EEE 558 (2016 Fall) Project 1: Diversity Techniques over Fading Channels

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1. Introduction:

This report is a Matlab simulation for general selection combining on a Rayleigh fading channel:

$$x_m = \sqrt{\bar{\gamma}} h_m s + v_m \qquad m = 1, \dots, M$$[1]

2. Generalized selection combining:

2.1 Assumptions and objectives:

2.1.1 Assumptions:

- total M antennas
- select L antennas with highest channel gain, denoted as set U_L, $(L = 1, \dots, M)$
- perform MRC on each element in U_L, $(L = 1, \dots, M)$
- modulation: K-PSK, elements are: $\left\{e^{\frac{j2\pi(k-1)}{8}}\right\}_{k=1}^{K}$
- channel: i.i.d. Rayleigh fading on each branch

2.1.2 Objectives:

- evaluate the performance $(\overline{P_b}, P_{out})$ on different L and M
- compare the simulation results with analytical results

2.2 analytical results:

channel model: $x_m = \sqrt{\bar{\gamma}} h_m s + v_m$, where $h_m \sim CN(0,1), v_m \sim CN(0,1), s = \left\{e^{j2\pi(k-1)/M}\right\}_{k=1}^{M}$

we take $\bar{\gamma}: 0dB \sim 30dB$

average probability of symbol error: $\overline{P_s}(\gamma_\Sigma) = \int_0^\infty P_s(\gamma) P_{\gamma_\Sigma}(\gamma) d\gamma$

- for K-PSK, $P_s(\gamma) = 2Q(\sqrt{2\gamma} \sin\left(\frac{\pi}{K}\right))$
- Consider one branch, for Rayleigh fading, $P_{\gamma_s}(\gamma) = \frac{1}{\gamma_s} e^{-\frac{\gamma}{\gamma_s}}$

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[1] $x_m = \sqrt{\bar{\gamma}} h_m s + v_m$

x_m: sampled output, $\bar{\gamma}$: average SNR of the system, h_m: complex channel gain, s: transmitted signal

v_m: the noise sampled at the m^{th} diversity branch

Then $\overline{P_s}(\overline{\gamma_s}) = \int_0^\infty 2Q(\sqrt{2\gamma}\sin\left(\frac{\pi}{K}\right))\frac{1}{\overline{\gamma_s}}e^{-\frac{\gamma}{\overline{\gamma_s}}}d\gamma = 1 - \sqrt{\frac{sin^2(\frac{\pi}{K})\overline{\gamma_s}}{1+sin^2(\frac{\pi}{K})\overline{\gamma_s}}}$

(derived from 6.61: $\overline{P_s} \approx \int_0^\infty \alpha_M Q(\sqrt{\beta_M\gamma})\cdot\frac{1}{\overline{\gamma_s}}e^{-\frac{\gamma}{\overline{\gamma_s}}}d\gamma = \frac{\alpha_M}{2}[1-\sqrt{\frac{0.5\beta_M\overline{\gamma_s}}{1+0.5\beta_M\overline{\gamma_s}}}] \approx \frac{\alpha_M}{2\beta_M\gamma}$,

Where $\alpha_M = 2, \beta_M = 2sin^2(\frac{\pi}{M})$)

- Assuming i.i.d. Rayleigh fading on each branch with same average branch SNR $\bar{\gamma}$,
- For L=1 case, it is Selection Combining (7.9):

$$p_{\gamma_\Sigma}(\gamma) = \frac{K}{\bar{\gamma}}\left[1-e^{-\frac{\gamma}{\bar{\gamma}}}\right]^{M-1}e^{-\frac{\gamma}{\bar{\gamma}}}$$

Then, $\overline{P_s}(\bar{\gamma}) = \int_0^\infty 2Q(\sqrt{2\gamma}\sin\left(\frac{\pi}{K}\right))\frac{K}{\bar{\gamma}}\left[1-e^{-\frac{\gamma}{\bar{\gamma}}}\right]^{M-1}e^{-\frac{\gamma}{\bar{\gamma}}}d\gamma$

- For L=M case, the General Selection Combining (GSC) becomes Maximum Ratio Combining (MRC) (7.18 in textbook):

