**RGPV SOLUTION MA-111 ENGINEERING MATHEMATICS 2 DEC-2017**

1. (a) Find rank and nullity of the following matrix by reducing it to the normal form



Solution: Given the matrix is



Applying, 



Applying, 



Applying, 



Applying, 



Applying, 



Applying, 



Clearly this is required normal form and rank  **Answer**

Nullity N(A) = order of square matrix-Rank of Matrix

N(A) = 3 – 3 = 0 **Answer**

**(b) Examine whether the following equations are consistent and solve them if they are consistent.**







Solution : Given the system of equation is



⇒ 

This is Homogeneous system of equation, and then augmented matrix is



Applying, 



Applying, 



Applying, 



Applying, 



Clearly  and ⇒ (No. of unknown variables)

 The system is consistent and having infinite many solutions.







Taking, z = k, then we get x = k and y = k

Hence the required solution is

 and  **Answer**

**2. (a) Find Eigen values and Eigen vectors of the matrix**



**Solution :** The characteristic equation is



⇒ 

⇒ 

⇒ 

⇒ 

⇒ 

⇒ 

⇒ 

⇒ 

⇒ 

**Case 1 :** Suppose  be Eigen vector corresponding to Eigen value  then



⇒ 

⇒  

Since all rows are non identical then taking and  rows as

Taking, 



 

⇒ 

i.e., 

 

Case 2 : Suppose  be Eigen vector corresponding to Eigen value  then



⇒ 

⇒ 

Since all rows are non identical then taking and  rows as

Taking, 



 

⇒ 

i.e., 

 

**Case 3 :** Suppose  be Eigen vector corresponding to Eigen value  then



⇒ 

⇒ 

Since all rows are non identical then taking and  rows as

Taking, 



 

⇒ 

i.e., 

 

(b) Verify Cayley-Hamilton theorem for the following matrix



Solution : Given the matrix is



The characteristics equation is,





⇒ 

⇒ 

⇒ 

⇒ 

⇒ 

⇒  …….. (1)

This is required characteristic equation.

**Eigen Values:**

Clearlysatisfy to equation (1), then  is a root of equation (1).

Now, 





⇒ 

[**Verification of Cayley-Hamilton theorem**](https://www.rgpvonline.com/)

By Cayley- Hamilton theorem every characteristic equation satisfy its characteristics equation, then from (1), we

Have

 ………(2)

Now, 

and 

*L.H.S.* = 



Hence verify Cayley-Hamilton theorem.

**3. (a) Solve** 

**Solution :** Given LDE is,

 ………(1)

Here, P = 1 and Q = 1

 

The solution is



⇒ 

⇒  **Answer**

**(b) Solve **

**Solution :** Given differential equation is,



The A.E. is,



Clearly  will satisfying the equation, then



⇒ 

⇒ 

⇒ 

The complete solution is,

y = C.F.

⇒  **Answer**

**4. (a) Solve the exact differential equation **

**Solution :** Given differential equation is

 …………(1)

Here,  and 

Now, and 

⇒ 

Therefore equation (1) is exact differential equation.

The solution of exact differential equation is,



⇒ 

⇒  **Answer**

**(b) Solve : **

**Solution :** Given : 

The A.E. is



⇒ 

⇒ 

 

Now, 

⇒ 

⇒ 

The required solution is,



⇒   **Answer**

**5. (a)** [**Solve the simultaneous differential equations :**](https://www.rgpvonline.com/)

** and **

**Solution :** Given differential equations are

 ………….(1)

and  ………….(2)

Let 

  ……………(3)

 ……………(4)

Eliminate y from equation (3) and (4), we get



⇒ 

The A.E. is,



⇒ 

⇒ 

   **Answer**

Differentiate w.r.t. *t* we get



From equation (1), we get



⇒ 

⇒ 

  **Answer**

**(b) Solve  if is one integral.**

**Solution :** Given,  …………(1)

Here,  and R = 0

Since  is a part of C.F., then Suppose the complete solution is

 …………..(2)

Where *v* is a function of *x*

Since 



⇒ 

Taking, 

∴ 

⇒ 

Integrating on both sides, we get



⇒ 

⇒ 

⇒ 

Integrating on both sides, we get



Putting in equation (2), we get

 **Answer**

**6. (a) Using method of removal of first derivative, solve the equation**

****

**Solution :** Given the differential equation is,

 ………..(1)

Here,  and 

Now this problem solve by Removable of first derivative method.

Suppose the complete solution is,

 ………..(2)

Where is a function of  only.

Now we can find the value of such as



and 

and 

The normal form of Removable of first derivative is,



⇒  …………..(3)

This is LDR of higher order.

The A.E. is



⇒ 

Therefore C.F.=

Now, 

The solution of equation (3) is,



Putting in equation (2), which our complete solution

 **Answer**

**(b) Using the method of variation of parameter, solve the equation:**

****

**Solution :** Given differential equation is



Here and 

The A.E. is



⇒ 

Therefore C.F. is



Suppose  and 

⇒  and 

and 

Suppose the complete solution of equation (1) is

 ……………(2)

Where A and B determine by the formula,



⇒ 

and 

⇒ 

Putting the values of A and B in equation (2), we get



⇒  **Answer**

**7. (a) Use Lagrange’s method solve the equation**



**Solution :** Given differential equation is …………(1)



This is Lagrange PDE.

Here 

The Lagrange A.E. is



Taking first two ratios, we get



Integrate both sides, we get



So that,  ………..(2)

Taking last two ratios, we get



⇒  [From (2)]

Integrate both sides, we get



⇒ 

The General solution of equation (1), we get

  **Answer**

**(b) Solve **

**Solution :** Given PDE is,

 …………(1)

The A.E. is



⇒ 

⇒ 

∴ 

Now, 

where  and [By Short-cut Method]





The complete solution is,



⇒  **Answer**

**8. (a) Solve : **

**Solution :** Given PDE is,



⇒  where  and 

The A.E. is,



⇒ 

⇒ 

∴ 

Now, 



















The Solution is,

  **Answer**

**(b)** [**Solve : **](https://www.rgpvonline.com/)

**Solution :** This is Lagrange LPDE of first order.

The A.E. is,



Taking, 

Integrating on both sides we get



⇒ 

Using the multiplier respectively we get





⇒ 

∴ 

Integrating on both sides we get



⇒ 

⇒ 

The general solution is,

** Answer**

**\*\*\*\*\*\*\*\*\*\***