

# Photon mapping



# Course announcements

- Programming assignment 4 posted, due Friday 4/1 at 23:59.
  - How many of you have looked at/started/finished it?
  - Any questions?
- Take-home quiz 8 will be posted tonight.
- Suggest topics for this week's reading group.
- No lecture on Thursday! Vote for when to reschedule.
- Talk by Nathan Matsuda in graphics lab meeting.
- Presentation by Yannis in imaging reading group.

# Overview of today's lecture

- Photon mapping.

# Slide credits

Most of these slides were directly adapted from:

- Wojciech Jarosz (Dartmouth).







# Today's Menu

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Difficult light paths

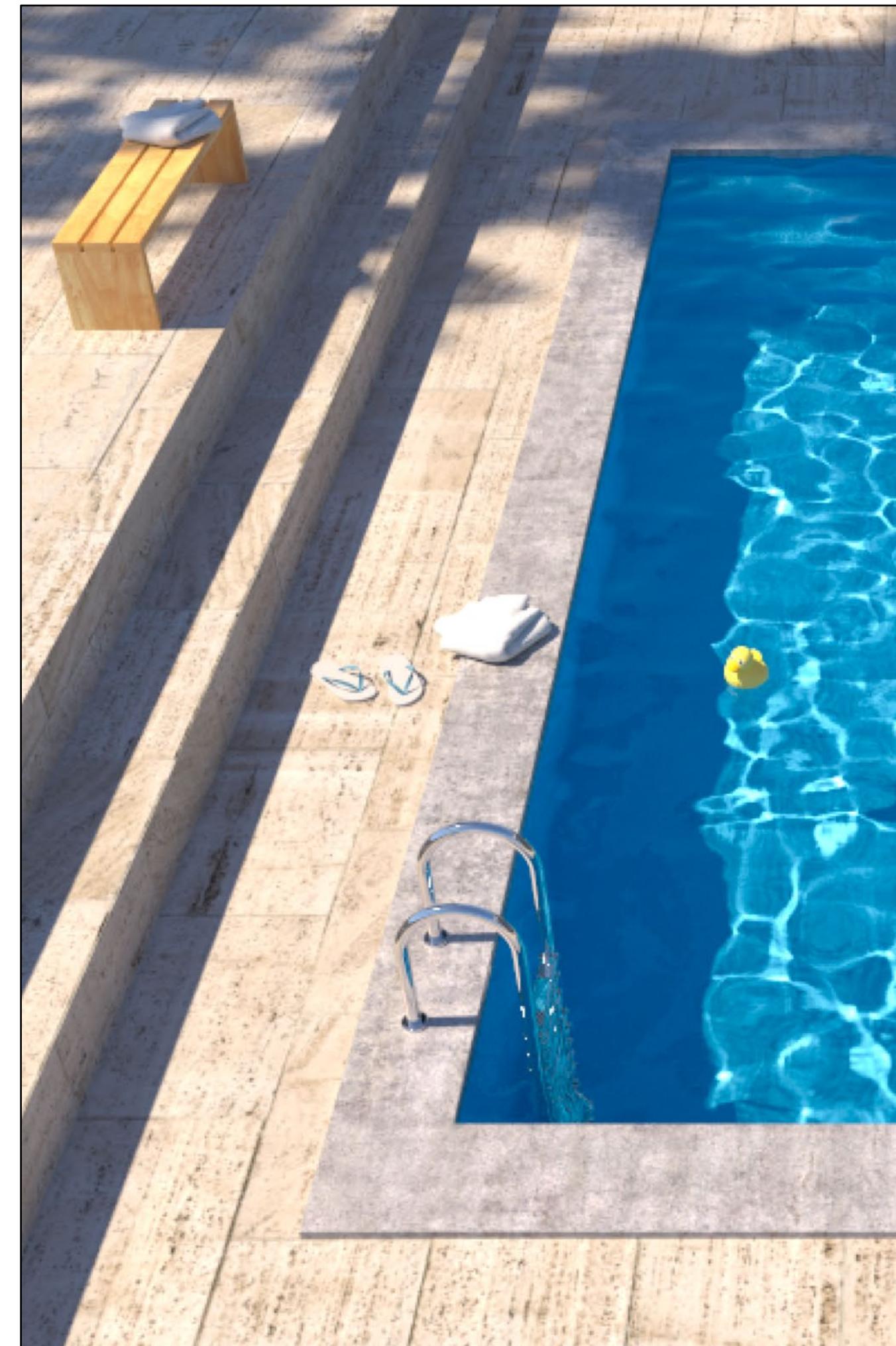
Photon Mapping

# Specular-Diffuse-Specular Paths



# Specular-Diffuse-Specular Paths

Reference

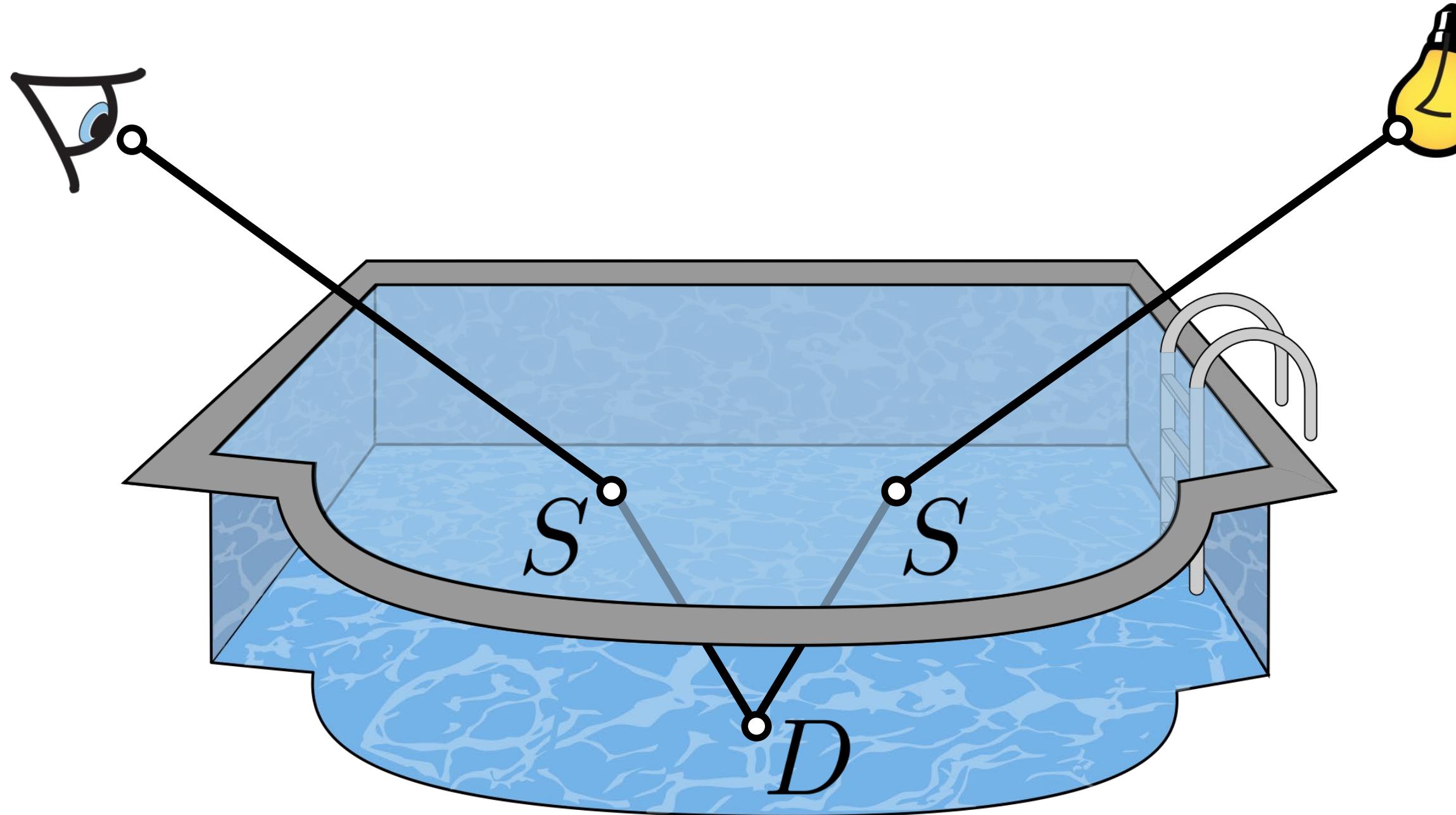


Bidirectional PT



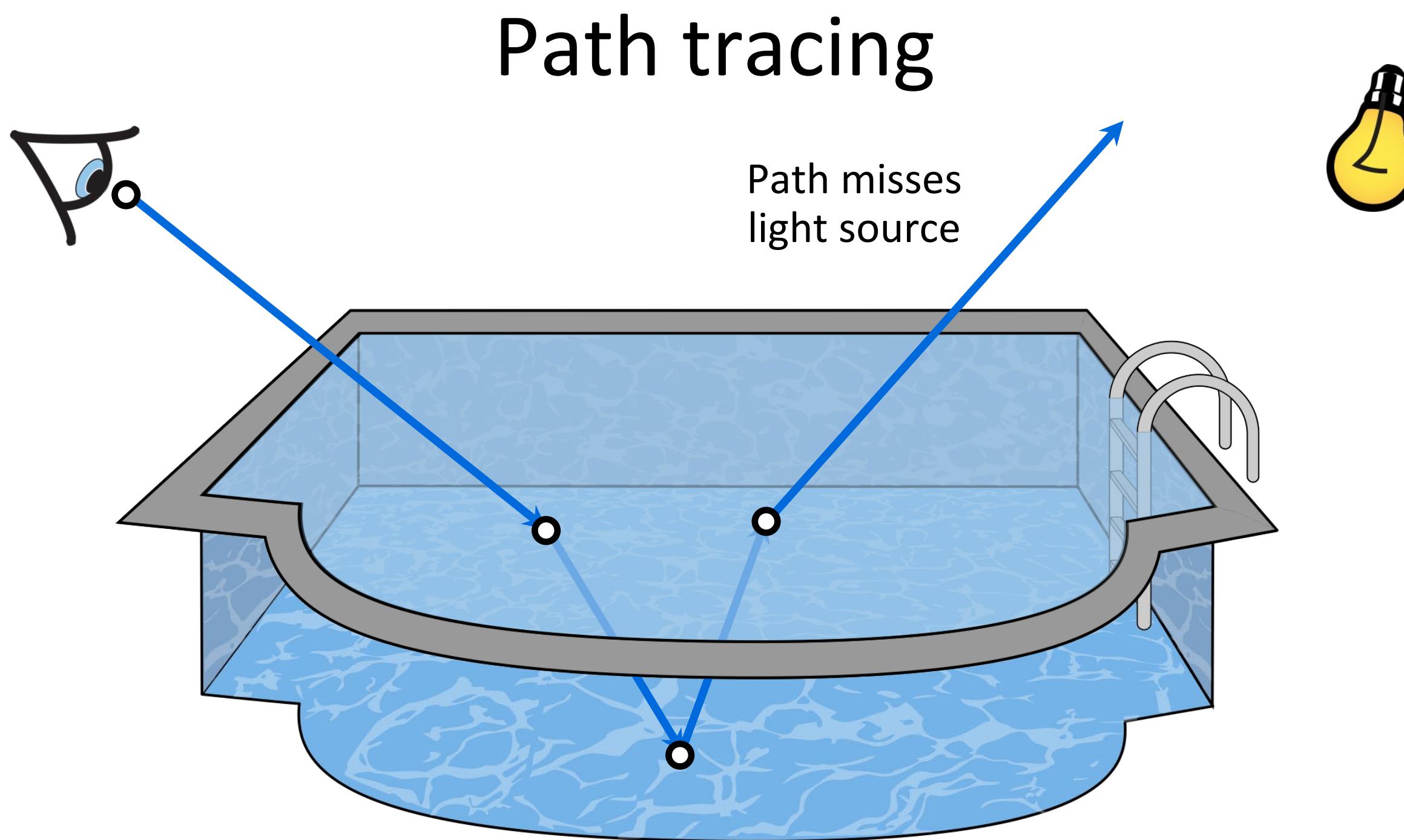
# Specular-Diffuse-Specular Paths

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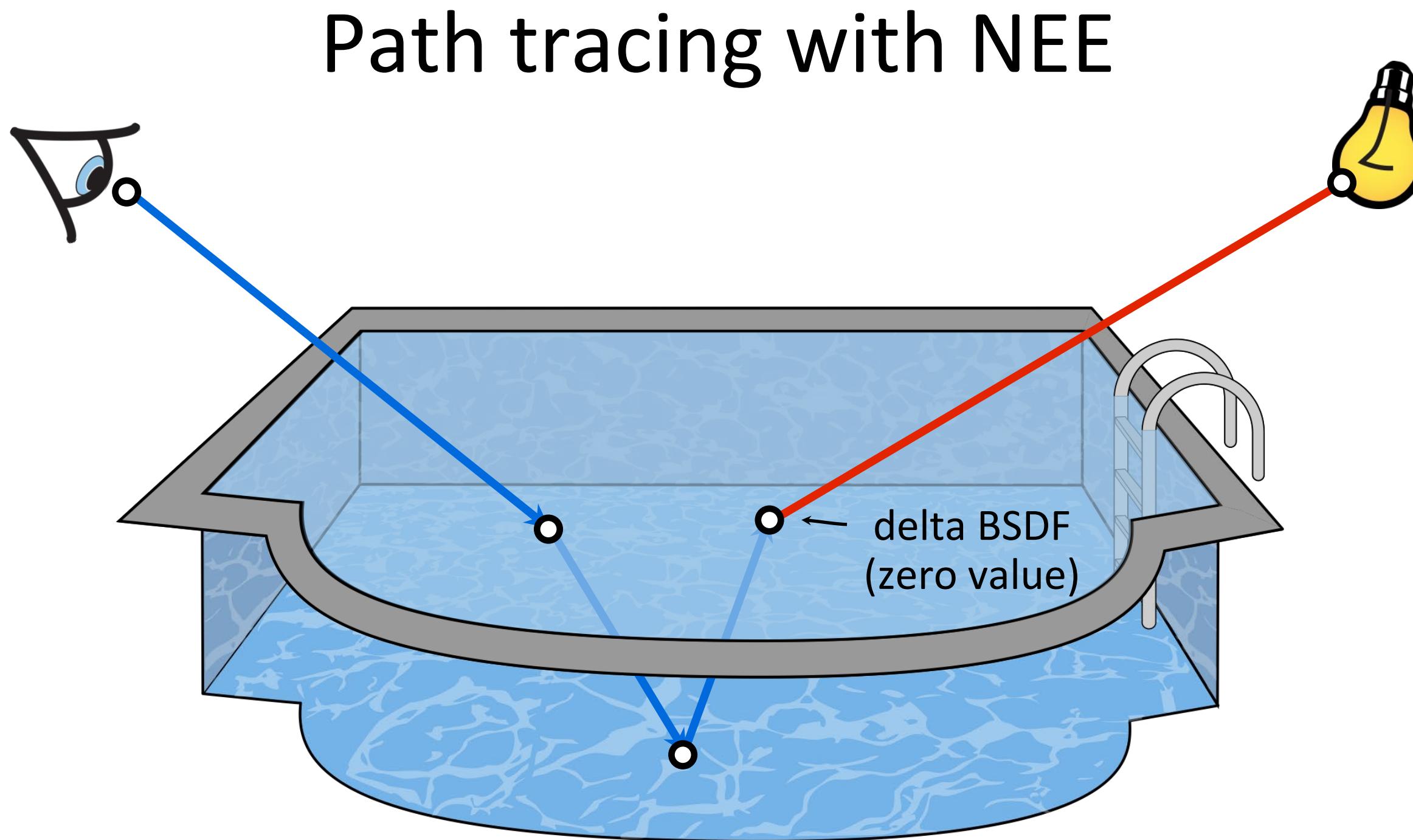
*LSDSE* paths are difficult for unbiased techniques

