

Lifetime Analysis System - Statistical Reliability Assessment

TECHNICAL ANALYSIS REPORT

1) Analysis title:	Weibull Distribution Lifetime Analysis
2) Data source file:	dataset-1.csv

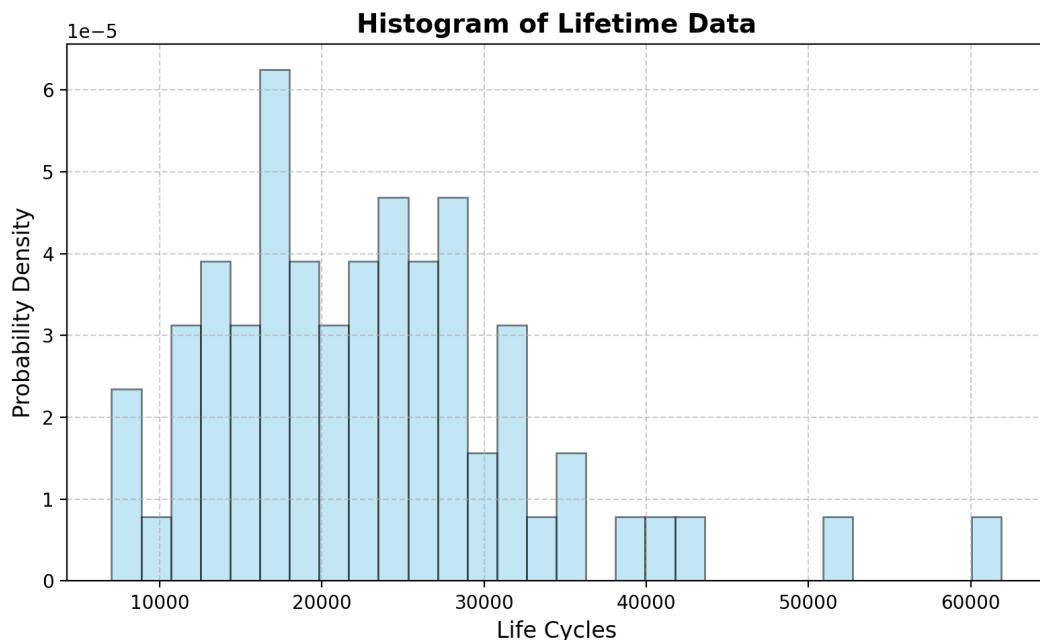
3) Main objectives:
<ul style="list-style-type: none"> - Perform statistical distribution fitting to identify optimal failure model - Estimate Weibull parameters (shape and scale) with confidence intervals - Generate reliability predictions and lifetime percentiles (B10, B50, B95) - Validate model accuracy through goodness-of-fit testing - Provide engineering recommendations for maintenance and design decisions

4) Statistical methods:
<ul style="list-style-type: none"> - Maximum Likelihood Estimation (MLE) for parameter fitting - Bootstrap resampling (1000 iterations) for uncertainty quantification - Monte Carlo simulation (10,000 samples) for lifetime prediction - Kolmogorov-Smirnov test for model validation - Weibull probability paper analysis for visual assessment

5) Summary of analysis results:		
Parameter	Estimated Value	Confidence Interval
Shape Parameter (Beta)	2.52	Bootstrap 95% CI
Scale Parameter (Eta)	25738.65	Bootstrap 95% CI
B10 Life (10% failure)	10395.90 cycles	Monte Carlo
B50 Life (median)	22187.58 cycles	Monte Carlo
B95 Life (95% failure)	39935.43 cycles	Monte Carlo
Mean Time To Failure	22783.98 cycles	Calculated
Model Validation	Kolmogorov-Smirnov	p-value: 0.7481

6) Statistical validation criteria met:	Status
Data quality check (outliers, completeness)	Yes
Distribution fitting convergence	Yes
Bootstrap parameter stability	Yes
Monte Carlo simulation convergence	Yes
Goodness-of-fit test acceptance	Yes
Confidence interval calculation	Yes
Model validation against actual data	Yes
Engineering reasonableness check	Yes

Figure 1: Histogram of Lifetime Data



Description

This figure illustrates the distribution of the observed failure times collected from the test samples. Each bar in the histogram represents the frequency of failures occurring within a specific range of lifetime cycles. By examining the overall shape of the histogram, we can identify how the failures are distributed over time for instance, whether most components fail early (indicating early-life issues), steadily (indicating random failures), or predominantly at later stages (indicating wear-out mechanisms).

