

## 阻尼振动与受迫振动（简要报告）

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### 1 实验目的

1. 观测阻尼振动，学习测量振动系统基本参数的方法。
2. 研究受迫振动的幅频特性和相频特性，观察共振现象。
3. 观测不同阻尼对受迫振动的影响。

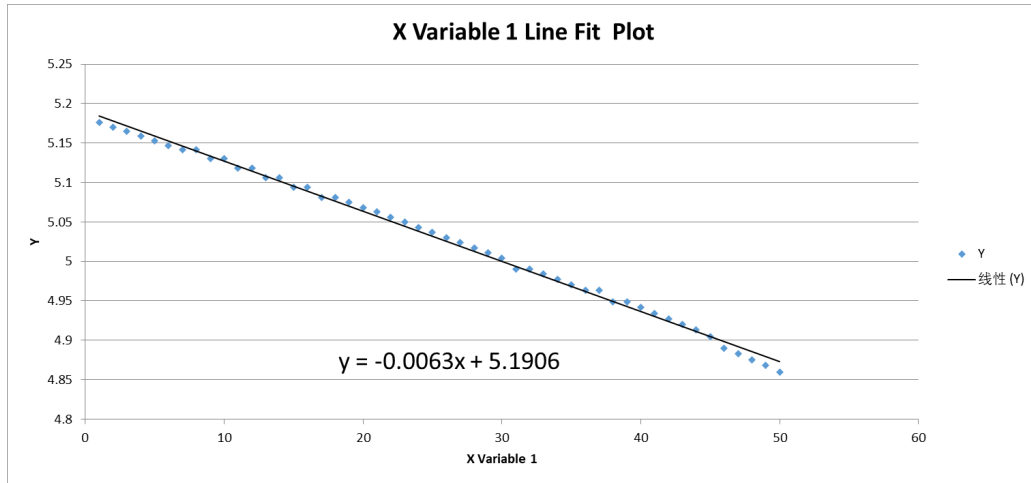
### 2 实验原理、仪器、步骤等请参看预习报告部分

### 3 数据处理

#### 1. 阻尼振动：

##### (1) 无电磁阻尼

序号	$T_i = 10\overline{T_d} / \text{s}$	$\theta_j / ^\circ$	$\ln\theta_j$	序号	$T_i = 10\overline{T_d} / \text{s}$	$\theta_j / ^\circ$	$\ln\theta_j$	$D_j = \ln\theta_{j+25} - \ln\theta_j$
1	14.520	177	5.176	26	(续左下)	153	5.030	-0.146
2		176	5.170	27		152	5.024	-0.146
3		175	5.165	28		151	5.017	-0.148
4		174	5.159	29		150	5.011	-0.148
5		173	5.153	30		149	5.004	-0.149
6		172	5.147	31	14.562	147	4.990	-0.157
7		171	5.142	32		147	4.990	-0.152
8		171	5.142	33		146	4.984	-0.158
9		169	5.130	34		145	4.977	-0.153
10		169	5.130	35		144	4.970	-0.160
11	14.539	167	5.118	36		143	4.963	-0.155
12		167	5.118	37		143	4.963	-0.155
13		165	5.106	38		141	4.949	-0.157
14		165	5.106	39		141	4.949	-0.157
15		163	5.094	40		140	4.942	-0.152
16		163	5.094	41	14.572	139	4.934	-0.160
17		161	5.081	42		138	4.927	-0.154
18		161	5.081	43		137	4.920	-0.161
19		160	5.075	44		136	4.913	-0.162
20		159	5.068	45		135	4.905	-0.163
21	14.551	158	5.063	46		133	4.890	-0.173
22		157	5.056	47		132	4.883	-0.183
23		156	5.050	48		131	4.875	-0.175
24		155	5.043	49		130	4.868	-0.175
25		154	5.037	50		129	4.860	-0.177



a. 对  $y_i = \ln \theta_i$  和  $i$  进行回归分析得到拟合直线:

$$y = -0.0063x + 5.1906$$

用逐差法计算阻尼比  $\zeta$ , 读取振幅值  $\theta_1, \theta_2 \dots \theta_n$  则,

$$b = \ln \theta_j - \ln \theta_{j-1} = -\beta T_d = -\beta \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}} = -\frac{2\pi}{\sqrt{\zeta^{-2} - 1}}$$

$$b = \frac{\bar{D}}{I} = \frac{\sum (y_{j+I} - y_j)}{I^2} = -0.0063456$$

$$S_b = \frac{\sqrt{\sum (D_j - \bar{D})^2 / (I - 1)}}{I}$$

$$\Delta b = S_b = \frac{1}{n} \sqrt{\frac{\sum (D_i - \bar{D})^2}{n - 1}} = \frac{S_D}{25} = 3.77 \times 10^{-4}$$

故

$$b = (-6.34 \pm 0.37) \times 10^{-3}$$

由于  $b = -\frac{2\pi}{\sqrt{\zeta^{-2} - 1}}$ ,

$$\zeta = \left( \left( \frac{2\pi}{b} \right)^2 + 1 \right)^{-\frac{1}{2}} = 0.00101$$

$$\frac{d\zeta}{db} = \frac{4\pi^2}{b^3} \left( \left( \frac{2\pi}{b} \right)^2 + 1 \right)^{-\frac{3}{2}}$$

$$\Delta \zeta = \left| \frac{d\zeta}{db} \right| \Delta b = 0.000161$$

故

$$\zeta = (1.01 \pm 0.16) \times 10^{-3}$$

$$S_{10\overline{T_d}} = \sqrt{\frac{\sum (T_{d_i} - \overline{T_d})^2}{n-1}} = 0.00041$$

