

Assignment 2

MACS 30000

Li Liu

Due Wednesday, Oct. 17 at 11:30 AM

1. Imputing age and gender

(a) Propose a strategy for imputing age (age) and gender (female) variables into the BestIncome.txt data by using information from the SurveyIncome.txt data. Describe your proposed method, including equations.

We want to replace the missing values for *age* and *gender* in *BestIncome* with substituted values. As *BestIncome* and *SurveyIncome* have common variables of *totinc* (sum of *lab_inc* and *cap_inc* in *BestIncome*) and *weight*, we could predict the missing values using the regression models trained by *SurveyIncome* data.

Linear regression for *age*:

$$Age_i = \alpha_0 + \alpha_1 totinc_i + \alpha_2 wgt_i + \epsilon_i$$

where:

α_0 is the intercept of the linear line on the y-axis

α_1 and α_2 represent the marginal effect on age when variables change 1 unit

ϵ_i is an error term (deviations of observations from the linear)

Logistic regression for dichotomous variable *gender*:

$$\log\left(\frac{p(X)}{1 - P(X)}\right) = \beta_0 + \beta_1 totinc_i + \beta_2 wgt_i + \epsilon_i$$

where $X=(totinc, wgt)$ are two predictors. The left-hand side is called the log-odds.

(b) Using your proposed method from part (a), impute the variables age (age) and gender (female) into the BestIncome.txt data.

```
In [1]: # Import packages
import numpy as np
import pandas as pd
import statsmodels.api as sm
import matplotlib.pyplot as plt
plt.style.use('seaborn')
import warnings
warnings.filterwarnings("ignore")
```

C:\Anaconda\lib\site-packages\statsmodels\compat\pandas.py:56: FutureWarning: The pandas.core.datetools module is deprecated and will be removed in a future version. Please use the pandas.tseries module instead.
 from pandas.core import datetools

```
In [2]: # Read in data, name four variables, descriptive statistics
Best = pd.read_csv('BestIncome.txt', header=None, names = ['lab_inc', 'cap_inc', 'hgt', 'wgt'])
Best.head()
```

Out[2]:

	lab_inc	cap_inc	hgt	wgt
0	52655.605507	9279.509829	64.568138	152.920634
1	70586.979225	9451.016902	65.727648	159.534414
2	53738.008339	8078.132315	66.268796	152.502405
3	55128.180903	12692.670403	62.910559	149.218189
4	44482.794867	9812.975746	68.678295	152.726358

```
In [3]: Surv = pd.read_csv('SurvIncome.txt', header = None, names = ['tot_inc', 'wgt', 'age', 'female'])
Surv.head()
```

Out[3]:

	tot_inc	wgt	age	female
0	63642.513655	134.998269	46.610021	1.0
1	49177.380692	134.392957	48.791349	1.0
2	67833.339128	126.482992	48.429894	1.0
3	62962.266217	128.038121	41.543926	1.0
4	58716.952597	126.211980	41.201245	1.0

Impute age in BestIncome using the models constructed from SurveyIncome

```
In [32]: #OLS Regression for predicting 'age' using 'tot_inc' and 'wgt' using SurveyIncome  
X, y = sm.add_constant(Surv[['wgt','tot_inc']], prepend=False), Surv['age']  
m= sm.OLS(y, X).fit()  
print(m.summary())
```

OLS Regression Results

```

=====
=
Dep. Variable:          age    R-squared:                0.00
1
Model:                  OLS    Adj. R-squared:            -0.00
1
Method:                 Least Squares    F-statistic:          0.632
6
Date:                   Wed, 17 Oct 2018    Prob (F-statistic):    0.53
1
Time:                   09:58:44    Log-Likelihood:       -3199.
4
No. Observations:      1000    AIC:                  640
5.
Df Residuals:          997    BIC:                  641
9.
Df Model:               2

