

CSE215

Foundations of Computer Science

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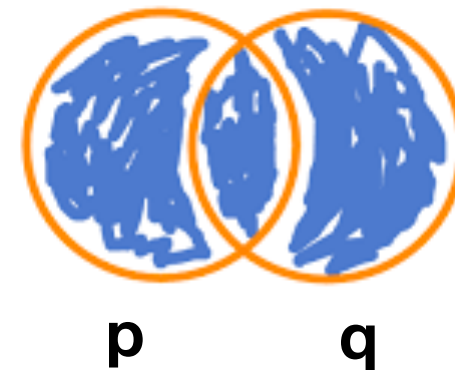
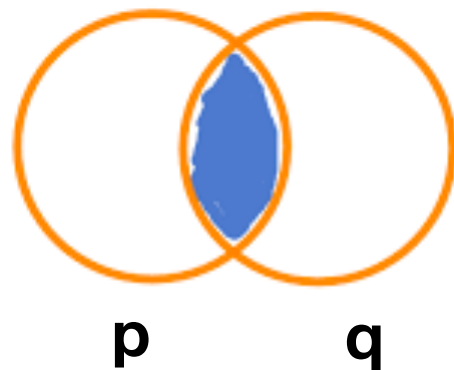
State University of New York, Korea

Today's plan

- Equivalence laws
- Valid arguments

Commutative Law

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$



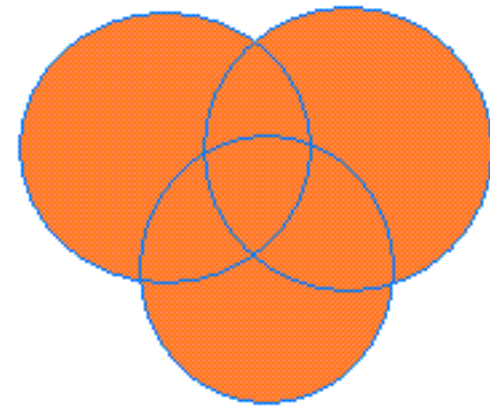
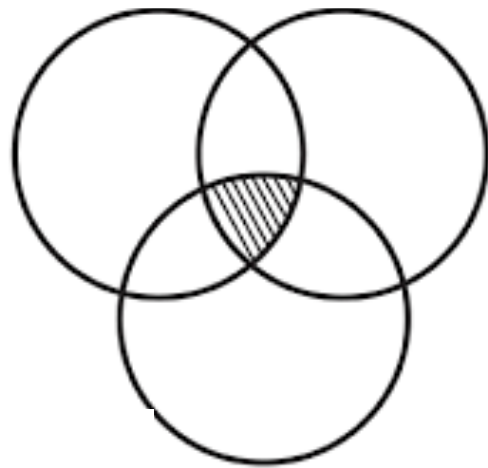
- Give some equivalent statement forms for $(p \wedge q) \vee (s \vee t)$

