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Mesh distribution system analysis in presence of distributed generation with time varying load model

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ABSTRACT

With the implementation of competitive electricity supply system all over the world, there is the need of optimal utilization of the existing generation sources at the same time to curtail the generation from the conventional power plants and add the renewable sources generation looking into the environmental concern and limitation of the fossil fuels. This has lead to the motivation for the studies on the integration of distributed resources to the grid. In this paper optimal locations and sizes for DG is determined for weakly meshed distribution networks based on the sensitivity method. Novel method based on loss sensitivity is used in this paper to determine optimal size and location of DGs. The modified Novel method is proposed for DG allocation. The main contribution of the paper is: (i) distributed generation allocation for mesh network using sensitivity approach, (ii) modified Novel method for DG allocation and sizes calculation for meshed distribution system with load variations, (iii) comparison of the results obtained with single and two DG placement with load variations, (iv) the loss savings and overall cost savings per annum with single and two DGs placement with load variations. In this paper we considered the impact of time varying load flow with realistic load model. The realistic ZIP load model has been considered for study. The results have been obtained for a distribution network of UK Distribution Corporation consisting of 38 buses. The results have also been obtained for radial distribution system for comparison.

Introduction

With technological improvements micro-turbines, fuel cells, mini-hydro, battery storage etc., this has provided an opportunity for large-scale integration of these generation sources called as distributed energy sources (DERs) into distribution systems. These distributed generation sources are modular in design and can be deployed near load sites to address increasing power demand of the current electric utilities [1–6]. The integration of the DGs may provide technical as well as economical benefits by supplying loads during peak load periods, when the cost of electricity is higher. DG can best serve as a price hedging mechanism in real time pricing mechanism in the new competitive electricity market regime. However, penetration and viability of DG at a particular location is influenced by technical as well as economic factors. The technical merits of DG integration include voltage support, energy-loss reduction, release of system capacity, and improve utility system reliability [1,4–6]. Economical merit, on the other hand, encompasses hedge against high electricity price. The distributed generation renewable resources such as; small hydro, wind, solar energy can be integrated into distribution systems with the several issues related to technical barriers that are challenging. Distributed Generation is electricity production that is on-site or close to the load center to avoid the need of the network expansions in order to cover new load centers and to support the increased energy transfer which would be necessary for satisfying consumers increased demand. DG can be an alternative for residential, commercial, and industrial applications. However, distributed generation can be defined in a variety of ways as reported in the literature [1–4]. The impact of DG on radial distribution network is explained i.e., voltage support, loss reduction, and distribution capacity release and power quality issues in [5]. There are many reasons behind the increasingly widespread use of DG deferring the Transmission and Distribution (T&D) costs, good efficiencies especially in cogeneration and in combined cycles, are reduced, creating opportunities for new utilities in the power generation sector, provides a flexible way to choose a wide range of combinations of cost and reliability [6]. DG impacts different parameters of a power system, comprising voltage profile, line losses, and short circuit current, amount of injected harmonic, and system reliability and stability and the installation of DG units should be allocated in an optimal way to maximize the system

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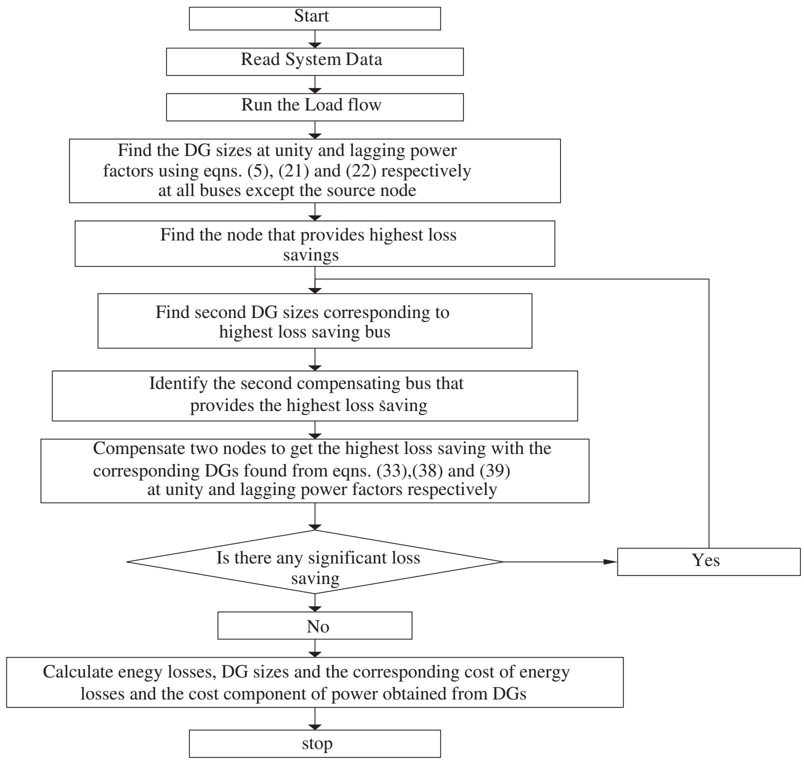

Fig. 1. Flow chart for DG allocation.

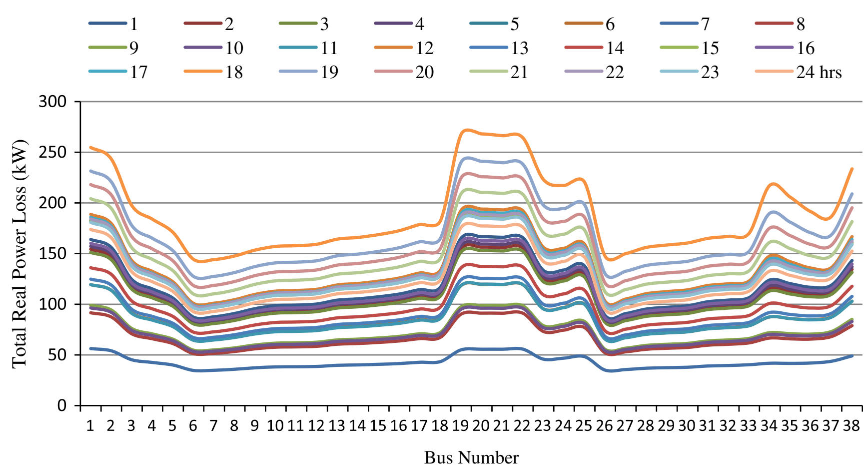

Fig. 2. Total real power loss variation with single.

efficiency. To analyze the distributed energy resources (DER) impacts, different types of 'generator groups' can be considered [6].

In [7] a new method has been suggested based on nodal pricing for optimally allocating DG in radial distribution system. Authors in [8] presented a method for optimal sitting and sizing of multiple distributed generators (DGs) using particle swarm optimization (PSO) based approach. A simple and effective cumulative performance index, utilizing voltage profile improvement, loss reduction, and voltage stability index improvement is considered. Loss sensitivity factor, based on equivalent current injection method for sizing and sitting of DG in radial distribution system is given in [9]. Calculation of cost of DG is given in [10] based on conventional, triangular, and complex power limit. Authors in [11] described a technique for selection of buses in a sub transmission system for location of distributed generation (DG) and determination of their optimum capacities by minimizing transmission losses. The buses have been selected based on incremental voltage (dV/dP) sensitivities. Ref. [12] presented two new methodologies for optimal placement of distributed generation sources using an optimal power flow (OPF) based model in real time wholesale electricity market. The problem of optimal placement, including size, is formulated for two different objectives, namely, social welfare maximization and profit maximization. The candidate locations for DG placement are identified on the basis of locational marginal price (LMP). Optimal sizing and sitting decisions for DG capacity planning using heuristic approach was proposed in [13]. A multi-objective optimization approach using evolutionary algorithm with an objective of minimizing cost of energy losses, network upgrading and service interruptions for sizing and sitting of DG in distribution systems has been presented in

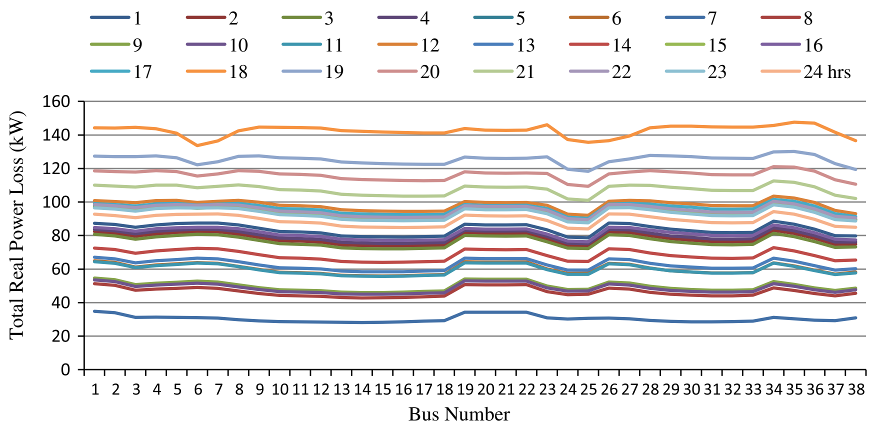

Fig. 3. Total real power loss variation with two DGs.

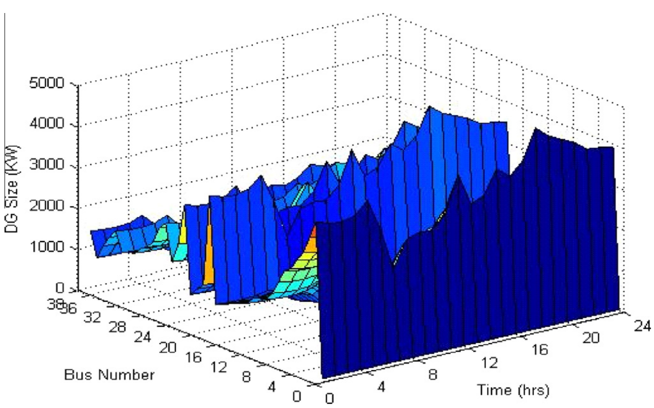

Fig. 4a. DG sizes for 38 bus radial system at unity power factor.

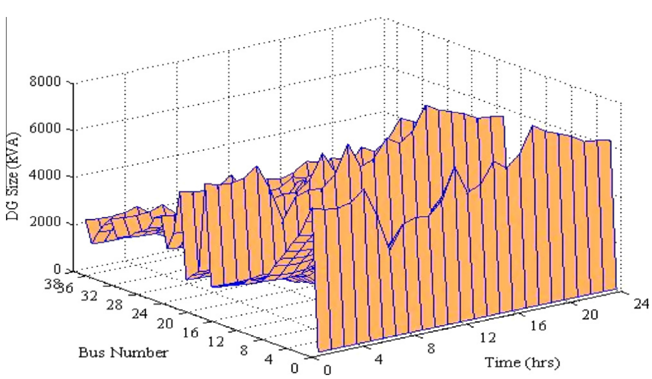

Fig. 4b. DG sizes for 38 bus radial system at 0.9 power factor lag.

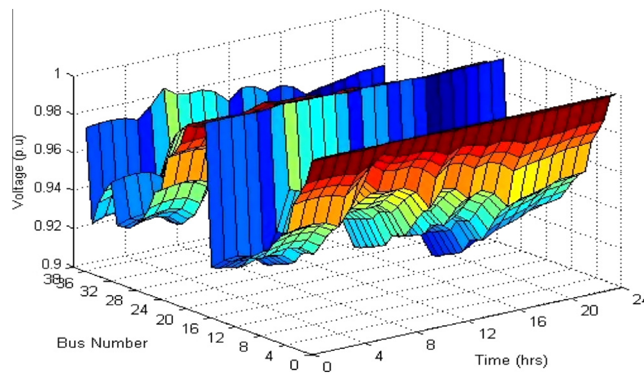

Fig. 5a. Voltage without DG for 38 bus radial system.

