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T1 ________________	**55869**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**E**	F4 ________________

2018
MCM/ICM
Summary Sheet

Summary

In recent years, with he rapid development in economy and explosive growth in population, environment pollution and depletion of resources have been aggravated day by day. Smart Growth was then put forward. Lots of cities have drawn up the development plans on basis of the combination of ten principles for Smart Growth and their own conditions. Our work is to evaluate the success of each city in Smart Growth and provide a guidance for plan making.Meanwhile, we need to measure the success of our smart growth plan. Therefore, we define the "Smart Growth Index" (SGI) to measure the success of smart growth and development plans of a city.

Our model establishes a Detail group of 52 indictors, which reflects the ten principles for smart growth and the three E's of sustainability. On basis of this, we can assess the success of smart growth of a city. In the construction of SGI model, we use linear aggregation to combine individual variables into the theme scores. Compensability of linear aggregation ensures fair assessment of different cities which have different emphasis in urban development. Then we use geometric aggregation to combine theme scores into SGI. Low compensability of geometric aggregation requires a city to develop evenly in all themes. We use entropy method to determine weights of indictors or themes, which avoid subjectivity. SGI model has an all-round and objective assessment of the success of smart growth of a city.

With the completion of these, we respectively select a city in China and America (Zhenjiang and Columbia.SC). We assess the success of city development and their smart growth on basis of SGI model. Finally, we proved the reasonability and applicability of our model.

Keywords: **Geometric and linear aggregation ; entropy method**
Smart Growth Index ; PCA

Contents

1 Introduction

1.1 Background

1.2 Our work

2 Assumptions

(1) Evaporation will result in no loss of water. The loss of water from evaporation is too little to consider, so we ignore the change of quantity of water.
(2) Consider the energy of water in 36 $^\circ C$ as the energy reference. For we can arbitrarily set our energy reference to be zero.

3 Symbol Description

Table 1:Constants

Symbol	uint	Meaning	value
P_v''	hP_a	Vapor pressure of thin saturated layer on water surface	test
P_v	hP_a	Water vapor partial pressure in wet air	
P_a	hP_a	Atmospheric pressure	test
L	kj/kg	heat of water vaporization	2500
ε		emissivity	0.97
C_p	$KJ/(kg\cdot K)$	Specific heat capacity of dry air at 300k	1.005

Table 2:Notation

Symbol	uint	Meaning
$V(h)$	m^3	The relationship between volume of water and
$S_1(h)$	m^2	The top surface area of the bathtub
$S_2(h)$	m^2	The bottom surface area of the bathtub
$S_3(h)$	m^2	The side surface area of the bathtub
S	m^2	The surface area of the body
h_p	cm	height of a person
w	kg	weight of a person
α	$W/(m^2 \cdot^\circ C)$	Heat dissipation coefficient
t	$^\circ C$	Surface temperature of water
θ	$^\circ C$	Dry bulb temperature of air
β	$W/(m^2 \cdot hP_a)$	Evaporation coefficient
Q_{upper}	J	Evaporation coefficient
Q_{side}	J	Bathtub side surface unit time heat dissipation
S_{upper}	m^2	Surface area of bathtub water
S_{side}	m^2	Bathtub Side surface Area

4 Heat Dissipation Model

In order to analyze the situation of heat dissipation. We first analyze the bathtub.

The effect of a bathtub on heat loss is mainly in material and shape.

Because different bathtubs have different shapes and attributes, We use functions to describe the relationship between the surface area, the surface height and the volume of the surface area.

$$\begin{aligned} h &= H(v) \\ S_1 &= S_1(v) \\ S_2 &= S_2(v) \end{aligned} \tag{1}$$

The water in the bathtub dissipates heat by contacting the air on the upper surface and the bathtub wall on the side surface.

4.1 Liquid surface heat loss model

5 Model of hot water addition

5.1 Changes in the volume of water in a bathtub

5.1.1 People take the initiative to drain through the drain of the bathtub

5.1.2 Water overflow through the upper drain

6 Factors that affect the model

6.1 The effect of human body and body temperature on the model

We improved our model in view of the human body influence. Our improvement is mainly in the following aspects.

