Quantum state discrimination

I. CLASSICAL INFORMATION OVER QUANTUM CHANNELS

$\Phi \in \mathsf{Chan}(\mathcal{X}, \mathcal{Y})$ and $\Psi \in \mathsf{Chan}(\mathcal{Z})$. The channel Φ emulates Ψ if

$$\exists \Xi_E \in \mathsf{Chan}(\mathcal{Z}, \mathcal{X}) \ \Xi_D \in \mathsf{Chan}(\mathcal{Y}, \mathcal{Z}) \ \Psi = \Xi_D \Phi \Xi_E \tag{1}$$

The channel Ξ_E is called an encoding channel and Ξ_D is called a decoding channel.

Definition 1 $\Psi_0, \Psi_1 \in \mathsf{Chan}(\mathcal{Z})$. Ψ_0 *is ϵ-approximation to* Ψ_1 *if*

$$|||\Psi_0 - \Psi_1||| < \epsilon \tag{2}$$

Definition 2 $\Phi \in \mathsf{Chan}(\mathcal{X}, \mathcal{Y})$. $\mathcal{Z} = \mathbb{C}^{\{0,1\}}$. $\Delta \in \mathsf{Chan}(\mathcal{Z})$ *is the completely dephasing channel.*

A value $\alpha > 0$ is achievable if $\forall \epsilon > 0$, for all but finitely many $n \in \mathbb{N}$, the channel $\Phi^{\otimes n}$ emulates an ϵ-approximation to the channel $\Delta^{\lfloor \alpha n \rfloor}$.

The classical capacity of Φ, denoted $\mathsf{C}(\Phi)$, is the supremum value of all achievable rates for classical information transmission through Φ.

Remark 1 *When considering an emulation of the m-fold tensor product $\Delta^{\otimes m}$ of this ideal classical channel by the channel $\Phi^{\otimes n}$, no generality is lost in restricting one's attention to classical-to-quantum encoding channels Ξ_E and quantum-to-classical decoding channels Ξ_D.*

$$\Xi_E = \Xi_E \Delta^{\otimes m} \qquad \Xi_D = \Delta^{\otimes m} \Xi_D \tag{3}$$

Theorem 1 $\Phi \in \mathsf{Chan}(\mathcal{X}, \mathcal{Y})$

$$\mathsf{C}(\Phi^{\otimes k}) = k \ \mathsf{C}(\Phi) \tag{4}$$

Definition 3 *Entanglement-assisted classical capacity*

$$Z \mapsto (\Xi_D(\Phi \Xi_E \otimes \mathbb{1}_{\mathsf{Lin}(\mathcal{W})}))(Z \otimes \xi) \tag{5}$$

A value $\alpha > 0$ is achievable if $\forall \epsilon > 0$, for all but finitely many $n \in \mathbb{N}$, the channel $\Phi^{\otimes n}$ emulates an ϵ-approximation to the channel $\Delta^{\lfloor \alpha n \rfloor}$.

The entanglement-assisted classical capacity of Φ, denoted $\mathsf{C}_E(\Phi)$, is the supremum value of all achievable rates for classical information transmission through Φ.

Theorem 2 $\Phi \in \mathsf{Chan}(\mathcal{X}, \mathcal{Y})$

$$\mathsf{C}_E(\Phi^{\otimes k}) = k \mathsf{C}_E(\Phi) \tag{6}$$

Definition 4 $\Phi \in \mathsf{Chan}(\mathcal{X}, \mathcal{Y})$. *The Holevo capacity of Φ is defined as*

$$\chi(\Phi) = \sup_{\eta} \chi(\Phi(\eta)) \tag{7}$$

Theorem 3 *It is enough to maximize Holevo information using pure states. And what's more, the size of the alphabet of the ensemble can be restricted to $\dim(\mathcal{X})^2$.*