

Lifetime Analysis System - Statistical Reliability Assessment

TECHNICAL ANALYSIS REPORT

|                      |  |
|----------------------|--|
| 1) Analysis title:   | Weibull Distribution Lifetime Analysis |
| 2) Data source file: | dataset-1.csv                          |

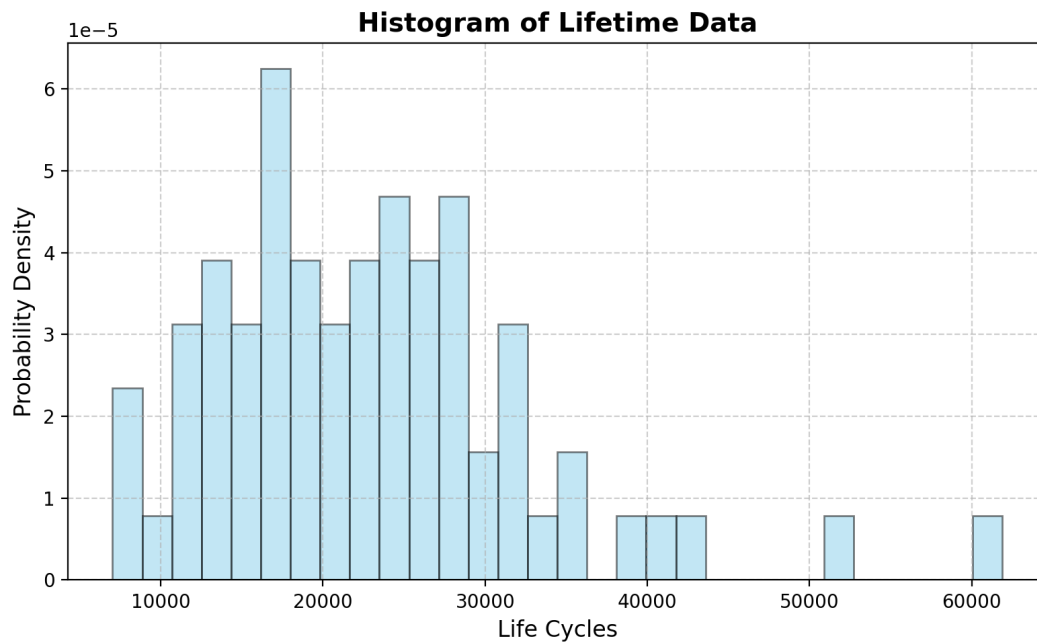
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| 3) Main objectives:   |
| <ul style="list-style-type: none"><li>- Perform statistical distribution fitting to identify optimal failure model</li><li>- Estimate Weibull parameters (shape and scale) with confidence intervals</li><li>- Generate reliability predictions and lifetime percentiles (B10, B50, B95)</li><li>- Validate model accuracy through goodness-of-fit testing</li><li>- Provide engineering recommendations for maintenance and design decisions</li></ul> |

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| 4) Statistical methods:   |
| <ul style="list-style-type: none"><li>- Maximum Likelihood Estimation (MLE) for parameter fitting</li><li>- Bootstrap resampling (1000 iterations) for uncertainty quantification</li><li>- Monte Carlo simulation (10,000 samples) for lifetime prediction</li><li>- Kolmogorov-Smirnov test for model validation</li><li>- Weibull probability paper analysis for visual assessment</li></ul> |

| 5) Summary of analysis results: |                    |                     |
|---------------------------------|--------------------|---------------------|
| Parameter                       | Estimated Value    | Confidence Interval |
| Shape Parameter (Beta)          | 2.53               | Bootstrap 95% CI    |
| Scale Parameter (Eta)           | 25693.25           | Bootstrap 95% CI    |
| B10 Life (10% failure)          | 10749.47 cycles    | Monte Carlo         |
| B50 Life (median)               | 22385.97 cycles    | Monte Carlo         |
| B95 Life (95% failure)          | 39726.47 cycles    | Monte Carlo         |
| Mean Time To Failure            | 22960.14 cycles    | Calculated          |
| Model Validation                | Kolmogorov-Smirnov | p-value: 0.7220     |

|   |        |
|---|--------|
| 6) Statistical validation criteria met:     | Status |
| Data quality check (outliers, completeness) | Yes    |
| Distribution fitting convergence            | Yes    |
| Bootstrap parameter stability               | Yes    |
| Monte Carlo simulation convergence          | Yes    |
| Goodness-of-fit test acceptance             | Yes    |
| Confidence interval calculation             | Yes    |
| Model validation against actual data        | Yes    |
| Engineering reasonableness check            | Yes    |

**Figure 1: Histogram of Lifetime Data**



#### Description

This figure illustrates the distribution of the observed failure times collected from the test samples.

