

IT/CS-4507

B. E. (Fourth Semester) EXAMINATION, Dec., 2002

(Common for IT & CS)

DISCRETE STRUCTURES

(IT/CS-4507)

Time : Three Hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : Answer any five questions. All questions carry equal marks.

1. (i) Define an uncountable set, Mathematical Induction, Equivalence relation, partial ordered set, a binary relation. 8
- (ii) Let R be an equivalence relation in a set A . Then the quotient set A/R is a partition of A . Prove it. 8
2. (i) Define a lattice, distributive lattice. For any a and b in A , prove that : 10

$$a \vee (a \wedge b) = a$$

$$a \wedge (a \vee b) = a$$
- (ii) Prove that in a distributive lattice, if an element has a complement, then this complement is unique. 6
3. (i) Obtain the converse, inverse and contrapositive of the conditional statement $p \rightarrow q$. 3

- (ii) Write the negation of the statement : 2
He swims if and only if the water is warm.
- (iii) Test the validity of the argument : 4
If two sides of a triangle are equal, then the opposite angles are equal :
Two sides of a triangle are not equal
The opposite angles are not equal
- (iv) Define normal form in propositional logic. 2
- (v) Show that $p \vee \sim (q \wedge r)$ is equivalent to $(p \vee \sim q) \vee \sim r$. 5
4. (i) Define Finite State Machine and Finite State Automation. Define the transitional diagram of the machine, language $L(M)$ determined by an automation M . 8
- (ii) Let $A = \{a, b\}$. Construct an automation M which will accept precisely those words from A which have an even number of a 's. 8
5. (i) Define a planner graph, Eulerian path, Hamiltonian circuit, a tree, a spanning tree and cut-set. 8
- (ii) Prove that : Let G be a linear graph of n vertices. If the sum of the degrees for each pair of vertices in G is $n - 1$ or larger, then there exists a hamiltonian path in G . 8
6. (i) Let a, b, c be numeric functions such that $a * b = c$. Given : 7

$$a_r = \begin{cases} 1 & r = 0 \\ 2 & r = 1 \\ 0 & r \geq 2 \end{cases}, \quad c_r = \begin{cases} 1 & r = 0 \\ 0 & r \geq 1 \end{cases}$$

Determine b .

- (ii) Solve the recurrence relation : 9

$$a_r - 7a_{r-1} + 10a_{r-2} = 0$$

given $a_0 = 0, a_1 = 3$.

7. (i) Prove that the order of the subgroup of a finite group divides the order of the group. 5
- (ii) Define normal subgroup, homomorphism, isomorphism, automorphism. 5
- (iii) Prove that a finite integral domain is a field. 6
8. Write short notes on any *three* of the following : 16
- (i) A relational model for databases
- (ii) A pigeon hole principle
- (iii) Shortest path in weighted graph
- (iv) Codes and group codes