PS2 Solution

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0.1 PS2—Q1

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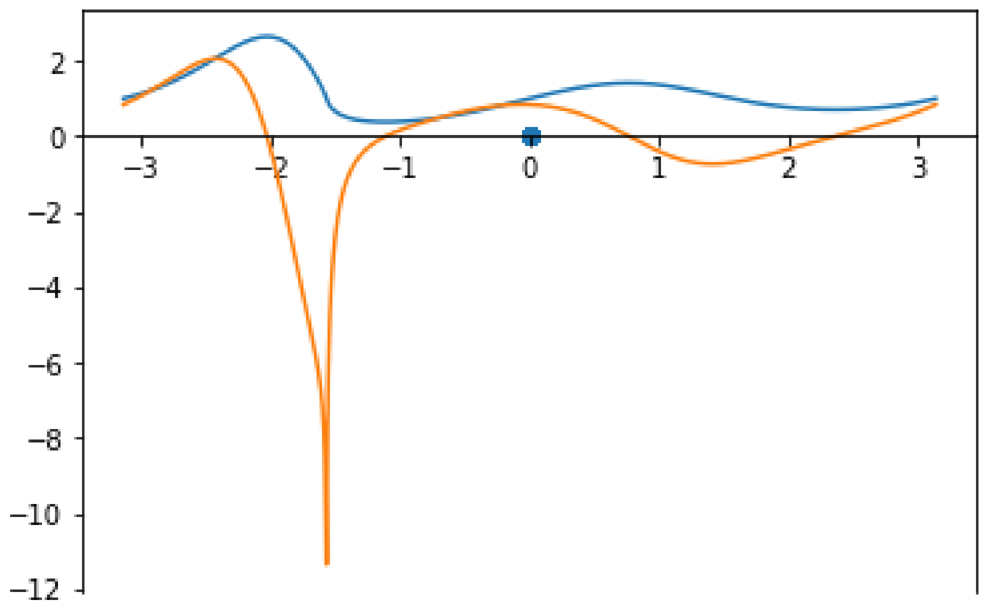
Pbm 1

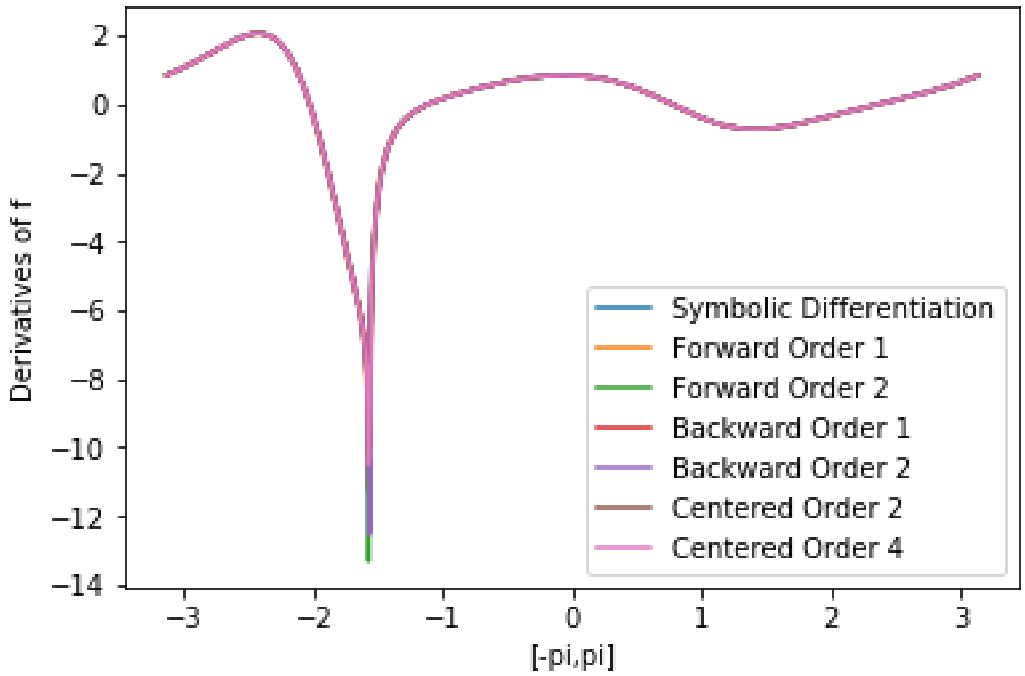
```
In [256]: from matplotlib import pyplot as plt
          from sympy import *
          from math import pi
          import numpy as np

          #Define the symbolic function
          x = Symbol('x')
          y=(sin(x)+1)**(sin(cos(x)))

          sym=lambda x: y.diff(x)
          print("Symbolic derivative of the function:")
          sym(x)

Symbolic derivative of the function:


Out[256]: (-log(sin(x) + 1)*sin(x)*cos(cos(x)) + sin(cos(x))*cos(x)/(sin(x) + 1))*(sin(x) + 1)*

In [34]: #Create the NumPy array and lambdify the functions
         xvec=np.linspace(-pi,pi,1000)
         f = lambdify(x, y, 'numpy')
         fprime=lambdify(x,sym(x),'numpy')

In [35]: #Plot f and f'
         ax = plt.gca()
         ax.spines["bottom"].set_position("zero")
         plt.plot(xvec, f(xvec))
         plt.plot(xvec, fprime(xvec))
         plt.show()
```



