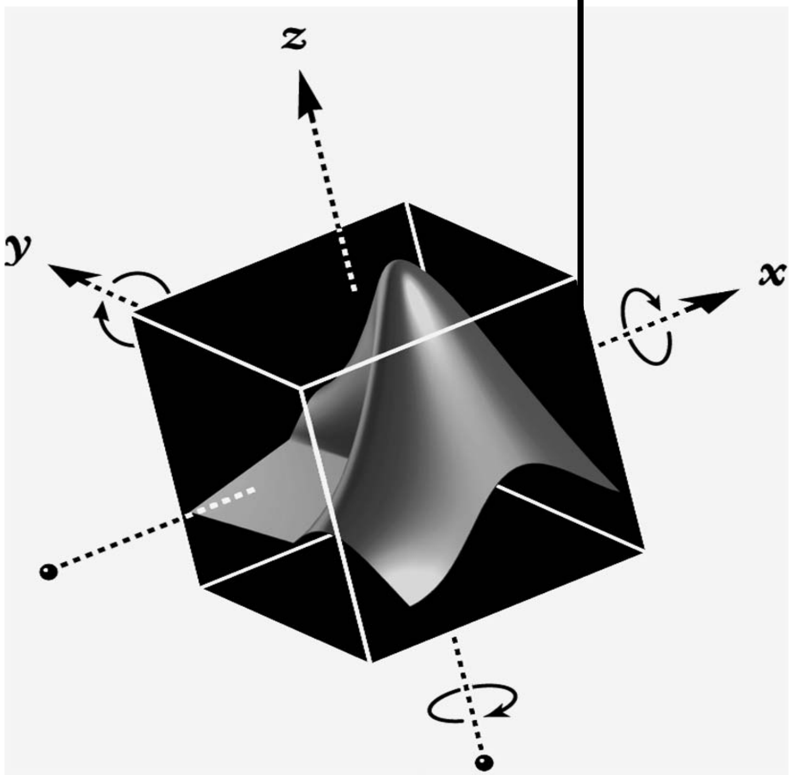
Generating Multivariate Gaussian



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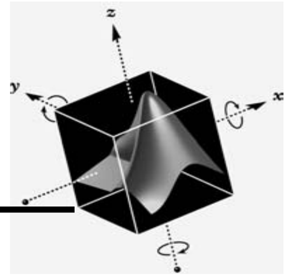
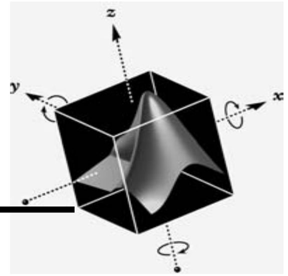


- We require n normal random numbers $\mathbf{X} = (X_1, X_2, ..., X_n)^T$ drawn from a multivariate normal distribution with

$$\boldsymbol{\mu}_{\mathrm{X}} = (\mu_1, \mu_1, ..., \mu_n)^T \qquad \mathbf{C}_{\mathrm{X}} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{12} & \sigma_2^2 & & \\ \vdots & & \ddots & \\ \sigma_{1n} & & & \sigma_n^2 \end{bmatrix}$$

Generating Multivariate Gaussian



- Generate n independent, zero-mean, unit variance normal random variables

$$\mathbf{Y} = (Y_1, Y_2, ..., Y_n)^T \qquad \boldsymbol{\mu}_Y = (0, 0, ..., 0)^T \qquad \mathbf{C}_Y = E\{\mathbf{Y}\mathbf{Y}^T\} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \end{bmatrix}$$

