

CSE215

Foundations of Computer Science

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Previous lecture

Argument
Premise ₁
Premise ₂
:
Premise _m
∴ Conclusion

- Use truth table to check if a logic argument is valid

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$$\begin{aligned} p \rightarrow (q \vee r) \\ \sim(p \rightarrow q) \\ ∴ r \end{aligned}$$

Final, 2020-1

Today

- Use inference rules to prove an argument is valid

Inference rules

Definition

- A **rule of inference** is a valid argument form that can be used to establish logical deductions

Modus Ponens

Definition

- It has the form:
If p , then q
 p
 $\therefore q$
- The term *modus ponens* in Latin means “method of affirming”

p	q	$p \rightarrow q$	p	q
T	T	T	T	T
T	F	F	T	
F	T	T	F	
F	F	T	F	

What is wrong?

- You are a detective investigating a crime scene with your partner, Sherlock. You come across a note that says: "If the butler did it, then the knife will be found in the kitchen."
- Sherlock responds, "Well, we just found the knife in the kitchen. Therefore, the butler did it."

Modus Tollens

Definition

- It has the form:
If p , then q
 $\sim q$
 $\therefore \sim p$
- The term *modus tollens* in Latin means “method of denying”

p	q	$p \rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

- If it rains, then the ground will be wet.”
- The ground is not wet
- ==> ?

Generalization

Definition

- It has the form:

p

$\therefore p \vee q$

p	q	p	$p \vee q$
T	T	T	T
T	F	T	T
F	T	F	
F	F	F	

Example

- 35 is odd.

\therefore (more generally) 35 is odd or 35 is even.

What is wrong?

- a friend who tells you, "I really like pizza." Based on this statement, you can apply the Generalization inference rule to make the following conclusion:
 - "All people like pizza."

Specialization

Definition

- It has the form:

$$p \wedge q$$

$$\therefore p$$

p	q	$p \wedge q$	p
T	T	T	T
T	F	F	
F	T	F	
F	F	F	

Example

- Ana knows numerical analysis and Ana knows graph algorithms.
 \therefore (in particular) Ana knows graph algorithms

Conjunction

Definition

- It has the form:

p

q

$\therefore p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	
F	T	
F	F	

Example

- Lily loves mathematics.
Lily loves algorithms.
 \therefore Lily loves both mathematics and algorithms.

Elimination

Definition

- It has the form:

$$p \vee q$$

$$\sim q$$

$$\therefore p$$

- Intuition: When you have only two possibilities and you can rule one out, the other must be the case

p	q	$p \vee q$	$\sim q$	p
T	T	T	F	T
T	F	T	T	T
F	T	T	F	F
F	F	F	T	F

Example

- Suppose $x - 3 = 0$ or $x + 2 = 0$.
Also, suppose x is nonnegative.
 $\therefore x = 3$.

Transitivity

Definition

- It has the form:

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\therefore p \rightarrow r$$

- Can be generalized to a chain with any number of conditionals

Example

- If 18,486 is divisible by 18, then 18,486 is divisible by 9.
If 18,486 is divisible by 9, then the sum of the digits of 18,486 is divisible by 9.
 \therefore If 18,486 is divisible by 18, then the sum of the digits of 18,486 is divisible by 9.

Division into cases

Definition

- It has the form:

$$p \vee q$$

$$p \rightarrow r$$

$$q \rightarrow r$$

$$\therefore r$$

Example

- x is positive or x is negative.

If x is positive, then $x^2 > 0$.

If x is negative, then $x^2 > 0$.

$$\therefore x^2 > 0.$$

Summary of Inference rules

Name	Rule	Name	Rule
Modus Ponens	$p \rightarrow q$ p $\therefore q$	Elimination	$p \vee q$ $\sim q$ $\therefore p$
Modus Tollens	$p \rightarrow q$ $\sim q$ $\therefore \sim p$	Transitivity	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$
Proof by division into cases	$p \vee q$ $p \rightarrow r$ $q \rightarrow r$ $\therefore r$	Generalization	p $\therefore p \vee q$ $\therefore p \vee q$
Conjunction	p q $\therefore p \wedge q$	Specialization	$p \wedge q$ $\therefore p$ $\therefore q$
		Contradiction	$\sim p \rightarrow c$ $\therefore p$

Some wrong inference

Definition

- A **fallacy** is an error in reasoning that results in an invalid argument

Fallacy: Converse error

Definition

- It has the form:

$$p \rightarrow q$$

$$q$$

$$\therefore p$$

- Superficially resembles modus ponens but is invalid

p	q	$p \rightarrow q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Example

- If $x > 2$, then $x^2 > 4$.

$$x^2 > 4.$$

$$\therefore x > 2.$$

Fallacy: Inverse error

Definition

- It has the form:

$$p \rightarrow q$$

$$\sim p$$

$$\therefore \sim q$$

- Superficially resembles modus tollens but is invalid

p	q	$p \rightarrow q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Example

- If $x > 2$, then $x^2 > 4$.