Pbm 2

```
In [14]: def Forward1(func,*args,h=0.01):
             return [(func(x+h)-func(x))/h for x in args][0]

In [15]: def Forward2(func,*args,h=0.01):
             return [(4*func(x+h)-func(x+2*h)-3*func(x))/(2*h) for x in args][0]

In [16]: def Backward1(func,*args,h=0.01):
             return [(func(x)-func(x-h))/h for x in args][0]

In [17]: def Backward2(func,*args,h=0.01):
             return [(-4*func(x-h)+func(x-2*h)+3*func(x))/(2*h) for x in args][0]

In [18]: def Centered2(func,*args,h=0.01):
             return [(func(x+h)-func(x-h))/(2*h) for x in args][0]

In [39]: def Centered4(func,*args,h=0.01):
             return [(func(x-2*h)-8*func(x-h)+8*func(x+h)-func(x+2*h))/(12*h) for x in args][0]

In [44]: plt.plot(xvec, fprime(xvec),label='Symbolic Differentiation')
         plt.plot(xvec, Forward1(f,xvec),label='Forward Order 1')
         plt.plot(xvec, Forward2(f,xvec),label='Forward Order 2')
         plt.plot(xvec, Backward1(f,xvec),label='Backward Order 1')
         plt.plot(xvec, Backward2(f,xvec),label='Backward Order 2')
         plt.plot(xvec, Centered2(f,xvec),label='Centered Order 2')
         plt.plot(xvec, Centered4(f,xvec),label='Centered Order 4')
         plt.legend(loc='lower right')
```

```
plt.xlabel("[-pi,pi]")
plt.ylabel("Derivatives of f")
plt.show()
```