由于  $\Delta_B = 0.001s$  故,

$$\Delta S_{10\overline{T_d}} = \sqrt{S_{10\overline{T_d}}^2 + \Delta_B^2} = 0.00108$$

$$\Delta T_d = \frac{\Delta T_d}{10} = 0.000108$$

故,

$$T_d = (1.4549 \pm 0.0001)s$$

$$\omega_0 = \frac{2\pi}{T_d\sqrt{1-\zeta^2}} = 4.3186$$

$$\frac{\Delta\omega_0}{\omega_0} = \sqrt{\left(\frac{\Delta T_d}{T_d}\right)^2 + \left(\frac{\zeta\Delta\zeta}{1-\zeta}\right)^2} = 0.000177$$

$$\Delta\omega_0 = \frac{\Delta\omega_0}{\omega_0}\omega_0 = 0.000765s^{-1}$$

故,

$$\omega_0 = (4.3186 \pm 0.0008)s^{-1}$$

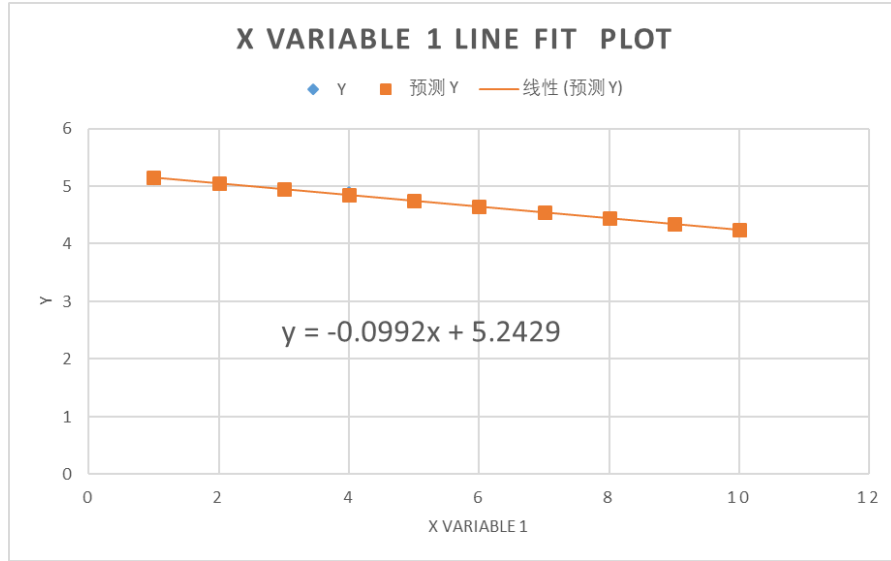
综上所述, 无阻尼时:

$$b = (-6.34 \pm 0.37) \times 10^{-3}, \quad \zeta = (1.01 \pm 0.16) \times 10^{-3},$$

$$T_d = (1.4549 \pm 0.0001)s, \quad \omega_0 = (4.3186 \pm 0.0008)s^{-1}$$

(2) 电磁阻尼 2 档时:

序号	$\theta_j / ^\circ$	$\ln\theta_j$	$\overline{T_d} / s$	序号	$\theta_j / ^\circ$	$\ln\theta_j$	$\overline{T_d} / s$	$D_j = \ln\theta_{j+5} - \ln\theta_j$
1	169	5.130	1.452	6	104	4.644	1.460	-0.486
2	154	5.037	1.454	7	95	4.554	1.462	-0.493
3	140	4.942	1.456	8	85	4.443	1.463	-0.499
4	127	4.884	1.458	9	77	4.344	1.463	-0.540
5	115	4.745	1.459	10	70	4.248	1.465	-0.497



a. 对  $y_i = \ln \theta_i$  和  $i$  进行回归分析得到拟合直线:

$$y = -0.0992x + 5.2429$$

用逐差法计算阻尼比  $\zeta$ , 读取振幅值  $\theta_1, \theta_2 \dots \theta_n$  则,

$$b = \ln \theta_j - \ln \theta_{j-1} = -\beta T_d = -\beta \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}} = -\frac{2\pi}{\sqrt{\zeta^{-2} - 1}}$$

$$b = \frac{\bar{D}}{I} = \frac{\sum (y_{j+I} - y_j)}{I^2} = -0.1002$$

$$S_b = \frac{\sqrt{\sum (D_j - \bar{D})^2 / (I - 1)}}{I}$$

$$\Delta b = S_b = \frac{1}{n} \sqrt{\frac{\sum (D_i - \bar{D})^2}{n - 1}} = \frac{S_D}{25} = 0.0004$$

故

$$b = (-0.1002 \pm 0.0004)$$

由于  $b = -\frac{2\pi}{\sqrt{\zeta^{-2} - 1}}$ ,

$$\zeta = \left( \left( \frac{2\pi}{b} \right)^2 + 1 \right)^{-\frac{1}{2}} = 0.0159$$

$$\frac{d\zeta}{db} = \frac{4\pi^2}{b^3} \left( \left( \frac{2\pi}{b} \right)^2 + 1 \right)^{-\frac{3}{2}}$$

$$\Delta \zeta = \left| \frac{d\zeta}{db} \right| \Delta b = 0.00006$$

故

$$\zeta = 0.01594 \pm 0.0007$$

$$S_{\overline{T_d}} = \sqrt{\frac{\sum (T_{d_i} - \overline{T_d})^2}{n-1}} = 0.00042$$

由于  $\Delta_B = 0.001s$  故,

$$\begin{aligned}\Delta S_{\overline{T_d}} &= \sqrt{S_{\overline{T_d}}^2 + \Delta_B^2} = 0.00108 \\ \Delta T_d &= 0.00108\end{aligned}$$