Associative Law

Associative laws

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$



- Think about an equivalent form for $(p \wedge q) \vee (s \vee t)$

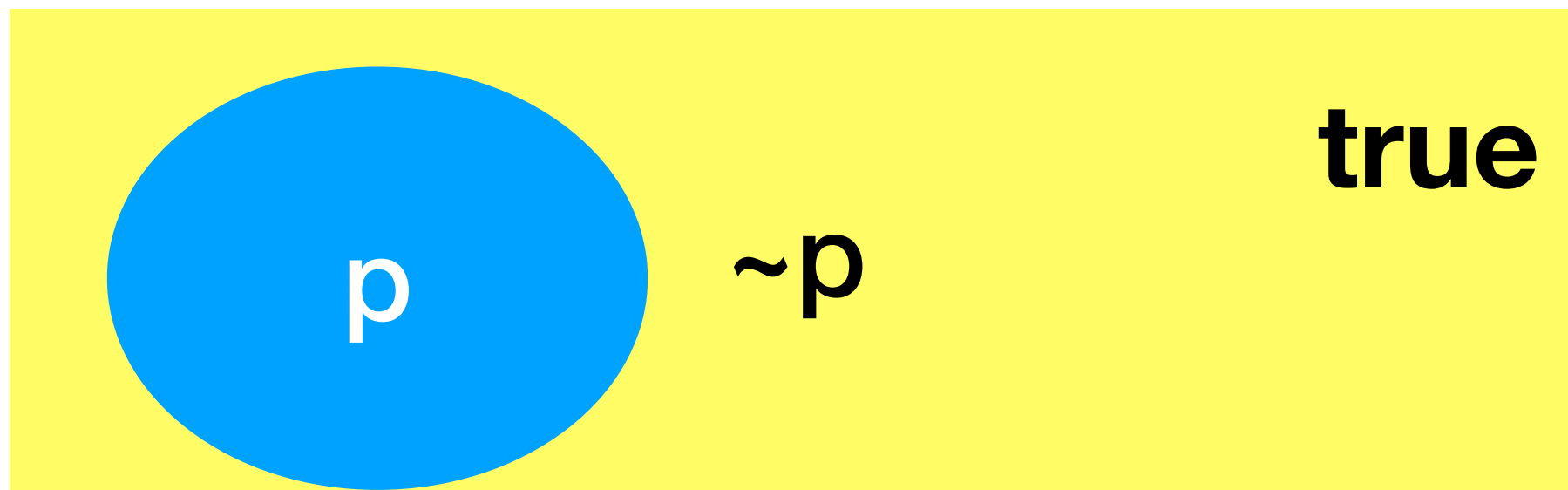
Distributive Law

Distributive laws $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- A bit like $a * (b + c) = a * b + a * c$
- Think about an equivalent forms for $(p \wedge q) \vee (s \vee t)$

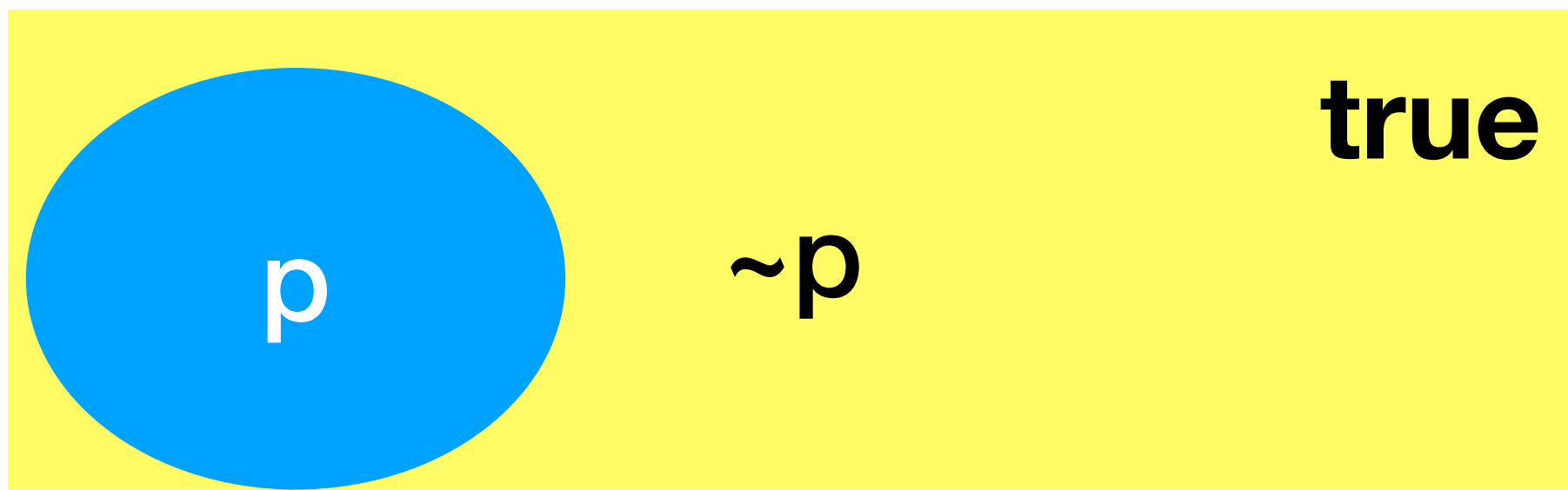
Laws with “true” and “false”

Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$



Double-negation Law

Double neg. law	$\sim (\sim p) \equiv p$
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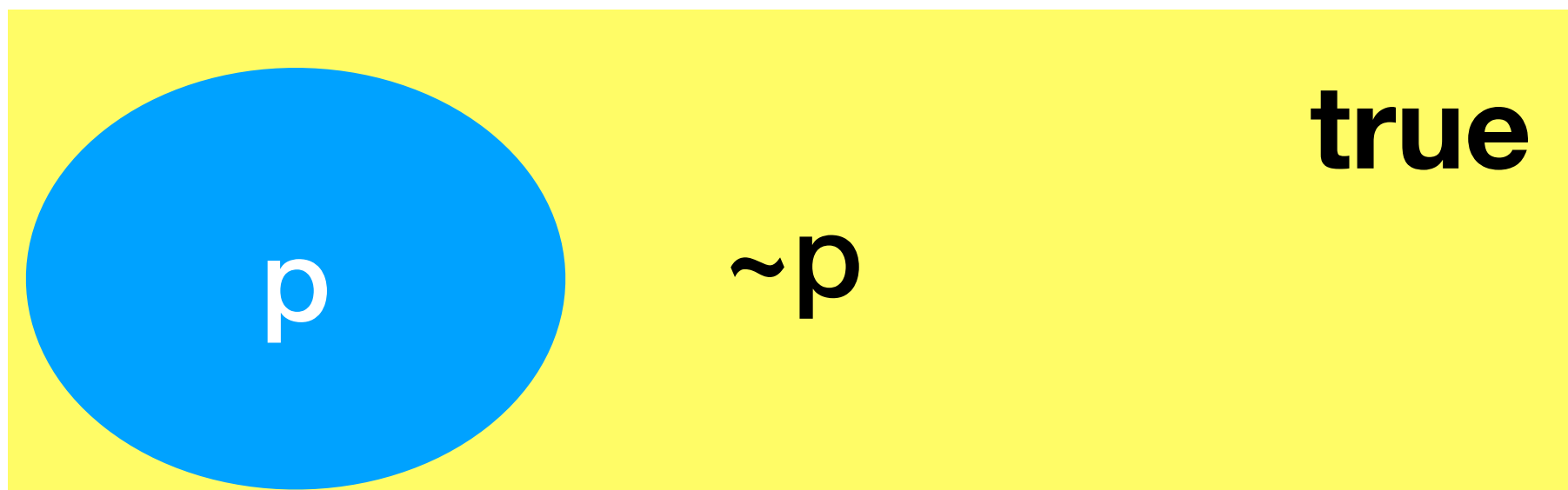


Idempotent Law

Idempotent laws

$$p \wedge p \equiv p$$

$$p \vee p \equiv p$$



De Morgan Law

De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
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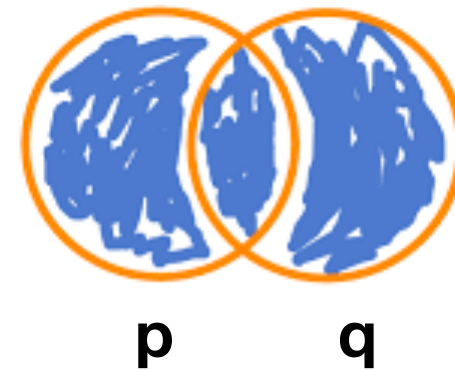
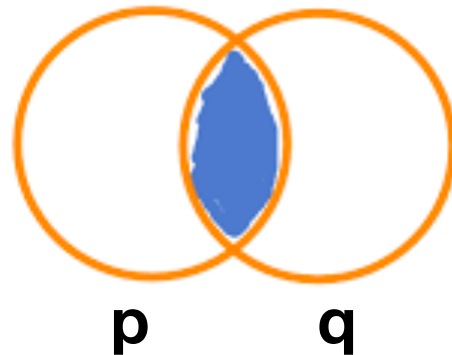
- p = student A is from Korea
- q = student B is from Korea
- $p \wedge q$ = Both student A and B are from Korea
- $\sim(p \wedge q)$ = Either A is not from Korea, or B is not from Korea
- $p \vee q$ = student A or student B is from Korea
- $\sim(p \vee q)$ = _____

Absorption Law

Absorption laws

$$p \vee (p \wedge q) \equiv p$$

$$p \wedge (p \vee q) \equiv p$$



Equivalence laws

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double neg. law	$\sim(\sim p) \equiv p$	
Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Exercise

Problem 3. [5 points]

Mention whether the following statements are true or false. Reasons are not needed.

- (a) [1 point] $p \vee \sim p \equiv \mathbf{c}$
- (b) [1 point] $p \vee (p \wedge q) \equiv p \wedge (p \vee q)$
- (c) [1 point] $\mathbf{c} \equiv p \vee \mathbf{t}$
- (d) [1 point] $p \wedge p \equiv p \vee p$
- (e) [1 point] $p \wedge \mathbf{c} \equiv \sim \mathbf{t}$

2021-Final

Exercise

Problem 2. [5 points]

Is conditional operator \rightarrow an associative operator? That is, is $(p \rightarrow q) \rightarrow r$ logically equivalent to $p \rightarrow (q \rightarrow r)$? Prove your answer.