Engineering Significance

Assess failure behavior

For example, a right-skewed distribution with a long tail may suggest a small portion of components last significantly longer than average, while a steep early peak could imply process or material inconsistency.

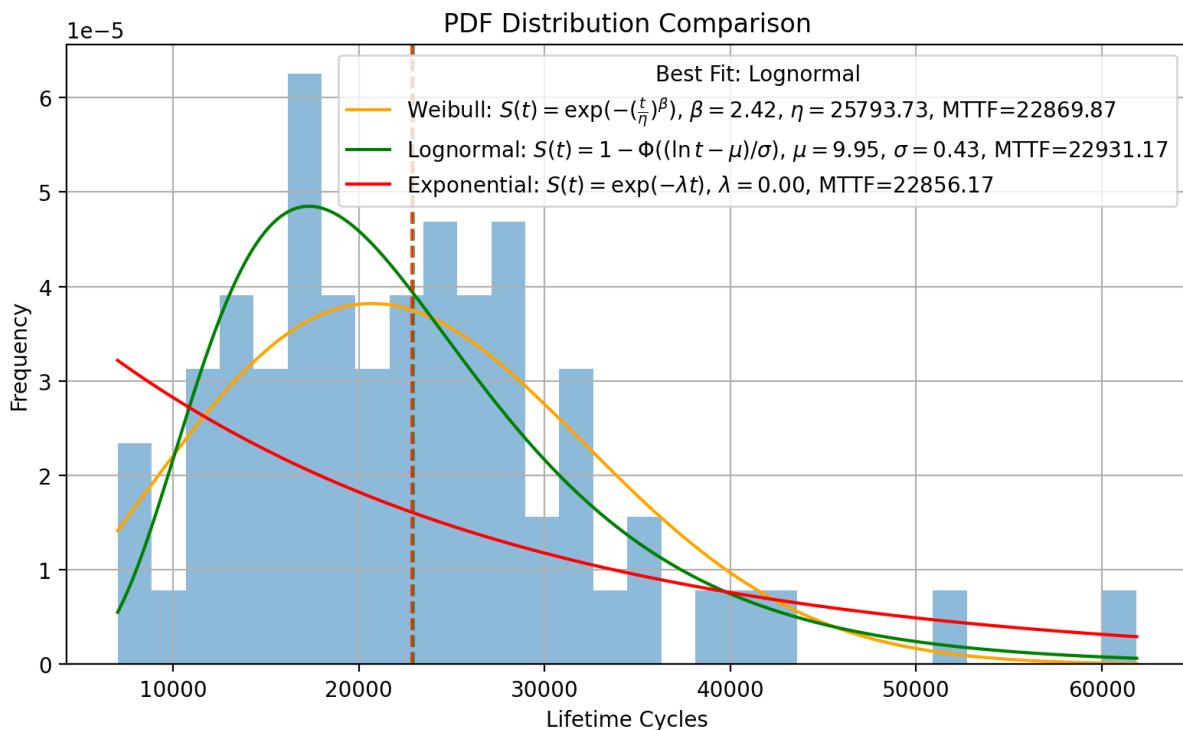
Choose an appropriate statistical model

The histogram's shape provides visual cues for model selection (e.g., Weibull for wear-out failures, Exponential for constant failure rates).

Guide reliability improvements

If failures are concentrated in early cycles, process control or burn-in testing may be needed; if they appear in later cycles, focus should shift toward material fatigue, corrosion, or long-term degradation mechanisms.

Figure 2: Probability Density Function (PDF) Comparison



Description

This figure presents a comparison among three statistical lifetime models—Weibull, Lognormal, and Exponential—each fitted to the observed data using the Maximum Likelihood Estimation (MLE) method. MLE is a mathematical approach that determines the parameter values making the observed data most likely under a given model. In other words, it automatically finds the curve that best describes how failures occur over time, rather than relying on visual guessing. Each curve shown represents a Probability Density Function (PDF), indicating how likely failures are to occur at different lifetime ranges. The closer a model's curve follows the histogram, the better it captures the true behavior of the product population. The three models used here represent distinct physical or statistical assumptions commonly encountered in reliability engineering:

