MCSE/MSE–101

M. E./M. Tech. (First Semester) EXAMINATION, June 2013

ADVANCED COMPUTATIONAL MATHEMATICS

Time : Three Hours

Maximum Marks : 70

Note : Attempt any *five* questions.

1. (a) (i) Let $T : R^3 \to R^2$ be a linear transformation defined by

$$T(x, y, z)\,\tau = \begin{pmatrix} y+z \\ y-x \end{pmatrix}.$$

Taking $\{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ as the basic in R^3, determine the matrix of T.

(ii) Let $A = \{\theta_1, \theta_2, \ldots, \theta_n\}$ be a finite set in a vector space V. Then show that either A is linearly independent or some UK is a linear combination of the preceeding vectors $\theta_1, \theta_2, \ldots, \theta_{k-1}$.

(b) Define Heaviside unit step function. Using this find

(i) $L(H(t-a))$ and $L\{H(t-a) - H(t-b)\}$

(ii) $L(t^3 - rt + 5 + 3\sin 2t)$.

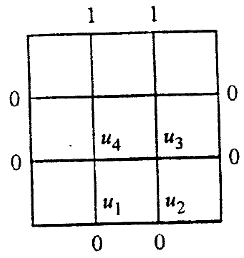
2. (a) Solve $\dfrac{\partial^2 z}{\partial x^2} + 4\dfrac{\partial^z}{\partial y^2} = 0$

using separation of variables method.

(b) (i) Solve the equation $u_{xx} + u_{yy} = 0$ in the domain of following fig

using Gaus-Seidel's method :

(upto 5 iterations)



(ii) Define Haar transform and give an example.

3. (a) (i) A bag contains 10 white and 15 black balls. Two balls are dra in succession without replacement. What is the probability t first is white and second is black ?

(ii) State the extension of multiplication theorem for probability of events of random nature.

(b) (i) Show that central moment of a Poisson distribution satisfy following recurrence relation :

$$\mu_{r+1} = \lambda\left(r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right).$$

(ii) A die is thrown repeatedly until an ace turns up. Find expected number of throws necessary.

4. (a) (i) Define Stochastic Process, Markov Process.

(ii) Consider the following Markov chain

$$P = \begin{pmatrix} 0{\cdot}2 & 0{\cdot}8 \\ 0{\cdot}6 & 0{\cdot}4 \end{pmatrix}$$

Determine $a^{(1)}, a^{(4)}$, given that $a^{(0)} = (0{\cdot}7, 0{\cdot}3)$.

(b) Customers at a box office window, being managed by a single m arrive according to a Poisson input process with a mean rate of 30 hour. The time required to serve a customer has an exponen distribution with a mean of 2 minutes. Find the average waiting ti of a customer.

5. (a) (i) Let $\tilde{A}(x) = \{(3, 0{\cdot}5), (5, 0{\cdot}4), (7, 0{\cdot}6)\}$

$\tilde{B}(x) = \{(3, 1), (5, 0{\cdot}6)\}$

Then find the fuzzy sets given by :

$$\tilde{A} \times \tilde{B}, \tilde{A} + \tilde{B}, \tilde{A} \oplus \tilde{B}, \tilde{A} - \tilde{B}.$$

(ii) Show that Yager's union operator satisfy

$$\mu_{\tilde{A} \cup \tilde{B}}(x) = \mu_{\tilde{A}(x)} \text{ for } \mu_{\tilde{B}}(x) = 0.$$

(b) Let $x = \{1, 2, 3, 4\}$ and "small integer" be defined $\tilde{A} = \{(1, 1), (2, 0{\cdot}5), (3, 0{\cdot}4), (4, 0{\cdot}2)\}$ and let the fuzzy relation "alm equal" be defined as :

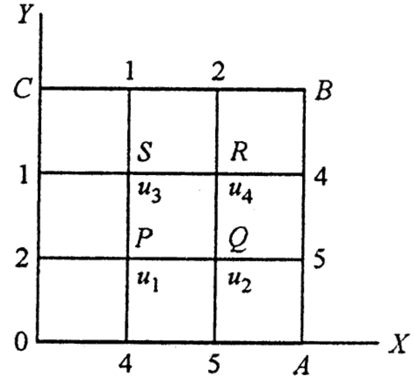
$$\tilde{R}: \quad \begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0.8 & 0 & 0 \\ 2 & 0.8 & 1 & 0.8 & 0 \\ 3 & 0 & 0.8 & 1 & 0.8 \\ 4 & 0 & 0 & 0.8 & 1 \\ \hline \end{array}$$

Find the membership of the fuzzy set B = "rather small integers".

6. (a) Use the method of separation of variables to solve the equation :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + 4, \text{ when } u(x, 0) = 6e^{-3x}.$$

(b) Solve Laplace's equation for the square region shown below the boundary values being indicated there



7. (a) Find the Fourier transform of

(i) $f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$

(ii) $p(x) = \dfrac{1}{x^2 + a^2}$.

(b) (i) Show that $F[f(ax)] = \dfrac{1}{a}F(w/a),\ a > 0$

(ii) A coin is tossed, then a die is thrown. Find the probability of obtaining a "6" given that head came up.

8. (a) Give different MATLAB functions and their applications.

(b) If $\lambda \in F$ is a characteristic root of TEA(V), then show that for any polynomial $q(x) \in F\{x\}$, $q(\lambda)$ is a characteristic root of $q(T)$.

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