数 学 作 业 纸

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1. 解：$f(t) = \sin t\cos 2t + 5\cos 3t\sin 4t$ 周期 $T=2\pi$ $\omega_0 = 1$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi}(\sin t\cos 2t + 5\cos 3t\sin 4t)\,dt$$
$$= \frac{1}{2\pi}\int_0^{2\pi}(2\sin t + \frac{1}{2}\sin 3t + \frac{5}{2}\sin 7t)\,dt$$
$$= 0$$

$$a_n = \frac{1}{2\pi}\int_0^{2\pi}(2\sin t + \frac{1}{2}\sin 3t + \frac{5}{2}\sin 7t)\cos(nt)\,dt$$
$$= \frac{1}{2\pi}\left(\int_0^{2\pi}[2\sin t\cos nt]\,dt + \int_0^{2\pi}\frac{1}{2}\sin 3t\cos nt\,dt + \int_0^{2\pi}\frac{5}{2}\sin 7t\cos nt\right)dt$$
$$= 0$$

$$b_n = \frac{1}{2\pi}\int_0^{2\pi}(2\sin t + \frac{1}{2}\sin 3t + \frac{5}{2}\sin 7t)\sin nt\,dt$$
$$= \begin{cases} 2 & n=1 \\ \frac{1}{2} & n=3 \\ \frac{5}{2} & n=7 \\ 0 & \text{other} \end{cases}$$

$\therefore$ $$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega_0 t + b_n \sin n\omega_0 t)$$
$$= 0 + 0 + 2\sin t + \frac{1}{2}\sin 3t + \frac{5}{2}\sin 7t$$
$$= 2\sin t + \frac{1}{2}\sin 3t + \frac{5}{2}\sin 7t \quad (t\in R)$$

$$\therefore \begin{cases} a_0 = 0 \\ a_n = 0 \quad (n\geq 1) \\ b_1 = 2 \quad b_3 = \frac{1}{2} \quad b_7 = \frac{5}{2} \quad \text{其它 } b_n = 0 \end{cases}$$