MVCT–101

M. E./M. Tech. (First Semester)
EXAMINATION, Dec., 2011

(Grading/Non-Grading System)

ADVANCED MATHEMATICS

(MVCT – 101)

Time : Three Hours

Maximum Marks : $\begin{cases} GS: & 70 \\ NGS: & 100 \end{cases}$

Note : Attempt any *five* questions. All questions carry equal marks.

1. (a) Solve the partial differential equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in the domain by Gauss-Seidel method, given :

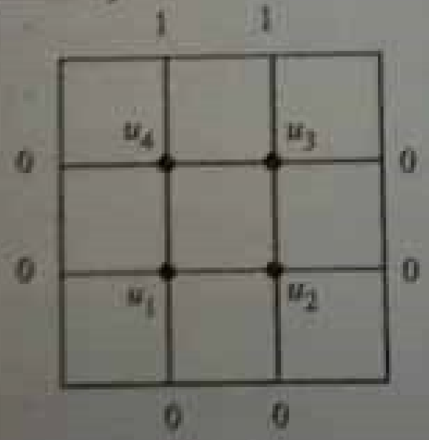

Fig. 1

(b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to initial condition $u = \sin \pi x$ at $t = 0$ for $0 \le x \le 1$ and $u = 0$ at $x = 0$ and $x = 1$, by Gauss-Seidel method.

2. (a) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh with boundary values as shown in the figure.

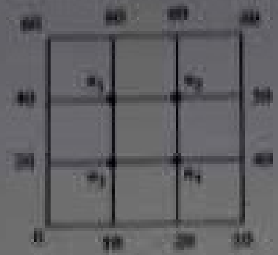

Fig. 2

(b) Show that if $n = 0$, the Hankel transform of :

$$H\left[\frac{\sin ax}{x}\right] = \begin{cases} 0 & \text{if } s > a \\ \dfrac{1}{\sqrt{a^2 - s^2}} & \text{if } 0 < s < a \end{cases}$$

3. (a) Find the Fourier transforms of :

$$F(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

and hence evaluate :

$$\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3}\right) \cos x/2 \, dx$$

(b) Prove that Mellin transform of :

(i) $M\{f(ax)\} = a^{-s} \bar{f}(s)$

(ii) $M\{x^a f(x)\} = \bar{f}(s + a)$

7. a) Solve the integral equation:

$$y(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)\, y(t)\, dt$$

b) Find the integral equation corresponding to the boundary value problem.

$$y''(x) + \lambda y(x) = 0,\ y(0) = y(1) = 0$$

8. a) Using the method of successive approximation Volterra integral equation:

$$y(x) = 1 + x + \int_0^x (x-t)\, y(t)\, dt$$

b) Define :

i) Abel's integral equation

ii) Integro Differential equation

iii) Green function.

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