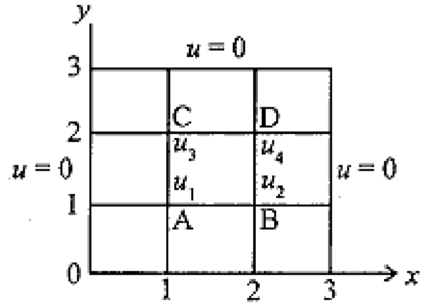
b) Solve the Poisson equation

$u_{xx}+u_{yy}=-10\,(x^2+y^2+10)$

in the domain mentioned below:



7\. a) Find Fourier transform of

i) $f(x)=\begin{cases}1 & 0<x<a\\ 0 & otherwise\end{cases}$

ii) Sine-transform of $f(x)=e^{-3x}$

b) i) Show that $F(f(x-a))=e^{-iwa}F(w)$

ii) If $P(A)=\dfrac{3}{8}$, $P(B)=\dfrac{1}{2}$ and $P(A\cap B)=\dfrac{1}{4}$, then find $P(\bar{A}/\bar{B})$.

8\. a) Write short notes on:

i) Hash function

ii) Hermite polynomial

iii) Modular mathematics.

b) If $\lambda_1,\lambda_2,\ldots\ldots,\lambda_K$ in Fare distinct characteristic roots of $T\in A(V)$ and $v_1,v_2,\ldots,v_K$ are characteristic vectors of T belonging to $\lambda_1,\lambda_2,\ldots\ldots,\lambda_K$ respectively, then show that $v_1,v_2,\ldots,v_K$ are LI over F.

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Total No. of Questions :8] *[Total No. of Printed Pages :4*

Roll No ....

MCSE/MSE - 101

M.E/M.Tech., I Semester

Examination, December 2013

Advanced Computational Mathematics

Time : Three Hours

Maximum Marks : 70

Note : Attempt any five questions.

1\. a) i) Let $x_1=(1,3)^T$, $x_2=(4,6)^T$ be a basis of R^2. Find a linear transformation $T:\mathbb{R}^2\to\mathbb{R}^3$ such that

$Tx_1=(-2,2,-7)^T, Tx_2=(-2,-4,-10)^T$

ii) If $v_1,v_2,\ldots\ldots\ldots,v_n$ are linearly independent in a vector space V over F. Then show that every element in their linear span has a unique representation in the form $\lambda_1 v_1+\lambda_2 v_2+\ldots\ldots+\lambda_n v_n$ with $\lambda_i\in F$.

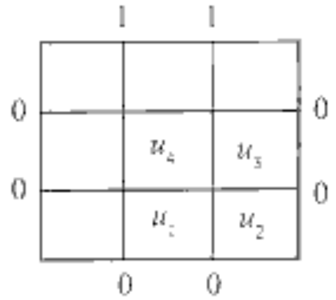
b) Find

i) The Fourier transform of $f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a\end{cases}$

ii) Using the Heaviside unit step function find the convolution $(f*f)\,(x)$ and hence find the Fourier transform of $(f*f)\,(x)$, where f is same as in (i) above.

2\. a) Solve $\dfrac{\partial^2 z}{\partial x^2}-2\dfrac{\partial z}{\partial x}+\dfrac{\partial z}{\partial y}=0$ using separation of variables method.

b) i) Solve the equation $u_{xx}+u_{yy}=0$ in the domain of following figure by using Jacobi's method upto 6 iterations.



ii) Define Haar wavelet and justify with an example.

3. a) i) If A and B are two events of a random experiment, then show that $P(\bar{A}/B)=1-P(A/B)$

ii) Two integers are selected at random from integers 1 through 11. If the sum is even, find the probability that both the numbers are odd.

b) i) Define normal distribution and give an example.

ii) If $X_1, \ldots\ldots, X_n$ are independent normal variables, X_i being $N(\mu_i, \sigma_i)$, then show that the variate

$Y=\sum_{i=1}^{n} a_i X_i$ is an $N(\mu', \sigma')$ random variable with

$\mu'=\sum_{i=1}^{n} a_i \mu_i, \sigma'=\sqrt{\sum_{i=1}^{n} a_i^2 \sigma_i^2}$.

4. a) i) What do you mean by steady state probability?

ii) Find the steady state probability for the following Markov chain:

$$P = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1/2 & 1/2 \end{pmatrix} \end{array}$$

b) A computer repairman finds that the time spent on his jobs has an exponential distribution with a mean of 30 minutes. If he repairs sets on the first-come-first-served basis and if the arrival of sets is with an average rate of 10 per 8-hours a day, find the repairman's expected idle time each day. Also find average number of units in the system.

5. a) i) Describe different averaging operators on fuzzy sets.

ii) Prove that the Yager union operator satisfies:

$\mu_{\tilde{A}\cup\tilde{B}}(x)=1 \; for \; \mu_{\tilde{B}}(x)=1$

$\mu_{\tilde{A}\cup\tilde{B}}(x)\geq \mu_{\tilde{A}}(x) \; for \; \mu_{\tilde{A}}(x)=\mu_{\tilde{B}}(x)$

b) Consider the linguistic variable "Age" and let the term "old" be defined as

$$\mu_{old}(x)=\begin{cases} 0 & if\ x\in[0,40] \\ \left(1+\dfrac{x-40}{5}\right)^{-1} & if\ x\in[40,100] \end{cases}$$

Determine the membership functions of the terms "very old", "not very old" and "more or less old".

6. a) Use the method of separation of variables to solve the equation :

$4\dfrac{\partial z}{\partial x}+\dfrac{\partial z}{\partial y}=3z$, when $x=0$, $z=3e^{-y}-e^{-5y}$.