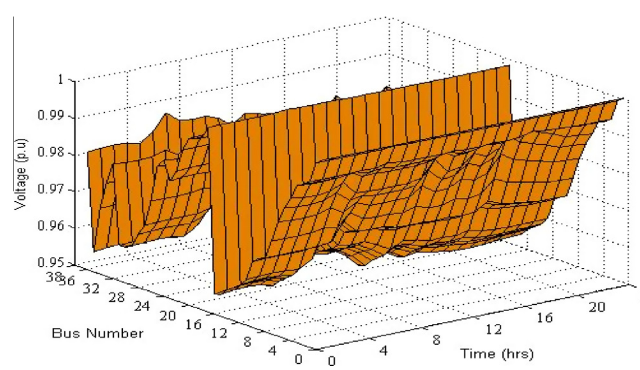

Fig. 5b. Voltage with single DG at unity pf for 38 bus radial system.

[14]. An analytical expression based on real power loss sensitivity to calculate optimal DG size and optimal location of DG minimizing power losses in a distribution network was proposed in [15]. A new methodology given in [16] using Fuzzy and Artificial Immune System (AIS) for the placement of Distributed Generators (DGs) in a radial distribution system to reduce the real power losses and to improve the voltage profile. In the first stage, the Fuzzy Set approach is used to find the optimal DG locations and in the second stage, Clonal Selection algorithm of AIS is used to size the DGs corresponding to maximum loss reduction. This algorithm is a new, population based, optimization method inspired by the cloning principle of the human body immune system. Paper [17] deals with impact of voltage dependent load models on the predicted energy losses in DG planning. A multi-objective optimization approach considering losses reduction and voltage profile improvement for DG allocation using GA was proposed in [18]. In [19] describes a novel methodology to calculate optimal DG sizes based on real power loss. An analytic method which can be used to determine the optimal placement and sizing of DG without use of admittance, impedance or Jacobian matrix with only one power flow for radial systems is presented in [20]. Ref. [21] presented an approach to determine the optimal sitting and sizing of DG with multi system constraints to achieve a single or multi-objectives using genetic algorithm (GA). It deals with the benefits (voltage profile improvement, spinning reserve increasing, power

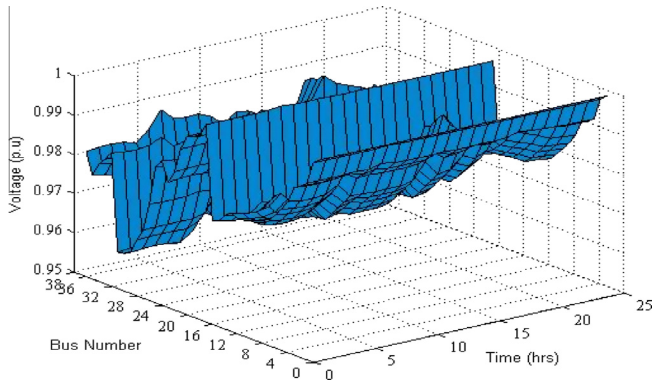

Fig. 5c. Voltage with two DGs at unity pf for 38 bus radial system.

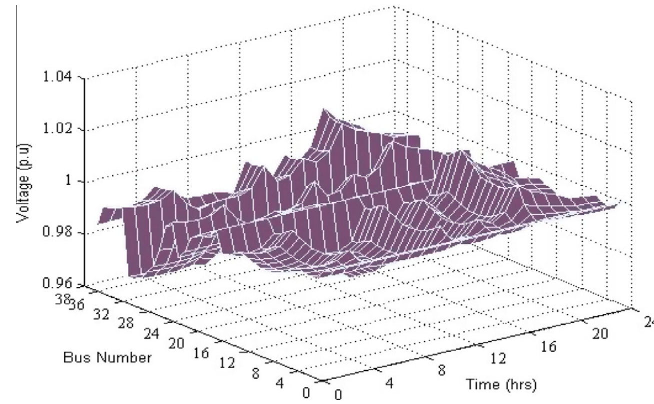

Fig. 5e. Voltage with two DGs at 0.9 pf lag for 38 bus radial system.

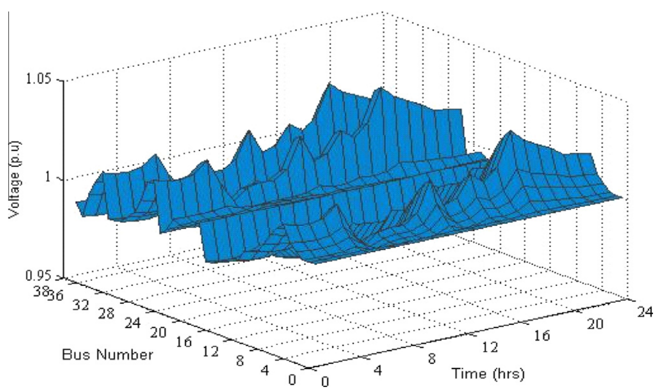

Fig. 5d. Voltage with single DG at 0.9 pf lag for 38 bus radial system.

flow reduction and total line loss reduction) obtained with optimal DG installation. Authors in [22] presented a simple method for investigating the problem of optimal location and capacity of DG in three-phase unbalanced radial distribution systems (URDS) for power loss minimization and to improve the voltage profile of the system using voltage index (VSI) analysis. Loss sensitivity factors (LSF) are used [23] to select the candidate locations for the multiple DG placements and Simulated Annealing (SA) is used to estimate the optimal size of DGs at the optimal locations. A new method for optimal sizing and sitting of DG in radial distribution systems was proposed in [24]. In this, optimal location for DG obtained by power loss sensitivity and optimal size is given by Harmony Search Algorithm (HSA). In [25] optimal placement of DG is given based on loss sensitivity and voltage stability index. An investigation into the effect of load models on the predicted energy losses in DG planning with time varying load demand is presented in [26]. It describes detailed voltage dependent load model, for DG planning use, which considers three categories of loads: residential, industrial and commercial. A value based DG placement for service quality improvement is proposed in [27]. Borges et al. proposed distributed generation allocation for reliability, losses, and voltage improvements [28]. Kumar and Gao proposed mixed integer programming based approach for distributed generation placement using criteria of nodal price reduction along with losses in pool and hybrid electricity markets [29]. However, the method has been proposed for transmission system network. Based on the literature survey, it is observed that distributed generation allocation has been obtained based on the sensitivity and optimal power flow based approaches for radial distribution system. Most of the authors have concentrated on solving radial distribution networks with time invariant load. This paper has presented an effective approach for determining the optimal capacity and location of DG units in meshed power systems. The objective functions considered in the study were maximization of the system loading margin as well as the DISCO's profit [30]. The time varying load models impact needs to be addressed for distributed generation allocation for mesh network also. The load flow for the mesh system is based on the concept discussed in [31].

Identification-based adaptive voltage regulation (I-BAVR) is proposed in [32] which uses real-time identification of the

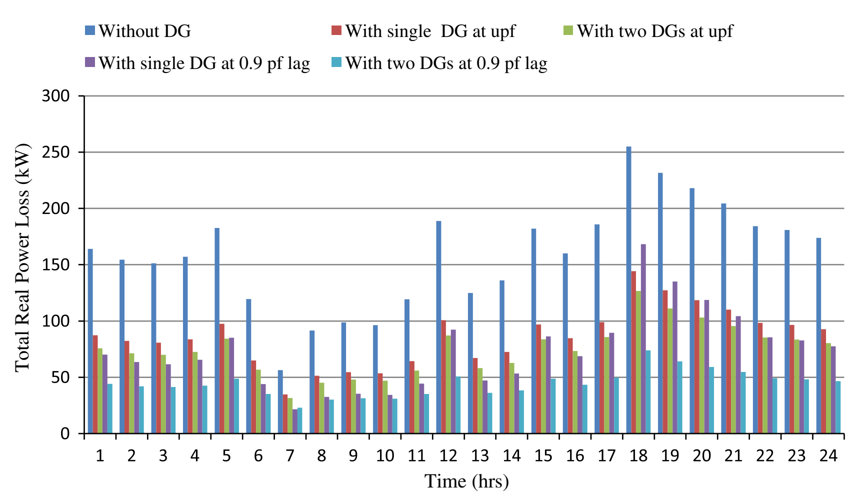

Fig. 6. Total real power loss for 38 bus radial system.

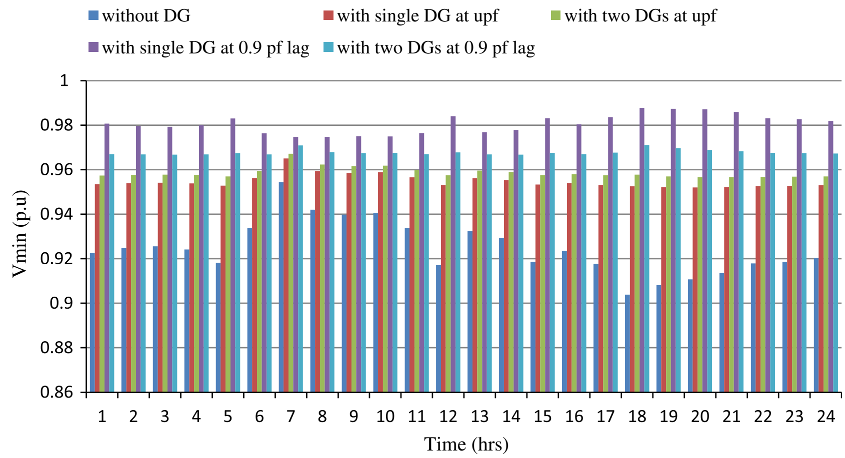

Fig. 7. Minimum bus voltage for 38 bus radial system.

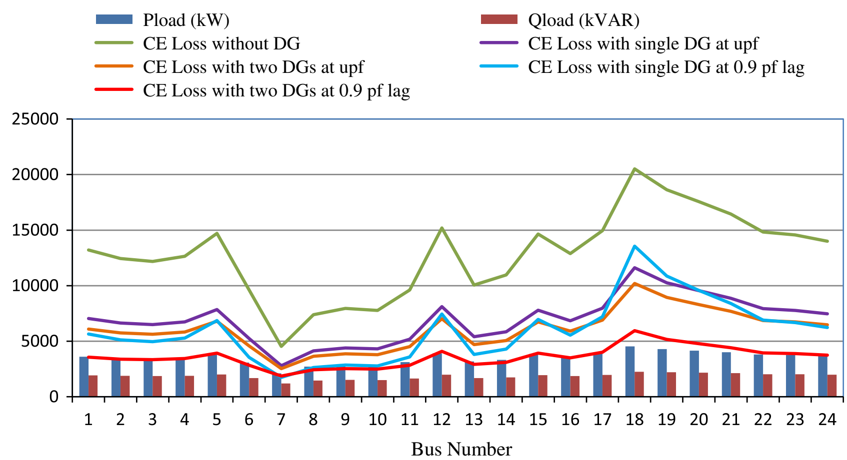

Fig. 8. Load variation and cost of energy loss for 38 bus radial system.

Table 1
Results without DG for 38 bus radial system.