故,

$$T_d = (1.4592 \pm 0.0011)s$$

$$\omega_0 = \frac{2\pi}{T_d \sqrt{1-\zeta^2}} = 4.3064$$

$$\begin{aligned}\frac{\Delta \omega_0}{\omega_0} &= \sqrt{\left(\frac{\Delta T_d}{T_d}\right)^2 + \left(\frac{\zeta \Delta \zeta}{1-\zeta^2}\right)^2} = 0.0026 \\ \Delta \omega_0 &= \frac{\Delta \omega_0}{\omega_0} \omega_0 = 0.0113s\end{aligned}$$

故,

$$\omega_0 = (4.3064 \pm 0.0113)s^{-1}$$

$$\tau = \frac{1}{\zeta \omega_0} = 14.563s$$

$$\frac{\Delta \tau}{\tau} = \sqrt{\left(\frac{\Delta \zeta}{\zeta}\right)^2 + \left(\frac{\Delta \omega_0}{\omega_0}\right)^2} = 0.004$$

故,

$$\tau = (14.563 \pm 0.072)s$$

$$Q = \frac{1}{2\zeta} = 31.357$$

$$\begin{aligned}\frac{\Delta Q}{Q} &= \sqrt{\left(\frac{\Delta \zeta}{\zeta}\right)^2} = 0.003 \\ Q &= (31.357 \pm 0.125)\end{aligned}$$

综上所述, 阻尼 2 档时:

$$b = (-0.1002 \pm 0.0004), \quad \zeta = 0.0159 \pm 0.0001,$$

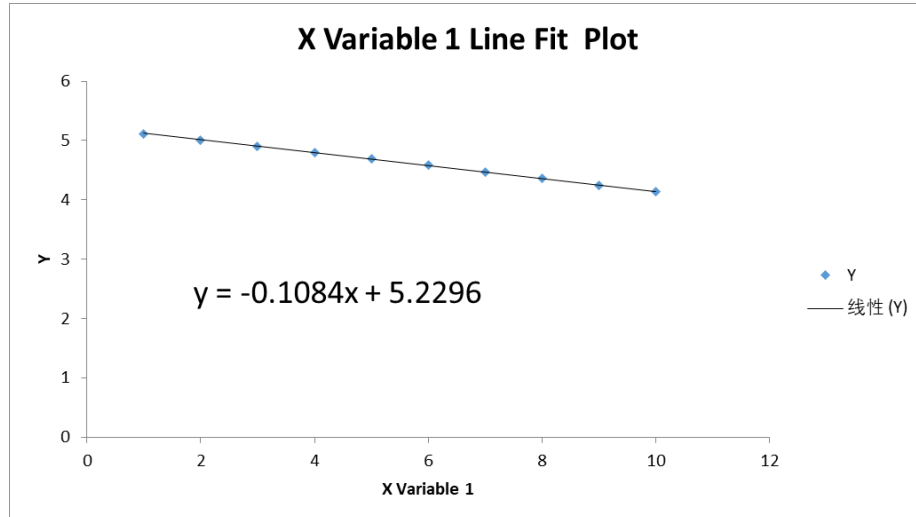
$$T_d = (1.4592 \pm 0.0042)s, \quad \omega_0 = (4.3064 \pm 0.0128)s^{-1}$$

$$\tau = (14.563 \pm 0.072)s, \quad Q = (31.357 \pm 0.125)$$

(3) 电磁阻尼 3 档时:

序号	$\theta_j / ^\circ$	$\ln \theta_j$	$\overline{T_d} / s$	序号	$\theta_j / ^\circ$	$\ln \theta_j$	$\overline{T_d} / s$	$D_j = \ln \theta_{j+5} - \ln \theta_j$
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1	167	5.118	1.452	6	98	4.585	1.462	-0.533
2	150	5.011	1.455	7	87	4.470	1.462	-0.541
3	135	4.905	1.457	8	79	4.369	1.464	-0.536
4	121	4.796	1.460	9	70	4.248	1.464	-0.548
5	109	4.691	1.460	10	63	4.143	1.465	-0.548



a. 对  $y_i = \ln \theta_i$  和  $i$  进行回归分析得到拟合直线:

$$y = -0.1084x + 5.2296$$

用逐差法计算阻尼比  $\zeta$ , 读取振幅值  $\theta_1, \theta_2 \dots \theta_n$  则,

$$b = \ln \theta_j - \ln \theta_{j-1} = -\beta T_d = -\beta \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}} = -\frac{2\pi}{\sqrt{\zeta^{-2} - 1}}$$

$$b = \frac{\bar{D}}{I} = \frac{\sum (y_{j+I} - y_j)}{I^2} = -0.1082$$

$$S_b = \frac{\sqrt{\sum (D_j - \bar{D})^2 / (I - 1)}}{I}$$

$$\Delta b = S_b = \frac{1}{n} \sqrt{\frac{\sum (D_i - \bar{D})^2}{n - 1}} = \frac{S_D}{25} = 0.0007$$

故

$$b = (-0.1082 \pm 0.0007)$$

由于  $b = -\frac{2\pi}{\sqrt{\zeta^{-2} - 1}}$ ,

$$\zeta = \left( \left( \frac{2\pi}{b} \right)^2 + 1 \right)^{-\frac{1}{2}} = 0.0172$$

$$\frac{d\zeta}{db} = \frac{4\pi^2}{b^3} \left( \left( \frac{2\pi}{b} \right)^2 + 1 \right)^{-\frac{3}{2}}$$

$$\Delta \zeta = \left| \frac{d\zeta}{db} \right| \Delta b = 0.0001$$

故

$$\zeta = 0.0172 \pm 0.0001$$

$$S_{\overline{T_d}} = \sqrt{\frac{\sum (T_{d_i} - \overline{T_d})^2}{n-1}} = 0.0042$$

