**Roll No** .....

# CS/IT - 302

## B.E. III Semester

Examination, June 2016

## Discrete Structure

***Time : Three Hours***

***Maximum Marks : 70***

***Note:*** i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.

ii) All parts of each questions are to be attempted at one place.

iii) All questions carry equal marks, out of which part A and B (Max.50 words) carry 2 marks, part C (Max.100 words) carry 3 marks, part D (Max.400 words) carry 7 marks.

iv) Except numericals, Derivation, Design and Drawing etc.

### Unit - I

1. a) Write down Demorgan's law for set.

   b) Let R = {(1, 1), (1, 3), (1, 4)} be a relation on A = {1, 2, 3, 4, 5}. It is not reflexive. Why?

   c) Check whether relation | of divisibility on the set N of positive integers is an equivalence relation or not? Justify your answer.

   d) If A be the set of all triangles in a plane and R = {(a, b) : $\Delta a = \Delta b$}, i.e. aRb $\leftrightarrow$ area of the triangle a = Area of the triangle, than prove that R is an equivalence relation.





OR

Thirty cars were assembled in a factory. The options available were a radio, an air conditioner and white wall tyres. It is known that fifteen of the cars have radios eight of them have air-conditioners and six of them have white wall tyres. Moreover, three of them have all the three options. At least how many of them have no options at all.

### Unit - II

2. a) Find the multiplication table for G = {1, 2, 3, 4, 5, 6} under multiplication modulo 7.

   b) Define ring with example.

   c) What do you mean by cyclic group' show that any subgroup of a cyclic group is cyclic group

   d) Let G be a group, and H is a normal subgroup of G. If K is a normal subgroup of G containing H i.e. $H \leq K$, then the quotient group K/H is a normal subgroup of the quotient group G/H. Conversely, if K/H is a normal subgroup of G/H, then K is a normal subgroup of G containing H.

OR

Prove that the set Q1 of all rational numbers other than –1 with the operation defined by : $a*b = a+b-ab$ is an abelian group.

## Unit - III

3. a) If p = Ramesh is a player and q = Mohan is an intelligent boy. Then, write down the following formulae into sentences. 

   i) $\neg p \leftrightarrow q$

   ii) $p \vee q$

   iii) $\neg q \leftrightarrow \neg p$

   iv) $q \leftrightarrow p$

   b) Prove that following is tautology or not.

   $(p \vee q \vee r) \leftrightarrow [(((p \rightarrow q) \rightarrow q) \rightarrow r) \rightarrow r]$

   c) Consider the following assertions about the sets A, B and C. Write them down in the language of predicate logic. Use only the constructions of predicate logic ($\forall, \exists, \neg, \Rightarrow, \wedge, \vee$) and the element-of symbol ($\in$). Do not use derived notions ($\cap, \cup, =$, etc.).

   Example : "A is a subset of B" can be formalized as $\forall x. x \in A \Rightarrow x \in B$.

   i) The sets A and B are equal.

   ii) Every element of A is in the set B or the set C.

   iii) If A is disjoint from B then B and C overlap.





   d) i) Write down a structured proof of the following sentence. $(\forall x.\neg P(x)) \Rightarrow \neg \exists x.P(x)$.

   ii) Which of the following formulas are tautologies? Explain what is meant by "tautology" and write down truth tables to justify your answers.

   a) $p \Rightarrow q$

   b) $(p \Leftrightarrow q) \Rightarrow p$

   c) $((p \Rightarrow q) \Rightarrow p) \Rightarrow p$

Or

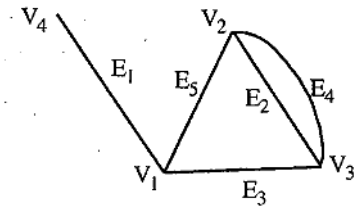
Minimize the following automata machine given below. With initial state S0 and final state S3.

| State | Input | |
|---|---|---|
| | 0 | 1 |
| S0 | S1 | S0 |
| S1 | S0 | S2 |
| S2 | S3 | S1 |
| S3 | S3 | S0 |
| S4 | S2 | S5 |
| S5 | S6 | S4 |
| S6 | S5 | S6 |
| S7 | S6 | S3 |

## Unit-IV

4. a) Explain any two application of coloring of graph.

   b) Explain adjacency matrix and incidence matrix for the graph representation using suitable example and find the incidence matrix for the following graph.



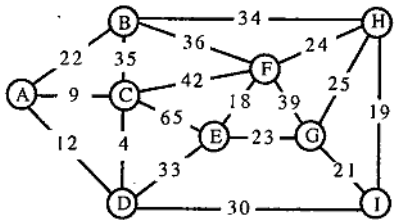

   c) Give an example of a graph and explain for the followings:

      i) A graph is having Hamiltonian circuit and Euler circuit.

      ii) A graph is having Hamiltonian circuit but not an Euler circuit.

      iii) A graph is having an Euler circuit but not a Hamiltonian circuit.

   d) Does the minimum spanning tree of a graph give the shortest distance between any two specified nodes? Convert the given graph with weighted edges to minimal spanning tree.








Or

Let G = {V, E} be an undirected graph with k-components and |V| = n, |E| = m. Prove that m ≥ n–k.

## Unit-V

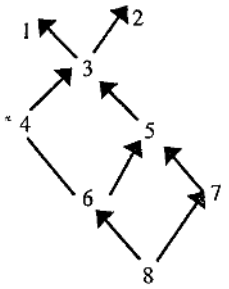
5. a) Let W = {1, 2, 3, 4, 5, 6, 7, 8} be ordered as shown in the adjoining figure consider the subset V = {4, 5, 6} of W.

      i) Find the set of upper bounds of V.

      ii) Find the set of lower bounds of V.

      iii) Does sup (V) exits.

      iv) Does inf (V) exits.




   b) For $f : R \to R$

$$F(x) = \begin{cases} 3x - 4, & x \geq 0 \\ -3x + 2, & x \leq 0 \end{cases}$$

Find $f^1(0)$s

c) How many integers between 1 to 300 (inclusive) are

i) Divisible by at least one of 3, 5, 7?

ii) Divisible by 3 and 5 but not by 7?

iii) Divisible by 5 but neither by 3 to 7?

d) Solve the following recurrence relation:

$a_r - 5a_{r-1} + 6a_{r-2} = 3^r$, given that $a_0 = 0$, $a_1 = 1$.

Or

Determine the discrete numeric function corresponding to each of the following generating functions.

i) $A(z) = (1 + z^2) / (4 - 4z - z^2)$

ii) $1/(5 - 6z + z^2)$

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