PS4 Solution

February 3, 2019

```
In [207]: import pandas as pd
          import matplotlib.pyplot as plt
          import numpy as np
          import scipy.stats as sts
          import scipy.optimize as opt
          import warnings
          warnings.filterwarnings("ignore")
```

0.0.1 Problem 1

```
In [208]: df=pd.read_csv("data/incomes.txt",names=['Income'])
          df.head()
```

```
Out[208]:           Income
          0   51253.497156
          1  100630.320241
          2   83009.276137
          3   82882.106543
          4   77338.294839
```

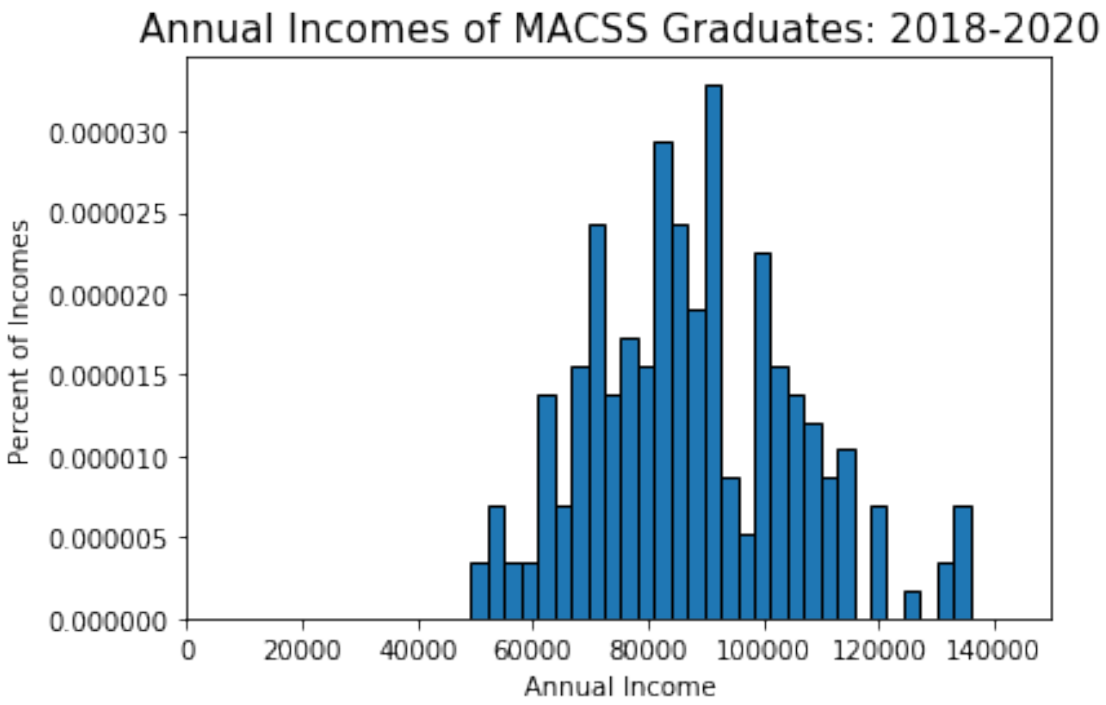
```
In [209]: df.describe()
```

```
Out[209]:              Income
          count     200.000000
          mean    87588.930769
          std     18081.166388
          min     49278.801938
          25%     74088.607447
          50%     85801.697366
          75%    100102.725206
          max    135865.026796
```

a

```
In [210]: num_bins = 30
          plt.hist(df['Income'], num_bins, normed=True, edgecolor='k')
          plt.title('Annual Incomes of MACSS Graduates: 2018-2020', fontsize=15)
          plt.xlabel('Annual Income')
```