由于  $\Delta_B = 0.001s$  故,

$$\begin{aligned}\Delta S_{\overline{T_d}} &= \sqrt{S_{\overline{T_d}}^2 + \Delta_B^2} = 0.0044 \\ \Delta T_d &= 0.0044\end{aligned}$$

故,

$$T_d = (1.4601 \pm 0.0044)s$$

$$\omega_0 = \frac{2\pi}{T_d \sqrt{1-\zeta^2}} = 4.3039$$

$$\begin{aligned}\frac{\Delta \omega_0}{\omega_0} &= \sqrt{\left(\frac{\Delta T_d}{T_d}\right)^2 + \left(\frac{\zeta \Delta \zeta}{1-\zeta^2}\right)^2} = 0.0030 \\ \Delta \omega_0 &= \frac{\Delta \omega_0}{\omega_0} \omega_0 = 0.0129s\end{aligned}$$

故,

$$\omega_0 = (4.3039 \pm 0.0129)s^{-1}$$

$$\tau = \frac{1}{\zeta \omega_0} = 13.494s$$

$$\frac{\Delta \tau}{\tau} = \sqrt{\left(\frac{\Delta \zeta}{\zeta}\right)^2 + \left(\frac{\Delta \omega_0}{\omega_0}\right)^2} = 0.007$$

故,

$$\tau = (13.494 \pm 0.007)s$$

$$Q = \frac{1}{2\zeta} = 29.039$$

$$\begin{aligned}\frac{\Delta Q}{Q} &= \sqrt{\left(\frac{\Delta \zeta}{\zeta}\right)^2} = 0.006 \\ Q &= (29.039 \pm 0.188)\end{aligned}$$

综上所述, 阻尼 3 档时:

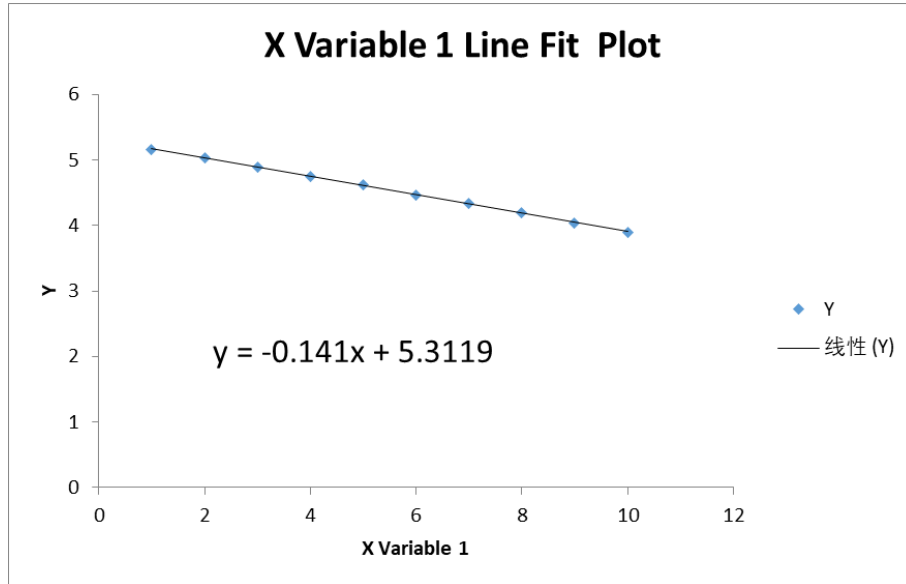
$$b = (-0.1082 \pm 0.0007), \quad \zeta = 0.0172 \pm 0.0001,$$

$$T_d = (1.4601 \pm 0.0044)s, \quad \omega_0 = (4.3039 \pm 0.0129)s^{-1}$$

$$\tau = (13.494 \pm 0.007)s, \quad Q = (29.039 \pm 0.188)$$

(4) 电磁阻尼 4 档时:

序号	$\theta_j / ^\circ$	$\ln \theta_j$	$\overline{T_d} / s$	序号	$\theta_j / ^\circ$	$\ln \theta_j$	$\overline{T_d} / s$	$D_j = \ln \theta_{j+5} - \ln \theta_j$
1	175	5.165	1.455	6	87	4.466	1.464	-0.699
2	153	5.030	1.457	7	76	4.331	1.465	-0.699
3	133	4.890	1.459	8	66	4.190	1.467	-0.700
4	115	4.745	1.462	9	57	4.043	1.467	-0.702
5	101	4.615	1.463	10	49	3.892	1.468	-0.723



a. 对  $y_i = \ln \theta_i$  和  $i$  进行回归分析得到拟合直线:

$$y = -0.1410x + 5.3119$$

用逐差法计算阻尼比  $\zeta$ , 读取振幅值  $\theta_1, \theta_2, \dots, \theta_n$  则,

$$b = \ln \theta_j - \ln \theta_{j-1} = -\beta T_d = -\beta \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}} = -\frac{2\pi}{\sqrt{\zeta^{-2} - 1}}$$

$$b = \frac{\overline{D}}{I} = \frac{\sum (y_{j+I} - y_j)}{I^2} = -0.1409$$

$$S_b = \frac{\sqrt{\sum (D_j - \overline{D})^2 / (I - 1)}}{I}$$

$$\Delta b = S_b = \frac{1}{n} \sqrt{\frac{\sum (D_i - \overline{D})^2}{n - 1}} = \frac{S_D}{25} = 0.0002$$

