

CSE215

Foundations of Computer Science

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Midterm 1 will take place after Proof part

**Today: mock exam for Midterm1
(35 min)**

Explanation and self-evaluation (15min)

- To finish by 3h30-4h25

* Exercise 1

Tautology, contradiction, or neither of them?

- $(p \text{ XOR } q) \wedge (p \leftrightarrow q)$
- $(p \text{ XOR } q) \vee (p \leftrightarrow q)$
- $(p \rightarrow q) \wedge (\sim p \rightarrow \sim q)$

Solution

- Contradiction
- Tautology
- Neither contradiction, or tautology

* Exercise 2

Consider the following statement form: $(p \oplus q) \rightarrow (\sim r \rightarrow (p \vee q))$.

1. How many rows would you need to construct its truth table?
2. How many columns would you need at least, to construct its truth table?
3. What is the truth value of the statement form when $p = T$, $q = F$, $r = T$?

Solution

- 9 rows including the header row
- 4 columns
- true

* Exercise 3

Show $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are **not** logical equivalent?

Solution

- Not equivalent. When $p = \text{false}$ and $r = \text{false}$, the first proposition $(p \rightarrow q) \rightarrow r$ is false whereas the second one, $p \rightarrow (q \rightarrow r)$ is true.

* Exercise 4

Write a negation for the following statement:

$\exists x \exists y$ such that $(0 < x \leq y^2 < 100)$.

Solution

- $\sim (\exists x \exists y \text{ such that } (0 < x \leq y^2 < 100))$ is the same as:
- $\sim (\exists x \exists y \text{ such that } (0 < x \wedge x \leq y^2 \text{ and } y^2 < 100))$,
which becomes:
- $\forall x \forall y, (0 \geq x \vee x >= y^2 \vee y^2 \geq 100),$

* Exercise 5

Determine if the following argument is valid:

$p \leftrightarrow q$

$q \oplus r$

$\therefore p \vee r$

Solution

- It is valid.
- Assume the premises are true.
- Namely, $p \leftrightarrow q$ is true and $q \text{ xor } r$ is true.
- Since $p \leftrightarrow q$ is true, we know p and q must be the same truth value.
- Since $q \text{ xor } r$ is true, we know q and r must be different truth values.
- Thus, p and r must be different truth values.
- Thus p and r can be either (true and false), or (false and true)
- Thus $q \vee r$, namely, the conclusion, is true.