```

Covariance Type: nonrobust

```

=====
=
               coef    std err          t      P>|t|      [0.025    0.97
5]
-----
-
wgt          -0.0067     0.010     -0.686     0.493     -0.026     0.01
3
tot_inc      2.52e-05    2.26e-05     1.114     0.266    -1.92e-05    6.96e-0
5
const       44.2097      1.490     29.666     0.000     41.285     47.13
4
=====
=

```

```

Omnibus:                2.460    Durbin-Watson:          1.92
1
Prob(Omnibus):           0.292    Jarque-Bera (JB):        2.32
2
Skew:                   -0.109    Prob(JB):                0.31
3
Kurtosis:                3.092    Cond. No.                5.20e+0
5
=====
=

```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 5.2e+05. This might indicate that there are strong multicollinearity or other numerical problems.

```
In [5]: # Apply the model to impute 'age' in BestIncome
#tot_inc = F(K,L) = YK + YL = capital income + labor income='lab_inc'+'cap_in
c'
Best['tot_inc']=Best['lab_inc']+Best['cap_inc']
Best['const'] = 1
Best['age'] = m.predict(Best[['wgt', 'tot_inc', 'const']])
```

Impute *gender* in *BestIncome* using the models constructed from *SurveyIncome*

```
In [33]: #Logistic Regression for predicting 'female' using 'tot_inc' and 'wgt' using SurveyIncome
X, y = sm.add_constant(Surv[['wgt','tot_inc']], prepend=False), Surv['female']
m2= sm.Logit(y, X).fit()
print(m2.summary())
```

Optimization terminated successfully.

Current function value: 0.036050

Iterations 11

Logit Regression Results

```
=====
=
Dep. Variable:                female    No. Observations:                100
0
Model:                        Logit     Df Residuals:                    99
7
Method:                       MLE      Df Model:
2
Date:                         Wed, 17 Oct 2018    Pseudo R-squ.:                0.948
0
Time:                         09:58:50    Log-Likelihood:                -36.05
0
converged:                    True      LL-Null:                        -693.1
5
                                LLR p-value:                4.232e-28
6
=====
=
                                coef      std err          z      P>|z|      [0.025      0.97
5]
-----
-
wgt                -0.4460         0.062     -7.219     0.000     -0.567     -0.32
5
tot_inc            -0.0002     4.25e-05     -3.660     0.000     -0.000     -7.22e-0
5
const              76.7929        10.569       7.266     0.000       56.078       97.50
8
=====
=
```

Possibly complete quasi-separation: A fraction 0.55 of observations can be perfectly predicted. This might indicate that there is complete quasi-separation. In this case some parameters will not be identified.

```
In [7]: # Apply the model to impute 'female' in BestIncome
#Define threshold of female=1 when p>=0.5, female=0 when p<0.5
Best['female'] = m2.predict(Best[['wgt','tot_inc', 'const']])
Best['female'][Best['female']>0.5]=1
Best['female'][Best['female']<0.5]=0
```

In [8]: `Best.head()`

Out[8]:

	lab_inc	cap_inc	hgt	wgt	tot_inc	const	age	female
0	52655.605507	9279.509829	64.568138	152.920634	61935.115336	1	44.742614	0
1	70586.979225	9451.016902	65.727648	159.534414	80037.996127	1	45.154387	0
2	53738.008339	8078.132315	66.268796	152.502405	61816.140654	1	44.742427	0
3	55128.180903	12692.670403	62.910559	149.218189	67820.851305	1	44.915836	0
4	44482.794867	9812.975746	68.678295	152.726358	54295.770612	1	44.551391	0

(c) Report the descriptive statistics for my new imputed variables

In [9]: `Best['age'].describe()[['mean', 'std', 'min', 'max', 'count']]`

Out[9]:

```

mean      44.890828
std        0.219150
min        43.976495
max        45.703819
count    10000.000000
Name: age, dtype: float64

```

In [10]: `Best['female'].describe()[['mean', 'std', 'min', 'max', 'count']]`

Out[10]:

```

mean      0.454600
std        0.497959
min         0.000000
max         1.000000
count    10000.000000
Name: female, dtype: float64

```

(d) Report the correlation matrix for the now six variables—labor income (lab inc, capital income cap inc, height (hgt), weight (wgt) age (age), and gender (female) —in the BestIncome.txt data.