故

$$b = (-0.1409 \pm 0.0002)$$

由于  $b = -\frac{2\pi}{\sqrt{\zeta^{-2} - 1}}$ ,

$$\zeta = \left( \left( \frac{2\pi}{b} \right)^2 + 1 \right)^{-\frac{1}{2}} = 0.0224$$

$$\frac{d\zeta}{db} = \frac{4\pi^2}{b^3} \left( \left( \frac{2\pi}{b} \right)^2 + 1 \right)^{-\frac{3}{2}}$$

$$\Delta \zeta = \left| \frac{d\zeta}{db} \right| \Delta b = 0.00003$$



故

$$\zeta = 0.0224 \pm 0.0001$$

$$S_{\overline{T_d}} = \sqrt{\frac{\sum (T_{d_i} - \overline{T_d})^2}{n-1}} = 0.0044$$

由于  $\Delta_B = 0.001s$  故,

$$\begin{aligned}\Delta S_{\overline{T_d}} &= \sqrt{S_{\overline{T_d}}^2 + \Delta_B^2} = 0.0045 \\ \Delta T_d &= 0.0045\end{aligned}$$

故,

$$T_d = (1.4627 \pm 0.0045)s$$

$$\omega_0 = \frac{2\pi}{T_d \sqrt{1-\zeta^2}} = 4.2967$$

$$\begin{aligned}\frac{\Delta \omega_0}{\omega_0} &= \sqrt{\left(\frac{\Delta T_d}{T_d}\right)^2 + \left(\frac{\zeta \Delta \zeta}{1-\zeta^2}\right)^2} = 0.0031 \\ \Delta \omega_0 &= \frac{\Delta \omega_0}{\omega_0} \omega_0 = 0.0134s\end{aligned}$$

故,

$$\omega_0 = (4.2967 \pm 0.0134)s^{-1}$$

$$\tau = \frac{1}{\zeta \omega_0} = 10.381s$$

$$\frac{\Delta \tau}{\tau} = \sqrt{\left(\frac{\Delta \zeta}{\zeta}\right)^2 + \left(\frac{\Delta \omega_0}{\omega_0}\right)^2} = 0.003$$

故,

$$\tau = (10.381 \pm 0.035)s$$

$$Q = \frac{1}{2\zeta} = 22.302$$

$$\begin{aligned}\frac{\Delta Q}{Q} &= \sqrt{\left(\frac{\Delta \zeta}{\zeta}\right)^2} = 0.001 \\ Q &= (22.302 \pm 0.032)\end{aligned}$$

综上所述, 阻尼 4 档时:

$$b = (-0.1409 \pm 0.0002), \quad \zeta = 0.0224 \pm 0.0001,$$

$$T_d = (1.4627 \pm 0.0045)s, \quad \omega_0 = (4.2967 \pm 0.0134)s^{-1}$$

$$\tau = (10.381 \pm 0.035)s, \quad Q = (22.302 \pm 0.032)$$

## 2. 受迫振动

## (1) 电磁阻尼 2 档时:

	T/s	$\omega/\omega_0$	$\theta_m/^\circ$	$\phi_1/^\circ$	$\phi_2/^\circ$	$\phi = (\phi_1 + \phi_2)/2/^\circ$	$\phi/^\circ$ 理论值	相 对 偏 差 /%
1	1.430	1.020	68	149.0	149.5	149.2	141.6	5.32
2	1.439	1.014	87	138.5	139.5	139.0	131.0	6.09
3	1.445	1.010	105	127.5	128.5	128.0	121.3	5.52
4	1.448	1.008	119	118.5	119.5	119.0	115.5	3.01
5	1.452	1.005	128	107.0	108.0	107.5	106.9	0.55
6	1.455	1.003	133	98.0	99.0	98.5	99.9	-1.38
7	1.458	1.001	134	89.5	90.5	90.0	92.5	-2.75
8	1.461	0.998	133	80.5	82.0	81.3	85.1	-4.53
9	1.466	0.995	128	70.5	72.0	71.3	73.3	-2.76
10	1.473	0.990	117	60.0	61.0	60.5	59.1	-2.41
11	1.482	0.984	103	48.0	49.0	48.5	45.5	6.56
12	1.487	0.981	93	42.0	43.0	42.5	39.9	6.40
13	1.500	0.972	75	33.0	34.0	33.5	29.9	12.18

计算过程取:

$$\omega_0 = 4.3064, \quad \zeta = 0.0159$$

$$\frac{\omega}{\omega_0} = \frac{2\pi}{T\omega_0}$$

$$\phi = \arctan \frac{2\zeta(\omega/\omega_0)}{1 - (\omega/\omega_0)^2}$$

## (2) 电磁阻尼 3 档时:

	T/s	$\omega/\omega_0$	$\theta_m/^\circ$	$\phi_1/^\circ$	$\phi_2/^\circ$	$\phi = (\phi_1 + \phi_2)/2/^\circ$	$\phi/^\circ$ 理论值	相 对 偏 差 /%
1	1.427	1.023	63	148.5	149.0	148.8	142.9	4.09
2	1.435	1.017	76	139.5	140.5	140.0	134.9	3.71
3	1.442	1.012	90	131.5	132.0	131.8	125.6	4.91
4	1.447	1.008	104	119.5	120.5	120.0	117.2	2.33
5	1.450	1.006	113	112.5	113.5	113.0	111.6	1.30
6	1.454	1.004	119	102.5	103.5	103.0	103.2	-0.20
7	1.458	1.001	122	90.5	91.5	91.0	94.3	-3.48
8	1.465	0.996	119	77.5	78.5	78.0	78.4	-0.63
9	1.472	0.991	113	65.5	66.5	66.0	64.3	2.59
10	1.477	0.988	105	58.5	59.0	58.8	55.9	5.24
11	1.482	0.985	95	51.5	52.5	52.0	48.8	6.47
12	1.496	0.975	77	39.0	39.5	39.3	35.1	11.85
13	1.504	0.970	69	33.0	33.5	33.3	30.0	10.95

