CSE215
Foundations of Computer Science

Instructor: Zhoulai Fu

State University of New York, Korea

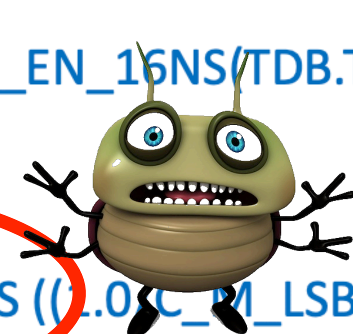
Course materials and Info available here:
https://github.com/zhoulaifu/23_cse215_spring

French Ariane 5 Rocket, 1996

What is Floating-Point ?

Ada code for Ariane 5 Rocket

```
if L_M_BV_32 > 32767 then
  P_M_DERIVE(T_ALG.E_BV) := 16#7FFF#;
elsif L_M_BV_32 < -32768 then
  P_M_DERIVE(T_ALG.E_BV) := 16#8000#;
else
  P_M_DERIVE(T_ALG.E_BV) := UC_16S_EN_16NS(TDB.T_ENTIER_16S(L_M_BV_32));
end if;
P_M_DERIVE(T_ALG.E_BH) :=
  UC_16S_EN_16NS (TDB.T_ENTIER_16S ((1.0/C_M_LSB_BH)*G_M_INFO_DERIVE(T_ALG.E_BH)));
```



$7 billion Software Disaster

Comparison:

SUNY Korea was awarded $0.05 billion for 10 years under an MKE grant
(Source: https://sunyk.cs.stonybrook.edu/)

From 2018 to 2020, South Korea GDP dropped $94 billion;
(Source: World bank)

Propositional Logic

Predicate Logic

Proof

Why does a computing system fail (or work)?

Sequences

Sets

Functions

Relations

Expected Learning Outcomes

- An ability to check if a mathematical argument is valid
- An ability to verify the correctness of proofs of some existing theorems and prove some new theorems
- An ability to use the mathematical concepts of sequences, functions, relations, and sets in solving computing problems

Meet the Instructor

Education

- B.Sc, M.Sc, Ecole Polytechnique, France
- M.Eng. Telecom Paris, France
- Ph.D. INRIA (National CS Lab), France

Teaching & Research

- University of California Davis, United States
- IT University of Copenhagen, Denmark
- SUNY Korea

TA CheaYoung Park

- <cheayoung dot park at stonybrook dot edu>

Team

You	TA	Instructor	ChatGPT
Lectures	Office hours	Office hours	Not do homework
Homework	Answer questions	Lectures	Answer questions
Ask questions		Grading	
		Answer questions	

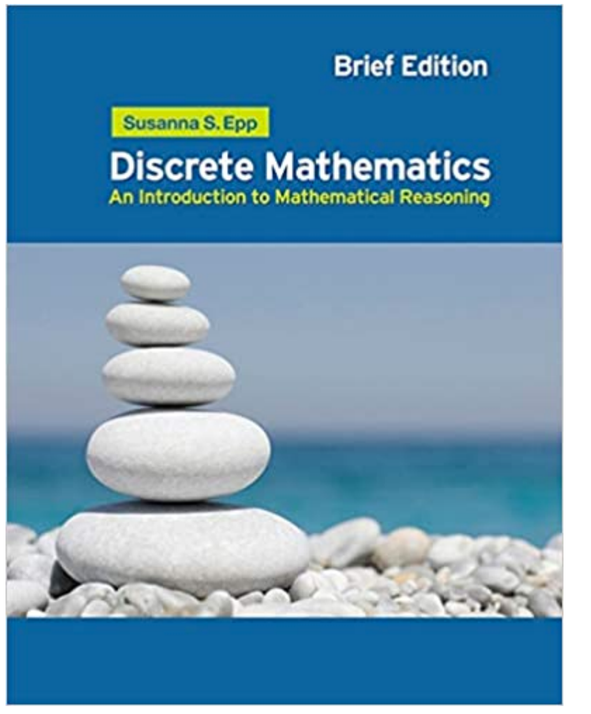
Practical matters

- COVID is gone, but…
- Textbook
- Schedule
- Homework
- Exams and grading
- Ask for help