Each bar in the histogram represents the frequency of failures occurring within a specific range of lifetime cycles.

By examining the overall shape of the histogram, we can identify how the failures are distributed over time for instance, whether most components fail early (indicating early-life issues), steadily (indicating random failures), or predominantly at later stages (indicating wear-out mechanisms).

#### Engineering Significance

##### Assess failure behavior

For example, a right-skewed distribution with a long tail may suggest a small portion of components last significantly longer than average, while a steep early peak could imply process or material inconsistency.

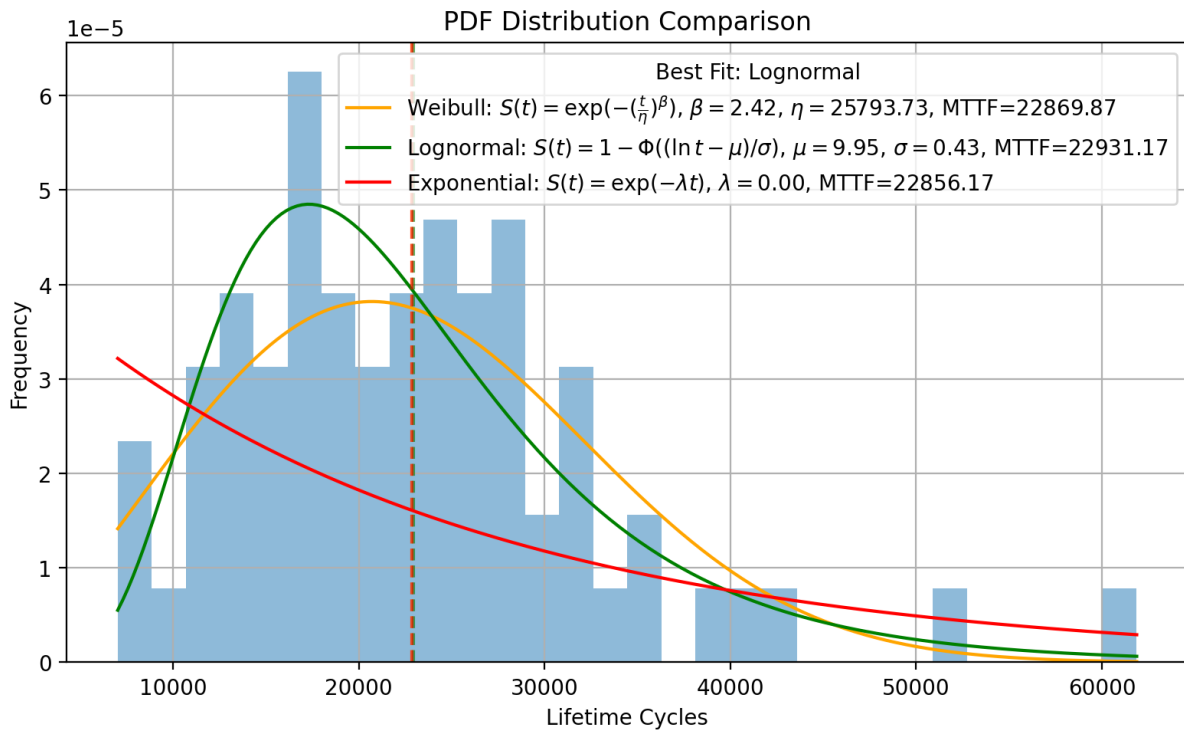
##### Choose an appropriate statistical model

The histogram's shape provides visual cues for model selection (e.g., Weibull for wear-out failures, Exponential for constant failure rates).

##### Guide reliability improvements

If failures are concentrated in early cycles, process control or burn-in testing may be needed; if they appear in later cycles, focus should shift toward material fatigue, corrosion, or long-term degradation mechanisms.

