General Physics Note

LittleCube

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3 Vectors

Assume two vectors $\vec{v}_1, \vec{v}_2$ are differ by θ radius, then the magnitude of $\vec{v}_1 + \vec{v}_2$ is

$$\sqrt{|\vec{v}_1|^2 + |\vec{v}_2|^2 - 2|\vec{v}_1||\vec{v}_2|\cos\boldsymbol{(\pi - \theta)}}.$$

It is better to decompose them along the axes and add them up.

5 Force and Motion – I

A Confusion I Have Made For a Long Time

The unit **Newton** is defined by

$$1\mathrm{N} = 1\mathrm{kg} \cdot \mathrm{m/s}^2$$

Drag Force

The formula to the drag force is

$$\frac{1}{2}C\rho Av^2,$$

where C is the **drag coefficient**, ρ is the **density**, A is the **effective cross-section area** (the area that is facing toward the front; the area that is perpendicular to the direction of velocity).

Centripedal Force Does Not Change the Speed

Let's say we have two vector functions: velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$ which satisfy $\langle \mathbf{v}(t), \mathbf{a}(t) \rangle = 0$. We want to show that

$$\|\mathbf{v}(t)\| = \text{constant}.$$

Proof. We know that

$$\langle \mathbf{v}(t), \mathbf{v}(t) \rangle = \|v(t)\|^2.$$

Differentiating both sides gives

$$\langle \mathbf{a}(t), \mathbf{v}(t) \rangle + \langle \mathbf{v}(t), \mathbf{a}(t) \rangle = \frac{d}{dt}\|v(t)\|^2$$

$$2\langle \mathbf{a}(t), \mathbf{v}(t) \rangle = 0 = \frac{d}{dt}\|v(t)\|^2$$

Therefore $\|v(t)\|$ is constant. ■

8 Potential Energy and Conservation of Energy

The mechanical energy E_{mec} of a system can only inclue

- Kinetic energy K
- Potential energy **between objects inside the system**

For example, a ball is free falling. If the system contains **only the ball**, then it has kinetic energy of $K = \frac{1}{2}mv^2$ and a **constant** potential energy U. The Earth is constantly exerting force on the system, causing the mechanical energy to increase.

37 Relativity

For special relativity, all discussions are based on inertial reference frames (i.e. we don't take acceleration into consideration).

Special relativity is based on two postulates:

1. **Laws of physics** stay the same.
2. The **speed of light** stays the same, $c = 299\ 792\ 458\ \mathrm{m/s}$.

Time Dilation

When two events occur at the **same location** in an inertial reference frame, the time interval between them measured in that frame is called the **proper time** Δt_0.

For any other reference frame move at speed v relative to that frame, the interval between the two event will be measured as

$$\Delta t = \gamma \Delta t_0, \text{where } \gamma = \frac{1}{1-\beta^2} \text{ is called } \textbf{Lorentz factor} \text{ with } \beta = \frac{v}{c}$$

Length Contraction

The length L_0 of an object measured in the rest frame of the object is its **proper length** or rest length.

For any other frame move at speed v relative to that frame, the object length will be measured as

$$L = \frac{L_0}{\gamma}$$

Length contraction only occurs along the direction of the relative speed.

Lorentz Transformation

A event E measured by two different inertial frame F and F', where F' moves at speed v along their common x axes, has different space-time coordinates. This pair of coordinates can be transformed as:

$$\begin{matrix} x \\ t \end{matrix} \longrightarrow \begin{matrix} x' & = & \gamma(x - vt) \\ t' & = & \gamma\left(t - \frac{vx}{c^2}\right) \end{matrix}$$

It can be derived from time dilation and length contraction.

For two different event, with different x and t, let $\Delta x, \Delta t$ be their x-coordinate and time difference, we shift the coordinates from F' to F:

$$\begin{matrix} \Delta x' & = & \gamma(\Delta x - v\Delta t) \\ \Delta t' & = & \gamma\left(\Delta t + \frac{v\Delta x}{c^2}\right) \end{matrix} \Rightarrow v' = \frac{\Delta x'}{\Delta t'} = \frac{u + v}{1 + \frac{uv}{c^2}}, \text{where } u = \frac{\Delta x}{\Delta t}$$

From which we derived the formula of adding velocities. Note when one of u, v is c, the result is always c.

Doppler Effect for Light

Assume the source and detector are moving **toward** each other with velocity v. Let f_0 be the **proper frequency** (frequency measured by the source) and λ_0 be the correspond **proper wavelength**. To the source, the time duration between two consecutive light wave arrives to the detector t_s is

$$\lambda_0 = ct_s - vt_s \Rightarrow t_s = \frac{\lambda_0}{c - v} = \frac{1}{f_0(1 - \beta)}$$

However, the time is actually *dilated*. The actual frequency and wavelength observed by the detector is

$$f = \frac{1}{t} = \frac{\gamma}{t_s} = \frac{f_0(1 - \beta)}{\sqrt{1 - \beta^2}} = f_0\sqrt{\frac{1 - \beta}{1 + \beta}} \Rightarrow \lambda = \lambda_0\sqrt{\frac{1 + \beta}{1 - \beta}}$$

Momentum and Energy

We start with the new definition:

$$p = mv = m\frac{\Delta x}{\Delta t_0}$$

Since the observer's time is *dilated*, so our observed time is actually longer, therefore

$$p = m\frac{\Delta x}{\Delta t_0} = m\frac{\Delta x}{\Delta t} \cdot \gamma = \gamma mv$$