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CS 512

(see next page for work)

A

1) $\begin{bmatrix} 3 \cdot 2 + 2 \cdot 0 \\ 3 \cdot -1 + 2 \cdot 3 \\ 3 \cdot 4 + 2 \cdot 5 \end{bmatrix} = \boxed{\begin{bmatrix} 6 \\ 3 \\ 22 \end{bmatrix}}$

2) $\|p\| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{21}$

$$\boxed{\hat{p} = \begin{bmatrix} \frac{2}{\sqrt{21}} \\ \frac{-1}{\sqrt{21}} \\ \frac{4}{\sqrt{21}} \end{bmatrix}}$$

3) $\|p\| = \sqrt{21}$ (see above)

Y axis unit vector $\hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$p \cdot \hat{j} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = -1$

$\cos\theta = \frac{p \cdot \hat{j}}{\|p\|} = \frac{-1}{\sqrt{21}}$ $\boxed{\theta = \cos^{-1}\left(\frac{-1}{\sqrt{21}}\right)}$

4) $\begin{bmatrix} \frac{2}{\sqrt{21}} \\ \frac{-1}{\sqrt{21}} \\ \frac{4}{\sqrt{21}} \end{bmatrix}$

5) $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|}$

$\cos\theta = \frac{p \cdot q}{\|p\| \cdot \|q\|} = \frac{2 \cdot 0 + -1 \cdot 3 + 4 \cdot 5}{\sqrt{21} \cdot \sqrt{34}} = \frac{17}{\sqrt{714}}$

6) $p \cdot q = q \cdot p = 2 \cdot 0 + -1 \cdot 3 + 4 \cdot 5 = \boxed{17}$

7) angle formula: $\cos\theta = \frac{p \cdot q}{\|p\| \cdot \|q\|} = \frac{17}{\sqrt{714}}$

$\theta = \cos^{-1}\left(\frac{17}{\sqrt{714}}\right)$

$p \cdot q = \|p\|\|q\| \cos\theta$

$= \sqrt{714} \cdot \cancel{\cos}\left(\cos^{-1}\left(\frac{17}{\sqrt{714}}\right)\right) = \boxed{17}$

8) Scalar projection q onto $\hat{p}$

$$q \cdot \hat{p} = \frac{q \cdot p}{\|p\|} = \boxed{\frac{17}{\sqrt{21}}}$$

9) $v \cdot p = 0$

$$2 \cdot v_x + -1 \cdot v_y + 4 \cdot v_z = 0$$

$$2 \cdot -1 + -1 \cdot 2 + 4 \cdot 1 = 0$$

$$\boxed{v = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}}$$

10) $p \times q = \begin{vmatrix} i & j & k \\ 2 & -1 & 4 \\ 0 & 3 & 5 \end{vmatrix} = i\big((-1)(5) - (4)(3)\big) - j\big((2)(5) - (4)(0)\big) + k\big((2)(3) - (-1)(0)\big)$

$$p \times q = \boxed{= \begin{bmatrix} -17 \\ -10 \\ 6 \end{bmatrix}} \quad \boxed{q \times p = -(p \times q) = \begin{bmatrix} 17 \\ 10 \\ -6 \end{bmatrix}}$$

11) $p \times q$ is perpendicular to p and q

$$\boxed{\begin{bmatrix} -17 \\ -10 \\ 6 \end{bmatrix}}$$

12) $c_1 \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ -1 & 3 & -2 & 0 \\ 4 & 5 & 2 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 \\ -1 & 3 & -2 & 0 \\ 4 & 5 & 2 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 3 & -\frac{3}{2} & 0 \\ 4 & 5 & 2 & 0 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 3 & -\frac{3}{2} & 0 \\ 0 & 5 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$c_1 = c_2 = c_3 = 0$$

$\boxed{\text{Thus linearly independent}}$

13) $p^T q = p \cdot q = \frac{17}{\sqrt{21}}$

$$pq^T = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \begin{bmatrix} 0 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 2\cdot 0 & 2\cdot 3 & 2\cdot 5 \\ -1\cdot 0 & -1\cdot 3 & -1\cdot 5 \\ 4\cdot 0 & 4\cdot 3 & 4\cdot 5 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 6 & 10 \\ 0 & -3 & -5 \\ 0 & 12 & 20 \end{bmatrix}}$$

Part B

1) $X + 2Y$

$$\begin{bmatrix} 2+8 & 1-2 & 0+4 \\ -1+6 & 3+0 & 4-6 \\ 3+2 & 2+4 & -2+2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 10 & -1 & 4 \\ 5 & 3 & -2 \\ 5 & 6 & 0 \end{bmatrix}}$$

