计量经济学：作业四

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1. (a)

$$P(y=0,x=0)=P(y=0|x=0)P(x=0)=(1-\frac{e^{\beta 0}}{1+e^{\beta 0}})(1-\alpha)=\frac{1}{2}(1-\alpha)$$

$$P(y=0,x=1)=P(y=0|x=1)P(x=1)=(1-\frac{e^{\beta}}{1+e^{\beta}})\alpha=\frac{\alpha}{1+e^{\beta}}$$

$$P(y=1,x=0)=P(y=1|x=0)P(x=0)=\frac{e^{\beta 0}}{1+e^{\beta 0}}(1-\alpha)=\frac{1}{2}(1-\alpha)$$

$$P(y=1,x=1)=P(y=1|x=1)P(x=1)=\frac{e^{\beta}}{1+e^{\beta}}\alpha$$

   (b) 似然函数:

$$\begin{aligned} f &= \prod_{i=1}^{N_1}P(y=0,x=0)\prod_{i=1}^{N_2}P(y=1,x=0)\prod_{i=1}^{N_3}P(y=0,x=1)\prod_{i=1}^{N_4}P(y=1,x=1) \\ &= \prod_{i=1}^{N_1}\frac{1}{2}(1-\alpha)\prod_{i=1}^{N_2}\frac{1}{2}(1-\alpha)\prod_{i=1}^{N_3}\frac{\alpha}{1+e^{\beta}}\prod_{i=1}^{N_4}\frac{e^{\beta}}{1+e^{\beta}}\alpha \\ &= \prod_{i=1}^{N_1+N_2}\frac{1}{2}(1-\alpha)\prod_{i=1}^{N_3+N_4}\frac{\alpha}{1+e^{\beta}}\prod_{i=1}^{N_4}e^{\beta} \end{aligned} \tag{1}$$

   对数似然:

$$L=\log f=(N_1+N_2)\log(\frac{1-\alpha}{2})+(N_3+N_4)\log\frac{\alpha}{1+e^{\beta}}+N_4\beta$$

   一阶条件:

$$\frac{\partial L}{\partial \alpha}=\frac{N_3+N_4}{\alpha}-\frac{N_1+N_2}{1-\alpha}=0$$

$$\frac{\partial L}{\partial \beta}=\frac{N_4}{1+e^{\beta}}-\frac{N_3e^{\beta}}{1+e^{\beta}}=0$$

   解得:

$$\alpha=\frac{N_3+N_4}{N}$$

$$\beta=\log N_4-\log N_3$$

2. (a) 令 $Q^s=Q^d$ 得到:

$$\alpha_1P+u^d=\alpha_2P+\beta_2z^s+u^s$$

解得:

$$P = \frac{\beta_2}{\alpha_1 - \alpha_2} z^s + \frac{u^s - u^d}{\alpha_1 - \alpha_2}$$

$$Q = \alpha_1 \frac{\beta_2}{\alpha_1 - \alpha_2} z^s + \frac{\alpha_1 u^s - \alpha_2 u^d}{\alpha_1 - \alpha_2}$$

令 $v_1 = \frac{u^s - u^d}{\alpha_1 - \alpha_2}, v_2 = \frac{\alpha_1 u^s - \alpha_2 u^d}{\alpha_1 - \alpha_2}$ 得到:

$$P = \frac{\beta_2}{\alpha_1 - \alpha_2} z^s + v_1$$

$$Q = \alpha_1 \frac{\beta_2}{\alpha_1 - \alpha_2} z^s + v_2$$

因为 z^s 是外生的，所以 z^s 和 v_1, v_2 不相关。

(b) 对上面两个式子进行 OLS 估计。得到参数分别为

$$\frac{\beta_2}{\alpha_1 - \alpha_2} = \pi_1$$

$$\alpha_1 \frac{\beta_2}{\alpha_1 - \alpha_2} = \pi_2$$

两个系数相除，可以得到:

$$\alpha_1 = \frac{\pi_2}{\pi_1}$$

所以如果观测到了 P, Q, z^s, 则我们可以识别 α_1.

(c) 如果需要识别 α_2，那么我们需要方程 (1) 中包含不在方程 (2) 中的外生变量，设为 z^d. 那么上述联立方程变为:

$$Q^d = \alpha_1 P + \beta_1 z^d + u^d$$

$$Q^s = \alpha_2 P + \beta_2 z^s + u^s$$

联立上述方程并化简可以得到:

$$P = \frac{\beta_2}{\alpha_1 - \alpha_2} z^s - \frac{\beta_1}{\alpha_1 - \alpha_2} z^d + v_1$$

$$Q = \alpha_1 \frac{\beta_2}{\alpha_1 - \alpha_2} z^s - \alpha_2 \frac{\beta_1}{\alpha_1 - \alpha_2} z^d + v_2$$

对上述两个方程分别进行 OLS 估计，得到的系数分别为 $\pi_1, \pi_2, \sigma_1, \sigma_2$. 则对应系数相除可以识别 α_1, α_2:

$$\alpha_1 = \frac{\sigma_1}{\pi_1}$$

$$\alpha_2 = \frac{\sigma_2}{\pi_2}$$

3. (a)

$$E(x_t) = E(e_t) - \frac{1}{4} E(e_{t-1}) + \frac{1}{2} E(e_{t-2}) = 0$$

$$Var(x_t) = Var(e_t) + \frac{1}{16} Var(e_{t-1}) + \frac{1}{4} Var(e_{t-2}) = \frac{21}{16}$$

(b)

$$cov(x_t, x_{t+1}) = cov(e_t - \frac{1}{4}e_{t-1} + \frac{1}{2}e_{t-2}, e_{t+1} - \frac{1}{4}e_t + \frac{1}{2}e_{t-1}) = -\frac{1}{4}Var(e_t) - \frac{1}{8}Var(e_{t-1}) = -\frac{3}{8}$$

$$cov(x_t, x_{t+2}) = cov(e_t - \frac{1}{4}e_{t-1} + \frac{1}{2}e_{t-2}, e_{t+2} - \frac{1}{4}e_{t+1} + \frac{1}{2}e_t) = \frac{1}{2}Var(e_t) = \frac{1}{2}$$

$$cov(x_t, x_{t+h}) = cov(e_t - \frac{1}{4}e_{t-1} + \frac{1}{2}e_{t-2}, e_{t+h} - \frac{1}{4}e_{t+h-1} + \frac{1}{2}e_{t+h-2}) = 0, \forall h > 2$$

因为 $E(x_t^2) < +\infty, E(x_t) = const, Var(x_t) = const$ 且 $\forall t, h \geq 1, cov(e_t, e_{t+h})$ 和 t 无关，所以 $\{x_t : t = 1, \cdots\}$ 是协方差平稳过程。

(c)

$$corr(x_t, x_{t+h}) = \frac{cov(x_t, x_{t+h})}{\sigma_{x_t}, \sigma_{x_{t+h}}} = 0, \forall h > 2$$

满足 $h \to \infty, corr(x_t, x_{x+h}) \to 0$ 且是协方差平稳过程，所以 $\{x_t : t = 1, \cdots\}$ 是渐进无关的。