Total No. of Questions : 10] [Total No. of Printed Pages : 4

Roll No. ....

MCA–102

M. C. A. (First Semester) EXAMINATION, June, 2008

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

(MCA – 102)

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt all questions. (i. e. *one* from each Unit). All questions carry equal marks.

Unit – I

1. (a) If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ and $U = \{1, 2, 3, 4, 5, 6\}$, verify that :

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

(b) Let $A = \{1, 2, 3\}$ and let R and S be relations on A. Suppose that the matrices of R and S are :

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Determine $M_{\overline{R}}$, $M_{R^{-1}}$, $M_{R \cap S}$ and $M_{R \cup S}$ where $\overline{R}$ and R^{-1} denote respectively the complementary and the inverse relation of R.

Or

2. (a) If $f : R \rightarrow R$ and $g : R \rightarrow R$ and $h : R \rightarrow R$ defined by :

$$f(x) = x^3 - 4x, g(x) = \frac{1}{x^2 + 1}, h(x) = x^4$$

find the following composition functions :

(i) (fogoh) (x)

(ii) (gog) (x)

(iii) (hogof) (x)

(b) Use principle of inclusion and exlusion to determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7.

Unit–II

3. (a) Write a short note on Quantifiers.

(b) Prove by truth table that the following are tautologies or contradictions :

(i) $(p \rightarrow q \wedge r) \rightarrow (\sim r \rightarrow \sim q)$

(ii) $(p \vee q) \wedge \{p \vee (\sim q)\} \wedge \{(\sim p) \vee q\} \wedge \{(\sim p) \vee (\sim q)\}$

Or

4. (a) Explain the following terms :

(i) Lattice

(ii) Distributive lattice

(iii) Complemented lattice

(b) Draw the switching circuit of the following function and replace that by simpler one :

Unit – III

5. (a) State and prove Lagrange's theorem for a finite group.
   (b) Prove that every finite integral domain is a field.

Or

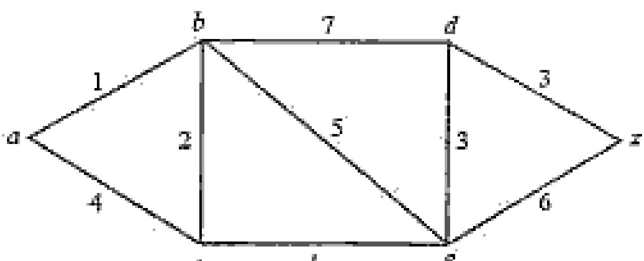
6. (a) The intersection of any *two* normal subgroups of a group is a normal subgroup.
   (b) Define the following :
   (i) Subgroup
   (ii) Co-set
   (iii) Polynomial roots
   (iv) Reducible polynomials
   (v) Primitive polynomials

Unit – IV

7. (a) Prove that the maximum number of edges in a graph with n vertices is $\frac{n(n-1)}{2}$.
   (b) Define rooted, binary and a spanning tree. Prove that every connected graph has at least *one* spanning tree.

Or

8. (a) Find the shortest path between a and z for the graph :



Unit – V

9. (a) Determine the generating function of the numeric function a_r, where :

$$a_r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ 2^{-r} & \text{if } r \text{ is odd} \end{cases}$$

(b) Solve the recurrence relation :

$$S(k) - 7S(k-1) + 10S(k-2) = 6 + 8k$$

with $S(0) = 1$ and $S(1) = 2$.

Or

10. (a) Write short notes on the following :

(i) discrete numeric functions

(ii) generating functions

(b) Solve the recurrence relation :

$$S(k) - 5S(k-1) + 6S(k-2) = 2^k + k, \; k \geq 2$$

with boundary conditions $S(0) = 1$ and $S(1) = 1$.

Total No. of Questions : 5] [Total No. of Printed Pages : 4

MCA-102(N)

M. C. A. (First Semester) EXAMINATION, May/June, 2006

(New Scheme)

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

[MCA−102 (N)]

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt any *two* parts from each question. All questions carry equal marks.

1\. (a) Use principle of inclusion and exclusion to determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7.

(b) Give the definitions of reflexive, irreflexive, symmetric, antisymmetric and asymmetric relation. Give the example of a relation :

(i) which is neither reflexive nor irreflexive

(ii) which is both symmetric and antisymmetric

(iii) which is both asymmetric and antisymmetric

(iv) which is reflexive and antisymmetric

P. T. O.

(c) Define one-one and onto function. Using mathematical induction, prove that :

$$n\,! < \left(\frac{n+1}{2}\right)^{n}, \text{ where } n > 1 \text{ and } n \in N.$$

2\. (a) Define the following :

(i) Lattice isomorphism

(ii) Complete lattice

(iii) Sub-lattice

(iv) Algebra of prepositional

(b) Define poset.

Let X be a set and $P = 2^X$ i. e., the family of all subsets of X. Let P be ordered by set inclusion C. Show that (P, C) is a lattice, where meet $\wedge$ and join $\vee$ are respectively defined by :

$A \wedge B = A \cap B$ and $A \vee B = A \cup B$ for all $A, B \in P$.

(c) (i) Prove that the following statement is logically equivalent :

$$p \vee (p \leftrightarrow r) \equiv (p \vee q) \leftrightarrow (p \vee r)$$

(ii) Construct the truth table of the following :

$$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$$

3\. (a) State and prove Lagrange's theorem for a finite group.

(b) Define normal subgroup with example. Let Z be the set of all integers. On Z define $*$ as follows :

$$a * b = a + b - 1$$

Show that $(Z, *)$ is a group.

(c) Describe the following with example :

(i) Primitive polynomial and Polynomial roots

(ii) Field

4. (a) Define the following :

    (i) Centre of tree

    (ii) Complete bipartite graph

    (iii) Planar graph

    (iv) Connected graph

    (v) Walk and bridge

(b) Define spanning tree for the complete graph. Find the minimum spanning tree for the following graph :