计算过程取：

$$\omega_0 = 4.3039, \quad \zeta = 0.0172$$

$$\frac{\omega}{\omega_0} = \frac{2\pi}{T\omega_0}$$

$$\phi = \arctan \frac{2\zeta(\omega/\omega_0)}{1 - (\omega/\omega_0)^2}$$

(4) 电磁阻尼 4 档时：

	T/s	$\omega/\omega_0$	$\theta_m/^\circ$	$\phi_1/^\circ$	$\phi_2/^\circ$	$\phi = (\phi_1 + \phi_2)/2/^\circ$	$\phi/^\circ$ 理论值	相对偏差/%
1	1.414	1.034	47	149.0	149.5	149.3	146.3	2.03
2	1.424	1.027	55	143.0	143.4	143.2	139.8	2.38
3	1.437	1.017	68	131.0	131.6	131.3	127.9	2.61
4	1.442	1.014	73	124.5	125.0	124.7	122.0	2.20
5	1.451	1.007	85	110.5	111.5	111.0	109.1	1.69
6	1.455	1.005	87	104.0	105.0	104.5	102.6	1.81
7	1.462	1.000	90	91.0	92.0	91.5	90.6	1.02
8	1.467	0.996	91	84.0	85.0	84.5	81.8	3.18
9	1.474	0.992	87	71.5	72.5	72.0	70.4	2.18
10	1.484	0.985	79	59.0	60.0	59.5	56.7	4.93
11	1.491	0.980	73	52.0	52.5	52.2	49.0	6.36
12	1.502	0.973	63	43.0	43.5	43.2	39.9	8.21
13	1.514	0.966	54	34.5	35.0	34.8	32.8	6.03