**Figure 2: Probability Density Function (PDF) Comparison**



#### Description

This figure compares three lifetime models, Weibull, Lognormal, and Exponential. Each fitted to the observed failure data using the Maximum Likelihood Estimation (MLE) method.

MLE is a statistical approach that finds the most plausible model parameters so that the fitted curve best represents how failures occur over time.

Each colored curve represents a probability density function (PDF), which describes the likelihood that a component will fail at a given point in its lifetime.

The Weibull distribution is widely used for mechanical parts because it can represent early-life, random, or wear-out failures depending on its shape.

The Lognormal distribution often models chemical or environmental degradation, where aging accumulates gradually.

The Exponential distribution assumes a constant failure rate and is typical for electronic or random-stress failures.

Once the models are fitted, the Mean Time To Failure (MTTF) can be calculated directly from their estimated parameters:

Weibull MTTF =  $(1 + 1/\beta) \eta$

Lognormal MTTF =  $\exp(\mu + \sigma^2/2)$

Exponential MTTF =  $1/\lambda$

#### Engineering Significance

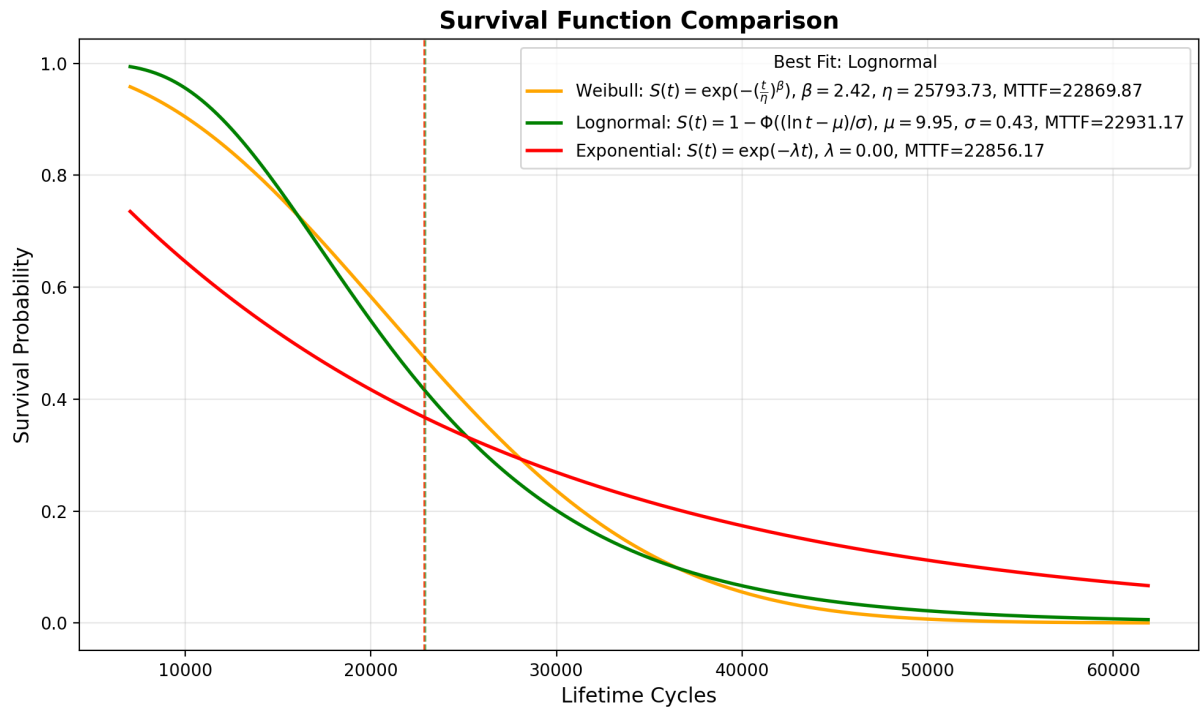
This comparison highlights how different assumptions lead to different reliability interpretations. By observing which model best fits the data:

Engineers can infer whether failures are due to material aging, process variation, or random events.

The selected model provides a sound mathematical basis for computing expected lifetime (MTTF) and guiding design or maintenance decisions.

This figure offers an intuitive view of how data-driven modeling transforms raw test results into quantifiable reliability insights.