Time (h)	Vmin (p.u)	TPL (kW)	TQL (kVAR)	Real power from the substation (kW)	Reactive power from the substation (kVAR)	Cost of energy losses ($)
1	0.9225	164	109.03	3759.8	2036.3	13201
2	0.92476	154.48	102.7	3645.8	1980.2	12435
3	0.92556	151.21	100.53	3607.8	1958.4	12172
4	0.92414	157.13	104.46	3683.7	1989.6	12648
5	0.91825	182.75	121.48	3987.6	2128	14711
6	0.93379	119.44	79.415	3190.8	1758.7	9614.4
7	0.95449	56.329	37.458	2170.6	1231.1	4534.2
8	0.94203	91.639	60.916	2811.7	1522.9	7376.5
9	0.93983	98.798	65.673	2925	1574.7	7952.8
10	0.94055	96.418	64.092	2887.2	1558.4	7761.2
11	0.93389	119.4	79.363	3227.7	1716.7	9611.1
12	0.91709	188.82	125.48	4100.4	2107.3	15199
13	0.93239	124.94	83.042	3303.5	1753.8	10057
14	0.92946	136.1	90.46	3455	1821.9	10955
15	0.91857	182.05	120.98	4024.3	2071.7	14654
16	0.92356	160.09	106.4	3758.5	1961.8	12886
17	0.91774	185.77	123.46	4062.4	2096	14954
18	0.90386	254.91	169.4	4786.3	2419.1	20519
19	0.90814	231.57	153.92	4520.3	2357	18640
20	0.91076	218.11	144.99	4368.3	2309.8	17557
21	0.91356	204.4	135.88	4216.2	2251	16453
22	0.91788	184.22	122.48	3988.1	2154.1	14829
23	0.91863	180.83	120.22	3950	2135.6	14556
24	0.92018	173.96	115.66	3873.9	2095.2	14003

thevenin equivalent circuit of the system, giving the X/R ratio to identify the active and reactive power dispatch of the DG unit. A new constrained multi objective Particle Swarm Optimization (PSO) is presented in [33], for Wind Turbine Generation Unit (WTGU) and photovoltaic (PV) array placement approach to minimize power loss and improve voltage stability in radial distribution system. A new two-layer simulation-based optimization (SBO) approach is proposed [34] to determine the optimal allocation and capacity of distributed energy resources (DER) in a power distribution system. In the first layer, a dynamic optimal power flow (DOPF) routine is embedded in a simulation algorithm to calculate the cost and reliability level of the system over one year. In the second layer, a particle swarm optimization (PSO) algorithm uses the outputs of the first layer to optimize the location and capacity of wind turbines, PV panels, and grid-scale batteries, in order to minimize cost while meeting reliability requirements. In ref. [35] authors presented a review of optimal DG planning in the distribution systems. In [36], a hybrid method based on improved particle swarm optimization (IPSO) algorithm and Monte Carlo simulation methodology for optimal distributed generation allocation and sizing in distribution systems is proposed to minimize the costs of active and reactive losses, and to improve the voltage profile and reliability of the distribution systems. Heuristic algorithm based on sensitivity indexes methodology is proposed in [37] for the optimal distributed generation allocation associated with the optimal reconfiguration in radial distribution networks to minimize energy losses. The data of the system which has been considered for the analysis is also provided in [38].

In this paper, locations and sizes for DG is determined for mesh distribution network based on the sensitivity method. The Novel

Table 2
Results with DG at unity and 0.9 pf (lag) for 38 bus radial system.

Time (h)	Single DG		Two DGs	
	Real power from the substation (kW)	Reactive power from the substation (kVAR)	Real power from the substation (kW)	Reactive power from the substation (kVAR)
1	1307.4	1864	1312.4	1874
2	1267.5	1818	1272.4	1827.4
3	1254.1	1799.7	1258.8	1808.8
4	1280.3	1824.7	1285.2	1834.3
5	1386.7	1935.7	1392.5	1946.9
6	1109.4	1632.8	1113.3	1639.8
7	756.1	1169.4	758.43	1172.2
8	976.55	1425.5	979.78	1430.7
9	1015.6	1470.2	1018.9	1475.8
10	1002.6	1456.2	1006	1461.7
11	1119.9	1591.1	1123.8	1598.2
12	1423.7	1908.4	1429.8	1920.1
13	1146.1	1622.6	1150.1	1630.1
14	1198.4	1679.1	1202.8	1687.4
15	1396.8	1880.2	1402.6	1891.5
16	1303.9	1793.9	1308.9	1803.8
17	1410.5	1900.4	1416.4	1911.9
18	1672	2144.3	1681.1	2159.5
19	1576.7	2110.2	1584.5	2124.3
20	1522.7	2078.4	1529.7	2091.7
21	1468.6	2035	1475	2047.5
22	1388	1960.2	1393.8	1971.5
23	1374.6	1945.4	1380.2	1956.4
24	1347.6	1912.4	1353	1923
Results with DG at 0.9 pf (lag) for 38 bus radial system				
	Single DG at 0.9 pf lag		Two DGs at 0.9 pf lag	
1	239.25	322.45	1209	783.23
2	224.64	314.41	1170.3	764.69
3	221.67	309.64	1158.7	756.02
4	234.47	310.39	1186.7	763.49
5	281.59	324.96	1294.7	804.43
6	170.9	287.92	1015.7	692.63
7	88.948	218.65	681.29	504.68
8	160.6	232.21	907.98	591.6
9	173.73	235.14	947.77	606.96
10	168.79	234.78	934.14	602.55
11	208.18	246.3	1052.3	650.47
12	338.19	277.94	1362.7	761.5
13	216.33	250.58	1077.7	662.64
14	236.74	254.85	1131.4	681.94
15	325.91	272.91	1335.9	750.41
16	278.62	268.11	1237.5	723.76
17	328.68	280.18	1346.9	761.2
18	474.64	330.03	1612.7	857.36
19	386.82	350.51	1492.4	867.1
20	344.99	355.11	1428.4	863.47
21	311.06	352.39	1369.3	850.76
22	267.74	343.94	1284.6	825.15
23	262.25	341.16	1271.5	819.04
24	253.08	333.06	1246.6	804.26

Table 3
Results with DG at unity pf and 0.9 (lag) pf 38 bus radial systems.

Time (h)	Single DG	Two DGs		Single DG	Two DGs
	kW	kW	kW	Cost of PDG ($/MWh)	Cost of PDG ($/MWh)
1	2375.8	1819.2	539.83	47.766	47.431
2	2306.3	1766	524.44	46.376	46.059
3	2283.2	1748.4	519.34	45.914	45.605
4	2329.9	1784.1	529.69	46.848	46.526
5	2515.6	1926	570.71	50.562	50.184
6	2026.9	1552.4	462.34	40.788	40.545
7	1393	1067.5	319.89	28.11	27.998
8	1794.8	1375	410.44	36.146	35.959
9	1865.3	1429	426.25	37.556	37.355
10	1841.8	1410.9	420.98	37.086	36.888
11	2052.8	1572.4	468.16	41.306	41.061
12	2588.7	1982.1	586.91	52.024	51.63
13	2099.5	1608.1	478.57	42.24	41.983
14	2193	1679.6	499.37	44.11	43.829
15	2542.4	1946.8	576.71	51.098	50.72
16	2379.3	1822.1	540.69	47.836	47.506
17	2565.2	1964.2	581.73	51.554	51.169
18	3003.7	2299.3	677.78	60.324	59.792
19	2839.5	2173.6	641.85	57.04	56.559
20	2746.1	2102.2	621.39	55.172	54.722
21	2653.3	2031.3	601.01	53.316	52.896
22	2514.2	1924.9	570.39	50.534	50.156
23	2491.1	1907.3	565.3	50.072	49.702
24	2445	1872.1	555.14	49.15	48.795

Time (h)	Single DG	Two DGs		Single DG		Two DGs	
	kVA	kVA	kVA	Cost of PDG ($/MWh)	Cost of QDG ($/MVARh)	Cost of PDG ($/MWh)	Cost of QDG ($/MVARh)
Results with DG at 0.9 pf lag 38 bus radial system							
1	3807.4	2130.1	571.02	68.783	6.6809	48.87	4.7464
2	3700.4	2070.6	555.17	66.857	6.4938	47.514	4.6147
3	3662.8	2049.5	549.74	66.18	6.4281	47.036	4.5683
4	3730.7	2087.1	560.08	67.403	6.5468	47.899	4.6521
5	4009.2	2241.5	601.73	72.416	7.0337	51.428	4.9949
6	3271.4	1831.8	491.27	59.135	5.7437	42.065	4.0853
7	2274.3	1275.4	342.49	41.187	4.000	29.372	2.8519
8	2880.1	1611.9	434.95	52.092	5.0594	37.093	3.6022
9	2986.6	1671	451.1	54.009	5.2457	38.448	3.7338
10	2951.7	1651.7	445.76	53.381	5.1846	38.004	3.6907
11	3271.8	1829.5	494.06	59.142	5.7444	42.074	4.0862
12	4073.1	2273.9	614.58	73.566	7.1455	52.243	5.0741
13	3343.8	1869.6	504.8	60.438	5.8703	42.989	4.1751
14	3483.9	1947.3	525.95	62.96	6.1152	44.769	4.348
15	4003.1	2235	604.17	72.306	7.0231	51.355	4.9878
16	3765.1	2103.5	568.1	68.022	6.6069	48.339	4.6948
17	4041.7	2256.7	609.61	73.001	7.0906	51.844	5.0353
18	4694.5	2618.4	706.72	84.751	8.232	60.102	5.8376
19	4485.6	2505.2	673.16	80.991	7.8667	57.46	5.581
20	4360	2436.5	653.53	78.73	7.6471	55.871	5.4265
21	4227.8	2363.6	633.41	76.35	7.416	54.196	5.2639
22	4024.1	2250.9	602.75	72.684	7.0598	51.616	5.0132
23	3988.7	2231.3	597.52	72.047	6.9979	51.169	4.9698
24	3915.9	2190.6	586.92	70.736	6.8706	50.245	4.88

method was proposed for unity power factor only [19], however, the Novel method has been modified at lagging power factor to obtain the location and size of DGs as DGs also supply reactive power also for better voltage profile meeting real power demand. The modified Novel method is proposed for DG allocation considering the loss reduction. The main contribution of the paper is: (i) distributed generation allocation for mesh network using sensitivity approach, (ii) modified Novel method for DG allocation and sizes calculation for mesh distribution system with load variations, (iii) comparison of the results obtained with single and two DG placement with load variations, (iv) the loss savings and overall cost savings with single and two DGs placement In this paper, we considered the impact of time varying load flow with realistic ZIP load model. The results have been obtained for the distribution network of UK Distribution Corporation consisting of 38 buses [17]. The results have also been obtained for radial distribution system for comparison.

Optimal location of DGs using Novel method

Distributed generation location based on Novel method was proposed for DG allocation at unity power factor based on real power loss minimization [19]. Only real power loss was considered in the proposed model [19]. The formulae were derived for unity power factor and real loss minimization. However, DG can also supply reactive power in a system and therefore it is essential to study the impact of reactive loss along with real power loss in the system. Load demand change not only impacts real power loss but also has impact on reactive power loss. In this paper, we have included the reactive power loss also along with real power loss for

Table 4
Results with DG at unity pf 38 bus radial system.

