Total No. of Questions : 5] [Total No. of Printed Pages : 4

Roll No. .....

BE–102(GS)

B. E. (First/Second Semester) EXAMINATION, June, 2012

(Grading System)

(Common for all Branches)

ENGINEERING MATHEMATICS – I

[BE – 102(GS)]

Time : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 22 (D Grade)

Note : Attempt all questions. All questions carry equal marks.

1. (a) Prove that :

$$\tan^{-1}(x+h) = \tan^{-1}x + h\sin z \cdot \frac{\sin z}{1} - \frac{(h\sin z)^2}{2}\cdot \sin 2z + ...$$

where $z = \cot^{-1}x$.

(b) If $u = x\phi(y/x) + \psi(y/x)$, then prove that :

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$

Or

(a) What error in the common logarithm of a number will be produced by an error of 1% in the number ?

(b) Find the maxima and minima of the following function :

$\sin x + \sin y + \sin(x+y)$ in $\left[0 \le x \le \frac{\pi}{2}, 0 \le y \le \frac{\pi}{2}\right]$.

2. (a) Find ab-initio the value of the integral :

$$\int_0^{\pi/2} \sin x\,dx$$

(b) Evaluate :

$$\int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{24}}\,dx$$

Or

(a) Evaluate :

$$\lim_{n\to\infty}\left\{\frac{\lfloor n}{n^n}\right\}^{1/n}$$

(b) Change the order of integration :

$$\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dx\,dy$$

Hence evaluate it.

3. (a) Solve :

$$y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$$

(b) Solve :

$$\frac{d^2y}{dx^2} + 4y = e^x + \sin 2x$$

Or

(a) Solve :

$$p(p \quad y) = x(x+y)$$

where $p \equiv \frac{dy}{dx}$.

(b) Solve :

$$\frac{dx}{dt} + y = \sin t$$

$$\frac{dy}{dt} + x = \cos t$$

Given that $x = 2$ and $y = 2$, when $t = 0$.

4. (a) Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 4 & 3 & 6 & 1 \\ 0 & 2 & 3 & 1 & 4 \\ 0 & 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{5 \times 5}$$

by defining it in Echelon form.

(b) Find the eigen values and eigen vectors of the matrix : 10

$$A = \begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$$

Or

(a) Find the values of k such that the system of equations :

$$x + ky + 3z = 0$$

$$4x + 3y + kz = 0$$

$$2x + y + 2z = 0$$

has non-trivial solution.

(b) Verify the Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

5. (a) Define the following terms for a graph :
   (i) Subgraph
   (ii) Degree of a vertex
   (iii) Composition and De-composition
   (iv) Rooted tree

(b) Define fuzzy logic and its applications in science and engineering.

Or

(a) "Prepare a truth table to get the negation of the statement "Sita is dull and careless."

(b) Prove that :

$$a \cdot b + b \cdot c + c \cdot a = (a + b) \cdot (b + c) \cdot (c + a) \ \forall\, a, b, c, \in B$$