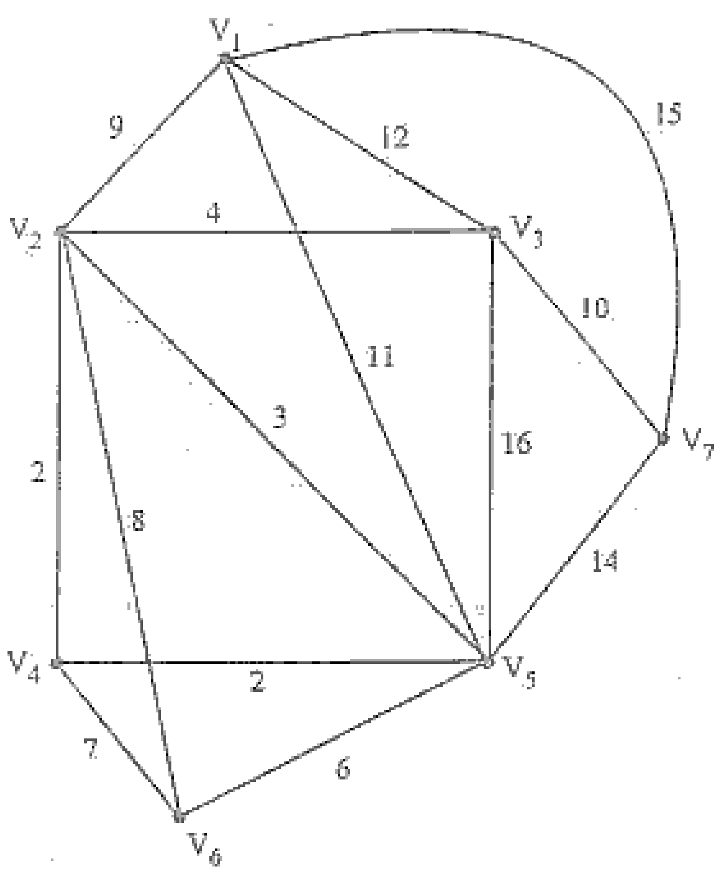


Fig. 1

(c) (i) Find the adjacency matrix of the digraph.

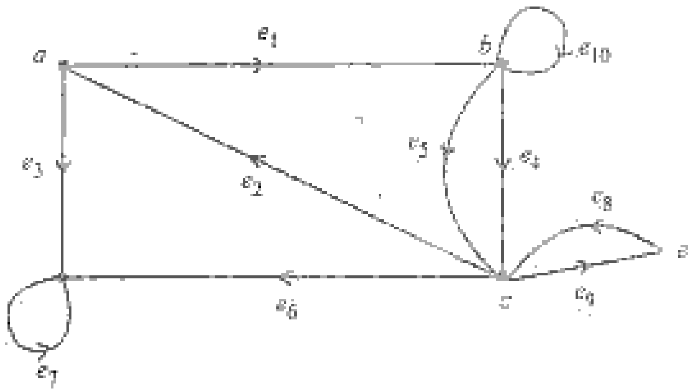


Fig. 2

(ii) Prove that a connected graph with n-vertices and $(n-1)$ edges is a tree.

5. (a) (i) Discuss recursive algorithms with example.

(ii) Let a be a numeric function, where :

$$a_r = r^3 - 3r^2 + 3r + 2$$

Determine Δa and $\Delta^2 a$, where Δ is forward difference operator.

(b) (i) Determine the discrete numeric function corresponding to the following generating function :

$$A(z) = \frac{(1+z)^2}{(1-z)^4}$$

(ii) Determine the generating function of the numeric function a_n where :

$$a_r = \begin{cases} 2^r; & \text{if } r \text{ is even} \\ 2^{-r}; & \text{if } r \text{ is odd} \end{cases}$$

(c) Solve the recurrence relation :

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)^2, r \geq 2.$$

Total No. of Questions : 8] [Total No. of Printed Pages : 4

MCA–102

M. C. A. (First Semester) EXAMINATION, June, 2005

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(MCA – 102)

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt any *five* questions. All questions carry equal marks.

1. (a) If A, B, C and D are any four sets then prove that : 8

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

(b) Prove that $2^n > n^2$ for each positive integer $n \geq 5$. 8

(c) Show that the function $f : N \times N \to N$ defined by : 4

$$f(x, y) = x + y$$

is onto but not one-to-one.

2. (a) Let :

$A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3), (3, 4), (2, 1)\}$

be a relation on A. Find the transitive closure of R using Warshall algorithm. 8

(b) Define distributive and modular lattices and show that the lattice shown in following fig. 1 is modular but not distributive. 8

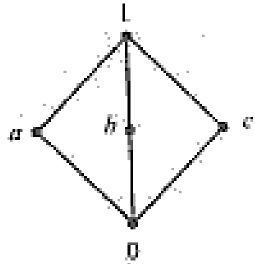


Fig. 1

(c) Determine whether the following proposition is contradiction or a tautology where p and q are propositions : 4

$$9 \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$$

3. (a) Let (B, $*$, $\oplus$, $'$, 0, 1) be a Boolean algebra. Define the operations $+$ and $\cdot$ on the elements of B by : 10

$$a + b = (a * b') \oplus (a' * b)$$
$$a \cdot b = a * b$$

Show that :

(i) $a + a = 0$

(ii) $a + 0 = a$

(iii) $(a + b) + b = a$

(iv) $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

(v) $a + 1 = a'$

(b) Show that the following Boolean experssions are equivalent and obtain their sum of products canonical form : 10

(i) $(x_1 \oplus x_2) * (x_1' \oplus x_3)$

(ii) $(x_1 * x_3) \oplus (x_1' * x_2) \oplus (x_2 * x_3)$

4. (a) In a Boolean algebra prove that : 10

(i) $a = b \Leftrightarrow (a * b') \oplus (a' * b) = 0$

(b) If a is an integer and m is a prime then prove that : 10

$$a^m \bmod m = a \bmod m$$

5. (a) Define group and show that if every element in a group is its own inverse, then the group must be abelian. 10

(b) Let $(G, *)$ and (H, Δ) be groups and $g : G \rightarrow H$ is a homomorphism. Prove that the kernel of g is a normal subgroup. 10

6. (a) Define the following terms giving examples : 10

(i) Simple graph
(ii) Multigraph
(iii) Indegree and outdegree
(iv) Complete graph
(v) Bipartite graph
(vi) Isomorphic graphs
(vii) Tree
(viii) Spanning tree
(ix) Connected graph
(x) Eulerian graph

(b) Find the shortest path between the vertices a and z in the following graph using Dijkstra's algorithm. 10

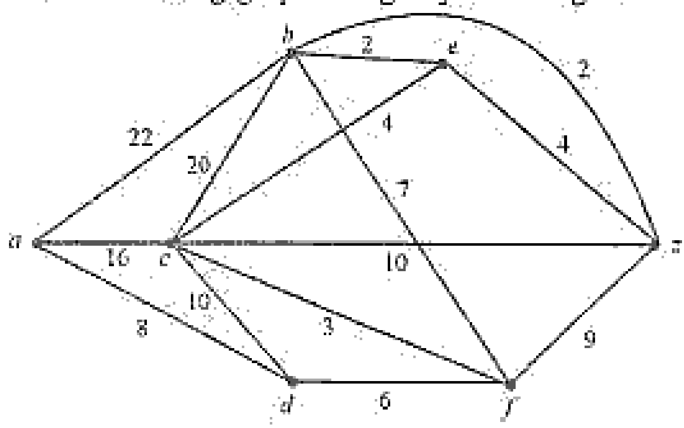


Fig. 2

7. (a) Prove that a graph G wih invertices, $(n - 1)$ edges and no circuit is connected. 10