6.1.1 The influence of the heat conduction

The interior of the body is basically maintained at a constant temperature, and the temperature of the skin is close to the water in bathtub. So we can simplify the question.The simplified question is illustrated as floows.

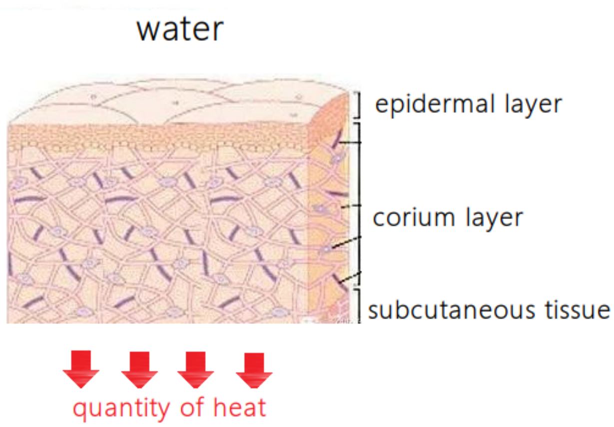

Figure 1: Heat conduction model of human body

We can assume that the temperature of the skin is the same as the water. We select the temperature of the rectum about 37.5 degrees as the body temperature. The problem can be simplified as the skin heat dissipating to the interior of the body.

So we can use **Newton's law of cooling** to calculate the speed of heat dissipation.

$$\begin{aligned} \Delta T &= |T_w - T_f| \\ q &= h\Delta T \\ \phi &= qA = Ah\Delta T \end{aligned} \tag{2}$$

Where ***q*** is the heat flux density. ***h*** is the convection heat transfer coefficient of matter. ϕ is heat transfer power (or heat transfer per unit time). ***A*** is the heat transfer area.

The surface area of the human body can be calculated by the following formula.

$$S = \begin{cases} 0.0057 * h + 0.0121 * w + 0.0820(\text{ man }) \\ 0.0073 * h + 0.0127 * w - 0.2106(\text{woman}) \end{cases} \tag{3}$$

According to formula 1, the quantity of heat dissipated(dQ) in unit time is:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \phi = qA = Sh\Delta T \tag{4}$$

where h=1.48,$T_f = 37.5°$.

6.1.2 The influence of the body shape

The volume of the person in the water affects the height of the liquid. In order to discuss the influence on the liquid surface conveniently. We use the weight to estimate the volume of the body. We use $\rho = 1.06 \times 10^3 kg/m^3$ as the density of the human body, so the volume can be calculated:

$$v = \frac{m}{\rho} = \frac{m}{1.06 \times 10^3}(m^3) \tag{5}$$

7 Solutions for tasks

7.1 Different shapes of the bathtub

We model and analysis several different shapes of bathtubs.

7.1.1 Bathtub based on ovals

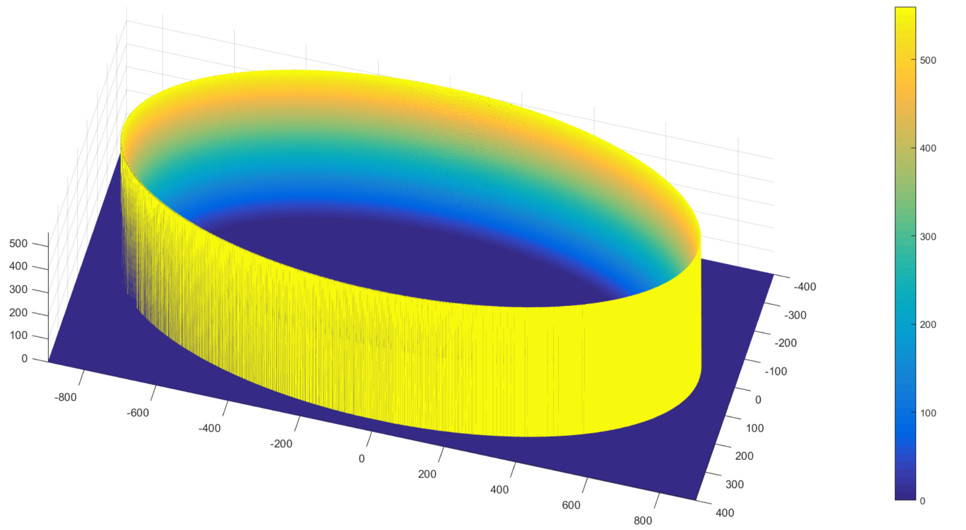

Figure 2: The shape of bathtub based on ovals

The elliptical long axis on the bottom of the bathtub is A, the short axis is B, the height of the whole bathtub is h, and the lower bottom long axis is $A - 2d$.