# Specular-Diffuse-Specular Paths



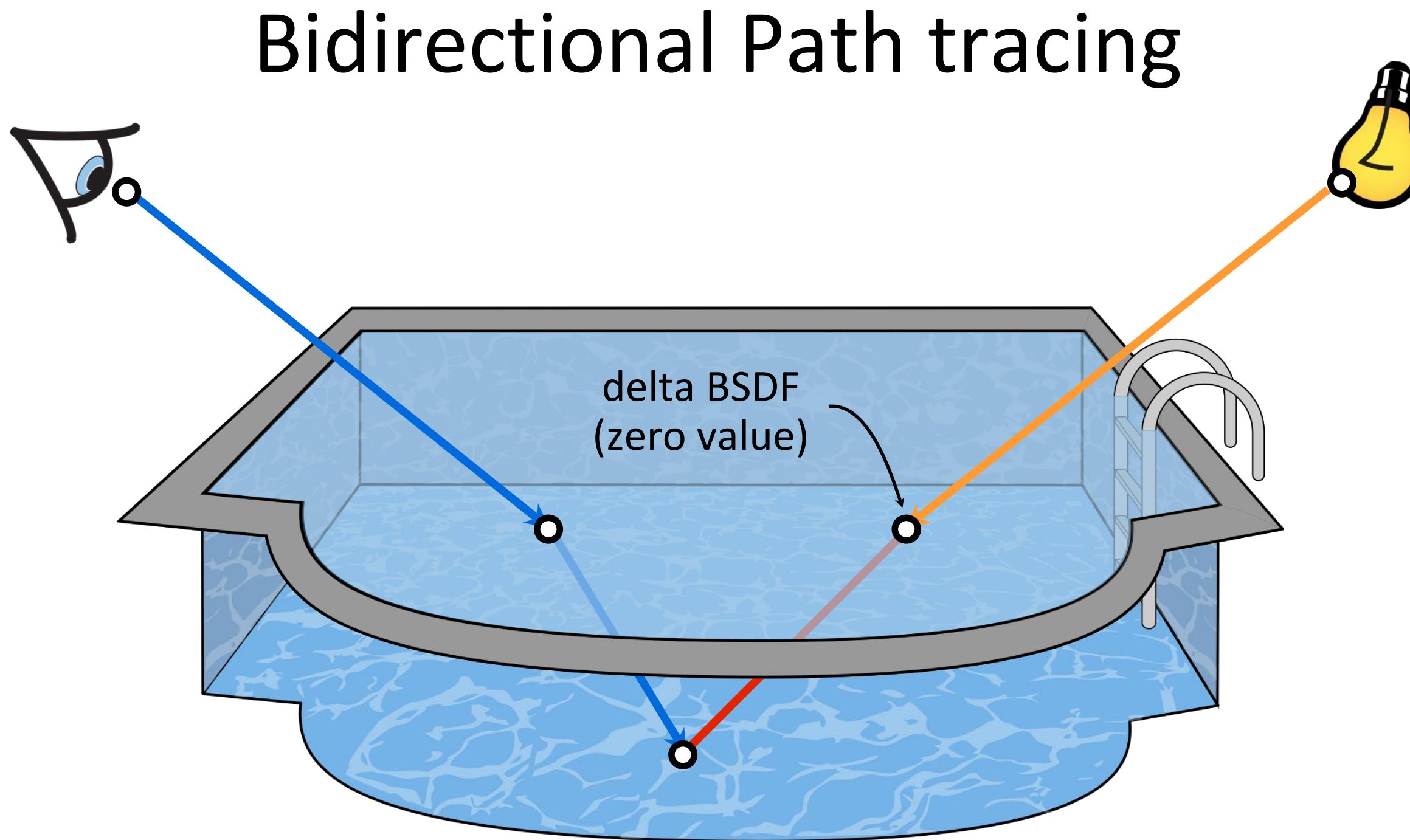
# Specular-Diffuse-Specular Paths

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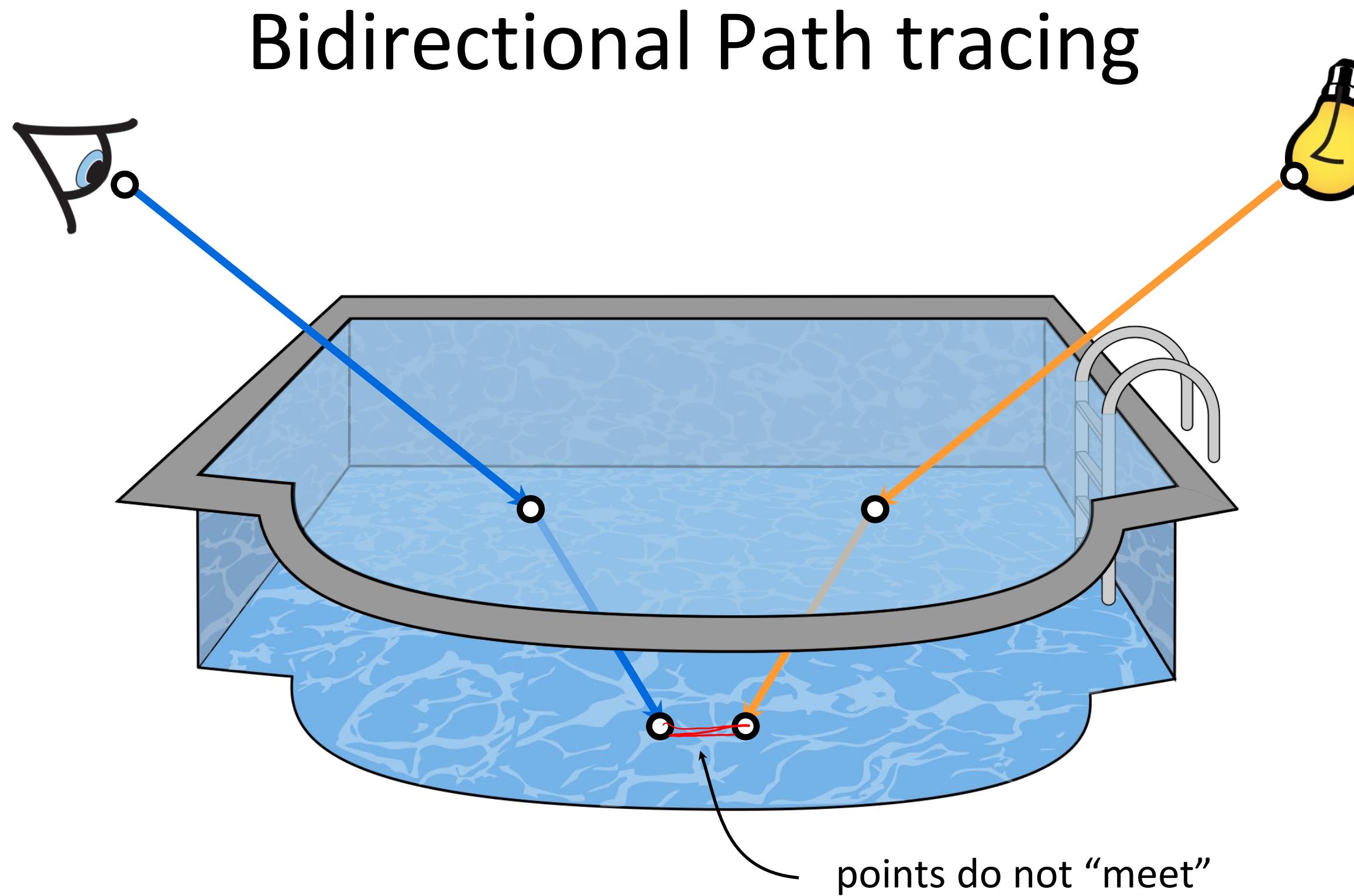


# Specular-Diffuse-Specular Paths

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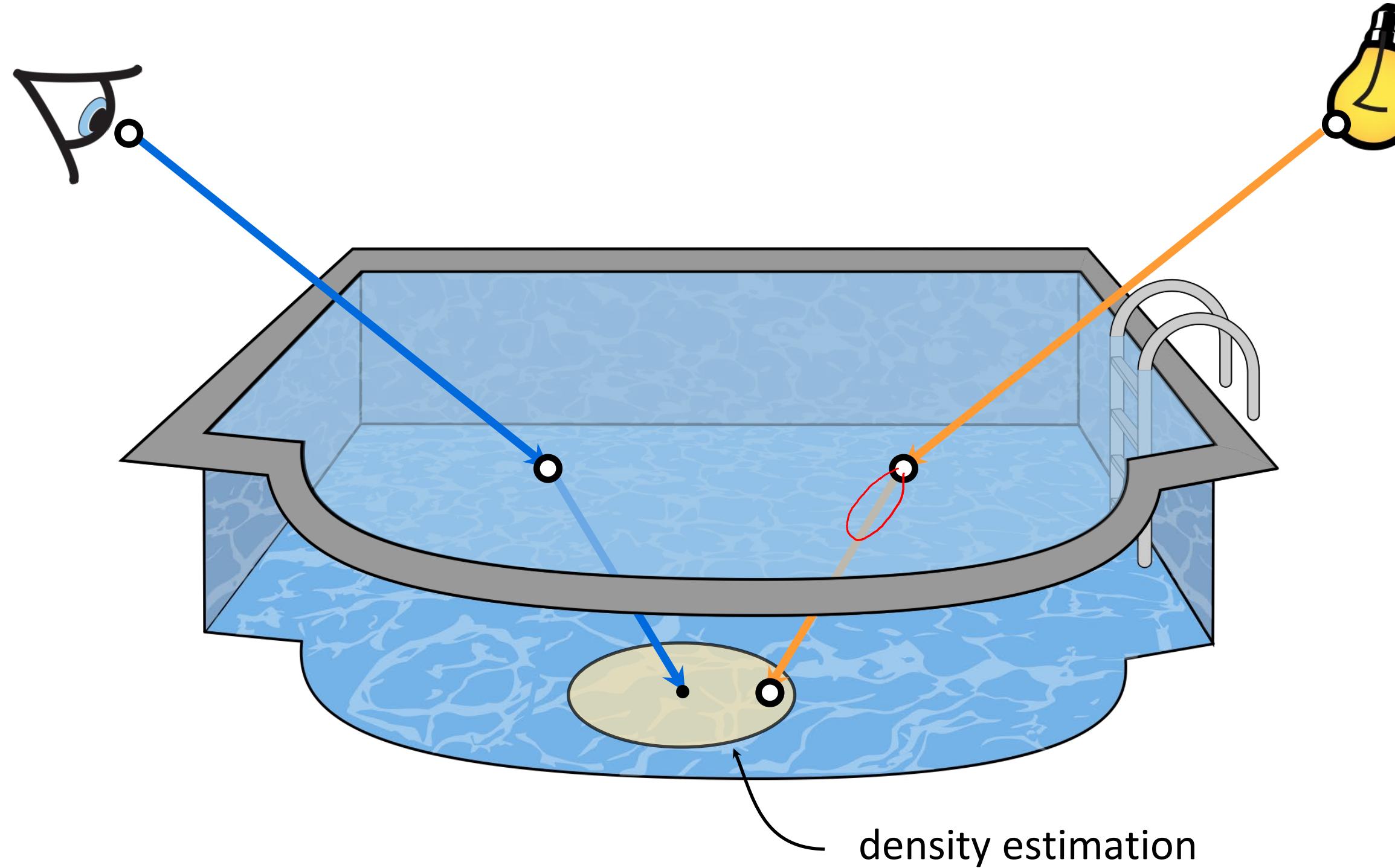


# Specular-Diffuse-Specular Paths



What now?

# Specular-Diffuse-Specular Paths



Regularize delta functions (path points):  
e.g., by employing kernel density estimation (blurring in space)

# “Backward” Ray Tracing

(predecessor of photon mapping)

# “Backward” Ray Tracing

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James Arvo. In *Developments in Ray Tracing*, SIGGRAPH ‘86  
Course Notes

Emit photons from light sources and store them in *illumination maps*

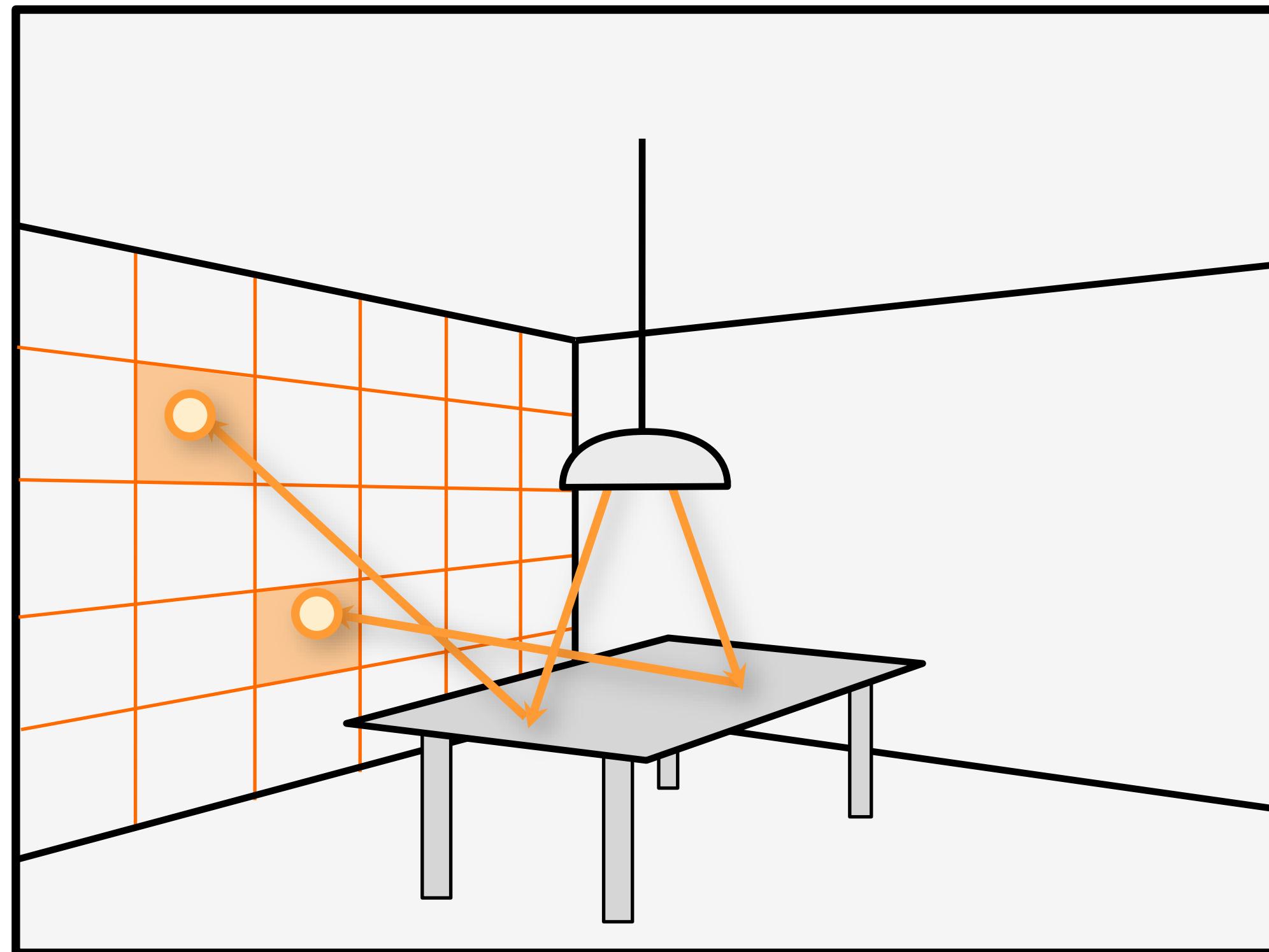
*Illumination map* = texture for accumulating irradiance

Note on the name of the technique: In retrospect, Arvo regretted using the term “backward” to refer to tracing light paths since many later publications use it in the opposite sense, i.e. tracing eye paths. To avoid confusion, he recommends terms such as *light tracing* and *eye tracing* as they are unambiguous.