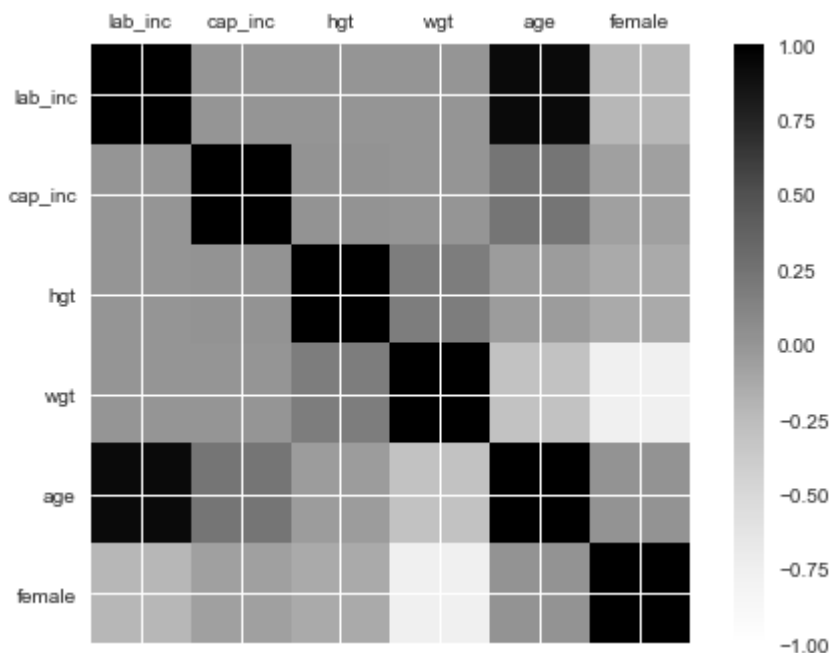
```
In [11]: # Correlation matrix

def corr_plot(df):

    names = df.columns
    N = len(names)

    correlations = df.corr()
    fig = plt.figure()
    ax = fig.add_subplot(111)
    cax = ax.matshow(correlations, vmin=-1, vmax=1)
    fig.colorbar(cax)
    ticks = np.arange(0,N,1)
    ax.set_xticks(ticks)
    ax.set_yticks(ticks)
    ax.set_xticklabels(names)
    ax.set_yticklabels(names)
    plt.show()

corr_plot((Best[['lab_inc', 'cap_inc', 'hgt', 'wgt', 'age', 'female']]))
corr=Best[['lab_inc', 'cap_inc', 'hgt', 'wgt', 'age', 'female']].corr()
print(corr)
```



	lab_inc	cap_inc	hgt	wgt	age	female
lab_inc	1.000000	0.005325	0.002790	0.004507	0.924053	-0.215469
cap_inc	0.005325	1.000000	0.021572	0.006299	0.234159	-0.062569
hgt	0.002790	0.021572	1.000000	0.172103	-0.045083	-0.127416
wgt	0.004507	0.006299	0.172103	1.000000	-0.300288	-0.763821
age	0.924053	0.234159	-0.045083	-0.300288	1.000000	0.020059
female	-0.215469	-0.062569	-0.127416	-0.763821	0.020059	1.000000

2. Stationarity and data drift

(a) Estimate by OLS and report coefficients

```
In [12]: # Read in my third data set
Data= pd.read_csv('IncomeIntel.txt', header=None, names = ['grad_year', 'gre_qnt', 'salary_p4'])
Data.head()
```