- Weibull Distribution** - A flexible model capable of describing early-life, random, or wear-out failures depending on its shape parameter (β). Often used for mechanical components such as seals, bearings, or polymers that degrade over time.
- Lognormal Distribution** - Assumes that the logarithm of lifetime follows a normal distribution. Typically applied to chemical or material degradation processes, where gradual accumulation of damage or oxidation leads to failure. It is well-suited when variability in lifetime arises from multiplicative effects (e.g., environmental or process variation).
- Exponential Distribution** - Represents a constant failure rate over time, implying no aging effect. Commonly used for electronic parts or components under random stress events, where the likelihood of failure does not depend on age.

After each model is fitted using MLE, we can calculate the Mean Time To Failure (MTTF). For different distributions, this integral has closed-form expressions once the fitted parameters are known:

- Weibull: $MTTF = \eta \Gamma(1 + 1/\beta)$
- Lognormal: $MTTF = \exp(\mu + \sigma^2/2)$
- Exponential: $MTTF = 1/\lambda$

These formulas show that MTTF is not a fixed value; it changes depending on the assumed failure model and the parameters estimated from data.

Engineering Significance

This figure demonstrates how different statistical assumptions lead to different interpretations of product reliability. By comparing the three distributions:

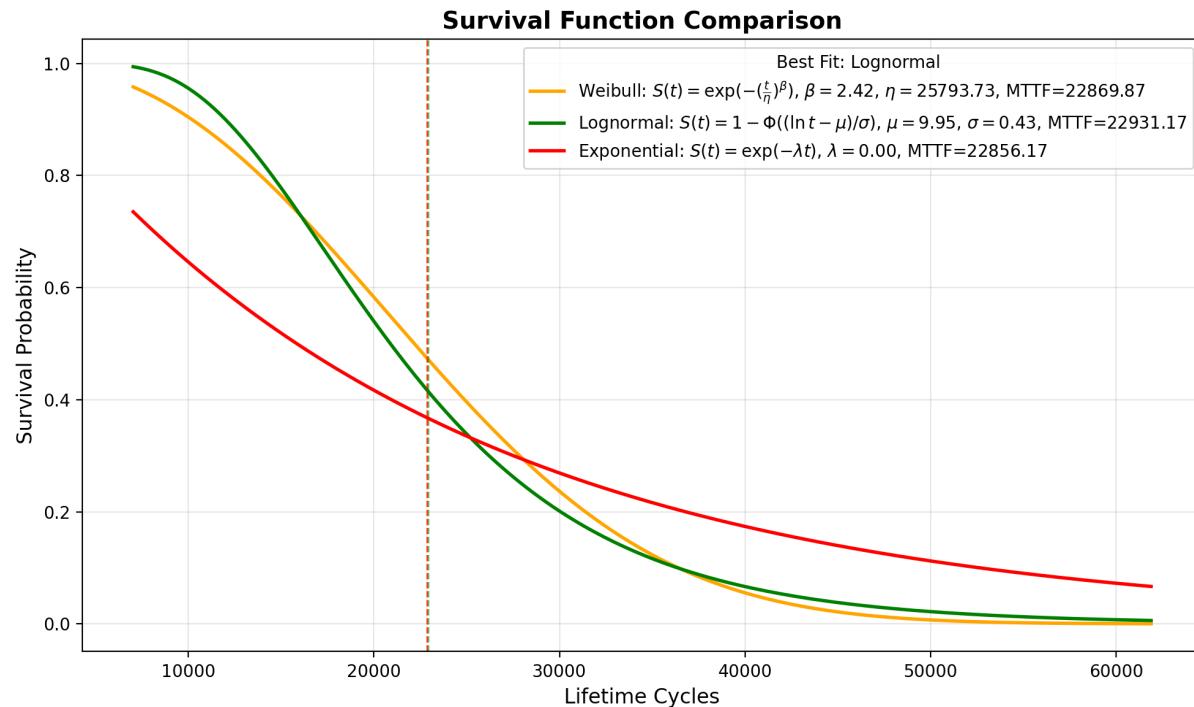
Engineers can identify which model best matches observed behavior

for example, mechanical wear (Weibull), environmental aging (Lognormal), or random electronic failure (Exponential).

The chosen model provides a mathematically consistent way to compute key reliability metrics such as MTTF
which quantifies the expected operational lifetime under normal conditions.

