B.E. (Fifth Semester) EXAMINATION, Dec., 2004

(Computer Science & Engg. Branch)

THEORY OF COMPUTATION

(CS–505)

Note : Attempt any *five* questions. All questions carry equal marks.

1. (a) Use induction on the size of S to show that if S is a finite set then $|2^S| = 2^{|S|}$.

(b) Show that the following order of magnitude holds :

(i) $n^2 + 5 \log n = o(n^2)$ (ii) $n! = o(n^n)$

(c) Construct an NFA that accepts the language

(abb) (a/b)*

convert it into DFA.

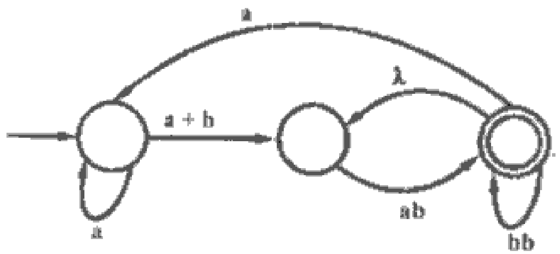
2. (a) Show that if L is regular, so is L^R.

(b) Prove or disprove the following conjecture. If $M = (Q, \in, \delta, q_0, p)$ is a minimal d.f.a. for a regular language L, then $\hat{M} = (Q, \Sigma, \delta, q_0, Q - F)$ is a minimal d.f.a. for $\overline{L}$.

(c) Show that if r_1 and r_2 are regular expressions, then :

(i) $L(r_1 . r_2) = L(r_1) L(r_2)$ (ii) $L(r_1^*) = (L(r_1))^*$

3. (a) What is the language accepted by the following generalized transition graph ?



(b) Find a regular expression for the following languages on {a, b} :

(i) $L = \{\omega : n_a(\omega)$ and $n_b(\omega)$ are both even$\}$

(ii) $L = \{\omega : (n_a(\omega) - n_b(\omega)) \bmod 3 \neq 0\}$

(c) The head of a language is the set of all prefixes of its strings, that is:

$$\text{head}(L) = \{x : xy \in L \text{ for some } y \in \Sigma^*\}$$

Show that the family of regular languages is closed under this operation.

4. (a) Let $L = \{a^n b^n : n \geq 0\}$

show that L^2 is context-free.

(b) Show that the following grammar is ambiguous :

$S \to AB \mid aa B$

$A \to a \mid Aa$

$B \to b$

(c) Remove all unit productions, all useless productions and all λ-productions from the grammar :

$S \to a A \mid a BB$

$A \to aa A \mid \lambda$

$B \to b B \mid bb C$

$C \to B$

What language does this grammar generate ?

5. (a) Find an npda on $\Sigma = \{a, b, c\}$ that accepts the language :

$$L = \{\omega_1 c \omega_2 : \omega_1, \omega_2 \in \{a, b\}^*, \omega_1 \neq \omega_2^R\}$$

(b) Find a context-free grammar that generates the language accepted by the npda $M = (\{q_0, q_1\}, \{a, b\}, \{A, Z\}, \delta, q_0, z, \{q_1\})$ with transitions :

$\delta(q_0, a, z) = \{(q_0, A_z)\}$

$\delta(q_0, b, A) = \{(q_0, AA)\}$

$\delta(q_0, a, A) = \{(q_1, \lambda)\}$

6. (a) Show that the family by linear languages is closed under union, but not closed under concatenation.

(b) Construct a turing machine to compute the function :

$f(w) = w^R$

7. (a) Show that for arbitrary context-free grammars G_1 and G_2, the problem "$L(G_1) \cap L(G_2)$ is context-free" is undecidable.

(b) Show that Ackermann's function is a total function in I × I.

(c) Show that for $|\Sigma| = 1$, the post correspondence problem is decidable that is, an algorithm that can decide whether or not (A, B) has a PC-solution for any given (A, B) on a single letter alphabet.

8. Write short notes on any *four* of the following :

(a) Various types of Turing Machines

(b) Deterministic Push Down Automata

(c) Pumping Lemma for Regular Languages

(d) Automata with Output

(e) Unrestricted Grammars

(f) CNF