Time (h)	Single DG Vmin (p.u)	Two DGs Vmin (p.u)	Single DG TPL (kW)	Two DGs TPL (kW)	Single DG TQL (kVAR)	Two DGs TQL (kVAR)	Single DG Cost of energy losses ($)	Two DGs Cost of energy losses ($)
1	0.95344	0.95733	87.376	75.673	63.192	53.213	7033.4	6091.3
2	0.95391	0.95766	82.397	71.406	59.481	50.124	6632.6	5747.8
3	0.9541	0.95781	80.692	69.942	58.217	49.07	6495.3	5630
4	0.95382	0.95763	83.709	72.502	60.482	50.933	6738.2	5836.1
5	0.95283	0.957	97.516	84.43	70.786	59.579	7849.6	6796.2
6	0.95629	0.95952	65.028	56.767	46.52	39.532	5234.4	4569.5
7	0.96508	0.9672	34.796	31.522	24.27	21.533	2800.9	2537.4
8	0.95942	0.96228	51.346	45.216	36.455	31.284	4133.1	3639.7
9	0.95859	0.96158	54.665	47.948	38.926	33.251	4400.3	3859.6
10	0.95885	0.96179	53.57	47.051	38.108	32.603	4312.1	3787.4
11	0.9566	0.95993	64.393	56.011	46.187	39.076	5183.3	4508.6
12	0.95312	0.95748	100.79	87.217	73.399	61.713	8113.1	7020.6
13	0.95615	0.95956	67.078	58.259	48.191	40.701	5399.5	4689.6
14	0.95537	0.95896	72.53	62.828	52.277	44.018	5838.3	5057.4
15	0.95331	0.95758	96.877	83.764	70.489	59.216	7798.1	6742.6
16	0.95405	0.958	84.839	73.313	61.483	51.625	6829.1	5901.3
17	0.95316	0.95747	99.035	85.671	72.077	60.586	7971.8	6896.1
18	0.95253	0.95773	144.28	126.76	105.45	90.16	11614	10204
19	0.9521	0.95694	127.38	111.1	92.928	78.844	10253	8943
20	0.95206	0.95669	118.55	103.1	86.371	73.053	9542.7	8299.1
21	0.95219	0.95663	110.11	95.548	80.094	67.589	8863.3	7691.2
22	0.95262	0.95678	98.431	85.27	71.401	60.145	7923.2	6863.8
23	0.95273	0.95684	96.539	83.619	69.995	58.951	7770.9	6730.9
24	0.953	0.95702	92.748	80.319	67.184	56.569	7465.8	6465.3
Results with DG at 0.9 pf lag 38 bus radial system								
1	0.98071	0.96701	70.151	44.192	54.852	33.434	5646.8	3557.2
2	0.97964	0.96685	63.638	42.08	49.915	31.69	5122.6	3387.2
3	0.9793	0.96681	61.573	41.387	48.342	31.118	4956.3	3331.5
4	0.97995	0.9669	65.569	42.675	51.391	32.186	5278	3435.1
5	0.98303	0.96751	85.09	48.826	66.056	37.229	6849.3	3930.3
6	0.97638	0.96686	43.902	35.178	34.616	25.934	3533.9	2831.7
7	0.97476	0.9709	21.548	23.088	16.288	16.196	1734.5	1858.5
8	0.97476	0.96783	32.7	30.176	25.608	21.8	2632.2	2429
9	0.97508	0.9675	35.428	31.47	27.841	22.865	2851.8	2533.2
10	0.97496	0.9676	34.493	31.042	27.078	22.511	2776.5	2498.7
11	0.97641	0.96692	44.476	35.201	35.103	25.974	3580.1	2833.5
12	0.98403	0.96781	92.387	50.72	71.538	38.796	7436.7	4082.7
13	0.97685	0.96684	47.233	36.231	37.278	26.837	3802	2916.4
14	0.97786	0.96679	53.345	38.383	42.053	28.64	4294	3089.6
15	0.98313	0.96758	86.414	48.885	67.108	37.311	6955.9	3935
16	0.98039	0.96701	68.82	43.461	53.905	32.863	5539.7	3498.4
17	0.9836	0.9677	89.483	49.861	69.381	38.099	7203	4013.6
18	0.9878	0.97113	168.34	73.925	126.63	57.056	13551	5950.6
19	0.98733	0.96966	135.02	64.105	102.61	49.4	10868	5160.1
20	0.98711	0.96893	118.79	59.249	90.812	45.57	9562	4769.3
21	0.9859	0.9683	104.26	54.838	80.191	42.061	8392.4	4414.2
22	0.98316	0.96753	85.641	49.117	66.441	37.454	6893.7	3953.7
23	0.98273	0.96743	82.846	48.241	64.362	36.743	6668.7	3883.2
24	0.98189	0.96724	77.48	46.537	60.358	35.356	6236.8	3746

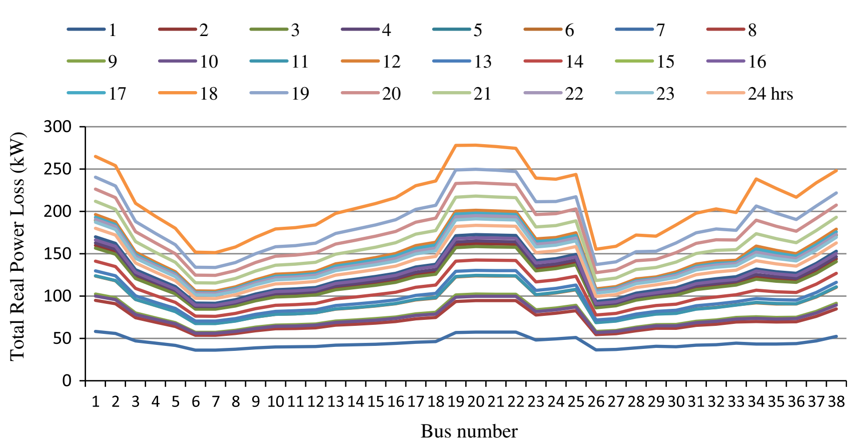

Fig. 9. Total real power loss variation with single DG for mesh system.

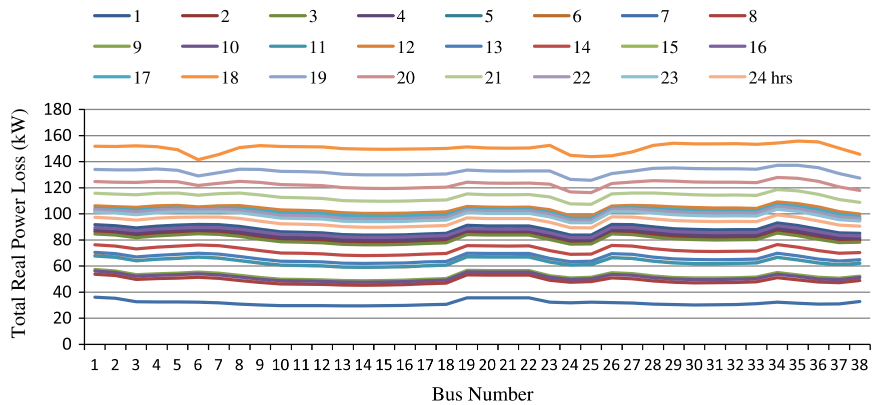

Fig. 10. Total real power loss variation with two DGs for mesh system.

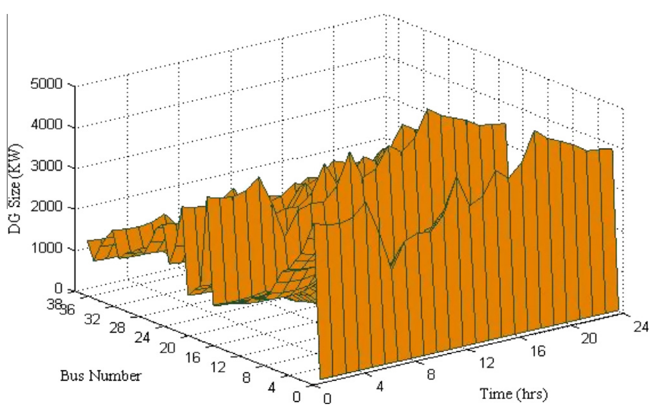

Fig. 11a. DG sizes for 38 bus mesh system at unity power factor.

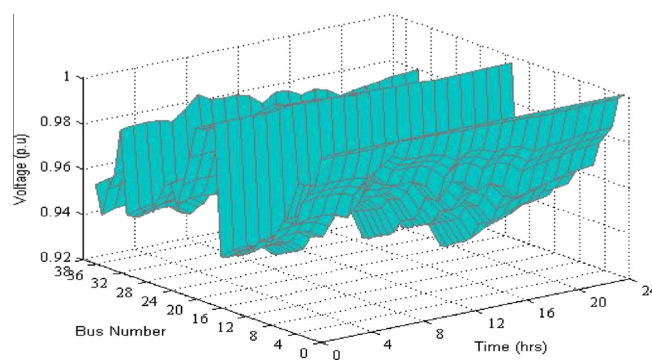

Fig. 12a. Voltage without DG for 38 bus for mesh system.

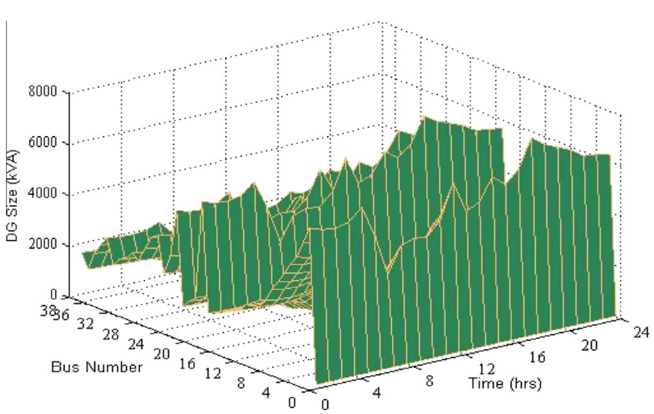

Fig. 11b. DG sizes for 38 bus mesh system at 0.9 power factor lag.

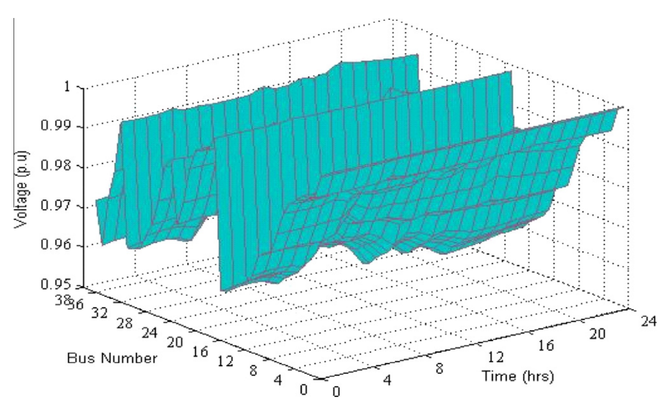

Fig. 12b. Voltage with single DG at unity pf for 38 bus mesh system.

calculating optimal DG sizes by considering both real and reactive power loss sensitivities into account. This method where both the real and reactive power losses have been included is named as "Modified Novel Method". Optimal DG sizes at unity and lagging power factors at each bus are obtained based on the minimum loss calculation. By placing these DG sizes at each bus except source node, run the load flow to plot the total real power loss variation with DG size. Then select the node at which loss saving is maximum and corresponding DG size is the optimal DG size.

TLP = total active power loss
TLQ = total reactive power loss
I_i = branch current
R_i = branch resistance
X_i = branch reactance
I_{ai} = active component of branch current
I_{ri} = reactive component of branch current
TLP_a = loss associated with active component of branch current
TLP_r = loss associated with reactive component of branch current

$$TLP = \sum_{i=1}^{br} I_i^2 R_i \quad (1)$$

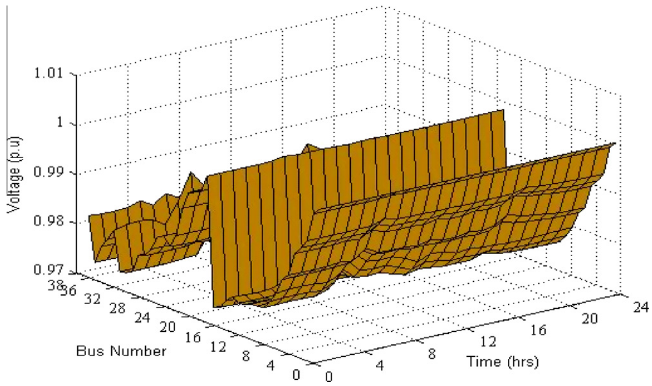

Fig. 12c. Voltage with two DGs at unity pf for 38 bus mesh system.

