MA-220

(AU/ME/CM/FT/MI)

B.E. IV Semester

Examination, June 2017

Choice Based Credit System (CBCS)

Mathematics - III

Time : Three Hours] [Maximum Marks : 60

Note: i) Attempt any five questions.

ii) All questions carry equal marks.

Find the Fourier series of $f(x)=\frac{1}{x}(x-x)$ in the interval $(0,2\pi)$ Hence deduce that

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+......$$

b) Find the half range cosine series of $f(x)=x(\pi-x)$ in the interval $(0,\pi)$ and hence deduce that

$$\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+......$$

2. a) Find the Fourier transform of

$$f(x)=\begin{cases}1 & |x|<9\\ 0 & |x|>9\end{cases}$$

Hence find the value of $\int_0^\infty \frac{\sin x}{x}dx$

b) Find the Fourier cosine transform of

$$f(x)=\begin{cases}x & 0<x<1\\ 2-x & 1<x<2\\ 0 & x>2\end{cases}$$

3. a) State and prove second shifting theorem.

(b) Show that $\int_0^\infty \frac{\cos 6t-\cos 4t}{t}=\log\frac{2}{3}$

4. a) (i) Find the Inverse Laplace transform of the $\tan^{-1}\frac{2}{p}$

(ii) Find $L^{-1}\left\{\frac{p+2}{p^2(p+3)}\right\}$

b) Solve $(D^2+9)y=18t, y(0)=0, y\left(\frac{\pi}{2}\right)=1$

5. a) Show that the function $e^x(\cos y+i\sin y)$ is Holomorphic and find its derivaties.

b) Evaluate the Integral

$$\int_0^{1+i} Z^2dz$$

6. a) State and prove Cauchy's theorem.

b) Determine the pole of the function $f(z) = \frac{z^2}{(z+1)^2(z+2)}$

and residue at each pole.

7. a) Use Picard's method of successive approximation obtain a solution upto third approximation of differential equation.

$\frac{dy}{dx} = x + y^2$ where $y(0) = 0$

b) Employ Taylor's series method to obtain approximate value of y at x = 0.2 for the differential equation $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$

8. a) Using Euler's method solve for y at x = 0.1 from $\frac{dy}{dx} = x + y + xy, y(0) = 1$ taking h = 0.025.

b) Apply Runge Kutta Fourth order method to find an approximate Value of y when x = 0.2 given that $\frac{dy}{dx} = x + y, y(0) = 1$

****