1. $f(x) = \frac{1}{1+x^2}$, $x \in [-5, 5]$

With equally spaced nodes, the interpolation error grows near the endpoints.

This is called Runge's phenomenon.

Error formula: $f(x) - P_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0}^{n} (x - x_i)$

The term becomes large at the boundaries, causing oscillations.

Remedies: (I) Use Chebyshev nodes instead of equally spaced nodes.

(II) Use piecewise interpolation or splines.

2. We take 10 equally spaced nodes: $x_i = \frac{i}{9}$, $i = 0, \dots, 9$, $y_i = \sin(x_i)$

Lagrange form: $P(x) = \sum_{i=0}^{n} y_i \prod_{j \neq i} \frac{x - x_j}{x_i - x_j}$

Newton form: $P(x) = y_0 + \sum_{k=1}^{n} f[x_0, \dots, x_k] \prod_{j=0}^{k-1} (x - x_j)$

Barycentric form is numerically stable, but equivalent in theory.