

CS/EI/IT-405**B. E. (Fourth Semester) EXAMINATION, Dec., 2005**

(Common for CS, EI & IT Engg.)

DISCRETE STRUCTURE*Time : Three Hours**Maximum Marks : 100**Minimum Pass Marks : 35***Note :** Attempt any five questions. All questions carry equal marks.

1. (a) Let A, B and C be any three sets, then prove that : 10

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$
- (b) If the function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = \cos x$ and the function $g: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $g(x) = x^3$, find $(g \circ f)(x)$ and $(f \circ g)(x)$ and prove that they are not equal. 10
2. (a) Prove by mathematical induction that $n^2 + n$ is an even number for all natural numbers $n \geq 1$. 10
- (b) Show that the relation : 10
 $R = \{(a, b) | a, b \in \mathbb{Z} \text{ and } a - b \text{ is divisible by } 3\}$
 is an equivalence relation, where \mathbb{Z} is the set of all integers.

P. T. O.

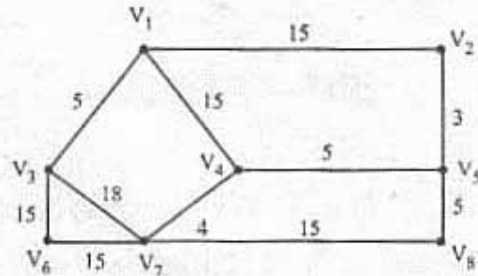
3. (a) Show that $(p \vee q) \wedge (\neg p \wedge (\neg p \wedge q))$ is equivalent to $\neg p \vee \neg p$. 6
- (b) Test the validity of the following argument : 7
 If I get the job and work hard, then I will get promoted.
 If I get promoted then I will be happy.
 I will not be happy.
 Therefore either I will not get the job or I will not work hard.
- (c) Obtain disjunctive normal form of : 7

$$p \vee (\neg p \Rightarrow (q \vee (q \Rightarrow \neg r)))$$
4. (a) For the finite state machine shown below, find all equivalent states and obtain an equivalent finite state machine with the smallest number of states : 10

State	Input		Output
	0	1	
\Rightarrow A	F	B	0
B	D	C	0
C	G	B	0
D	E	A	1
E	D	A	0
F	A	G	1
G	C	H	1
H	A	H	1

- (b) Construct a finite-state acceptor that will accept the set of natural numbers x which are divisible by 3. 10
5. (a) Write and explain an algorithm to find the shortest path from a specified vertex to another specified vertex of a graph. 10

- (b) Find the minimum weight spanning tree for the graph shown in fig. using Kruskal's algorithm. 10



6. (a) Solve the recurrence relation : 10

$$a_r - 6a_{r-1} + 8a_{r-2} = r4^r$$

where $a_0 = 8$ and $a_1 = 22$

- (b) Prove that any two right cosets of a subgroup H of a group G are either disjoint or identical. 10
7. (a) Write the generating function for the sequence $\{a_r\}_{r=0}^{\infty}$ defined by : 5 each
- $a_r = (r+1)3^r$
 - $a_r = 5^r + (-1)^r 3^r + 8^r + {}^3C_r$
- (b) Define normal subgroup and show that the intersection of two normal subgroups of a group is a normal subgroup. 10
8. (a) Define field and show that the set of real numbers of the form $a + b\sqrt{3}$ where a and b are rational numbers, is a field with respect to addition and multiplication. 10
- (b) Write short notes on the following : 5 each
- Group codes
 - Homomorphism