# “Backward” Ray Tracing

Preprocess:

- shoot light from light sources
- deposit photon energy in illumination maps



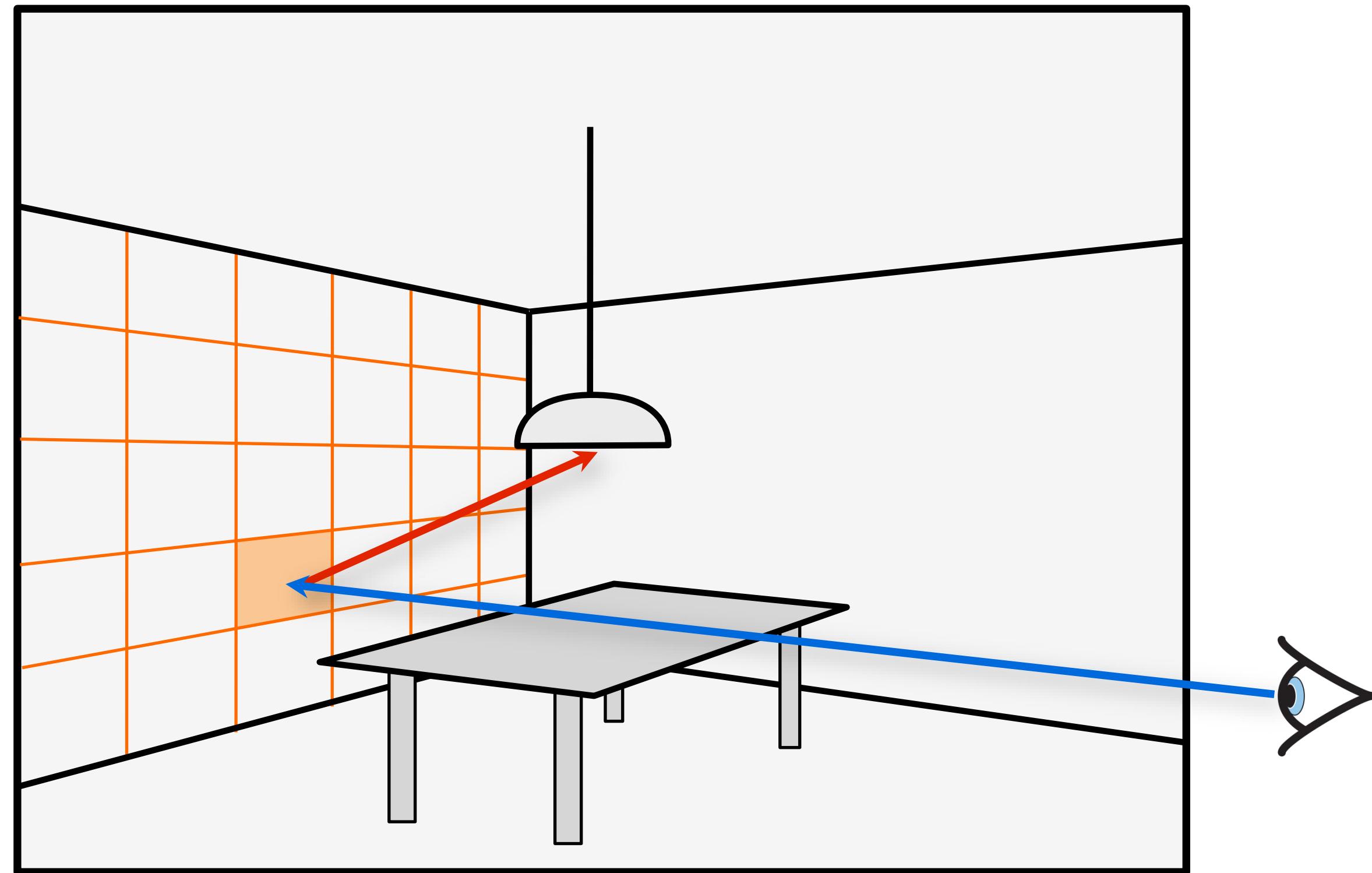
**Irradiance:** “number of photons hitting a small patch of a wall per second, divided by size of patch”

# “Backward” Ray Tracing

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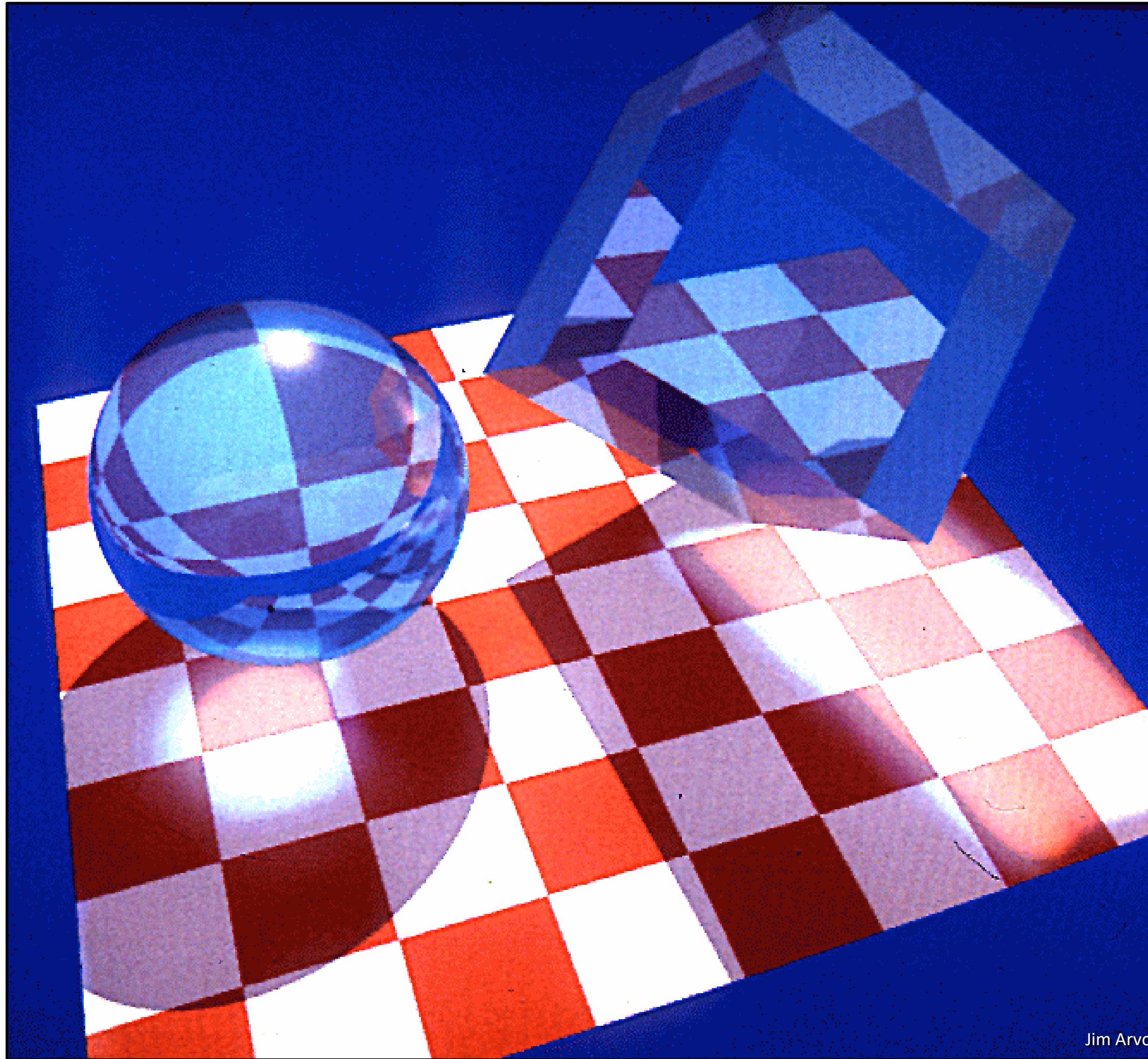
For each shading point

- compute direct lighting
- lookup indirect lighting from illumination maps



# “Backward” Ray Tracing

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"Backward Ray Tracing", by James Arvo. In *Developments in Ray Tracing*, SIGGRAPH '86 Course Notes, Volume 12.

# “Backward” Ray Tracing

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✓ One of the first techniques to simulate caustics!

✗ Requires parametrizing surfaces or meshing

- Difficult to handle complex or procedural geometry

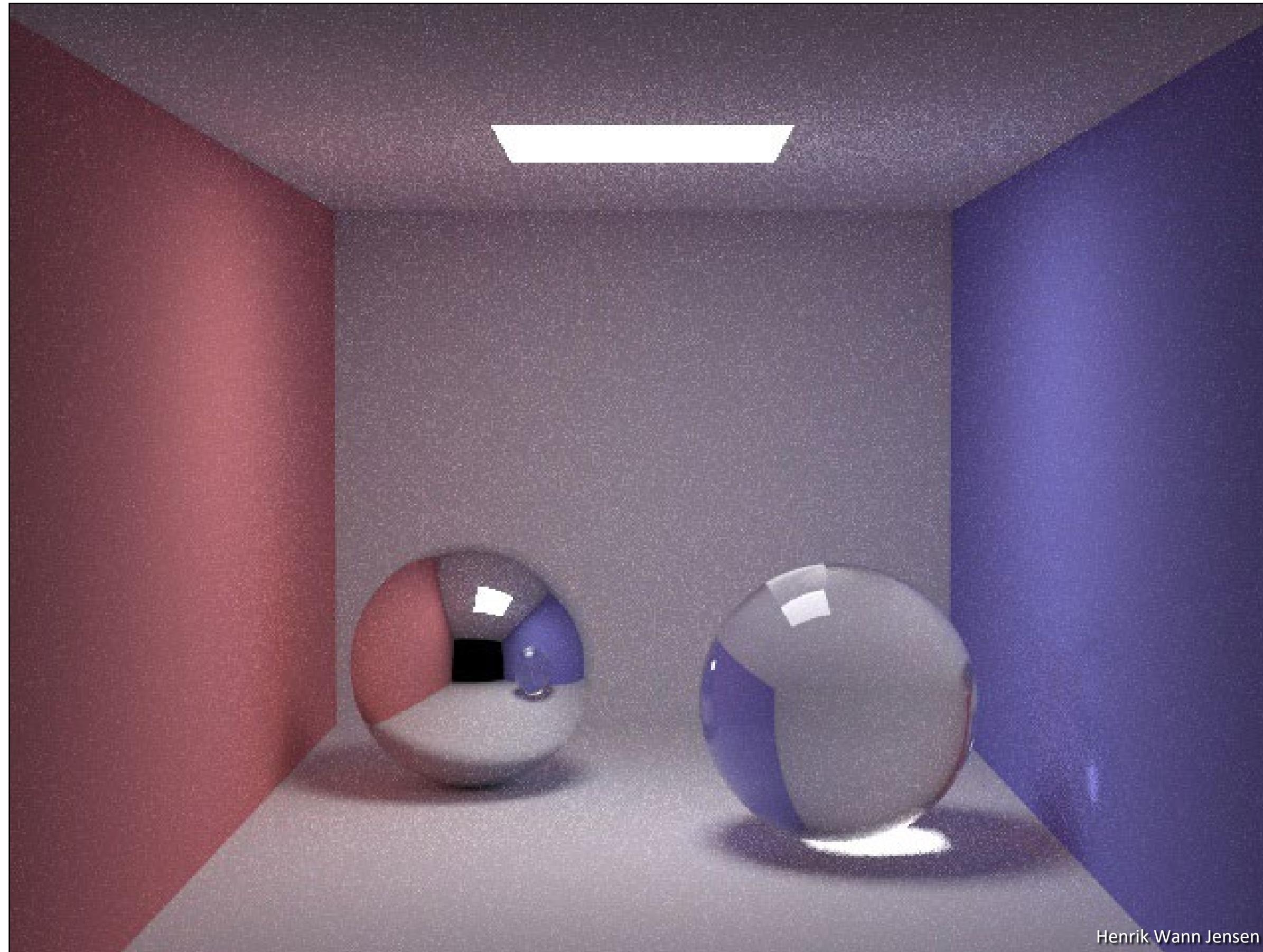
✗ Hard to choose illumination map resolution

- high resolution with few photons: high-frequency noise
- low resolution with many photons: blurred illumination

# Photon Mapping

# Path Tracing

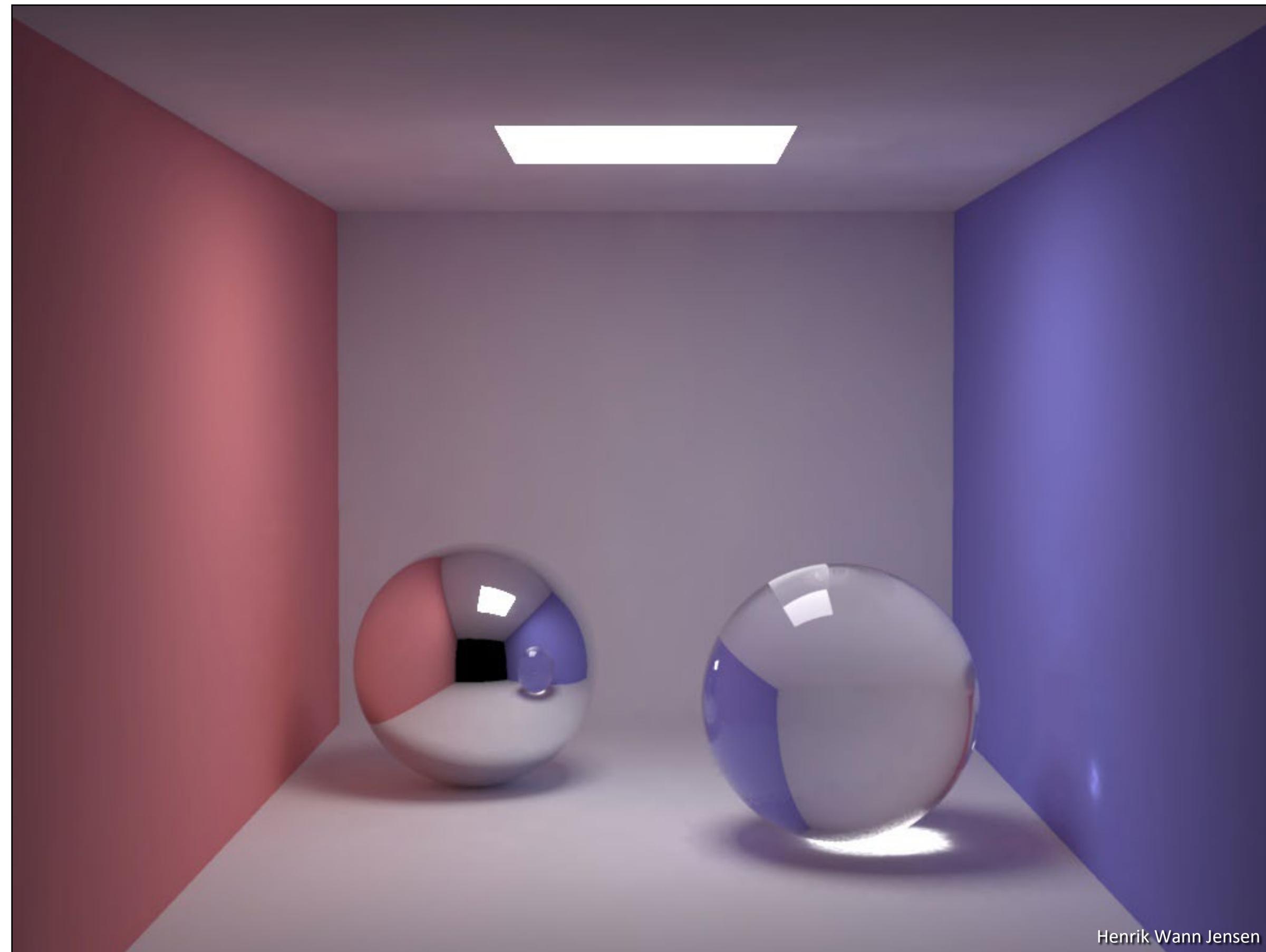
---



100 paths/pixel (5 minutes)