(b) Determine the minimum weight spanning tree for the following graph using prism algorithm : 10

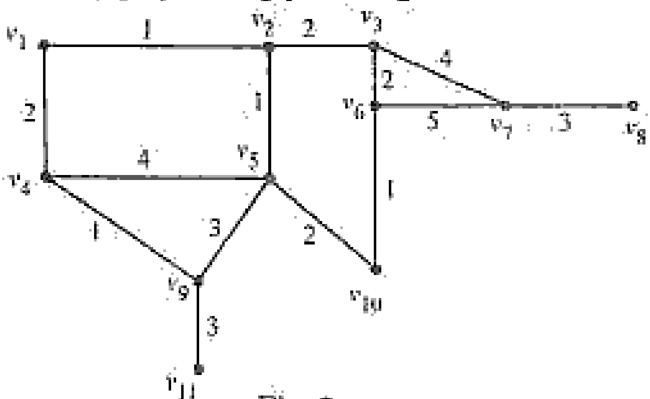


Fig. 3

8. (a) Write down the following : 10

(i) Adjacency matrix

(ii) Incidence matrix

(iii) Circuit matrix

(iv) Cutset matrix

of the graph given below :

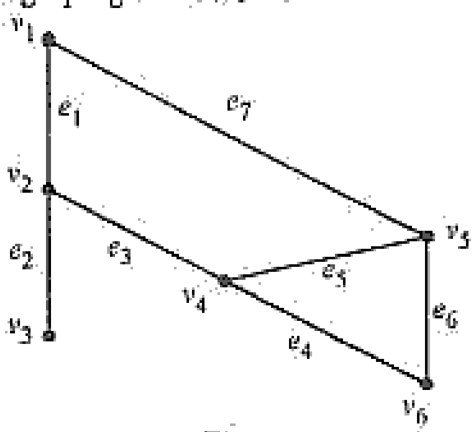


Fig. 3

(b) Write short notes on the following : 10

(i) Group Codes

(ii) Applications of Graphs

Total No. of Questions : 8] [Total No. of Printed Pages : 4

MCA-102(O)

M. C. A. (First Semester) EXAMINATION, Dec., 2005

(Old Scheme)

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

[MCA–102(O)]

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt any *five* questions. All questions carry equal marks.

1. (a) Prove the following :

   (i) $(A \cap B) \cup (A \cap B') = A$

   (ii) $B \Delta A = (B - A) \cup (A - B)$

   (b) By principle of mathematical induction prove that $11^{n+2} + 12^{2n+1}$ is divisible by 133, $n \in N$.

2. (a) Consider the set $N \times N$, the set of ordered pairs of natural numbers. Let R be the relation in $N \times N$ which is defined by :

$$(a, b) \, R \, (c, d) \Leftrightarrow a + d = b + c.$$

   Prove that R is an equivalence relation.

   (b) Let $f : R \to R$ and $g : R \to R$, where R is the set of real numbers. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 - 2$ and

P. T. O.

$g(x) = x + 4$. State whether these functions are injective, surjective and bijective ?

3. (a) Let $< L, \leq >$ be a lattice. For any $a, b, c \in L$ the following holds :

$$a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$$

(b) Let $(A, \leq)$ and $(B, \leq)$ are two posets, then show that $(A \times B, \leq)$ is a poset, where the relation $\leq$ on $A \times B$ is defined by :

$$(a, b) \leq (a', b') \Leftrightarrow a \leq a'$$

in A and $b \leq b'$ in B.

4. (a) Using truth table, determine whether each of the following is a tautology, a contingency or an absurdity ?

(i) $(p \to q) \wedge (p \vee q)$

(ii) $(p \to q) \Leftrightarrow (\sim q \to \sim p)$

(b) Show that the following propositions are tautologies :

(i) $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

(ii) $\{(p \vee \sim q) \wedge (\sim p \vee \sim q)\} \vee q$

5. (a) Let $(A, *)$ be a group and B is subset of A. If B is a finite set then $(B, *)$ is a subgroup of $(A, *)$ if $*$ is a closed operation on B.

(b) The order of a subgroup H of a group is a divisor of the order of G. Prove it.

6. (a) Show that the set of numbers of the form $a + b\sqrt{2}$, with a and b as rational numbers is a field.

(b) Define the following with examples :

(i) Conjunction and Disjunction

(ii) Proposition

(iii) Complete graph and multigraph

(iv) Rooted tree and Binary tree

(v) Hamiltonian graph

7. (a) Determine shortest path between 'a' and 'z' in the graph shown below, where the numbers associated with the edges are distance between vertices.

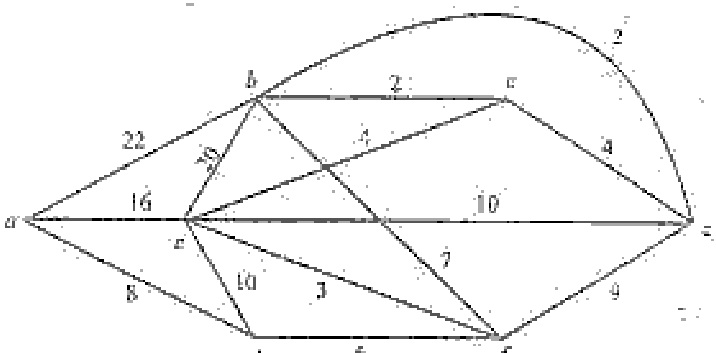


Fig. 1

(b) Define a Eulerian path in a graph and prove that an undirected graph possesses a Eulerian path if and only if it is connected and has either zero or two vertices of odd degree.

8. (a) Determine a minimum weight spanning tree for the following graph.

