Roll No ....

MVSE/MVCT-101

M.E./M.Tech. I Semester

Examination, December 2012

Advance Mathematics and Numerical Analysis

Time : Three Hours

Maximum Marks : 70

Note: Attempt any five questions. All questions carry equal marks.

1. a) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ subject to condition:

   $u(x, 0) = 6\, e^{-3x}$

   b) Find the numerical solution of Poisson's equation:

   $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$ using finite difference method.

2. a) Find the Fourier transform of

   $F(x) = \begin{Bmatrix} 1 - x^2 & if\ |x| < 1 \\ 0 & if\ |x| > 1 \end{Bmatrix}$ and use it to evaluate

   $\int_0^\infty \left(\frac{x\cos x - x\sin x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx$

   b) Find the Fourier Sine Transform of $\frac{e^{-ax}}{x}$

3. a) Find the Hankel Transform of $\frac{\cos ax}{x}$ taking $x J_0(sx)$ as the kernel.

   b) Define Hankel Transform and prove that :

   $H\{f(ax)\} = a^{-2} H\left(\frac{s}{a}\right)$

4. a) Define

   i) Functionals

   ii) Extremal

   b) Find the extremals of the functional

   $I\left[{}^{y}\!/_{x}\right] = \int_{x_0}^{x_1} \frac{1 + y^2}{y'^2}\,dx$

5. a) Solve the Euler's Equation for $\int_{x_0}^{x_1} (x + y')\, y'\,dx$.

   b) Prove that the shortest distance between two points is along a straight line.

6. a) Solve the boundary value problem :

   $y'' - y' + x = 0\ (0 \le x \le 1)\ y(0) = y(1) = 0$ by Rayleigh - Ritz method.

   b) Explain finite elements method for one dimensional problems considering suitable example.

7. a) Solve the integral equation:

$$y(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)\, y(t)\, dt$$

b) Find the integral equation corresponding to the boundary value problem.

$$y''(x) + \lambda y(x) = 0,\ y(0) = y(1) = 0$$

8. a) Using the method of successive approximation Volterra integral equation:

$$y(x) = 1 + x + \int_0^{x} (x-t)\, y(t)\, dt$$

b) Define :

i) Abel's integral equation

ii) Integro Differential equation

iii) Green function.

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