# Photon Mapping

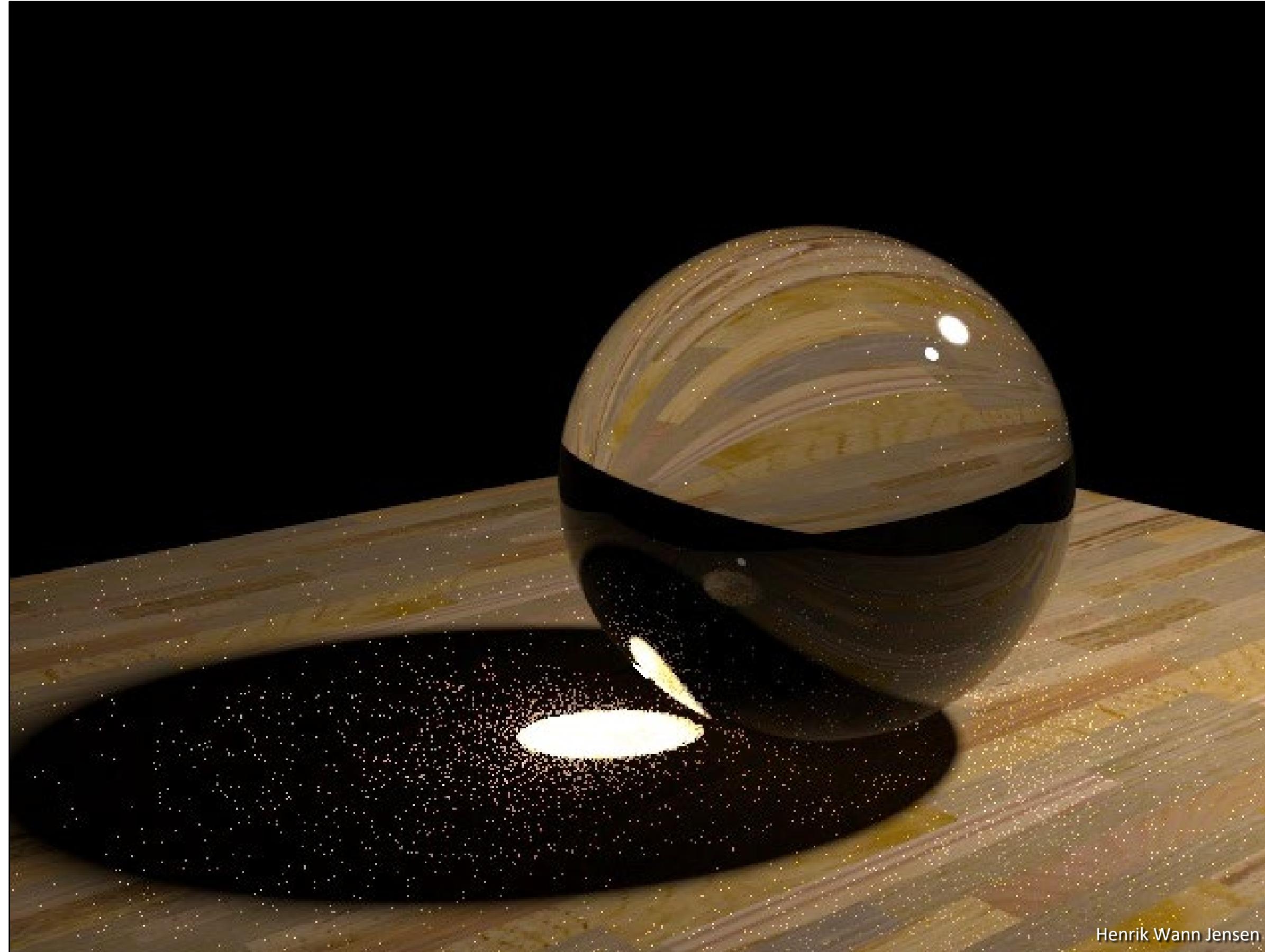
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10 rays/pixel (5 seconds)

# Path Tracing

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1000 paths/pixel

# Photon Mapping

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# Photon Mapping

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A two-pass algorithm:

- Pass 1: Tracing photons from light sources, and caching them in a *photon map*
- Pass 2: Tracing from the eye and approximating indirect illumination using the photons

Similar to “backward” ray tracing, but different way of storing photons & computing density

# Photon Mapping

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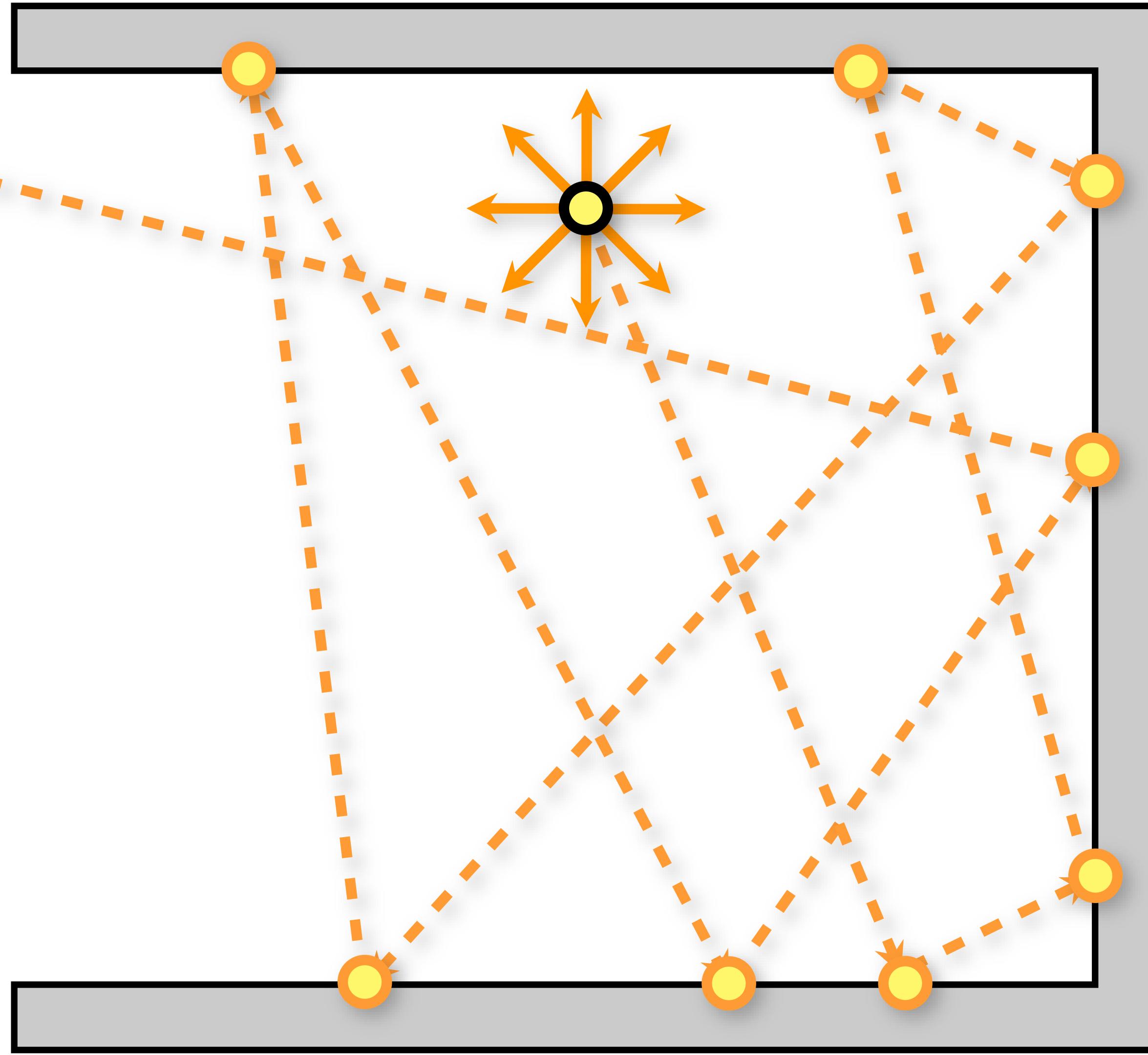
A two-pass algorithm:

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Similar to “backward” ray tracing, but different way of storing photons & computing density

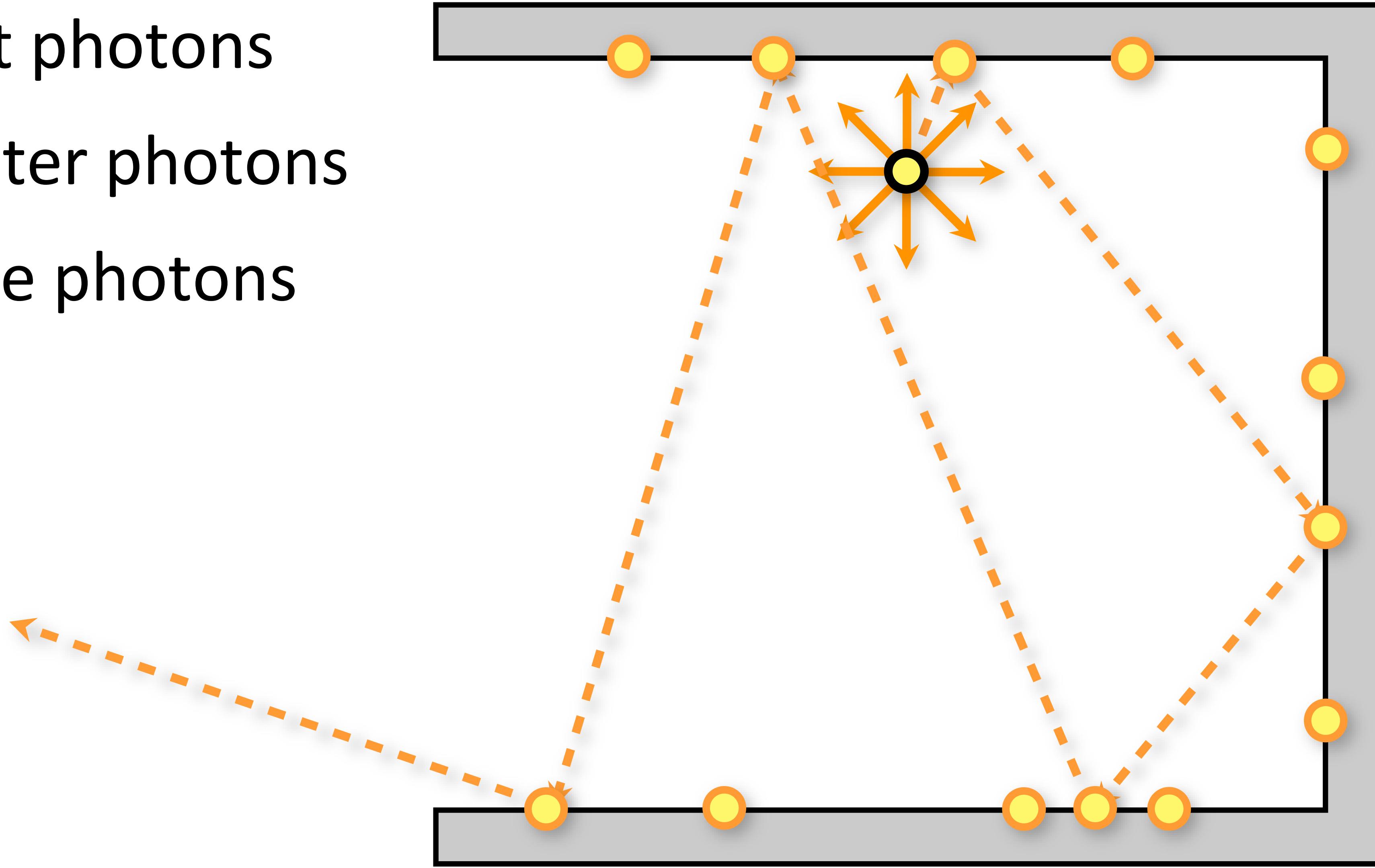
# Photon Tracing

- 1) Emit photons
- 2) Scatter photons
- 3) Store photons



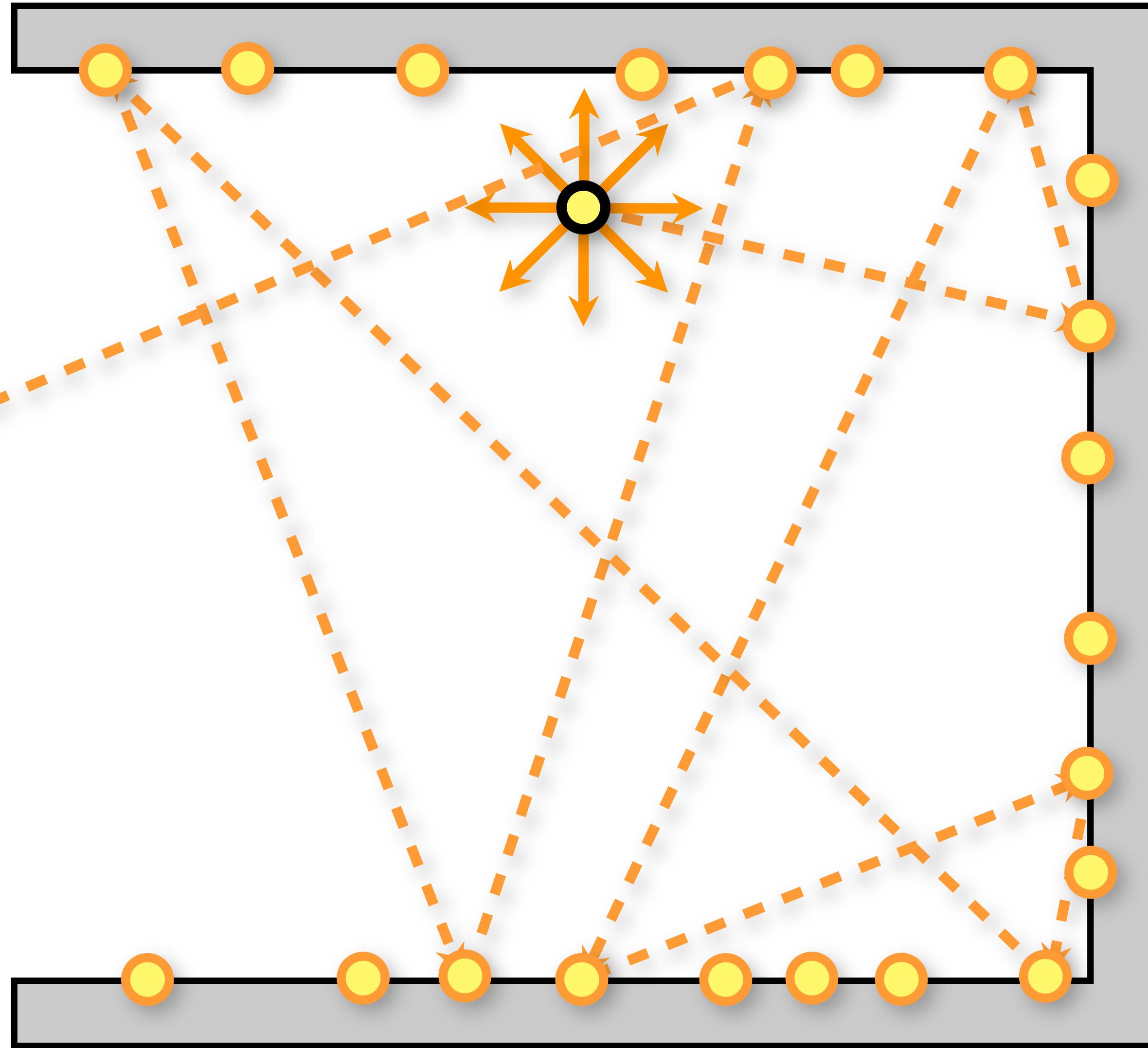
# Photon Tracing

- 1) Emit photons
- 2) Scatter photons
- 3) Store photons



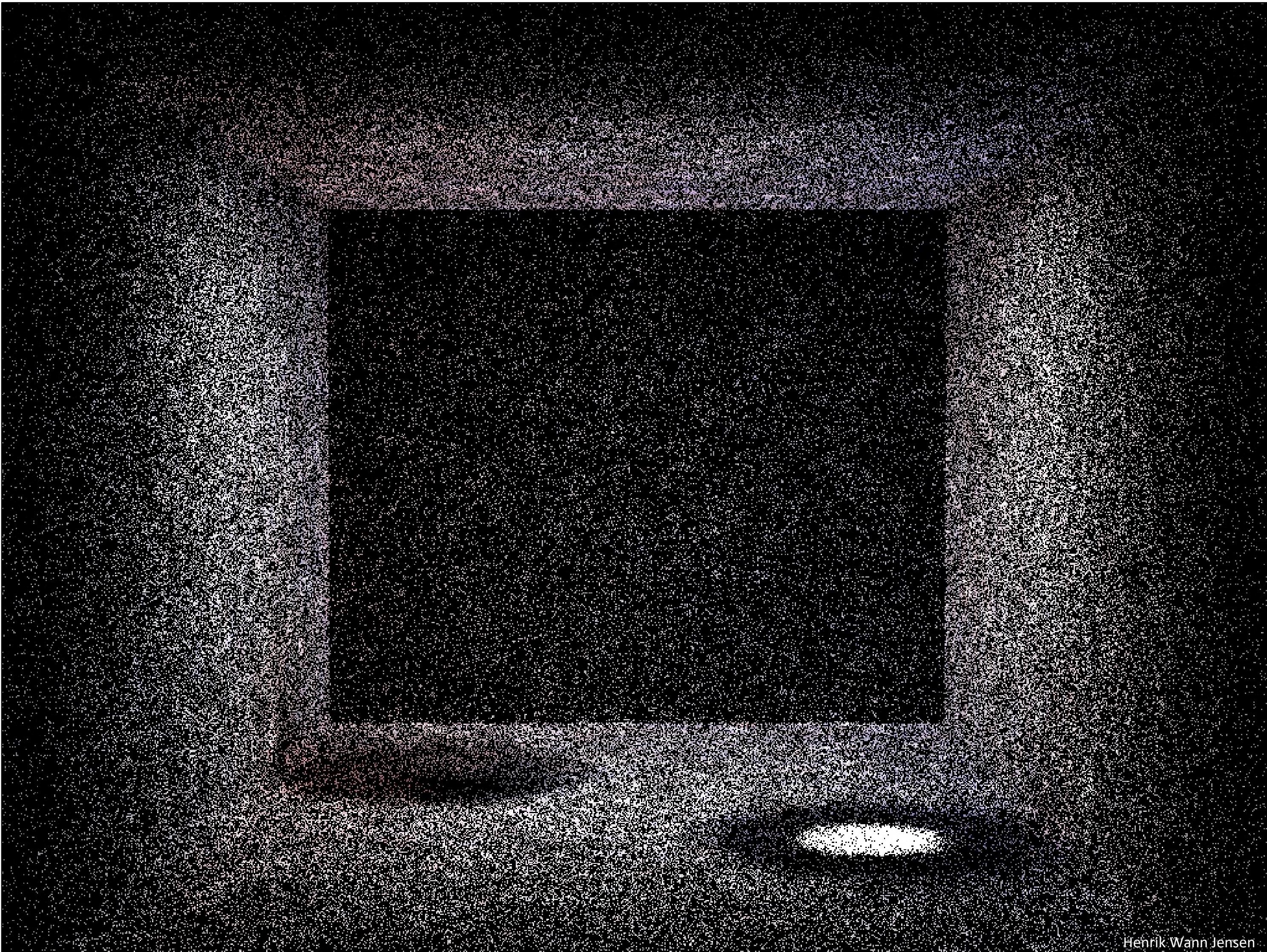
# Photon Tracing

- 1) Emit photons
- 2) Scatter photons
- 3) Store photons



# Visualization of the Photon Map

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Henrik Wann Jensen

# Photon Emission

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Photons carry power (flux) not radiance!