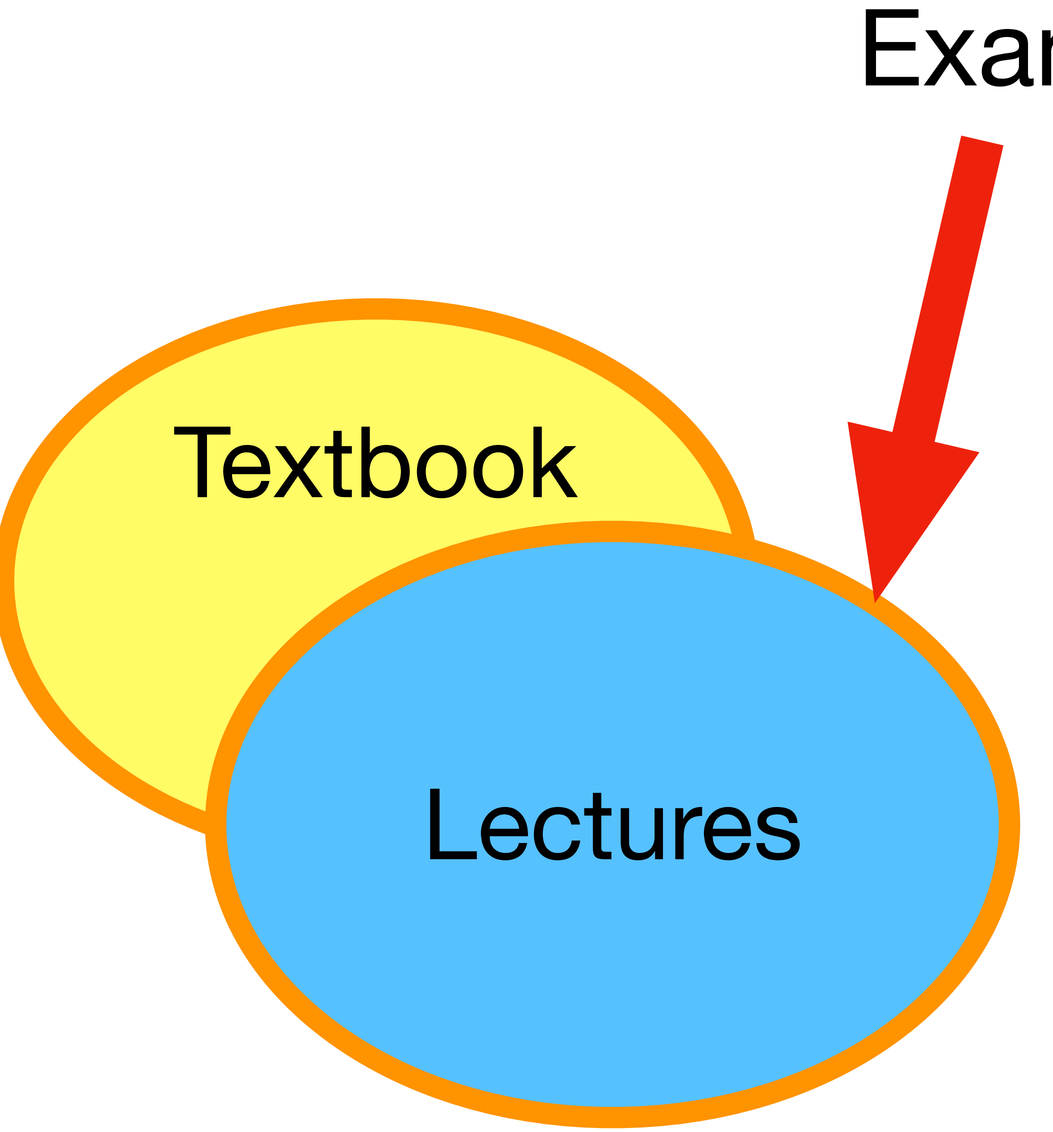
Covid

- Inform instructor immediately of the date of a positive test. Your absences will be excused
- Follow government guidelines including a 7-day quarantine.
- Return to the class after quarantine. Negative test not needed.

