homework 04 for design of experiment   106354005 余佑駿

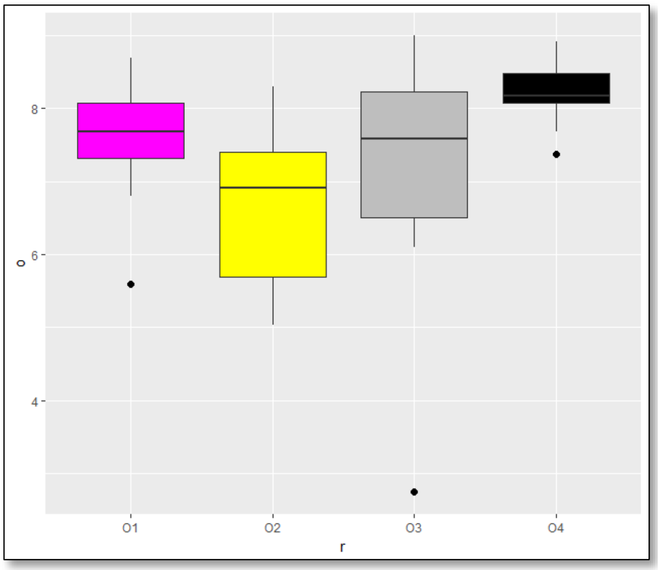
introduction and comparison with experimental study and observation study

problem 3.1

Cardiac pacemakers contain electrical connections that are platinum pins soldered onto a substrate. The question of interest is whether different operators produce solder joints with the same strength. Twelve substrates are randomly assigned to four operators. Each operator solders four pins on each substrate, and then these solder joints are assessed by measuring the shear strength of the pins. Data from T. Kerkow. Analyze these data to determine if there is any evidence that the operators produce different mean shear strengths. (Hint: what are the experimental units?)

Strength (lb)												
Operator	Substrate 1				Substrate 2				Substrate 3			
1	5.60	6.80	8.32	8.70	7.64	7.44	7.48	7.80	7.72	8.40	6.98	8.00
2	5.04	7.38	5.56	6.96	8.30	6.86	5.62	7.22	5.72	6.40	7.54	7.50
3	8.36	7.04	6.92	8.18	6.20	6.10	2.75	8.14	9.00	8.64	6.60	8.18
4	8.30	8.54	7.68	8.92	8.46	7.38	8.08	8.12	8.68	8.24	8.09	8.06

先對數據做敘述性統計的觀察，可以看到，就中位數而言，Operator 1、3 較為相近；但就四分位差(Q3-Q1)而言，Operator1、4 較為相近，Operator 2、3 較為相近。



右下表可以看到四個 Operator 在 12 個焊接強度值的特徵值。

```
   Min. 1st Qu. Median     Mean 3rd Qu. Max.
O1 5.60   7.325   7.68 7.573333   8.080 8.70
O2 5.04   5.695   6.91 6.675000   7.410 8.30
O3 2.75   6.500   7.59 7.175833   8.225 9.00
O4 7.38   8.075   8.18 8.212500   8.480 8.92
```

現在要做檢定看看每個 Operator 的焊接強度，是否不相同。

假設：

48 個焊接強度的變量獨立且服從常態，且四組 Operator 變量的母體變異數相等。

如果 Operator 對焊接的強度沒有顯著影響，則四位作業員焊接點強度的母體平均應沒有顯著差異，檢定假設如下：$\begin{cases} H_0: \mu_1. = \mu_2. = \mu_3. = \mu_4. \\ H_1: \mu_1., \mu_2., \mu_3., \mu_4. \ are \ \ not \ \ all \ \ the \ \ same \end{cases}$