We divide the bathtub into an infinite number of elliptical columns with a height of dz, with a long semi-axis of each elliptical element

$$a(z) = \frac{A}{2} - d + \frac{d}{H^2}z^2 \tag{6}$$

Short semi-axis of each elliptical element is

$$b(z) = \frac{B}{2} - d\frac{B}{A} + \frac{Bd}{AH^2}z^2 \tag{7}$$

Calculation of the volume of water

The volume of the small cylinder is

$$dV = Sdz = \pi a(z)b(z) \tag{8}$$

The relationship between the volume of water in the bathtub and the height of the liquid surface is

$$\begin{aligned} V(h) &= \int dV = \int Sdz = \int_0^h \pi a(z)b(z)dz \\ &= \int_0^h \pi(\frac{A}{2} - d + \frac{dz^2}{H^2})(\frac{B}{2} - d\frac{B}{A} + \frac{dBz^2}{AH^2})dz \end{aligned} \tag{9}$$

We can get relationship between the volume of water and the height of liquid surface by replacing the bathtub size into the formula (9).

Calculation of the contact area of water with the bathtub and air

Without considering the effect of person, the upper and lower surface of the liquid contact are standard ovals.

The area of upper surface is

$$S_1 = \frac{\pi AB}{4} \tag{10}$$

And area of the lower surface is

$$S_1 = \frac{\pi(A-2d)(B-2d\frac{B}{A})}{4} \tag{11}$$

In order to calculate the contact area between the liquid and side of the bathtub, we use the calculus method.

7.1.2 Bathtub based on circles

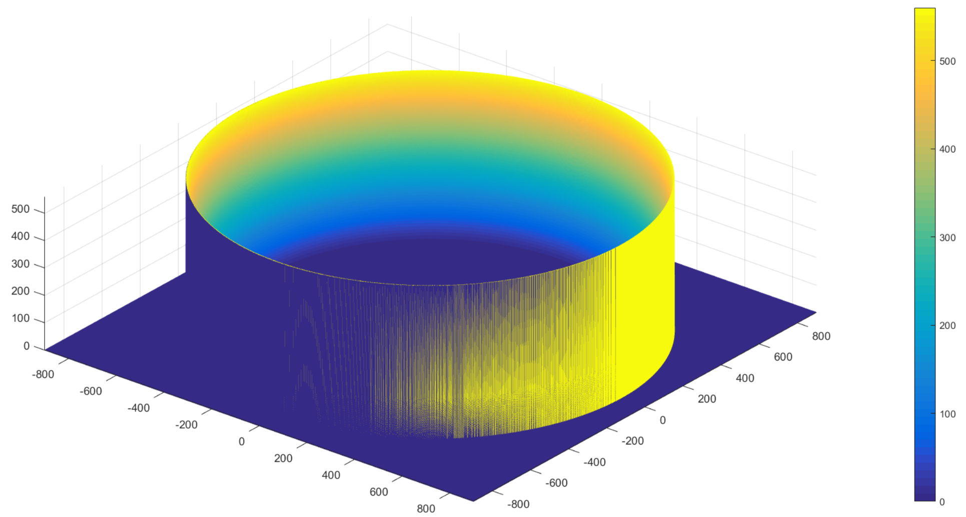

Figure 3: The shape of bathtub based on circles

7.2 Task 2

According to the current researches on urban planning of our chosen cities (Columbia.SC and Zhenjiang), we adopt quantization analysis of plans or policies. Then we obtain the following policy conclusion:

Using SGI model, we can obtain SGI of two cities after implementing the current plan. Also, we can obtain original SGI on the basis of original data. Based on the method in section 4, we can get the following result by way of matlab.

