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**B. E. (Fourth Semester) EXAMINATION, June, 2004**

(Common for CS/EI/IT)

**DISCRETE STRUCTURE**

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 35

**Note :** Attempt any five questions. All questions carry equal marks.

1. (a) Show by induction, every third fibonacci number is even : 10

1, 1, 2, 3, 5, 8, 13, 21, 34 .....

Also explain induction.

- (b) Explain the principle of inclusion and exclusion. Find the number of integers between 1 and 250 that are divisible by any of integers 2, 3, 5 and 7. 10
2. (a) Explain partially ordered relation with example.  
Show that the set of integers which are divisors of 60 is partially ordered set. Also draw its Hasse diagram. 10
- (b) Explain Pigeon hole principle. Prove that among 1,00,000 people there are two who are born on same time ? 10

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3. (a) Construct the truth table for the following : 10

$$(i) (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$$

$$(ii) p \leftrightarrow (\bar{p} \vee \bar{q})$$

- (b) Design an FSM whose input sequence is a sequence  $(x_1, x_2, x_3, \dots)$  of 0's and 1's and whose output sequence  $(y_1, y_2, y_3, \dots)$  indicates the arrival of every other input 1 i. e. :

$$Y_n = \begin{cases} 1 \rightarrow \text{if } x_n = 1 \text{ and if there have been even number of input is so far} \\ 0 \rightarrow \text{in all other cases} \end{cases} \quad 10$$

4. (a) Consider the FSM in the given table. Find the equivalent classes and construct the reduced machine : 15

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
A	F	C	0	0
B	E	G	1	0
C	F	B	1	1
D	G	E	0	1
E	B	D	1	0
F	G	F	0	0
G	D	B	0	1
H	E	B	0	0

- (b) Explain Planar Graphs with example. 5

5. (a) Apply Dijkstra's algorithm to determine the shortest path between A to B : 15

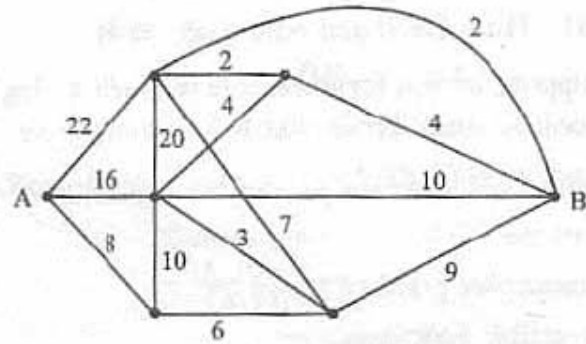


Fig. 1

- (b) Represent the given algebraic expression as a binary tree and then write the expression in reverse polish notation : 5

$$((A + B) * (C + D)) \div (((A - B) * C) + D)$$

6. (a) Find the generating function of the sequence  $Y_0, Y_1, Y_2, \dots$  defined as follows :

$$Y_0 = 0, Y_1 = 1, Y_n + 2Y_{n-1} - 15Y_{n-2} = 0 \quad \text{for } n \geq 2 \quad 10$$

- (b) Find the minimal spanning tree for the weighted graph by Kruskal algo. 10

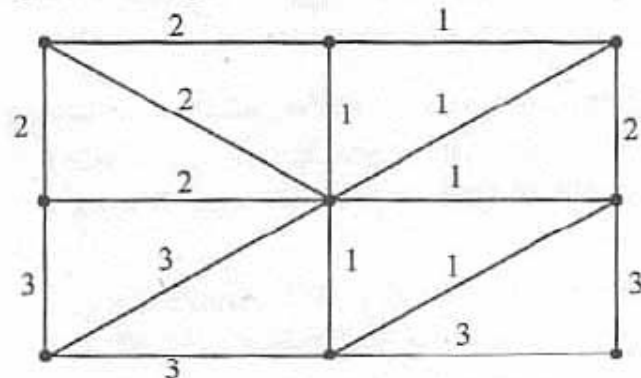


Fig. 2

7. (a) Let  $H$  be a subgroup of  $G$ . Prove : 19
- $H = Ha$  if and only if  $a \in H$
  - $Ha = Hb$  if and only if  $ab^{-1} \in H$
- (b) Suppose  $a^2 = a$  for every  $a \in R$  (such a ring is called Boolean ring). Prove that  $R$  is commutative. 10
8. Write short notes on any four of the following : 5 each
- Lattices
  - Isomorphic graphs
  - Invertible functions
  - Finite and infinite sets
  - Fields