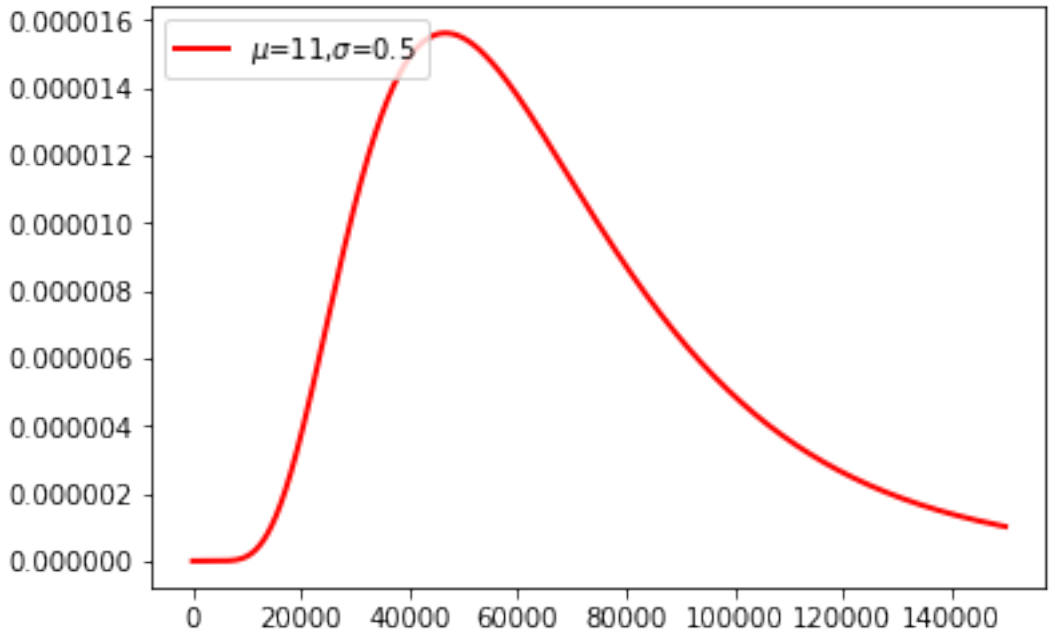
```
plt.ylabel('Percent of Incomes')
plt.xlim([1, 150000])
plt.show()
```



b

```
In [211]: # Define function that generates values of a lognormal pdf
          def trunc_lognorm_pdf(xvals, mu, sigma, cut_lb, cut_ub):

              if cut_ub == 'None' and cut_lb == 'None':
                  prob_notcut = 1.0
              elif cut_ub == 'None' and cut_lb != 'None':
                  prob_notcut = 1.0 - sts.lognorm.cdf(cut_lb, sigma, scale=np.exp(mu))
              elif cut_ub != 'None' and cut_lb == 'None':
                  prob_notcut = sts.lognorm.cdf(cut_ub, sigma,scale=np.exp(mu))
              elif cut_ub != 'None' and cut_lb != 'None':
                  prob_notcut = (sts.lognorm.cdf(cut_ub, sigma,scale=np.exp(mu)) -
                                 sts.lognorm.cdf(cut_lb, sigma,scale=np.exp(mu)))

              pdf_vals    = ((1/(xvals*sigma * np.sqrt(2 * np.pi)) *
                              np.exp( - (np.log(xvals) - mu)**2 / (2 * sigma**2))) /
                              prob_notcut)

              return pdf_vals
```

In [212]:
```
# Plot smooth line with distribution (mu_1 = 11, sig_1 = 0.5)
dist_pts = np.linspace(1, 150000, 3000)
mu_1 = 11
sig_1 = 0.5
plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_1,sig_1, 0, 150000),
         linewidth=2, color='r', label='$\mu$=11,$\sigma$=0.5')
plt.legend(loc='upper left')
plt.show()
```