Selecting the right model improves decision accuracy in warranty prediction, maintenance scheduling, and design validation since over- or under-estimating lifetime can directly affect cost and safety.

Figure 3: Survival Function Comparison



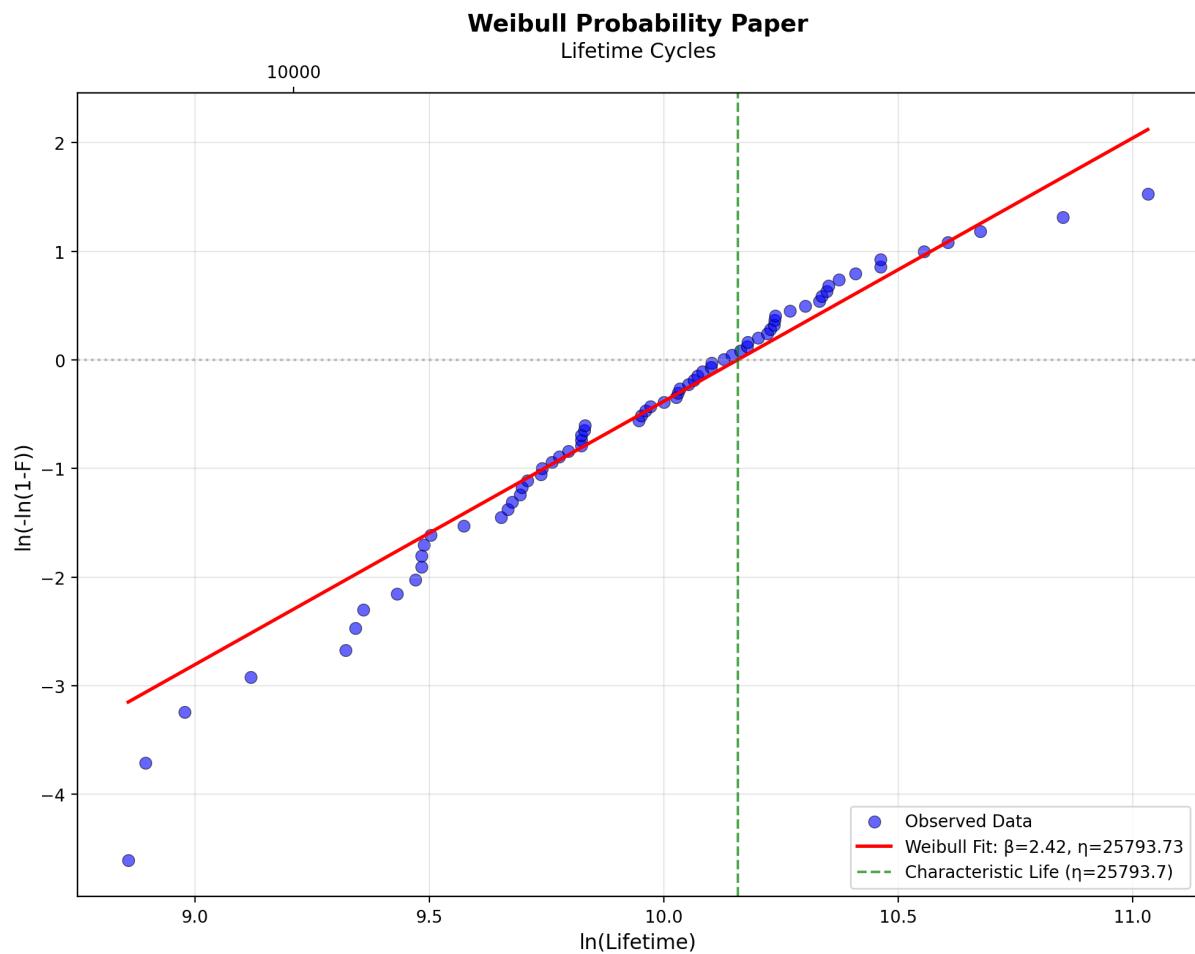
Description and Technical Analysis:

Survival functions $S(t)$ represent the probability that a component will survive beyond time t without failure. This plot compares the survival curves for all fitted distributions.

Engineering Significance:

Critical for determining warranty periods, maintenance schedules, and replacement strategies in reliability engineering.