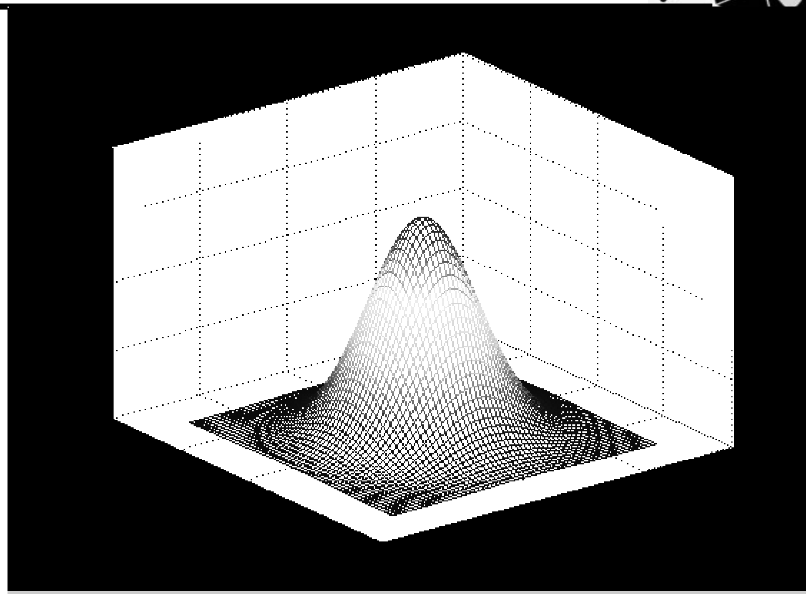
- Take $\mathbf{X} = \mathbf{S}\mathbf{Y} + \boldsymbol{\mu}_X$ where $\mathbf{C}_X = \mathbf{S}\mathbf{S}^T$ (Cholesky factorization)

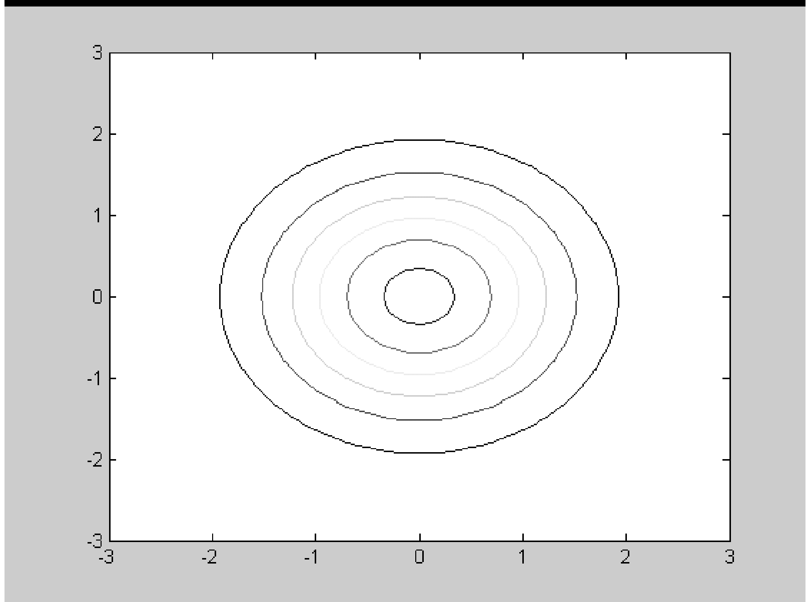
$$\mathbf{C}_X = E\{(\mathbf{X} - \boldsymbol{\mu}_{\mathrm{X}})(\mathbf{X} - \boldsymbol{\mu}_{\mathrm{X}})^T\} = E\{(\mathbf{S}\mathbf{Y})(\mathbf{S}\mathbf{Y})^T\} = E\{\mathbf{S}\mathbf{Y}\mathbf{Y}^T\mathbf{S}^T\} = \mathbf{S}E\{\mathbf{Y}\mathbf{Y}^T\}\mathbf{S}^T = \mathbf{S}\mathbf{S}^T$$

Generating Multivariate Gaussian

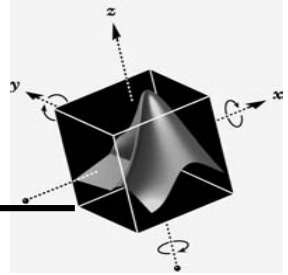
$$\mu_X = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad C_X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \rho = 0$$

```
mx=[0 0]';
Cx=[1 0; 0 1];
x1=-3:0.1:3;
x2=-3:0.1:3;
for i=1:length(x1),
    for j=1:length(x2),

   f(i,j)=(1/(2*pi*det(Cx)^1/2))*exp((
   -1/2)*([x1(i) x2(j)]-
   mx')*inv(Cx)*([x1(i);x2(j)]-mx));
    end
end
mesh(x1,x2,f)
pause;
contour(x1,x2,f)
pause
```