- not a physical photon
- just a fraction of the light source power
- in most practical implementations, each photon carries multiple wavelengths (e.g. RGB)

# Photon Emission

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Define initial:

- $\mathbf{x}_p$ : position  
→
- $\omega_p$ : direction
- $\Phi_p$ : photon power

General recipe:

- Sample position on surface area of light with  $p(\mathbf{x}_p)$   
→
- Sample direction with  $p(\omega_p \mid \mathbf{x}_p)$

$$\Phi_p = \frac{1}{M} \frac{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos \theta_p}{p(\mathbf{x}_p) p(\vec{\omega}_p \mid \mathbf{x}_p)}$$

# of emitted photons

# Photon Emission

Interesting derivation:

- if PDFs are proportional to the emission:

$$p(\mathbf{x}_p) = \frac{\int_{H^2} L_e(\mathbf{x}_p, \vec{\omega}) \cos \theta d\vec{\omega}}{\int_A \int_{H^2} L_e(\mathbf{x}, \vec{\omega}) \cos \theta d\vec{\omega} d\mathbf{x}} \quad p(\vec{\omega}_p | \mathbf{x}_p) = \frac{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos \theta_p}{\int_{H^2} L_e(\mathbf{x}_p, \vec{\omega}) \cos \theta d\vec{\omega}}$$

- then:

$$\begin{aligned} \Phi_p &= \frac{1}{M} \frac{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos \theta_p}{p(\mathbf{x}_p) p(\vec{\omega}_p | \mathbf{x}_p)} \\ &= \frac{1}{M} \frac{\cancel{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos \theta_p}}{\cancel{\int_{H^2} L_e(\mathbf{x}_p, \vec{\omega}) \cos \theta d\vec{\omega}}} = \frac{\cancel{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos \theta_p}}{\cancel{\int_A \int_{H^2} L_e(\mathbf{x}, \vec{\omega}) \cos \theta d\vec{\omega} d\mathbf{x}}} = \frac{\cancel{L_e(\mathbf{x}_p, \vec{\omega}_p) \cos \theta_p}}{\cancel{\int_{H^2} L_e(\mathbf{x}_p, \vec{\omega}) \cos \theta d\vec{\omega}}} \end{aligned}$$

Total power of  
the light source

$\Phi$

If you *perfectly importance sample* the emitted radiance,  
just take the *total power* and divide by # of emitted photons.

# Photon Emission Examples

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Isotropic point light:

- Generate uniform random direction over sphere

Spotlight:

- Generate uniform random direction within spherical cap

Diffuse area light:

- Generate uniform random position on surface
- Generate cosine-weighted direction over hemisphere

# Pseudocode

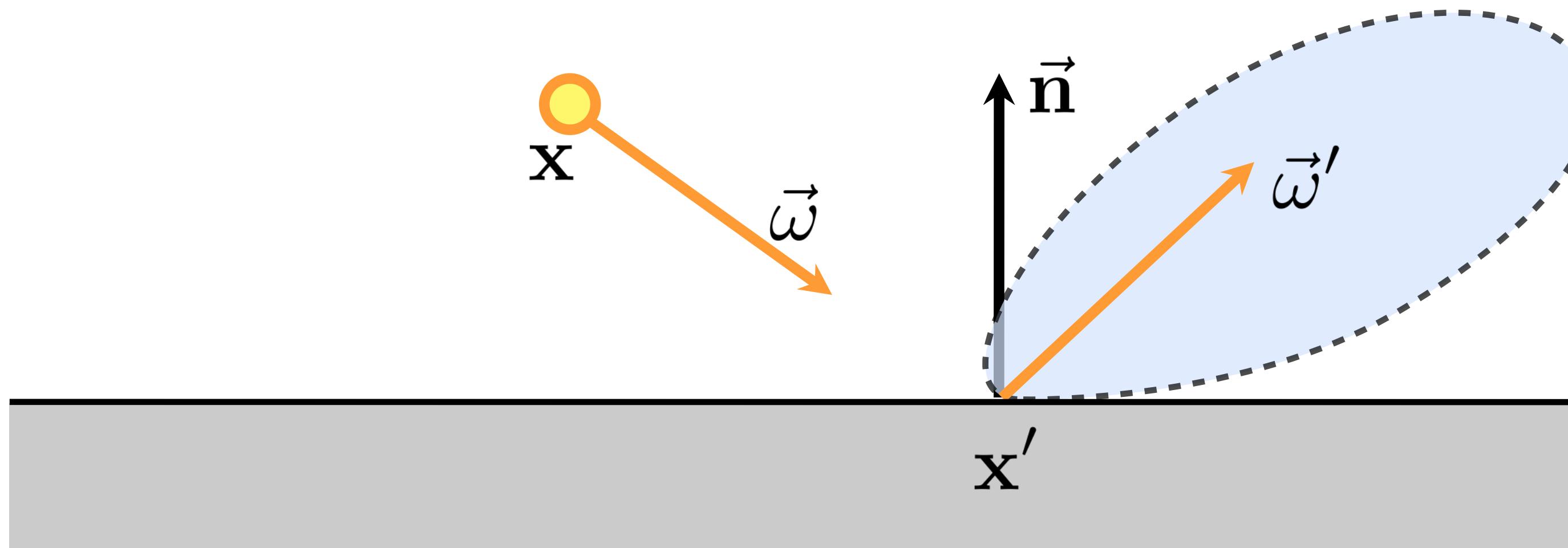
---

```
void generatePhotonMap()  
repeat:  
    (l, Probl) = chooseRandomLight()  
    (x, ω, Φ) = emitPhotonFromLight(l)  
    tracePhoton(x, ω, Φ / Probl)  
until we have enough photons;  
divide all photon powers by number of emitted photons
```

```
void tracePhoton(x, ω, Φ)
```

# Pseudocode

---



```
void tracePhoton(x, ω, Φ)
(x', n) = nearestSurfaceHit(x, ω)
possiblyStorePhoton(x', ω, Φ)
(ω', pdf) = sampleBSDF(x', -ω)
Φ' = Φ * absDot(n, ω') * BSDF(x', -ω, ω') / pdf
tracePhoton(x', ω', Φ')
```

# Storing Photons

---

Store only on diffuse (or moderately glossy) surfaces

- Specular surfaces need to be handled using path tracing from the camera

Stored data: [36 bytes]

```
struct Photon
{
    float position[3];
    float power[3];
    float direction[3];
};
```

# Storing Photons

---

Store only on diffuse (or moderately glossy) surfaces

- Specular surfaces need to be handled using path tracing from the camera

Stored data:

```
struct Photon
{
    float position[3];
    char power[4];          // Packed RGBE format
    char phi, theta; // Packed direction
};
```

# Scattering of Photons

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Photons can be:

- absorbed or scattered (reflected or refracted)
- BSDF sampling chooses either reflection or refraction
- the power of the scattered photon is lowered to account for absorption

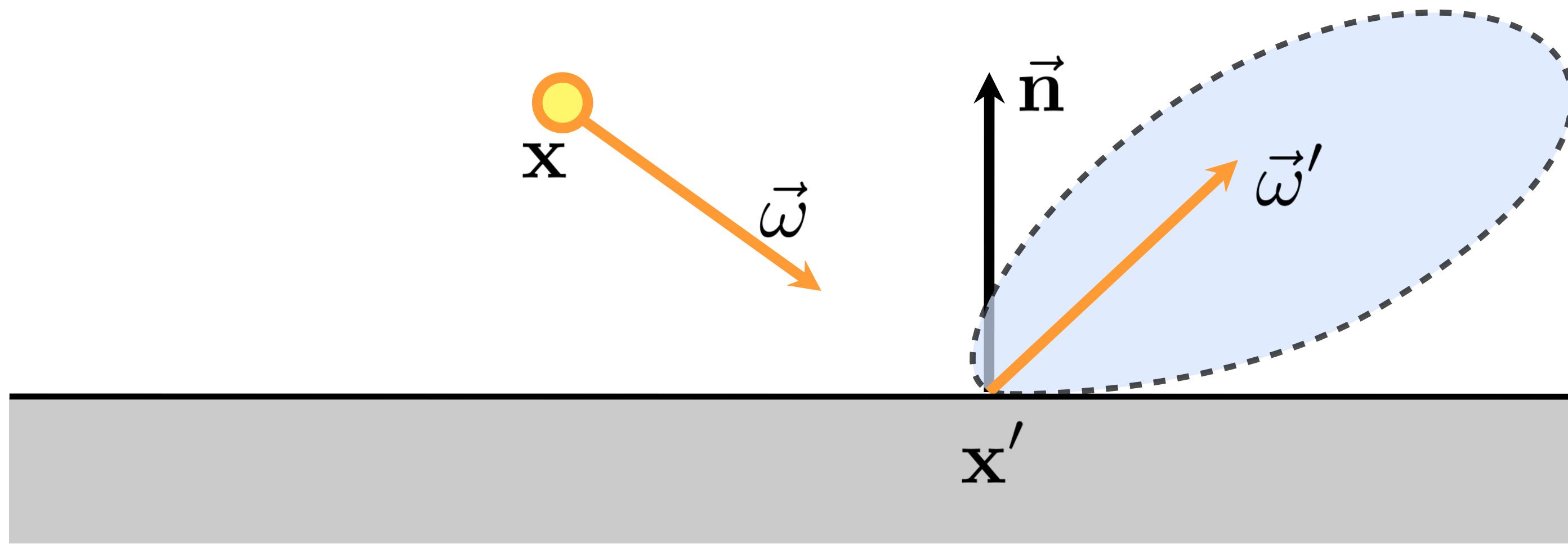
Problem:

- as photons bounce they carry less and less power
- ideally all stored photons would have the same power
- also, when should we terminate the recursion?

Solution: Russian roulette

# Pseudocode

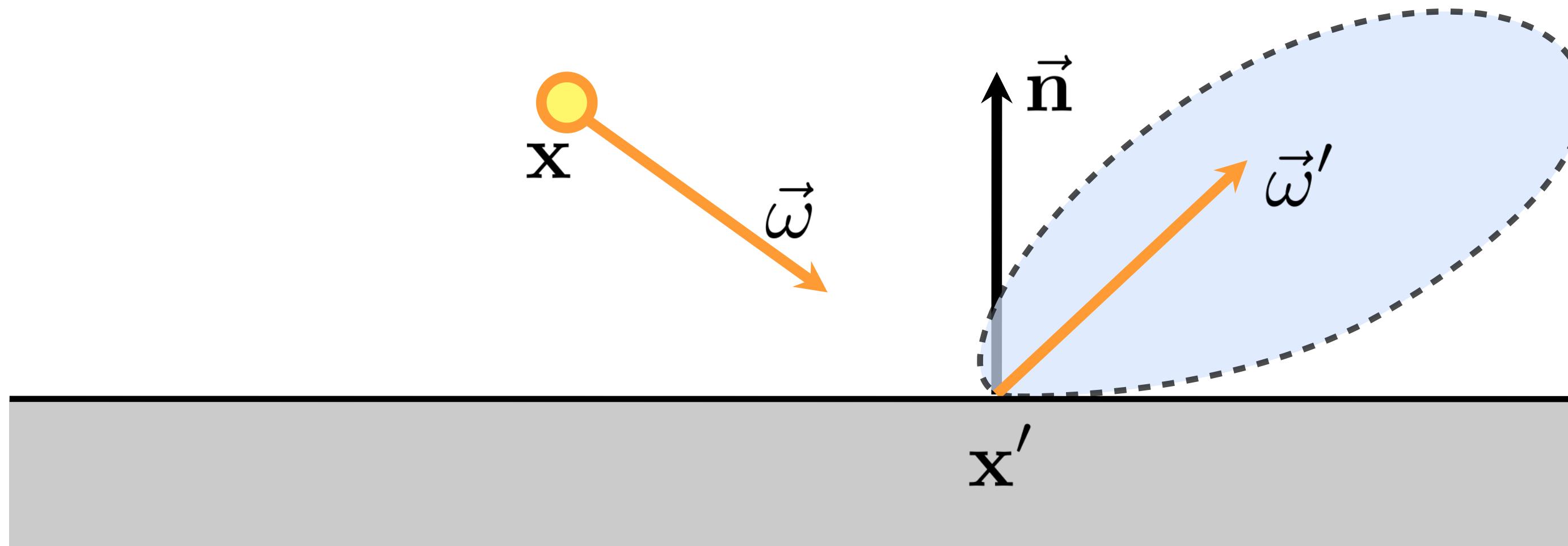
---



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tracePhoton(x', ω', Φ')
```

# Pseudocode

---



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void tracePhoton(x, ω, Φ)
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possiblyStorePhoton(x', ω, Φ)
(ω', pdf) = sampleBSDF(x', -ω)
Φ' = Φ * absDot(n, ω') * BSDF(x', -ω, ω') / pdf
if survivedRussianRoulette(Φ, Φ')
    tracePhoton(x', ω', Φ')
```

# Photon Path Termination

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Probabilistically terminate the photon walk using Russian roulette (continue with prob.  $p$ )

$$E[F'] = (1 - p) \cdot 0 + p \cdot \frac{E[F]}{p} = E[F]$$

Option 1: local termination probability:

$$p = \min\left(1, \frac{\Phi'}{\Phi}\right)$$

# Photon Path Termination

---

```
bool survivedRussianRoulette( $\Phi$ ,  $\Phi'$ )  
  
p = min(1,  $\Phi' / \Phi$ )  
  
if rand() > p:  
    // terminate  
  
return false  
  
else:  
    // continue with re-weighted power  
  
 $\Phi' /= p$   
  
return true
```

if  $\Phi' / \Phi$  is smaller than 1, then  $\Phi' = \Phi' / p = \Phi$   
i.e., the scattered photon has the same power!