Figure 4: Hazard Rate Function Comparison



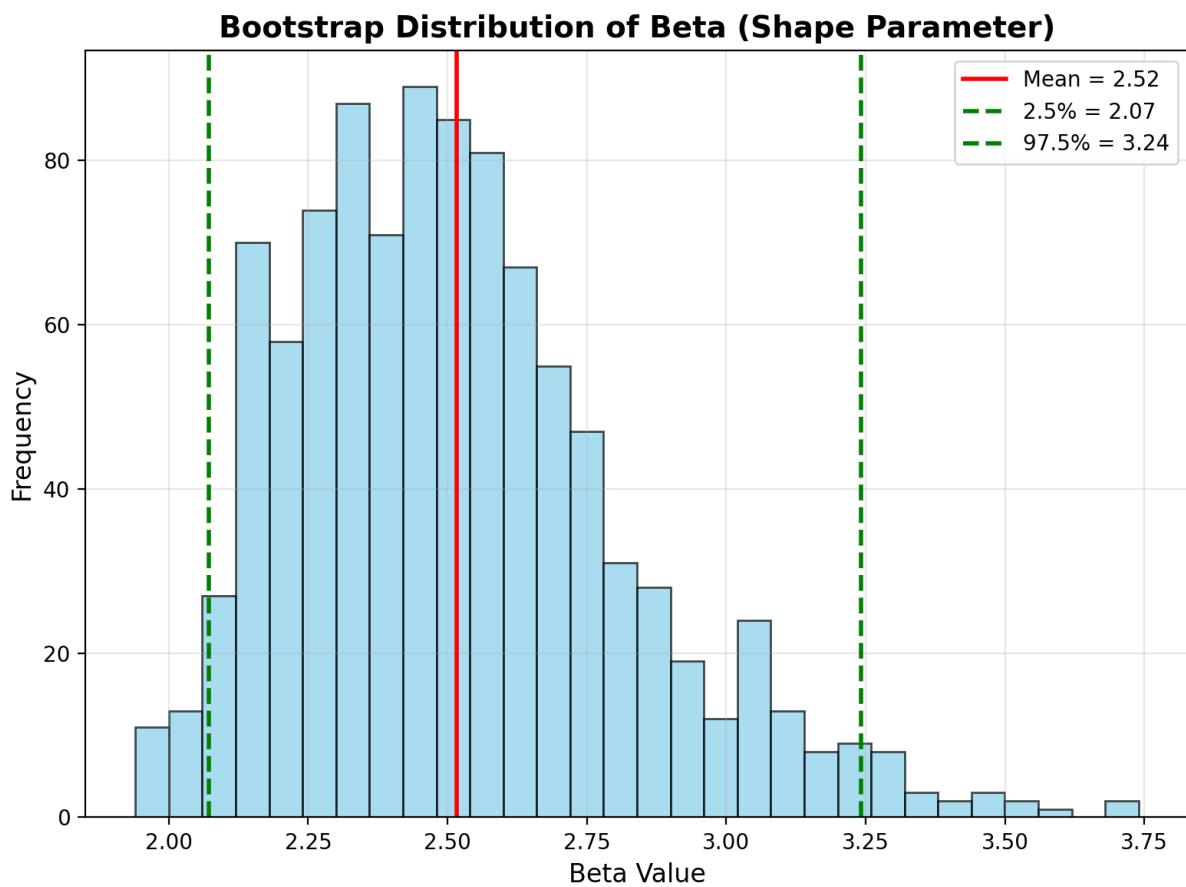
Description and Technical Analysis:

Hazard rate functions $h(t)$ represent the instantaneous failure rate at time t , given that the component has survived up to time t .

Engineering Significance:

Shape indicates failure modes: decreasing (infant mortality), constant (random), increasing (wear-out).