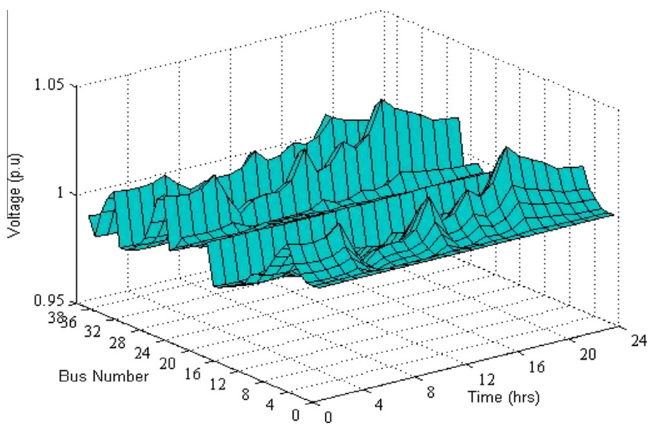

Fig. 12d. Voltage with single DG at 0.9 pf lag for 38 bus mesh system.

$$TLP = \sum_{i=1}^{br} I_a^2 R_i + \sum_{i=1}^{br} I_r^2 R_i \tag{2}$$

$$TLP = TLP_a + TLP_r \tag{3}$$

DG at unity power factor placed at bus 'k'(Novel Method)

I_{adgk} = active component of current supplied by DG at node 'k'.

$$TLP = \sum_{i=1}^{k} [I_{ai} + I_{adgk}]^2 R_i + \sum_{i=k+1}^{br} I_{ai}^2 R_i + \sum_{i=1}^{br} I_r^2 R_i \tag{4}$$

Subtract Eq. (2) from Eq. (4)

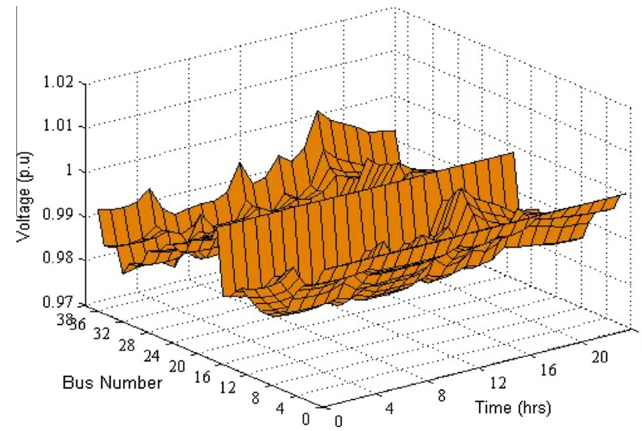

Fig. 12e. Voltage with two DGs at 0.9 pf lag for 38 bus mesh system.

$$\Delta TLP = \sum_{i=1}^{br} I_a^2 R_i + \sum_{i=1}^{br} I_r^2 R_i - \sum_{i=1}^{k} [I_{ai} + I_{adgk}]^2 R_i + \sum_{i=k+1}^{br} I_{ai}^2 R_i + \sum_{i=1}^{br} I_r^2 R_i$$
$$= \sum_{i=1}^{br} I_a^2 R_i - \sum_{i=1}^{k} [I_{ai} + I_{adgk}]^2 R_i$$

$$\frac{\partial \Delta TLP}{\partial I_{adgk}} = 0 \text{ or } -2I_{adgk} \sum_{i=1}^{k} R_i - 2\sum_{i=1}^{k} I_{ai} R_i = 0$$

For maximum loss saving required current to be supplied by DG is given by

$$I_{adgk} = -\frac{\sum_{i=1}^{k} I_{ai} R_i}{\sum_{i=1}^{k} R_i}$$

V_{dgk}= is the voltage magnitude of DG at node 'k'.

Optimal size of DG at unity power factor is given as

$$P_{dgk} = I_{adgk} * V_{dgk} \tag{5}$$

DG at lagging power factor placed at bus 'k' (proposed Modified Novel Method)

$$TLP = \sum_{i=1}^{k} [I_{ai} + I_{adgkp}]^2 R_i + \sum_{i=k+1}^{br} I_{ai}^2 R_i + \sum_{i=1}^{k} [I_{ri} + I_{rdgkp}]^2 R_i + \sum_{i=k+1}^{br} I_r^2 R_i \tag{6}$$

Subtract Eq. (2) from Eq. (6)

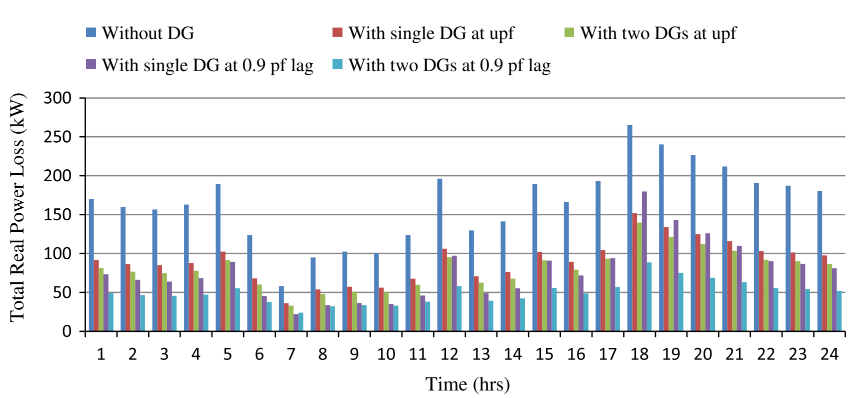

Fig. 13. Total real power loss for 38-bus mesh system.

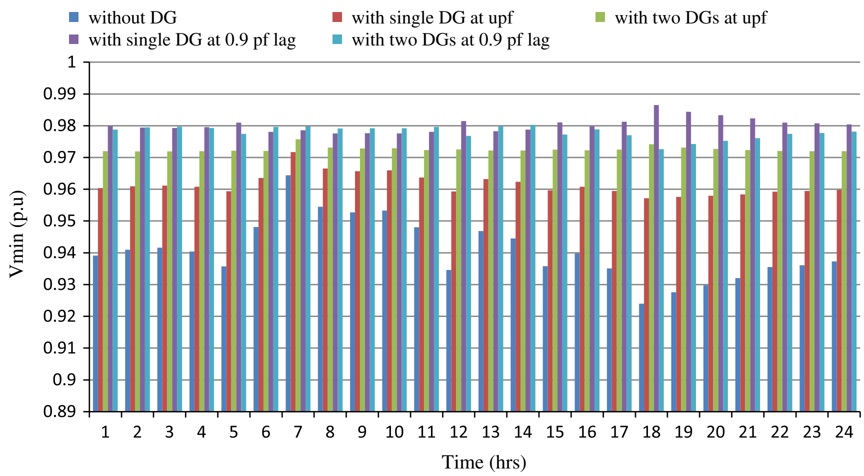

Fig. 14. Minimum bus voltage for 38-bus mesh system.

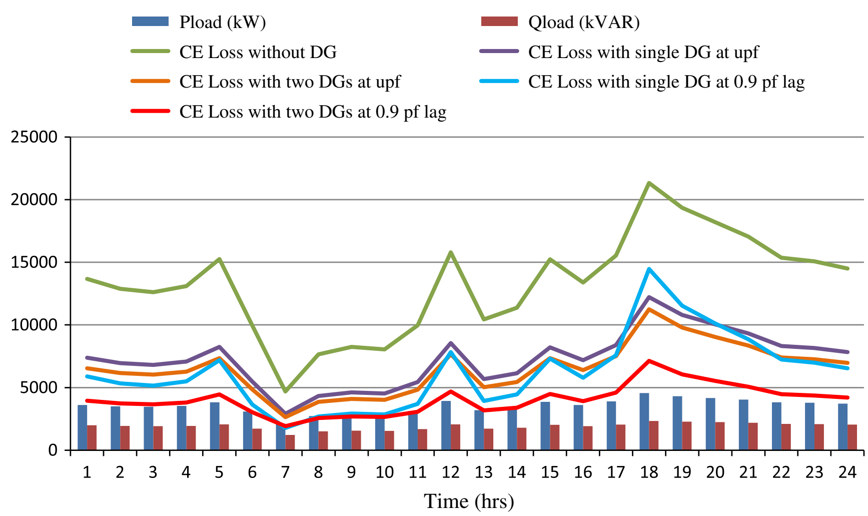

Fig. 15. Load variation and cost of energy loss for 38-bus mesh system.

$$\Delta TLP = \sum_{i=1}^{br} I_a^2 R_i + \sum_{i=1}^{br} I_r^2 R_i - \sum_{i=1}^{k} [I_{ai} + I_{adgkp}]^2 R_i - \sum_{i=k+1}^{br} I_{ai}^2 R_i - \sum_{i=1}^{k} [I_{ri} + I_{rdgkp}]^2 R_i - \sum_{i=k+1}^{br} I_r^2 R_i \tag{7}$$

$$\frac{\partial \Delta TLP}{\partial I_{adgk} p} = 0 \tag{8}$$

$$\text{and } \frac{\partial \Delta TLP}{\partial I_{rdgk} p} = 0 \tag{9}$$

From Eq. (7)

$$I_{adgkp} = -\frac{\sum_{i=1}^{k} I_{ai} R_i}{\sum_{i=1}^{k} R_i} \tag{10}$$

$$I_{rdgkp} = -\frac{\sum_{i=1}^{k} I_{ri} R_i}{\sum_{i=1}^{k} R_i} \tag{11}$$

$$TLQ = \sum_{i=1}^{br} I_i^2 X_i$$

$$TLQ = TLQ_a + TLQ_r, TLQ = \sum_{i=1}^{br} I_a^2 X_i + \sum_{i=1}^{br} I_r^2 X_i \tag{12}$$

$$TLQ = \sum_{i=1}^{k} [I_{ai} + I_{adgkq}]^2 X_i + \sum_{i=k+1}^{br} I_{ai}^2 X_i + \sum_{i=1}^{k} [I_{ri} + I_{rdgkq}]^2 X_i + \sum_{i=k+1}^{br} I_r^2 X_i \tag{13}$$

Eq. (13) can be written as:

$$\Delta TLQ = \sum_{i=1}^{br} I_a^2 X_i + \sum_{i=1}^{br} I_r^2 X_i - \sum_{i=1}^{k} [I_{ai} + I_{adgkq}]^2 X_i - \sum_{i=k+1}^{br} I_{ai}^2 X_i - \sum_{i=1}^{k} [I_{ri} + I_{rdgkq}]^2 X_i - \sum_{i=k+1}^{br} I_r^2 X_i \tag{14}$$

From Eq. (14), the equations can be derived for maximum loss savings as:

$$\frac{\partial \Delta TLQ}{\partial I_{adgk} q} = 0 \tag{15}$$

Table 5
Results without DG for 38-bus mesh system.