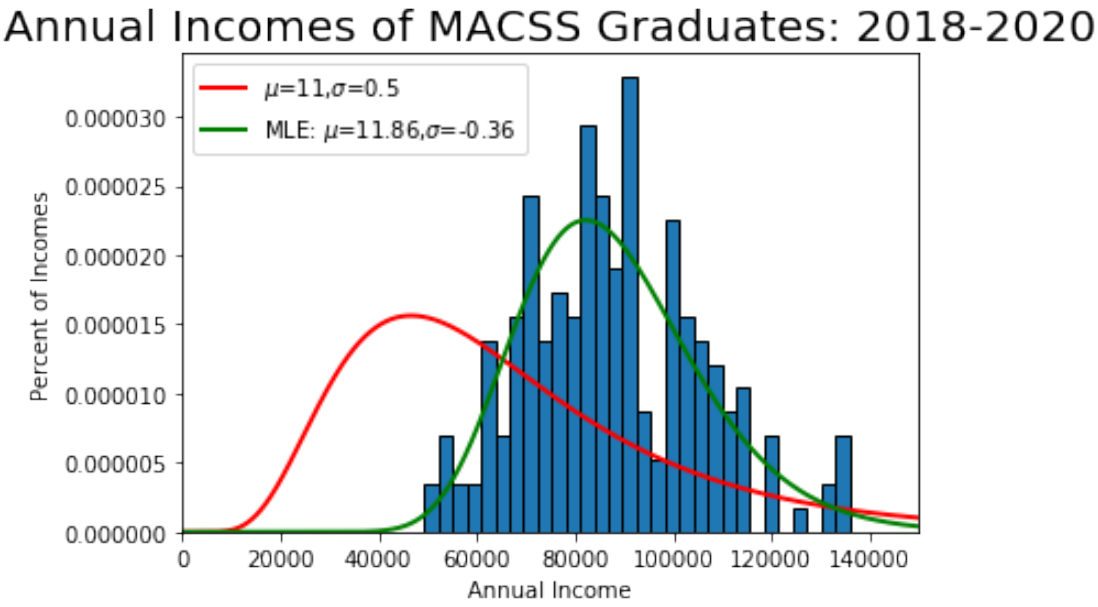


In [213]:
```
# Define log likelihood function for the log normal distribution
def log_lik_trunclognorm(xvals, mu, sigma, cut_lb, cut_ub):

    pdf_vals = trunc_lognorm_pdf(xvals, mu, sigma, cut_lb, cut_ub)
    ln_pdf_vals = np.log(pdf_vals)
    log_lik_val = ln_pdf_vals.sum()

    return log_lik_val

print('Log-likelihood:', log_lik_trunclognorm(df['Income'], mu_1, sig_1, 0, 150000))
```

```
Log-likelihood: -2379.120591931827
```

c

```
In [214]: def crit(params, *args):

              mu, sigma = params
              xvals, cut_lb, cut_ub = args
              log_lik_val = log_lik_trunclognorm(xvals, mu, abs(sigma), cut_lb, cut_ub)
              neg_log_lik_val = -log_lik_val

              return neg_log_lik_val

In [215]: mu_init = 10
          sig_init = 0.5
          params_init = np.array([mu_init, sig_init])
          mle_args = (df['Income'], 0, 150000)
          results_uncstr = opt.minimize(crit, params_init, args=(mle_args))
          mu_MLE= results_uncstr.x[0]
          sig_MLE = -results_uncstr.x[1]
          print('ML estimates for ţ=', mu_MLE)
          print('ML estimates for ', sig_MLE)
          print("Maximized Log Likelihood:",log_lik_trunclognorm(df['Income'], mu_MLE, sig_MLE

ML estimates for ţ= 11.361699975021795
ML estimates for  0.2117432788726898
Maximized Log Likelihood: -2240.934337511636


In [216]: # Plot the histogram of the data
          plt.hist(df['Income'], num_bins, normed=True, edgecolor='k')

          # Plot the PDF from (b)
          plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_1, sig_1, 0, 150000),
                   linewidth=2, color='r', label='$\mu$=11,$\sigma$=0.5')
          plt.legend(loc='upper left')

          # Plot the MLE estimated distribution
          plt.plot(dist_pts, trunc_lognorm_pdf(dist_pts, mu_MLE,sig_MLE, 0, 150000),
                   linewidth=2, color='g', label='MLE: $\mu$=11.86,$\sigma$=-0.36')
          plt.legend(loc='upper left')

          plt.title('Annual Incomes of MACSS Graduates: 2018-2020', fontsize=20)
          plt.xlabel('Annual Income')
          plt.ylabel('Percent of Incomes')
          plt.xlim([0, 150000])
          plt.show()
```




```
In [217]: vcv_mle = results_uncstr.hess_inv

          stderr_mu_mle = np.sqrt(vcv_mle[0,0])
          stderr_sig_mle = np.sqrt(vcv_mle[1,1])
          print('VCV(MLE) = ', vcv_mle)
          print('Standard error for mu estimate = ', stderr_mu_mle)
          print('Standard error for sigma estimate = ', stderr_sig_mle)
          print('mu percent', 100*stderr_mu_mle/mu_MLE,"%")
          print('sig percent', 100*stderr_sig_mle/sig_MLE,"%")
```

```
VCV(MLE) =  [[ 2.14293375e-04 -1.20173290e-05]
 [-1.20173290e-05  1.30638926e-04]]
Standard error for mu estimate =  0.014638762766080225
Standard error for sigma estimate =  0.011429738647692086
mu percent 0.1288430674834128 %
sig percent 5.397922762197421 %
```