# Photon Path Termination

---

Probabilistically terminate the photon walk using Russian roulette (continue with prob.  $p$ )

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Option 1: local termination probability:

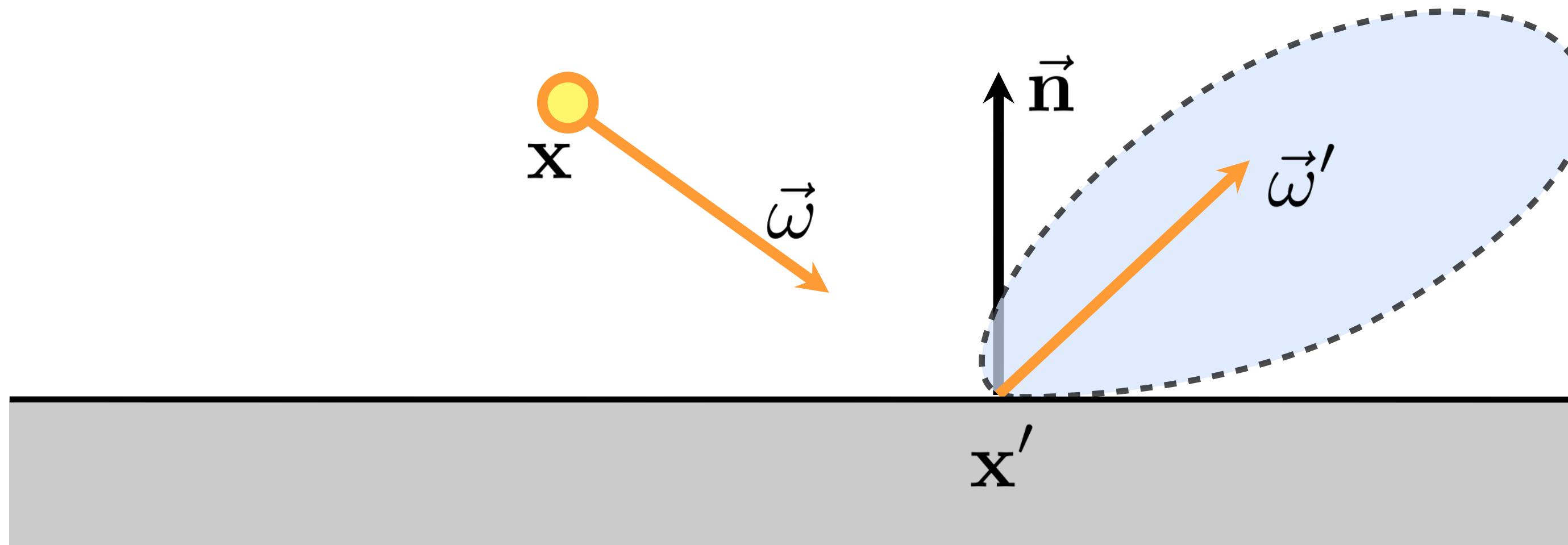
$$p = \min\left(1, \frac{\Phi'}{\Phi}\right)$$

Option 2: history-aware termination probability:

- try to keep each photon same power

# Pseudocode

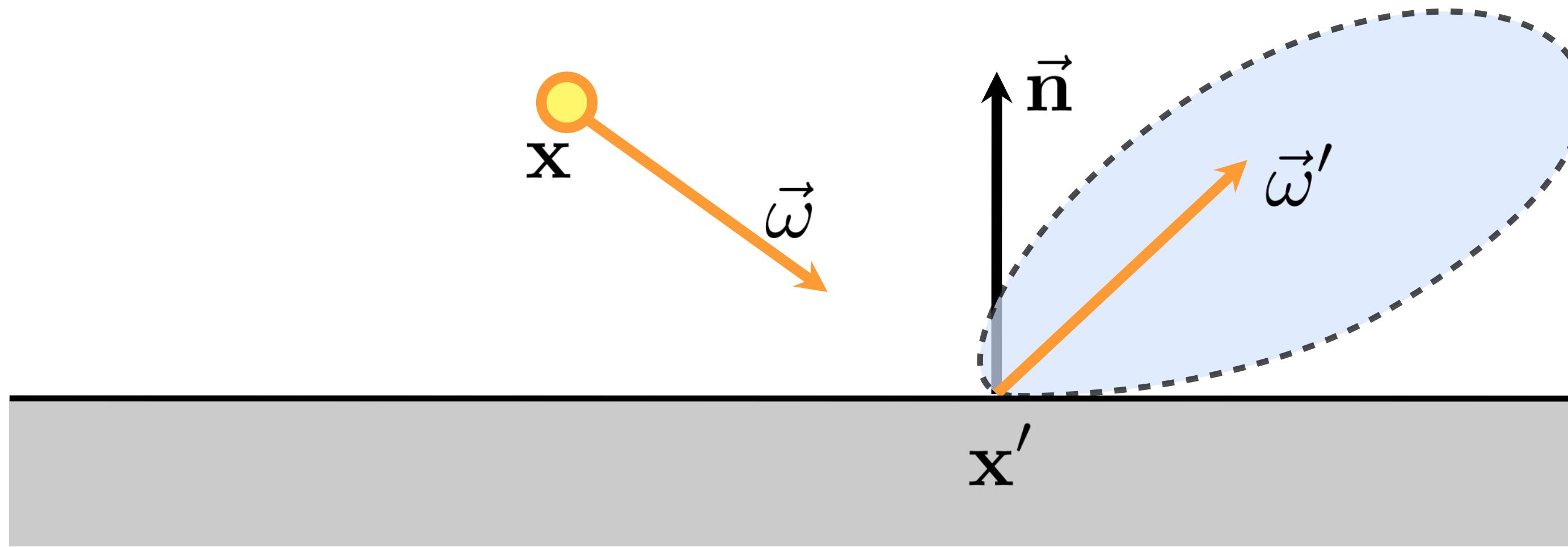
---



```
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possiblyStorePhoton(x', ω, Φ)
(ω', pdf) = sampleBSDF(x', -ω)
Φ' = Φ * absDot(n, ω') * BSDF(x', -ω, ω') / pdf
if survivedRussianRoulette(Φ, Φ')
    tracePhoton(x', ω', Φ', Φ)
```

# Pseudocode

---



```
void tracePhoton(x, ω, Φ, Φorig)
(x', n) = nearestSurfaceHit(x, ω)
possiblyStorePhoton(x', ω, Φ)
(ω', pdf) = sampleBSDF(x', -ω)
Φ' = Φ * absDot(n, ω') * BSDF(x', -ω, ω') / pdf
if survivedRussianRoulette(Φorig, Φ')
    tracePhoton(x', ω', Φ', Φorig)
```

# Russian Roulette Example

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300 photons with power 1.0 W hit a surface with reflectance 50%

Instead of reflecting 300 photons with power 0.5 W, RR will make ~150 photons continue with power 1.0 W

Very important!

# Photon Mapping

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A two-pass algorithm:

- Pass 1: Tracing of photons from light sources, and caching them in a photon map
- Pass 2: Tracing from the eye and approximating indirect illumination using the photons

# Photon Mapping

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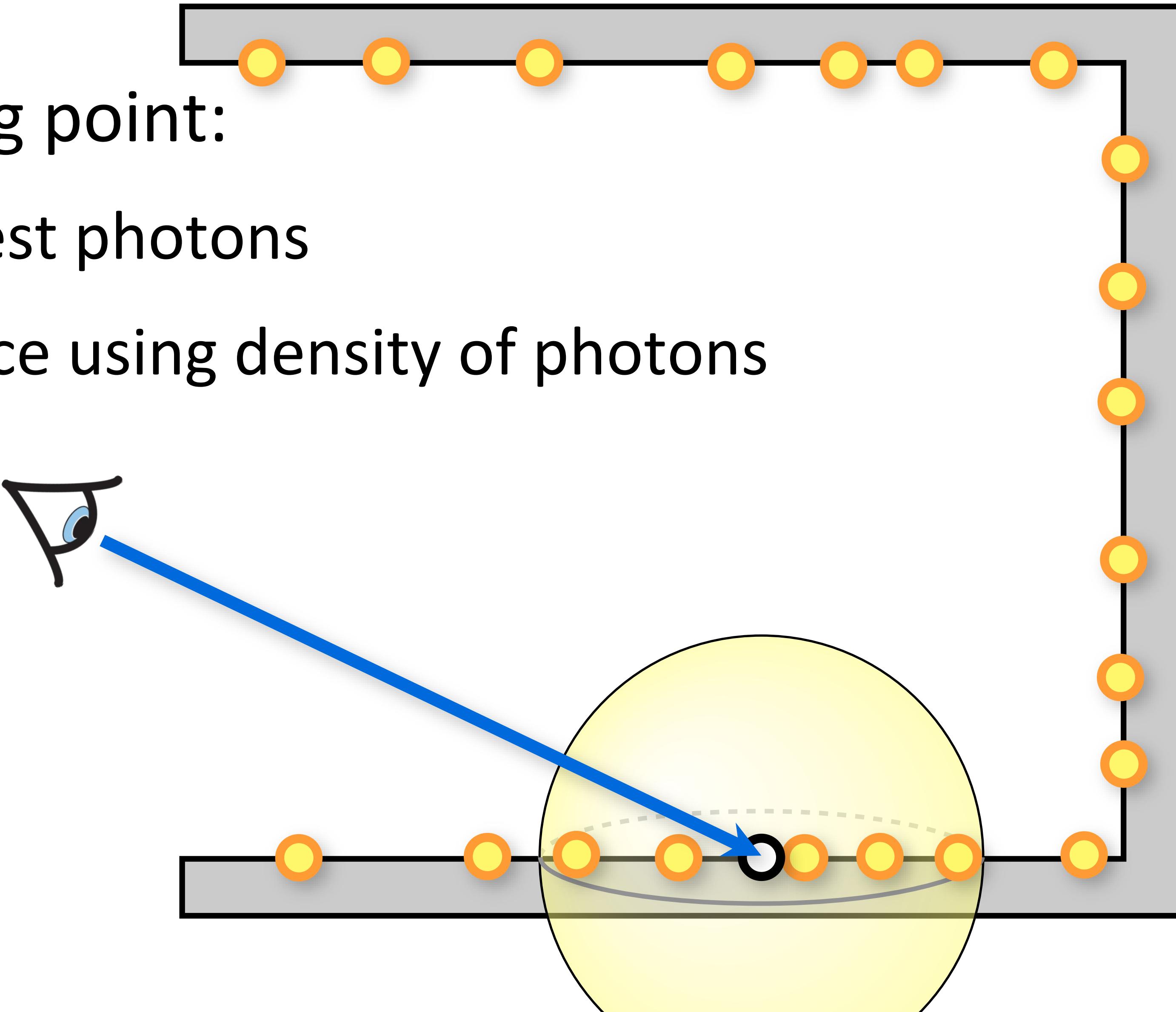
A two-pass algorithm:

- Pass 1: Tracing of photons from light sources, and caching them in a photon map
- Pass 2: Tracing from the eye and approximating indirect illumination using the photons

# Rendering

For each shading point:

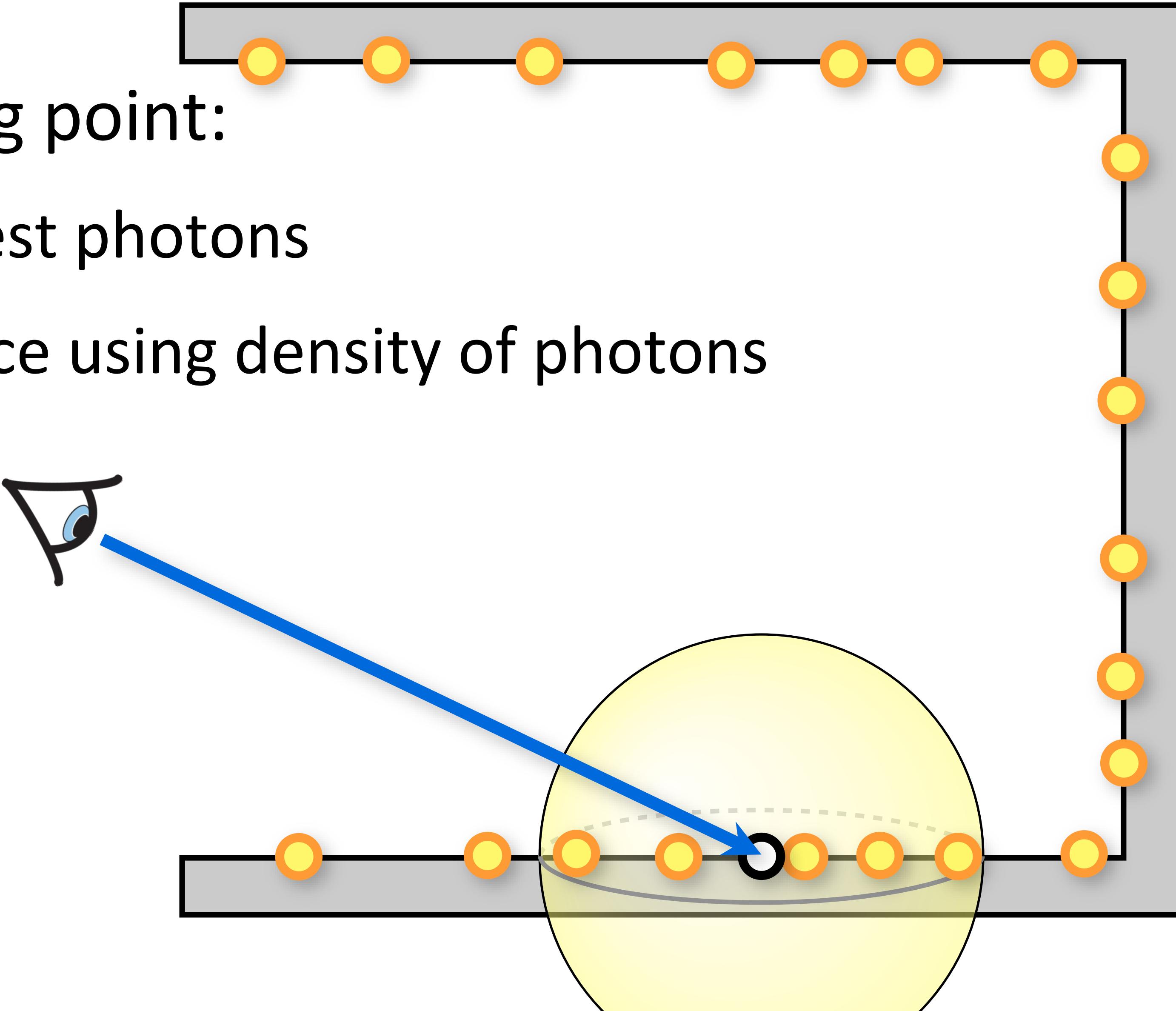
- Find the k closest photons
- Approx. radiance using density of photons



# Rendering

For each shading point:

- Find the k closest photons
- Approx. radiance using density of photons

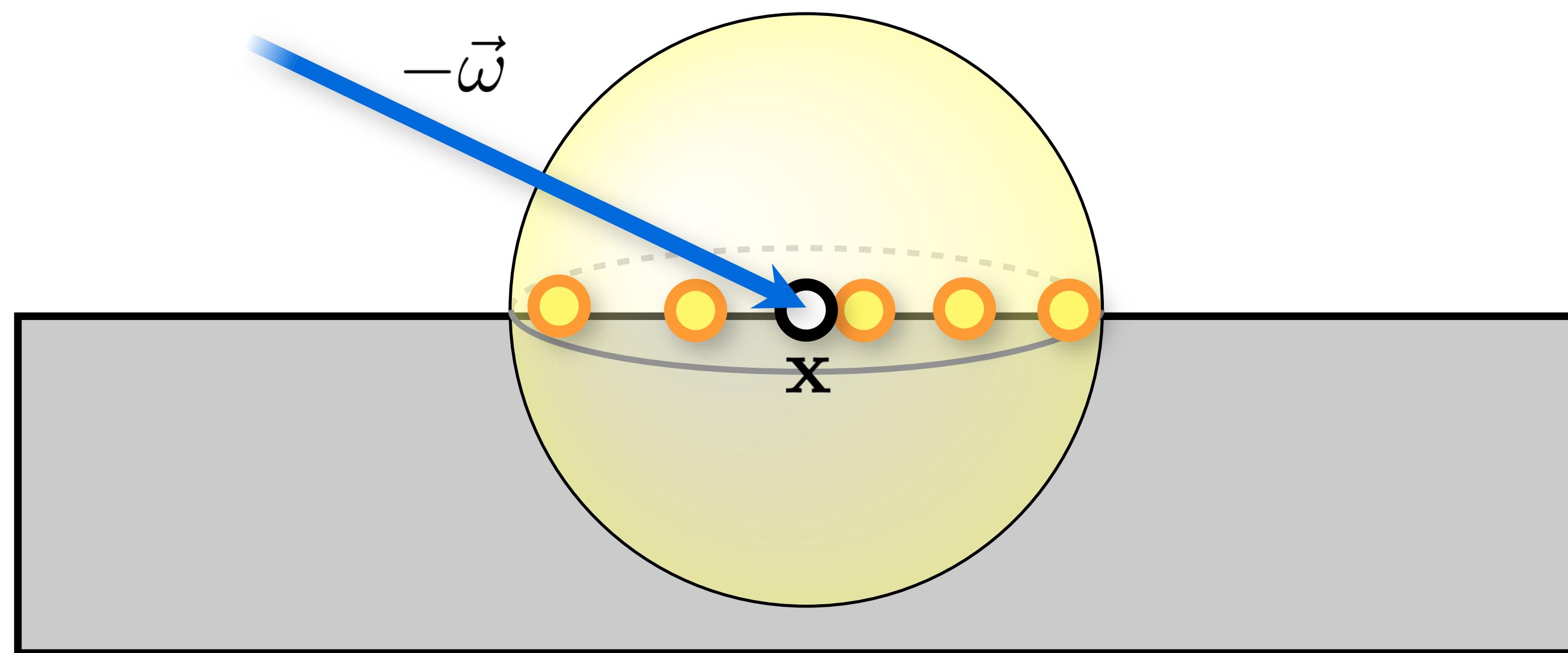


# The Radiance Estimate

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Based on kernel density estimation

