Part 1.

For the same filtering windows on two different images f_1 and f_2, we get:

$$\alpha \sum_{s_1=-a}^{a} \sum_{t_1=-b}^{b} w(s_1,t_1) f_1(x+s_1, y+t_1) + \beta \sum_{s_2=-a}^{a} \sum_{t_2=-b}^{b} w(s_2,t_2) f_2(x+s_2, y+t_2)$$

Since $\sum_{s_1=-a}^{a} \sum_{t_1=-b}^{b} w(s_1,t_1) = \sum_{s_2=-a}^{a} \sum_{t_2=-b}^{b} w(s_2,t_2)$,

we can get:

$$\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) \left[\alpha f_1(x+s, y+t) + \beta f_2(x+s, y+t) \right]$$

So, the operator is a linear operator.

Part 2.

$$\begin{aligned}(A \cdot B)^c &= ((A \oplus B) \ominus B)^c \\ &= (A \oplus B)^c \oplus \hat{B} \\ &= (A^c \ominus \hat{B}) \oplus \hat{B} \\ &= A^c \circ \hat{B}\end{aligned}$$

Part 3.

According to the defiations:

$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \phi \}$ $\quad A \ominus B = \{ z \mid (B)_z \subseteq A \}$.

$\hat{B_1} = B_1$ $\quad \hat{B_2} = B_2$

Then,

$$A \oplus B_1 = \begin{matrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{matrix}$$

$$A \ominus B_1 = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$A \oplus B_2 = \begin{matrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{matrix}$$

$$A \ominus B_2 = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$