Figure 5: Weibull Probability Paper



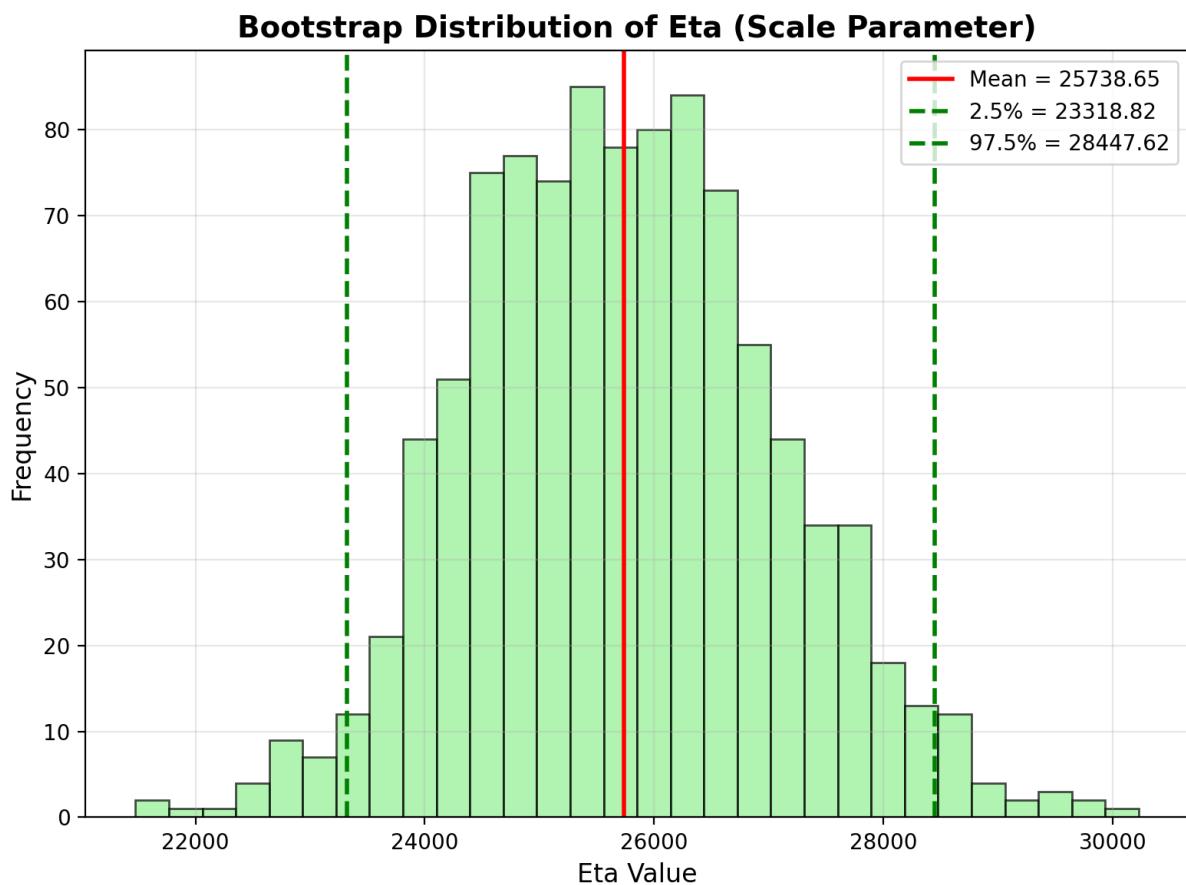
Description and Technical Analysis:

The Weibull probability paper is a specialized plot where Weibull-distributed data appears as a straight line. Data points represent empirical failure probabilities.

Engineering Significance:

Classical reliability tool for visual assessment of Weibull model adequacy and parameter estimation.

Figure 6: Bootstrap Distribution of Beta (Shape Parameter)



