Show that the following matrix satisfies Cayley-Hamilton theorem :

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

Unit – V

9. (a) In a Boolean algebra B $a, b \in$ B prove that :

   (i) $a + b = a \Rightarrow a \cdot b' = 0$

   (ii) $a' + a \cdot b = a' + b$

   (b) Show that a tree with n-vertices has $(n - 1)$ edges.

Or

10. (a) Define the following terms giving examples :

   (i) Support of fuzzy set

   (ii) Complement of a fuzzy set

   (iii) Union of two fuzzy sets

   (iv) Intersection of two fuzzy sets

   (b) Prove that the maximum number of edges in a simple graph with n-vertices is $\frac{n(n-1)}{2}$.