$P_{\gamma_\Sigma}(\gamma)$ is chi-squared distribution: $P_{\gamma_\Sigma}(\gamma) = \frac{\gamma^{M-1}e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}^M(M-1)!}$, $\gamma \geq 0$

Then, $\overline{P_s}(\bar{\gamma}) = \int_0^\infty 2Q(\sqrt{2\gamma}\sin\left(\frac{\pi}{K}\right))\frac{\gamma^{M-1}e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}^M(M-1)!}d\gamma$

- For general cases (1<L<M):

$P_{\gamma_\Sigma}(\gamma) = M(M-1)\frac{1}{\bar{\gamma}}e^{-\frac{\gamma}{\bar{\gamma}}}\cdot[\frac{\gamma}{2\bar{\gamma}} + \sum_{k=1}^{M-2}\binom{M-2}{k}(-1)^k\frac{1}{k}(1-e^{-\frac{k\gamma}{2\bar{\gamma}}})]$ Then,

$$\overline{P_s}(\overline{\gamma_s}) = \int_0^\infty 2Q(\sqrt{2\gamma}\sin\left(\frac{\pi}{K}\right))M(M-1)\frac{1}{\bar{\gamma}}e^{-\frac{\gamma}{\bar{\gamma}}}\cdot[\frac{\gamma}{2\bar{\gamma}} + \sum_{k=1}^{M-2}\binom{M-2}{k}(-1)^k\frac{1}{k}(1-e^{-\frac{k\gamma}{2\bar{\gamma}}})]d\gamma$$

Let K=2(BPSK), M=3:

$$\overline{P_s}(\overline{\gamma_s}) = \int_0^\infty 2Q(\sqrt{2\gamma})\cdot 6\frac{1}{\bar{\gamma}}e^{-\frac{\gamma}{\bar{\gamma}}}\cdot(\frac{\gamma}{2\bar{\gamma}} + e^{-\frac{\gamma}{2\bar{\gamma}}} - 1)d\gamma$$

We plot P_s versus $\overline{\gamma_s}$ in i.i.d. Rayleigh fading, with K-PSK modulations, obtained by a numerical evaluation of the above integration.

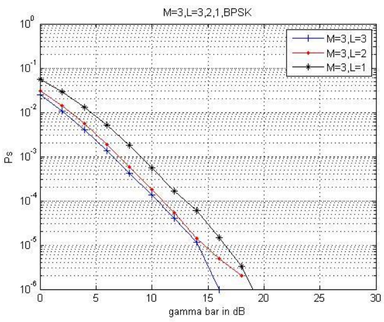
2.3 Monte Carlo simulation:

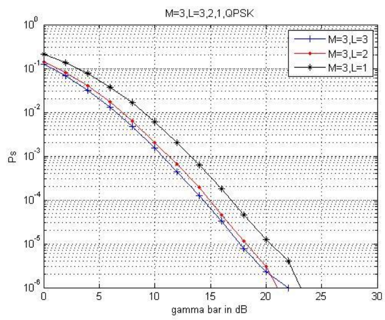
we take $\bar{\gamma}: 0dB \sim 30dB$, simulation length of 3,000,000.

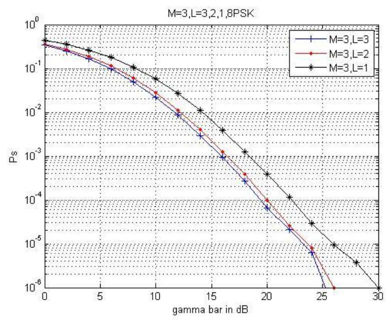
2.3.1 Independent channels:

First, for a given M (total number of antennas), we varied the value of L (number of antennas chosen). The symbol error probabilities (P_b) versus average SNR per symbol ($\bar{\gamma}$) are as follows:

- M=3, L=1,2,3:

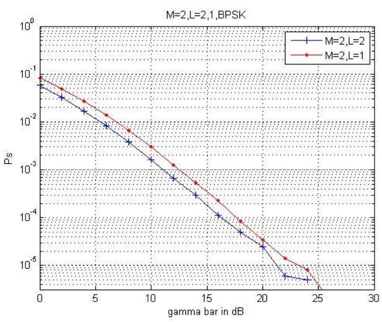


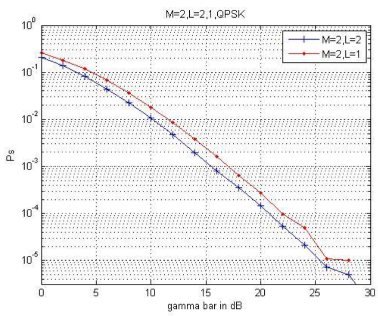


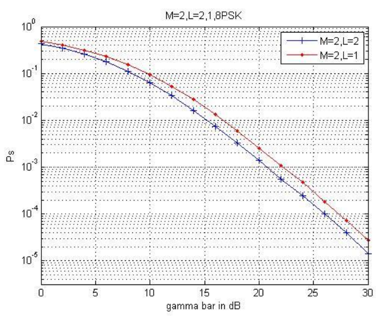


From the figures above, we see that there is dramatic improvement even with just two-branch selection combining: going from *L* = 1 to *L* = 2, for symbol error probability, there is an approximate 2 dB reduction in required SNR. However, going from *L* = 2 to *L* = 3, diversity results in an additional reduction of approximately 1 dB, which is smaller than 2dB. Clearly the power savings is most substantial going from *L* = 1 to *L* = 2.

- M=2, L=1,2:

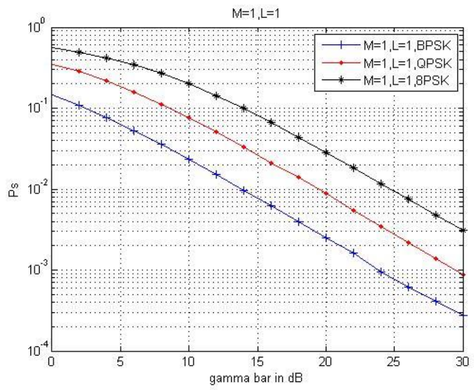






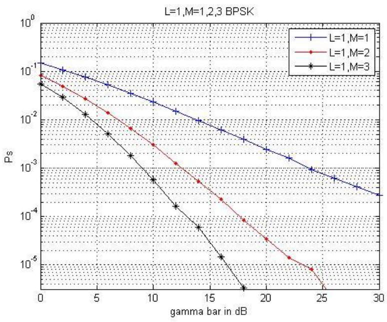
Comparing the diversity gain performance between M=3 (L=1,2,3) with the M=2 (L=1,2), diversity with 3 branches generally performances better (approximately 7 B) than that of 2 branches.

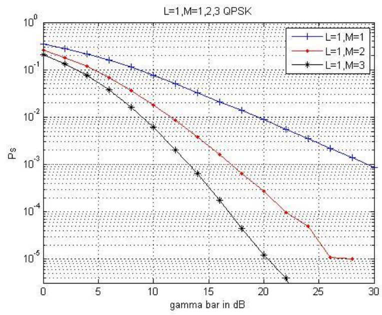
- M=1, L=1:

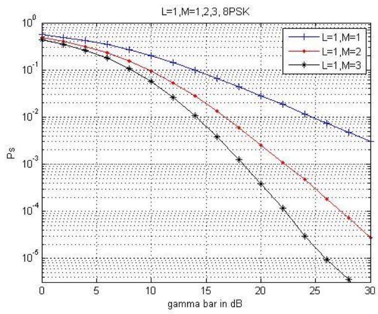


As we plot the no diversity case (M=1, L=1) for the BPSK, QPSK, 8PSK all in the same figure, it can be clearly seen that different modulation techniques result in about 7 B increase in required SNR, going from BPSK to QPSK and from QPSK to 8PSK. The higher modulation order (i.e. K in K-PSK), the higher requirement in SNR, in order to achieve the same symbol error probability.