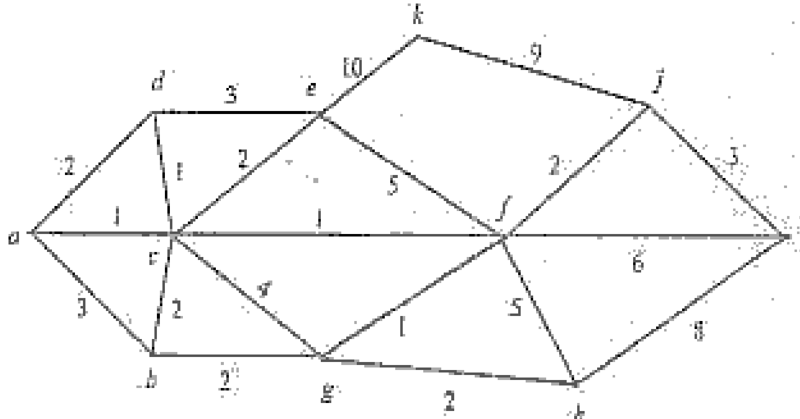


Fig. 2

P. T. O.

(b) Find the adjacency matrix of the digraph show below.

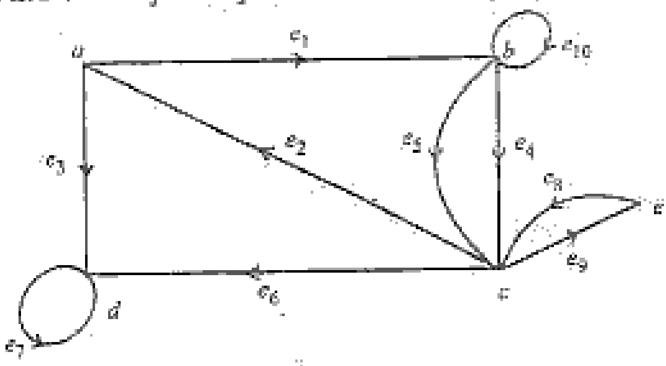


Fig. 3

Total No. of Questions : 8] [Total No. of Printed Pages : 4

MCA–102

M. C. A. (First Semester) EXAMINATION, Dec., 2004

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(MCA – 102)

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt any *five* questions. All questions carry equal marks. Answer to a new question should start from a fresh page.

1\. (a) Show that in a Boolean algebra, every element has a unique complement, while it is possible for an element of a complemented lattice to have more than one complements.

(b) Simplify the following circuit :

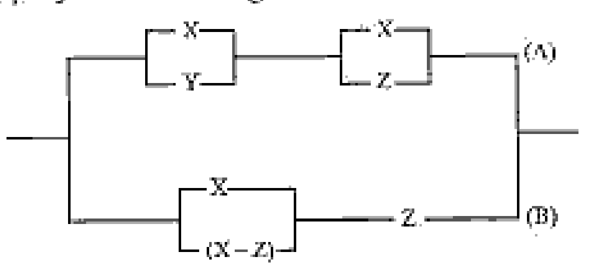


Fig. 1

2\. (a) Let F = $Z_n = \{0, 1, 2, ..., n-1\}$ under the operations of addition and multiplication modulo n,

P. T. O.

where n is a positive integer. Prove that Z_n is a field if and only if n is a prime number.

(b) Solve the equation $X^2 + 2X - 1 = 0$ in the following fields :

(i) z_3

(ii) z_7

(iii) z_2

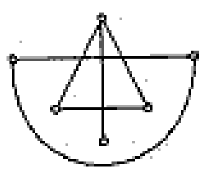
(iv) set of rational numbers

3\. (a) Define the terms 'group', 'subgroup' and 'semi-group'. Show that a group of order $n \le 4$ is abelian.

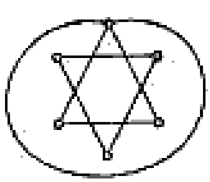
(b) If H_1 and H_2 are subgroups of a group G, then prove that $H_1 \cap H_2$ is a subgroup of G.

4\. (a) Define 'multigraph' and 'connected graph'. Prove that every connected graph has at least one spanning tree. Which of the following maps are a multigraph as well as connected ?

(i)



(ii)



(iii)

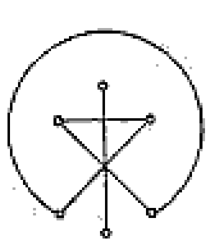


Fig. 2

(b) Define an 'Eulerian graph' and an 'Eulerian trail'. Give an example of an Eulerian graph with six vertices. For this graph, verify Euler's formula which relates the number of vertices, the number of edges and the number of regions in such graphs.

5. (a) Define an incidence matrix and an adjacency matrix for a graph. Which representation should be used and why to store a graph in a digital computer in the following cases ?

(i) the graph has no self loops.

(ii) the graph has no parallel edges.

(iii) the graph is simple, i. e., it has neither self loops nor parallel edges.

(b) Define a 'rooted tree' and hence a 'binary tree'. Show that a binary tree with n vertices has $\frac{n+1}{2}$ pendant vertices. Also show that the maximum number of vertices in a binary tree of height h is $(2^{h+1} - 1)$.

6. (a) Let z be the set of integers. Let $f : z \to z$ be $f(x) = 5x + 1$. Show that f is one to one but not 'onto'. Show how to define a new function $g : z \to z$., which has the same pairs as f, but has restricted co-domain in such a way that g is 'onto' ?

(b) For a Pascal's triangle to be defined, the following information is given to establish its pattern :

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

(i) Write the next three rows of Pascal's triangle.

(ii) Give a rule for building the next row from the previous rows.

7. (a) (i) What is meant by 'cardinality' of union of two sets ? Explain.

(ii) Define the 'characteristic function' (call it f_c) of a set A. Write the characteristic function of A^C in terms of f_c (A^C is complement of the set A).

(b) Let σ be a permutation of $\{1, 2, ..., 9\}$ given by :

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 5 & 6 & 8 & 7 & 3 & 4 & 2 & 1 \end{pmatrix}$$

Express σ as a product of disjoint cycles, illustrating the cycle decomposition by a diagram. Hence, find the order of σ and the order of σ^2 in the permutation group.

8. (a) If $\{ \{a, c, e\}, \{b, d, f\} \}$ is a partition of the set $A = \{a, b, c, d, e, f\}$, determine the corresponding equivalence relation R.

(b) Let $A = \{1, 2, 3\}$ and let R and S be relations on A. Suppose that the matrices of R and S are :

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Determine $M_{\overline{R}}$, $M_{R^{-1}}$, $M_{R \cap S}$ and $M_{R \cup S}$ where $\overline{R}$ and R^{-1} denote respectively the complementary relation and the inverse relation of R.

Total No. of Questions : 5] [Total No. of Printed Pages : 4

MCA–102

M. C. A. (First Semester) EXAMINATION, June, 2004

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(MCA – 102)

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt all questions by selecting any *three* parts from each question. All questions carry equal marks.

