*Total No. of Questions :8]* *[Total No. of Printed Pages :3*

**Roll No** ...................................

# MVSE/MVCT/MBCT/MVCP-101

## M.E./M.Tech., I Semester

Examination, December 2013

## Adv. Mathematics & Numerical Analysis /Adv. Mathematics

***Time : Three Hours***

***Maximum Marks : 70***

*Note:* 1. Attempt any five questions.
2. All questions carry equal marks.

1. a) Solve the Poisson's equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-10(x^2+y^2+10)$$

Over the square with sides $x=0, y=0, x=3, y=3$ with $u(x,0)=0$ on the boundary and mesh length = 1.

b) Solve $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$ under condition

$u(0,t) = u(4,t) = 0$

and $u(x,0)=x(4-x)$ taking $h=1$, find value upto $t=5$.

2. a) Find the Fourier Cosine transform of

$$F(x)=\begin{Bmatrix} x & \text{for } 0<x<1 \\ 2-x & \text{for } 1<x<2 \\ 0 & \text{for } \quad x>2 \end{Bmatrix}$$

b) Using finite Fourier transform solve

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ given}$$

$u(0,t) = u(4,t) = 0$

and $u(x,0) = 2x$ where $0 < x < 4$, $t > 0$.

3\. a) Define Hankel Transform and find Hankel Transform of

$$F(x) = \begin{cases} a^2 - x^2 & 0 < x < a \quad n = 0 \\ 0 & x > a \quad n = 0 \end{cases}$$

b) Prove that

$$H\left(\frac{\sin ax}{a}\right)_{n=1} = \frac{a}{s(s^2 - a^2)^{1/2}}$$

4\. a) Solve the Euler's Equation for

$$\int_{x_0}^{x_1} \left(1 + x^2 y'\right) y' \, dx$$

b) Find the extremals of the functional and extremum value of the following :

$$I\left[\frac{y}{x}\right] = \int_{x_0}^{x_1} \frac{1 + y^2}{y'^2} \, dx$$

5\. a) Explain discretization in finite element method.

b) Use Rayleigh-Ritz method to solve the equation:

$$\frac{d^2 y}{dx^2} + y = x, \; y(0) = 0, y(1) = 1.$$





6\. a) Use Galerkin's method to solve the equation :

$$\frac{d^2 y}{dx^2} - y + x = 0, \; y(0) = 0, y(1) = 1$$

b) What is the difference between FEM and DFT.

7\. a) Convert the differential equation

$$Y''(x) - 3y'(x) + 2y(x) = 5\sin x, \; y(0) = 1, y'(0) = -2$$

into an integral equation.

b) Find the green function for the boundary value problem:

$$\frac{d^2 y}{dx^2} + \mu^2 x^2 = 0 \quad y(0) = 0, y(1) = 0$$

8\. a) Solve the Abel's integral equation:

$$\int_0^x \frac{y(t)}{\sqrt{(x-t)}} dt = 1 + 2x - x^2$$

b) Using the method of successive approximations, solve the Volterra integral equation:

$$Y(x) = 1 + x + \int_0^x (x-t)\, y(t)\, dt.$$

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