$$x \leq 2.$$

$$\therefore x^2 \leq 4.$$

Break;

Exercises

To finish by 11:50am

Supply the missing statements using inference rules

1. 1. $p \rightarrow \neg q$ Premise
2. p Premise
3. - - - - 1,2 Modus Ponens

2. 1. $\neg p \rightarrow q$ Premise
2. $\neg p$ Premise
3. - - - - 1,2 Modus Ponens

3. 1. $(\neg p \vee q) \rightarrow \neg(q \wedge r)$ Premise
2. $\neg p \vee q$ Premise
3. - - - - 1,2 Modus Ponens

4. 1. $(\neg p \wedge q) \rightarrow (q \wedge \neg r)$ Premise
2. $\neg p \wedge q$ Premise
3. - - - - 1,2 Modus Ponens

5. 1. $(\neg p \vee q) \rightarrow \neg(q \wedge r)$ Premise
2. $q \wedge r$ Premise
3. - - - - 1,2 Modus Tollens

6. 1. $(\neg p \wedge q) \rightarrow (q \wedge \neg r)$ Premise
2. $\neg(q \wedge \neg r)$ Premise
3. - - - - 1,2 Modus Tollens

7. 1. $\neg(\neg p \vee q)$ Premise
2. - - - - De Morgan

8. $\neg(p \wedge \neg q)$ Premise
2. - - - - De Morgan

9. 1. $(p \wedge r) \rightarrow \neg q$ Premise
2. $\neg q \rightarrow r$ Premise
3. - - - - 1,2 Transitive Law

10. 1. $(\neg p \wedge q) \rightarrow (q \wedge \neg r)$ Premise
2. $(q \wedge \neg r) \rightarrow s$ Premise
3. - - - - 1,2 Transitive Law

Solution (from our class)

1. 1. $p \rightarrow \neg q$
2. p
3. $\neg q$

Premise \vdash
Premise
1,2 Modus Ponens

3. 1. $(\neg p \vee q) \rightarrow \neg(q \wedge r)$ Premise \leftarrow
2. $\neg p \vee q$ Premise \leftarrow
3. $\neg(q \wedge r)$ 1,2 Modus Ponens

2. 1. $\neg p \rightarrow q$ Premise
2. $\neg p$ Premise
3. q

Premise
Premise
1,2 Modus Ponens

5. 1. $(\neg p \vee q) \rightarrow \neg(q \wedge r)$ Premise
2. $q \wedge r$ Premise ~~$\vdash \neg(p \vee q)$~~
3. $\neg(\neg p \vee q)$ 1,2 Modus Tollens

6. 1. $(\neg p \wedge q) \rightarrow (q \wedge \neg r)$ Premise
2. $\neg(q \wedge \neg r)$ Premise
3. $\neg(\neg p \wedge q)$ 1,2 Modus Tollens

7. 1. $\neg(\neg p \vee q)$ Premise
2. $\neg\neg p \wedge \neg q$ De Morgan

8. $\neg(p \wedge \neg q)$ Premise
2. $\neg\neg p \vee q$ De Morgan

9. 1. $(p \wedge r) \rightarrow \neg q$ Premise \vdash
2. $\neg q \rightarrow r$ Premise \vdash
3. r
 $(p \wedge r) \rightarrow r$

10. 1. $(\neg p \wedge q) \rightarrow (q \wedge \neg r)$ Premise
2. $(q \wedge \neg r) \rightarrow s$ Premise
3. s 1,2 Transitive Law

- 11.** 1. $(p \wedge r) \rightarrow \neg q$ Premise
 2. $\neg q \rightarrow r$ Premise
 3. $\neg r$ Premise
 4. - - - - 1,2 Transitive Law
 5. - - - - 3,4 Modus Tollens

- 12.** 1. $(\neg p \wedge q) \rightarrow (q \wedge \neg r)$ Premise
 2. $(q \wedge \neg r) \rightarrow s$ Premise
 3. $\neg s$ Premise
 4. - - - - 1,2 Transitive Law
 5. - - - - 3,4 Modus Tollens

- 15.** 1. $p \rightarrow (r \wedge q)$ Premise
 2. $\neg r$ Premise
 3. - - - - 2, Addition of $\neg q$
 4. - - - - 3, De Morgan
 5. - - - - 1,4 Modus Tollens

- 17.** 1. $(p \wedge q) \rightarrow r$ Premise
 2. q Premise
 3. p Premise
 4. - - - - 3,2 Rule C
 5. - - - - 1,4 Modus Ponens

- 18.** 1. $p \rightarrow r$ Premise
 2. p Premise
 3. s Premise
 4. - - - - 1,2 Modus Ponens
 5. - - - - 3,4 Rule C

These great exercises are taken from
<https://www.zweigmedia.com/RealWorld/logic/logicex5.html>

Solution (from our class)