Textbook



- Our course relates to Chapters 2-7
- Very helpful, though optional
- Suggestion: Skim the related chapter before the lecture; read deeper after the lecture
- Textbook may not cover everything in the exams; lectures do



Schedule

- Lectures: Monday and Wednesday 10h30am - 11h50am, at C105
- Recitation: Wednesday 5pm - 5h50pm, at C105
- Office hours: Monday 4pm - 5pm and Wednesday 7h00pm - 8h00pm, at B424
- TA office hours: TBA
- course website: https://github.com/zhoulaifu/23_cse215_spring

Classes

- Propositional logic
- Predicate logic
- Proof techniques
- Midterm 1
- Sequences
- Sets
- Midterm 2
- Function
- Final exam

Grading

- Attendance: 5%
- take-home assignment (homework): 25%
- In-class assignments (quiz): 10%
- Midterms: 30%
- Final exam: 30%
- Students with regular participation get 1% bonus

Cont.

- In-class assignments take place in recitation classes (Wednesday). Format: Paper-based, open-notes. No Internet.
- In-class assignments are basic exercises.
- Take-home assignments take place at the end of each “chapter”, usually on Thursday
- Take-home assignments are harder and may need reflection, but you can ask for help.
- Exam format = In-class assignment format.

Questions so far?

Course overview

A personal story

The story

- Once upon a time, I worked for a project involving financial calculation
- I needed to sum up a number of floating-point values like
  - 0.1 + 0.2 + 0.3 + 0.7 + 0.9 + 1.2 + 3.5…
- There were billion of numbers like this, so performance was a key for the project's success
- We decided to use the state-of-the-art multi-core, parallel computing
- Parallel computing works like a divide-and-conquer:
  - (0.1 + 0.2) + (0.3 + 0.7) + (0.9 + 1.2 + 3.5) + …
- Now, let us think why it looks reasonable le to use parallel computing for this task??
- The reason is associative law. (a + b) + c = a + (b + c)

A problem

We get different results for each round, if we put parentheses differently each time.

- (0.1 + 0.2) + (0.3 + 0.7) + (0.9 + 1.2 + 3.5) + …
- becomes different from
- (0.1 + 0.2 + 0.3) + (0.7+ 0.9)+ (1.2 + 3.5) + …

Why?

- We made this assumption:
  - for any numbers a, b, c, (a + b) + c = a + (b+ c)
- This is a statement that can be assigned with true or false value, we call it a **proposition**
- The inner part has variables, and can be denoted as a statement with parameters (a, b, c). We call it a **predicate**.
- Many CS work involves determining if a proposition is true or false. To show the truth is called **to prove**.
- The reason for the problem is that the proposition above is false.

Summary for the story

The whole is called a proposition,
to which we can assign a truth value

$$\forall a, b, c \in I, \underbrace{(a+b)+c = a+(b+c)}_{\text{Predicate}}$$

quantifier

variables

set

Summary so far

- The ultimate goal of this course is to learn fundamentals for understanding why our digital world works or fails.
- We will study logic (propositions, and predicates), proof, and math structures like sets as a language to reason about computer science

Break
Part 2: Exam overview

To know a list of key concepts
that will be covered in the exams

To finish by 11h50

Selected exam problems

book chapter	Topics	Exam problems
2	Propositional logic	2021-final, pb 1
3	Predicate logic	2021-midterm1, pb3
4	Proof	2021-final, pb4
5	Sequences	2021-final, pb7
6	Sets	2021-midterm2, pb2
7	Functions	2021-final, pb9
8	Relations	2021-final, pb11

We will proceed in 2 passes

- We first go over the problems to highlight the “key” concepts.
- We will then go over the solutions.
- No worry if you do not understand the details.

Key concepts

Propositional Logic
Final 2021

Problem 1. [5 points]
Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

Key: Truth Table

Truth table for p ^ q

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Predicate Logic — Midterm 1, 2021

Problem 3. [10 points]
Give negations of the following statements. Reasoning is not required.