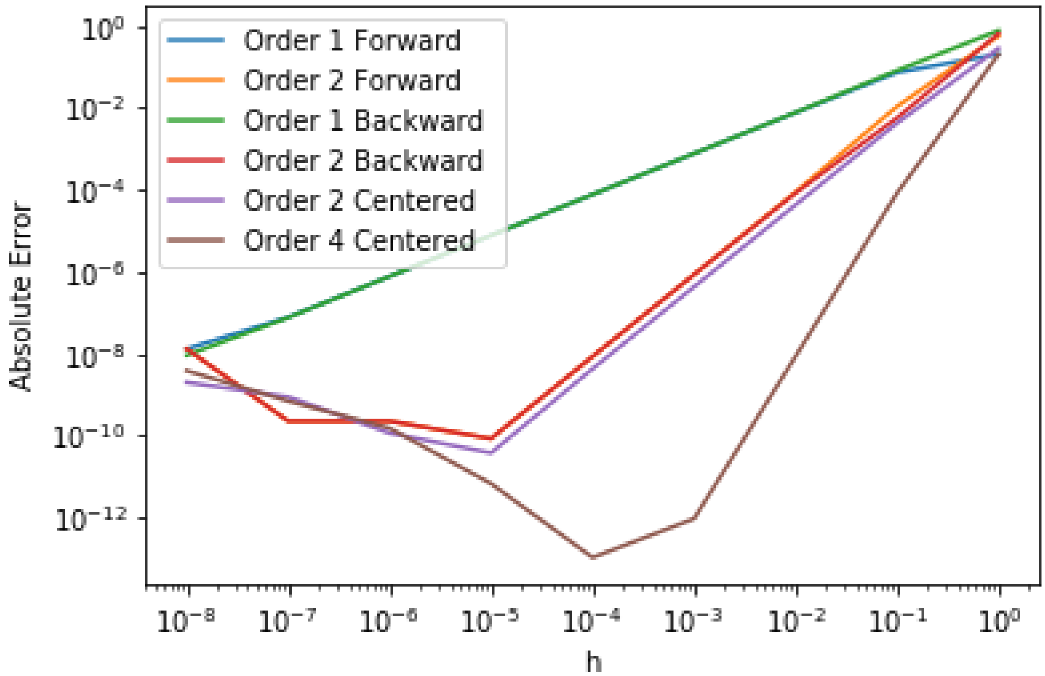


The seven plots are almost the same!

Pbm 3

```
In [45]: #Function for computing absolute error against h of exact and approximate f'(x)
         def convergence(x):
             x0=fprime(x)
             hvec=np.logspace(-8,0,9)
             plt.plot(hvec,abs([Forward1(f,x,h=i) for i in hvec]-x0),label='Order 1 Forward')
             plt.plot(hvec,abs([Forward2(f,x,h=i) for i in hvec]-x0),label='Order 2 Forward')
             plt.plot(hvec,abs([Backward1(f,x,h=i) for i in hvec]-x0),label='Order 1 Backward'
             plt.plot(hvec,abs([Backward2(f,x,h=i) for i in hvec]-x0),label='Order 2 Backward'
             plt.plot(hvec,abs([Centered2(f,x,h=i) for i in hvec]-x0),label='Order 2 Centered'
             plt.plot(hvec,abs([Centered4(f,x,h=i) for i in hvec]-x0),label='Order 4 Centered'
             plt.legend(loc='upper left')
             plt.xlabel("h")
             plt.ylabel("Absolute Error")
             plt.loglog()
             plt.show()
```

```
In [46]: #Example plot at x=1
         convergence(1)
```



Pbm 4

```
In [266]: #Transform the matrix data into DataFrame for better represenation
          import pandas as pd
          import warnings
          warnings.filterwarnings("ignore")
          df=np.load("plane.npy")
          radar=pd.DataFrame(df,columns=['t','alpha','beta'])

          #Degree to radian
          radar['alpha']=np.deg2rad(radar['alpha'])
          radar['beta']=np.deg2rad(radar['beta'])
          a=500

          #Calculate the plane location on the Cartesian coordinates
          radar['x(t)']=(a*np.tan(radar['beta']))/(np.tan(radar['beta'])-np.tan(radar['alpha'])
          radar['y(t)']=(a*np.tan(radar['beta'])*np.tan(radar['alpha']))\
                          /(np.tan(radar['beta'])-np.tan(radar['alpha']))

          #Approximate x'(t) and y'(t)
          radar['x_prime(t)']=0
          radar['x_prime(t)'][0]=radar['x(t)'][1]-radar['x(t)'][0]
          radar['x_prime(t)'][7]=radar['x(t)'][7]-radar['x(t)'][6]
          radar['y_prime(t)']=0
          radar['y_prime(t)'][0]=radar['y(t)'][1]-radar['y(t)'][0]
```

```
radar['y_prime(t)'][7]=radar['y(t)'][7]-radar['y(t)'][6]
for i in range(1,7):
    radar['x_prime(t)'][i]=0.5*(radar['x(t)'][i+1]-radar['x(t)'][i-1])
    radar['y_prime(t)'][i]=0.5*(radar['y(t)'][i+1]-radar['y(t)'][i-1])

radar['speed']=np.sqrt(radar['x_prime(t)']**2+radar['y_prime(t)']**2).round(2)
radar
```