用 Two-way ANOVA (二因子變異數分析)來試圖檢定 Operator 是否有顯著影響，結果如下

```
Analysis of Variance Table

Response: Strength
                   Df Sum Sq Mean Sq F value   Pr(>F)
Operator            3 15.189  5.0630  4.7001 0.00719 **
Substrate           2  3.232  1.6158  1.4999 0.23676
Operator:Substrate  6 10.496  1.7493  1.6239 0.16891
Residuals          36 38.779  1.0772
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Two-way ANOVA(Strength ～Operator * Substrate)

「Operator：Substrate」，代表 Operator 跟 Substrate 的交互作用項

在服從 ANOVA 假設下，可以有得知：

(1) Operator 對 Strength 有顯著影響，推論不同的作業員焊接的強度在統計上有顯著差異。

(2) Substrate 對 Strength 沒有顯著影響，推論在不同種類的基板，對焊接的強度並無統計上的顯著影響。

(3) Operator 及 Substrate 的交互作用對 Strength 沒有顯著影響，推論不同作業員在不同的基板上，對於焊接技術並無特別的依賴關係。

problem 3.2 (Descriptive Analysis)

Scientists are interested in whether the energy costs involved in reproduction affect longevity. In this experiment, 125 male fruit flies were divided at random into five sets of 25. In one group, the males were kept by themselves. In two groups, the males were supplied with one or eight receptive virgin female fruit flies per day. In the final two groups, the males were supplied with one or eight unreceptive (pregnant) female fruit flies per day.

Other than the number and type of companions, the males were treated identically. The longevity of the flies was observed. Data from Hanley and Shapiro (1994).

Analyze these data to test the null hypothesis that reproductive activity does not affect longevity. Write a report on your analysis. Be sure to describe the experiment as well as your results.

Companions	Longevity(days)												
None	35	37	49	46	63	39	46	56	63	65	56	65	70
	63	65	70	77	81	86	70	70	77	77	81	77	
1 pregnant	40	37	44	47	47	47	68	47	54	61	71	75	89
	58	59	62	79	96	58	62	70	72	75	96	75	
1virgin	46	42	65	46	58	42	48	58	50	80	63	65	70
	70	72	97	46	56	70	70	72	76	90	76	92	
8 pregnant	21	40	44	54	36	40	56	60	48	53	60	60	65
	68	60	81	81	48	48	56	68	75	81	45	68	
8virgin	16	19	19	32	33	33	30	42	42	33	26	30	40
	54	34	34	47	47	42	47	54	54	56	60	44	

```
Anderson.Darling
       0.4151358
```

想做壽命(long)對不同陪伴數量(comp)的變異數分析表(ANOVA Table)，必須先檢測壽命(long)是否服從常態，利用 ad.test 可知$p-value \approx 0.42 > 0.05$，因此可以利用 ANOVA 做初步分析。

```
Analysis of Variance Table

Response: long
           Df Sum Sq Mean Sq F value    Pr(>F)
comp        4  11939 2984.82  13.612 3.516e-09
Residuals 120  26314  219.28