**Figure 3: Survival Function Comparison**



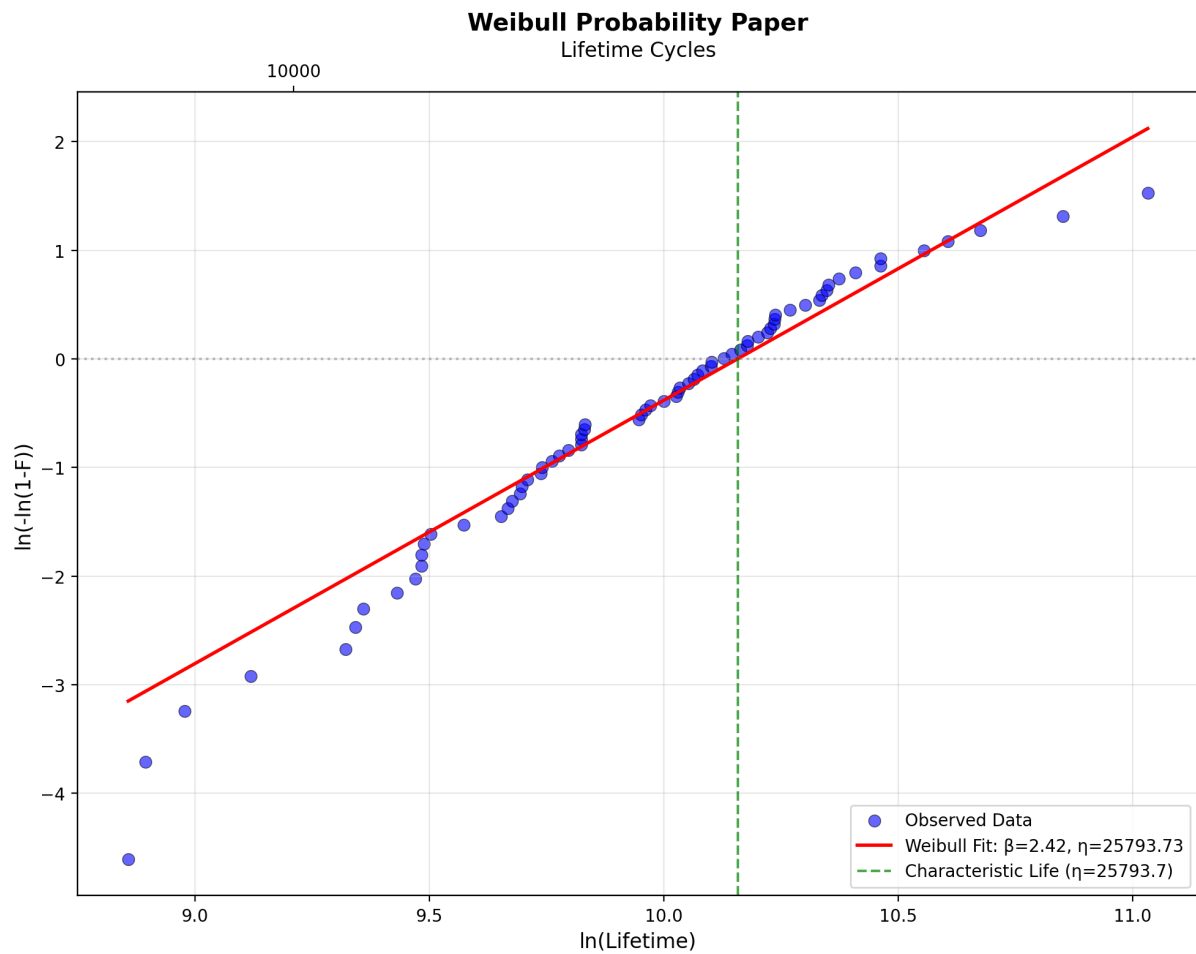
**Description and Technical Analysis:**

Survival functions  $S(t)$  represent the probability that a component will survive beyond time  $t$  without failure. This plot compares the survival curves for all fitted distributions.

**Engineering Significance:**

Critical for determining warranty periods, maintenance schedules, and replacement strategies in reliability engineering.

