CSE215
Foundations of Computer Science

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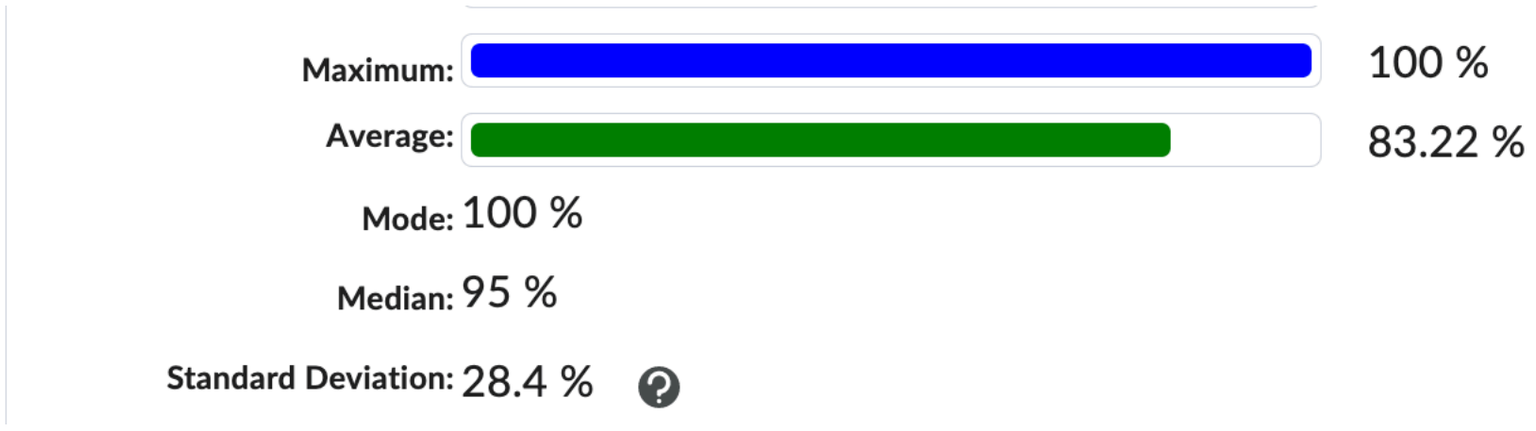
Today

- homework the week 02 (mistakenly named as homework01.md on Brightspace)

Last homework



Exercise 1. (score = 5)

Check if the two statement forms below are logically equivalent. Explanation is needed (e.g. based on the truth table).

- p ⋁ q -> r
- (p -> r) ⋀ (q -> r)

Solution

p	q	r	p->q	q->r	r->p	p->q ∧ q->r	p->q ∧ q->r ∧ r->p	p ∧ q ∧ r
T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	F	F
T	F	T	F	T	T	F	F	F
T	F	F	F	T	T	F	F	F
F	F	F	T	T	T	T	T	F
F	F	T	T	T	F	T	F	F
F	T	F	T	F	T	F	F	F
F	T	T	T	T	F	T	F	F

Not equivalent

Exercise 2 (score = 5)

Check if the two statement form are equivalent. Explanation is needed (e.g. based on the truth table).

- (p -> q) /\ (q -> r) /\ (r -> p)
- p /\ q /\ r

Solution

p	q	r	~p	~q	~r	p->q	q->r	r->p	(p->q)/\(q->r)/\(r->p)	p /\ q /\ r
T	T	T	F	F	F	T	T	T	T	T
T	T	F	F	F	T	T	F	T	F	F
T	F	T	F	T	F	F	T	T	F	F
T	F	F	F	T	T	F	T	T	F	F
F	T	T	T	F	F	T	T	F	F	F
F	T	F	T	F	T	T	F	T	F	F
F	F	T	T	T	F	T	T	F	F	F
F	F	F	T	T	T	T	T	T	T	F

The two are not equivalent, as shown when p, q, r are all False (last row).

Exercise 3 (score = 10)

Explanation can be omitted for this exercise.

Consider six statement forms (a-f):

- (a) p -> q
- (b) q -> p
- (c) ~p ⋁ q
- (d) ~q ⋁ p
- (e) ~q -> ~p
- (f) ~p -> ~q

1. Find all statement forms that are equivalent to (a), expect (a) itself.
2. Find all statement forms that are equivalent to (b), expect (b) itself.

Solution

- (a) and (c) (e)
- (b) and (d) (f)

Exercise 4 (score = 15)

Consider the proposition ~P ∧ (Q → P). What can you conclude about P and Q if you know the statement is true? Explanation is needed (e.g. based on the truth table).

Solution

P	Q	~P	Q->P	~P ∧ (Q->P)
T	T	F	T	F
T	F	F	T	F
F	F	T	T	T
F	T	T	F	F

P and Q are false.

Exercise 5 (score = 15)

For each statement form below, determine if it is a tautology, contradiction, or neither. Explanation is needed (e.g. based on the truth table).

