MMPD/MMCM/MMMD/MMTP/MMIE–101

M. E./M. Tech. (First Semester)

EXAMINATION, June 2013

ADVANCED MATHEMATICS

Time : Three Hours

Maximum Marks : 70

Note : Attempt any *five* questions.

All the questions carry equal marks.

1. (a) Show that the mapping $f: V_3(\mathbf{R}) \to V_3(\mathbf{R})$ defined as below :

$$f(a, b, c) = (c, a+b) \text{ is linear.}$$

(b) Let W be the set of vectors of the form $(x, 2x, -3x, x)$ then prove that W is a sub-space of $V_4(f)$.

2. (a) What is Modular mathematics and solve

$$y = 2x + 3 \pmod 8 \text{ if } x = 4.$$

(b) Express $H(x) = x^4 + 2x^3 + 2x^2 - x - 3$, in terms of Hermite polynomials.

3. (a) Solve $u \frac{\partial 4}{\partial x} + \frac{\partial 4}{\partial y} = 3u$ subject to the condition that $u(0, y) = 3e^{-y} - 5e^{-5y}$.

(b) Solve $4_{xx} + 4_{yy} = 0$ in $0 \le x \le 4$, $0 \le y \le 4$ given that $u(0, y) = 0$, $y = (4, y) = 8 + 2y$,

$u(x, 0) = \frac{x^2}{2}$ and $u(x, y) = x^2$ take $h = k = 1$ and obtain the result correct to one decimal place.

4. (a) Solve the Poisson's equation :

$u_{xx} + u_{yy} = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0$, $y = 0$, $x = y = 3$ with $u = 0$ on the boundary and mesh length 1.

(b) Define following :

(i) FT (ii) DFT.

5. (a) Discuss and criticize the following :

$P(A) = \frac{2}{3}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{6}$, for the probabilities of three mutually exclusive events A, B, C.

(b) The probability function of a discrete random variable is as follows :

$X = x$	$P(x) = P(X = x)$
0	0
1	K
2	$2K$
3	$2K$
4	$3K$
5	K^2
6	$2K^2$
7	$K^2 + K$

Find (i) K (ii) $P(X < b)$ and $P(X \ge b)$.

6. (a) In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target.

(b) A book of 500 pages contains 500 misprints, estimate the probability that a given page contains at least three misprints.

7. (a) Define stochastic process and Markov Process.

(b) A system can be one of two possible states. Initially the chance is the same for each state and at each transition the stochastic matrix P is as follows :

i/j	1	2
1	1/3	2/3
2	1/2	1/2

What are the probabilities for the two states after three steps ?

8. (a) Write a note on finite elements method for one dimensional problem.

(b) State and prove Euler-Lagrange's equation of motion.