8. a) Prove that a graph G with n vertices and (n–1) edges is a tree.

   b) Prove that a tree with two or more vertices has at least two pendant vertices.

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Total No. of Questions : 8] *[Total No. of Printed Pages : 4*

Roll No .....

MCTA-101

M.E./M.Tech. I Semester

Examination, December 2016

Mathematical Foundations of Computer Application

Time : Three Hours

Maximum Marks: 70

Note : i) Attempt any five questions out of eight.
ii) All questions carry equal marks.

1. a) If A, B and C are three sets, then prove that

   i) $A \cup (B \cup C) = (A \cup B) \cup C$ and

   ii) $A \cap (B \cap C) = (A \cap B) \cap C$

   b) Let R be a binary relation on the set of all positive integers such that

   $R = \{(a,b) / |a-b| \leq 1, a,b \text{ are positive integers}\}$

   Is R an equivalence relation.

2. a) Define lattice. Prove that in a lattice $(L, \leq)$, for any $a, b \in L$ the following hold:

   i) $a \leq b \Leftrightarrow a \wedge b = a$ and

   ii) $a \leq b \Leftrightarrow a \vee b = b$

   b) Let Q be the set of all rational numbers and $f: Q \to Q$ be a function defined by $f(x) = 3x + 5, x \in s$.

   Then prove that f is bijective and also find its inverse.

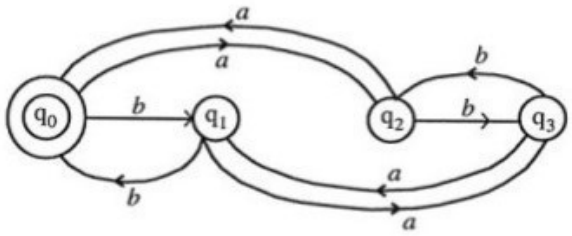
3. a) Obtain the following into conjunctive normal form :

$$\sim(p \vee q) \leftrightarrow (p \wedge q)$$

b) Given a DFA, $m = \{\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_0\}\}$. The transition diagram for the DFA is given below. Check whether strings

i) aababa and

ii) bababa are accepted by DFA.



4. a) Show that $L = \{a^n b^n / n \geq 0\}$ is not regular.

b) Find disjunctive normal form of the following proposition.

$$(\sim p \vee \sim q) \rightarrow (p \leftrightarrow \vee q)$$

5. a) Solve the following recurrence relation:

$$a_r + 6a_{r-1} + 9a_{r-2} = 3$$

given that $a_0 = 0$ and $a_1 = 1$

b) Let S = {a, b, c} and P(s) be the power set. Then show that $(P(s), \subseteq)$ is a partially ordered set. Draw the Hasse diagram also.

6. a) Determine the discrete numeric function corresponding to the following generating functions:

i) $A(z) = \dfrac{2}{1-4z^2}$

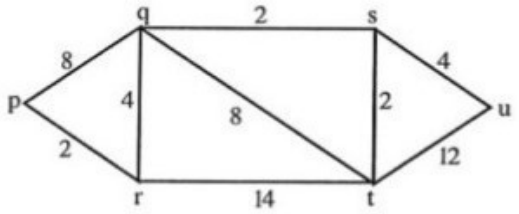
ii) $A(z) = \dfrac{z^4}{1-2z}$

b) Solve the recurrence relation:

$$a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r, \quad r \geq 2$$

with boundary conditions $a_0 = 1$ and $a_1 = 1$.

7. a) Using Dijkstra's algorithm find the shortest path from p to u in the following weighted graph:



b) A graph G has the following adjacency matrix check whether it is connected? Also, draw the graph

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$