Time (h)	Vmin (p.u)	TPL (kW)	TQL (kVAR)	Real power from the substation (kW)	Reactive power from the substation (kVAR)	Cost of energy losses ($)
1	0.93915	169.88	114.17	3765.7	2041.4	13675
2	0.94094	160	107.53	3651.4	1985	12879
3	0.94157	156.63	105.26	3613.2	1963.2	12608
4	0.94042	162.82	109.42	3689.3	1994.6	13106
5	0.93572	189.5	127.34	3994.3	2133.8	15254
6	0.94811	123.59	83.071	3194.9	1762.4	9948.4
7	0.96442	58.169	39.105	2172.5	1232.8	4682.3
8	0.95449	95.026	63.858	2815	1525.9	7649.1
9	0.95274	102.49	68.873	2928.7	1578	8250
10	0.95332	100.01	67.202	2890.8	1561.5	8050.3
11	0.94802	123.95	83.286	3232.3	1720.6	9977.4
12	0.93458	196.3	131.87	4107.9	2113.7	15801
13	0.94683	129.71	87.152	3308.2	1758	10441
14	0.94449	141.35	94.969	3460.3	1826.4	11378
15	0.93577	189.26	127.14	4031.6	2077.8	15235
16	0.93978	166.33	111.75	3764.7	1967.2	13389
17	0.93512	193.09	129.72	4069.8	2102.2	15543
18	0.92399	265.06	178.04	4796.5	2427.7	21336
19	0.9276	240.41	161.52	4529.2	2364.6	19352
20	0.92976	226.24	152.02	4376.4	2316.8	18211
21	0.93203	211.9	142.39	4223.7	2257.5	17057
22	0.9355	190.84	128.25	3994.6	2159.8	15362
23	0.9361	187.32	125.89	3956.5	2141.3	15078
24	0.93732	180.21	121.11	3880.1	2100.7	14506

$$\frac{\partial \Delta TLQ}{\partial I_{rdgkq}} = 0 \tag{16}$$

Solving Eqs. (15) and (16), we get components of currents as:

$$I_{adgkq} = -\frac{\sum_{i=1}^{k} I_{ai} X_i}{\sum_{i=1}^{k} R_i} \tag{17}$$

$$I_{rdgkq} = -\frac{\sum_{i=1}^{k} I_{ri} X_i}{\sum_{i=1}^{k} R_i} \tag{18}$$

I_{adgk} = active component of the current to be supplied by DG for maximum loss saving at node 'k'.
I_{rdgk} = reactive component of the current to be supplied by DG for maximum loss saving at node 'k'.

$$I_{adgk} = \sqrt{I_{adgkp}^2 + I_{adgkq}^2} \tag{19}$$

$$I_{rdgk} = \sqrt{I_{rdgkp}^2 + I_{rdgkq}^2} \tag{20}$$

P_{dgk} = optimal real power supplied by DG at power factor cos ∅ at bus 'k'.
Q_{dgk} = optimal reactive power supplied by DG at power factor cos ∅ at bus 'k'.
S_{dgk} = optimal complex power supplied by DG at bus 'k'.

$$P_{dgk} = I_{adgk} * V_{dgk} * \cos\emptyset \tag{21}$$

$$Q_{dgk} = I_{rdgk} * V_{dgk} * \sin\emptyset \tag{22}$$

$$S_{dgk} = \sqrt{P_{dgk}^2 + Q_{dgk}^2} \tag{23}$$

Using Eqs. 5, 21, and 22, optimal DG sizes can be obtained for unity and lagging power factor respectively. Practical UK 38 bus test system is shown in Fig. 1.

Multiple DG allocation (proposed Modified Novel Method)

kdg = number of buses compensated by DG
I_{dg} = kdg dimensional vector consisting of DG currents
$\propto_j$ = Set of branches from the source bus to the jth DG bus (j = 1, 2… k)

If three DGs (k = 3) are placed at buses 6, 15 and 30, the branch set

$$\alpha_1 = [1 \quad 2 \quad 3 \quad 4 \quad 5]$$
$$\alpha_2 = [1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14]$$
$$\alpha_3 = [1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 25 \quad 26 \quad 27 \quad 28 \quad 29]$$

where G and H are $kdg \times kdg$ square matrix and B, C, D and E are kdg-dimensional vector. The elements of G and H are given by

$$G = \sum_{i \in \alpha_j} R_i \tag{24}$$

$$G_{jm} = \sum_{i \in (\alpha_j \cap \alpha_m)} R_i \tag{25}$$

$$H_{jj} = \sum_{i \in \alpha_j} X_i \tag{26}$$

$$H_{jm} = \sum_{i \in (\alpha_j \cap \alpha_m)} X_i \tag{27}$$

$$B_j = \sum_{i \in \alpha_j} I_{ri} R_i \tag{28}$$

$$C_j = \sum_{i \in \alpha_j} I_{mi} R_i \tag{29}$$

$$D_j = \sum_{i \in \alpha_j} I_{ri} X_i \tag{30}$$

$$E_j = \sum_{i \in \alpha_j} I_{mi} X_i \tag{31}$$

The DG currents at unity pf for the highest loss saving can be obtained

$$[I_{dg}] = [G]^{-1}[B] \tag{32}$$

By calculating DG currents at unity pf using Eq. (32) real power supplied by DG can be calculated as:

$$[P_{dg}] = [V_{dg}][I_{dg}] \tag{33}$$

The DG currents at lagging power factor for the loss reduction can be obtained

$$I_{g1} = \sqrt{B^2 + C^2} \tag{34}$$

$$I_{g2} = \sqrt{D^2 + E^2} \tag{35}$$

$$I_p = [G]^{-1}[I_{g1}] \tag{36}$$

$$I_q = [H]^{-1}[I_{g2}] \tag{37}$$

By calculating DG currents at lagging pf using Eqs. (36) and (37) real and reactive powers supplied by DG can be calculated as:

$$P_{dgk} = I_p * V_{dg} * \cos\emptyset \tag{38}$$

$$Q_{dgk} = I_q * V_{dg} * \sin\emptyset \tag{39}$$

$$S_{dgk} = \sqrt{P_{dgk}^2 + Q_{dgk}^2} \tag{40}$$

Here V_{dg} is the voltage magnitude vector of DG buses.

Cost of energy loss and cost of power obtained from DGs

The cost of energy losses and cost component of DG power has been calculated based on the mathematical model represented as:

(i) Cost of Energy Losses (CL): The annual cost of energy loss is given by [32]

$$\mathrm{CL} = (\text{Total Real power Loss}) * (\mathrm{Kp} + \mathrm{Ke} * \mathrm{Lsf} * 8760)\$ \tag{41}$$

where

Table 6
Results with DG at unity pf 38-bus mesh network.

Time (h)	Single DG		Two DGs	
	Real power from the substation (kW)	Reactive power from the substation (kVAR)	Real power from the substation (kW)	Reactive power from the substation (kVAR)
1	1304.8	1993.9	1307.3	1985.2
2	1264.9	1940.3	1267.1	1932
3	1251.5	1919.4	1253.7	1911.2
4	1278.1	1949.1	1280.5	1940.7
5	1385.4	2081.3	1388.7	2072
6	1106.3	1728.3	1107.9	1721.8
7	753.74	1219.1	754.78	1216.3
8	975.17	1500.5	976.49	1495.5
9	1014.3	1550.2	1015.7	1544.8
10	1001.2	1534.5	1002.6	1529.3
11	1119.1	1686.1	1120.9	1679.6
12	1425.9	2059.5	1430.1	2050.3
13	1145.5	1721.8	1147.3	1715
14	1198.2	1786.7	1200.3	1779.4
15	1398.8	2025.3	1402.6	2016.3
16	1304.5	1920.5	1307.4	1912.1
17	1412.3	2048.8	1416.3	2039.6
18	1677.1	2361.1	1685.6	2351
19	1578	2301.4	1584	2291
20	1522.3	2256.1	1527.1	2245.8
21	1467.1	2199.8	1471.3	2189.7
22	1385.5	2107	1388.7	2097.6
23	1372	2089.3	1375.1	2080
24	1345	2050.6	1347.8	2041.5
Results with DG at 0.9 pf lag for 38 bus mesh system				
1	180.55	295.01	1166.5	764.91
2	168.67	288.22	1129.7	747.03
3	166.26	283.74	1118.5	738.5
4	176.85	283.5	1145.1	745.52
5	216.43	294.75	1248	784.71
6	125.87	266.66	982.78	677.94
7	65.49	207.56	663.93	496.69
8	120.5	213.2	878.86	578.34
9	130.75	214.74	916.61	592.81
10	126.87	214.94	903.79	588.79
11	157.89	222.48	1016	634.19
12	266.58	244.7	1312	740.28
13	164.31	225.98	1040.3	645.99
14	180.91	228.56	1091.2	664.22
15	256.03	240.43	1286.4	729.49
16	215.48	238.53	1192.3	704.19
17	258.24	247.48	1297	740.11
18	389	291.57	1553.2	834.64
19	308.8	315.02	1437.5	845.29
20	271.35	321.51	1376.2	842.32
21	241.51	320.5	1319.6	830.36
22	204.09	314.55	1238.8	805.88
23	199.48	312.02	1226.4	800.04
24	191.91	304.61	1202.5	785.42

Table 7
DG output at unity pf and 0.9 (lag) pf for 38-bus mesh system.

Time (h)	Single DG	Two DGs		Single DG	Two DGs
	kW	kW	kW	Cost of PDG ($/MWh)	Cost of PDG ($/MWh)
1	2382.7	1826.6	543.2	47.904	47.646
2	2313	1773.2	527.64	46.51	46.267
3	2289.8	1755.4	522.46	46.046	45.807
4	2336.3	1791	532.87	46.976	46.727
5	2521.9	1933.2	574.26	50.688	50.399
6	2033	1558.7	464.94	40.91	40.723
7	1396.7	1071.1	321.22	28.184	28.096
8	1798.6	1379.1	412.28	36.222	36.078
9	1869.2	1433.2	428.2	37.634	37.478
10	1845.7	1415.2	422.9	37.164	37.012
11	2056.8	1576.9	470.42	41.386	41.196
12	2591.9	1986.6	590.05	52.088	51.783
13	2103.5	1612.7	480.91	42.32	42.122
14	2197	1684.3	501.87	44.19	43.973
15	2545.6	1951.2	579.76	51.162	50.869
16	2383.2	1826.9	543.52	47.914	47.658
17	2568.7	1968.9	584.87	51.624	51.325
18	3006.1	2303.7	681.76	60.372	59.959
19	2844.9	2180.4	645.97	57.148	56.777
20	2752.7	2109.9	625.49	55.304	54.958
21	2660.5	2039.2	604.99	53.46	53.134
22	2521.7	1933	574.14	50.684	50.393
23	2498.6	1915.3	568.99	50.222	49.936
24	2452.3	1879.9	558.69	49.296	49.022

Time (h)	Single DG	Two DGs		Single DG		Two DGs	
	kVA	kVA	kVA	Cost of PDG ($/MWh)	Cost of QDG ($/MVARh)	Cost of PDG ($/MWh)	Cost of QDG ($/MVARh)
DG output at 0.9 pf lag for 38-bus mesh system							
1	3876.1	2171.1	582.68	70.02	6.801	49.818	4.8385
2	3765.6	2109.5	566.2	68.031	6.6078	48.413	4.702
3	3727.1	2087.9	560.58	67.338	6.5405	47.923	4.6544
4	3797.8	2127.1	571.36	68.61	6.6641	48.822	4.7418
5	4086.3	2287.5	614.74	73.803	7.1685	52.49	5.0981
6	3323	1862.7	499.93	60.064	5.8339	42.777	4.1545
7	2300.9	1291.3	346.79	41.666	4.0465	29.736	2.8872
8	2925.5	1639.1	442.09	52.909	5.1388	37.711	3.6623
9	3035.3	1700.2	458.77	54.885	5.3308	39.111	3.7983
10	2999.2	1680.1	453.25	54.236	5.2677	38.65	3.7535
11	3329.2	1863.9	503.18	60.176	5.8447	42.857	4.1623
12	4158.1	2324.7	628.38	75.096	7.2941	53.405	5.187
13	3403.3	1905.2	514.29	61.509	5.9743	43.801	4.2539
14	3548.1	1985.8	536.22	64.116	6.2275	45.646	4.4332
15	4085.7	2284.4	617.55	73.793	7.1675	52.485	5.0976
16	3838.7	2147.5	579.97	69.347	6.7356	49.344	4.7925
17	4125.1	2306.6	623.19	74.502	7.2364	52.986	5.1463
18	4802.3	2682.8	724.65	86.691	8.4205	61.584	5.9816
19	4581.6	2562.4	689.4	82.719	8.0346	58.782	5.7094
20	4449.6	2489.9	668.81	80.343	7.8038	57.107	5.5466
21	4311.5	2413.5	647.74	77.857	7.5623	55.352	5.3762
22	4099.5	2295.9	615.69	74.041	7.1916	52.659	5.1145
23	4062.9	2275.5	610.24	73.382	7.1276	52.193	5.0693
24	3987.9	2233.6	599.21	72.032	6.9965	51.241	4.9767

Kp: annual demand cost of power loss ($/kW).
Ke: annual cost of energy loss ($/kWh).
Lsf: loss factor.

