Total No. of Questions : 5] [Total No. of Printed Pages : 3

Roll No. 0502MC101111

BE-102(GS)

B. E. (First/Second Semester) EXAMINATION, June, 2011

(Common for all Branches)

ENGINEERING MATHEMATICS – I

Time : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 22 (D Grade)

Note : Attempt all questions. All questions carry equal marks. Internal choice are also given.

1. (a) Find the Taylor's series expansion of the function about the point $\pi/3$:

$$f(x) = \log \cos x$$

(b) Find the maximum value of u, where :

$$u = \sin x \sin y \sin (x + y)$$

Or

(a) If :

$$z(x + y) = (x^2 + y^2)$$

show that :

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

(b) Show that the radius of curvature at any point on the cardioid $r = a(1 - \cos\theta)$ is $2/3\sqrt{2ar}$.

P. T. O.

2. (a) Evaluate :

$$\lim_{n \to \infty} \left\{ \left(1 + \frac{1}{n^2}\right) \left(1 + \frac{2^2}{n^2}\right) \left(1 + \frac{3^2}{n^2}\right) \ldots\ldots \left(1 + \frac{n^2}{n^2}\right) \right\}^{1/n}$$

(b) Change the order of integration in :

$$I = \int_0^{2a} \int_{\sqrt{2ax - x^2}}^{\sqrt{2ax}} V \, dx \, dy$$

Or

(a) Find the volume bounded by the paraboloid $x^2 + y^2 = az$, the cylinder $x^2 + y^2 = 2ay$ and the plane $z = 0$.

(b) Prove that :

$$\int_0^b (x - a)^{m-1} (b - x)^{n-1} dx = (b - a)\, x^{m+n-1} \beta (m, n)$$

3. (a) Solve the following differential equations :

(i) $x\,dy - y\,dx + 2x^3 dx = 0$

(ii) $y = 2px + y^2 p^3$

(b) Solve :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + 3e^x + \sin 2x$$

Or

(a) Solve :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x$$

(b) Solve the following simultaneous equations :

$$\frac{dx}{dy} = 2x + 6y \; ; \; \frac{dy}{dt} = x + y$$

4. (a) Find the rank of the matrix :

$$A = \begin{bmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{bmatrix}$$

(b) Find the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

Or

(a) Test for consistency and solve the following linear equations :

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

(b) Find the characteristic equation of the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

5. (a) Show that the following proposition is tautology :

$$\{(p \vee \sim q) \wedge (\sim p \vee \sim q)\} \vee q$$

(b) In a Boolean algebra [B, +, . , '], prove that :

$$(xy' + yz)(xz + yz') = (xy' + y)(xy' + z)(xz + y)(xz + z')$$

Or

(a) Draw the circuit for the following Boolean function and replace by simpler one :

$$F(x, y, z) = xz + y(y + z)(x + y)z$$

(b) Define the following :
Degree of vertex, Spanning tree, Fuzzy proposition, Euler graph.