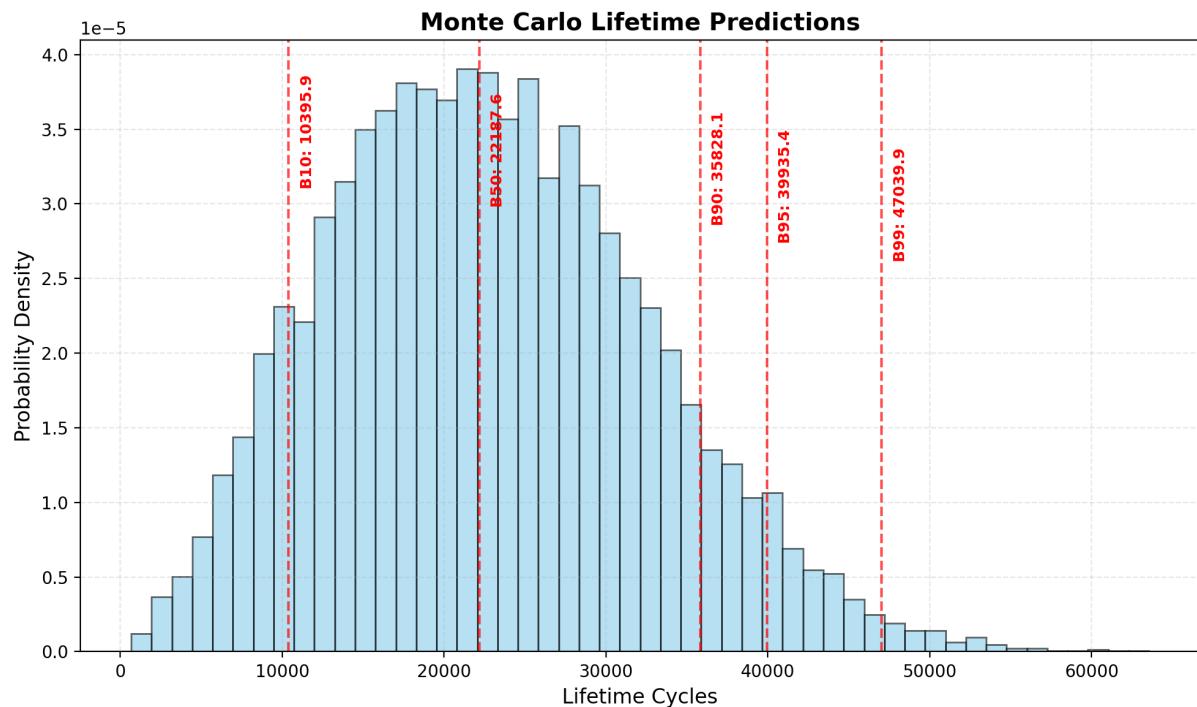
Description and Technical Analysis:

This histogram shows the distribution of the Weibull shape parameter (Beta) obtained through 1000 bootstrap resampling iterations.

Engineering Significance:

Provides non-parametric uncertainty estimation for the shape parameter critical to failure mode identification.

Figure 7: Bootstrap Distribution of Eta (Scale Parameter)



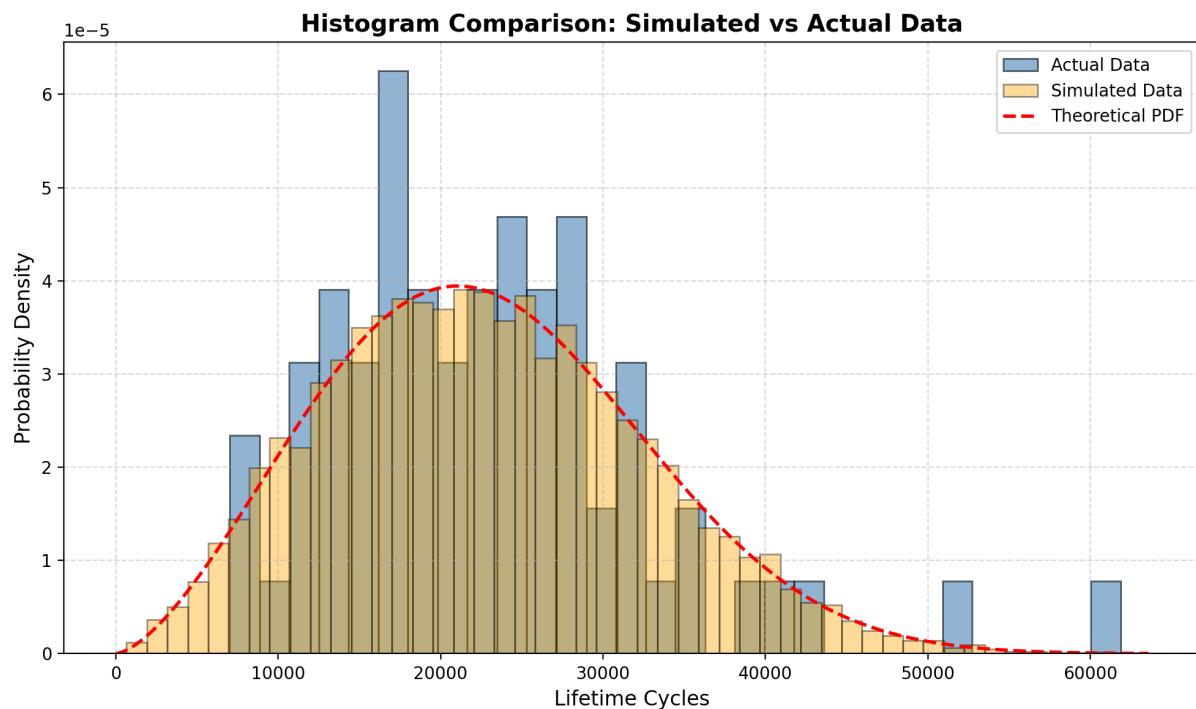
Description and Technical Analysis:

This histogram displays the distribution of the Weibull scale parameter (Eta) from bootstrap analysis. Eta represents the characteristic life.

Engineering Significance:

Essential for setting warranty periods and maintenance schedules with quantified uncertainty bounds.

Figure 8: Monte Carlo Lifetime Predictions



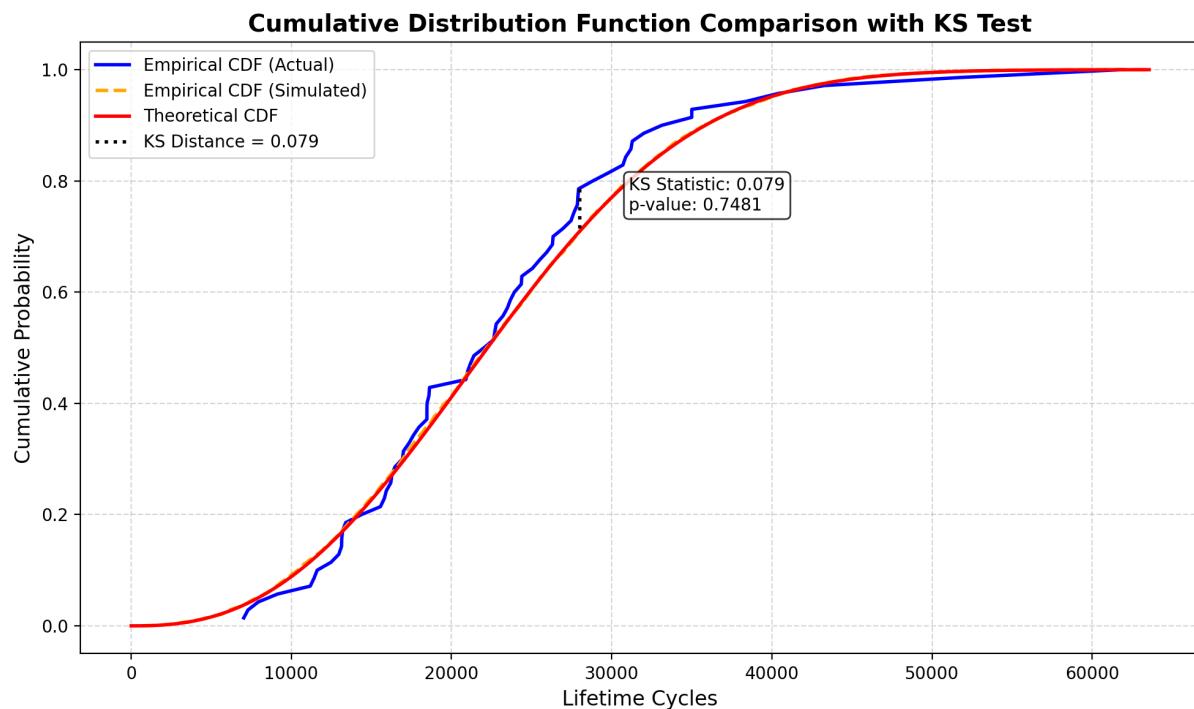
Description and Technical Analysis:

This histogram shows the distribution of 10,000 simulated lifetime values generated using the fitted Weibull parameters with key reliability percentiles marked.

Engineering Significance:

Incorporates parameter uncertainty providing robust lifetime predictions for engineering decision-making.

Figure 9: Histogram Comparison: Simulated vs Actual Data



Description and Technical Analysis:

This histogram comparison overlays the actual observed failure data with simulated data generated from the fitted Weibull model.

Engineering Significance:

Validates model accuracy and ensures the fitted Weibull distribution represents actual failure behavior.