Out[12]:

	grad_year	gre_qnt	salary_p4
0	2001.0	739.737072	67400.475185
1	2001.0	721.811673	67600.584142
2	2001.0	736.277908	58704.880589
3	2001.0	770.498485	64707.290345
4	2001.0	735.002861	51737.324165

```
In [13]: # Run regression model
X, y = sm.add_constant(Data['gre_qnt'], prepend=False), Data['salary_p4']
reg= sm.OLS(y, X).fit()
print(reg.summary())
```

OLS Regression Results

```
=====
=
Dep. Variable:          salary_p4    R-squared:                0.26
3
Model:                  OLS         Adj. R-squared:            0.26
2
Method:                 Least Squares   F-statistic:              356.
3
Date:                   Wed, 17 Oct 2018   Prob (F-statistic):       3.43e-6
8
Time:                   09:44:59         Log-Likelihood:           -1067
3.
No. Observations:       1000          AIC:                     2.135e+0
4
Df Residuals:           998          BIC:                     2.136e+0
4
Df Model:                1
```

Covariance Type: nonrobust

```
=====
=
               coef      std err          t      P>|t|      [0.025      0.97
5]
-----
-
gre_qnt      -25.7632      1.365      -18.875      0.000      -28.442      -23.08
5
const       8.954e+04     878.764     101.895      0.000      8.78e+04     9.13e+0
4
=====
```

```
=====
=
Omnibus:                9.118    Durbin-Watson:              1.42
4
Prob(Omnibus):           0.010    Jarque-Bera (JB):          9.10
0
Skew:                   0.230    Prob(JB):                  0.010
6
Kurtosis:               3.077    Cond. No.                  1.71e+0
3
=====
```

```
=====
=
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correc
tly specified.
[2] The condition number is large, 1.71e+03. This might indicate that there a
re
strong multicollinearity or other numerical problems.
```

The estimated regression model is :

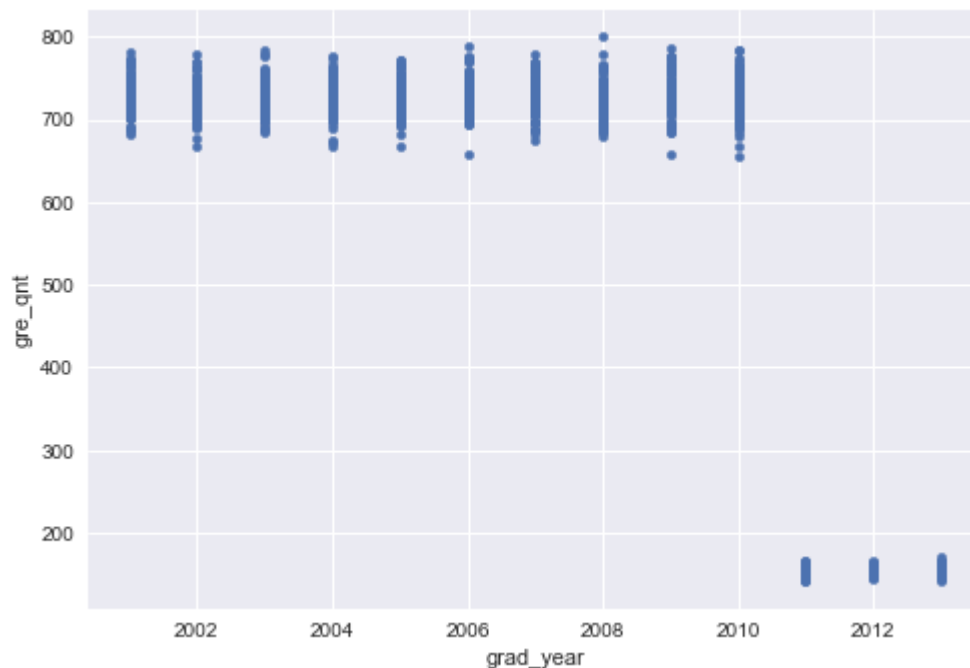
$$\widehat{salary_p4}_i = 8.954 * 10^4 - 25.7632 * \widehat{gre_qnt}_i$$

where $\beta_0 = 8.954e+04$ and $\beta_1 = -25.7632$

The standard errors are 878.764 for β_0 and 1.365 for β_1 .