- $(p \rightarrow q) \rightarrow r = (\sim p \vee q) \rightarrow r = \sim(\sim p \vee q) \vee r = (p \wedge \sim q) \vee r$
- $p \rightarrow (q \rightarrow r) = \sim p \vee (q \rightarrow r) = \sim p \vee (\sim q \vee r) = (\sim p \vee \sim q) \vee r$
- To show the two differ, consider $r = \text{false}$, $\sim q = \text{false}$, $p = \text{false}$
- Alternatively, we could use a truth table

Break if $T < 1\text{h}15\text{ pm}$

Part 2. Valid arguments

Final, 2020-1

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$$p \rightarrow (q \vee r)$$

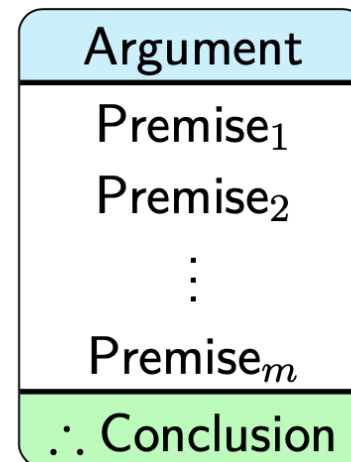
$$\sim (p \rightarrow q)$$

$$\therefore r$$

What is a logical argument?

Definitions

- **Logical argument.** Sequence of statements aimed at demonstrating the truth of an assertion
- **Conclusion.** Last statement in an argument
- **Premises.** Last-but-one statements in an argument



If Premise₁ and Premise₂ and \dots and Premise_m, then Conclusion.

What is a valid argument?

Definition

- An argument is **valid** if the conclusion follows necessarily from the premises

<https://youglish.com/pronounce/%22Valid%20Argument%22/english/us?>

- Every person will die
- Socrates is a person
- So, Socrates will die
- Congressmen/women own classified info
- Investors can take profit from classified info
- So, congressmen/women should not be allowed to actively do investment

Exercise: Valid or Not?

- All cups are blue
- Socrates is cup
- So, Socrates is blue

Valid or not?

Examples

Valid	<ul style="list-style-type: none">• If Socrates is a man, then Socrates is mortal. Socrates is a man. Therefore, Socrates is mortal.	$\text{If } p, \text{ then } q.$ $p.$ Therefore, $q.$
Invalid	<ul style="list-style-type: none">• If Socrates is a man, then Socrates is mortal. Socrates is mortal. Therefore, Socrates is a man.	$\text{If } p, \text{ then } q.$ $q.$ Therefore, $p.$
Valid	<ul style="list-style-type: none">• If Socrates is a man, then Socrates is mortal. Socrates is not mortal. Therefore, Socrates is not a man.	$\text{If } p, \text{ then } q.$ $\sim q.$ Therefore, $\sim p.$
Invalid	<ul style="list-style-type: none">• If Socrates is a man, then Socrates is mortal. Socrates is not a man. Therefore, Socrates is not mortal.	$\text{If } p, \text{ then } q.$ $\sim p.$ Therefore, $\sim q.$

How to mathematically check if an argument is valid?

- Truth table
- Inference rules

Method 1: Truth table

1. Identify the premises and conclusion
2. Construct a truth table for premises and conclusion
3. A row of the **truth table** in which all the premises are true is called a **critical row**.

If there is a critical row in which the conclusion is false, then the argument is **invalid**. If the conclusion in every critical row is true, then the argument is **valid**.

Importantly, if there are no critical rows, then the arguments is considered valid

Example

Problem

- Determine the validity of the argument:

$$p \rightarrow q \vee \sim r$$

$$q \rightarrow p \wedge r$$

$$\therefore p \rightarrow r$$

p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \rightarrow q \vee \sim r$	$q \rightarrow p \wedge r$	$p \rightarrow r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Exercise

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$$p \rightarrow (q \vee r)$$

$$\sim (p \rightarrow q)$$

$$\therefore r$$

2020 Final-1

Summary

- Logical equivalence
- Valid arguments
- Check validity using truth tables

Thank you for your attention!