1. (a) Define the following terms giving examples :

   (i) Symmetric difference

   (ii) Irreflexive relation

   (iii) Transitive closure

   (iv) Recursive set

   (v) Cardinality

   (vi) Partial order relation

   (b) Write the principle of Mathematical induction and prove that $3^n > n^3$ for all $n \geq 4$.

   (c) Prove that the function :

   $$f(x, y) = x^y$$

   is primitive recursive.

   (d) Prove that the set $N \times N$ is denumerable.

2. (a) Prove that in a distributive lattice, if an element has a complement, then this complement is unique.

(b) Define a modular lattice and prove that every distributive lattice is modular but not conversely.

(c) Given an expression $\alpha(x_1, x_2, x_3)$ defined to be Σ 0, 2, 3, 5, determine the value of $\alpha(a, 1, b)$, where $a, 1, b \in B$ and $(B, *, \oplus, ', 0, 1)$ is the boolean algebra given in the following fig. 1.

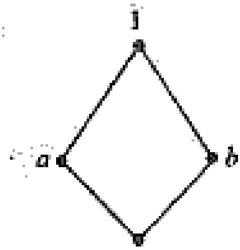


Fig. 1

(d) Draw the switching circuit of the function :

$$F(x, y, z) = x \cdot y' \cdot (z + x) + y \cdot (y' + z)$$

and replace it by a simplified one.

3. (a) Show that :

$$((P \vee Q) \wedge \rceil (\rceil P \wedge (\rceil Q \vee \rceil R))) \vee (\rceil P \wedge \rceil Q) \vee (\rceil P \wedge \rceil P)$$

is a tautology.

(b) Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\rceil R \vee P$ and Q.

(c) Test the validity of the argument :

If two sides of a triangle are equal, then the opposite angles are equal :

Two sides of triangle are not equal

The opposite angles are not equal

(d) Prove that the set of idempotent elements of a commutative monoid forms a submonoid.

4. (a) Show that the set of all matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, where $a, b \in R$ is a field with respect to matrix addition and matrix multiplication.

(b) Prove that every cyclic group is an abelian group.

(c) Prove that the maximum number of edges in a graph with n vertices is $\frac{n(n-1)}{2}$.

(d) Construct a graph whose adjacency matrix is given below :

$$X(G) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

5. (a) Write down circuit matrix of the following graph.

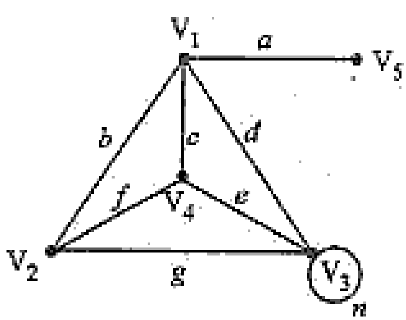


Fig. 2

(b) Prove that a tree with n vertices has $n-1$ edges.

(c) Define Isomorphic graphs and prove that the ahead graphs are isomorphic.

Fig. 3

(d) Write and explain Kruskel's Algorithm for finding a minimal spanning tree of weighted graph.

Total No. of Questions : 5] [Total No. of Printed Pages : 4

MCA–102

M. C. A. (First Semester) EXAMINATION, Dec., 2003

MATHEMATICAL FOUNDATIONS
OF COMPUTER SCIENCE

(MCA – 102)

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt all questions. Attempt any *three* parts of each question. All questions carry equal marks.

1. (a) Prove that :

   (i) $(A-B) \cup (B-A) = (A \cup B)-(A \cap B)$

   (ii) $(A-B) \cap (A-C) = A-(B \cup C)$

   (b) Prove that the set of divisors of a positive integer n is recursive.

   (c) Let $A = \{a, b, c, d\}$ and

   $R = \{(a, b), (b, c), (c, d), (b, a)\}$

   be a relation on A. Find the transitive closure of R using Warshall's algorithm.

   (d) Prove by mathematical induction that :

   $$2^n < \lfloor n \quad \text{for} \quad n \geq 4$$

2. (a) Define the following terms giving examples :

   (i) Lattice

P. T. O.

(ii) Distributive lattice

(iii) Modular lattice

(iv) Complemented lattice

(v) Sublattice

(vi) Chain

(b) In a complemented distributive lattice $(L, *, \oplus, ', 0, 1)$ prove that :

(i) $(a * b)' = a' \oplus b'$

(ii) $(a \oplus b)' = a' * b'$

(c) Prove the following Boolean identities :

(i) $(a \oplus b) * (a' \oplus c) * (b \oplus c) = (a * c) \oplus (a' \oplus b) \oplus (b * c)$

(ii) $(a * b) \oplus (a * b') = a$

(d) Find the value of :

$$x_1 * x_2 * [(x_1 * x_4) \oplus x_2' \oplus (x_3 * x_1')]$$

for $x_1 = a, x_2 = 1, x_3 = b$ and $x_4 = 1$, where a, b, $1 \in B$ and the Boolean algebra $(B, *, \oplus, ', 0, 1)$ is shown in the following fig. 1.

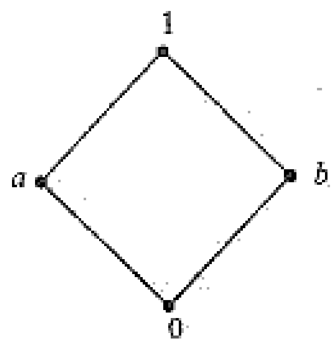


Fig. 1

3. (a) Obtain principal disjunctive normal form of :

$$(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$$

(b) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.

(c) Prove that the composition of two semigroup homomorphism is also a semigroup homomorphism.

(d) Show that if every element in a group is its own inverse, then group must be abelian.

4. (a) Define field and prove that the set of complex numbers is a field with respect to ordinary addition and multiplication.

(b) Prove that every finite integral domain is a field.

(c) Prove that the number of vertices of odd degree in a graph is always even.

(d) Write down the graph corresponding to the following incidence matrix :

$$A(G) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. (a) Prove that a graph with n vertices, $n - 1$ edges, and no circuits is connected.

(b) Write and explain an algorithm to find the shortest path from a specified vertex to another specified vertex of a graph.

(c) Find the minimal spanning tree of the weighted given ahead in fig. 2 using Kruskals algorithm.

