b) Find the least Eigen value of $y'' + \lambda y = 0$, given $y'(0) = 0$, $y(1) = 0$.

8. a) Derive the finite element equation from one dimensional second order equation by variational approach.

b) Using Ritz's method, find the approximate solutions of the problem $y'' - y + 4xe^x = 0$, with $y'(0) - y(0) = 1$, $y'(1) + y(1) = -e$

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Total No. of Questions : 8] *[Total No. of Printed Pages : 4*

Roll No .....

MVSE-101

M.E./M.Tech., I Semester

Examination, June 2016

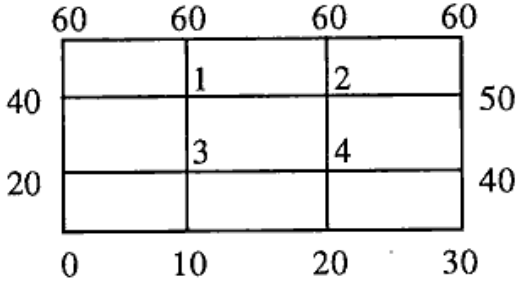
Advance Mathematics And Numerical Analysis

Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions.
ii) All questions carry equal marks.

1. a) Solve Laplace equation $\nabla^2 u = 0$ at mesh points:



with given conditions.

b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

2. a) Solve the boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0, t) = u(1, t) = 0$ and $u(x, 0) = \sin \pi x$, $0 \le x \le 1$ using $h = 0.2$ and $k = 0.02$.

b) Prove that Mellin transform of

i) $m\{x^a f(x)\} = \bar{f}(s+a)$

ii) $m\{f(ax)\} = \bar{a}^s \bar{f}(s)$

3. a) Find Fourier transform of $F(x)$ defined by

$$F(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

and hence evaluate $\int_0^\infty \frac{\sin s}{s} ds$.

b) Show that if n = 0, the Hankel transform of

$$H\left\{\frac{\sin ax}{x}\right\} = \begin{cases} 0, & \text{if } s > a \\ \frac{1}{\sqrt{a^2 - s^2}}, & \text{if } 0 < s < a \end{cases}$$

4. a) Show that the function $\phi(x) = xe^x$ is a solution of the volterra integral equation

$$\phi(x) = \sin x + 2\int_0^x \cos(x,t)\phi(t)\,dt$$

b) Transform the following boundary value problem into corresponding integral equation:

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - xy = \sin x,$$

given that $y = 1, y' = -1, y'' = \frac{1}{2}$ at $x = 0$.

5. a) Show that $y(x) = 1$ is a solution of the Fredholm integral equation

$$y(x) + \int_0^1 x\left(e^{tx} - 1\right) y(t)\,dt = e^x - x$$

b) Using literative method, solve the volterra integral equation $y(x) = 2\left(1 + x^2\right) - \int_0^x xy(t)\,dt$

6. a) Find the extremal of the functional

$$\int_1^2 \frac{\sqrt{1 + y'^2}}{x} dx,\ y(1) = 0,\ y(2) = 1$$

b) Solve $y'' + y = -x,\ 0 \le x < 1,\ y(0) = y(1) = 0$ by Galerkin method.

7. a) Find the extremal of

$$\int_{x_0}^{x_1} \left(16y^2 - (y'')^2 + x^2\right) dx$$