Total No. of Questions : 8] [Total No. of Printed Pages : 4

CS–4507

B. E. (Fourth Semester) EXAMINATION, June, 2002

(Computer Science & Engg.)

DISCRETE STRUCTURES

(CS–4507)

Time : Three Hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : Attempt any *five* questions. All questions carry equal marks.

1. (a) Let R be a binary relation on the set of non-negative integers such that :

$$R = \{(a, b) \mid 2 \text{ divides } (a - b)\}$$

Is R an equivalence relation ? 5

(b) Explain Pigeon hole principle. 4

(c) By principle of mathematical induction prove that $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$ is divisible by 25. 7

2. (a) Let $A = \{a, b, c\}$ and P (A) its power set. Draw Hasse diagram of $(P(A), \subseteq)$. 7

(b) Explain any *three* of the following with examples : 3, 3, 3

(i) Finite and Infinite sets

(ii) Countable and uncountable sets

(iii) Principles of Inclusion and Exclusion

(iv) Multisets

(v) Lattice

3. (a) Write following statements in terms of p, q and r and logical connectives :

(i) If I am not in good mood then I will go to movie.

(ii) I will not go to a movie, and I will study discrete structure.

(iii) I will go to a movie only if I will not study discrete structure.

(iv) If I will not study discrete structure then I am not in a good mood. 4

(b) State whether the argument given below is valid or not valid ? If it is valid, identify the tautology or tautologies used.

If I drive to work, then I will arrive tired.

I drive to work.

$\therefore$ I will arrive tired. 6

(c) Define the following terms with examples : 2 each

(i) Absurdity

(ii) Tautology

(iii) Contingency

4. (a) Show that the language :

$$L = \{a^k b^k \mid k \geq 1\}$$

is not a finite state language. 6

(b) Show that following two machines are equivalent : 10

	State	Input 1	Input 2	Output
⇒	A	B	C	0
	B	F	D	0
	C	G	E	0
	D	H	B	0
	E	B	F	1
	F	D	H	0
	G	E	B	0
	H	B	C	1

(i)

	State	Input 1	Input 2	Output
⇒	A	B	C	0
	B	C	D	0
	C	D	E	0
	D	E	B	0
	E	B	C	1

(ii)

5. (a) Explain any *two* of the following with examples : 6
   (i) Multigraph and weighted graphs
   (ii) Hamilton path and circuit
   (iii) Spanning tree and cut set

   (b) Write an algorithm for shortest path in weighted graph and use it to find shortest path from a to z in the graph shown in fig. 1 where the number associated with edges are the weights : 4, 6

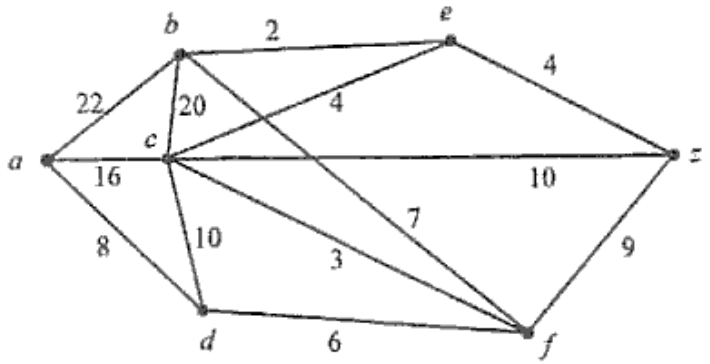

Fig. 1

6. (a) Write an algorithm to find minimum spanning tree and use it to determine a minimum spanning tree for the graph shown in fig. 2 : 4, 6

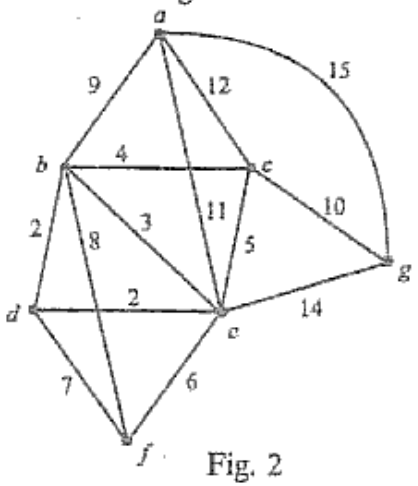

Fig. 2

   (b) Detemine the discrete numeric function corresponding to the following generating function : 6

$$A(z) = \frac{z^5}{5 - 6z + z^2}$$

7. (a) Solve the following recurrence relation :

$$a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r,\ r \geq 2$$

with boundary condition $a_0 = 1$ and $a_1 = 1$. 8

   (b) Write short notes on any *two* of the following : 4 each
   (i) Homomorphism
   (ii) Codes and Group codes
   (iii) Rings

8. (a) Prove that any two right cosets of a subgroup are either identical or disjoint. 6
   (b) Prove that every field is an integral domain. 6
   (c) Prove that every cyclic group is an abelian group. 4