BE - 301

B.E. III Semester

Examination, December 2012

Engineering Mathematics - II

(Common for all Branches)

Time : Three Hours

Maximum Marks : 70/100

Note: Attempt one question from each unit. All questions carry equal marks.

Unit - I

1. a) Expand $f(x) = x \sin x,\ 0 \le x \le 2\pi$ in a Fourier series.

   b) Find the Fourier transform of the function

$$f(t)=\begin{cases}1-\frac{|t|}{a}, & for\ |t|\le a\\ a & otherwise\end{cases}$$

OR

2. a) Determine half range sine series for the function f defined by $f(x) = x^2+x,\ 0 \le x \le \pi$.

   b) Find Fourier cosine and sine transform of the function f defined by $f(t)=e^{-at}$, a is a constant.

   Deduce the value of $\int_0^\infty \frac{\cos nx}{a^2+x^2}dx$.

Unit - II

3. a) Find Laplace transforms of the following functions

   i) $\frac{e^{-t}\sin t}{t}$ ii) $\sin at \sin bt$

   b) Using convolution theorem find the inverse Laplace transform of $\left(\frac{1}{s^2+a^2}\right)^2$.

OR

4. a) Find inverse Laplace transform of the following functions:

   i) $\tan^{-1}\left(1/\beta\right)$ ii) $\log\left(\frac{\beta+3}{\beta+2}\right)$

   b) Solve the ordinary differential equation using Laplace transform method $(D^4+2D^2+1)\,y=0$.

   where $y(0)=0, y'(0)=1, y''(0)=2$ and $y'''(0)=-3$.

Unit - III

5. a) Solve the differential equation

$$\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}-(1-\cot x)\,y=e^x\sin x$$

   given that $y=e^x$ is a past of its complementary function.

   b) Find the power series solution of the differential equation.

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$$

about $x=0$.

OR

6. a) Solve the differential equation

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=x^5$$

by changing the independent variable.

b) Solve the differential equation $(D^2+a^2)y=\tan ax$ by the method of variation of parameters.

Unit - IV

7. a) Solve the following differential equations

i) $p(1+q)=qz$ ii) $x^2p^2+y^2q^2=z^2$

b) Solve by Charpit's method $(p^2+q^2)y=qz$.

OR

8. a) Solve the pastial differential equation

$$\frac{\partial^3 z}{\partial x^3}-\frac{\partial^3 z}{\partial y^3}=x^3y^3$$

b) Find the solution of the wave equation

$$\frac{\partial^2 y}{\partial t^2}=C^2\frac{\partial^2 y}{\partial x^2}$$

such that $y=p_0\cos pt$ (p_0 is a constant) when $x=l$ anc $y=0$ when $x=0$.

Unit - V

9. a) Prove that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$.

b) Apply stoke's theorem to evaluate

$$\int_C\left[(x+y)dx+(2x-z)dy+(y+z)dz\right],$$

Where C is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6).

OR

10. a) Prove that

$$\left(y^2-z^2+3yz-2x\right)\hat{i}+(3xz+2xy)\hat{j}+(3xy-2xz+2z)\hat{k}$$

is both solenoidal and irrotational.

b) Using Gauss's divergence theorem evaluate $\iint_S \vec{f}.d\vec{s}$ where $\vec{f}=yz\hat{i}+2y^2\hat{j}+xz^2\hat{k}$ and S is the surface of cylinder $x^2+y^2=g$ contained in the first octant between the planes $z=0$ and $z=2$.