(b) Create a scatterplot of GRE score and graduation year.

```
In [14]: Data.plot(x='grad_year', y='gre_qnt', kind='scatter')
plt.show()
```



The scoring scale for GRE quantitative part was changed to a 130-170 scale from the 200-800 scale in 2011. As a result, the values of scores since 2011 are much smaller than the ones before. This is a system drift (change in the system itself). If we don't recognize this issue, we'll probably overestimate the marginal effect of `gre_qnt` on predicting `salary_p4`. The hypothesis that the estimated coefficient β_1 is zero will likely not be accepted using this raw data.

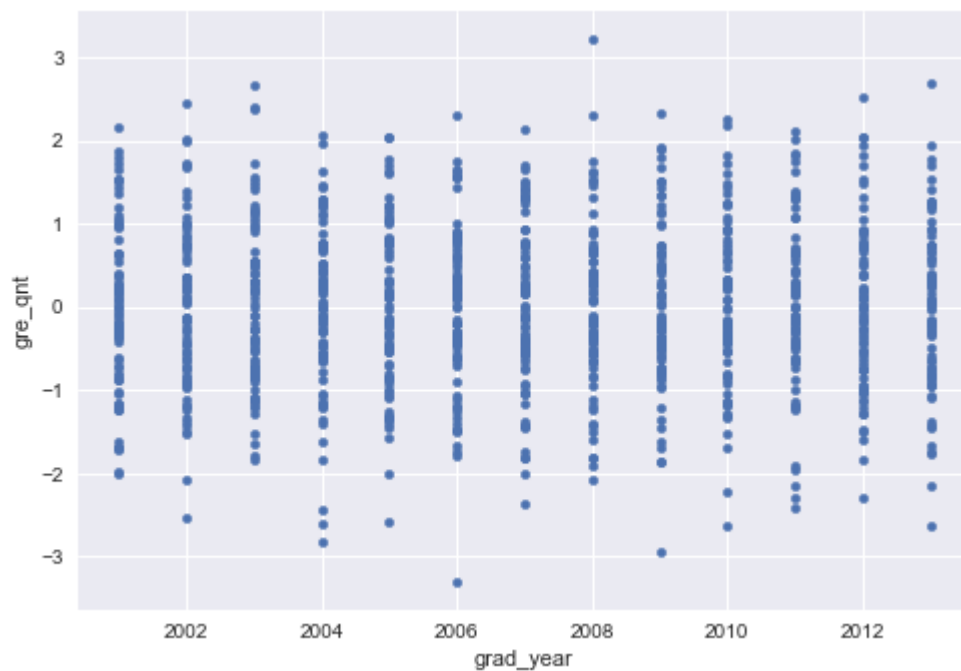
One solution is to normalize the `gre_qnt` using z-score with each year. The standard scores are not affected by different scales.

$$z_{i,j} = \frac{gre_qnt_{i,j} - \mu_j}{\sigma_j}$$

where i means different observations within j -th year.