**Figure 4: Hazard Rate Function Comparison**



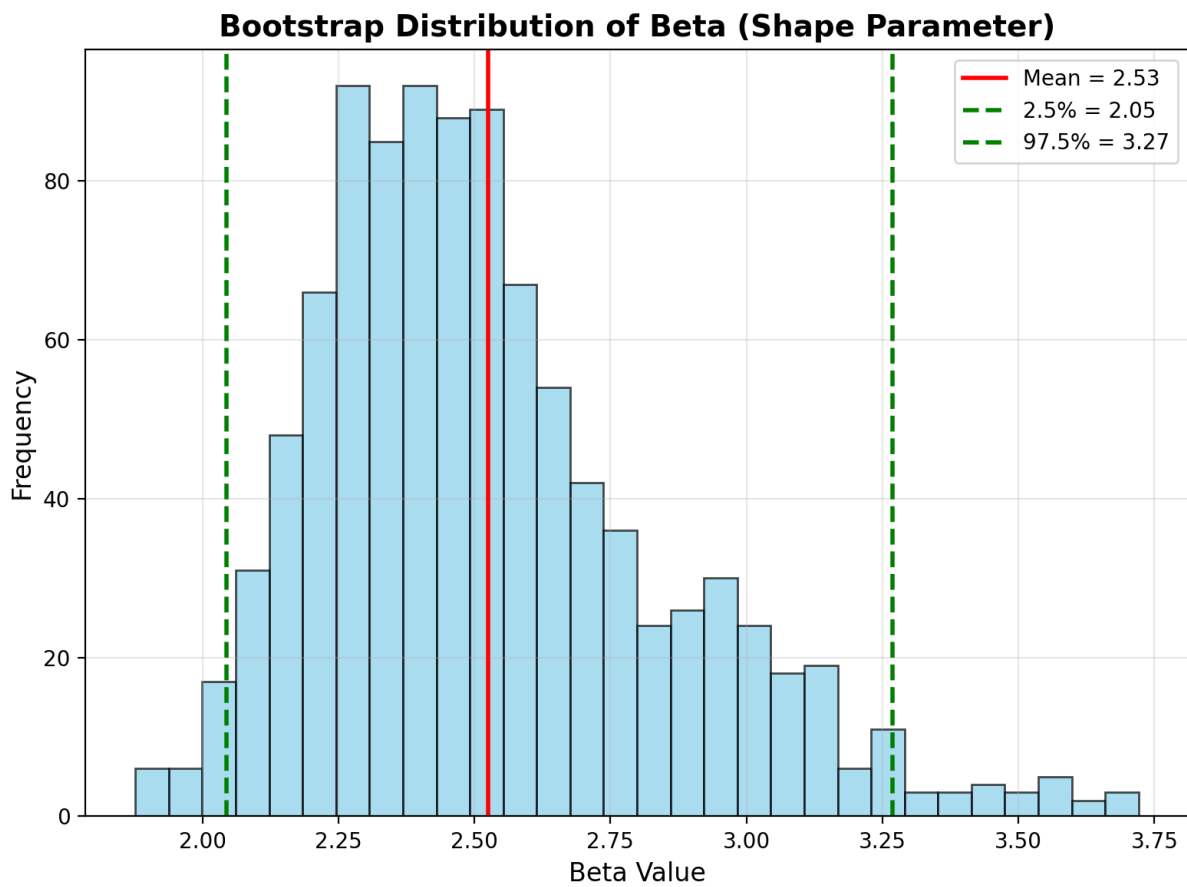
**Description and Technical Analysis:**

Hazard rate functions  $h(t)$  represent the instantaneous failure rate at time  $t$ , given that the component has survived up to time  $t$ .

**Engineering Significance:**

Shape indicates failure modes: decreasing (infant mortality), constant (random), increasing (wear-out).