(f) [1 point] $\forall x, \forall y$ such that $p(x, y)$

(g) [1 point] $\forall x, \exists y$ such that $p(x, y)$

Key: Negation & quantifiers

- $\sim(\forall \boldsymbol{x}, \boldsymbol{P}(\boldsymbol{x})) \equiv \exists \boldsymbol{x}, \sim\boldsymbol{P}(\boldsymbol{x})$
- $\sim(\exists \boldsymbol{x}, \boldsymbol{P}(\boldsymbol{x})) \equiv \forall \boldsymbol{x}, \sim\boldsymbol{P}(\boldsymbol{x})$

Proof — Final 2021

Problem 4. [5 points]
Prove that the sum of the squares of any two consecutive odd integers is even.

Key: Prove propositions about integers using basic facts

Example of basic facts:

- an even integer can be written as 2*n;
- (x+y)^2= x^2 + 2xy + y^2

Sequences - Final 2021

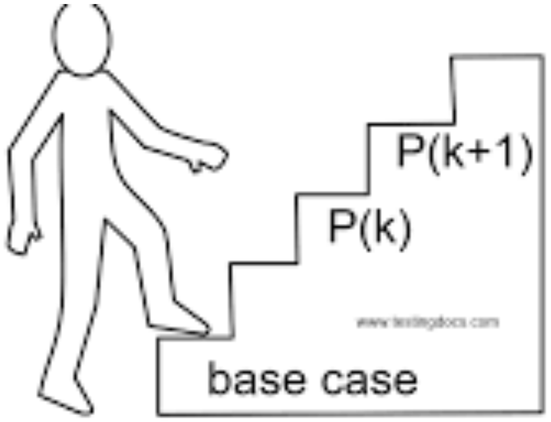
Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

Key: Use Mathematical Induction to show facts about integers



Sets — Midterm 2, 2021

Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons. Assume all sets are subsets of a universal set U.

(a) **[1 point]** $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$

Key: Union and intersection on Sets

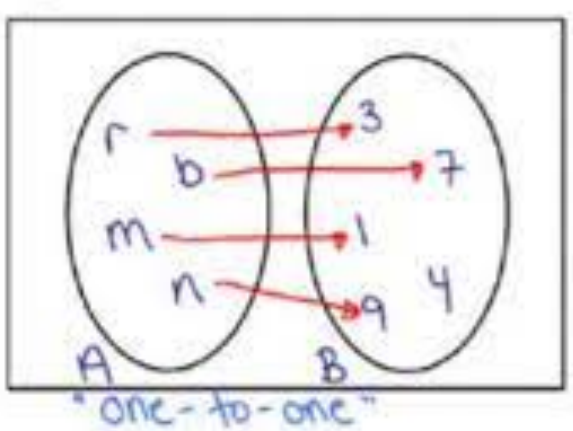
Functions — Final 2021

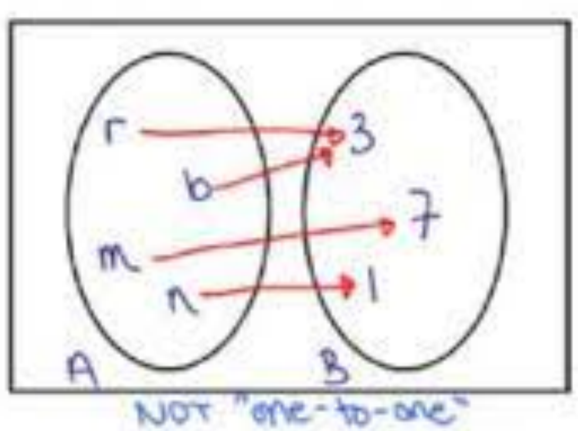
Problem 9. [5 points]

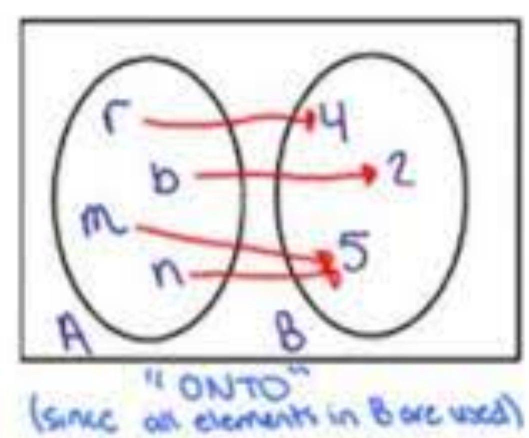
Write and fill the table with ✓ or ✗. If a function is one-to-one or onto, then use ✓. On the other hand, if a function is not one-to-one or not onto, then use ✗.

