CSE215
Foundations of Computer Science

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Previous lecture

Argument
Premise$_1$
Premise$_2$
$\vdots$
Premise$_m$
$\therefore$ Conclusion

- Use truth table to check if a logic argument is valid

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$p \to (q \vee r)$

$\sim (p \to q)$

$\therefore\ r$

Final, 2020-1

Today

- Use inference rules to prove an argument is valid

Inference rules

Definition

- A rule of inference is a valid argument form that can be used to establish logical deductions

Modus Ponens

Definition

- It has the form:
  If p, then q
  p
  $\therefore q$
- The term *modus ponens* in Latin means "method of affirming"

p	q	$p \to q$	p	q
T	T	T	T	T
T	F	F	T	
F	T	T	F	
F	F	T	F	

Example

- If the sum of the digits of 371,487 is divisible by 3, then 371,487 is divisible by 3.
  The sum of the digits of 371,487 is divisible by 3.
  $\therefore$ 371,487 is divisible by 3.

Modus Tollens

Definition

- It has the form:
  If p, then q
  $\sim q$
  $\therefore \sim p$
- The term *modus tollens* in Latin means "method of denying"

p	q	$p \rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

Example

- If Zeus is human, then Zeus is mortal.
  Zeus is not mortal.
  $\therefore$ Zeus is not human.

Generalization

Definition

- It has the form:

  p

  $\therefore p \vee q$

p	q	p	$p \vee q$
T	T	T	T
T	F	T	T
F	T	F	
F	F	F	

Example

- 35 is odd.

  $\therefore$ (more generally) 35 is odd or 35 is even.

Specialization

Definition

- It has the form:
  $p \wedge q$
  $\therefore p$

p	q	$p \wedge q$	p
T	T	T	T
T	F	F	
F	T	F	
F	F	F	

Example

- Ana knows numerical analysis and Ana knows graph algorithms.
  $\therefore$ (in particular) Ana knows graph algorithms

Conjunction

Definition

- It has the form:

  p

  q

  $\therefore p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	
F	T	
F	F	

Example

- Lily loves mathematics.

  Lily loves algorithms.

  $\therefore$ Lily loves both mathematics and algorithms.

Elimination

Definition

- It has the form:
  $p \vee q$
  $\sim q$
  $\therefore p$
- Intuition: When you have only two possibilities and you can rule one out, the other must be the case

p	q	$p \vee q$	$\sim q$	p
T	T	T	F	T
T	F	T	T	T
F	T	T	F	F
F	F	F	T	F

Example

- Suppose $x - 3 = 0$ or $x + 2 = 0$.
  Also, suppose x is nonnegative.
  $\therefore x = 3$.

Transitivity

Definition

- It has the form:

  $p \to q$

  $q \to r$

  $\therefore p \to r$
- Can be generalized to a chain with any number of conditionals

Example

- If 18,486 is divisible by 18, then 18,486 is divisible by 9.

  If 18,486 is divisible by 9, then the sum of the digits of 18,486 is divisible by 9.

  $\therefore$ If 18,486 is divisible by 18, then the sum of the digits of 18,486 is divisible by 9.

Division into cases

Definition

- It has the form:
  $p \vee q$
  $p \rightarrow r$
  $q \rightarrow r$
  $\therefore r$

Example

- x is positive or x is negative.
  If x is positive, then $x^2 > 0$.
  If x is negative, then $x^2 > 0$.
  $\therefore x^2 > 0$.

Summary of Inference rules

Name	Rule		Name	Rule	
Modus Ponens	$p \to q$		Elimination	$p \vee q$	$p \vee q$
	p			$\sim q$	$\sim p$
	$\therefore q$			$\therefore p$	$\therefore q$
Modus Tollens	$p \to q$		Transitivity	$p \to q$	
	$\sim q$			$q \to r$	
	$\therefore \sim p$			$\therefore p \to r$	
Proof by division into cases	$p \vee q$		Generalization	p	q
	$p \to r$			$\therefore p \vee q$	$\therefore p \vee q$
	$q \to r$		Specialization	$p \wedge q$	$p \wedge q$
	$\therefore r$			$\therefore p$	$\therefore q$
Conjunction	p		Contradiction	$\sim p \to c$	
	q			$\therefore p$	
	$\therefore p \wedge q$				

Some wrong inference

Definition

- A fallacy is an error in reasoning that results in an invalid argument

Fallacy: Converse error

Definition

- It has the form:
  $p \rightarrow q$
  q
  $\therefore p$

- Superficially resembles modus ponens but is invalid

p	q	$p \rightarrow q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Example

- If $x > 2$, then $x^2 > 4$.
  $x^2 > 4$.
  $\therefore x > 2$.

