Homework 6

Dec. 17, 2020

NOTE: Homework 6 is due next Thursday (Dec. 24, 2020). The questions started with * are for Exercise only, and you are not required to submit the answers.

1. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from binomial distribution $B(1, \theta)$.
   - Considering the test problem $H_0 : \theta \leq 0.01 \leftrightarrow \ H_1 : \theta > 0.01$, construct a test with level of significance $\alpha = 0.05$.
   - Determine the sample size n such that the type II error of the above test is no more than 0.05 at $\theta = 0.1$.
2. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from normal distribution $N(\mu, \sigma^2)$ where μ is unknown and σ is known.
   - For testing the hypothesis $H_0 : \mu \leq 0 \ \leftrightarrow \ H_1 : \mu > 0$, show that the sample size n can be determined to achieve a given level of significance α and given power $\pi(1)$.
   - What is the numerical value of n for $\alpha = 0.05, \pi(1) = 0.95$ when $\sigma = 1$?
3. The breaking powers of certain steel bars produced by processes A and B are independent r.v.'s distributed at Normal with possibly different means but the same variance. A random sample of size 25 is taken from bars produced by each one of the processes, and it is found that $\bar{x} = 60, s_x = 6, \bar{y} = 70, s_y = 5$. Test whether there is a difference between the two processes at the level of significance $\alpha = 0.05$.
4. An advertisement manager for a radio station claims that over $100p_0\%$ $(0 < p_0 < 1)$ of all young adults in the city listen to a weekend music program. To establish this conjecture, a random sample of size n is taken from among the target population and t of them listen to the weekend music program. Let $X_i = 1$ if the ith young adult listens to the program and $X_i = 0$ otherwise.
   (i) Decide a suitable probability model describing the number of young adults who listen to the weekend music program.
   (ii) For $p_0 = 5\%$, $n = 100$, $t = 20$, check whether the claim made the manager is supported or not (at level of significance $\alpha = 0.01$).
5. Two testers (A and B) analyzed the same product samples and obtained the following results:

tester	sample 1	sample 2	sample 3	sample 4	sample 5	sample 6	sample 7	sample 8
A	4.4	3.3	3.8	3.5	3.6	4.8	3.3	3.9
B	3.8	4.2	3.8	3.8	4.6	3.9	2.9	4.4

   Test whether there is a significant difference (in means) of the analysis results of testers A and B. Take $\alpha = 0.05$.
6. A coin, with probability θ of falling heads, is tossed independently 100 times and 60 heads are observed. At level of significance $\alpha = 0.01$:

(i) Use the likelihood ratio test in order to test the hypothesis $H_0 : \theta = 1/2 \leftrightarrow H_1 : \theta \neq 1/2$.

(ii) Employ the appropriate approximation to determine the critical value.

7. A medical researcher wishes to determine whether a pill has the undesirable side effect of reducing the blood pressure of the user. The study requires recording the initial blood pressure of n college-age women. After the use of the pill regularly for 6 months, their blood pressures are again recorded. With μ denoting the difference of blood pressure after the usage of the pill and before it, the claim is that $\mu < 0$.

   (i) Check this claim by testing the hypothesis $H_0 : \mu \geq 0 \leftrightarrow H_1 : \mu < 0$ at level of significance α, by using the likelihood ratio test.

   (ii) Carry out the test if $n = 90$ and $\alpha = 0.05$.

8. The diameters of certain cylindrical items produced by a machine are r.v.'s distributed as $N(\mu, 0.01)$. A sample of size 16 is taken and it is found that $\bar{x} = 2.48$ inches.

   (i) If the desired value for μ is 2.5 inches, formulate the appropriate testing hypothesis problem and carry out the likelihood ratio test at level of significance $\alpha = 0.05$.

   (ii) Determine the power of the test.

9. Let $X_i, i = 1, \cdots, 9$ and $Y_j, j = 1, \cdots, 10$ be independent r.v.'s from the distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively. Suppose that the observed values of the sample variance are $s_x^2 = 4$, $s_y^2 = 9$.

   (i) At level of significance $\alpha = 0.05$, test the hypothesis $H_0 : \sigma_1 = \sigma_2, \leftrightarrow H_1 : \sigma_1 \neq \sigma_2$ by using likelihood ratio test.

   (ii) Find an expression for the computation of the power of the test for $\sigma_1 = 2$ and $\sigma_2 = 3$.

10. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from uniform distribution $U(\theta, 1)$ where $\theta < 1$ is unknown.

   (i) At level of significance α, carry out the likelihood ratio test of the hypothesis

   $$H_0 : \theta \geq \theta_0 \longleftrightarrow H_1 : \theta < \theta_0,$$

   where $\theta_0 < 1$ is given.

   (ii) Determine the power function of the test.

11. Consider the data set "data.csv" in the previous homework. For simplicity, suppose that the dependent variable SalePrice (denoted as y_i) and the independent variables Gr_Liv_Area (denoted as x_i, and x_i is a fixed quantity, not random variable) follow a linear regression model with intercept, i.e.,

   $$y_i = \beta_0 + \beta_1 x_i + \epsilon_i.$$

   Testing the following hypotheses:

   (i) $H_0 : \beta_1 = 0 \longleftrightarrow H_1 : \beta_1 \neq 0$

   (ii) Let σ^2 be the variance of ϵ_i,

   $$H_0 : \sigma = 53000 \longleftrightarrow H_1 : \sigma \neq 53000.$$

*1. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from Normal distribution $N(\mu, \sigma^2)$ where μ is unknown and σ^2 is known. For testing the hypothesis $H_0 : \mu \geq \mu_0 \longleftrightarrow H_1 : \mu < \mu_0$. Show that the rejection region with level of significance α is given by

$$D_3 = \left\{ \boldsymbol{X} = (X_1, \cdots, X_n) : Z = \frac{\sqrt{n}(\bar{X} - \mu_0)}{\sigma} < -z_\alpha \right\}$$