

Roll No

MEIC - 102**M.E./M.Tech., I Semester**

Examination, December 2015

Linear Control System**Time : Three Hours****Maximum Marks : 70**

- Note :** i) Attempt any five questions.
 ii) All questions carry equal marks.

1. a) Explain the concept and significance of Eigen values, Eigen vectors and generalized Eigen vectors.
 b) Derive the expression for state transition matrix of continuous and discrete time control systems.

2. a) Construct a state model for a system characterised by differential equation

$$\ddot{y} + 7\dot{y} + 5y + u = 0.$$

- b) Consider a system described by the state equation : (time variant)

$$\dot{X}(t) = A(t)X(t) + bu(t)$$

where,

$$A(t) = \begin{bmatrix} 1 & e^{-t} \\ 0 & -1 \end{bmatrix}; b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Is this system controllable at $t = 0$? If yes, find the minimum energy control to derive it from

$$x(0) = 0 \text{ to } X = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ at } t = 1.$$

3. a) Define controllability and observability. Explain both of them with the help of Kalman's test.
- b) Obtain the Jordan canonical form realizations for the following transfer function :

$$\frac{Y(z)}{R(z)} = \frac{3z^2 - 4z + 6}{\left(z - \frac{1}{3}\right)^3}.$$

4. a) State and explain Lyapunov's stability theorems.
- b) A linear autonomous system is described by

$$\dot{X} = AX$$

where,

$$A = \begin{bmatrix} -4k & 4k \\ 2k & -6k \end{bmatrix}$$

Find the restrictions on the parameter k to guarantee stability of the system.

5. a) What is state observer? Explain with block diagram.
- b) Investigate the controllability and observability of the system described by

$$X_{(K+1)} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} X_{(K)} + \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} U(K)$$

$$Y_{(K)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X_{(K)}$$

6. a) Find $X_1(t)$ and $X_2(t)$ of the system described by

$$\begin{bmatrix} \dot{x}_1 & (t) \\ \dot{x}_2 & (t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where the initial conditions are

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

- b) Obtain the state model of the system whose transfer function is given as

$$\frac{s^2 + 4s + 3}{s^3 + 9s + 20}$$

7. a) Explain stability, asymptotic stability and instability in the sense of Lyapunov.
- b) Evaluate the stability of the following system by direct method of Lyapunov

$$\dot{X} = \begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix} X.$$

8. Write short notes on any two of the following :

- a) Controllability in continuous and discrete time
- b) Stability of distributed parameter systems
- c) Generation of Lyapunov function
