b) Solve the boundary value problem $y''+y+x=0$, $(0 \le x \le 1)$, $y(0) = y(1) = 0$ by Rayleigh-Ritz method compare your solution with its exact solution.

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Total No. of Questions :8] *[Total No. of Printed Pages :4*

Roll No .....

MVCT/MBCT/MVCP/MVSE - 101(Old)

M.E./M.Tech., I Semester

Examination, June 2016

Advanced Mathematics

Time : Three Hours

Maximum Marks : 70

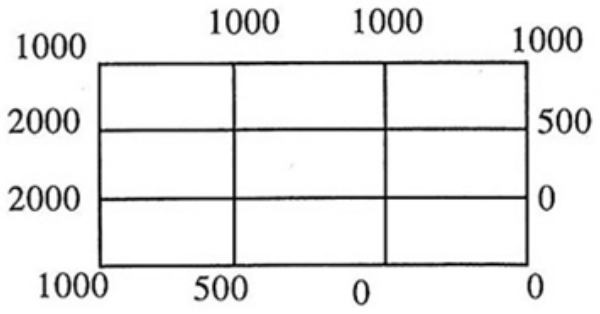
Note : Attempt any five questions. All questions carry equal marks.

1. a) Classify the following partial differential equations:

i) $$x\frac{\partial^2 u}{\partial x \partial y} - y\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial y}$$

ii) $$\left(1+x^2\right)\frac{\partial^2 u}{\partial x^2} + \left(5+2x^2\right)\frac{\partial^2 u}{\partial t \partial x} + \left(4+x^2\right)\frac{\partial^2 u}{\partial t^2} = 0$$

b) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown:



2. a) Solve the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0, y = 0, x = 3, y = 3$ with $x = 0$ on the boundary and mesh length = 1.

   b) Find the Hankel transform of $\frac{\cos ax}{x}$, taking Kernel of transform $x_0 J_0(px)$.

3. a) Prove that $M\{x^n f^n(x)\} = (-1)^n \frac{\lceil s+n}{\lceil s} \bar{f}(s)$ where $\bar{f}(s)$ is the mellin transform of $f(x)$.

   b) Using Fourier sine transform, solve the differential equation $\frac{\partial U}{\partial t} = k\frac{\partial^2 U}{\partial x^2}$, for $x > 0, t > 0$

   under the boundary conditions $U = U_0$

   when $x = 0, t > 0$ and the initial condition U = 0 when $t = 0, x > 0$.

4. a) Solve the integral equation

$$\int_0^\infty f(x)\cos 5x\,dx = \begin{cases} 1-s, & 0 \le s \le 1 \\ 0, & s > 0 \end{cases}.$$

   Hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$.

   b) Form an integral equation corresponding to the differential equation $\frac{d^3y}{dx^3} + x\frac{d^2y}{dx^2} + (x^2 - x)y = xe^x + 1$

   with initial conditions $y(0) = 1 = y'(0), y''(0) = 0$

5. a) Find the resolvent Kernel of the volterra integral equation with the Kernel $k(x, \varepsilon) = \frac{2 + \cos x}{2 + \cos \varepsilon}$

   b) Solve the integral equation

$$\phi(x) = (1 + x) + \lambda \int_0^x (x - \varepsilon)\phi(\varepsilon)\,d\varepsilon$$

6. a) Solve the Fredholm integral equation of the second kind by the method of successive approximation to the third order:

$$\phi(x) = 2x + \lambda \int_0^1 (x + \varepsilon)\phi(\varepsilon)\,d\varepsilon,\ \phi_0(x) = 1$$

   b) Solve the variational problem

$$\int_1^2 [x^2 y'^2 + 2y(x + y)]\,dx \text{ given } y(1) = y(2) = 0$$

7. a) Find the extremal of the functional

$$v(x) = \int_0^{\pi/2} \left(x_1'^2 + x_2'^2 + 2x_1 x_2 \right) dt$$

   b) Using Galerkin's method, solve the boundary value problem $y'' = 3x + 4y,\ y(0) = 0,\ y(1) = 1$

8. a) Derive the finite element equation from one dimensional second order equation by variational approach.