Table 2:Current Plan

Zheiang 5 years	**change**	**Columbia 5 years**	**change**
per capita GDP(yuan/person)	+8%	per capita disposable income(yuan)	+5%
production of tertiary industry ratio(%)	to 50%	per capita green spaces(m^2)	+4%
per capita disposable income(yuan)	+8.5%	average wages of staff(yuan)	+5%
tourist income ratio in GDP(%)	+100%	sewage treatment ratia(%)	to 99%
total export(100 million dollars)	+50%	high school enrollment(10000 person)	+8%
amount of doctors (per ten thousand)	+10%	per capita living space(m^2)	+7%
per captia area of roads(urban,m^2)	+10%	green cover in built up area(%)	+5%

Table 3:SGI of Current Plan

City	SGI_0 (2014)	SGI (2020)	ΔSGI
Zhenjiang	12.5126	14.3771	1.8645
columbia	13.962	14.1089	0.1469

Through the analysis of the relative change in SGI of two cities, we can assess the success of the growth plan.
Apparently, SGI of two cities both increases after implementing their own current growth plan.
Thus, their current growth plan is both successful.

7.3 Task 3

7.3.1 Our growth plans

Combining the current plans of two cities and ten principles of smart growth, we make the following growth plans for our cities over few decades.

7.3.2 Reasons for choosing these initiatives

Ten principles of smart growth can be reflected in the detailed of indictors. Thus, in the process of making our smart growth plans, we just need to choose appropriate indictors supported by policies on basis of geography and economic development.

For Zhenjiang, as manufacturing industry basement in middle and lower reaches of Yangtze River in China, adept in manufacturing and export, we can stimulate its export in our plan, which can accelerate the development of city. Even though economic development is on the agenda, the environment and medical level is as important as economic development. Thus, we also give priority to relative indictors of these elements

Table 4:Our Plan

Zhenjiang 10 years	**change**	**Columbia 10 years**	**change**
per capita GDP(yuan/person)	+100%	per captia area of roads(urban,m^2)	+15%
production of tertiary industry ratio(%)	to 55%	ability of harmless treatment planet(ton)	+100%
total export(100 million dollars)	+100%	tourist income ratio in GDP(%)	+100%
per capita disposable income(yuan)	+100%	amount of professionals(10000 person)	+50%
per capita green spaces(m^2)	+20%	expenditure in education ratio in GDP	to 10%
high school enrollment(10000 person)	+100%	high school enrollment(10000 person)	+50%
total social labor productivity(yuan)	+200%	amount of ward beds(per ten thousand)	+30%
amount of ward beds(per ten thousand)	+20%	per capita green spaces(m^2)	+10%

in our smart growth plan.
For Columbia.SC, its economy and livelihood have higher level, thus we put its emphasis on education, technology and protection of environment. In the meanwhile, tertiary industry should be developed rapidly, which plays important role in urban development.
Therefore, we make these smart growth plans above for both cities.

7.3.3 Assessment of the success of our smart growth plans

Based on the method in task 2, we can obtain the changes in SGI of two cities by way of matlab after implementing our growth plans.

Table 5:SGI of Our Plan

City	SGI_0 (2014)	SGI (2025)	ΔSGI
Zhenjiang	12.5126	15.4329	2.9023
columbia	13.962	15.092	1.13

Apparently, SGI of two cities both increases after implementing our smart growth plan. Therefore, our smart growth plans are successful in the chosen cities through analysis.

7.4 Task 4

7.4.1 Assessment of the importance of each initiative

In this task, we need to rank our chosen initiatives of our smart growth plan. Thus, we can compare the importance of each initiative to complete this task.

On basis of the method mentioned in 4.2 of section 4 and SGI model, we can obtain the following ranking of each city by way of matlab.

