# 1 证明中心化协方差矩阵公式

假设为 Z 为去中心化后的二维矩阵 $\mathrm{Z}^T = \begin{pmatrix} x_1 & \cdots & x_m \\ y_1 & \cdots & y_m \end{pmatrix}$

$\frac{1}{m}\ \mathrm{Z}^T Z = \frac{1}{m} \begin{pmatrix} \sum_{i=1}^{m} x_i^2 & \sum_{i=1}^{m} x_i y_i \\ \sum_{i=1}^{m} y_i x_i & \sum_{i=1}^{m} y_i^2 \end{pmatrix}$

$\mathrm{D} = \begin{pmatrix} cov(X,X) & cov(X,Y) \\ cov(Y,X) & cov(Y,Y) \end{pmatrix} = \begin{pmatrix} \frac{1}{m-1}\sum_{i=1}^{m} x_i^2 & \frac{1}{m-1}\sum_{i=1}^{m} x_i y_i \\ \frac{1}{m-1}\sum_{i=1}^{m} y_i x_i & \frac{1}{m-1}\sum_{i=1}^{m} y_i^2 \end{pmatrix}$

样本足够大的时候，m = m-1, $\mathrm{D} = \frac{1}{m}\ \mathrm{Z}^T Z$