

Advance Mathematics and Numerical Analysis

Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions out of eight.

ii) All questions carry equal marks.

1. Find the solution of the parabolic equation $u_{xx} = 2ut$, when $u(0,t) = u(4,t) = 0$ and $u(x,0) = x(4-x)$, taking $h = 1$ Find the values upto $t = 5$.
2. Evaluate the pirotal values of the equation $u_{tt} = 16u_{xx}$ taking $Dx = 1$ upto $t = 1.25$. The boundary conditions are $u(0,t) = u(5,t) = 0$, $u_t(x,0) = 0$ and $u(x,0) = x^2(5-x)$.

3. a) Find the Hankel transform of $\frac{e^{-ax}}{x}$ taking $x J_0(px)$ as the Kernel of the transform.
- b) Find Mellin transform of $\sin x$.

4. a) Find Fourier sine transform of $f(n) = \frac{1}{x}$.
- b) Find Fourier cosine transform of $f(x) = e^{-x}$.
5. a) Show that the function $y = xe^x$ is a solution of the volterra integral equation

$$y(x) = \sin x + 2 \int_0^x \cos(x-t)y(t)dt$$
- b) Form an integral equation corresponding to the differential equation $y'' + xy' + y = 0$ with the initial condition $y(0) = 1$; $y'(0) = 1$.

6. Test for the extremum of the functional

$$I[y(x)] = \int_0^{\pi/2} (y'^2 - y^2)dx; y(0) = 0; y(\pi/2) = 1$$

7. Solve the boundary value problem

$$y'' - y + x = 0, (0 \leq x \leq 1), y(0) = 0, y(1) = 0 \text{ by Rayleigh-Ritz method.}$$

8. Write a short note on the followings:

- a) Green's function
- b) Integro differential equation
- c) Finite element method
- d) Functional