d

```
In [218]: mu_new, sig_new = np.array([420, 130])
          log_lik_h0 = log_lik_trunclognorm(df['Income'], mu_1, sig_1, 0, 150000)
          print('hypothesis value log likelihood', log_lik_h0)
          log_lik_mle = log_lik_trunclognorm(df['Income'], mu_MLE, sig_MLE, 0, 150000)
          print('MLE log likelihood', log_lik_mle)
          LR_val = 2 * (log_lik_mle - log_lik_h0)
          print('likelihood ratio value', LR_val)
```

```
pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 2)
print('chi squared of H0 with 2 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2379.120591931827
MLE log likelihood -2240.934337511636
likelihood ratio value 276.3725088403826
chi squared of H0 with 2 degrees of freedom p-value =  0.0
```

p-value is very small, so we can reject the null hypothesis that the probability that the data comes from is f(x | mu=11,sigma=0.5).

e

```
In [219]: 1-sts.lognorm.cdf(100000, sig_MLE, scale=np.exp(mu_MLE))
```

```
Out[219]: 0.2375540357115089
```

There is 24% of the chance that I will earn more than $100000.

```
In [220]: sts.lognorm.cdf(75000, sig_MLE, scale=np.exp(mu_MLE))
```

```
Out[220]: 0.25964393792847373
```

There is 26% of the chance that I will earn less than $75000.

0.0.2 Problem 2

a

```
In [228]: df=pd.read_csv("data/sick.txt").astype('float64')
          df.head()
```

```
Out[228]:    sick    age  children  avgtemp_winter
          0  1.67  57.47      3.04           54.10
          1  0.71  26.77      1.20           36.54
          2  1.39  41.85      2.31           32.38
          3  1.37  51.27      2.46           52.94
          4  1.45  44.22      2.72           45.90
```