11.	<p>1. $(p \wedge r) \rightarrow \neg q$</p> <p>2. $\neg q \rightarrow r$</p> <p>3. $\neg r$</p> <p>4. $(p \wedge r) \rightarrow r$</p> <p>5. $\neg(p \wedge r)$</p>	Premise Premise Premise <u>1,2 Transitive Law</u> 3,4 Modus Tollens
12.	<p>1. $(\neg p \wedge q) \rightarrow (q \wedge \neg r)$</p> <p>2. $(q \wedge \neg r) \rightarrow s$</p> <p>3. $\neg s$</p> <p>4. $(\neg p \wedge q) \rightarrow s$</p> <p>5. $\neg(\neg p \wedge q)$</p>	Premise Premise Premise <u>1,2 Transitive Law</u> 3,4 Modus Tollens

15.	$\begin{cases} 1. p \rightarrow (r \wedge q) \\ 2. \neg r \\ 3. \neg r \vee \neg q \\ 4. \neg (\neg r \wedge \neg q) \\ 5. \neg \neg p \end{cases}$	Premise Premise 2, Addition of $\neg q$ Generalization 3, De Morgan 1,4 Modus Tollens
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<u>17.</u>	1. $(p \wedge q) \rightarrow r$	Premise	18.	1. $p \rightarrow r$	Premise
	2. q	Premise		2. p	Premise
	3. p	Premise		3. s	Premise
	4. $\frac{\frac{p \wedge q}{p}}{p \wedge q}$	3,2 Rule C <i>disjunction</i>		4. $\frac{p}{\neg r}$	1,2 Modus Ponens
	5. $\frac{\frac{p \wedge q}{p} \quad \frac{p}{\neg r}}{\neg r}$	1,4 Modus Ponens		5. $\frac{\frac{p}{\neg r}}{SAR}$	3,4 Rule C

Prove the following is valid
using logical inference

Problem 1. [5 points]

~~Determine if the following deduction rule is valid.~~

$$p \rightarrow (q \vee r)$$

$$\sim(p \rightarrow q)$$

$$\therefore r$$

Solution

$$\begin{aligned} p &\rightarrow (q \vee r) \\ \sim(p \rightarrow q) &\\ \therefore r & \end{aligned}$$

- We assume $p \rightarrow (q \vee r)$, $\sim(p \rightarrow q)$ are true. We need to prove r is true.
- Thus, $p \rightarrow (q \vee r)$, $\sim(\sim p \vee q)$ // since $p \rightarrow q = \sim p \vee q$
- Thus, $p \rightarrow (q \vee r)$, $p \wedge \sim q$ //double negation & De Morgen laws
- Thus, $p \rightarrow (q \vee r)$, p , $\sim q$ //specification
- Thus, $q \vee r$, $\sim q$ // modus ponens
- Thus, r must be true // elimination
- We have proved the conclusion from the premises.

2020 Mid-exam-2

Problem 9. [5 points]

A set of premises and a conclusion are given. Use the valid arguments forms to deduce the conclusion from the premises, giving a reason for each step. Assume all variables are statement variables.

1. $b \vee \sim a \rightarrow c$
2. $\sim b \vee d$
3. $\sim e$
4. $c \wedge \sim a \rightarrow \sim d$
5. $a \rightarrow e$
6. $\therefore \sim b$

Solution

1. $b \vee \sim a \rightarrow c$
2. $\sim b \vee d$
3. $\sim e$
4. $c \wedge \sim a \rightarrow \sim d$
5. $a \rightarrow e$
6. $\therefore \sim b$

- From premises “3. $\sim e$ ” and “5. $a \rightarrow e$ ”, we know $\sim a$ is true, using Modus Tollens
- From “1. $b \vee \sim a \rightarrow c$ ” and the fact that $\sim a$ is true, we know c is true using Modus Ponens
- From “4. $c \wedge \sim a \rightarrow \sim d$ ” and the facts $\sim a$ is true and c is true”, we know $\sim d$ is true using Modus Ponens
- From “2. $\sim b \vee d$ ” and the fact that $\sim d$ is true, we know $\sim b$ is true using Elimination.
- So, we have proved the conclusion from the premises, and the argument is valid following inference rules.

Problem of truth tellers and liars

Problem

- There is an island containing two types of people: truth tellers who always tell the truth and liars who always lie. You visit the island and are approached by two natives who speak to you as follows:
A says: *B* is a truth teller.
B says: *A* and I are of opposite type.
- What are *A* and *B*?

Education is what remains after one has forgotten what one has learned in school. — Albert Einstein

Solution

A says: *B* is a truth teller.
B says: *A* and I are of opposite type.

- Assume *A* is a truth teller.
- Then, *B* is a truth teller from what *A* says.
- Then, *A* and *B* are of the opposite type. This is a contradiction!
- So, *A* must be a liar.
- Then, *B* must be a liar from what *A* says.
- This is consistent with what *B* says (which is a lie).
- Conclusion: Both *A* and *B* are liars.

Summary

- Prove validity using inference rules

추석 잘 보내세요 !