

## ADVANCED COMPUTATIONAL MATHEMATICS

Time : Three Hours

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Maximum Marks : 100

**Note :** Attempt five question in all. One question from each Unit is compulsory. All questions carry equal marks.

## Unit-I

1. Let  $F$  be a subfield of complex numbers and let  $T$  be the function from  $F^3$  to  $F^3$  defined by

$$T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, x_1 - 2x_2 + 2x_3).$$

- (a) Verify that  $T$  is a linear transformation.  
 (b) If  $(a, b, c)$  is a vector in  $F^3$ , what are the conditions on  $a, b$  and  $c$  that the vector be in the range of  $T$ ? What is the rank of  $T$ ?  
 (c) What are the conditions on  $a, b$  and  $c$  than  $(a, b, c)$  be in the null space of  $T$ ? What is the nullity of  $T$ ?

Or

2. Define Hash function with examine and write the Hamiltonian differential equation and Mathematician form of Mean value polynomial. Check whether the following function of a  $R^2$  into  $R^2$  are when transfer is on is not :

- (a)  $T(x_1, x_2) = (1 + x_1, x_2)$  (b)  $T(x_1, x_2) = (x_2, x_1)$   
 (c)  $T(x_1, x_2) = (x_1, x_2)$  (d)  $T(x_1, x_2) = (\sin x_1, x_2)$   
 (d)  $T(x_1, x_2) = (x_1 - x_2, 0).$

## Unit-II

3. The longitudinal displacement in a vibrating elastic but can be described by the boundary value problem :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < l, t > 0;$$

$$\frac{\partial u}{\partial x}(0, t) = 0,$$

$$\frac{\partial u}{\partial x}(l, t) = 0, \quad t > 0$$

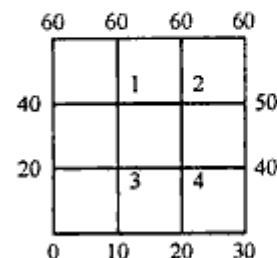
$$u(x, 0) = x^2 (1 - \sin x),$$

$$\frac{\partial u}{\partial t}(x, 0) = 0, \quad 0 < x < l.$$

Find the solution of the above boundary value problems by using method of separation of variables.

Or

4. (a) Explain the following with one example in each case where it is applied :  
 (i) DFT (ii) WFT.  
 (b) Solve Laplace equation at mesh points :



with given conditions.

## Unit III

5. Define discrete and continuous random variables. Let  $X$  have the density function :

$$f(x) = 0.75(1 - x^2), \quad \text{if } -1 \leq x \leq 1$$

$$= 0 \quad \text{otherwise.}$$

Find the distribution function. Find the probabilities  $P\left(-\frac{1}{2} < x \leq \frac{1}{2}\right)$  and  $P\left(\frac{1}{4} \leq x \leq 2\right)$ . Find  $x$  s.t.

$$P(X \leq x) = 0.95.$$

Also compute the probability of obtaining at least two 4's in rolling a fair die 4 times.

Or

6. Define null and alternative hypothesis. Define confidence interval.  
 If the probability of producing a defective screw is  $p = 0.01$ , what is the probability that a lot of 100 screws will contain more than 2 defectives?

## Unit-IV

7. Distinguish between discrete parameter Markov chain and continuous parameter Markov chain. A man either drives his car or takes a train to work each day. Suppose he covers takes the train two days in a row, but he drives to work, then the next day he is just likely to drive again as he is to take the train. Find the transition matrix. Also find the probability that he changes from going by train to driving exactly in four days.

Or

8. A one-person barber shop has six chairs to accommodate people waiting for haircut. Assume a customer arrives when all six chairs are full leaves without entering the barber shop. Customer's arrive at the average rate of 3 per hour and spend an average of 15 minutes in the barber shop. Find the probability that the shop is empty.

- (a) The probabilities a customer can get directly into the barber chair upon arrival.
- (b) Expected number of customer waiting for a hair cut.
- (c) Effective arrival rate.
- (d) The time a customer can expect to spend in the barber shop.

**Unit-V**

9. (a) What is fuzzy membership function ? Explain the triangular, trapezoidal and Gaussian membership function with their mathematical form.
- (b) Explain different defuzzification methods.

**Or**

10. Explain various features of MATLAB in detail. Give *five* applications where it is used.

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