2) $XY = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{bmatrix} \times \begin{bmatrix} 4 & -1 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2\cdot 4 + 1\cdot 3 + 0\cdot 1 & 2\cdot -1 \quad 1\cdot 0 \quad 0\cdot 2 & \cdots \\ \vdots & \ddots & \\ \vdots & & \end{bmatrix}$$

$$\boxed{XY = \begin{bmatrix} 11 & -2 & 1 \\ 9 & 9 & -7 \\ 16 & -7 & -2 \end{bmatrix}}$$

$YX = \begin{bmatrix} 4 & -1 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 4\cdot 2 + -1\cdot -1 + 2\cdot 3 & 4\cdot 1 + -1\cdot 3 + 2\cdot 2 & \cdots \\ 3\cdot 2 + 0\cdot -1 + -3\cdot 3 & \ddots & \\ \vdots & & \end{bmatrix}$$

$$\boxed{YX = \begin{bmatrix} 15 & 5 & -8 \\ -3 & -3 & 6 \\ 3 & 9 & 6 \end{bmatrix}}$$

3) $\underline{(XY)^T} = \begin{bmatrix} 11 & -2 & 1 \\ 9 & 9 & -7 \\ 16 & -7 & -2 \end{bmatrix}^T$

$$\boxed{= \begin{bmatrix} 11 & 9 & 16 \\ -2 & 9 & -7 \\ 1 & -7 & -2 \end{bmatrix}}$$

$\underline{Y^T X^T} = \begin{bmatrix} 4 & 3 & 1 \\ -1 & 0 & 2 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & 2 \\ 0 & 4 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 4\cdot 2 + 3\cdot 1 + 1\cdot 0 & 4\cdot -1 + 3\cdot 3 + 1\cdot 4 & \cdots \\ \vdots & \ddots & \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 11 & 9 & 16 \\ -2 & 9 & -7 \\ 1 & -7 & -2 \end{bmatrix}}$$

4) $\det X = 2\big((3)(-2) - (4)(2)\big) - 1\big((-1)(2) - (3)(4)\big) + 0$

$= 2(-14) - (-10)$

$= -28 + 10$

$\boxed{|X| = -18}$

$\det Z = 2(4\cdot 2 - 1\cdot 5) + -1(1\cdot 1 - 3\cdot 4)$

$6 + 11$

$\boxed{|Z| = 17}$

$\boxed{17 \neq 0 \quad 18 \neq 0 \quad \text{No linear dependency}}$

5)

X: $R_1 R_2 = 1$

$R_1 R_3 = 8$ Not ortho

$R_2 R_3 = -5$

Y: $R_1 R_2 = 6$

$R_1 R_3 = 4$ Not ortho

$R_2 R_3 = 0$

Z: $R_1 R_2 = -3$

$R_1 R_3 = 4$ Not ortho

$R_2 R_3 = 17$

None of these matrixes are orthogonal

6)

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$$X^{-1} = \frac{1}{|X|} \operatorname{adj}(X)$$

$$= \frac{1}{-18} \operatorname{adj}(X)$$

$$= \begin{bmatrix} \frac{14}{18} & \frac{2}{-18} & -\frac{4}{18} \\ \frac{10}{-18} & \frac{-4}{-18} & \frac{-8}{-18} \\ \frac{-11}{-18} & \frac{-1}{-18} & \frac{7}{-18} \end{bmatrix}$$

* using cofactor calculator and transposed

$$\operatorname{adj}(X) = \begin{bmatrix} -14 & 2 & 4 \\ 10 & -4 & -8 \\ -11 & -1 & 7 \end{bmatrix}$$

$$Y^{-1} = \frac{1}{42} \operatorname{adj}(Y)$$

$$= \begin{bmatrix} \frac{6}{42} & 5/42 & 3/42 \\ -6/42 & 2/42 & 18/42 \\ 6/42 & -9/42 & 3/42 \end{bmatrix}$$

7) $\begin{bmatrix} \frac{3}{17} & -\frac{1}{17} & \frac{4}{17} \\ \frac{13}{17} & \frac{2}{17} & -\frac{11}{17} \\ -\frac{11}{17} & -\frac{2}{7} & \frac{8}{17} \end{bmatrix}$

$|Z| = 17 \neq 0$

$\boxed{\text{linearly independent}}$

8) $\begin{bmatrix} 2\cdot -1 + 1\cdot 4 + 0\cdot 0 \\ -1\cdot -1 + 3\cdot 4 + 4\cdot 0 \\ 3\cdot -1 + 2\cdot 4 + -2\cdot 0 \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ 13 \\ 5 \end{bmatrix}}$

9) Scalar projection

$\frac{a\cdot b}{\|b\|}$

$\|s\| = \sqrt{-1^2+4^2+0^2} = \sqrt{17}$

$\hat{s} = \begin{bmatrix} \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \\ 0 \end{bmatrix}$

