Total No. of Questions :8] [Total No. of Printed Pages :3

Roll No .....

MVCT/MBCT/MVCP-101(Old)

M.E/M.Tech. I Semester

Examination, June 2017

Advance Mathematics

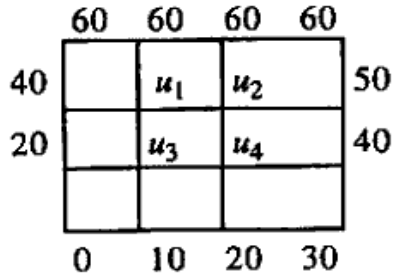
Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions.
ii) All questions carry equal marks.

1. a) Solve the boundary value problem $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0,t)=u(1,t)=0$ and $u(x,0)=\sin \pi x$, $0\le x\le 1$ taking $h=0.2$ and $k=0.02$.

b) Solve the elliptic equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$ for the square mesh with the boundary values shown in figure



2. a) Solve the wave equation $\frac{\partial^2 u}{\partial t^2}+\frac{\partial^2 u}{\partial x^2}$ with the conditions

$u(0,t)=u(1,t)=0, u(x,0)=\frac{1}{2}x(1-x)$ and $u_t(x,0)=0$

taking $h=k=0.1$ for $0\le t\le 0.4$.

b) Solve the partial differential equation $\nabla^2 u=-10(x^2+y^2+10)$ over the square with sides $x=0=y, x=3=y$ with $u=0$ on the boundary and mesh length 1.

3. a) Use Fourier transform to solve the boundary value problem

$\frac{\partial^2 u}{\partial t^2}=9\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t)=0$, $u(2,t)=0$, $u(x,0)=0.05x(2-x)$ and $u_t(x,0)=0$ where $0<x<2, t>0$.

b) Find the Fourier transform of e^{-ax^2}, where $a>0$.

4. a) Show that the function $u(x)=xe^x$ is a solution of the Volterra integral equation

$u(x)=\sin x+2\int_0^x \cos(x-\xi)u(\xi)d\xi$.

b) Form an integral equation corresponding to the differential equation

$y''+xy'+2y=0$, $y(0)=1$, $y'(0)=0$ into an integral equation.

5. a) Solve, by using method of successive approximations, the integral equation $y(x) = 1 + \lambda \int_0^1 xt\, y(t)dt$.

   b) Solve the integral equation

   $$y(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)\, y(t)dt.$$

6. a) Solve the Euler's equation for the functional

   $$\int_{x_1}^{x_2} (1 + x^2 y')\, y'\, dx.$$

   b) Using Rayleigh-Ritz method, solve the boundary value problem $y'' - y + x = 0;\ (0 \le x \le 1),\ y(0) = 0, y(1) = 0$.

7. a) Show that the shortest distance between two points in a plane is a straight line.

   b) Solve the integral equation $\int_0^x y(t)\, y(x-t)dt = 4\sin 9x$

8. Write short note on each of the followings:
   a) Green's function
   b) Finite difference method
   c) Abel's integral equations

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