Out[266]:

```
      t     alpha      beta         x(t)         y(t)  x_prime(t)  y_prime(t)  \
0   7.0  0.981748  1.178795  1311.271337  1962.456239          44          12
1   8.0  0.969181  1.161866  1355.936476  1975.114505          45          12
2   9.0  0.956440  1.144761  1401.918398  1987.346016          47          12
3  10.0  0.943525  1.127308  1450.497006  2000.840713          48          13
4  11.0  0.930959  1.110378  1498.640350  2013.512411          46          12
5  12.0  0.919614  1.095020  1543.798955  2025.792234          49          13
6  13.0  0.906524  1.077217  1598.041382  2040.990583          51          14
7  14.0  0.895005  1.061509  1647.596093  2055.065571          49          14

   speed
0  45.61
1  46.57
2  48.51
3  49.73
4  47.54
5  50.70
6  52.89
7  50.96
```

In [267]:

```
#Return the speed at each t
radar[['t','speed']]
```

Out[267]:

```
      t  speed
0   7.0  45.61
1   8.0  46.57
2   9.0  48.51
3  10.0  49.73
4  11.0  47.54
5  12.0  50.70
6  13.0  52.89
7  14.0  50.96
```

Pbm 5

In [65]:

```
def Jacobian(func,pt,h):
    n=len(func)
    dim=len(pt)
    I=np.identity(dim)
    J = zeros(n,dim)
    for i,fu in enumerate(func):
```

```
            for j,s in enumerate(pt):
                f= lambdify((x,y), fu, 'numpy')
                right=pt+h*I[:,j]
                left=pt-h*I[:,j]
                J[i,j]=(f(right[0],right[1])-f(left[0],left[1]))/(2*h)
        return J
```

In [68]:
```
#Test the Jacobian calculation function
x = Symbol('x')
y = Symbol('y')
func1=x**2
func2=x**3-y
func=[func1,func2]
pt=[1,1]
h=0.01
Jacobian(func,pt,h)
```

Out[68]:
```
Matrix([
[              2.0,  0.0],
[3.00010000000001, -1.0]])
```

Pbm 7

In [86]:
```
from autograd import numpy as anp
from autograd import grad
```

In [111]:
```
#Play around with Autograd
yy=lambda x: (anp.sin(x)+1)**(anp.sin(anp.cos(x)))
fauto=grad(yy)
print("Results from symbolic differentiation:",fprime(0.0))
print("Results from Autograd differentiation",fauto(0.)) #Take floats
```

```
Results from symbolic differentiation: 0.8414709848078965
Results from Autograd differentiation 0.8414709848078965
```

In [268]:
```
import time

def experiment(N):

    global CT1,CT2,CT3,abse1,abse2
    CT1,CT2,CT3=[],[],[]
    abse1,abse2=[],[]
    for i in range(N):
        xr=np.random.uniform(-pi,pi)
        t0=time.clock()
        sym=lambda x: y.diff(x)
        fprime=lambdify(x,sym(x),'numpy')
        x0=fprime(xr)
```

```
        t1=time.clock()
        CT1.append(t1-t0)

        t2=time.clock()
        xapp=Centered4(f,xr,h=0.01)
        t3=time.clock()
        CT2.append(t3-t2)
        abse1.append(abs(xapp-x0))

        t4=time.clock()
        xauto=fauto(xr)
        t5=time.clock()
        CT3.append(t5-t4)
        abse2.append(abs(xauto-x0))

    plt.scatter(np.array(CT1),np.array([1e-18] * N), alpha=0.8,label="SymPy")
    plt.scatter(np.array(CT2),np.array(abse1),alpha=0.8,label="Difference Quotients")
    plt.scatter(np.array(CT3),np.array(abse2), alpha=0.8,label="Autograd")
    plt.legend(loc='upper right')
    plt.xlabel("Computation Time (seconds)")
    plt.ylabel("Absolute Error")
    plt.xlim(10**-5,10**-2)
    plt.ylim(10**-19,10**-7)
    plt.loglog()
    plt.show()
```

```
In [269]: experiment(200)
```