X: $r_1 = [2\ 1\ 0]$

$\frac{r_1 \cdot \hat{s}}{\|\hat{s}\|} = \frac{2\cdot -1}{\sqrt{17}} + 1\cdot\frac{4}{\sqrt{17}} + 0 = \boxed{\frac{2}{\sqrt{17}}}$

$r_2 = [-1\ 3\ 4]$

$= \frac{1}{\sqrt{17}} + \frac{12}{\sqrt{17}} + 0 = \boxed{\frac{13}{\sqrt{17}}}$

$r_3 = [3\ 2\ -2]$

$= 3\cdot\frac{-1}{\sqrt{17}} + 2\cdot\frac{4}{\sqrt{17}} + 0 = \boxed{\frac{5}{\sqrt{17}}}$

10) vector Projection

$$\left(\frac{a \cdot b}{\|b\|^2}\right) b \rightarrow \left(\frac{r \cdot \hat{s}}{\|\hat{s}\|^2}\right) \hat{s} \rightarrow (r \cdot \hat{s}) \hat{s}$$

x: $r_1 = [2 \ 1 \ 0]$

$r_1 \cdot \hat{s} = \frac{2}{\sqrt{17}}$

$$(r_1 \cdot \hat{s}) \hat{s} = \frac{2}{\sqrt{17}} \begin{bmatrix} \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{2}{17} \\ \frac{8}{17} \\ 0 \end{bmatrix}}$$

$$(r_2 \cdot \hat{s}) \hat{s} = \frac{13}{\sqrt{17}} \begin{bmatrix} \frac{-1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{-13}{17} \\ \frac{52}{17} \\ 0 \end{bmatrix}}$$

$$(r_3 \cdot \hat{s}) \hat{s} = \frac{5}{\sqrt{17}} \begin{bmatrix} \frac{-1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} -\frac{5}{17} \\ \frac{20}{17} \\ 0 \end{bmatrix}}$$

11) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ -2 \end{bmatrix}$

$$-1 \cdot \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + 4 \begin{bmatrix} 4 \\ 12 \\ 8 \end{bmatrix} + 0 \cdot \begin{bmatrix} 0 \\ 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ 12 \\ 8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ 13 \\ 5 \end{bmatrix}}$$

12) $$\begin{bmatrix} 4 & -1 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}$$

$t_1 = \frac{1}{3}$

$t_2 = \frac{1}{3}$

$t_3 = -1$

$$\boxed{t = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ -1 \end{bmatrix}}$$

13) $$\begin{bmatrix} 2 & 0 & -1 \\ 1 & 4 & 5 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}$$

$$t = \begin{bmatrix} \frac{-7}{17} \\ \frac{15}{17} \\ \frac{3}{17} \end{bmatrix}$$

the determinant for Z is 17, which means it is linearly independent and only has 1 solution.

$t = Z^{-1} s$

since Z^{-1} involves division by determinant 17, the solution of t will include $\frac{1}{17}$.

Part C

1) $M - \lambda I = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-\lambda & 2 \\ -1 & 4-\lambda \end{bmatrix}$

$(3-\lambda)(4-\lambda) - (-1)(2) = 12 - 7\lambda + \lambda^2 + 2$

$= \lambda^2 - 7\lambda + 14 = 0$

$$\lambda = \frac{7 \pm \sqrt{49 - 4(14)}}{2} = \frac{7 \pm \sqrt{-7}}{2}$$

$$= \frac{7 \pm i\sqrt{7}}{2}$$

$$\boxed{\lambda_1 = \frac{7 + i\sqrt{7}}{2} \qquad \lambda_2 = \frac{7 - i\sqrt{7}}{2}}$$

$$\begin{bmatrix} 3 - \frac{7+i\sqrt{7}}{2} & 2 \\ -1 & 4 - \frac{7+i\sqrt{7}}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(3 - \frac{7+i\sqrt{7}}{2}\right) v_1 + 2 v_2 = 0$$

$v_2 = \left(-\frac{3}{2} + \frac{7}{4} + \frac{i}{4}\sqrt{7}\right) v_1$

$v_2 = \left(\frac{1}{4} + \frac{i}{4}\sqrt{7}\right) v_1$

$$\boxed{v^1 = \begin{bmatrix} 4 \\ 1 + i\sqrt{7} \end{bmatrix} \qquad v^2 = \begin{bmatrix} 4 \\ 1 - i\sqrt{7} \end{bmatrix}}$$

