## Homework: Verify for a double exponential (or Laplace) distribution, $ARE\left(\mathrm{Med}_n, \bar{X}_n\right) = 2$ and $ARE\left(HL_n, \bar{X}_n\right) = 1.5$

For Laplace

$$egin{aligned} f_x(x) &= rac{\lambda}{2}e^{-\lambda|x- heta|} \ E(x) &= heta \ \mathrm{Var}(x) = E\left[(x- heta)^2
ight] = 2\int_0^{+\infty} x^2rac{\lambda}{2}e^{-\lambda x}dx = x\int_0^{+\infty} x^2e^{-\lambda x}dx \ &= \lambda\left(-rac{1}{\lambda}x^2 - rac{1}{\lambda^2}2x - rac{2}{\lambda^3}
ight)e^{-\lambda x}igg|_0^{+\infty} = \lambda\left[0 - \left(-rac{2}{\lambda^3}
ight)
ight] = rac{2}{\lambda^2} \end{aligned}$$

By CLT,

$$ar{X}_n \stackrel{d}{ o} N\left( heta, rac{2}{n\lambda^2}
ight) \ \mathrm{Med}_n \stackrel{d}{ o} N\left( heta, rac{1}{4[f( heta)]^2 n}
ight) \Rightarrow \mathrm{Med}_n \stackrel{d}{ o} N\left( heta, rac{1}{\lambda^2 n}
ight)$$

So

$$ARE\left(\operatorname{Med}_{\operatorname{n}},ar{X}_{n}
ight)=rac{rac{2}{n\lambda^{2}}}{rac{1}{n\lambda^{2}}}=2$$

We note

$$\int_{-\infty}^{-\infty} f^2(x) dx = 2 \int_0^{+\infty} \frac{\lambda^2}{4} e^{-2\lambda x} dx = \frac{\lambda^2}{2} \times \left( -\frac{1}{2\lambda} e^{-2\lambda x} \Big|_0^{+\infty} \right) = \frac{\lambda}{4}$$

Then

$$HL_{n}\stackrel{d}{
ightarrow}N\left( heta,rac{1}{12\left[\int f^{2}(x)dx
ight]^{2}n}
ight)\Rightarrow HL_{n}\stackrel{d}{
ightarrow}N\left( heta,rac{4}{3n\lambda^{2}}
ight)$$

So

$$ARE\left(HL_{n},ar{X}_{n}
ight)=rac{rac{2}{n\lambda^{2}}}{rac{4}{3n\lambda^{2}}}=rac{3}{2}$$