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National Cheng Kung University

數值方法 HW3 E94111114 劉家瑋

$$1. f(0.75) = \cos(0.750) = 0.7317$$

$$f(0.698) = 0.7661$$

$$f(0.733) = 0.7432$$

$$f(0.768) = 0.7193$$

$$f(0.803) = 0.6946$$

degree 1:

$$f(x) = 0.7193 \frac{x-0.733}{0.768-0.733} + 0.7432 \frac{x-0.768}{0.733-0.768}$$

$$f(0.75) \approx 0.73169$$

$$|f(x) - p_1(x)| \leq \frac{M}{2!} |(x-x_0)(x-x_1)|$$

degree 2:

$$\text{error bound} = \frac{1}{2} |(0.75-0.768)(0.75-0.733)| = 1.53 \times 10^{-4}$$

$$f(x) = 0.7661 \times \frac{(x-0.733)(x-0.768)}{(0.698-0.733)(0.698-0.768)}$$

$$+ 0.7432 \times \frac{(x-0.698)(x-0.768)}{(0.733-0.698)(0.733-0.768)} + 0.7193 \times \frac{(x-0.698)(x-0.733)}{(0.768-0.698)(0.768-0.733)}$$

$$f(0.75) \approx 0.73172$$

$$|f(x) - p_2(x)| \leq \frac{M}{3!} |(x-x_0)(x-x_1)(x-x_2)|$$

$$\text{error bound} = \frac{1}{6} |(0.75-0.698)(0.75-0.733)(0.75-0.768)| = 2.652 \times 10^{-6}$$

⇒ 續 (next page)

degree 3:

$$f(x) = 0.7661 \cdot \frac{(x-0.733)(x-0.768)(x-0.803)}{(0.698-0.733)(0.698-0.768)(0.698-0.803)} + 0.7432 \cdot \frac{(x-0.698)(x-0.768)(x-0.803)}{(0.733-0.698)(0.733-0.768)(0.733-0.803)} \\ + 0.7193 \cdot \frac{(x-0.698)(x-0.733)(x-0.803)}{(0.768-0.698)(0.768-0.733)(0.768-0.803)} + 0.6946 \cdot \frac{(x-0.698)(x-0.733)(x-0.768)}{(0.803-0.698)(0.803-0.733)(0.803-0.768)}$$

$$f(0.75) \approx 0.73170$$

$$|f(x) - p_3(x)| \leq \frac{M}{4!} |(x-x_0)(x-x_1)(x-x_2)(x-x_3)|$$

$$\text{error bound} = \frac{1}{24} |(0.75-0.698)(0.75-0.733)(0.75-0.768)(0.75-0.803)| = 3.51 \times 10^{-8}$$

degree 4:

∴ only 4 個點, degree 4 需要 5 個點



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$$f(x) = x - e^{-x} = 0 = y$$

2. $f(0.3) = 0.3 - e^{-0.3} = 0.3 - 0.740818 = -0.440818$
 $f(0.4) = 0.4 - e^{-0.4} = 0.4 - 0.670320 = -0.270320$
 $f(0.5) = 0.5 - e^{-0.5} = 0.5 - 0.606531 = -0.106531$
 $f(\underbrace{0.6}_{\text{output}}) = 0.6 - e^{-0.6} = 0.6 - 0.548812 = 0.051188$
input

$$f^{-1}(0) = P(0)$$

$f^{-1}(y)$ 建立 Lagrange $P(y)$

$$\begin{aligned} P(0) &= 0.3 \times \frac{(0 + 0.270320)(0 + 0.106531)(0 - 0.051188)}{(-0.440818 + 0.270320)(-0.440818 + 0.106531)(-0.440818 - 0.051188)} \\ &+ 0.4 \times \frac{(0 + 0.440818)(0 + 0.106531)(0 - 0.051188)}{(-0.270320 + 0.440818)(-0.270320 + 0.106531)(-0.270320 - 0.051188)} \\ &+ 0.5 \times \frac{(0 + 0.440818)(0 + 0.270320)(0 - 0.051188)}{(-0.106531 + 0.440818)(-0.106531 + 0.270320)(-0.106531 - 0.051188)} \\ &+ 0.6 \times \frac{(0 + 0.440818)(0 + 0.270320)(0 + 0.106531)}{(0.051188 + 0.440818)(0.051188 + 0.270320)(0.051188 + 0.106531)} \\ &= 0.567143 = x^* \end{aligned}$$



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3.

	T	t_0	t_1	t_2	t_3	t_4	
		0	3	5	8	13	(s)
$f(t)$	D	0	200	375	620	990	(feet)
$f'(t)$	V	75	77	80	74	72	(feet per second)

$f(t)$	1 st	2 nd	3 rd	4 th	5 th	6 th	7 th	8 th
$t_0 z_0$	0							
$t_0 z_1$	0	75						
$t_1 z_2$	200	66.6	-2.7					
$t_1 z_3$	200	77	3.4	2.074				
$t_2 z_4$	375	87.5	5.25	-3.7	-0.108			
$t_2 z_5$	375	80	-3.75	-4.5	-0.972	-0.135		
$t_3 z_6$	620	81.6	0.5	0.861	0.25	0.0476851852		
$t_3 z_7$	620	74	-2.5	-1.037	1.072	-0.0682407		
$t_4 z_8$	990	74	0	0.3194	-0.41	0.0345289352		
$t_4 z_9$	990	72	-0.4	-0.08	-0.719	-0.0274363426	-0.0082355324	

$$\frac{200-0}{3-0} = 66.6$$

$$\frac{375-200}{5-3} = 87.5$$

$$\frac{620-375}{8-5} = 81.6$$

$$\frac{990-620}{13-8} = 74$$

(a) $D(10) = 768.96$ feet ($t=10$ 秒時)

$V(10) = 74.64$ ft/s

(b) 第一次超速 $V = 80.69$ ft/s $\Rightarrow t = 3.15$ s

(c) $V_{\max} = 92.04$ ft/s