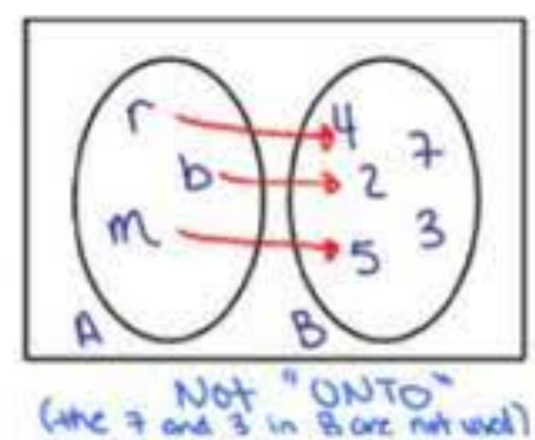
Function	Domains	One-to-one function?	Onto function?
$f(x) = 3x$	$f : \mathbb{Z} \to \mathbb{Z}$		

Key: One-to-one and onto functions







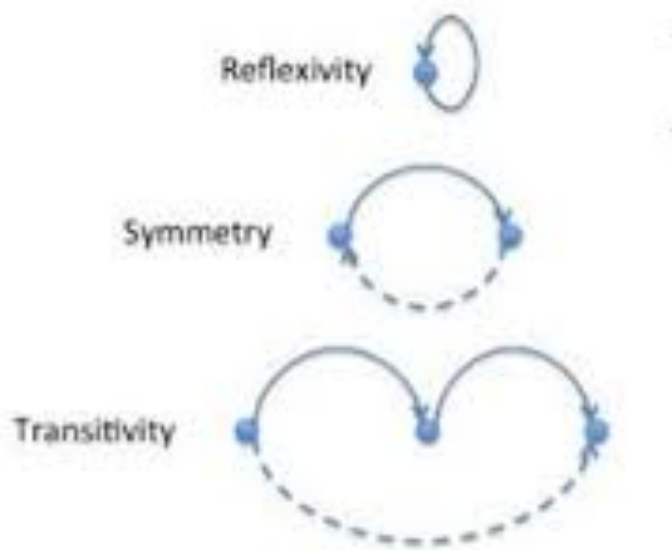


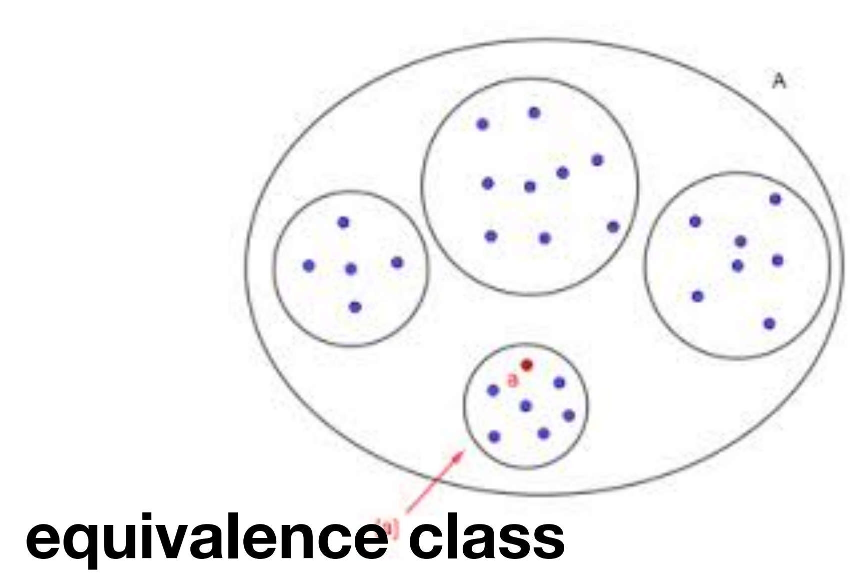
Relations - Final 2021

Problem 11. [5 points]

Let A be the set of all people. Let R be the relation defined on A as follows: For persons p and q in A, we have $p\ R\ q \Leftrightarrow p$ has the same birthday as q.
Is R an equivalence relation? Prove your answer. If R is an equivalence relation, what are the distinct equivalence classes of the relation?