Loss factor is expressed in terms of load factor (Lf) as below

$$Lsf = k * Lf + (1 - k) * Lf^2 \tag{42}$$

The values taken for the coefficients in the loss factor calculation are:
k = 0.2, Lf = 0.47, Kp = 57.6923 $/kW, Ke = 0.00961538 $/kWh

(ii) Cost component of DG for real and reactive power

Cost characteristic of DG is selected as per the data available in [13]

$$C(Pdg) = a * Pdg^2 + b * Pdg + c \quad \$/\mathrm{MWh} \tag{43}$$

Cost coefficients are taken as:

$$\mathrm{a} = 0 \quad \mathrm{b} = 20 \quad \mathrm{c} = 0.25$$

Cost of reactive power supplied by DG is calculated based on maximum complex power supplied by DG as [11]

$$C(Qdg) = \left[Cost(Sgmax) - Cost\left(\sqrt{Sgmax^2 - Qg^2}\right)\right] * k \tag{44}$$

$$Sgmax = \frac{Pgmax}{\cos\emptyset}$$

$$Pmax = 1.1 * Pg$$

Table 8
Results with DG at unity pf for 38 bus mesh system.

Time (h)	Single DG Vmin (p.u)	Two DGs Vmin (p.u)	Single DG TPL (kW)	Two DGs TPL (kW)	Single DG TQL (kVAR)	Two DGs TQL (kVAR)	Single DG Cost of energy losses ($)	Two DGs Cost of energy losses ($)
1	0.96035	0.97197	91.732	81.308	66.738	58.046	7384	6544.9
2	0.96093	0.97188	86.456	76.532	62.799	54.512	6959.3	6160.5
3	0.96115	0.97188	84.662	74.927	61.464	53.329	6814.9	6031.3
4	0.96078	0.97195	87.884	77.849	63.884	55.5	7074.2	6266.5
5	0.95939	0.97214	102.53	91.326	74.831	65.492	8253.2	7351.3
6	0.96357	0.97205	68.03	60.198	49.018	42.456	5476.1	4845.7
7	0.97173	0.97574	36.143	32.804	25.44	22.629	2909.3	2640.6
8	0.96651	0.97312	53.767	47.869	38.466	33.491	4328	3853.2
9	0.9657	0.97283	57.299	50.91	41.104	35.713	4612.3	4098
10	0.96596	0.97292	56.13	49.902	40.228	34.974	4518.2	4016.9
11	0.96369	0.97231	67.633	59.903	48.836	42.318	5444.1	4821.9
12	0.95932	0.97255	106.2	95.172	77.709	68.453	8548.6	7660.9
13	0.96321	0.97223	70.476	62.405	50.964	44.161	5673	5023.3
14	0.96234	0.97217	76.273	67.558	55.314	47.973	6139.6	5438.1
15	0.95963	0.97247	102.07	91.281	74.637	65.571	8216.1	7347.7
16	0.96076	0.97229	89.316	79.374	65.086	56.725	7189.5	6389.2
17	0.95944	0.97248	104.33	93.371	76.306	67.109	8398.1	7515.9
18	0.95718	0.97416	151.8	139.67	111.39	101.33	12219	11243
19	0.95758	0.97314	134.09	121.56	98.26	87.893	10794	9785
20	0.95793	0.97269	124.76	112.34	91.325	81.039	10043	9042.8
21	0.9584	0.97236	115.81	103.7	84.67	74.634	9322.2	8347.4
22	0.95925	0.97203	103.42	92.012	75.435	65.961	8324.8	7406.5
23	0.95942	0.97201	101.41	90.158	73.943	64.588	8163	7257.3
24	0.95978	0.97198	97.409	86.485	66.738	58.046	7841	6961.6
Results with DG at 0.9 pf lag for 38 bus mesh system								
1	0.97989	0.97881	73.249	49.148	57.407	38.11	5896.2	3956.2
2	0.97944	0.97947	66.272	46.449	52.123	35.831	5334.6	3738.9
3	0.97931	0.97969	64.065	45.586	50.444	35.099	5156.9	3669.5
4	0.97957	0.97925	68.346	47.263	53.705	36.518	5501.5	3804.4
5	0.98094	0.97747	89.329	55.243	69.455	43.21	7190.6	4446.8
6	0.97806	0.97964	45.266	37.777	35.856	28.442	3643.7	3040.9
7	0.97859	0.97976	21.99	23.931	16.756	17.018	1770.1	1926.3
8	0.97758	0.97915	33.498	31.962	26.401	23.506	2696.4	2572.8
9	0.97763	0.9792	36.351	33.507	28.737	24.804	2926.1	2697.2
10	0.9776	0.97918	35.372	32.989	27.937	24.368	2847.3	2655.5
11	0.9781	0.97963	45.887	38.077	36.383	28.689	3693.7	3065
12	0.98146	0.97679	97.282	58.232	75.405	45.681	7830.7	4687.4
13	0.9783	0.97981	48.81	39.361	38.685	29.785	3929	3168.4
14	0.97878	0.98024	55.315	42.103	43.759	32.124	4452.6	3389.1
15	0.98103	0.97724	90.832	55.831	70.627	43.689	7311.5	4494.1
16	0.97979	0.97882	71.879	48.661	56.425	37.687	5785.9	3917
17	0.98125	0.97702	94.138	57.063	73.075	44.712	7577.7	4593.3
18	0.98653	0.9726	179.7	88.56	135.17	70.244	14465	7128.7
19	0.98438	0.97428	143.4	75.265	109.02	59.588	11543	6058.5
20	0.98329	0.97521	125.75	68.757	96.207	54.318	10122	5534.6
21	0.98229	0.97612	110.01	62.93	84.7	49.561	8855.3	5065.6
22	0.98098	0.97747	89.892	55.438	69.853	43.376	7235.9	4462.5
23	0.98078	0.97769	86.883	54.31	67.616	42.436	6993.7	4371.7
24	0.9804	0.97814	81.114	52.139	63.313	40.622	6529.3	4196.9

$k = 0.05–0.1$.

In this paper work, the value of factor k is taken as 0.1.

Load model

For 38 bus systems load is time varying and it is combination of residential, industrial and commercial. The ZIP load model considering the exponent values for residential, commercial and industrial have been expressed as:

Let α, β and γ the percentages of residential, commercial and industrial load at each load node respectively [27] and the values taken are given in Appendix A, Table A1.

$$P = P_o\left[\alpha\left(\frac{V}{V_o}\right)^{n_{pr}} + \beta\left(\frac{V}{V_o}\right)^{n_{pc}} + \gamma\left(\frac{V}{V_o}\right)^{n_{pi}}\right] \tag{45}$$

$$Q = Q_o\left[\alpha\left(\frac{V}{V_o}\right)^{n_{qr}} + \beta\left(\frac{V}{V_o}\right)^{n_{qc}} + \gamma\left(\frac{V}{V_o}\right)^{n_{qi}}\right] \tag{46}$$

P_o and Q_o are the nominal real and reactive power consumed at a reference voltage V_o. where V_o is equals to 1.0 p.u.

Load types and exponent values taken are:

Load type	Residential		Commercial		Industrial	
	n_{pr}	n_{qr}	n_{pc}	n_{qc}	n_{pi}	n_{qi}
Values of coefficients taken	0.72	2.96	1.25	3.50	0.18	6.00

Algorithm

The computational steps involved in finding the optimal DG size and location to minimize the loss in distribution system are summarized as:

1. Run the load flow program and obtain the base case losses. Find the DG sizes at each bus using Eqs. (5), (21), and (22) at unity and lagging power factors respectively. Select a node and find the loss saving for the case of singly located DG. Repeat this step for all Nodes in the system, except the source node. Identify the first node that provides the highest loss saving.
2. Find second DG sizes corresponding to highest loss saving bus. Identify the second compensating bus that provides the highest loss saving.
3. Compensate two nodes to get the highest loss saving with the corresponding DGs found from Eqs. (33), (38), and (39) at unity and lagging power factors respectively.
4. Repeat step 2 with three compensated nodes until it is found that no significant loss saving can be achieved by further DG placement.
5. Calculate energy losses, DG sizes and the corresponding cost of energy losses and the cost component of power obtained from DGs.

The flow chart for implementation of the algorithm is presented in Fig. 1.

Results and discussions

The results have been obtained for practical 38 bus system using proposed modified Novel method for mesh system with realistic time varying loads. The results have also been determined for a radial distribution system for comparison using both proposed Novel method and existing Novel method. The location of single and multiple DGs have been obtained based on the minimum loss obtained corresponding to the DGs location at a particular bus based on the algorithm explained in the previous section. The cost of energy loss and energy savings have been determined with single and multiple DGs. In the next section, the results obtained for radial system using Novel method and modified Novel method.

Results for 38 bus radial system with time varying load using Novel method and modified Novel method

The results have been obtained for practical 38 bus system using Novel method with DG at unity and lagging power factor (pf) for mesh network. The results are also obtained for radial system for comparison. In this system 24 h load variation has been considered. It is found that total loss is minimum at bus 6 with single DG. With placement of second DG, it is found that total loss is minimum at bus 15. Therefore, the compensating buses for DG allocation are 6 and 15 respectively. The losses obtained for 24 h load variation with DGs is shown in Figs. 2 and 3. The DG sizes obtained at unity and lagging pf for radial network are shown in Figs. 4a and 4b. The output of DGs corresponding to 24 h load variation is also depicted in the figures. The DG sizes have been taken corresponding to the minimum loss at the buses 6 and 15.

It is observed that the DG output will not be same throughout the entire day due to time varying load. Peak demand occurred at 18th hour and minimum peak occurred at 7th hour in a day as shown in Fig. 8. Cost of energy losses, DG outputs and cost of power supplied by DG are computed for each hour. A detailed voltage dependent load model has been considered for DG outputs calculations, and the three categories of loads considered are: residential, industrial and commercial. The time variation of these loads is also considered for 24 h. The voltage profile obtained for radial 38 bus test system without DG and with single and multiple DGs at unity and lagging power factor are shown in Figs. 5a–5e. It is observed from the figures that the voltage profile obtained with single and two DGs at lagging power factor is better compared to the voltage profile obtained unity power factor and voltage profile without DG. This is due to the fact of reactive power support obtained from DGs at lagging power factor and thereby reduction of losses in the network. Total real power loss with single and two DGs at unity and lagging power factor for 24 h load variation is shown in Fig. 6. It can be observed from the figure that at maximum load, the loss is also maximum and the losses reduces at each hr with DGs placement and is observed minimum with two DGs at lagging power factor. The minimum voltage obtained for the system with DGs corresponding to load variation is also shown in Fig. 7. Voltage profiles improve with two DGs having better minimum voltage corresponding to the case without DG and DGs at unity power factor. The cost of energy loss (CEloss) with DG and without installation of DG corresponding to the load variation is also determined and is shown in Fig. 8. The variation curve for the cost of energy losses follows the pattern of the load variation and it higher without DGs. With DGs, the cost of energy losses reduces and is obtained lower with two DGs at lagging power factor due to the losses reduction. Overall cost of energy loss with single and two DGs is lower compare to the case without DGs. Results obtained without and with DGs are also given in Tables 1–4.