2.) Dot product between eigenvectors of M

$$4 \cdot 4 + (1+i\sqrt{7})(1-i\sqrt{7})$$

$$16 + (8)$$

$$\boxed{= 24}$$

3) $\begin{bmatrix} 5 & -3 \\ -3 & 6 \end{bmatrix}$ $\begin{bmatrix} 5-\lambda & -3 \\ -3 & 6-\lambda \end{bmatrix}$

$$30 - 11\lambda + \lambda^2 - 9$$

$$\lambda^2 - 11\lambda + 21$$

$$\frac{11 \pm \sqrt{121 - 4(21)}}{2} \to \frac{11 \pm \sqrt{37}}{2}$$

$$\begin{bmatrix} 5 - \left(\frac{11}{2} - \frac{\sqrt{37}}{2}\right) & -3 \\ -3 & 6 - \left(\frac{11}{2} - \frac{\sqrt{37}}{2}\right) \end{bmatrix}$$

$$-\frac{1}{2} + \frac{\sqrt{37}}{2} = \frac{-1 + \sqrt{37}}{2}$$

$$\frac{1}{2} + \frac{\sqrt{37}}{2} = \frac{1 + \sqrt{37}}{2}$$

$$\begin{bmatrix} \frac{-1+\sqrt{37}}{2} & -3 \\ -3 & \frac{1-\sqrt{37}}{2} \end{bmatrix}$$

$$\begin{bmatrix} 2.541 & -3 \\ -3 & 3.541 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_1 = \frac{3}{2.541} v_2$$

let $v_2 = 1$ $v_1 = 1.181$ normalized

$$v_1 = \begin{bmatrix} 1.181 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.763 \\ 0.646 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -0.763 \\ 0.646 \end{bmatrix}$$

4) Since N is symmetric, it has real eigenvalues and the eigenvectors are orthogonal.

5) $t = \boxed{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}$

6) $\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$t_1 + 2t_2 = 0$ let $t_2 = 1$ $t_1 = -2$

$t_1 = -2t_2$ $t_2 = -1$ $t_1 = 2$

$\boxed{t = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix}}$

7) $\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\hookrightarrow \begin{bmatrix} 3 & 2 & 0 \\ -1 & 4 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2/3 & 0 \\ -1 & 4 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2/3 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\hookrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\boxed{t = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$

$\det(M) = 3 \cdot 4 - 2 \cdot (-1) = 14 \neq 0$

since $\det(M) \neq 0$, it is invertible thus there is only one solution.

Part D

1) $x_1 = 0.5 \quad w_1 = 0.4$
$x_2 = -0.8 \quad w_2 = -0.6 \quad b = 0.2$
$x_3 = 0.3 \quad w_3 = 0.9$

$0.4 \cdot 0.5 + -0.8 \cdot -0.6 + 0.3 \cdot 0.9 =$

$0.2 + 0.48 + 0.27$

$= 0.95 + 0.2 = \boxed{1.15}$

2) Sigmoid activation function

$\sigma(1.15) = \frac{1}{1+e^{-1.15}} = \boxed{0.76}$

3) $\boxed{\text{ReLU}(1.15) = 1.15}$

4) $\begin{bmatrix} 0.4 \cdot 1 + 0.3 \cdot 2 \\ 0.2 \cdot 1 + 0.7 \cdot 2 \end{bmatrix} + \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 1.5 \end{bmatrix}$

$\text{reLU}\left(\begin{bmatrix} 1.1 \\ 1.5 \end{bmatrix}\right) = \boxed{\begin{bmatrix} 1.1 \\ 1.5 \end{bmatrix}}$

5) $\begin{bmatrix} 0.5 & -0.3 \end{bmatrix} \begin{bmatrix} 1.1 \\ 1.5 \end{bmatrix} = (0.5)(1.1) + (-0.3)(1.5)$

$= 0.1$

$0.1 + 0.1 = 0.2$

$\frac{1}{1+e^{-0.2}} = \frac{1}{1.8187} = \boxed{0.55}$

6) $\boxed{\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x_1}}$

part E

$f(x) = 2x^2 - 1$

$g(x) = 3x^2 + 4$

$h(x, y) = x^2 + y^2 + xy$

1) $\boxed{f'(x) = 4x \quad f''(x) = 4}$

2) $\frac{\partial h}{\partial x} = 2x + y$

$\frac{\partial h}{\partial y} = 2y + x$

3) $\nabla h = \left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y} \right)$

$\boxed{= \begin{pmatrix} 2x+y \\ 2y+x \end{pmatrix}}$

4) With chain rule

$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

$= 4(3x^2+4) \cdot 6x$

$= 24x(3x^2 + 4)$

$\boxed{= 72x^3 + 96x}$

Without chain rule

$\frac{d}{dx}\left(2(3x^2+4)^2 - 1\right) = \frac{d}{dx}\left(18x^4 + 48x^2 + 31\right)$

$\boxed{= 72x^3 + 96x}$