- Non-parametric way of estimating the probability density of a random variable (photon density)



# The Radiance Estimate

---

Based on kernel density estimation

- Non-parametric way of estimating the probability density of a random variable (photon density)

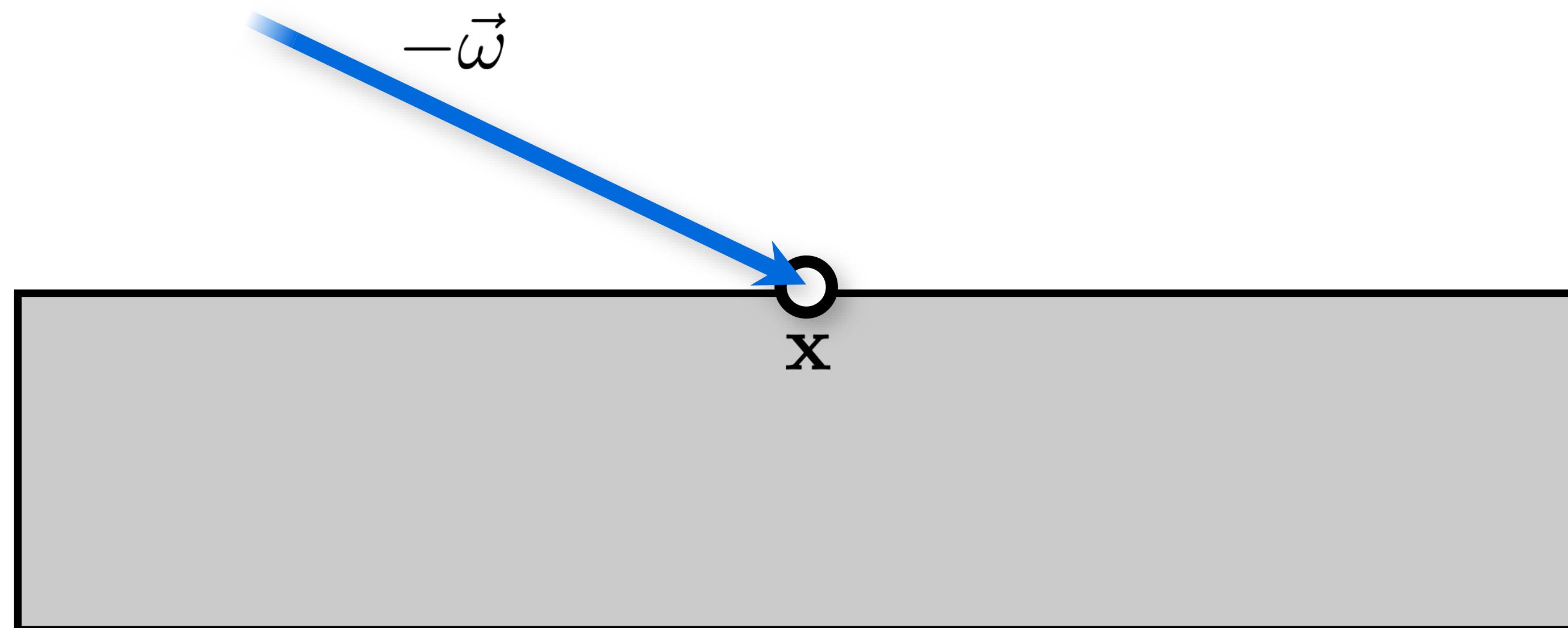
$$\begin{aligned} L_r(\mathbf{x}, \vec{\omega}) &= \int_{H^2} f_r(\mathbf{x}, \vec{\omega}', \vec{\omega}) L_i(\mathbf{x}, \vec{\omega}') \cos \theta' d\vec{\omega}' \\ &= \int_{H^2} f_r(\mathbf{x}, \vec{\omega}', \vec{\omega}) \frac{d\Phi^2(\mathbf{x}, \vec{\omega}')}{\cos \theta' d\vec{\omega}' dA} \cos \theta' d\vec{\omega}' \\ &= \int_{H^2} f_r(\mathbf{x}, \vec{\omega}', \vec{\omega}) \frac{d\Phi^2(\mathbf{x}, \vec{\omega}')}{dA} \\ &\approx \sum_{p=1}^n f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \frac{\Delta\Phi_p(\mathbf{x}, \vec{\omega}_p)}{\Delta A} \end{aligned}$$

# The Radiance Estimate

---

Approach 1: first define area, then find photons

$$L_r(\mathbf{x}, \vec{\omega}) \approx \sum_{p=1}^k f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \frac{\Phi_p}{A}$$

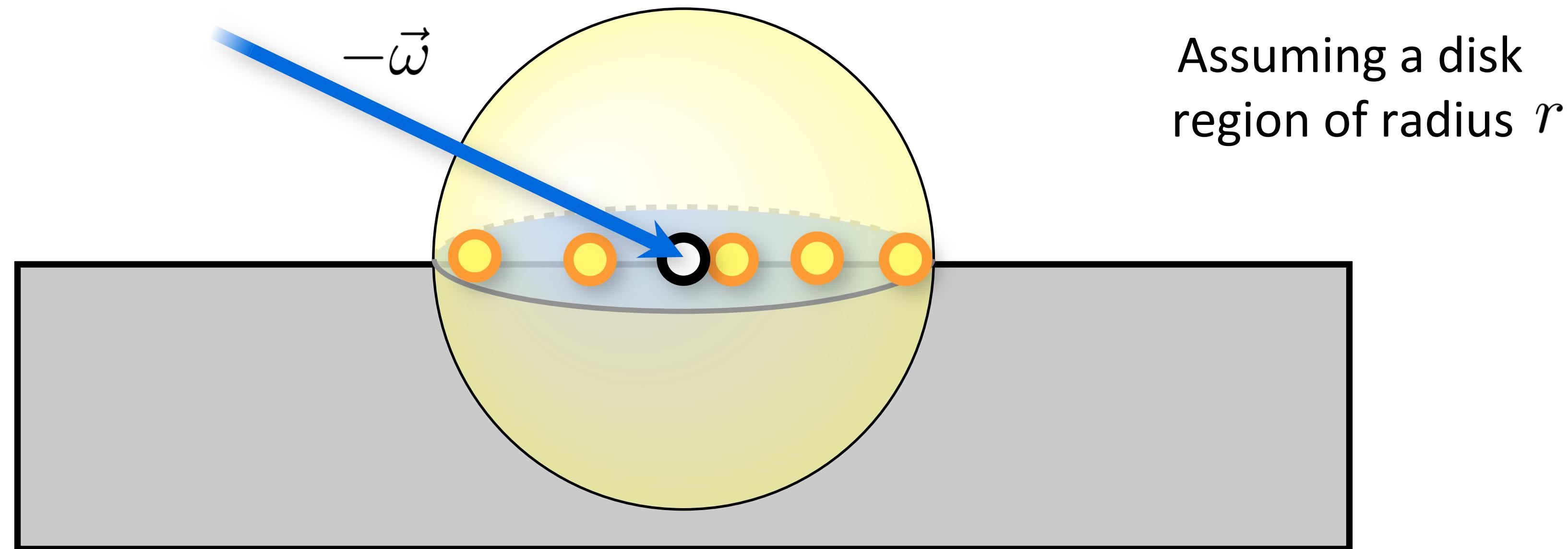


# The Radiance Estimate

Approach 1: first define area, then find photons

$$L_r(\mathbf{x}, \vec{\omega}) \approx \sum_{p=1}^k f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \frac{\Phi_p}{\pi r^2}$$

# of photons within disk

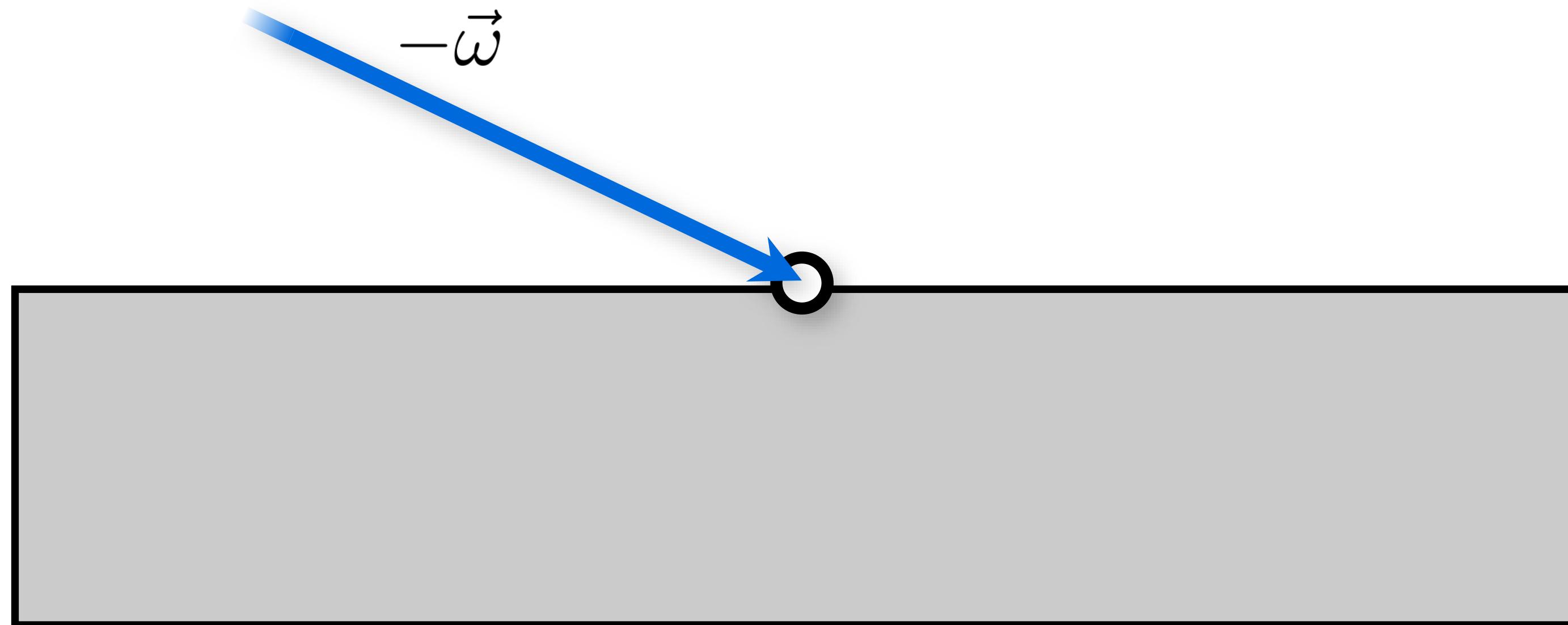


# The Radiance Estimate

---

Approach 2: first find  $k$  nearest photons, then define area

$$L_r(\mathbf{x}, \vec{\omega}) \approx \sum_{p=1}^k f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \frac{\Phi_p}{A}$$

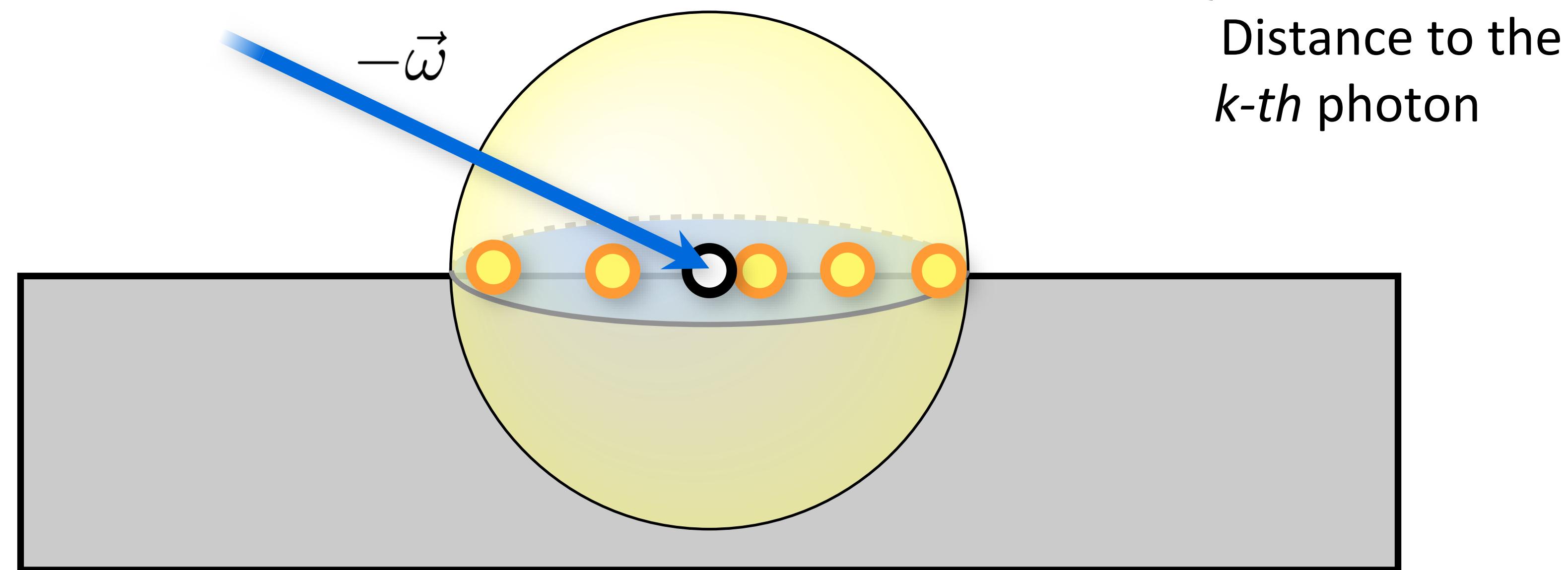


# The Radiance Estimate

Approach 2: first find  $k$  nearest photons, then define area

$$L_r(\mathbf{x}, \vec{\omega}) \approx \sum_{p=1}^{k-1} f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \frac{\Phi_p}{\pi r_k^2}$$

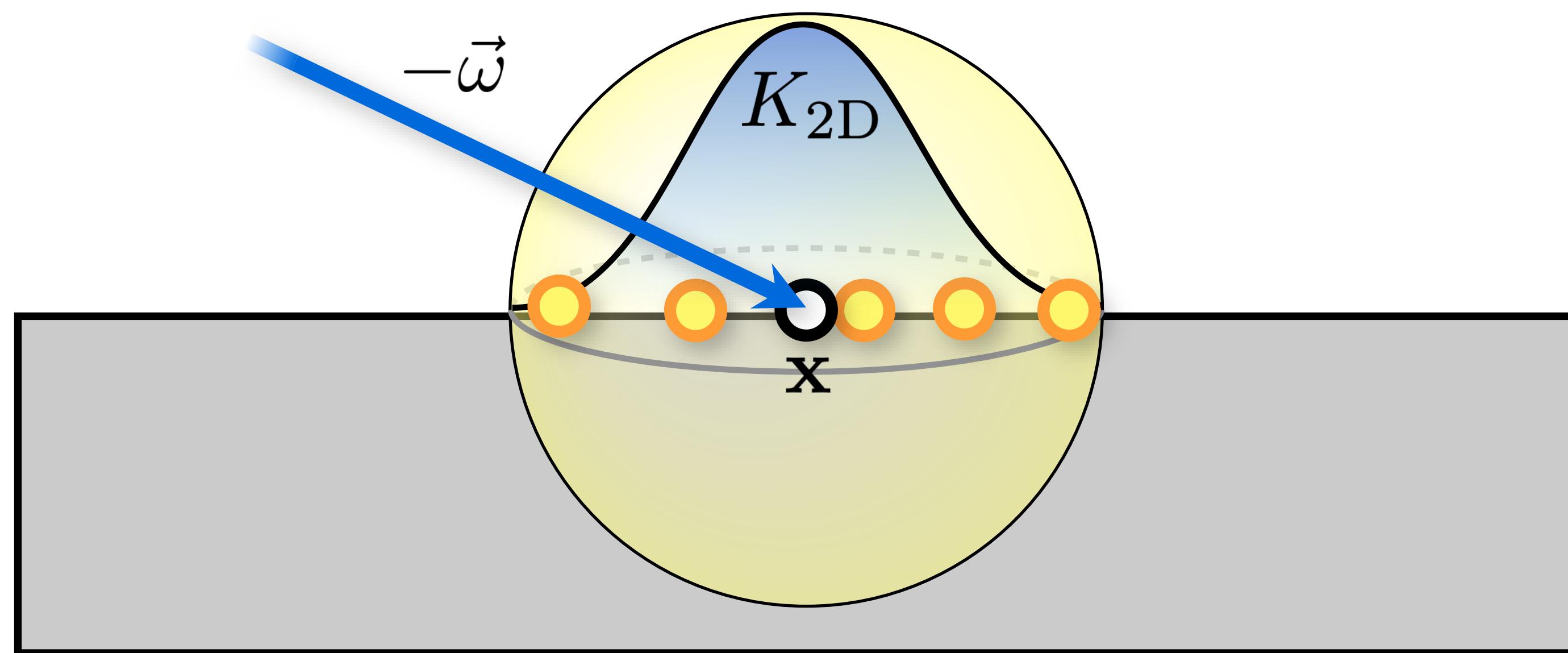
Ignore the  $k$ -th photon  
[García et al. 2012]



