Roll No .....

BE - 102

B.E. I & II Semester

Examination, June 2013

Engineering Mathematics-I

Time : Three Hours

Maximum Marks : 70

Note : 1. *Attempt five questions.*
2. *Select one question from each unit.*
3. *All questions carry equal marks.*

Unit - I

1. a) Prove that

$$\left(\sin^{-1} x\right)^2 = \frac{2}{2!}x^2 + \frac{2.2^2}{4!}x^4 + \frac{2.2^2.4^2}{6!}x^6 + ...$$

and hence deduce

$$\theta^2 = 2\frac{\sin^2\theta}{2!} + 2^2\frac{2\sin^4\theta}{4!} + 2^2.4^2\frac{2\sin^6\theta}{6!} + ...$$

b) If $u(x, y, z) = \log(\tan x + \tan y + \tan z)$, prove that

$$\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 2.$$

OR

2. a) Prove that if the perimeter of a triangle is constant, its area is maximum when the triangle is equilateral.

b) Determine the curvature of the parabola $y^2 = 2px$ at

(i) an arbitrary point (*x, y*).

(ii) the point $\left(\frac{p}{2}, p\right)$ and

(iii) the point (0,0).

Unit - II

3. a) Evaluate by expressing the limit of a sum in the form of a definite integral:

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)....\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$$

b) Define $B(m,n)$. Prove that

$$B(m,n)=B(m+1,n)+B(m,n+1)m,n>0.$$

OR

4. a) Evaluate the following integral by changing the order of integration:

$$\int_0^1\int_x^{\sqrt{2-x^2}}\frac{xdydx}{\sqrt{x^2+y^2}}$$

b) Find the volume cut from the sphere $x^2+y^2+z^2=a^2$ by the cylinder $x^2+y^2=ax$.

Unit - III

5. a) Solve $\left(3x^2y^4+2xy\right)dx+\left(2x^3y^3-x^2\right)dy=0$.

b) Solve $y-x=x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2$.

OR

6. a) Solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+13y=8e^{3x}\sin 4x+2^x$.

b) Solve $\frac{dx}{dt}+4x+3y=t$

$$\frac{dy}{dt}+2x+5y=e^t$$

Unit - IV

7. a) Define rank of a matrix. Find the rank of matrix A, where

$$A=\begin{bmatrix}1^2 & 2^2 & 3^2 & 4^2\\ 2^2 & 3^2 & 4^2 & 5^2\\ 3^2 & 4^2 & 5^2 & 6^2\\ 4^2 & 5^2 & 6^2 & 7^2\end{bmatrix}$$

b) Solve completely the system of equations $2w+3x-y-z=0, 4w-6x-2y+2z=0,$ $-6w+12x+3y-4z=0$.

OR

8. a) Determine the eigen values and eigen vectors of the matrix

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$

b) Show that Caley-Hamilton theorem is satisfied by the matrix A.

where $A=\begin{bmatrix}0 & 0 & 1\\ 3 & 1 & 0\\ -2 & 1 & 4\end{bmatrix}$.

Hence find A^{-1}.

Unit - V

9. a) Write the following function into disjunctive normal form of 3 variables *x,y,z*:
i) $x' + y'$ ii) $xy' + x'y$.

b) In a Boolean algebra B. Prove that the identity elements $0,1 \in B$ are unique and prove $0' = 1,1' = 0$.

OR

10. a) Define the following terms giving examples:
i) Support of fuzzy set.
ii) Complement of a fuzzy set.
iii) Union of two fuzzy sets.
iv) Intersection of two fuzzy sets.

b) Prove that the number of vertices of odd degree in a graph is always even.

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