刘程华　计91　2018011687

Prob 1　ANSM 2.4

H_0: $\theta = 225$　　H_1: $\theta \neq 225$

```
> binom.test(3,12)

        Exact binomial test

data:  3 and 12
number of successes = 3, number of trials = 12, p-value = 0.146
```

$p = 0.146$ 在 $\alpha = 0.05$ 的水平下，没有理由拒绝 H_0

替代假设：中位数不大于 225

$X \sim B(12, 0.5)$

$P(X=0) \approx 0.0002$，$P(X=1) \approx 0.0030$，$P(X=2) \approx 0.0161$

$P(X=3) \approx 0.0537$

$P(X \leq 3) = \sum_{i=0}^{3} P(X=i) = 0.0730$　　双尾 $2P(X \leq 3) = 0.146 > 0.05$

$P(X \leq 2) = \sum_{i=0}^{2} P(X=i) = 0.0193$　　双尾 $2P(X \leq 2) = 0.0386 \leq 0.05$

所以　$P < 0.05$ 的检验临界区域 $\{0, 1, 2, 12, 11, 10\}$

相应的 $p = 0.0386$

Prob 2.　ANSM 2.5

```
> SIGN.test(X,
+           md = 225)

        One-sample Sign-Test

data:  X
s = 9, p-value = 0.146
alternative hypothesis: true median is not equal to 225
95 percent confidence interval:
 163.6582 451.7664
sample estimates:
median of x
       308

Achieved and Interpolated Confidence Intervals:

                  Conf.Level   L.E.pt   U.E.pt
Lower Achieved CI     0.8540 228.0000 433.0000
Interpolated CI       0.9500 163.6582 451.7664
Upper Achieved CI     0.9614 156.0000 454.0000
```

由 Prob 1 中的计算 $P(X \leq 2)$ 是满足的，我们所找的研区间为第3到第10，即 (156, 454)

用程序验证如左图

Prob 3 ANSM 2.6

由Prob 1中计算知，$(-\infty, 454)$有 0.980的覆盖率，

$(-\infty, 433)$ 有 0.927的覆盖率 小于 0.95

时员数不能为负 实际区间为 $[0, 454)$

Prob 4

$$F_{(i)}(t) = P(X_{(i)} \le t)$$
$$= \sum_{k=i}^{n} C_n^k F(t)^k (1-F(t))^{n-k}$$

对 $F_{(i)}(t)$ 关于 t 求导得

$$f_{(i)}(t) = \left[\sum_{k=i}^{n} C_n^k \cdot k \cdot F(t)^{k-1} (1-F(t))^{n-k} - \sum_{k=i}^{n-1} C_n^k F(t)^k (n-k)(1-F(t))^{n-k-1}\right] f(t)$$

注意到：$C_n^k \cdot k - C_n^{k+1}(n-(k+1)) = 0$，$k = i+1, i+2 \cdots n$，故第一个和式中第 $i+1$ 到 n 项与后一和式对应抵消

$$f_{(i)}(t) = C_n^i F(t)^{i-1} (1-F(t))^{n-i} f(t)$$ 得证

```r
binom.test(3,12)
X<-c(126,142,156,228,245,246,370,419,433,454,478,503)
if(!require(BSDA)){install.packages("BSDA")}
if(!require(DescTools)){install.packages("DescTools")}
library(BSDA)
SIGN.test(X,
          md = 225)
```