Figure 5: Weibull Probability Paper



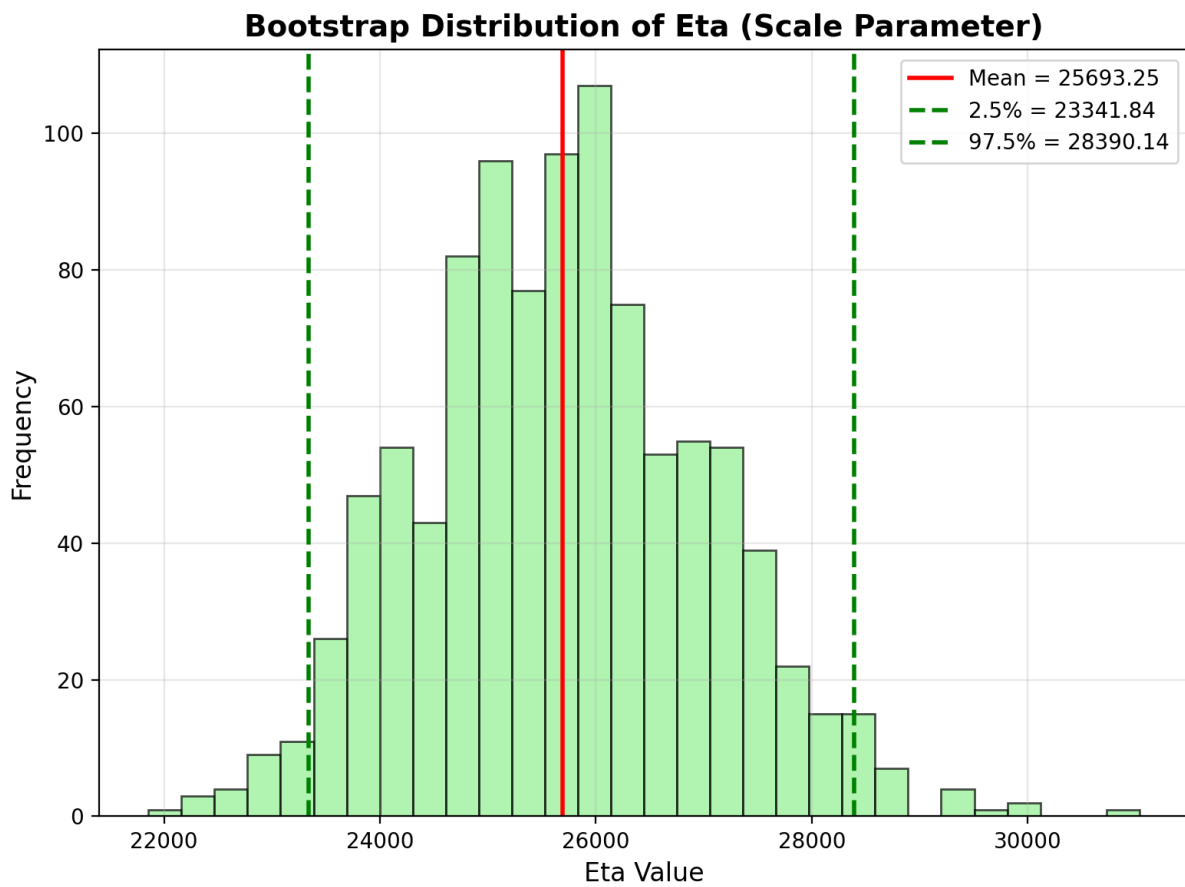
**Description and Technical Analysis:**

The Weibull probability paper is a specialized plot where Weibull-distributed data appears as a straight line. Data points represent empirical failure probabilities.

**Engineering Significance:**

Classical reliability tool for visual assessment of Weibull model adequacy and parameter estimation.