comp         ***
Residuals
```

p-value 遠小於 0.01，樣本與虛無假設有顯著差異，傾向拒絕虛無假設，推論 5 組平均壽命不相等，代表無求偶活動力(None)與有求偶活動力(其他四組)平均不完全相同，因此雄果蠅的繁殖行為所造成的能量耗損，可能會影響壽命。

Problem 4.2

Consider the data in Problem 3.2.

Design a set of contrasts that seem meaningful.

For each contrast, outline its purpose and test the null hypothesis that the contrast has expected value zero.

ANOVA 分析可以知道壽命與繁殖行為有關，再進一步針對兩兩組別之間做多重比較：

(1) 《None $v.s.$ 1 Pregnant》

1 隻已懷孕的雌果蠅傾向不接受 25 隻雄果蠅的求偶，假設排除生殖行為影響，推論 25 隻雄果蠅的競爭性求偶行為，這樣的能量耗損是否影響壽命。

(2) 《1 Pregnant $v.s.$ 1 Virgin》

在相同的競爭求偶壓力下，有無交配成功是否會影響壽命的長短。

(3) 《1 Virgin $v.s.$ 8 Virgin》

25 隻雄果蠅與 8 隻願意接受求偶的雌果蠅們相處，相較於(2)降低的競爭壓力，可觀察增加生殖行為對壽命的影響。

將上述 3 項設計，做出係數矩陣，利用$\binom{4}{2} = 6$種所有可能組合進行 Turkey 多重比較法。

	None	preg1	vir1	preg8	vir8
w1	1	1	0	0	0
w2	0	1	1	0	0
w3	0	0	1	0	1

由結果可以得知：

《None $v.s.$ 1 Pregnant》，區間包含 0，無顯著差異，代表競爭性求偶行為對壽命無顯著影響。

《1 Pregnant $v.s.$ 1 Virgin》，區間包含 0，無顯著差異，兩組都有求偶行為，代表有無生殖行為不是決定壽命的關鍵。

《1 Virgin $v.s.$ 8 Virgin》區間小於 0，v8 組壽命顯著較短，減少競爭壓力、增加了交配行為後，壽命顯著減少。(縱慾是不好的 XD?)

根據實驗設計與模擬結果，可以推論「增加交配次數」可能會影響壽命長短，交配次數增加，可能會導致壽命變短。

Exercise 4.3

Refer to the data in Problem 3.1. Workers 1 and 2 were experienced(Operator 1、2 是老手), Exercise 4.3 whereas workers 3 and 4 were novices(Operator 1、2 是親手啦).

Find a contrast to compare the experienced and novice workers and test the null hypothesis that experienced and novice works produce the same average shear strength.

要看老手和新手，在焊接平均強度上是否有差異，設定檢定假設$\frac{\mu_1+\mu_2}{2} = \frac{\mu_3+\mu_4}{2}$，利用線性多重比較法(linear contrasts)，令線性組合的加權係數$w = \begin{pmatrix} 0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{pmatrix}$，每位 Operator 作業員所焊接強度的母體平均$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$，則檢定假設可改寫為：$H_0$: $\boldsymbol{w^T\mu = 0}$ H_1: $\boldsymbol{w^T\mu \neq 0}$，再令$\bar{\mathbf{O}} = (\bar{O}_1,\bar{O}_2,\bar{O}_3,\bar{O}_4)$，$\bar{O}_i$為第 i 個作業員的所有 12 個焊接強度數據的平均值，並以$\bar{\boldsymbol{O}}$估計$\boldsymbol{\mu}$

觀察統計量$\boldsymbol{w^T\bar{O}}$分布：$\boldsymbol{w^T\bar{O}} \sim N(\boldsymbol{w^T\mu}, \frac{1}{12}\sigma^2\boldsymbol{w^Tw})$，但由於共同母體變異$\sigma^2$未知，因此將統計量做適當轉換$\frac{\boldsymbol{w^T\bar{O}} - \boldsymbol{w^T\mu}}{\sqrt{\frac{1}{12}\boldsymbol{MSE \cdot w^Tw}}} \sim t(\mathrm{N-g})$，其中$N$為總數據個數 $= 12\times4 = 48$，g 為 Operator 的人數 $= 4$，可利用信賴區間$\boldsymbol{w^T\bar{O}} \pm \mathbf{t}_{\frac{\varepsilon}{2};\mathbf{N-g}}\sqrt{\frac{1}{12}\boldsymbol{MSE \cdot w^Tw}}$，或是直接以$\frac{\boldsymbol{w^T\bar{O}} - \boldsymbol{w^T\mu}}{\sqrt{\frac{1}{12}\boldsymbol{MSE \cdot w^Tw}}}$作為檢定統計量，來進行 two-sided t – test 檢定，訂定 α=0.05，檢定結果如下：

