

**Prob1. Write a R function to return a  $100 \times (1-\alpha)\%$  confidence interval with the exact confidence level for any data set  $X$ , then use it to validate the results for two samples on p.55**

```
1 f=function(x){
2   l=length(x)
3   x=sort(x)
4   s=0
5   al=2^l
6   for(i in 0:l){
7     s=s+choose(l,i)
8     if(s*20>al){
9       return(c(x[i],x[l-i+1],(al-s*2+2*choose(l,i))/al))
10    }
11  }
12 }
13 x1=c(5.5 ,6.0 ,6.5 ,7.6 ,7.6 ,7.7 ,8.0 ,8.2 ,9.1 ,15.1)
14 x2=c(5.6 ,6.1 ,6.3 ,6.3 ,6.5 ,6.6 ,7.0 ,7.5 , 7.9, 8.0 ,8.0, 8.1, 8.1 ,8.2
    ,8.4 ,8.5 ,8.7 ,9.4,14.3, 26.0)
```

```
1 > f(x1)
2 [1] 6.0000000 9.1000000 0.9785156
3 > f(x2)
4 [1] 6.6000000 8.4000000 0.9586105
```

**3.2** In Comment 4 on Example 3.10 we asserted that the usual  $t$ -statistic could be used in place of  $S_+$  as the test statistic for the Pitman test because there was a one-to-one correspondence between the ordering of the two statistics. Establish that this is so. (Hint: show that the denominator of the  $t$ -statistic is invariant under all permutations of the signs of the deviations  $d_i$ .)

The raw data as scores indicate the variance occurring in the denominator of the  $t$  statistic is the same for all permutations of the signs attached to the  $x_i$ ; it follows therefore that the denominator is invariant under permutation.

**3.5** Establish that the permutation distribution of the Wilcoxon signed-rank statistic for testing the hypothesis  $H_0 : \theta = 6$ , given the observations 4,4,8,8,8,8 has a distribution equivalent to that for the sign test of the same hypothesis. Would this equivalence hold if the null hypothesis was changed to  $H_0 : \theta = 7$ ?

All deviations from 6 are either +2 or -2. Thus Wilcoxon test easily seen to be equivalent to sign test.

This equivalence will not hold when  $H_0 : \theta = 7$ , because that the signed deviations are of magnitudes 1 and 3

### 3. 8 The numbers of pages in the sample of 12 books given in Exercise 2.5 were

126 142 156 228 245 246 370 419 433 454 478 503

Use the Wilcoxon signed-rank test to test the hypothesis that the mean number of pages in the statistics books in the library from which the sample was taken is 400. Obtain a 95 percent confidence interval for the mean number of pages based on the Wilcoxon test and compare it with the interval obtained using a  $t$ -test under an assumption of normality.

$P = 0.13$  (two-tail).

95 percent Wilcoxon interval is (200.5, 443).

Normal theory interval is (227.4, 405.9)

```
1 > x<-c(126 , 142 , 156 , 228 , 245 , 246 , 370 , 419 , 433 , 454 , 478 ,
2 503)
3
4 Wilcoxon signed rank test
5
6 data: x
7 V = 19, p-value = 0.1294
8 alternative hypothesis: true location is not equal to 400
9 95 percent confidence interval:
10 200.5 433.0
11 sample estimates:
12 (pseudo)median
13 315.75
14 > t.test(x)
15
16 One Sample t-test
17
18 data: x
19 t = 7.8079, df = 11, p-value = 8.225e-06
20 alternative hypothesis: true mean is not equal to 0
21 95 percent confidence interval:
22 227.4005 405.9328
23 sample estimates:
24 mean of x
25 316.6667
```