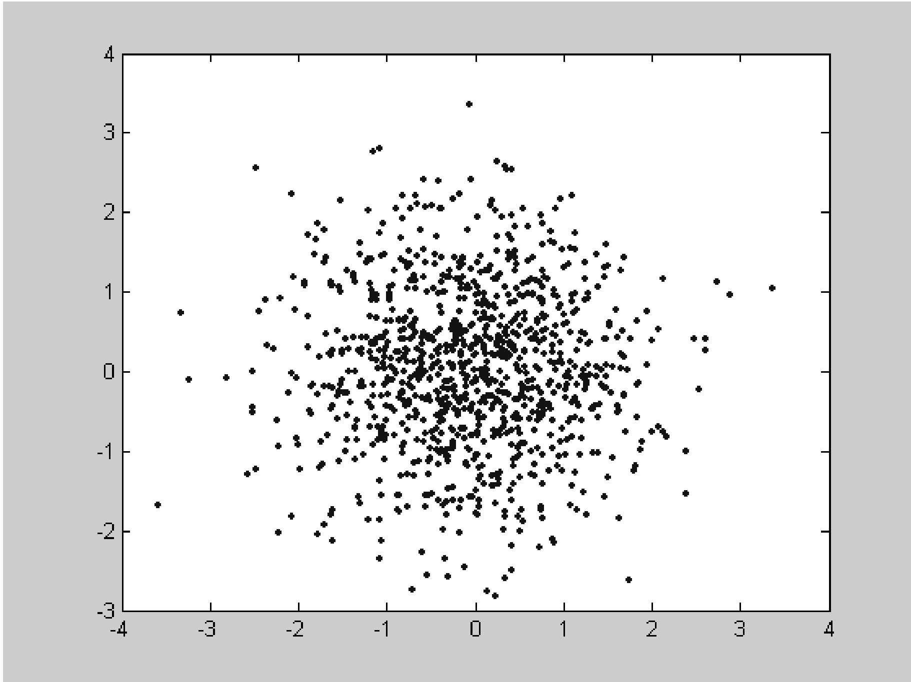


Generating Multivariate Gaussian



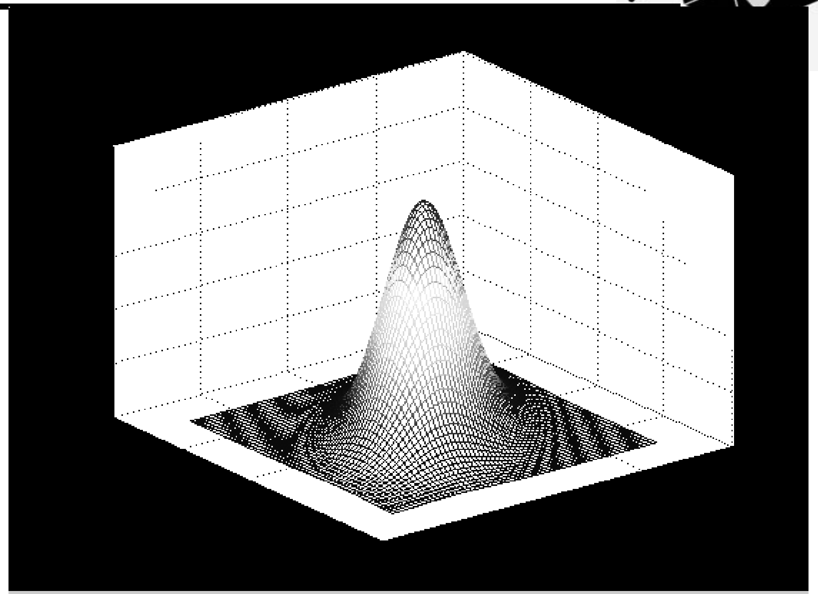
$$\mu_X = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad C_X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

```
N=1000;
[y1,y2]=gennormal(N,0,1);
plot(y1,y2,'.')
```