- L=1, M=1,2,3:

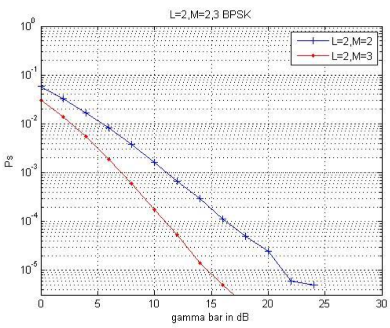


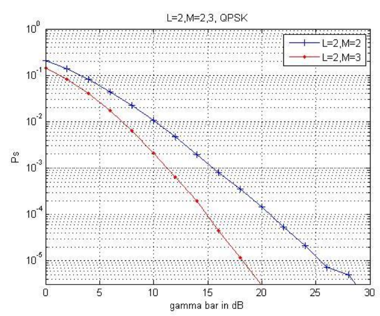


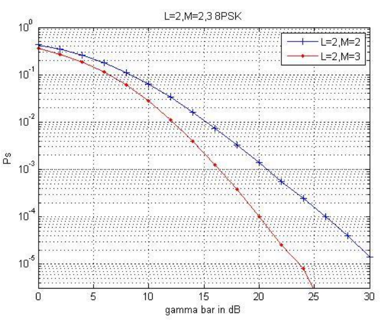


With a fixed number of selected antennas, the larger number of antennas we can choose from, the better performance in probability of errors we can get. For the P_b at 10^{-2}, easily seen there is 7 B decrease in required SNR going from M=1 to M=2, meanwhile, there is 2 B decrease in required SNR from M=2 to M=3.

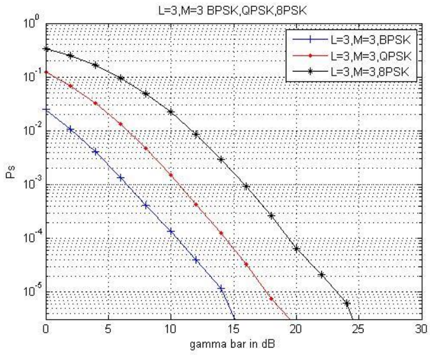
- L=2, M=2,3:

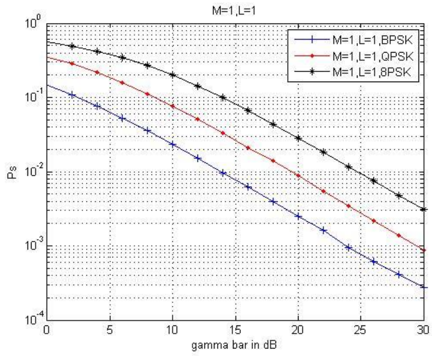






- L=3, M=3:



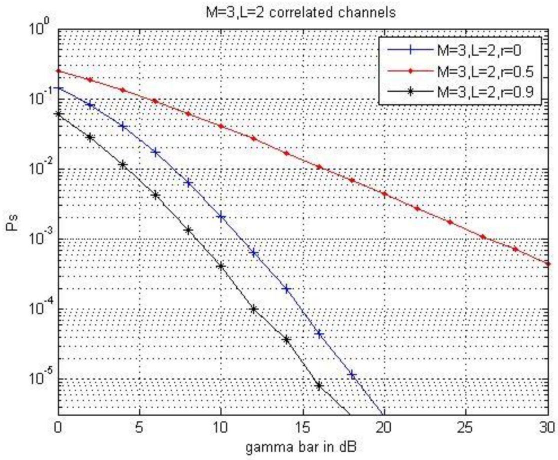


For the diversity with total 3 branches all selected, which is the same as Maximum Ratio Combining (MRC). Again we can see the performance differences regarding different modulation techniques. Moreover, here we take the above figure with M=1, L=1 to compare with M=3, L=3, we can see a great improvement of diversity.

2.3.2 correlated channels:

In the above discussions, we assume that there is no correlation between the branch amplitudes. If the correlation is nonzero, then there is a slight degradation in performance which is almost negligible for correlations below 0.5.

- M=3, L=2, r=0.9,0.5,0



3. Conclusion:

In Selection Combining (SC), the combiner outputs the signal on the branch with the highest SNR. The output from the combiner has an SNR equal to the maximum SNR of all the branches.

In maximal ratio combining (MRC), the output is a weighted sum of all branches.

In our General Selection Combining (GSC), we choose the first L antennas with highest SNRs, out of total M antennas, on each antenna the MRC technique is performed. Then we combine the L branches to reach an optimized combined output.

From the above analysis, we got an idea on how diversity techniques improve the performance, by evaluating probabilities on symbol error.