Homework 3

Nov. 26, 2020

NOTE: Homework 3 is due next Thursday (Dec. 3, 2020). The questions started with * are for Exercise only, and you are not required to submit the answers.

1. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from the distribution with p.d.f.

$$f(x; \mu, \sigma) = \frac{1}{\sigma} \exp\Big\{ - \frac{x-\mu}{\sigma} \Big\}, \quad x \geq \mu.$$

   - When μ is known, derive the moment estimator and MLE of σ;
   - When σ is known, derive the moment estimator and MLE of μ;
   - When both μ and σ are unknown, derive the moment estimators and MLEs of μ, σ and $P(X_1 \geq t)$ ($t > \mu$ and t is known).

2. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from uniform distribution $U(\theta/2, \theta)$, $0 < \theta < +\infty$.
   - Derive the MLE of θ;
   - Is the MLE unbiased? If not, find an unbiased estimate based on the MLE.
   - Is the MLE weakly consistent? Why?

3. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from Geometric distribution:

$$P(X_1 = i) = \theta(1-\theta)^{i-1}, \ i = 1, 2, \cdots, \ 0 < \theta < 1.$$

   Derive the UMVUE of θ^{-1} and θ.

4. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from normal distribution $N(\mu, \sigma^2)$, where both μ and σ^2 are unknown. Derive the UMVUE of (1) $\mu + \sigma^2$, and (2) μ^2/σ^2.

5. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from normal distribution $N(1, \sigma^2)$. Derive the UMVUE of σ.

6. Let $X_1, \cdots, X_m$ i.i.d. $\sim N(\mu, \sigma^2)$, $Y_1, \cdots, Y_n$ i.i.d. $\sim N(2\mu, \sigma^2)$, and suppose that X_i's and Y_j's are independent, derive the UMVUE of μ and σ^2.

7. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from normal distribution $N(0, \sigma^2)$, $\sigma^2 > 0$.
   - Derive the moment estimator and MLE of σ^2;
   - Derive the C-R lower bound for the variance of the unbiased estimator of σ^2;
   - Derive the UMVUE of σ^2.

8. Let $(X_1, Y_1), \cdots, (X_n, Y_n)$ i.i.d. drawn from a Bivariate Normal distribution $N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$.
   - Derive the MLEs of $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$;
   - Calculate the observed values of the above MLEs using the data in Homework 2.

*1. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from the distribution with p.d.f.

$$f(x;\theta) = \frac{1}{2\sigma}\exp\{-|x-a|/\sigma\},$$

where $\sigma > 0$, $-\infty < a < +\infty$. Find the MLE of a and σ.

*2. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from the Weibull distribution with p.d.f.

$$f(x) = \alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}}, \quad x > 0 \quad (\alpha, \beta > 0).$$

Suppose β is known, determine the MLE of α.