Numerical Methods Homework 4

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1. Determine the values $\int_1^2 e^x \sin(4x)dx$ with $h = 0.1$ by

a. Use the composite trapezoidal rule

b. Use the composite Simpsons' method

c. Use the composite midpoint rule

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(a) Composite Trapezoidal Rule: 0.3961475922149011
(b) Composite Simpson's Rule:   0.38566359602374456
(c) Composite Midpoint Rule:    0.36469560364143166
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2. Approximate $\int_1^{1.5} x^2 \ln x dx$ using Gaussian Quadrature with $n = 3$ and $n = 4$. Then compare the result to the exact value of the integral.

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Gaussian Quadrature (n=3): 0.19225937725687903
Absolute Error:            1.9524082989219593e-08
Gaussian Quadrature (n=4): 0.1922593578048632
Absolute Error:            7.206715779695116e-11
Exact Integral Value:      0.19225935773279604
```

3. Approximate $\int_0^{\pi/4} \int_{\sin x}^{\cos x} (2y \sin x + \cos^2 x)dydx$ using

a. Simpson's rule for $n = 4$ and $m = 4$

b. Gaussian Quadrature, $n = 3$ and $m = 3$

c. Compare these results with the exact value.

```
(a) Simpson's Rule (n=4, m=4):        0.5119875440121252
(b) Gaussian Quadrature (n=3, m=3):   0.5118655399452959
(c) The Exact Value:                  0.5118446353109126
    Error of Simpson's Rule:          0.00014290870121258514
    Error of Gaussian Quadrature:     2.0904634383311915e-05
```

4. Use the composite Simpson's rule and $n = 4$ to approximate the improper integral a) $\int_0^1 x^{-1/4} \sin x dx$, b) $\int_1^\infty x^{-4} \sin x dx$ by use the transform $t = x^{-1}$

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Approximate value of (a): 0.5259288092
Approximate value of (b): 0.2744816127
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