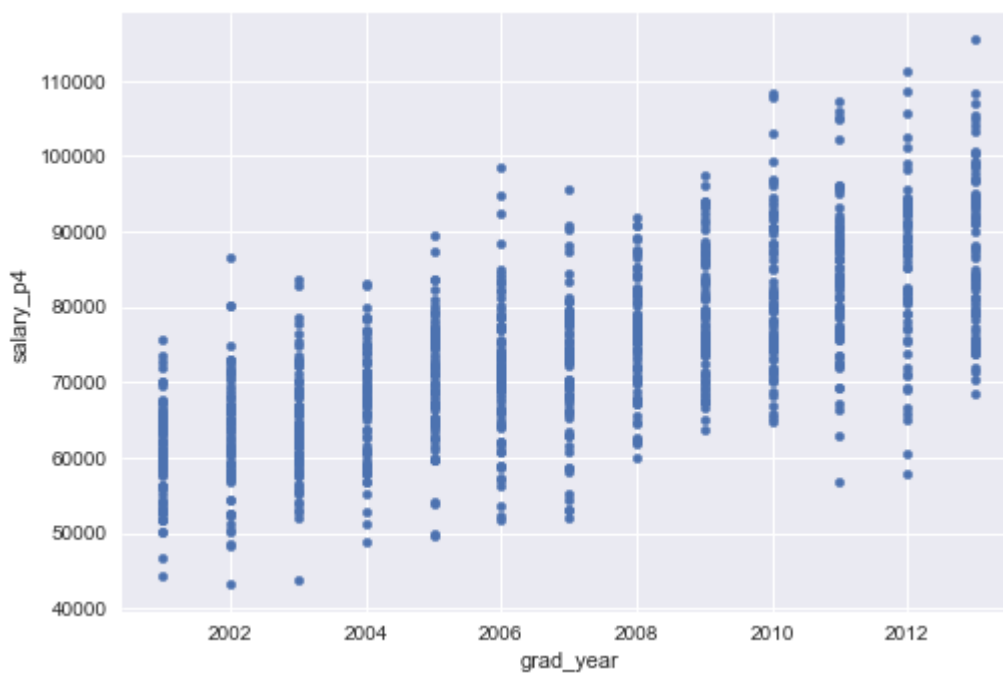
```
In [29]: Data['gre_qnt'] = Data.groupby('grad_year').transform(lambda x: (x - x.mean()) / x.std())['gre_qnt']
```

```
In [23]: #Scatter plot with z-score of 'gre_qnt`  
Data.plot(x='grad_year', y='gre_qnt', kind='scatter')  
plt.show()
```



(c) Create a scatterplot of income and graduation year

```
In [17]: Data.plot(x='grad_year', y='salary_p4', kind='scatter')  
plt.show()
```



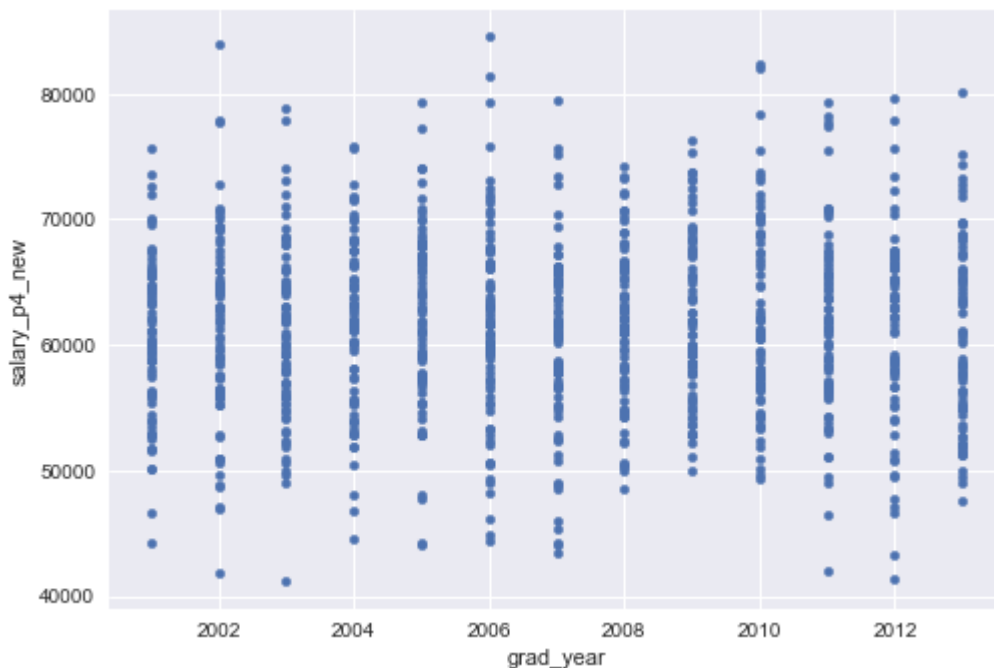
Across different years, the means of income seem to be growing. Thus the salary_p4 is not stationary. We need to de-trend this variable for the analysis. This could be done by calculating the average growth rate in salaries across all 13 years and dividing each salary by $(1 + \text{avg_growth_rate})^{(\text{grad_year} - 2001)}$.

```
In [30]: #Calculate the mean salary each year
avg_inc_by_year = Data['salary_p4'].groupby(Data['grad_year']).mean().values

#Calculate the average growth rate in salaries across all 13 years
avg_growth_rate = ((avg_inc_by_year[1:] - avg_inc_by_year[:-1]) / avg_inc_by_year[:-1]).mean()
#avg_growth_rate=0.0308

#Divide each salary by (1 + avg_growth_rate) ** (grad_year - 2001)
Data['rate']=(1 + avg_growth_rate) ** (Data['grad_year']-2001)
Data['salary_p4_new']=Data['salary_p4']/Data['rate']
```

```
In [31]: Data.plot(x='grad_year', y='salary_p4_new', kind='scatter')
plt.show()
```