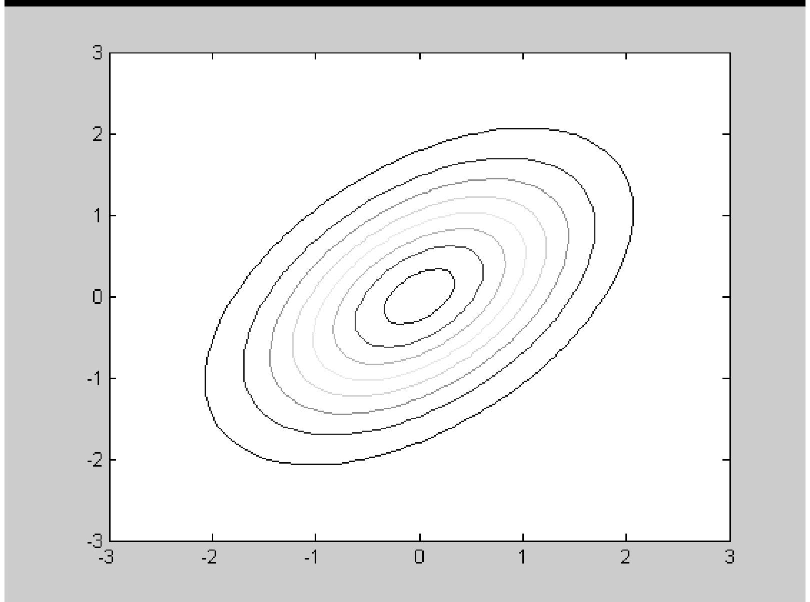


Generating Multivariate Gaussian

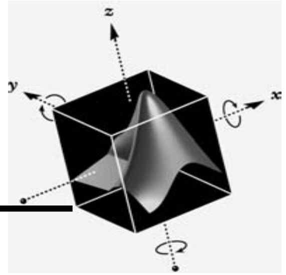
$$\mu_X = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad C_X = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} \qquad \rho = \frac{1}{2}$$



```
mx=[0 0]';
Cx=[1 1/2; 1/2 1];
```

