


General Physics Note

LittleCube 

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3 Vectors

Assume two vectors \vec{v}_1, \vec{v}_2 are differ by θ radius, then the magnitude of $\vec{v}_1 + \vec{v}_2$ is

$$\sqrt{|\vec{v}_1|^2 + |\vec{v}_2|^2 - 2|\vec{v}_1||\vec{v}_2|\cos(\pi - \theta)}.$$

It is better to decompose them along the axes and add them up.

4 Motion in Two and Three Dimensions

Proof of Uniform Circular Motion Acceleration

A particle in two dimension is moving counterclockwise in a circular motion revolving the origin with radius R , its velocity is constant and tangent to the circle, therefore

$$v_x = -v \sin \theta, v_y = v \cos \theta.$$

Substituting $\cos \theta, \sin \theta$ with displacement gives

$$v_x = -v \cdot \frac{x_y}{R}, v_y = v \cdot \frac{x_x}{R}.$$

Differentiating them yields the acceleration,

$$a_x = -v \cdot \frac{v_y}{R} = -v^2 \cdot \frac{\sin \theta}{R}, a_y = v \cdot \frac{v_x}{R} = -v^2 \cdot \frac{\cos \theta}{R}.$$

Therefore the magnitude of the acceleration is

$$a = \frac{v^2}{R}$$

5 Force and Motion – I

A Confusion I Have Made For a Long Time

The unit **Newton** is defined by

$$1\text{N} = 1\text{kg} \cdot \text{m/s}^2$$

Therefore, an object with mass m has the weight mg where g is the gravitational acceleration at the place. 1 N is **not** defined as 9.8 kg. It is a **misunderstand** (since in most cases we are dealing with objects on the Earth).

Drag Force

The formula to the drag force is

$$\frac{1}{2}C\rho Av^2,$$

where C is the **drag coefficient**, ρ is the **density**, A is the **effective cross-section area** (the area that is facing toward the front; the area that is perpendicular to the direction of velocity).

Centripetal Force Does Not Change the Speed

Let's say we have two vector functions: velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$ which satisfy $\langle \mathbf{v}(t), \mathbf{a}(t) \rangle = 0$. We want to show that

$$\|\mathbf{v}(t)\| = \text{constant}.$$

Proof. We know that

$$\langle \mathbf{v}(t), \mathbf{v}(t) \rangle = \|v(t)\|^2.$$

Differentiating both sides gives

$$\langle \mathbf{a}(t), \mathbf{v}(t) \rangle + \langle \mathbf{v}(t), \mathbf{a}(t) \rangle = \frac{d}{dt}\|v(t)\|^2$$

$$2\langle \mathbf{a}(t), \mathbf{v}(t) \rangle = 0 = \frac{d}{dt}\|v(t)\|^2$$

Therefore $\|v(t)\|$ is constant. ■

8 Potential Energy and Conservation of Energy

The mechanical energy E_{mec} of a system can only include

- Kinetic energy K
- Potential energy **between objects inside the system**

For example, a ball is free falling. If the system contains **only the ball**, then it has kinetic energy of $K = \frac{1}{2}mv^2$ and a **constant** potential energy U . The Earth is constantly exerting force on the system, causing the mechanical energy to increase.