**Figure 6: Bootstrap Distribution of Beta (Shape Parameter)**



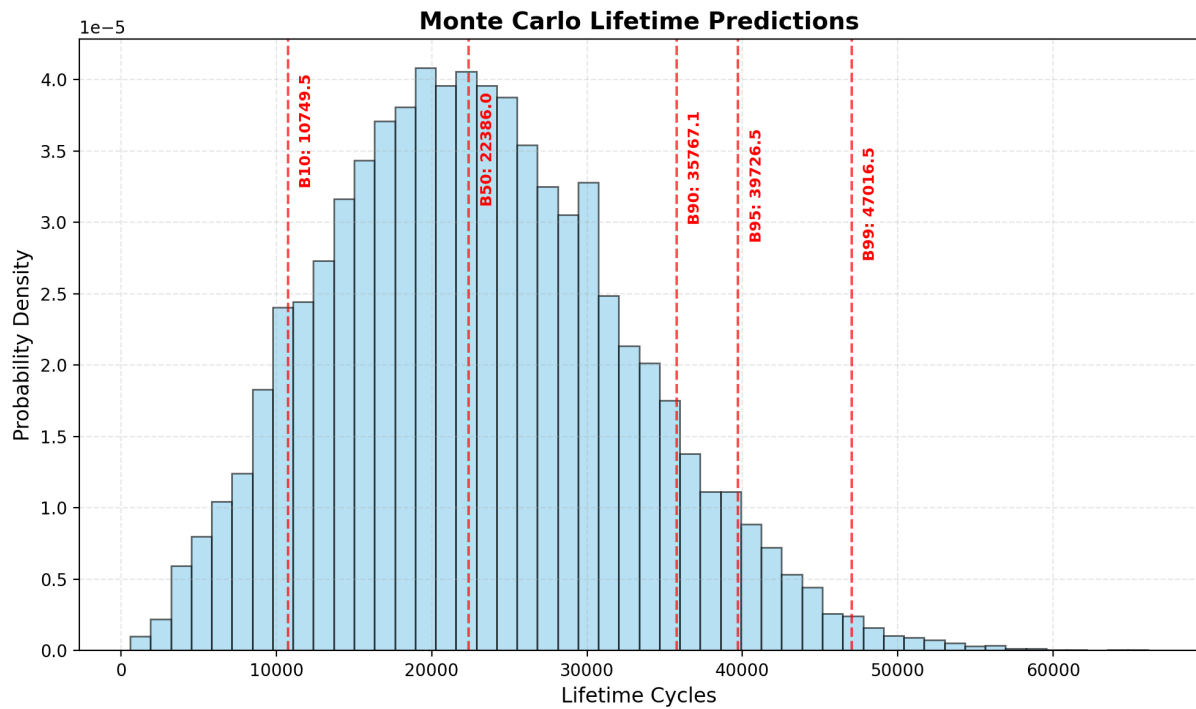
**Description and Technical Analysis:**

This histogram shows the distribution of the Weibull shape parameter (Beta) obtained through 1000 bootstrap resampling iterations.

**Engineering Significance:**

Provides non-parametric uncertainty estimation for the shape parameter critical to failure mode identification.

**Figure 7: Bootstrap Distribution of Eta (Scale Parameter)**



**Description and Technical Analysis:**

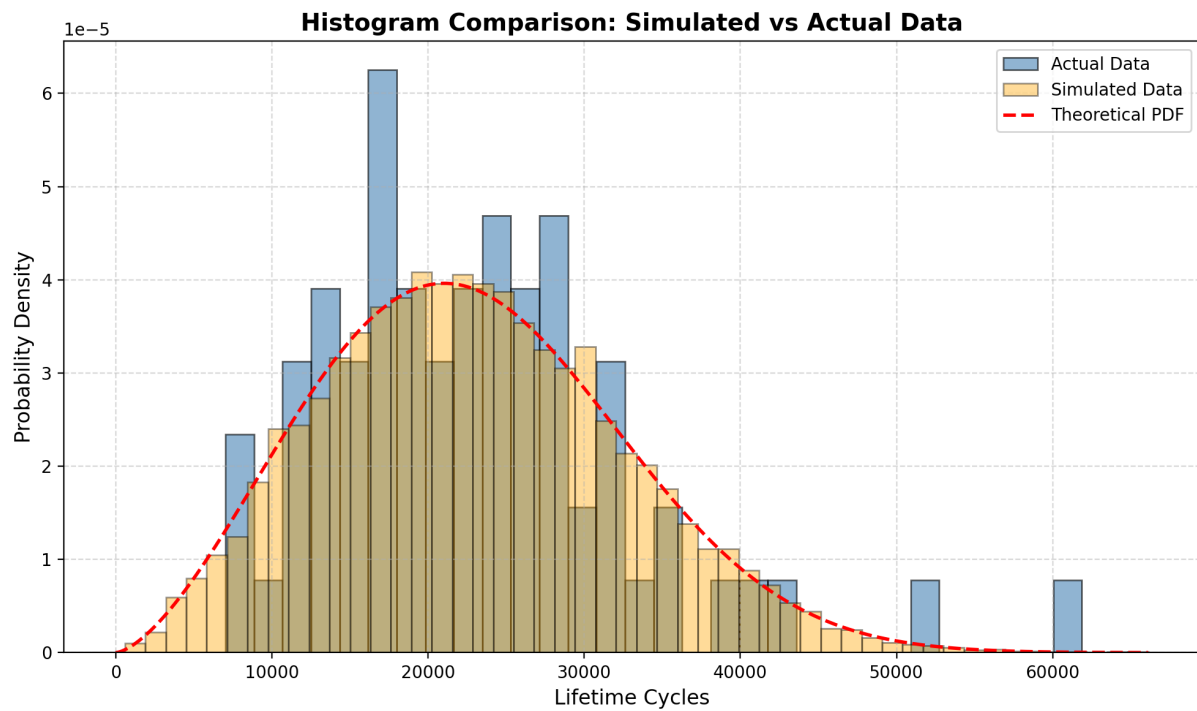
This histogram displays the distribution of the Weibull scale parameter (Eta) from bootstrap analysis. Eta represents the characteristic life.