(d) Re-estimate coefficients with updated variables.

```
In [27]: # Code to re-estimate, output of new coefficients
X, y = sm.add_constant(Data['gre_qnt'], prepend=False), Data['salary_p4_new']
reg2= sm.OLS(y,X).fit()
print(reg2.summary())
```

OLS Regression Results

```
=====
Dep. Variable:          salary_p4_new    R-squared:                0.00
Model:                  OLS              Adj. R-squared:          -0.00
Method:                 Least Squares    F-statistic:              0.439
Date:                   Wed, 17 Oct 2018  Prob (F-statistic):       0.50
Time:                   09:45:35         Log-Likelihood:          -1029
No. Observations:      1000             AIC:                    2.059e+0
Df Residuals:          998              BIC:                    2.060e+0
Df Model:               1
```

Covariance Type: nonrobust

```
=====
coef    std err          t    P>|t|    [0.025    0.975]
-----
gre_qnt    -150.6097    227.193     -0.663    0.508    -596.440    295.22
const      6.142e+04    225.711    272.117    0.000     6.1e+04    6.19e+04
=====
```

```
Omnibus:            0.776    Durbin-Watson:           2.02
Prob(Omnibus):      0.678    Jarque-Bera (JB):        0.68
Skew:               0.059    Prob(JB):                0.70
Kurtosis:           3.049    Cond. No.                1.0
=====
```

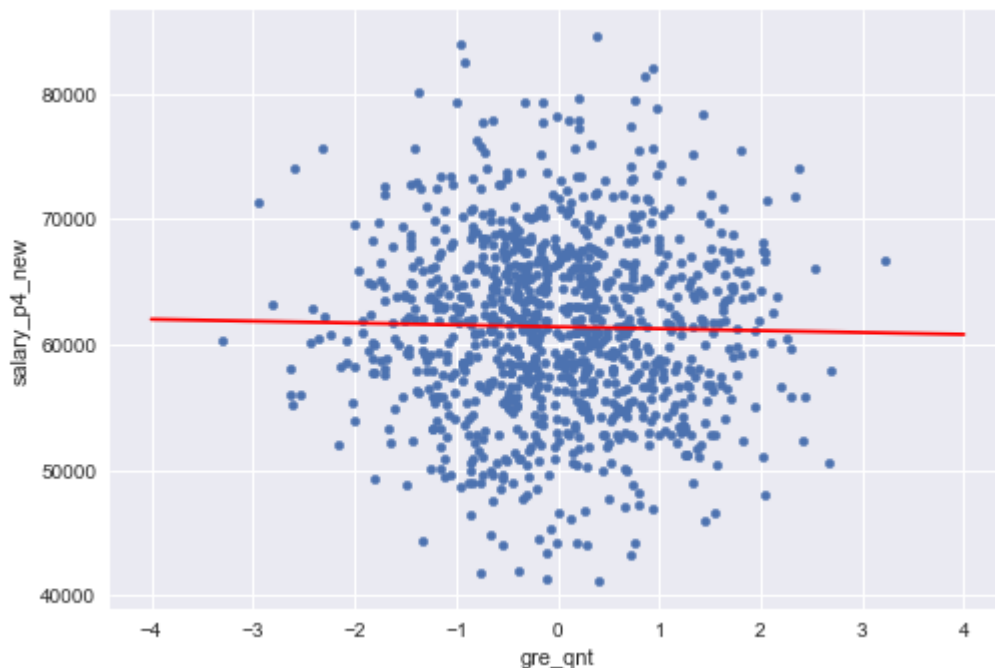
Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The estimated coefficients are β_0 is 6.142e+04 and β_1 is -150.6097. The standard error is 225.711 for β_0 and 227.193 for β_1 .

