Total No. of Questions :8] *[Total No. of Printed Pages :2*

Roll No ....

MVCT/MBCT/MVCP - 101

M.E./M.Tech., I Semester

Examination, December 2015

Advanced Mathematics

Time : Three Hours

Maximum Marks : 70

Note : Attempt any five questions. All questions carry equal marks.

1. a) Solve the Poisson equation $u_{xx}+u_{yy}=-81xy$, $0 < x < 1$, $0 < y < 1$ given that $u(0, y) = 0 = u(x, 0)$, $u(1, y) = 100 = u(x, 1)$ and $h = \frac{1}{3}$.

   b) Find the values of $u(x, t)$ satisfying the parabolic equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ and the boundary conditions $u(0, t) = u(8, t) = 0$ and $u(x, 0) = 4x - \frac{1}{2}x^2$ at the points $x = i : i = 0, 1, 2 ...., 8$ and $t = \frac{1}{8}j : j = 0, 1, 2, ...5$.

2. a) Find the Fourier transform of $f(x) = \begin{cases} 1, & for |x| < 1 \\ 0, & for |x| > 1 \end{cases}$, Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$

   b) Using finite Fourier transform, solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ given that $u(0, t) = 0$, $u(4, t) = 0$ and $u(x, 0) = 2x$, where $0 < x < 4$, $t > 0$.

3. a) Define Hankel transform. Find the Hankel transform of
   i) e^{-x} and
   ii) $\frac{e^{-ax}}{x}$ taking $xJ_0(px)$ as the kernel of the transformation.

   b) Find the Fourier sine transform of $f(x) = \frac{1}{x}$.

4. a) Show that the function $u(x) = e^x(2x - 2/3)$ is solution of the Fredholm integral equation $2xe^x = u(x) + 2\int_0^1 e^{x-\xi}u(\xi)d\xi$

   b) Form an integral equation corresponding to the differential equation $y'' + xy' + y = 0$ with the initial conditions: y(0) = 1, y′(0) = 0.

5. a) Solve the Fredholm integral equation $u(x) = e^x + \lambda\int_0^1 2e^x e^t u(t)dt$.

   b) Using the method of successive approximations, solve the Volterra integral equation $y(x) = 1 + x + \int_0^x (x - t)\, y(t)dt$.

6. a) Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x – axis gives a minimum surface area.

   b) Find the curves on which the functional $\int_0^1 \left[(y')^2 + 12xy\right]dx$ with y(0) = 0 and y(1) = 1 can be extremised.

7. Solve the boundary value problem $y'' - y + x = 0$, $0 \le x \le 1$, $y(0) = 0$, $y(1)=0$ by
   i) Galerkin's method
   ii) Rayleigh-Ritz method and compare your solution with the exact solution.

8. Write short note on each of the following:
   i) Mellin Transform
   ii) Green functions
   iii) Integral equations and their types

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