Table 6:Ranking of initiatives

Columbia				
rank	**initiatives**		**SGI**	ΔSGI
1	tourist income ratio in GDP(%)	+100%	14.3194	0.3574
2	high school enrollment(10000 person)	+50%	14.2334	0.2714
3	ability of harmless treatment planet(ton)	+100%	14.1711	0.2090
4	amount of professionals(10000 person)	+50%	14.0586	0.0966
5	expenditure in education ratio in GDP	to 10%	13.9929	0.0309
6	amount of ward beds(per ten thousand)	+30%	13.9838	0.0218
7	per capita green spaces(m^2)	+10%	13.9811	0.0190
8	per captia area of roads(urban,m^2)	+15%	13.9726	0.0106
Zhenjiang				
rank	**initiatives**		**SGI**	ΔSGI
1	total export(100 million dollars)	+100%	14.4935	1.9809
2	high school enrollment(10000 person)	+100%	13.0236	0.5109
3	per capita disposable income(yuan)	+100%	12.6755	0.1628
4	total social labor productivity(yuan)	+200%	12.6102	0.0975
5	per capita GDP(yuan/person)	+100%	12.5504	0.0377
6	amount of ward beds(per ten thousand)	+20%	12.5304	0.0177
7	per capita green spaces(m^2)	+20%	12.5288	0.0161
8	production of tertiary industry ratio(%)	to 55%	12.5128	0.0002

7.4.2 Analysis of the ranking

Evidently, the ranking proves the reasonability of our plans' emphasis.
The ranking of the same initiatives of both cities is the same (education, medical level and green spaces). Thus, each of these indictors play the same role in urban development. Giving thought to national conditions of both cities, Zhenjiang's emphasis is economic development while Columbia's is soft strength of society like education, as well as technology and tertiary industry. The ranking of each city's initiatives is correspond to our reasons for choosing these initiatives.

7.5 Task 5

In the case of that the population of city will increase 50% by 2050, we analyze our group of indictors. We find that the increase of population has negative effects on population density, per capita green spaces and per capita living space so on.
Due to population density and per capita living space not under the control of governmental policies or plans, we neglect these indictors in our smart growth plan.
For Columbia.SC and Zhenjiang there are several initiatives like $per\ capita\ green\ spaces$ and $amount\ of\ ward\ beds(per\ ten\ thousand)$

These implements all can effectively curb the negative impacts of the increase of population.

8 Sensitivity analysis

8.1 Selection of indictors

The aim of this analysis is to determine whether a single variable had an unduly large impact on the overall ranking. To test this, we randomly exclude variables/themes from the index several times. The rest of variables/themes can use the same method to aggregation smart growth index (SGI). Observe and analyze the distribution of SGI of two cities.

