刘程华 2018011687 计91

Homework: Verify for a double exponential (or Laplace) distribution, $ARE\left(\mathrm{Med}_n, \bar{X}_n\right)=2$ and $ARE\left(HL_n, \bar{X}_n\right)=1.5$

For Laplace

$$
\begin{gathered}
f_x(x)=\tfrac{\lambda}{2} e^{-\lambda|x-\theta|} \\
E(x)=\theta \\
\operatorname{Var}(x)=E\left[(x-\theta)^2\right]=2 \int_0^{+\infty} x^2 \tfrac{\lambda}{2} e^{-\lambda x} d x=x \int_0^{+\infty} x^2 e^{-\lambda x} d x \\
=\lambda\left(-\tfrac{1}{\lambda} x^2-\tfrac{1}{\lambda^2} 2 x-\tfrac{2}{\lambda^3}\right) e^{-\lambda x}\Big|_0^{+\infty}=\lambda\left[0-\left(-\tfrac{2}{\lambda^3}\right)\right]=\tfrac{2}{\lambda^2}
\end{gathered}
$$

By CLT,

$$
\begin{gathered}
\bar{X}_n \stackrel{d}{\rightarrow} N\left(\theta, \frac{2}{n \lambda^2}\right) \\
\mathrm{Med}_n \stackrel{d}{\rightarrow} N\left(\theta, \frac{1}{4[f(\theta)]^2 n}\right) \Rightarrow \mathrm{Med}_n \stackrel{d}{\rightarrow} N\left(\theta, \frac{1}{\lambda^2 n}\right)
\end{gathered}
$$

So

$$
ARE\left(\mathrm{Med}_{\mathrm{n}}, \bar{X}_n\right)=\frac{\frac{2}{n \lambda^2}}{\frac{1}{n \lambda^2}}=2
$$

We note

$$
\int_{-\infty}^{-\infty} f^2(x) d x=2 \int_0^{+\infty} \frac{\lambda^2}{4} e^{-2 \lambda x} d x=\frac{\lambda^2}{2} \times\left(-\frac{1}{2 \lambda} e^{-2 \lambda x}\Big|_0^{+\infty}\right)=\frac{\lambda}{4}
$$

Then

$$
HL_n \stackrel{d}{\rightarrow} N\left(\theta, \frac{1}{12\left[\int f^2(x) d x\right]^2 n}\right) \Rightarrow HL_n \stackrel{d}{\rightarrow} N\left(\theta, \frac{4}{3 n \lambda^2}\right)
$$

So

$$
ARE\left(HL_n, \bar{X}_n\right)=\frac{\frac{2}{n \lambda^2}}{\frac{4}{3 n \lambda^2}}=\frac{3}{2}
$$