M. E./M. Tech. (First Semester)

EXAMINATION, June, 2012

(Grading/Non-Grading)

ADVANCED MATHEMATICS AND NUMERICAL ANALYSIS

(MVSE/MVCT–101)
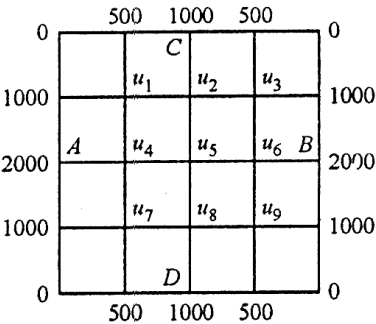
Time : Three Hours

Maximum Marks : $\begin{cases} GS : 70 \\ NGS : 100 \end{cases}$

Note : Attempt any *five* questions. All questions carry equal marks.

1. (a) Solve the elliptic equation :

$$u_{xx} + u_{yy} = 0$$

for the square mesh of the following fig. with boundary values as shown :

(b) Find the numerical solution of Poisson's equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

using finite difference method.

2. (a) Find the Hankel transform of $(a^2 - r^2)$, if $r J_0(P_r)$ is the kernel of the transform.

(b) Find the Fourier transform of :

$$f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

Hence evaluate :

$$\int_0^\infty \frac{\sin x}{x} \, dx.$$

3. (a) Convert the differentiate equation :

$$y''(x) - 3y'(x) + 2y(x) = 5 \sin x, \ y(0) = 1, \ y'(0) = -2$$

into an integral equation.

(b) Solve the Aber's integral equation :

$$\int_0^x \frac{y(t)}{\sqrt{x - t}} \, dt = 1 + 2x - x^2.$$

4. (a) Define Integro-differential equation and solve :

$$\frac{dy}{dx} + 3y + 2 \int_0^x y \, dx = x$$

given $y(0) = 1$.

(b) Solve by using the method of successive approximations, the integral equation :

$$y(x) = 1 + \lambda \int_0^1 xt \, y(t) \, dt.$$

5. (a) Solve the Euler's equation for :

$$\int_{x_0}^{x_1} (1 + x^2 y') \, y' \, dx.$$

(b) Prove that the shortest distance between two points is along a straight line.

6. (a) Find the extremal of :

$$\int_{x_0}^{x_1} (16y^2 - (y'')^2 + x^2) \, dx.$$

(b) Show that an isosceles triangle has the smallest perimeter for a given area and given base.

7. (a) Explain discretization in finite elements method.

(b) Use Rayleigh-Ritz method to solve the equation :

$$\frac{\partial^2 y}{\partial x^2} + y = x$$

$$y(0) = 0, \ y(1) = 1.$$

8. Write short notes on any *two* of the following :

(i) State and prove linear property of Fourier transform

(ii) Galerkin's method

(iii) Extremal

(iv) Mellin transform.