1. (~p ⋁ q) ⋁ (p ⋀ ~q)
2. (p ⋀ ~q) ∧ (~p ⋁ q)
3. (p ⋀ q) ⋁ (~p ⋁ (p ⋀ ~q))

5.

P	Q	~P∨Q	P∧~Q	(~P∨Q) ∨(P∧~Q)
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

(~P∨Q)∨(P∧~Q) is a tautology.

P	Q	P∧~Q	~P∨Q	(P∧~Q) ∧(~P∨Q)
T	T	F	T	F
T	F	T	F	F
F	T	F	T	F
F	F	F	T	F

(P∧~Q)∧(~P∨Q) is a contradiction.

P	Q	P∧Q	P∧~Q	~P∨(P∧~Q)	(P∧Q)∨(~P∨(P∧~Q))
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	F	T	T
F	F	F	F	T	T

(P∧Q)∨(~P∨(P∧~Q)) is a tautology.

Exercise 6 (score = 20)

Determine whether the argument form below is valid. Explanation is needed (e.g. based on the truth table).

(1)

- premises: p -> q, q
- conclusion: p

(2)

- premises: p-> q, ~p
- conclusion: ~q

(3)

- premises: p-> q, p
- conclusion: q

(4)

- premises: p-> q, ~q
- conclusion: ~p

(1)

- premises: p -> q, q
- conclusion: p
- **Not valid**

p	q	p->q	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

(2)

- premises: p-> q, ~p
- conclusion: ~q
- **Not valid**

p	q	p->q	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

(3)

- premises: p-> q, p
- conclusion: q
- **Valid**

p	q	p->q	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

(4)

- premises: p-> q, ~q
- conclusion: ~p
- **Valid**

p	q	p->q	~q	~p
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Exercise 7 (score = 30)

Use truth tables to determine whether the argument form below is valid. Explanation is needed (e.g. based on the truth table).

(1)

- Premises: p -> q, ~p -> ~q
- Conclusion: p \/ q

(2)

- Premises: p \/ q, p -> ~q, ~r -> ~p
- Conclusion: r

(3)

- Premises: p, ~q -> ~p, ~q \/ r
- Conclusion r

(4)

- Premises: p/\q -> ~r, p \/ ~q, ~q -> p
- Conclusion: ~r

(5)

- Premises: p -> r, q -> r
- Conclusion: (p \/ q) -> r

(6)

- Premises: p -> (q \/ r), ~q \/ ~r
- Conclusion: ~p \/ ~r

Solution 7.1

(1)

- Premises: p -> q, ~p -> ~q
- Conclusion: p \/ q
- **Not valid**

p	q	~p	~q	p->q	~p->~q	p \/ q
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	T	F

Solution 7.2

(2)

- Premises: p ∨ q, p -> ~q, ~r -> ~p
- Conclusion: r
- **Not valid**

p	q	r	~p	~q	~r	p ∨ q	p->~q	~r->~p	r
T	T	T	F	F	F	T	F	T	T
T	T	F	F	F	T	T	F	F	F
T	F	T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T	F	F
F	T	T	T	F	F	T	T	T	T
F	T	F	T	F	T	T	T	T	F
F	F	T	T	T	F	F	T	T	T
F	F	F	T	T	T	F	T	T	F

Solution 7.3

(3)

- Premises: p, ~q -> ~p, ~q ∨ r
- Conclusion: r
- **Valid**

p	q	r	~q	~p	~q->~p	~q ∨ r	p	r
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	F	T	F
T	F	T	T	F	F	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	F	T	T	T	F	T
F	T	F	F	T	T	F	F	F
F	F	T	T	T	T	T	F	T
F	F	F	T	T	T	T	F	F

Solution 7.4

(4)

- Premises: p/\q -> ~r, p \/ ~q, ~q -> p
- Conclusion: ~r
- **Not valid**

p	q	r	~q	~r	p /\ q	p/\q ->~r	p\/~q	~q->p	~r
T	T	T	F	F	T	F	T	T	F
T	T	F	F	T	T	T	T	T	T
T	F	T	T	F	F	T	T	T	F
T	F	F	T	T	F	T	T	T	T
F	T	T	F	F	F	T	F	T	F
F	T	F	F	T	F	T	F	T	T
F	F	T	T	F	F	T	T	F	F
F	F	F	T	T	F	T	T	F	T

Solution 7.5

(5)

- Premises: p -> r, q -> r
- Conclusion: (p ∨ q) -> r
- **Valid**

p	q	r	p ∨ q	p -> r	q -> r	p ∨ q -> r
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Solution 7.6

(6)

- Premises: p -> (q ∨ r), ~q ∨ ~r
- Conclusion: ~p ∨ ~r
- **Not valid**

p	q	r	~p	~q	~r	q ∨ r	p->(q∨r)	~q ∨ ~r	~p ∨ ~r
T	T	T	F	F	F	T	T	F	F
T	T	F	F	F	T	T	T	T	T
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	T	F	F	T	T
F	T	T	T	F	F	T	T	F	T
F	T	F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T	T	T
F	F	F	T	T	T	F	T	T	T