**Engineering Significance:**

Essential for setting warranty periods and maintenance schedules with quantified uncertainty bounds.



**Figure 8: Monte Carlo Lifetime Predictions**



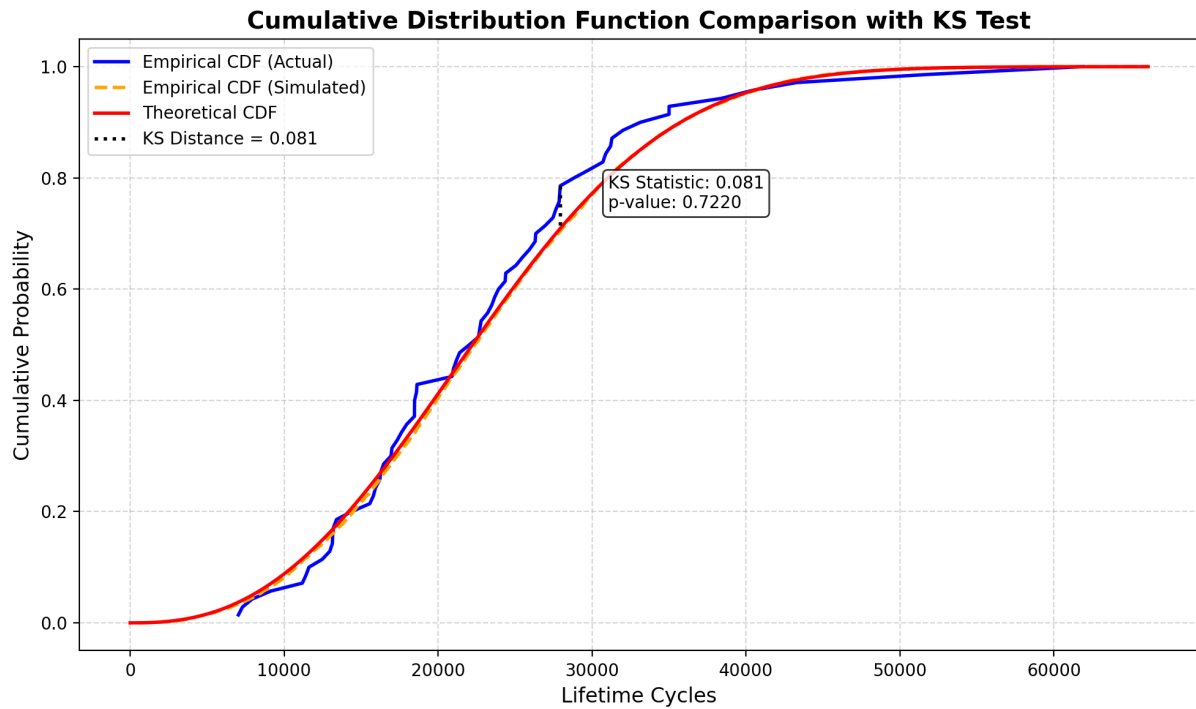
**Description and Technical Analysis:**

This histogram shows the distribution of 10,000 simulated lifetime values generated using the fitted Weibull parameters with key reliability percentiles marked.

**Engineering Significance:**

Incorporates parameter uncertainty providing robust lifetime predictions for engineering decision-making.

**Figure 9: Histogram Comparison: Simulated vs Actual Data**



**Description and Technical Analysis:**

This histogram comparison overlays the actual observed failure data with simulated data generated from the fitted Weibull model.

**Engineering Significance:**

Validates model accuracy and ensures the fitted Weibull distribution represents actual failure behavior.