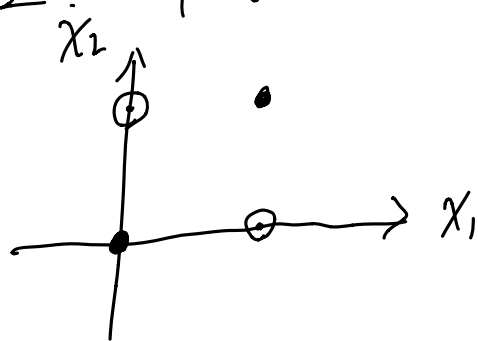


1. 证: 异或问题.



实心圆表示 $y=0$

空心圆表示 $y=1$

感知器, 即单层神经网络. $o = \text{sgn}(W^T X - T)$

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad W = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

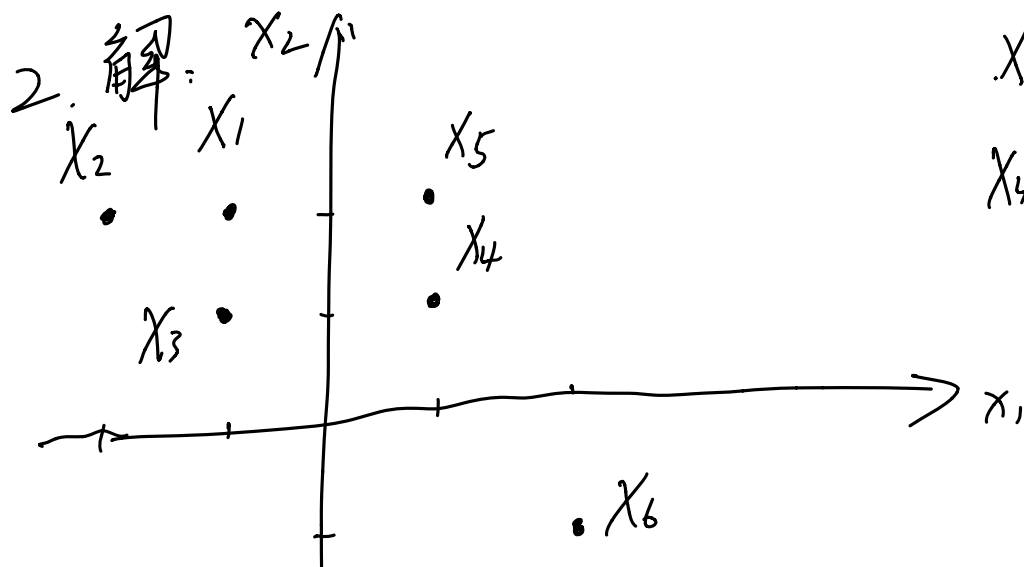
$$y = \text{sgn}(w_1 x_1 + w_2 x_2 - T)$$

代入4个值 (假设若能分类).

$$\begin{cases} 0 = \text{sign}(-T) \\ 1 = \text{sign}(w_2 - T) \\ 1 = \text{sign}(w_1 - T) \\ 0 = \text{sign}(w_1 + w_2 - T) \end{cases} \Rightarrow \begin{cases} T \geq 0 \\ w_2 - T > 0 & T < w_2 \quad (1) \\ w_1 - T > 0 & T < w_1 \quad (2) \\ w_1 + w_2 - T \leq 0, \quad T \geq w_1 + w_2 \end{cases}$$

①+② $2T < w_1 + w_2 \leq T \Rightarrow T < 0$ 与 $T \geq 0$ 矛盾.

故单层感知器不能解决“异或”问题.



$$x_1, x_2, x_3 \quad y = 1$$

$$x_4, x_5, x_6 \quad y = 0.$$

1) 感知器方程 $0 = \text{sgn}(W^T X - T)$, $\text{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$

$$0 = \text{sgn}(w_1 x_1 + w_2 x_2 - T) \quad \text{参数 } W = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, T$$

随便设计一个, 令 $w_1 = -1, w_2 = 0, T = 0$

$$\text{即 } 0 = -\text{sgn}(-x_1)$$

12) 输入验证即可.

13) $X^7 = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \Rightarrow d^7 = 1$

$$X^8 = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \Rightarrow d^8 = 0$$

3. 解: $W = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}, \quad 0 = \text{sgn}(w_1 x_1 + w_2 x_2 + w_3 x_3)$