General Physics Note

LittleCube

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4 Motion in Two and Three Dimensions

Proof of Uniform Circular Motion Acceleration

A particle in two dimension is moving countercolockwise in a circular motion revolving the origin with radius R, its velocity is constant and tangent to the circle, therefore

$$v_x = -v \sin\theta, v_y = v \cos\theta.$$

Substituting $\cos\theta, \sin\theta$ with displacement gives

$$v_x = -v \cdot \frac{x_y}{R}, v_y = v \cdot \frac{x_x}{R}.$$

Differentiating them yields the acceleration,

$$a_x = -v \cdot \frac{v_y}{R} = -v^2 \cdot \frac{\sin\theta}{R}, a_y = v \cdot \frac{v_x}{R} = -v^2 \cdot \frac{\cos\theta}{R}.$$

Therefore the magnitude of the acceleration is

$$a = \frac{v^2}{R}$$

5 Force and Motion – I

A Confusion I Have Made For a Long Time

The unit **Newton** is defined by

$$1\mathrm{N} = 1\mathrm{kg} \cdot \mathrm{m/s}^2$$

Therefore, an object with mass m has the weight mg where g is the gravitational acceleration at the place. 1 N is **not** defined as 9.8 kg. It is a **misunderstand** (since in most cases we are dealing with objects on the Earth).

Drag Force

The formula to the drag force is

$$\frac{1}{2}C\rho Av^2,$$

where C is the **drag coefficient**, ρ is the **density**, A is the **effective cross-section area** (the area that is facing toward the front; the area that is perpendicular to the direction of velocity).

Centripedal Force Does Not Change the Speed

Let's say we have two vector functions: velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$ which satisfy $\langle \mathbf{v}(t), \mathbf{a}(t) \rangle = 0$. We want to show that

$$\|\mathbf{v}(t)\| = \text{constant}.$$

Proof. We know that

$$\langle \mathbf{v}(t), \mathbf{v}(t) \rangle = \|v(t)\|^2.$$

Differentiating both sides gives

$$\langle \mathbf{a}(t), \mathbf{v}(t) \rangle + \langle \mathbf{v}(t), \mathbf{a}(t) \rangle = \frac{d}{dt}\|v(t)\|^2$$

$$2\langle \mathbf{a}(t), \mathbf{v}(t) \rangle = 0 = \frac{d}{dt}\|v(t)\|^2$$

Therefore $\|v(t)\|$ is constant. ■