Fallacy: Inverse error

Definition

- It has the form:

  $p \to q$

  $\sim p$

  $\therefore \sim q$

- Superficially resembles modus tollens but is invalid

p	q	$p \to q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Example

- If $x > 2$, then $x^2 > 4$.

  $x \leq 2$.

  $\therefore x^2 \leq 4$.

Break;

Exercises

To finish by 1h50

Supply the missing statements using inference rules

1.	1. $p \to \sim q$	Premise	**2.**	1. $\sim p \to q$	Premise
	2. p	Premise		2. $\sim p$	Premise
	3. - - - -	1,2 Modus Ponens		3. - - - -	1,2 Modus Ponens
3.	1. $(\sim p \vee q) \to \sim(q \wedge r)$	Premise	**4.**	1. $(\sim p \wedge q) \to (q \wedge \sim r)$	Premise
	2. $\sim p \vee q$	Premise		2. $\sim p \wedge q$	Premise
	3. - - - -	1,2 Modus Ponens		3. - - - -	1,2 Modus Ponens
5.	1. $(\sim p \vee q) \to \sim(q \wedge r)$	Premise	**6.**	1. $(\sim p \wedge q) \to (q \wedge \sim r)$	Premise
	2. $q \wedge r$	Premise		2. $\sim(q \wedge \sim r)$	Premise
	3. - - - -	1,2 Modus Tollens		3. - - - -	1,2 Modus Tollens
7.	1. $\sim(\sim p \vee q)$	Premise	**8.**	$\sim(p \wedge \sim q)$	Premise
	2. - - - -	De Morgan		2. - - - -	De Morgan
9.	1. $(p \wedge r) \to \sim q$	Premise	**10.**	1. $(\sim p \wedge q) \to (q \wedge \sim r)$	Premise
	2. $\sim q \to r$	Premise		2. $(q \wedge \sim r) \to s$	Premise
	3. - - - -	1,2 Transitive Law		3. - - - -	1,2 Transitive Law

Solution (from our class)

1.

1. $p \to \sim q$	Premise
2. p	Premise
3. $\sim q$	1,2 Modus Ponens

2.

1. $\sim p \to q$	Premise
2. $\sim p$	Premise
3. q	1,2 Modus Ponens

3.

1. $(\sim p \vee q) \to \sim(q \wedge r)$	Premise
2. $\sim p \vee q$	Premise
3. $\sim(q \wedge r)$	1,2 Modus Ponens

4.

1. $(\sim p \wedge q) \to (q \wedge \sim r)$	Premise
2. $\sim p \wedge q$	Premise
3. $q \wedge \sim r$	1,2 Modus Ponens

5.

1. $(\sim p \vee q) \to \sim(q \wedge r)$	Premise
2. $q \wedge r$	Premise $p \wedge \sim q$
3. $\sim(\sim p \vee q)$	1,2 Modus Tollens

6.

1. $(\sim p \wedge q) \to (q \wedge \sim r)$	Premise
2. $\sim(q \wedge \sim r)$	Premise
3. $\sim(\sim p \wedge q)$	1,2 Modus Tollens

7.

1. $\sim(\sim p \vee q)$	Premise
2. $p \wedge \sim q$	De Morgan

8.

$\sim(p \wedge \sim q)$	Premise
2. $\sim p \vee q$	De Morgan

9.

1. $(p \wedge r) \to \sim q$	Premise
2. $\sim q \to r$	Premise
3. r X	1,2 Transitive Law

$(p \wedge r) \to r$

10.

1. $(\sim p \wedge q) \to (q \wedge \sim r)$	Premise
2. $(q \wedge \sim r) \to s$	Premise
3. $(\sim p \wedge q) \to s$	1,2 Transitive Law

11.

1. $(p\wedge r)\to\sim q$	Premise
2. $\sim q\to r$	Premise
3. $\sim r$	Premise
4. - - - -	1,2 Transitive Law
5. - - - -	3,4 Modus Tollens

12.

1. $(\sim p\wedge q)\to(q\wedge\sim r)$	Premise
2. $(q\wedge\sim r)\to s$	Premise
3. $\sim s$	Premise
4. - - - -	1,2 Transitive Law
5. - - - -	3,4 Modus Tollens

15.

1. $p\to(r\wedge q)$	Premise
2. $\sim r$	Premise
3. - - - -	2, Addition of $\sim q$
4. - - - -	3, De Morgan
5. - - - -	1,4 Modus Tollens

17.

