

CSE215

Foundations of Computer Science

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Today's Objectives

- A comprehensive coverage of related SBU exam problems in 2020 & 2021

2021 Final

Problem 1. [5 points]

Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

Solution

p	q	r	p AND (q OR r)	p AND (q AND r)	p AND (q OR r) \Leftrightarrow p AND (q AND r)
T	T	T	T	T	T
T	T	F	T	F	F
T	F	F	F	F	F
T	F	T	T	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	F	F	F	F
F	F	T	F	F	F

Errata: row #2 (T T F row), column “p AND (q AND r)” should be F, and the last column should be F too.

2021 Final

Problem 2. [5 points]

Construct a truth table for the following statement form: $(p \rightarrow q) \vee ((q \oplus r) \rightarrow \sim p)$.

Solution

p	q	r	$p \rightarrow q$	$q \text{ xor } r$	$(q \text{ xor } r) \rightarrow \sim p$	$(p \rightarrow q) \setminus / ((q \text{ xor } r) \rightarrow \sim p)$
t	t	t	t	f	t	t
t	t	f	t			t
t	f	t	f	t	f	f
t	f	f	f	f	t	t
f	t	t	t			t
f	t	f	t			t
f	f	t	t			t
f	f	f	t			t

2021 Final

Problem 3. [5 points]

Verify using truth tables if the following two logical expressions are equivalent.

$(p \rightarrow q) \wedge (\sim p \rightarrow \sim q)$ and $\sim p \leftrightarrow \sim q$

Solution

p	q	$p \rightarrow q$	$\sim p \rightarrow \sim q$	$(p \rightarrow q) \wedge (\sim p \rightarrow \sim q)$	$\sim p \leftrightarrow \sim q$
t	t	t	t	t	t
t	f	f	t	f	f
f	t	t	f	f	f
f	f	t	t	t	t

2020 Final-a

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$$p \rightarrow (q \vee r)$$

$$\sim (p \rightarrow q)$$

$$\therefore r$$

Solution

p, q, r	$p \rightarrow (q \vee r)$	$\sim (p \rightarrow q)$	r
t t t	t	f	t
t t f	t	f	f
t f t	t	t	t
t f f	f	t	f
f t t	t	f	t
f t f	t	f	f
f f t	t	f	t
f f f	t	f	f

So, it is valid — the critical row being Row #3 where premises are true and conclusion is true

2020 final-b

Problem 2. [5 points]

Is conditional operator \rightarrow an associative operator? That is, is $(p \rightarrow q) \rightarrow r$ logically equivalent to $p \rightarrow (q \rightarrow r)$? Prove your answer.

Solution

p, q, r	$(p \rightarrow q) \rightarrow r$	$p \rightarrow (q \rightarrow r)$
t t t	t	t
t t f	f	f
t f t	t	t
t f f	t	t
f t t	t	t
f t f	f	t
f f t	t	t
f f f	f	t

So, the two are not equivalent, considering
the rows with p,q,r being f, t, f or f, f, f

Alternative solution with logical equivalence

Problem 2. [5 points]

Is conditional operator \rightarrow an associative operator? That is, is $(p \rightarrow q) \rightarrow r$ logically equivalent to $p \rightarrow (q \rightarrow r)$? Prove your answer.

- $(p \rightarrow q) \rightarrow r = (\sim p \vee q) \rightarrow r = \sim(\sim p \vee q) \vee r = (p \wedge \sim q) \vee r$
- $p \rightarrow (q \rightarrow r) = \sim p \vee (q \rightarrow r) = \sim p \vee (\sim q \vee r) = (\sim p \vee \sim q) \vee r$
- To show the two differ, consider $r = \text{false}$, $\sim q = \text{false}$, $p = \text{false}$

2020-final-b

Problem 3. [5 points]

Verify using truth tables if the following two logical expressions are equivalent.

$$\sim p \leftrightarrow \sim q \text{ and } \sim (p \oplus q)$$

Solution

p	q	$\sim p \leftrightarrow \sim q$	$\sim(p \text{ xor } q)$
t	t	t	t
t	f	f	f
f	t	f	f
f	f	t	t

Thus, the two are equivalent.

Summary

- Check validity and equivalence using truth tables

Thank you for your attention!