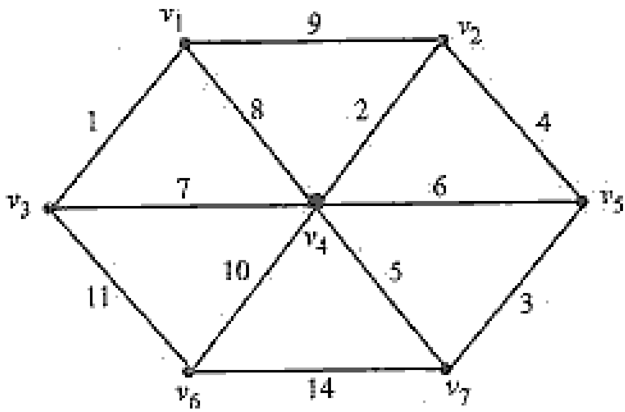


Fig. 2

(d) Explain the following terms giving examples :

(i) Eulerian Graphs

(ii) Isomorphic Graphs

(iii) Connected Graph

Total No. of Questions : 8]　　[Total No. of Printed Pages : 4

MCA – 102

M. C. A. (First Semester) EXAMINATION, June, 2003

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(MCA – 102)

Time : Three Hours

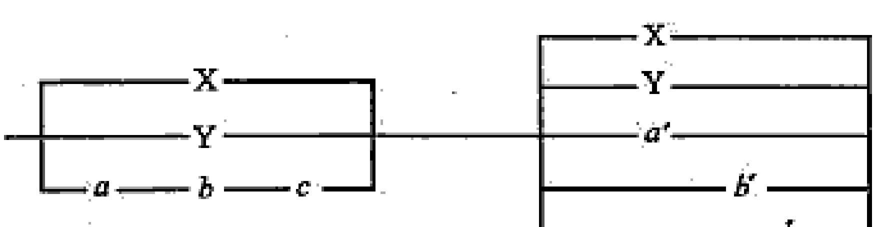
Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt any *five* questions. All questions carry equal marks. Answer to a new question should start from a fresh page.

1. (a) Define a 'lattice' and hence a 'Boolean Algebra'. Are there any Boolean algebras having three elements ? Why or why not ?

   (b) Simplify the following circuit :



2. (a) Let R be the field of all real numbers. Show that $R \times R$ is a field with respect to addition and multiplication defined as :

$$(a, b) + (c, d) = (a + c, b + d)$$
$$(a, b) \cdot (c, d) = (ac - bd, ad + bc)$$

(b) Define an 'irreducible polynomial' over the field (F, +, ·). Show that $f(x) = x^2 + x + 4$ is irreducible over the field $(F, +_{11}, \odot_{11})$ of integer modulo 11.

3. (a) Define a normal sub-group. Show that any sub-group of a commutative group is normal.

(b) Let A_n be the set of all binary sequences of length n and let $\oplus$ be a binary operation on A such that for X, Y in A, $X \oplus Y$ is a sequence of length n that has 1's (ones) in those positions where X and Y differ and 0's (zeros) in those positions where X and Y are same. (Hint : for example, $A_2 = \{00, 01, 10, 11\}$ is the set of all binary sequences of length 2.) Show that the algebraic system $(A_n, \oplus)$ is a group.

4. (a) Define a 'graph' and hence define a 'tree'. Show that a tree with n vertices has $(n - 1)$ edges.

(b) State and prove Euler's formula which relates the number of vertices, the number of edges and the number of regions in any connected map. Also verify the formula for the following map :

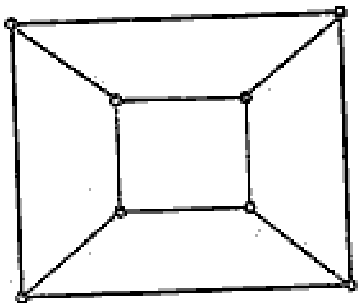


Fig. 2

5. (a) Prove that if a graph has exactly two vertices of odd degree then there must be a path joining these two vertices.

(b) Define adjacency matrix. Identify from the following, the adjacency matrix which corresponds to an undirected graph. Draw the graph so identified :

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

6. (a) Let $f : R \to R$ be a function on the set of real numbers defined by $f(x) = 2x - 3$. Find a formula for $((f \circ f)^{-1} \circ f)$ Here $(\ )^{-1}$ represents the inverse function and 'o' denotes composite operation.

(b) Prove by mathematical induction on $p \geq 4$ that $p^3 < 3^p$.

7. (a) Let $R = \{(5, 5), (6, 5) (5, 6), (7, 7)\}$ be a relation on the set $S = \{5, 6, 7\}$. Show that R is an equivalence relation. Determine the partition, S/R, of the set S induced by R.

(b) Consider the theorem : For all sets A, B; $A^C \cup B^C \subseteq (A \cup B)^C$. Find the mistake in the following proof : Suppose that A and B are sets and $x \in A^C \cup B^C$. Then $x \in A^C$ or $x \in B^C$ by the definition of 'union'. It follows that $x \notin A$ or $x \notin B$ by the definition of 'complement'; and so $x \notin A \cup B$ by the definition of 'union'. Thus $x \in (A \cup B)^C$ by the definition of 'complement'. Hence :

$$A^C \cup B^C \subseteq (A \cup B)^C.$$

8. (a) How do the following differ from one another :
   (i) Partially ordered set
   (ii) Totally ordered set
   (iii) Linearly ordered set

   (b) Let $A = \{a, b, c, d, e, f, g, h\}$ and let R be the relation defined by :

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   Show that the poset (A, R) is complemented and give all pairs of complements.

Total No. of Questions : 8] [Total No. of Printed Pages : 5

MCA–102

M. C. A. (First Semester) EXAMINATION, Dec., 2002

MATHEMATICAL FOUNDATIONS
OF COMPUTER SCIENCE

(MCA–102)

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt any *five* questions. All questions carry equal marks.

1. (a) If A and B are any two non-empty sets then prove that :

   (i) $B - A = B \cap A'$

   (ii) $B \Delta A = (B - A) \cup (A - B)$

   (b) Show that the mapping $f : Z^+ \to Z^+$ defined by $f(x) = x^2, \forall x \in Z^+$ where Z^+ is a set of positive integers, is one to one and into.

2. (a) Let R_1 and R_2 be any two Partial Order Relations. Then show that $R_1 \cap R_2$ is also a Partial Order Relation.