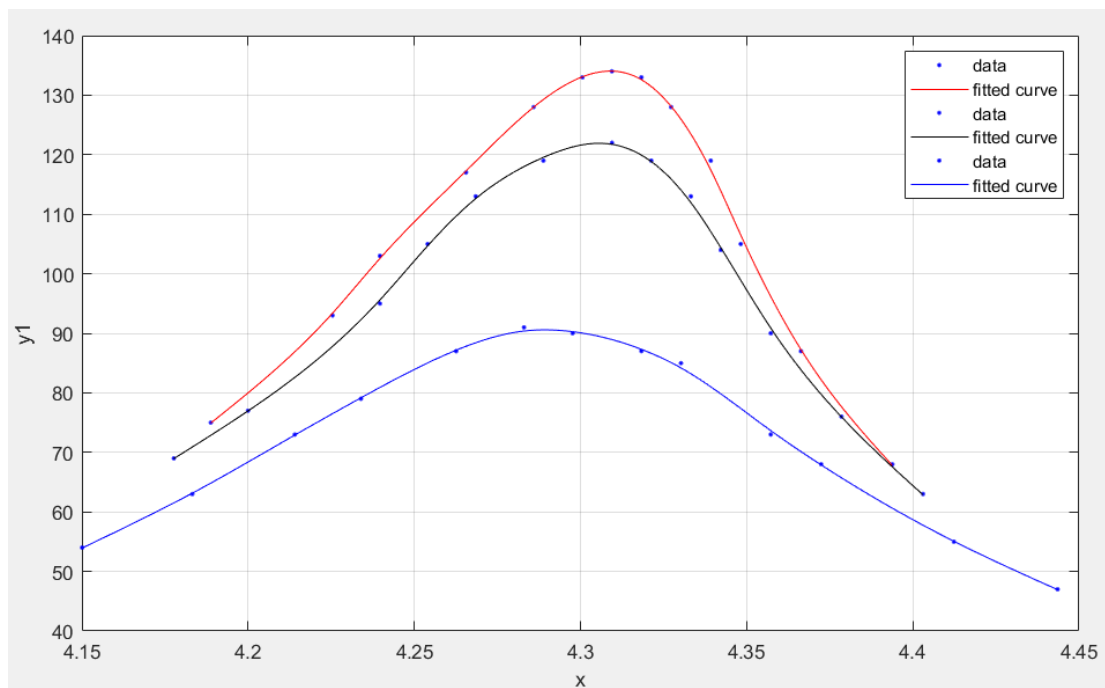
计算过程取：

$$\omega_0 = 4.2967, \quad \zeta = 0.0224$$

$$\frac{\omega}{\omega_0} = \frac{2\pi}{T\omega_0}$$

$$\phi = \arctan \frac{2\zeta(\omega/\omega_0)}{1 - (\omega/\omega_0)^2}$$

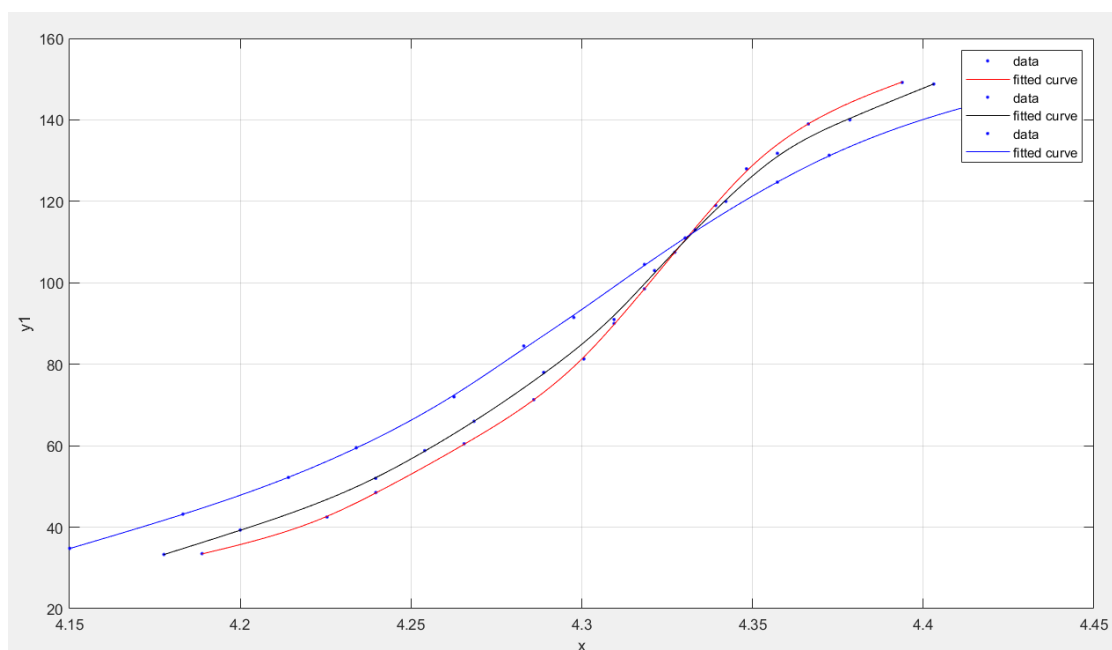
## 4 幅频特性曲线



红色：阻尼 2 档；黑色：阻尼 3 档；蓝色：阻尼 4 档

横坐标：角频率  $\omega$ ；纵坐标：幅值  $\theta$

## 5 相频特性曲线



红色：阻尼 2 档；黑色：阻尼 3 档；蓝色：阻尼 4 档

横坐标：角频率  $\omega$ ；纵坐标：相位差  $\phi$

## 6 思考题

1. 如何判断受迫振动已经处于稳定状态？

周期测量位于摆轮时，当显示窗中周期和振幅的示数都稳定时，受迫振动处于稳定状态。

2. 从幅频曲线的相对振幅比为  $1/2$  的点，也可求出  $\beta$  值。试用你的幅频特性曲线进行计算，把结果与练习 2 进行比较。

$$\frac{\theta_1}{\theta_2} = \frac{1}{2} \text{ 时,}$$

$$\frac{(\omega_0^2 - \omega_2^2)^2 + 4\beta^2\omega_2^2}{(\omega_0^2 - \omega_1^2)^2 + 4\beta^2\omega_1^2} = \frac{1}{4}$$

阻尼为 2 档时，取  $\theta_1 = 134$      $\omega_1 = 4.3158s^{-1}$ ,  $\theta_2 = 67$      $\omega_2 = 4.3925s^{-1}$ , 计算得：

$$\beta = 0.055$$

实验二中求得  $\beta = 0.068$ , 在误差允许范围内，与本题计算结果相符。

3. 实验中如何判断达到共振？共振频率是多少？

测得相位差，即闪光灯亮时有机玻璃盘上的读数为  $90$  度（相位差  $\pi/2$ ）时，达到共振。共振频率  $f$  与  $\omega_0$  近似相等。

阻尼为 2 时，约为  $0.6853\text{Hz} = 4.3064 \text{ rad/s}$

阻尼为 3 时，约为  $0.6850\text{Hz} = 4.3039 \text{ rad/s}$

阻尼为 4 时，约为  $0.6838\text{Hz} = 4.2967 \text{ rad/s}$

可以看到共振频率随着阻尼的差别而略有差别。

## 7 实验小结

本次实验进行十分顺利。通过本次实验，我巩固了物理课程中阻尼振动的相关知识，锻炼了自己数据处理分析的能力。在实验中，我曾有一个点与其他点的间隔大于  $10$  度，我在添加数据时，没有在原始数据下方添加，而是划去原来的错误数据，老师的提醒使我获得了经验：实验中的任何数据都要如实记录，不要因为多测了一组数据就抹去之前的数据。

谢谢老师的耐心指导！

#14

## 实验原始数据记录表格

1. 测量最小阻尼时的阻尼比  $\zeta$  和固有角频率  $\omega_0$ 

序号	$T_i = 10\overline{T_d} / s$	$\theta_j / ^\circ$	$\ln\theta_j$	序号	$T_i = 10\overline{T_d} / s$	$\theta_j / ^\circ$	$\ln\theta_j$	$D_j = \ln\theta_{j+25} - \ln\theta_j$
1	14.520	177	5.176	26	(续左下)	153	5.030	-0.146
2		176	5.170	27		152	5.024	-0.146
3		175	5.165	28		151	5.017	-0.148
4		174	5.159	29		150	5.011	-0.148
5		173	5.153	30		149	5.004	-0.149
6		172	5.147	31		147	4.990	-0.157
7		171	5.142	32		147	4.990	-0.152
8		171	5.142	33		146	4.984	-0.158
9		169	5.130	34		145	4.977	-0.153
10		169	5.130	35		144	4.970	-0.160
11	14.539	167	5.118	36	14.562	143	4.963	-0.155
12		167	5.118	37		143	4.963	-0.155
13		165	5.106	38		141	4.949	-0.157
14		165	5.106	39		141	4.949	-0.157
15		163	5.094	40		140	4.942	-0.152
16		163	5.094	41		139	4.934	-0.160
17		161	5.081	42		138	4.927	-0.154
18		161	5.081	43		137	4.920	-0.161
19		160	5.075	44		136	4.913	-0.162
20		159	5.068	45		135	4.905	-0.163
21	14.551	158	5.062	46	14.572	133	4.890	-0.173
22		157	5.056	47		132	4.883	-0.183
23		156	5.050	48		131	4.875	-0.175
24		155	5.043	49		130	4.868	-0.175
25		154	5.037	50		129	4.860	-0.177

2. 测量其他 2 种或 3 种阻尼状态的振幅, 求出  $\zeta$ .

