

Roll No

MVSE/MVCT/MBCT/MVCP - 101**M.E./M.Tech., I Semester**

Examination, June 2014

**Advanced Mathematics /Advanced Mathematics
And Numerical Analysis***Time : Three Hours**Maximum Marks : 70***Note :** Total number of questions eight. Attempt any five questions.

1. Classify the partial differential equation. Use the difference method, find numerical solution to the following parabolic partial differential equation.

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < 2, t > 0$$

$$\text{B.C. : } u(0, t) = u(2, t) = 0, \quad t > 0$$

$$\text{I.C. : } u(x, 0) = \sin(2\pi x), \quad 0 \leq x \leq 2$$

Use $h = 0.4$ and $k = 0.1$.

2. a) Explain Mellin transform and its applications.
b) Find the Fourier transform of the following function, if

$$f(x) = \begin{cases} 1, & \text{when } |x| < a \\ 0, & \text{when } |x| > a \end{cases}$$

$$\text{Hence evaluate } \int_0^\infty \frac{\sin ax}{x} dx.$$

3. Write the short notes on:
i) Finite difference method
ii) Hankel transform
iii) Fast Fourier transform
iv) Fourier sine and cosine transforms

4. a) Define Fredholm and Volterra integral equations.
b) Convert the differential equation

$$\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = 5 \sin x, \quad \text{with}$$

$$y(0) = 1, y'(0) = -2$$

into an integral equation.

5. a) Explain in brief integro-differential equations with example.
b) Find the curves on which the functional
- $$\int_0^1 [(y')^2 + 12xy] dx, \quad \text{with } y(0) = 0 \text{ and } y(1) = 1$$
- can be extremised.

6. a) Explain in brief method of successive approximations to solve integral equation.

- b) Show that the curve which extremizes the functional

$$I = \int_0^{\pi/4} (y'' - y^2 + x^2) dx, \quad \text{under the conditions } y(0) = 0,$$

$$y'(\pi/4) = 1, y(\pi/4) = y'(\pi/4) = \frac{1}{\sqrt{2}} \text{ is } y = \sin x.$$

7. Discuss the following:

- i) Galerkin method
ii) Rayleigh-Ritz method.

8. Solve the boundary value problem:

$$y''(x) - y(x) + x = 0, \quad (0 \leq x \leq 1)$$

with $y(0) = y(1) = 0$ by Rayleigh-Ritz method.