   (b) Let D (15) is a set of elements which are divisors of 15. Then determine with respect to meet and join operations whether :

   (i) {D (15), divides} is a Lattice

P. T. O.

(ii) {D (15), divides} is a Complemented Lattice

(iii) {D (15), divides} is a Distributive Lattice

(iv) {D (15), divides} is a chain

(v) {D (15), divides} is a Boolean Algebra

3. (a) Show that the set B = {0, 1} with two binary operations + and · and a unary operation, defined on B by the following operation tables is a Boolean Algebra :

+	0	1
0	0	1
1	1	1

.	0	1
0	0	0
1	0	1

a	a'
0	1
1	0

(b) Using Boolean Algebra show that :

(i) $a.b + a.b' + a'.b + a'.b' = 1$

(ii) $(a + b)(b + c)(c + a) = a.b + b.c + c.a$

4. (a) (i) Determine whether the following proposition is contradiction or a tautology where p and q are propositions :

$(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

(ii) If p and q are any two propositions then show that :

$$\sim (p \wedge q) = \sim p \vee \sim q$$

(b) Explain the following terms with the help of suitable examples :

(i) Conjunction

(ii) Disjunction

(iii) Conditional propositions

(iv) Converse propositions

(v) Inverse propositions

(vi) Contrapositive propositions

(vii) Boolean Algebra

(viii) Proposition

(ix) Biconditional Proposition

(x) Tautology

5. (a) Define Group. Let G be set of all non-singular $n \times n$ matrices. Does G form a group under matrix multiplication ? If G is a group then what is the identity element and what are inverses ?

(b) Let Z be the group of integers under addition and let H be the subgroup of Z consisting of multiples of 5. Show that H is a normal subgroup of Z and find the quotient group Z/H.

6. (a) Define field. Let S be the set of real numbers of the form $a + b\sqrt{3}$ where a and b are rational numbers. Show that S is a field with respect to addition and multiplication.

(b) (i) Suppose $f(t) = t^4 - 2t^3 + 11t - 10$. Find all the real roots of $f(t)$ assuming that there are two integer roots.

(ii) Discuss briefly the error correction codes sequence generation.

P. T. O.

7. (a) Find the shortest path for the following graph using Dijkstra's algorithm.

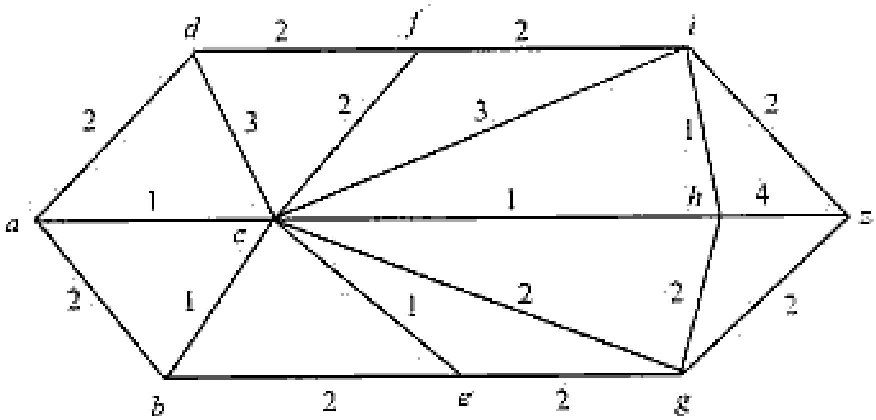


Fig. 1

(b) Write down the adjacency and incidence matrix of the graph given below :

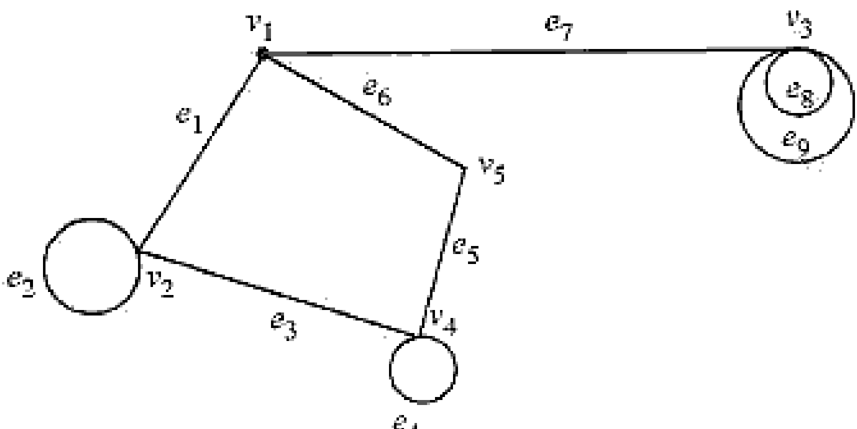


Fig. 2

8. (a) Determine the minimum weight spanning tree for the ahead graph using Prim's algorithm.

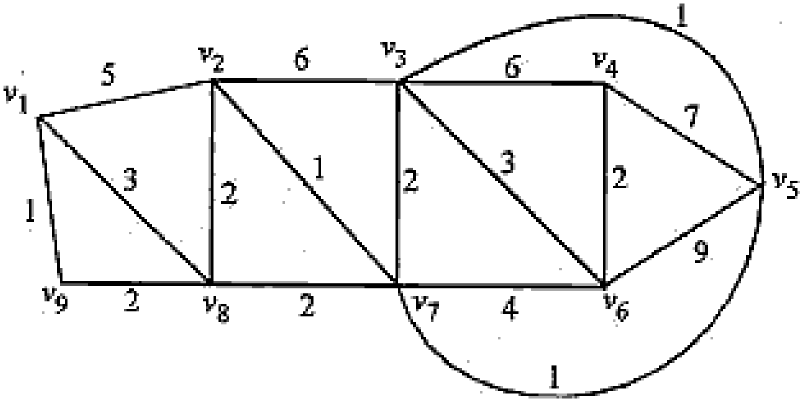


Fig. 3

(b) Explain the following terms using suitable examples :

(i) Multigraph

(ii) Spanning subgraph of a graph G

(iii) Path

(iv) Connected graph

(v) Eulerian graph

(vi) Walk

(vii) Circuit

(viii) Tree

(ix) Spanning Tree

(x) Cut vertices

Total No. of Questions : 5] [Total No. of Printed Pages : 3

MCA–102

M. C. A. (First Semester) EXAMINATION, June, 2002

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(MCA – 102)

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt any *two* parts from each of the following questions. All questions carry equal marks. Both parts of a question be attempted at one place. Answer to a new question must start from a fresh page.

1. (a) Define an equivalence relation. If R is an equivalence relation in X then prove that R^{-1} is also an equivalence relation in X.

   (b) Let $f : X \to Y$ and A, B be subsets of X. Prove that :

   (i) $f(A \cup B) = f(A) \cup f(B)$

   (ii) $f(A \cap B) \subseteq f(A) \cap f(B)$

   (c) Prove by mathematical induction that $n^3 + 2n$ is divisible by 3.