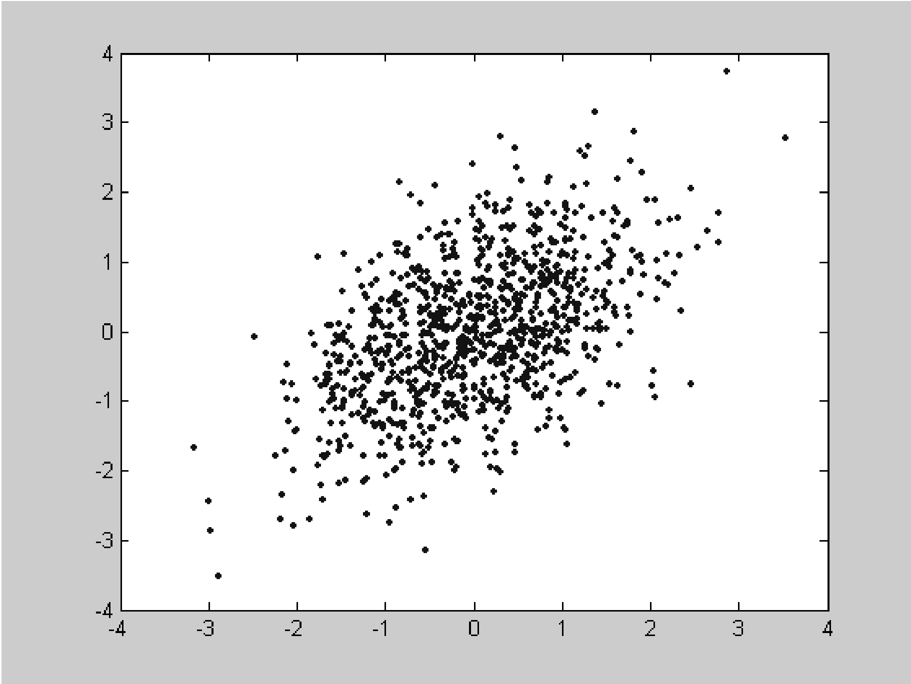


Generating Multivariate Gaussian



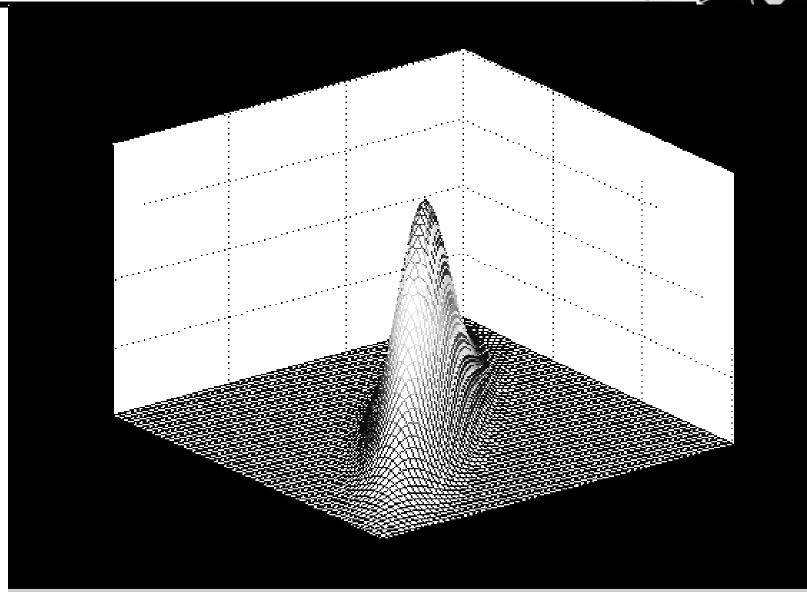
$$C_X = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

```
N=1000;
mx=zeros(2,N);
[y1,y2]=gennormal(N,0,1);

y=[y1;y2];
S=[1, 0; 1/2, sqrt(3)/2];
x=S*y+mx;
x1=x(1,:);
x2=x(2,:);
plot(x1,x2,'.')
r=corrcoef(x1',x2')

r =
    1.0000    0.5056
    0.5056    1.0000
```

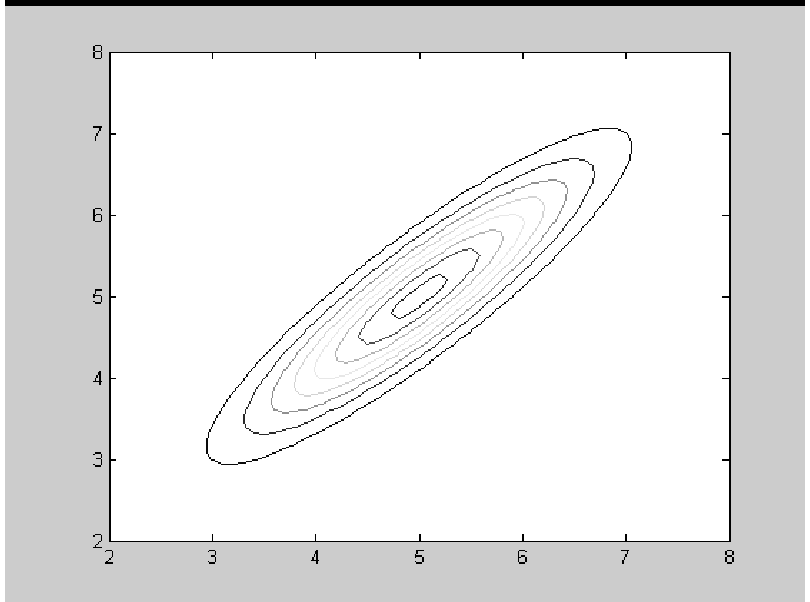


Generating Multivariate Gaussian

$$\mu_X = \begin{bmatrix} 5 \\ 5 \end{bmatrix} \quad C_X = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix} \quad \rho = 0.9$$



```
mx=[5 5]';
Cx=[1 9/10; 9/10 1];
```



Generating Multivariate Gaussian

$$C_X = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix} \quad S = \begin{bmatrix} 1 & 0 \\ \dfrac{9}{10} & \dfrac{\sqrt{19}}{10} \end{bmatrix}$$

```
N=1000;
mx=5*ones(2,N);
[y1,y2]=gennormal(N,0,1);

y=[y1;y2];
S=[1, 0; 9/10, sqrt(19)/10];
x=S*y+mx;
x1=x(1,:);
x2=x(2,:);
plot(x1,x2,'.')
r=corrcoef(x1',x2')

r =
    1.0000    0.9041
    0.9041    1.0000
```