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Prob 1. Prove that for the Multiplicative Congruential Generator (MCG)

$$X_{i+1} = aX_i \bmod m$$

is equivalent to

$$X_{i+1} = a^{i+1}X_0 \bmod m$$

proof

$$\begin{aligned} X_{i+1} &= (aX_i) \bmod m = (a \times (aX_{i-1}) \mod m) \bmod m = (a^2X_{i-1}) \bmod m \\ &= (a^3X_{i-2}) \bmod m = (a^4X_{i-3}) \bmod m = \cdots = (a^{i+1}X_0) \bmod m \end{aligned}$$

Q.E.D.

Prob 2. Analyze the following LCG:

$$X_{i+1} = (11X_i + 5) \bmod 16, X_0 = 1$$

What is the maximum possible period length for this generator?

The maximum possible period of an MCG is m=16

by Hull and Dobell,

$$\begin{gathered} (m, c) = 1 \\ 2|m \Rightarrow 2|a-1 \\ 4|m \Rightarrow 4|a-1 \end{gathered}$$

we let a=9, c=1,then:

```
> x<-c()
> i=1
> x[i]=1
> while(i<100){
+     x[i+1]=(x[i]*9+1)%%16
+     i=i+1
+ }
```

```
> x
  [1]  1 10 11  4  5 14 15  8  9  2  3 12 13  6  7  0
 [17]  1 10 11  4  5 14 15  8  9  2  3 12 13  6  7  0
 [33]  1 10 11  4  5 14 15  8  9  2  3 12 13  6  7  0
 [49]  1 10 11  4  5 14 15  8  9  2  3 12 13  6  7  0
 [65]  1 10 11  4  5 14 15  8  9  2  3 12 13  6  7  0
 [81]  1 10 11  4  5 14 15  8  9  2  3 12 13  6  7  0
 [97]  1 10 11  4
```

period length is 16.

Does this generator achieve the maximum possible period length? Justify your answer.

No, we find that the maximum possible period length for this generator is 8.

```
x<-c()
i=1
x[i]=1
while(i<100){
  x[i+1]=(x[i]*11+5)%%16
  i=i+1
}
```

```
> x
  [1]  1  0  5 12  9  8 13  4  1  0  5 12  9  8 13  4
 [17]  1  0  5 12  9  8 13  4  1  0  5 12  9  8 13  4
 [33]  1  0  5 12  9  8 13  4  1  0  5 12  9  8 13  4
 [49]  1  0  5 12  9  8 13  4  1  0  5 12  9  8 13  4
 [65]  1  0  5 12  9  8 13  4  1  0  5 12  9  8 13  4
 [81]  1  0  5 12  9  8 13  4  1  0  5 12  9  8 13  4
 [97]  1  0  5 12
```

Prob 3. Let X be discrete with $P(X=0)=0.6$ and $P(X=1)=0.4$ Use at least two methods to generate this random variable.

the first method

generate $T \sim U(0,1)$ if $T \leq 0.4$ we let $X=1$, else we let $X=0$.

the second method

generate $T_1, T_2 \sim_{iid} U(0,1)$ if $T_1 \leq 0.5$ and $T_2 \leq 0.8$ we let $X=1$, else we let $X=0$.