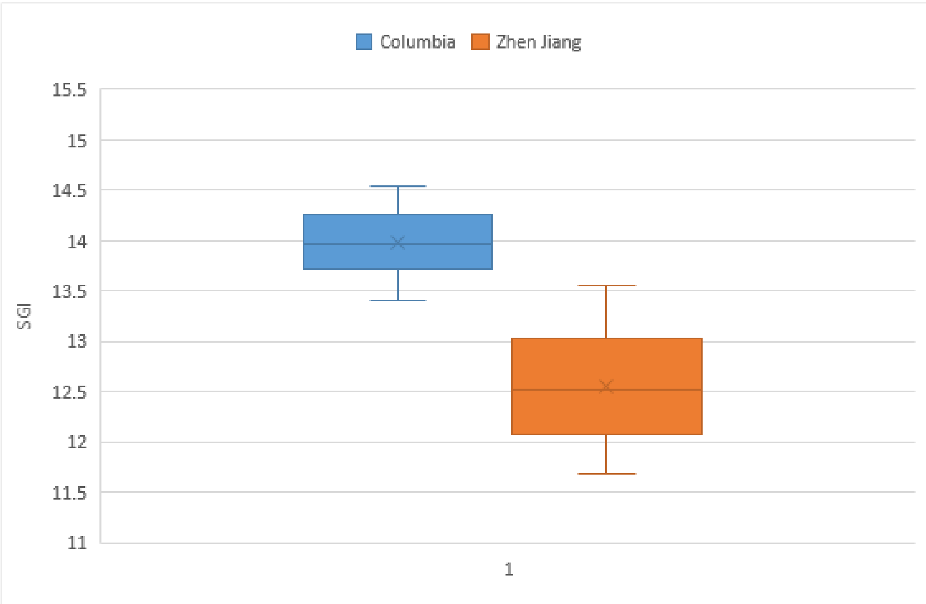

Figure2:boxplot

As is shown in the figure of statistics, the SGI of Columbia.SC is generally higher than that of Zhenjiang. The SGI of Zhenjiang is more sensitive to that of Columbia.SC. The result proves that SGI of Zhenjiang benefits from some indictors in degrees. In other words, the development of Columbia.SC is more balanced than that of Zhenjiang.

8.2 Selection of aggregation method

In Task 1, we compare linear aggregation with geometric aggregation in advantages. For that reason, we finally decided to use linear aggregation to combine individual variables into the theme scores and geometric aggregation to combine theme scores into SGI.

We tried to use geometric aggregation to combine individual variables into the theme scores. And we observe that the calculated SGI using different data may varies in difference.

Our model is sensitive to the selection of aggregation method.

9 Strengths and weaknesses

9.1 Strengths

- Our model establishes a detailed group of 52 indictors, which reflects the ten principles for smart growth and the three E's of sustainability. On basis of this, we can assess the success of smart growth of a city.
- In the construction of SGI model, we use linear aggregation to combine individual variables into the theme scores. Compensability of linear aggregation ensures fair assessment of different cities which have different emphasis in urban development. Then we use geometric aggregation to combine theme scores into SGI.Low compensability of geometric aggregation requires a city to develop evenly in all themes.
- We use entropy method to determine weights of indictors or themes, which avoid subjectivity.
- SGI model has an all-round and objective assessment of the success of smart growth of a city.

9.2 Weakness

- In our model, we regard the relationship between initiative and indictor as one to one. In reality, a plan can result in the change in other indictors(not included in plan) because of correlation between indictors. The real SGI may be different with our calculated SGI.
- In our model, we neglect the government's executive force.

10 References

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Appendices

Here are programmes we used in our model as follow.

calculation of SGI matlab source:

```
function [SGI]=model(X,Y)
A=[7.59925 7.27817 2.11033 3.71549 0.70123 11.5444 1.7815
   0.8213 2.37957 11.6974 19.5481 6.9475 9.46227 14.4136];
B=[0.01095 7.54951 0.38819 2.81685 1.38275 8.926287 0.90891 31.16803
   9.938145 1.82491 2.61773 1.30663 8.64347 2.89988 14.8049 4.812845];
C=[2.927595 4.052684 40.98544 1.645014 1.032296 1.580738
   2.971568 15.33443 1.924922 4.205913 3.414808 12.40645 7.518134];
D=[7.126399 0.604674 30.61872 3.974089 3.31051
   21.84158 11.70999 19.88107 0.932972];
E=[22.925 32.8844 22.6476 21,543];
H=X(1,1:14);
I=X(1,15:30);
J=X(1,31:43);
K=X(1,44:52);
L=Y(1,1:14);
M=Y(1,15:30);
N=Y(1,31:43);
O=Y(1,44:52);
AA=A./H;
BB=B./I;
CC=C./J;
DD=D./K;
a=sum(AA.*L)./sum(AA);
b=sum(BB.*M)./sum(BB);
c=sum(CC.*N)./sum(CC);
d=sum(DD.*O)./sum(DD);
SGI=a^(E(1)/100)*b^(E(2)/100)*c^(E(3)/100)*d^(E(4)/100);
```

calculation of weight by the entropy method matlab source:

```
function [s,w]=shang(x)
[n,m]=size(x);
[X,ps]=mapminmax(x');
ps.ymin=0.002; -
ps.ymax=0.996;
ps.yrange=ps.ymax-ps.ymin;
X=mapminmax(x',ps);
X=X';
for i=1:n
    for j=1:m
        p(i,j)=X(i,j)/sum(X(:,j));
```

```
    end
end
k=1/log(n);
for j=1:m
    e(j)=-k*sum(p(:,j).*log(p(:,j)));
end
d=ones(1,m)-e; -
w=d./sum(d);
```