

Roll No

MMTP/MMCM/MMMD/MMPD/MMIE-101

M.E./M.Tech. I Semester

Examination, December 2016

Advanced Mathematics

Time : Three Hours

Maximum Marks: 70

Note : Attempt any five questions. All questions carry equal marks.

1. a) Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ forms a basis of R^3 .
b) Find whether the set $S = \{(1, 2, 1), (3, 1, 5), (3, -4, 7)\}$ is linearly independent or dependent in $V_3(R)$.
2. a) Define linear transformation. Show that the mapping $T : V_2(R) \rightarrow V_2(R)$ defined by $T(x, y) = (2x + 3y, 3x - 4y)$ is a linear transformation.
b) Write a short note on each of the following with suitable example :
 - i) Vector space
 - ii) Vector subspace
 - iii) Linear transformation
3. a) Solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ by method of separation of variables
where $u(x, 0) = 6e^{-3x}$.

- b) Solve the partial differential equation

$\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y, x = 3 = y$ with $u = 0$ on the boundary and mesh length 1.

4. a) If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals :
- Exactly 3,
 - More than 2 individuals,
 - None,
 - More than one individual will suffer a bad reaction.

- b) A sample of 100 dry battery cells, tested to find the length of life, produced the following results: $\bar{x} = 12$ hours, $\sigma = 3$ hours. Assuming the data to be normally distributed, find what percentage of battery cells are expected to have life :
- More than 15 hours
 - Between 10 and 14 hours. (Given that the area under the standard normal curve between $x = 0$ and $x = 1$ is .3413 and between $x = 0$ and $x = 0.67$ is .2485)

5. a) A continuous random variable has the density function

$f(x) = 3x^2, 0 \leq x \leq 1$. Find a and b when

- $P(X \leq a) = P(X > a)$
 - $P(X > b) = 0.05$
- b) Ten individuals are chosen at random from a population and their heights are found to be in inches 63, 63, 64, 65, 66, 69, 69, 70, 70, 71. Discuss the proposal that mean height in the universe is 65 inches given that for 9 degrees of freedom the value of Student's t at 5 percent level of significance is 2.262.

6. a) Obtain the steady state difference equation for queueing model $\{(M / M / 1) : (N / FCFS)\}$.

- b) In a railway yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time (the time taken to hump a train) distribution is also exponential with an average 36 minutes. Calculate the followings :

- The average number of trains in a queue
- The probability that the queue size exceeds 10

If the input of trains increases to an average 33 per day, what will be change in (i) and (ii).

7. a) Solve the Euler's equation for the functional

$$\int_{x_1}^{x_2} (x + y') y' dx.$$

- b) Write short note on the following :

- t-test
- χ^2 -test

8. a) Write a short note on the following :

- Markov process
- Stochastic process with examples

- b) Using Galerkin's method; solve the boundary value problem $y'' = 3x + 4y$; $y(0) = 0, y(1) = 1$