Results obtained for TPL, TQL, real and reactive power received from the substation and cost of energy losses without DGs for 24 h load variation is also given in Table 1. In Table 2, the results of real and reactive power received from the substation with single and two DGs at unity and lagging power factor are given. Table 3 gives the results obtained for DG output and cost of DG power in $/MWh at unity and 0.9 power factor lagging. Results of minimum voltage, TPL, TQL, with DGs at unity and lagging power factor are given in Table 4.

Results for 38 bus mesh system with time varying load

The results have been obtained for practical 38 bus system using Novel method and modified Novel method at unity and lagging power factor for mesh network. In this system also load variation has been considered. Based on the minimum loss obtained at bus 6 and 15, compensating buses for DG allocation are taken as 6 and 15 respectively. The losses obtained for 24 h load variation with DGs is shown in Figs. 9 and 10. The optimal DG sizes obtained at unity and lagging pf for radial and mesh network are shown in Figs. 11a and 11b. The output of DGs corresponding to 24 h load variation is also depicted in the figures.

It is observed that the DG output will not be same throughout the entire day due to time varying load. Peak demand occurred at 18th hour and minimum peak occurred at 7th hour in a day as shown in Fig. 15. Cost of energy losses, DG outputs, and cost of power supplied by DG are computed for each hour. A detailed voltage dependent load model has been considered for DG outputs calculations, and the three categories of loads considered are: residential, industrial and commercial. The load variation is also considered with respect to time. The voltage profile obtained for mesh 38 bus test system with load variation at unity and lagging power factor without and with DGs are shown in Figs. 12a–12e. Voltage profile obtained with single and two DGs at lagging power factor are better compared to the voltage profile obtained unity power factor. This is due to the fact of reactive power support obtained from DGs at lagging power factor and thereby reduction of losses in the network. Total real power loss with single and two DGs at unity and lagging power factor for 24 h load variation is shown in Fig. 13. It can be observed from the figure that at maximum load, the loss is also maximum and the losses reduces at each hr with DGs placement and is observed minimum with two DGs at lagging power factor. The minimum voltage obtained for the system with DGs corresponding to load variation is also shown in Fig. 14. Voltage profile improvement with two DGs is better than the case without DG and DGs at unity power factor. The cost of energy loss (CEloss) corresponding to the load variation is also determined and is shown in Fig. 15. The variation curve for the cost of energy losses follows the pattern of the load variation and it is higher without DGs. With DGs, the cost of energy losses reduces and is obtained lower with two DGs at lagging power factor due to the losses reduction. Overall cost of energy loss with single and two DGs is lower compared to the case without DGs. Results obtained without and with DGs are also given in Tables 5–8.

Results obtained for TPL, TQL, real and reactive power received from the substation and cost of energy losses without DGs for 24 h load variation is also given in Table 5. In table 6, the results of real and reactive power received from the substation with single and two DGs at unity and lagging power factor are given. Table 7 gives the results obtained for DG output and cost of DG power in \$/MWh at unity and 0.9 power factor lag. Results of minimum voltage, TPL, TQL, with DGs at unity and lagging power factor are given in Table 8.

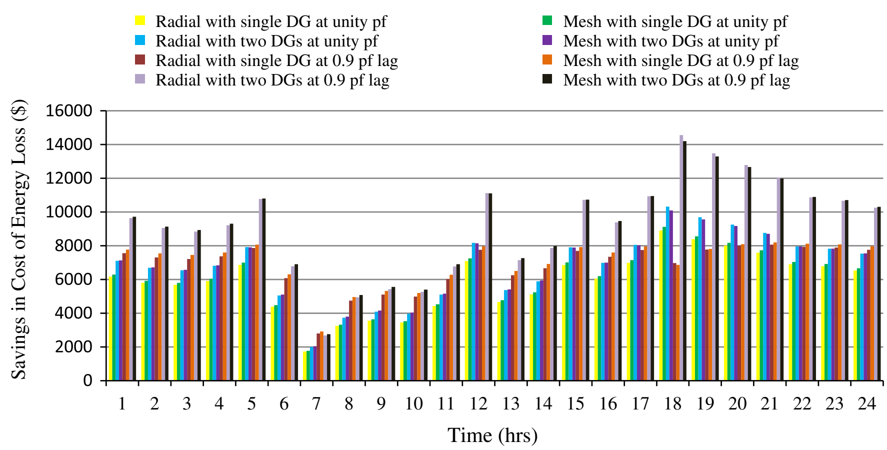

Fig. 16. Savings in cost of energy loss ($) for 38-bus system.

Table 9
Energy loss savings ($) for 38-bus system.

Time (h)	At unity pf				At 0.9 lag pf			
	Single DG		Two DGs		Single DG		Two DGs	
	Radial	Mesh	Radial	Mesh	Radial	Mesh	Radial	Mesh
1	6167.6	6291	7109.7	7130.1	7554.2	7778.8	9643.8	9718.8
2	5802.4	5919.7	6687.2	6718.5	7312.4	7544.4	9047.8	9140.1
3	5676.7	5793.1	6542	6576.7	7215.7	7451.1	8840.5	8938.5
4	5909.8	6031.8	6811.9	6839.5	7370	7604.5	9212.9	9301.6
5	6861.4	7000.8	7914.8	7902.7	7861.7	8063.4	10781	10807
6	4380	4472.3	5044.9	5102.7	6080.5	6304.7	6782.7	6907.5
7	1733.3	1773	1996.8	2041.7	2799.7	2912.2	2675.7	2756
8	3243.4	3321.1	3736.8	3795.9	4744.3	4952.7	4947.5	5076.3
9	3552.5	3637.7	4093.2	4152	5101	5323.9	5419.6	5552.8
10	3449.1	3532.1	3973.8	4033.4	4984.7	5203	5262.5	5394.8
11	4427.8	4533.3	5102.5	5155.5	6031	6283.7	6777.6	6912.4
12	7085.9	7252.4	8178.4	8140.1	7762.3	7970.3	11116	11114
13	4657.5	4768	5367.4	5417.7	6255	6512	7140.6	7272.6
14	5116.7	5238.4	5897.6	5939.9	6661	6925.4	7865.4	7988.9
15	6855.9	7018.9	7911.4	7887.3	7698.1	7923.5	10719	10741
16	6056.9	6199.5	6984.7	6999.8	7346.3	7603.1	9387.6	9472
17	6982.2	7144.9	8057.9	8027.1	7751	7965.3	10940	10950
18	8905	9117	10315	10093	6968	6871	14568	14207
19	8387	8558	9697	9567	7772	7809	13480	13294
20	8014.3	8168	9257.9	9168.2	7995	8089	12788	12676
21	7589.7	7734.8	8761.8	8709.6	8060.6	8201.7	12039	11991
22	6905.8	7037.2	7965.2	7955.5	7935.3	8126.1	10875	10900
23	6785.1	6915	7825.1	7820.7	7887.3	8084.3	10673	10706
24	6537.2	6665	7537.7	7544.4	7766.2	7976.7	10257	10309

Cost of energy savings has also been computed for practical 38 bus test system at unity and lagging power factor for both radial and mesh distribution systems with single and two DGs. It is observed that cost of energy savings in each hr is higher with two DGs at lagging power factor due more reduction in losses in the system. In the 24 h load variation, the load is lowest at bus 7 and highest at bus 18. It is observed from the table that the savings are higher at maximum load hr and is lowest at minimum load hr is day. The savings due to loss reductions are also plotted and shown in Fig. 16 (see Table 9).

Conclusions

In this paper, the sensitivity based method has been applied for DG allocation in the mesh network. In the proposed novel power loss sensitivity method, analytical expressions are derived to determine the optimal DG sizes at unity and lagging power factors. Also the proposed approach is extended for optimal multiple DG placement in distribution system. The analysis has been carried out for practical 38 bus test system with time varying ZIP load model. The results have been obtained for both radial and mesh network. The analysis has been carried out under different scenarios of unity and lagging power factor with time varying ZIP load model. The cost of energy loss, cost of powers obtained from DG and savings due to reduction in the losses has been obtained for both radial and mesh network.

Based on the results obtained:

1. It is observed that two DG placement is more effective in loss reduction and the voltage profile obtained is better for lagging power factor than at unity power factor. This is due to the fact that DGs also supply reactive power locally and causes more loss reduction.
2. It is observed that the losses reduce with DGs placement and reduction is higher for two DGs case.
3. It is observed that the voltage profile is better with two DGs at lagging power factor for both radial and mesh distribution system.
4. It is observed that two DG placement gives better results than single DG placement in terms of cost of energy loss and savings per annum savings due to loss reduction.

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Appendix A.

Fig. A1. Figure for 38 bus mesh system with two loops.

Base kV = 12.66 kV
Base MVA = 1 MVA

Two loops has taken

Loop-1: between buses 33 and 37 tie line impedance = 0.003113 + j^*0.003113 p.u.
Loop-2: between buses 29 and 38 tie line impedance = 0.003113 + j^*0.003113 p.u.

Table A1. Coefficients α, β and γ the percentages of residential, commercial and industrial load for each hour.

Table A1
Coefficients α, β and γ the percentages of residential, commercial and industrial load for each hour.

Hour	Residential (α)	Commercial (β)	Industrial (γ)
1	0.66	0.17	0.17
2	0.63	0.17	0.2
3	0.6	0.18	0.22
4	0.58	0.2	0.22
5	0.6	0.23	0.17
6	0.55	0.15	0.3
7	0.3	0.14	0.56
8	0.11	0.32	0.57
9	0.10	0.34	0.56
10	0.11	0.33	0.56
11	0.12	0.37	0.51
12	0.17	0.46	0.37
13	0.14	0.37	0.49
14	0.14	0.39	0.47
15	0.15	0.46	0.39
16	0.18	0.41	0.41
17	0.2	0.44	0.36
18	0.33	0.47	0.2
19	0.6	0.3	0.1
20	0.7	0.23	0.06
21	0.74	0.19	0.07
22	0.76	0.15	0.09
23	0.75	0.15	0.1
24	0.71	0.16	0.13

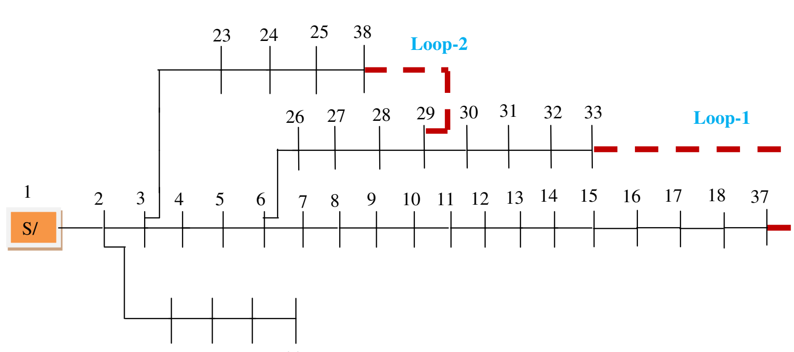

Fig. A1. Figure for 38 bus mesh system with two loops.

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