## M. E./M. Tech. (First Semester)

## EXAMINATION, June, 2012

(Grading/Non-Grading)

# ADVANCED COMPUTATIONAL MATHEMATICS

(MEIC/MEDC/MEMT/MEPS/MEPE/MEVD–101)

*Time : Three Hours*

*Maximum Marks* : $\begin{cases} GS : 70 \\ NGS : 100 \end{cases}$

**Note :** Attempt any *five* questions. All questions carry equal marks.

**1.** (a) Solve the wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ where :

$$u = \begin{cases} p_0 \cos pt & ; \; x = t \\ 0 & ; \; x = 0 \end{cases}$$

where $p_0$ is a constant.

(b) Solve the Poisson's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10\,(x^2 + y^2 + 10)$$

over the square with sides $x = 0, y = 0, x = 3 = y$ with $u(x, 0) = 0$ on the boundary and mesh length = 1.

**2.** (a) Find the Fourier cosine series expansion of the periodic function defined by :

$$f(t) = \frac{\sin(\pi t)}{l}$$

where $0 < t < l$.

(b) Express the function $f(x) = \begin{Bmatrix} 1 & ; & |x| \le 1 \\ 0 & ; & |x| > 1 \end{Bmatrix}$ as a Fourier integral and hence evaluate the following integral :

$$\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda}\, d\lambda.$$

**3.** (a) In a city $A$, 20% of a random sample of 900 school boys had a certain slight physical defect. In another city $B$, 18·5% of a random sample of 1600 school boys had the same defect. Is the difference between the properties significant ?

(b) Define Binomial distribution and discuss the properties of binomial distribution.

**4.** (a) Show that normal distribution as the limiting case of Binomial distribution when $p = q$.

(b) Assuming that the diameter of 1000 brass plugs taken consecutively from a machine from a normal distribution with mean 0·7515 and standard deviation 0·0020 cm, how many of the plugs are likely to be rejected if the approved diameter is $0{\cdot}752 \pm 0{\cdot}004$ cm ?

**5.** (a) Explain Markov chain. Draw transition diagram and write down the properties of Markov chain.

(b) The number of units of an item that are withdrawn from inventory on a day to day basis is a Markov chain process in which requirements for tomorrow depend on today's requirements. A one-day transition matrix is given below : number of units withdrawn from inventory :

$$\begin{array}{cc} & \text{Tomorrow} \\ & \begin{array}{ccc} 5 & 10 & 12 \end{array} \\ \text{Today} \begin{array}{c} 5 \\ 10 \\ 12 \end{array} & \begin{bmatrix} 0{\cdot}6 & 0{\cdot}4 & 0{\cdot}0 \\ 0{\cdot}3 & 0{\cdot}3 & 0{\cdot}4 \\ 0{\cdot}1 & 0{\cdot}3 & 0{\cdot}6 \end{bmatrix} \end{array}$$

Construct a tree diagram showing inventory requirements on two consecutive days. Develop a two-day transition matrix.

**6.** (a) In a railway marshalling yard, goods trains arrive at rate of 30 trains per day. Assuming that the inter-arrival time follows a exponential distribution and the service time distribution is also exponential with an average 36 minutes. Then calculate :

(i) the mean queue size

(ii) the probability that the queue size exceeds.

If the input of trains increases to an average 33 per day, what will be the change in (i) and (ii) ?

(b) Obtain the steady state equation for the queuiung model :

(M/M/1) : ($\infty$/FCFS)

**7.** (a) Let $A$ and $B$ be fuzzy sets defined on a universal set $X$. Then prove that :

$$|A| + |B| = |A \cup B| + |A \cap B|$$

where $\cup$ and $\cap$ are the standard union and intersection respectively.

(b) How fuzzy tool box works ? Explain different functions which MATLAB provides in fuzzy tools box.

**8.** (a) Define failure rates and failure rate distribution.

(b) The density function of the time to failure of an appliance is :

$$f(t) = \frac{4}{(t+2)^3}\,; \quad t > 0$$

is in years find :

(i) the reliability function $R(t)$

(ii) the failure rate $\lambda(t)$

(iii) the MTTF.

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