1. $(p\wedge q)\to r$	Premise
2. q	Premise
3. p	Premise
4. - - - -	3,2 Rule C
5. - - - -	1,4 Modus Ponens

18.

1. $p\to r$	Premise
2. p	Premise
3. s	Premise
4. - - - -	1,2 Modus Ponens
5. - - - -	3,4 Rule C

These great exercises are taken from https://www.zweigmedia.com/RealWorld/logic/logicex5.html

Solution (from our class)

11.

1.	$(p\wedge r)\to\sim q$	Premise
2.	$\sim q\to r$	Premise
3.	$\sim r$	Premise
4.	$(p\wedge r)\to r$	1,2 Transitive Law
5.	$\sim(p\wedge r)$	3,4 Modus Tollens

12.

1.	$(\sim p\wedge q)\to(q\wedge\sim r)$	Premise
2.	$(q\wedge\sim r)\to s$	Premise
3.	$\sim s$	Premise
4.	$(\sim p\wedge q)\to s$	1,2 Transitive Law
5.	$\sim(\sim p\wedge q)$	3,4 Modus Tollens

15.

1.	$p\to(r\wedge q)$	Premise
2.	$\sim r$	Premise
3.	$\sim r\vee\sim q$	2, Addition of ~q Generalization
4.	$\sim(r\wedge q)$	3, De Morgan
5.	$\sim p$	1,4 Modus Tollens ... $p\to q,\ \sim q \Rightarrow \sim p$

17.

1.	$(p\wedge q)\to r$	Premise
2.	q	Premise
3.	p	Premise
4.	$p\wedge q$	3,2 Rule C conjunction
5.	r	1,4 Modus Ponens

18.

1.	$p\to r$	Premise
2.	p	Premise
3.	s	Premise
4.	r	1,2 Modus Ponens
5.	$s\wedge r$	3,4 Rule C

Prove the following is valid using logical inference

Problem 1. [5 points]

~~Determine if the following deduction rule is valid.~~

$p \to (q \vee r)$

$\sim (p \to q)$

$\therefore\ r$

Solution

$$p \rightarrow (q \vee r)$$
$$\sim (p \rightarrow q)$$
$$\therefore \; r$$

- We assume p -> (q ∨ r), ~(p -> q) are true. We need to prove r is true.
- Thus, p -> (q ∨ r), ~(~p ∨ q) // since p->q = ~p ∨ q
- Thus, p -> (q ∨ r), p ∧ ~q //double negation & De Morgen laws
- Thus, p -> (q ∨ r), p, ~q //specification
- Thus, q ∨ r, ~q // modus ponens
- Thus, r must be true // elimination
- We have proved the conclusion from the premises.

2020 Mid-exam-2

Problem 9. [5 points]

A set of premises and a conclusion are given. Use the valid arguments forms to deduce the conclusion from the premises, giving a reason for each step. Assume all variables are statement variables.

1. $b \vee \sim a \rightarrow c$
2. $\sim b \vee d$
3. $\sim e$
4. $c \wedge \sim a \rightarrow \sim d$
5. $a \rightarrow e$
6. $\therefore \sim b$

Solution

1. $b \vee \sim a \rightarrow c$
2. $\sim b \vee d$
3. $\sim e$
4. $c \wedge \sim a \rightarrow \sim d$
5. $a \rightarrow e$
6. $\therefore \sim b$

- From premises “3. ~e” and “5. a ->e”, we know ~a is true, using Modus Tollens
- From “1. b⋁~a -> c” and the fact that ~a is true, we know c is true using Modus Ponens
- From “4. c⋀~a -> ~d” and the facts ~a is true and c is true”, we know ~d is true using Modus Ponens
- From “2. ~b⋁d” and the fact that ~d is true, we know ~b is true using Elimination.
- So, we have proved the conclusion from the premises, and the argument is valid following inference rules.

Problem of truth tellers and liars

Problem

- There is an island containing two types of people: truth tellers who always tell the truth and liars who always lie. You visit the island and are approached by two natives who speak to you as follows:
  A says: B is a truth teller.
  B says: A and I are of opposite type.
- What are A and B?

Education is what remains after one has forgotten what one has learned in school. — Albert Einstein

Solution

A says: B is a truth teller.
B says: A and I are of opposite type.

- Assume A is a truth teller.
- Then, B is a truth teller from what A says.
- Then, A and B are of the opposite type. This is a contradiction!
- So, A must be a liar.
- Then, B must be a liar from what A says.
- This is consistent with what B says (which is a lie).
- Conclusion: Both A and B are liars.

Summary

- Prove validity using inference rules

추석 잘 보내세요 !