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Prob 1. Using the randomized decision rules on p. 80, make exact $\gamma = 0.90$ this time and compare your results with that of exact $\alpha = 0.05$

Start from the smallest n and k_γ:

make exact $\gamma = 0.90$:

$$\sum_{k \geq k_\gamma} \binom{n}{k} 0.8413^k 0.1587^{n-k} + p \binom{n}{k_\alpha - 1} 0.8413^{k_\gamma - 1} 0.1587^{n-k_\gamma+1} = 0.90$$

the probability of rejection p :

$$\sum_{k \geq k_\gamma} \binom{n}{k} 0.5^n + p \binom{n}{k_\gamma - 1} 0.5^n$$

Increase k_γ and n until the probability of rejection p not more than 0.05

```
[1] "n=16 ka=12 p=0.00907666311316642 alpha=0.0384063720703125"
[1] "n=18 ka=13 p=0.158098277754089 alpha=0.048126220703125"
[1] "n=19 ka=14 p=0.0747484243798748 alpha=0.0317840576171875"
[1] "n=20 ka=15 p=0.0256087805102992 alpha=0.0206947326660156"
```

Compared results with that of exact $\alpha = 0.05$ we could find that the α calculated based on the $\gamma = 0.90$ standard will be larger.This is because the following inequality exists：

$$0.8413^{k-1} 0.1587^{n-k+1} > 0.5^n \quad when\ k > \frac{n}{2}$$

Code:

```
theta <- 1-pnorm(-1) #0.8413
error <- function(n, ka, theta = .5) {
  1 - pbinom(q = ka-1, size = n, p = theta) }
for (n in 10:20){
  for(ka in round((n/2)+1):n){
    ran <- (error(n = n, ka = ka,theta=theta)-0.9)/(1-theta)^(n-ka+1)
    p<-ran/choose(n=n ,k=ka-1)
    if(p<0|p>1){
      ka=ka+1
    }else{
      al<-error(n=n,ka=ka)+ran*0.05^n
      if(al>0.05| al < 0){
        ka=ka+1
      }else{
        print(paste0('n=',n,' ka=',ka,
                   ' p=',p, ' alpha=',al))
      }
      }
```

```
    }
}
```

Prob 2. Finish the exercises on p.88

On ANSM p.40: " If our sample comes from the double exponential distribution, which has much longer tails than the normal, the Pitman efficiency of the sign test relative to the t -test is 2 "

Verify this conclusion where the pdf of a double exponential distribution is

$$f_X(x) = \frac{\lambda}{2} e^{-\lambda|x-\theta|}$$

How about X following a uniform distribution $U(\theta - 1/2, \theta + 1/2)$?

(1)

sign test : $\quad H_0 : \theta = 0 \quad \text{vs} \quad H_1 : \theta > 0$

θ is the median of the population F_X

$E\left(T_n^{(1)}\right) = np \quad \operatorname{Var}\left(T_n^{(1)}\right) = np(1-p)$

$p = P(x > 0) = 1 - F(-\theta)$

$\frac{dp}{d\theta}|_{\theta=0} = f(-\theta)|_{\theta=0} = f(0)$

$\left.\frac{dE\left(T_n^{(1)}\right)}{d\theta}\right|_{\theta=0} = nf(0) = \frac{n\lambda}{2}$

$\operatorname{Var}\left(T_n^{(1)}\right)\Big|_{\theta=0} = n/4$

So, $e\left(T_n^{(1)}\right) = \frac{\left[dE\left(T_n^{(1)}\right)/d\theta|_{\theta=0}\right]^2}{Var\left(T_n^{(1)}\right)\Big|_{\theta=0}} = 4nf(0)^2 = 4n \times \frac{\lambda^2}{4} = n\lambda^2$

t test : $\quad H_0 : \theta = 0 \quad \text{vs} \quad H_1 : \theta > 0$

For a single random sample of size n from any continuous population F_X with mean θ and variance σ^2, the t test statistic:

$$T_n^{(2)} = \frac{\sqrt{n}\bar{X}_n}{S_n} = \left[\frac{\sqrt{n}\left(\bar{X}_n - \theta\right)}{\sigma} + \frac{\sqrt{n}\theta}{\sigma}\right]\frac{\sigma}{S_n}$$

$\sigma^2 = \frac{2}{\lambda^2}$

$\lim_{n\to+\infty} E\left(T_n^{(2)}\right) = \frac{\sqrt{n}\theta}{\sigma}$

$\operatorname{Var}\left(T_n^{(2)}\right)\Big|_{\theta=0} = \frac{n\operatorname{Var}(\bar{x}_n)}{\sigma^2} = 1$

$$\left.\frac{dE\left(T_n^{(2)}\right)}{d\theta}\right|_{\theta=0}=\frac{\sqrt{n}}{\sigma}$$

So, $e\left(T_n^{(2)}\right)=\frac{\left[dE\left(T_n^{(2)}\right)/d\theta|_{\theta=0}\right]^2}{Var\left(T_n^{(2)}\right)\Big|_{\theta=0}}=\frac{n}{(\sigma^2)}/1=\frac{n}{\frac{2}{\lambda^2}}=\frac{n^2\lambda}{2}$

$$ARE\left(T^{(1)},T^{(2)}\right)=\frac{e\left(T_n^{(1)}\right)}{e\left(T_n^{(2)}\right)}=2$$

(2)

sign test : $\quad H_0:\theta=0 \quad \text{vs} \quad H_1:\theta>0$

θ is the median of the population F_X

$$\left.\frac{dE\left(T_n^{(1)}\right)}{d\theta}\right|_{\theta=0}=nf(0)=n$$

$$\mathrm{Var}\left(T_n^{(1)}\right)\Big|_{\theta=0}=n/12$$

So, $e\left(T_n^{(1)}\right)=\frac{\left[dE\left(T_n^{(1)}\right)/d\theta|_{\theta=0}\right]^2}{Var\left(T_n^{(1)}\right)\Big|_{\theta=0}}=12n$

t test : $\quad H_0:\theta=0 \quad \text{vs} \quad H_1:\theta>0$

$\lim_{n\to+\infty}E\left(T_n^{(2)}\right)=\frac{\sqrt{n}\theta}{\sigma}$

$$\mathrm{Var}\left(T_n^{(2)}\right)\Big|_{\theta=0}=\frac{n\,\mathrm{Var}(\bar{x}_n)}{\sigma^2}=1$$

$$\left.\frac{dE\left(T_n^{(2)}\right)}{d\theta}\right|_{\theta=0}=\frac{\sqrt{n}}{\sigma}$$

So, $e\left(T_n^{(2)}\right)=\frac{\left[dE\left(T_n^{(2)}\right)/d\theta|_{\theta=0}\right]^2}{Var\left(T_n^{(2)}\right)\Big|_{\theta=0}}=\frac{n}{(\sigma^2)}/1=12n$

$$ARE\left(T^{(1)},T^{(2)}\right)=\frac{e\left(T_n^{(1)}\right)}{e\left(T_n^{(2)}\right)}=1$$