*Total No. of Questions : 8]* *[Total No. of Printed Pages : 2*

Roll No .....

# MVCT/MBCT/MVCP-101 (Old)

## M.E./M.Tech., I Semester

Examination, December 2016

## Advance Mathematics

***Time : Three Hours***

***Maximum Marks : 70***

*Note :* i) Attempt any five questions.
ii) All questions carry equal marks.

1. Solve the hyperbolic equation $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$ taking $h = 1$ upto $t = 1.25$ under the conditions $u(0, t) = u(5, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x^2(5 - x)$.

2. Solve the boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0, t) = u(1, t) = 0$ and $u(x, 0) = \sin\pi x$, $0 \le x \le 1$, taking $h = 0.2$ and $k = 0.02$.

3. a) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$. Hence, find Fourier sine transform of $\frac{1}{x}$.

   b) Find $f(x)$ if
   i) Its sine transform is $e^{-ax}$,
   ii) Its cosine transform is $e^{-as}$.







4. a) Define Mellin transform. Find the Mellin transform of
   i) $e^{-x}$ and
   ii) $\sin x$
   b) Find Hankel transform of $x^{-2}e^{-x}$, taking ${}_xJ_1(px)$ as the kernel.

5. a) Verify that the function $u(x) = 1 - x$ is solution of the integral equation $x = \int_0^x e^{x-\xi} u(\xi)\, d\xi$.

   b) Convert the differential equation $y'' - 2xy' - 3y = 0$ with the initial conditions $y(0) = 1$ and $y'(0) = 0$ to integral equation.

6. a) Solve the Freedhlom integral equation
   $u(x) = \cos x + \lambda \int_0^{\pi} \sin xu(t)\, dt$

   b) Using the method of successive approximations, solve the integral equation $y(x) = 1 + \int_0^x y(t)\, dt$.

7. a) Prove that the shortest distance between two points in a plane is a straight line.

   b) On which curve the functional $\int_0^{\frac{\pi}{2}} \left(y'^2 - y^2 + 2xy\right) dx$ with $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$ can be extremised.

8. Using Galerkin's method, solve the boundary value problem $y'' = 3x + 4y$; $y(0) = 0$, $y(1) = 1$.

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