9. a) Solve the difference equation $a_r - 4a_{r-1} + 4a_{r-2} = 0$ and find the particular solution, given that $a_0=1$ and $a_1=6$. 7

b) Define the following :

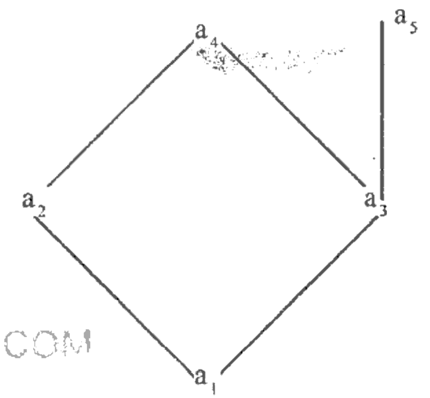
i) Complete lattice ii) Distributive lattice

iii) Complemented lattice iv) Bounded lattice. 7

OR

10. a) Find the total solution of the recurrence relation: $a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r, r \geq 2$ with boundary condition $a_0=1$ and $a_1=1$. 7

b) Consider the Hasse diagram of the Poset as shown in fig.



i) Determine the least and greatest element of Poset, if they exist.

ii) Determine L.V.B. of all pair of elements.

iii) Determine G.L.B. of all pair of elements. 7

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Total No. of Questions :10] [Total No. of Printed Pages :4

Roll No ....

CS/IT - 302

B.E. III Semester

Examination, December 2013

Discrete Structure

Time : Three Hours

Maximum Marks : 70

Note: 1. Attempt all questions.
2. All questions carry equal marks.

1. a) Prove that :

i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ 7

b) Prove by mathematical induction :

$$1^2 - 2^2 + 3^2 - \ldots\ldots + (-1)^{n-1} n^2 = \frac{(-1)^{n+1} n(n+1)}{2}$$ 7

OR

2. a) Among 100 students, 32 study Maths, 20 study Physics, 45 study Biology, 15 study Maths and Biology, 7 study Maths and Physics, 10 study Physics and Biology and 30 do not study any of the three subjects.

i) Find the number of students studying all the three subjects.

ii) Find the number of students studying exactly one of the three subjects. 7

b) Show that the relation

$R=\{(a,b)\,/\,a,b\in z \text{ and } a-b \text{ is divisible by } 3\}$

is on equivalence relation where z is set of all integer. 7

3. a) Define the following :

i) Semi group ii) Monoids iii) Sub group 7

b) Consider an algebraic system $(\theta,*)$, where θ is the set of rational numbers and $*$ is a binary operation defined by

$a*b=a+b-ab\ \forall\, a,b\in\theta$

Determine whether $(\theta,+)$ is a group. 7

OR

4. a) Consider a ring $(R,+,*)$ defined by $a*a=a$. Determine whether the ring is commutative or not. 7

b) Define normal subgroup and show that the intersection of two normal subgroups of a group is a normal subgroup. 7

5. a) Show that the following propositions are tautologies.

i) $(P\wedge Q)\vee(P\wedge \sim Q)\vee(\sim P\wedge Q)\vee(\sim P\wedge \sim Q)$

ii) $\{(P\vee \sim Q)\wedge(\sim P\vee \sim P)\}\vee Q$ 7

b) Show that the following language is not a finite state language:

$L=\{1^i\,0^j\,1^{i+j}\,/\,i\geq 1, j\geq 1\}$ 7

OR

6. a) Prove that the following statement is logically equivalent

$(p\vee q)\vee r\equiv(p\vee r)\Rightarrow(q\vee r)$. 7

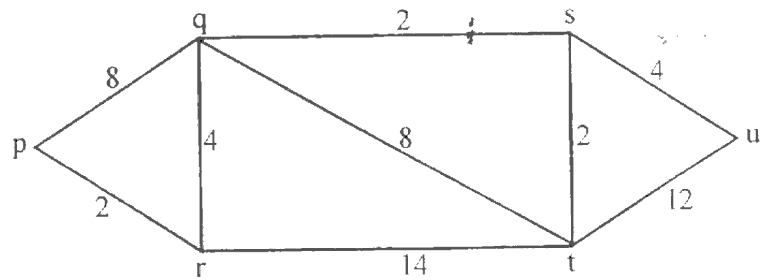
b) Determine the negation of the following statements.

i) $\exists_x\forall_y\left(p(x)\vee q(y)\right)$

ii) $\forall_x\exists_y\left(p(x,y)\rightarrow q(x,y)\right)$

iii) $\forall_x\forall_y\left(p(x)\wedge q(y)\right)$ 7

7. a) Using Dijkstra's algorithm find the shortest path between p to u in the following weighted graph: 7



b) Define the following :

i) Eulerian path ii) Hamiltonian circuit. 7

OR

8. a) Show that the maximum number of edges in a simple graph with n vertices is n(n-1)/2. 7

b) Draw the undirected graph for its incidence matrix given below. 7

$M_1=$

	e_1	e_2	e_3	e_4	e_5
v_1	1	0	0	1	0
v_2	0	1	1	0	0
v_3	1	1	0	0	1