Total No. of Questions : 10] [Total No. of Printed Pages : 4

Roll No. .....

CS/IT–402(N)

B. E. (Fourth Semester) EXAMINATION, Dec., 2009

(New Scheme)

(Common for CS & IT Engg. Branch)

DISCRETE STRUCTURE

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 35

Note : Attempt any *one* question from each Unit. All questions carry equal marks.

Unit–I

1. (a) Write the principle of mathematical induction and prove by it that $n(n^2 + 5)$ is an integer multiple of 6 for all positive integers n. 10

   (b) Consider the following relation on $\{1,2,3,4,5,6\}$, $R = \{(i,j) : |i - j| = 2\}$: 10

   (i) Is 'R' is reflexive.

   (ii) Is 'R' is symmetric.

   (iii) Is 'R' is transitive.

   (iv) Draw a graph of R.

Or

2. (a) If A, B, C, D are any four sets then prove that : 10

$$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$$

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(b) If $f: X \to Y$ is one-one and onto, then prove that $f^{-1}: Y \to X$ is also one-one and onto. 10

Unit – II

3. (a) Let G be the set of the non-zero real numbers and let $a * b = \frac{ab}{2}$, then show that $(G, *)$ is an abelian group. 10

(b) Write short notes on the following : 5 each

(i) Homomorphism and isomorphism

(ii) Cosets

Or

4. (a) If H_1 and H_2 are normal subgroups of a group G, then prove that $H_1 \cap H_2$ is also a normal subgroup of G. 10

(b) Define field and prove that the set $F = \{0, 1, 2, ..., 6\}$ under addition and multiplication modulo 7 is a field. 10

Unit – III

5. (a) Show that the following language is not a finite state language : 10

$$L = \{1^i 0^j 1^{i+j} \mid i \geq 1, j \geq 1\}$$

(b) Prove that the following are tautologies or contradiction or contingency : 10

(i) $(p \to (q \wedge r)) \to (\sim r \to \sim q)$

(ii) $(p \vee q) \wedge \{p \vee (\sim q)\} \wedge \{(\sim p) \vee q\} \wedge \{(\sim p) \vee (\sim q)\}$

Or

6. (a) How a finite state machine can be designed to add positive integers of any size ? 10

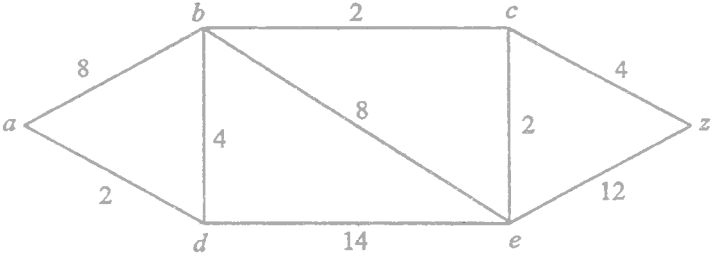
(b) Write the following in disjunctive normal form : 10

$$((x' \vee y') \wedge z) \vee (x' \wedge (x \vee z))$$

Unit – IV

7. (a) Explain Eulerian graph. Show that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. 10

(b) Find the shortest path between '*a*' to '*z*' in the graph shown below, where the numbers associated with the edges are the distance between vertices : 10



Or

8. (a) Write short notes on the following : 10

(i) Planer graphs

(ii) Isomorphism of graphs

(iii) Graph coloring

(iv) Chromatic number

(b) Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in V_1 and the other in V_2. 10

Unit – V

9. (a) Let L = { 1, 2, 3, 4, 6, 8, 9, 12, 18, 24 } be ordered by the relation '|' where *x*/*y* mean '*x* divides *y*'. Show

P. T. O.

that D_{24} the set of all divisors of the integer 24 of L is a sublattice of the lattice (L, |). 10

(b) Find the general solution of : 10

$$s(k) - 3s(k-1) - 4s(k-2) = 4^k$$

Or

10. (a) In how many ways can 12 students be partitioned into four teams A_1, A_2, A_3 and A_4, so that each team contains three students ? 10

(b) Determine the discrete numeric function corresponding to the following generating function : 10

$$A(z) = \frac{z^5}{5 - 6z + z^2}$$