The new β_0 is a bit smaller to the previous one. But the new β_1 is much smaller. Also we notice that the estimate of old β_1 is statistically significant, while the new β_1 is not. If we ignore the stationarity and data drift problems, we might falsely reject the null hypothesis that higher intelligence is not associated with higher income. Now we cannot reject the null hypothesis. We might want to conclude that higher intelligence is not associated with higher income.

```
In [21]: Data.plot(x='gre_qnt', y='salary_p4_new', kind='scatter')
x1=np.linspace(-4,4)
y1=-150.6097*x1+6.142e+04
plt.plot(x1,y1,'-r')
plt.show()
```



3. Assessment of Kossinets and Watts.

The question proposed and answered by the Kossinets & Watts (2009) paper is: What could explain the mechanism of homophily generation in a dynamic social network?

Kossinets & Watts (2009) merged three databases to cover "interaction, affiliation, and attribute-type longitudinal data" of 30396 individuals. These persons are students, professors, and staff affiliated with one large American university, who were active school email users in that academic year (Kossinets & Watts, 2009, p. 410).

The three parts of the database are the past records of email exchanges in one year, individual's characteristics, and course registration (Kossinets & Watts, 2009, p. 410).

Kossinets & Watts (2009) choose the following variables for the network modeling: "personal characteristics, organizational affiliations, course-related variables, and e-mail-related variables".

After the data cleaning, the authors obtained "7,156,162 messages exchanged by 30,396 stable e-mail users during the 270 days period" (Kossinets & Watts, 2009, p. 410). Appendix A of the paper provides a description and definition of these variables.

One potential problem about the dataset is that they exclusively select the email accounts linked with the central server. Kossinets & Watts (2009) address the problem that department-specified email accounts such as "xyz@department.university.edu" are hard to match with the individuals, thus they provide no value for the analysis. Another fact is that many people might choose to use their own email account such as "@gmail.com" for communications with others for better functionalities. Although these records are much harder to obtain, excluding them from the analysis might make the analysis of small social network incomplete and biased.

Matching the email-logs and characteristics of the senders and receivers to construct "social relationship" is a challenge task. Cortes et al. (2003) argue that E-mail exchanges comprise discontinuous "spike trains" which don't reflect the reality. It probably couldn't track the exact timing of the tie formation. To address this issue, Kossinets & Watts (2009, p. 413) introduced the "sliding window filter" model to approximate short-time network from "discrete dyadic interactions".

Reference:

Cortes, C., Daryl, D., & Chris, V. (2003). Computational Methods for Dynamic Graphs. *Journal of Computational and Graphical Statistics*, 12(950–70).

Kossinets, G., & Watts, D. (2009). Origins of Homophily in an Evolving Social Network. *American Journal of Sociology*, 115(2), 405-450. doi:10.1086/599247