2. (a) Define Lattice. Show that in a lattice if $a \leq b \leq c$, then :

   (i) $a \oplus b = b * c$

   (ii) $(a * b) \oplus (b * c) = (a \oplus b) * (a \oplus c)$

P. T. O.

(b) Let (B, $*$, $\oplus$, $'$, 0, 1) be a boolean algebra. Define the operation $+$ and $\cdot$ on the elements of B by :

$$a + b = (a * b') \oplus (a' * b)$$
$$a \cdot b = a * b$$

Prove that :

(i) $(a + b) + b = a$

(ii) $a + 1 = a'$

(c) Find the sum of products canonical forms of the following boolean expressions in three variables x_1, x_2 and x_3 :

(i) $(x_1 \oplus x_2) * (x'_1 \oplus x_3)$

(ii) $x_1 \oplus x_2$

3. (a) Define semi-group, monoid and group. Give examples :

(i) a semi-group which is not a monoid

(ii) a monoid which is not a group

(b) Define even and odd permutations giving examples. Find the inverse of the premutation :

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$$

(c) Prove that the order of subgroup of a finite group divides the order of group.

4. (a) Define field. Prove that the set of all rational numbers with composition $\oplus$ and $\odot$ defined by :

$$a \oplus b = a + b - 1$$
$$a \odot b = a + b - ab$$

is a field.

(b) Prove that the polynomial $2x^3 + x^2 + 2x + 2$ is

(c) Define the following terms giving examples :

(i) Distance and centre in a tree

(ii) Connected graph

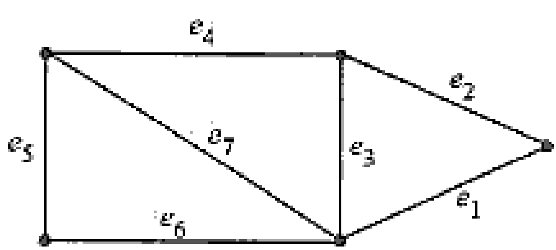
(iii) Rooted binary tree

(iv) Incidence matrix

5. (a) Prove that the number of vertices of odd degree in a graph is always even.

(b) Prove that every connected graph has at least one spanning tree.

(c) Define fundamental circuit matrix of the graph G and write the fundamental circuit matrix of the graph G in the following figure with respect to spanning tree $\{e_1, e_4, e_5, e_7\}$.



Total No. of Questions : 5] [Total No. of Printed Pages : 3

MCA-102

M. C. A. (First Semester) EXAMINATION, Dec., 2001

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(MCA—102)

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt all questions by selecting any *two* parts from each question. All questions carry equal marks. Both parts of a question be attempted at one place. Answer to a new question must start from a fresh page.

1. (a) Define irreflexive, symmetric, antisymmetric and asymmetric relations. 4, 6

Let A = {1, 2, 3, 4,}

Give an example of a relation R in A which is :

(i) neither symmetric nor antisymmetric

(ii) antisymmetric and reflexive but not transitive

(iii) transitive and reflexive but not antisymmetric

(b) Prove by mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43. 10

(c) Define characteristic function of a set. Prove the following set identities using characteristic function : 2, 8

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $(A \cup B)' = A' \cap B'$

P. T. O.

2. (a) Define lattice and boolean algebra. Show that the partially ordered set whose Hasse diagram is given in following figure is a lattice but not a boolean algebra. 4, 6

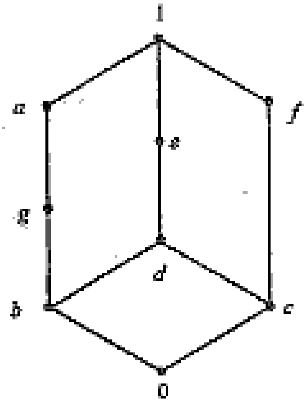


Fig. 1

(b) Find the product of sums canonical form of the following boolean expressions in three variables x_1, x_2 and x_3. 5, 5

(i) $x_1 * x_2$ (ii) $(x_1 * x_3) \oplus (x_1' * x_2)$

(c) Given an expression $\alpha(x_1, x_2, x_3) = \sum 0, 3, 5, 7$, determine the value of $\alpha(a, b, 1)$ where $a, b, 1 \in B$ and $(B, *, \oplus, ', 0, 1)$ is the boolean algebra given in the following figure. 10

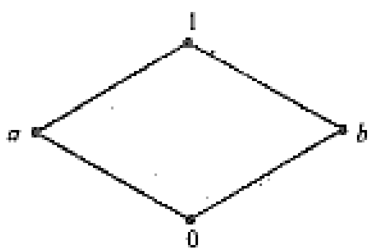


Fig. 2

3. (a) Let G be the set of all non-zero real numbers and $*$ be a binary operation defined on G by $a * b = \frac{a.b}{2}$. Prove that $(G, *)$ is an abelian group. 10

(b) Let $(G, *)$ and (H, Δ) be groups and $g : G \to H$ is a homomorphism. Prove that the kernel of g is a normal subgroup. 10

(c) If a is an integer and m is prime, prove that $a^m \bmod m = a \bmod m$. 10

4. (a) Define field. Prove that the set {0, 1, 2} (mod 3) is a field with respect to addition and multiplication (mod 3). 2, 8

(b) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 10

(c) Define the following terms giving examples : 10

(i) Eulerian graph

(ii) Cut set

(iii) Pendant vertices in a tree

(iv) Path matrix of a graph

5. (a) Find the minimum spanning tree of the following weighted graph. 10

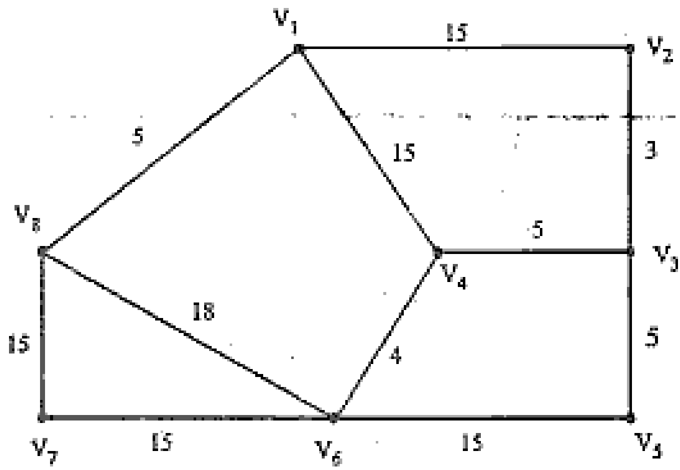


Fig. 3

(b) Prove that every circuit has even number of edges in common with any cut set. 10

(c) Write short notes on the following : 10

(i) Group Codes

(ii) Irreducible polynomials