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CS/IT – 405

B. E. (Fourth Semester) EXAMINATION, Dec., 2003

(Common for CS & IT Engg.)

DISCRETE STRUCTURES

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : Answer any *five* questions. All questions carry equal marks.

1. (i) Prove that if A and B are finite sets :

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

(ii) Define Mathematical Induction. Use it to prove :

$$2 + 4 + 6 + \ldots + 2n = n(n+1).$$

(iii) Given $A = \{1, 2\}$, $B = \{x, y, z\}$ and $C = \{3, 4\}$. Find $A \times B \times C$.

2. (i) Consider the relation R on the set Z of integers :

$$x \equiv y \pmod{m},\ m \text{ is an integer} > 1.$$

Show that it defines an equivalence relation on Z.

(ii) Consider the set Z of integers. Define $a R b$ by $b = a^r$ for some positive integer r. Show that R is a partial order on Z.

3. (i) Obtain the converse, inverse and contrapositive of the conditional statement $p \rightarrow q$.

(ii) Write the negation of the statement :

If she works, she will earn money.

(iii) Test the validity of the argument :

If I study, then I will not fail in Mathematics.

If I do not play basket ball, then I will study.

But I failed in Mathematics

Therefore, I must have played basket ball.

(iv) Show that $p \vee \sim(p \wedge r)$ is equivalent to $(p \vee \sim q) \vee \sim r$.

(v) Ajoy loves Nina.

Ajoy loves Anita too.

Does Ajoy really loves Nina ?

4. (i) Let $A = \{a, b\}$. Construct an automation M which will accept the language $L(M) = \{a^r b^s : r > 0, s > 0\}$.

(ii) Let M be the finite state machine with state table :

F	a	b
S_0	S_1, x	S_2, y
S_1	S_3, y	S_1, z
S_2	S_1, z	S_0, x
S_3	S_0, z	S_2, x

(a) Find the input set A, the state set S the output set Z and the initial state.

(b) Draw the transitional diagram of M.

(c) Let $w = aababaabbab$ is an input word. Find the output word V.

5. (i) Prove that a finite connected graph G is Eulerian if and only if each vertex has even degree.

   (ii) Define planar graph, regular and bipartite graph. Draw the graph $K_{2,5}$.

   (iii) Define a tree, spanning tree, minimum spanning tree.

6. (i) Let a, b, c be numeric functions such that $a * b = c$. Given :

$$a_n = \begin{cases} 1 & r = 0 \\ 2 & r = 1, \\ 0 & r \geq 2 \end{cases} \quad C_r = \begin{cases} 1 & r = 0 \\ 0 & r \geq 1 \end{cases}$$

   Determine b.

   (ii) Solve the recurrence relation :

$$a_r - 7\,a_{r-1} + 10\,a_{r-2} = 0, \text{ given } a_0 = 0, a_1 = b.$$

7. (i) Let H be a normal subgroup of group G. Then the cosets of H in G form a group under coset multiplication :

$$(a\,H)\,(b\,H) = ab\,H.$$

   Prove it.

   (ii) Prove that a finite integral domain is a field.

8. Write short notes on any *three* of the following :

   (i) A relational model for data bases

   (ii) A pigeon hole principle

   (iii) Shortest path in weighted graph

   (iv) Lattice, distributive lattice, complement of an element

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