```
        One Sample t-test

data:  wO
t = -1.7336, df = 11, p-value = 0.1109
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -1.2936663  0.1536663
sample estimates:
mean of x
    -0.57
```

95%信賴區間範圍涵蓋 0，不拒絕虛無假設。p-value = 0.1109 > 0.05 ，樣本不足以拒絕虛無假設

透過上述多重比較檢定法，由於虛無假設為$w^T\mu = 0$，等價於$\frac{\mu1+\ \mu2}{2} = \frac{\mu3+\ \mu4}{2}$，等價於 Operator 1、2 的平均焊接強度 = Operator 3、4 的平均焊接強度，不拒絕虛無假設暗示著老手的作業員們(experienced worker：Operator 1、2)跟新手作業員們(novices：Operator 3、4)在焊接技術上(以焊接點強度 Strength 衡量)並「無」統計上的顯著差異。

Exercise 4.4

Consider an experiment taste-testing six types of chocolate chip cookies:

	Type of chocolate					
	1	2	3	4	5	6
brand	A	A	B	B	C	D
Taste	chewy	crispy	chewy	crispy	chewy	crispy
Spend	expensive	expensive	inexpensive	inexpensive	expensive	inexpensive

We will use twenty different raters randomly assigned to each type (120 total raters).

(a) Design contrasts to compare chewy with crispy, and expensive with inexpensive.

(b) Are your contrasts in part (a) orthogonal? Why or why not?

Answer：

For (a)

We design $《contrast\ \ 1》 = \left(\frac{1}{6}, \frac{-1}{6}, \frac{1}{6}, \frac{-1}{6}, \frac{1}{6}, \frac{-1}{6}\right)$ and $《contrast\ \ 2》 = (1,1,-1,-1,1,-1)$

For (b)

$$《contrast\ \ 1》 \times 《contrast\ \ 2》 = \frac{1}{6} + \frac{-1}{6} - \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3} \neq 0$$

Question 3.1

Prove that $0 = \sum_{i=1}^{g} \sum_{j=1}^{n_i} \widehat{\alpha_i} r_{ij}$

Notation：

1°

$$\sum_{j=1}^{n_i} y_{ij} = n_i \bar{y}_{i\cdot}$$

$$y_{i\cdot} = \sum_{j=1}^{n_i} y_{ij}\,, \qquad \overline{y_{i\cdot}} = \frac{\sum_{j=1}^{n_i} y_{ij}}{n_i} = treatment\ \ mean$$

$$y_{\cdot\cdot} = \sum_{i=1}^{g} \sum_{j=1}^{n_i} y_{ij}\,, \qquad \bar{y}_{\cdot\cdot} = \frac{\sum_{i=1}^{g} \sum_{j=1}^{n_i} y_{ij}}{N} = grand\ \ mean$$

2°

$$\hat{\alpha}_i = \bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot} \quad , r_{ij} = y_{ij} - \bar{y}_{i\cdot}$$

$$(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})(y_{ij} - \bar{y}_{i\cdot}) = \bar{y}_{i\cdot}\, y_{ij} - (\bar{y}_{i\cdot})^2 - \bar{y}_{\cdot\cdot}\, y_{ij} + \bar{y}_{\cdot\cdot}\, \bar{y}_{i\cdot}$$

3°

$$\sum_{i=1}^{g} \sum_{j=1}^{n_i} \hat{\alpha}_i r_{ij} = \sum_{i=1}^{g} \sum_{j=1}^{n_i} (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})(y_{ij} - \bar{y}_{i\cdot})$$

$$= \sum_{i=1}^{g} \sum_{j=1}^{n_i} \bar{y}_{i\cdot}\, y_{ij} - \sum_{i=1}^{g} \sum_{j=1}^{n_i} (\bar{y}_{i\cdot})^2 - \sum_{i=1}^{g} \sum_{j=1}^{n_i} \bar{y}_{\cdot\cdot}\, y_{ij} + \sum_{i=1}^{g} \sum_{j=1}^{n_i} \bar{y}_{\cdot\cdot}\, \bar{y}_{i\cdot}$$

$$= \sum_{i=1}^{g} n_i (\bar{y}_{i\cdot})^2 - \sum_{i=1}^{g} n_i (\bar{y}_{i\cdot})^2 - \sum_{i=1}^{g} \bar{y}_{\cdot\cdot} (n_i \bar{y}_{i\cdot}) + \sum_{i=1}^{g} n_i \bar{y}_{\cdot\cdot}\, \bar{y}_{i\cdot} = 0$$

Question 4.1

Show that orthogonal contrasts in the observed treatment means are uncorrelated random variables.

Answer：

Two contrasts with coefficients $\{ci\}$ and $\{di\}$ are orthogonal if $\sum_{i=1}^{a} c_i d_i = 0$

Recall that contrast coefficients have zero sum.

Therefore, it means a zero mean.($\sum_{i=1}^{a} c_i = 0, \sum_{i=1}^{a} d_i = 0$)

So, Covariance $\mathrm{Cov}(A, \mathrm{B}) = (c_1 d_1 + \cdots + c_n d_n)\left(\frac{\sigma^2}{n}\right) + (\ldots) cov(\bar{y}_i . \bar{y}_j .)$

因為 ANOVA 假設的關係$cov(\bar{y}_i . \bar{y}_j .) = 0$，加上題目假設，所以得證。

That is that orthogonal contrasts in the observed treatment means are uncorrelated random variables!!