阻尼 2:

电磁阻尼 2 档。

序号	$\theta_j / ^\circ$	$\ln\theta_j$	$\overline{T_d} / s$	序号	$\theta_j / ^\circ$	$\ln\theta_j$	$\overline{T_d} / s$	$D_j = \ln\theta_{j+6} - \ln\theta_j$
1	169	5.130	1.452	7	95	4.554	1.462	
2	154	5.037	1.454	8	85	4.443	1.463	
3	140	4.942	1.456	9	77	4.344	1.463	
4	127	4.844	1.458	10	70	4.248	1.465	
5	115	4.745	1.459	11				
6	<del>104</del>	4.644	1.460	12				

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## 阻尼 3:

电磁阻尼 3 档。

序号	$\theta_j / ^\circ$	$\ln \theta_j$	$\overline{T_d} / s$	序号	$\theta_j / ^\circ$	$\ln \theta_j$	$\overline{T_d} / s$	$D_j = \ln \theta_{j+6} - \ln \theta_j$
1	167	5.118	1.452	7	87	4.470	1.462	
2	150	5.011	1.455	8	79	4.369	1.464	
3	135	4.905	1.458	9	70	4.248	1.464	
4	121	4.796	1.460	10	63	4.143	1.465	
5	109	4.691	1.460	11				
6	98	4.585	1.462	12				

## 阻尼 4:

电磁阻尼 4 档。

序号	$\theta_j / ^\circ$	$\ln \theta_j$	$\overline{T_d} / s$	序号	$\theta_j / ^\circ$	$\ln \theta_j$	$\overline{T_d} / s$	$D_j = \ln \theta_{j+6} - \ln \theta_j$
1	175	5.165	1.455	7	76	4.331	1.465	
2	153	5.030	1.457	8	66	4.190	1.467	
3	133	4.890	1.459	9	57	4.043	1.467	
4	115	4.745	1.462	10	49	3.892	1.468	
5	101	4.615	1.463	11				
6	87	4.466	1.464	12				

## 3. 测定受迫振动的幅频特性和相频特性曲线

## 阻尼 2:

电磁阻尼 2 档。

T/s	$\omega/\omega_0$	$\theta_m / ^\circ$	$\phi_1 / ^\circ$	$\phi_2 / ^\circ$	$\phi = (\phi_1 + \phi_2)/2 / ^\circ$	$\phi / ^\circ$ 理论值	相对偏差 /%
1.414		47	149.0	149.5	149.3		
1.424		55	143.0	143.4	143.2		
1.437		68	131.0	131.6	131.3		
1.442		73	124.5	125.0	124.7		
1.451		85	110.5	111.5	111.0		
1.455		81	104.0	105.0	104.5		
1.462		90	91.0	92.0	91.5		
1.467		91	84.0	85.0	84.5		
1.474		87	71.5	72.5	72.0		
1.484		89	59.0	60.0	59.5		
1.491		73	52.0	52.5	52.2		
1.502		63	43.0	43.5	43.2		
1.514		54	34.5	35.0	34.8		

## 阻尼 3:

电磁阻尼 3 档。

T/s	$\omega/\omega_0$	$\theta_m/^\circ$	$\phi_1/^\circ$	$\phi_2/^\circ$	$\phi = (\phi_1 + \phi_2)/2/^\circ$	$\phi/^\circ$ 理论值	相对偏差 /%
1.427		63	148.5	149.0	148.8		
1.435		76	139.5	140.5	140.0		
1.442		90	131.5	132.0	131.8		
1.447		<del>103</del> 104	119.5	120.5	120.0		
1.450		113	112.5	113.5	113.0		
1.454		119	102.5	94.5	94.0	102.0	
1.458		122	90.5	91.5	91.0		
1.465		119	77.5	78.5	78.0		
1.472		113	65.5	66.5	66.0		
1.477		105	58.5	59.0	58.8		
1.482		95	51.5	52.5	52.0		
1.496		77	39.0	39.5	39.3		
1.504		69	33.0	33.5	33.3		

## 阻尼 4:

电磁阻尼 2 档。

T/s	$\omega/\omega_0$	$\theta_m/^\circ$	$\phi_1/^\circ$	$\phi_2/^\circ$	$\phi = (\phi_1 + \phi_2)/2/^\circ$	$\phi/^\circ$ 理论值	相对偏差 /%
1.430		68	149.0	149.5	149.2		
1.439		87	138.5	139.5	139.0		
1.445		105	127.5	128.5	128.0		
1.448		119	118.5	119.5	119.0		
1.452		128	107.0	108.0	107.5		
1.455		133	98.0	99.0	98.5		
1.458		<del>133</del> 134	<del>98.5</del> 99.5	<del>99.5</del> 100.5	99.5	90.0	
1.461		133	88.5	82.0	81.3		
1.466		128	70.5	72.0	71.3		
1.473		117	60.0	61.0	60.5		
1.482		103	48.0	49.0	48.5		
1.487		93	42.0	43.0	42.5		
1.500		75	33.0	34.0	33.5		

10.31 陈 J19