Key: Equivalence relations and Equivalence classes





Solution overview

Propositional Logic Final 2021

Problem 1. [5 points]

Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

p	q	r	p AND (q OR r)	p AND (q AND r)	p AND (q OR r) <==> p AND (q AND r)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	F	F	F	T
T	F	T	T	F	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	F	F	F	T
F	F	T	F	F	T

Predicate Logic — Midterm 1, 2021

Problem 3. [10 points]

Give negations of the following statements. Reasoning is not required.

(f) **[1 point]** $\forall x, \forall y$ **such that** $p(x, y)$

(g) **[1 point]** $\forall x, \exists y$ **such that** $p(x, y)$

- There exists x, there exists y, ~p(x,y)
- There exists x, for all y, ~p(x,y)

Proof — Final 2021

Problem 4. [5 points]

Prove that the sum of the squares of any two consecutive odd integers is even.

Proof.

- We need to show: for any integer n, (2n+1)^2 + (2n+3)^2 is even.
- That is to say, we need to show the following proposition holds:
- for any integer n, 8n^2 + 16n + 10 is even.
- The formula above can be rewritten as 2 (4n^2 + 8n + 5) which must be even.

QED.

Sequences - Final 2021

Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

- Proof.
  - Let P(n) be the predicate 1* 1! + 2 * 2! + … n* n! = (n+1)! -1
  - Base step: We prove P(1).
  - Inductive step: We prove for any integer k>=1, P(k) -> P(k+1)
    - Let k be an arbitrary integer and k>=1.
    - Assume P(k) holds. That is 1* 1! + 2 * 2! + … k* k! = (k+1)! -1
    - We need to prove P(k+1), namely, 1* 1! + 2 * 2! + … (k+1) * (k+1)! = (k+2)! -1
    - Following assumption P(k), Left-hand-side above = (k+1)! -1 + (k+1) * (k+1)! which equals to Right-hand-side above.
- QED.

Sets — Midterm 2, 2021

Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons. Assume all sets are subsets of a universal set U.

(a) **[1 point]** $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$

- The statement is false. As a counter example:
  - A={1}, B={2}, C={1,2}.
  - Left-hand-side becomes empty set
  - Right-hand-side becomes {1}

Functions — Final 2021

Problem 9. [5 points]

Write and fill the table with ✓ or ✗. If a function is one-to-one or onto, then use ✓. On the other hand, if a function is not one-to-one or not onto, then use ✗.

Function	Domains	One-to-one function?	Onto function?
$f(x) = 3x$	$f : \mathbb{Z} \to \mathbb{Z}$		

- Yes: One to one function
- No: Onto function

Relations - Final 2021

Problem 11. [5 points]

Let A be the set of all people. Let R be the relation defined on A as follows: For persons p and q in A, we have $p\ R\ q \Leftrightarrow p$ has the same birthday as q.
Is R an equivalence relation? Prove your answer. If R is an equivalence relation, what are the distinct equivalence classes of the relation?

- R is an equivalence relation, as it is
  - reflective (p R p),
  - symmetric p R q <==> q R p
  - transitive p R q, q R r ==> p R r
- Equivalence classes is the set of the sets of people of A that have the same birthday.

Today’s take-away

book chapter	Topics	Exam problems	Key
2	Propositional logic	2021–final, pb 1	truth table
3	Predicate logic	2021–midterm1, pb3	negation on quantifiers
4	Proof	2021–final, pb4	facts about integers
5	Sequences	2021–final, pb7	math induction
6	Sets	2021–midterm2, pb2	unions and intersections
7	Functions	2021–final, pb9	1–1 and onto
8	Relations	2021–final, pb11	equiv. rel. and classes

Thank you for your attention!