Total No. of Questions : 8] Total No. of Printed Pages : 2

Roll No .....

MA-110

B.E. (All Branches), First Semester

Examination, December 2016

Choice Based Credit System (CBCS)

Mathematics - I

Time : Three Hours

Maximum Marks: 60

Note: i) Attempt any five out of eight questions.
ii) All questions carry equal marks.

1. a) Verify the Lagrange's mean value theorem for the function $f(x) = x^2 - 2x + 4$ in the interval [1, 5].

   b) Using integration by parts, evaluate $\int x.\tan^{-1} x\,dx$

2. a) Define tangent line of a curve. Find equation of the tangent line at point (3, 1) on the curve $4x^2 + 9y^2 = 45$

   b) Find the Taylor series expansion of log cos x about the point $x = 0$.

3. a) If $u = \log_e\left(\frac{x^4 + y^4}{x + y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.

   b) Discuss maxima and minima of the function $f(x, y) = x^3 - 4xy + 2y^2$.

4. a) Find radius of curvature at point 't' of the curve $x = at^2$, $y = 2at$.

   b) Evaluate $\int_a^b e^x dx$ from the definition of integral as limit of sum.

5. a) Evaluate $\lim_{n\to\infty}\left[\frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots + \frac{1}{2n}\right]$

   b) Evaluate $\int_1^2 \int_1^3 xy^2 dx\,dy$

6. a) Evaluate $\int_{-1}^{1} \int_0^z \int_{x-z}^{x+z} (x + y + z)\,dx\,dy\,dz$

   b) Change the order of integration in $\int_0^2 \int_0^{2-x} xy\,dx\,dy$ and hence evaluate it.

7. a) Express the integral $\int_0^1 x^3\left(1 - x^2\right)^4 dx$ in terms of gamma function and hence evaluate.

   b) Prove that $\Gamma n\Gamma(1-n) = \frac{\pi}{\sin n\pi}$, where $0 < n < 1$. From this result deduce the value of $\Gamma\left(\frac{1}{2}\right)$

8. a) Using triple integral, find volume of sphere $x^2 + y^2 + z^2 = a^2$.

   b) Trace the curve $ay^2 = x^3$

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