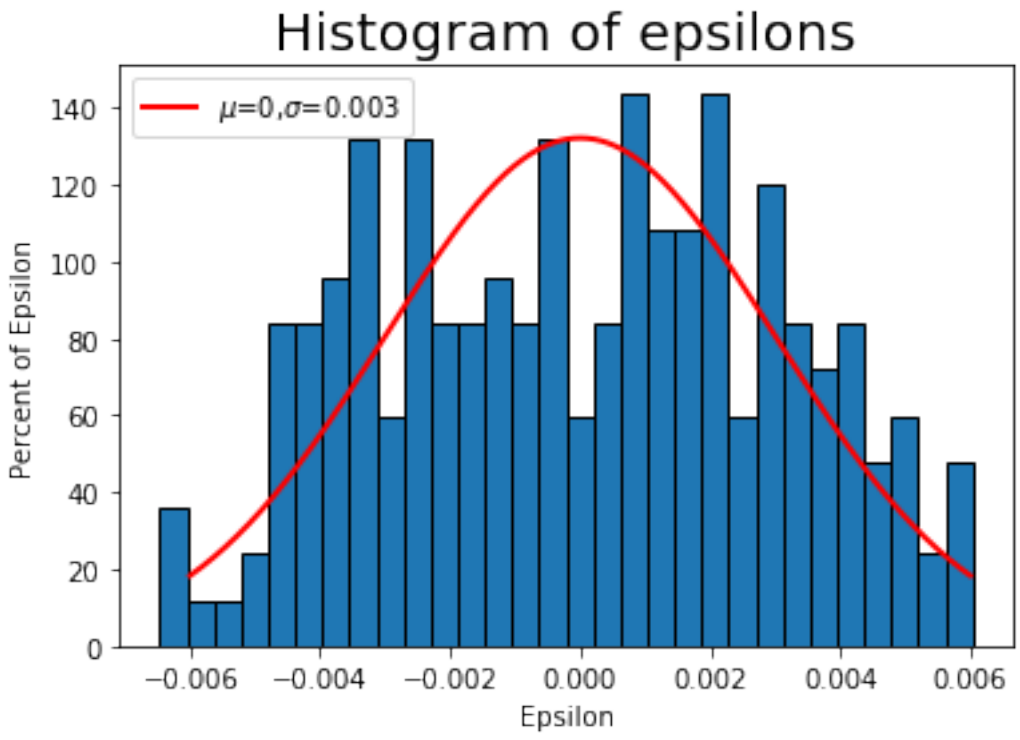
```
In [237]: df.describe()
```

```
Out[237]:              sick         age    children  avgtemp_winter
          count  200.000000  200.000000  200.000000      200.000000
          mean     1.008600   40.683850    1.674950       44.041250
          std      0.504222   11.268686    0.969761       11.101977
          min      0.040000   12.810000    0.000000       16.500000
          25%      0.650000   33.967500    0.970000       36.112500
          50%      0.960000   41.015000    1.560000       43.300000
          75%      1.322500   47.750000    2.322500       52.172500
          max      2.800000   74.890000    4.960000       68.600000
```

```
In [230]: def norm_pdf(xvals, sig):
              sig=abs(sig)
              pdf_vals = (1/(sig*np.sqrt(2*np.pi)))*np.exp(-(xvals)**2 / (2*sig**2))
              return pdf_vals

In [223]: def log_lik_norm(y, x1, x2, x3, b0, b1, b2, b3, sig):

              err=y-b0-b1*x1-b2*x2-b3*x3
              pdf_vals = norm_pdf(err, sig)
              ln_pdf_vals = np.log(pdf_vals)
              log_lik_val = ln_pdf_vals.sum()

              return log_lik_val

In [224]: def crit2(params,*args):

              b0, b1, b2, b3, sig = params
              y, x1, x2, x3 = args
              log_lik_val = log_lik_norm(y, x1, x2, x3, b0, b1, b2, b3, sig)
              neg_log_lik_val = -log_lik_val

              return neg_log_lik_val

In [225]: b0_init, b1_init, b2_init, b3_init, sig_init = (0.2,0,0,0,1)
          y=df['sick']
          x1, x2, x3 = df['age'], df['children'], df['avgtemp_winter']
          params_init = np.array([b0_init, b1_init, b2_init, b3_init, sig_init])
          results = opt.minimize(crit2, params_init,(y, x1, x2, x3))
          b0_MLE, b1_MLE, b2_MLE, b3_MLE, sig_MLE = results.x
          print('beta 0=', b0_MLE)
          print('beta 1=', b1_MLE)
          print('beta 2=', b2_MLE)
          print('beta 3=', b3_MLE)
          print('sigma=', sig_MLE)
          print("Value of the log likelihood function:",-results.fun)

beta 0= 0.2516462863009143
beta 1= 0.012933350241651839
beta 2= 0.4005020598812094
beta 3= -0.009991671522641493
sigma= 0.0030177004489735067
Value of the log likelihood function: 876.8650468414168


In [226]: vcv_mle = results.hess_inv
          print('VCV(MLE) = ', vcv_mle)

VCV(MLE) =  [[ 1.08579401e-06  4.52195731e-09 -1.47000525e-07 -2.20502350e-08
   5.57249528e-09]
```

```
 [ 4.52195731e-09  4.20615273e-09 -3.71445246e-08 -2.57941093e-09
  -1.28317610e-10]
 [-1.47000525e-07 -3.71445246e-08  3.77442634e-07  2.33178375e-08
    6.82198342e-10]
 [-2.20502350e-08 -2.57941093e-09  2.33178375e-08  1.99610060e-09
  -2.42837085e-11]
 [ 5.57249528e-09 -1.28317610e-10  6.82198342e-10 -2.42837085e-11
    2.31215973e-08]]
```

In [245]:
```
# Plot the histogram of the data
plt.hist(y-b0_MLE-b1_MLE*x1-b2_MLE*x2-b3_MLE*x3, num_bins, normed=True, edgecolor='k

dist_pts=np.linspace(-0.006,0.006,1000)
plt.plot(dist_pts, norm_pdf(dist_pts, sig_MLE),
         linewidth=2, color='r', label='$\mu$=0,$\sigma$=0.003')
plt.legend(loc='upper left', fontsize=10)

plt.title('Histogram of epsilons', fontsize=20)
plt.xlabel('Epsilon')
plt.ylabel('Percent of Epsilon')
plt.show()
```



b

```
In [249]: b0_t, b1_t, b2_t, b3_t, sig_t = 1, 0, 0, 0, 0.1
          log_lik_h0 = log_lik_norm(y, x1, x2, x3, b0_t, b1_t, b2_t, b3_t, sig_t)
          print('hypothesis value log likelihood', log_lik_h0)
          print('MLE log likelihood', -results.fun)
          LR_val = 2 * ((-results.fun)-log_lik_h0)
          print('likelihood ratio value', LR_val)
          pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 5)
          print('Chi squared of H0 with 5 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2253.700688042125
MLE log likelihood 876.8650468414168
likelihood ratio value 6261.131469767083
Chi squared of H0 with 5 degrees of freedom p-value =  0.0
```

We could reject the null hypothesis that age, number of children, and average winter temperature have no effect on the number of sick days. So there is 0% that age, number of children, and average winter temperature have no effect on the number of sick days.