# The Radiance Estimate

Using a non-constant kernel:

$$L_r(\mathbf{x}, \vec{\omega}) \approx \sum_{p=1}^{k-1} f_r(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \Phi_p K_{2D}(r_p, r_k)$$

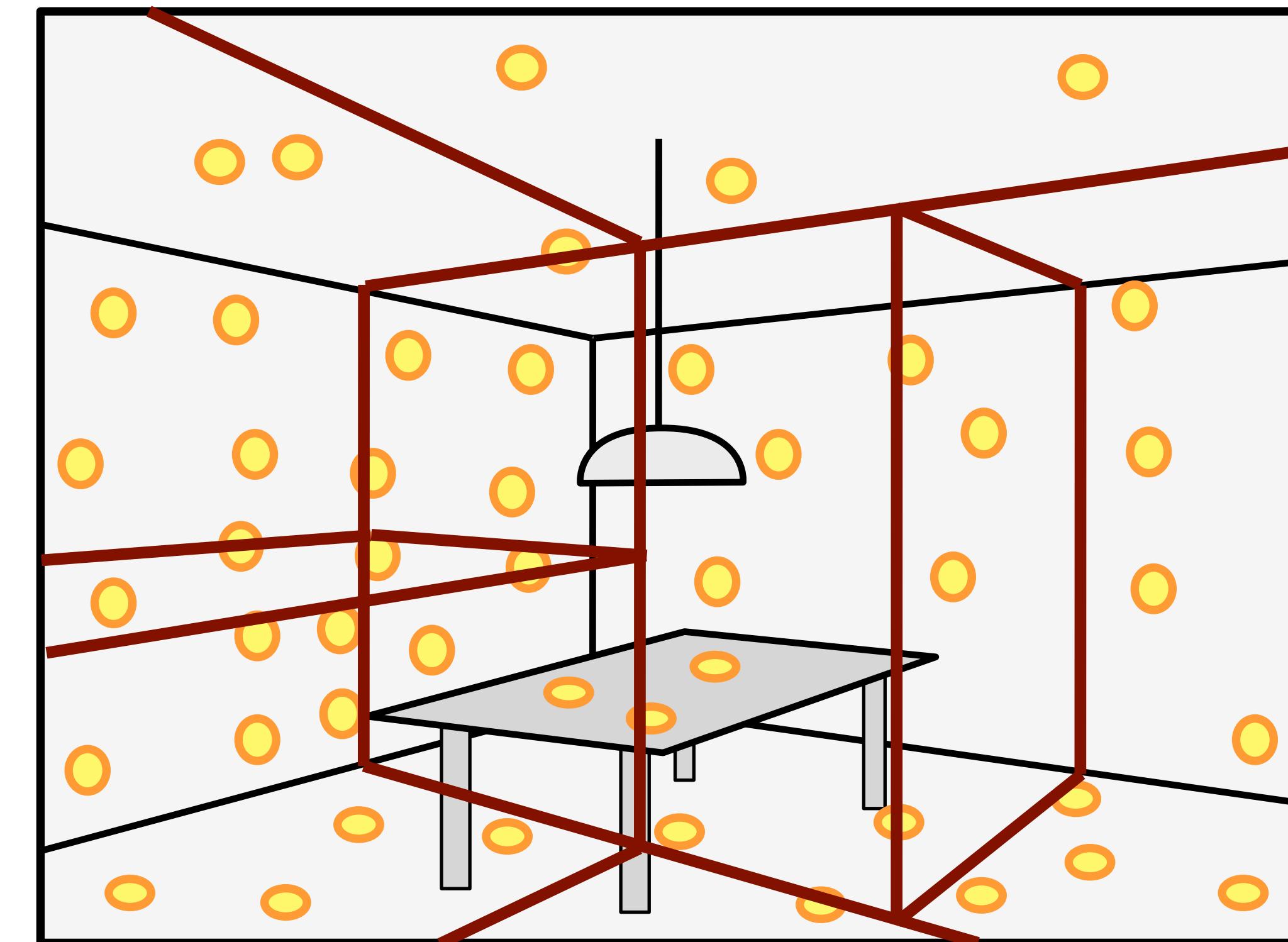


# The Photon Map Data Structure

## Requirements:

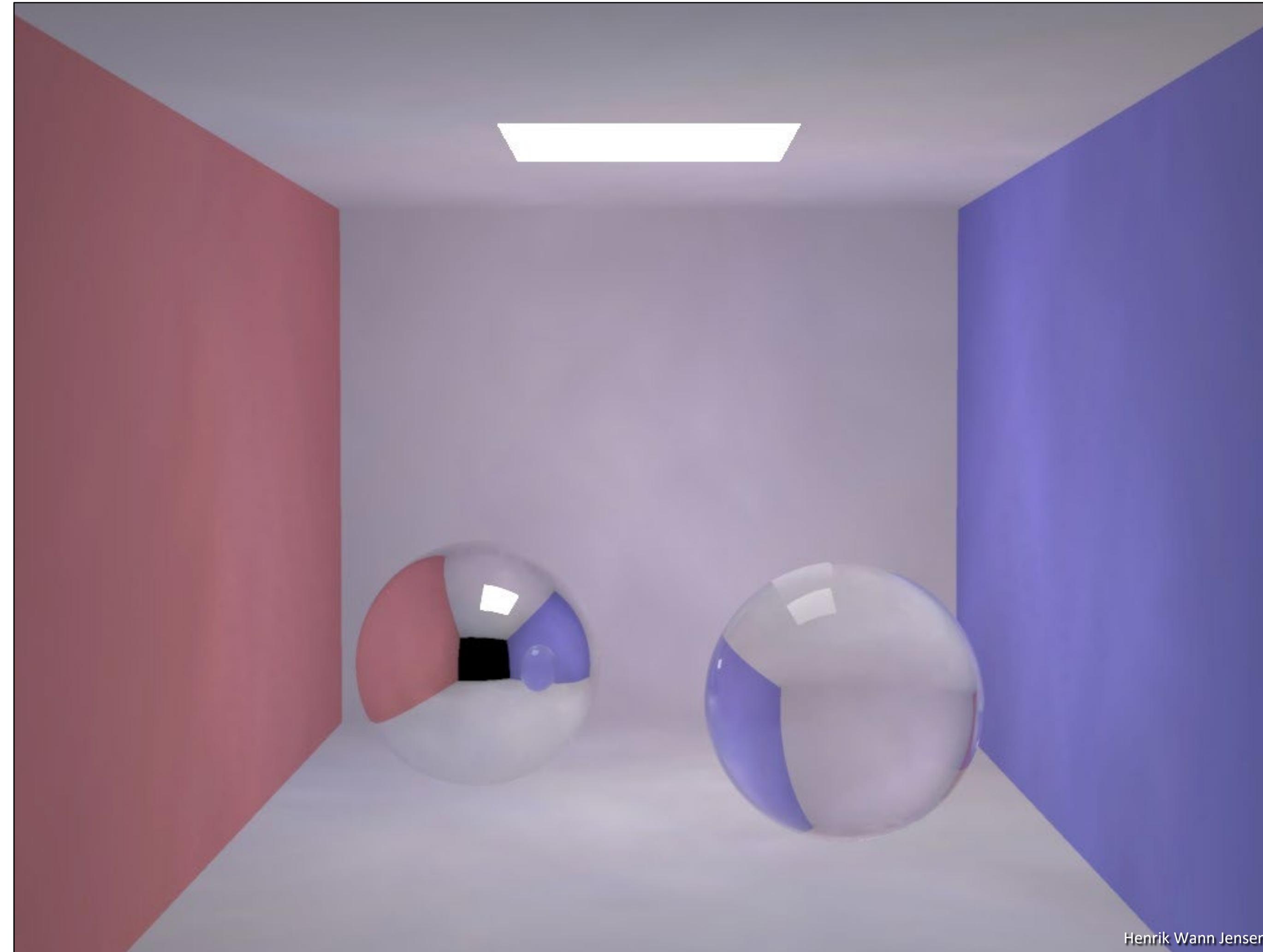
- Compact (we want many photons)
- Fast nearest neighbor search

## KD-tree



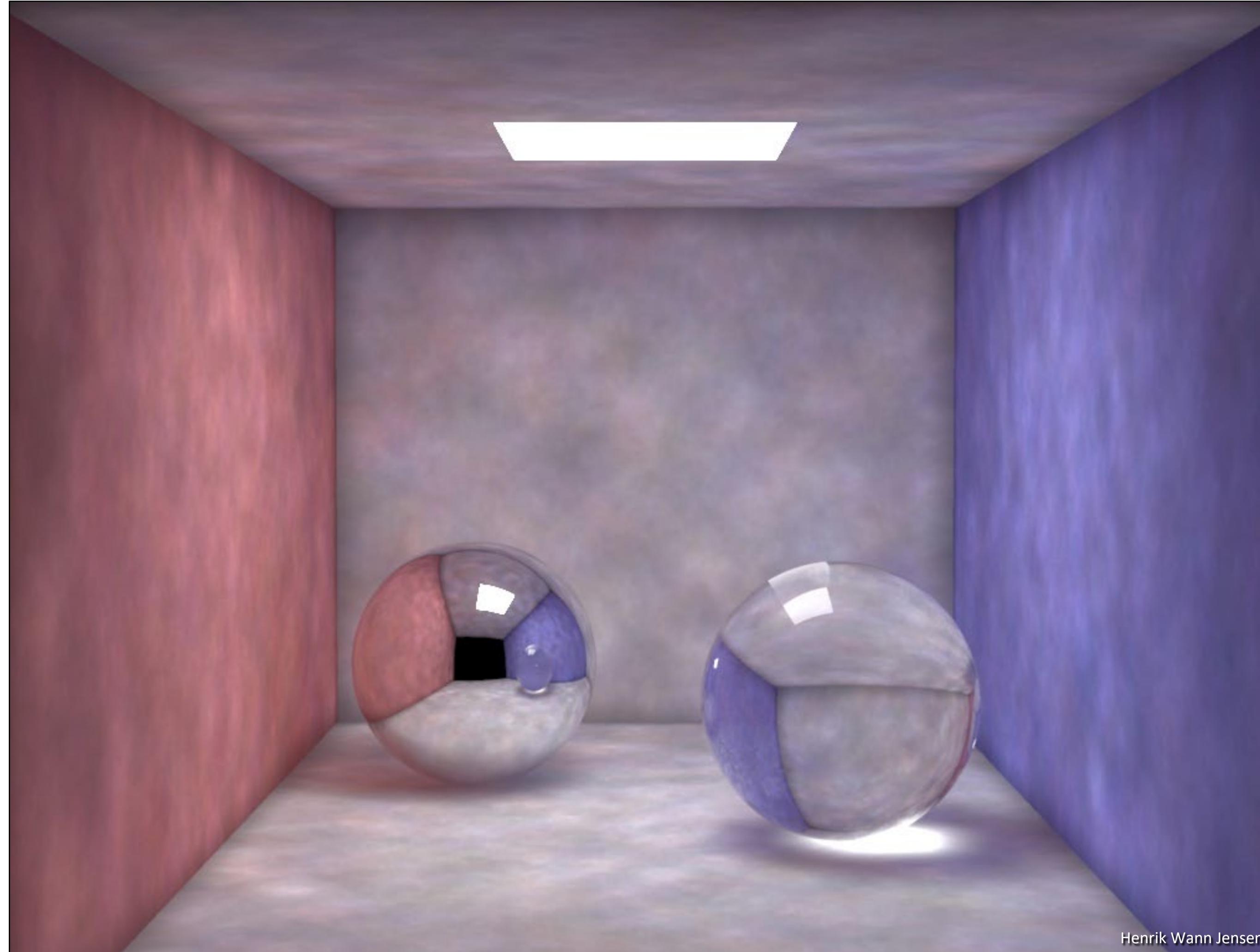
# Photon Mapping

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# Photon Mapping

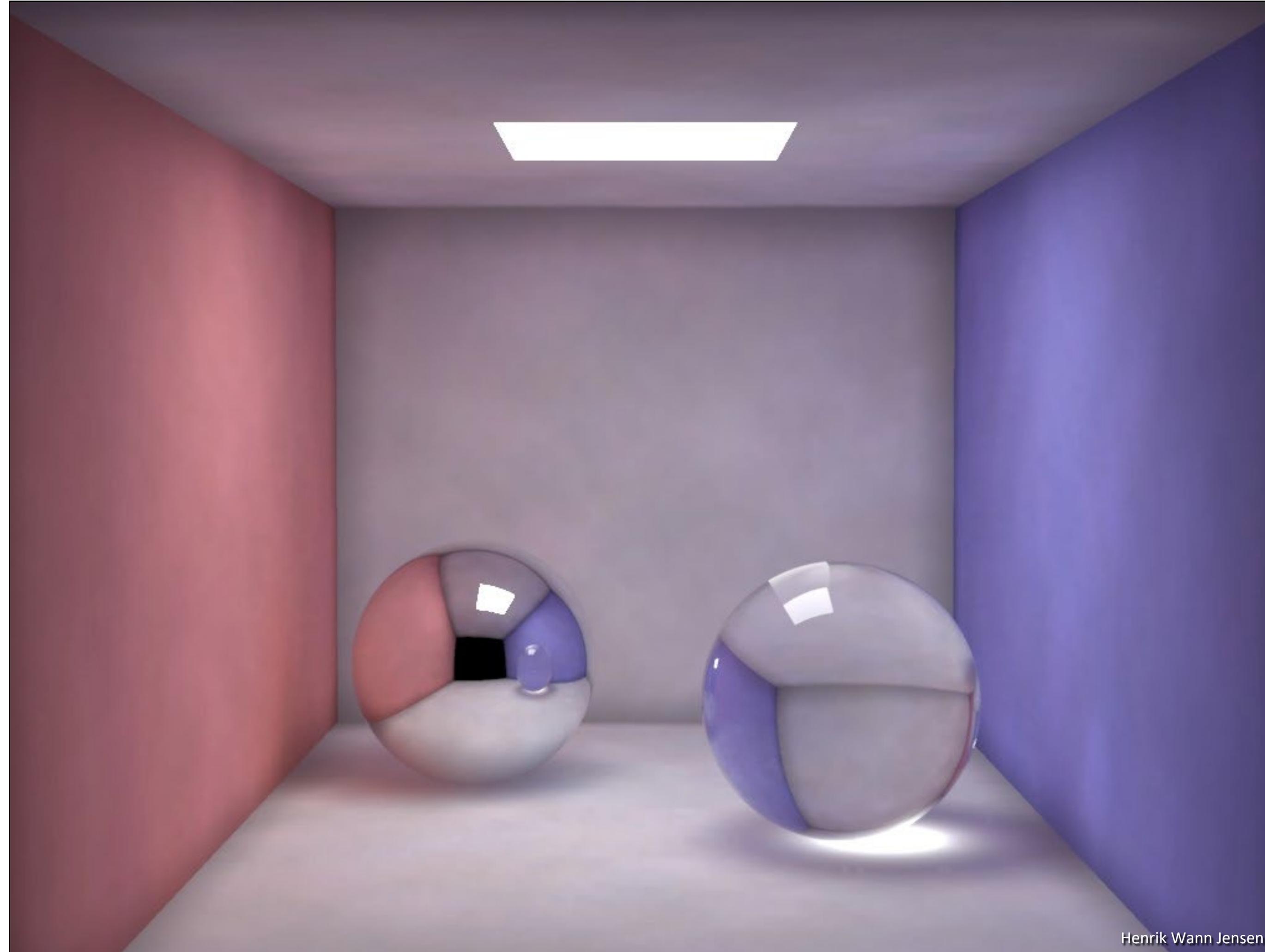
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100,000 photons / 50 photons in radiance estimate

# Photon Mapping

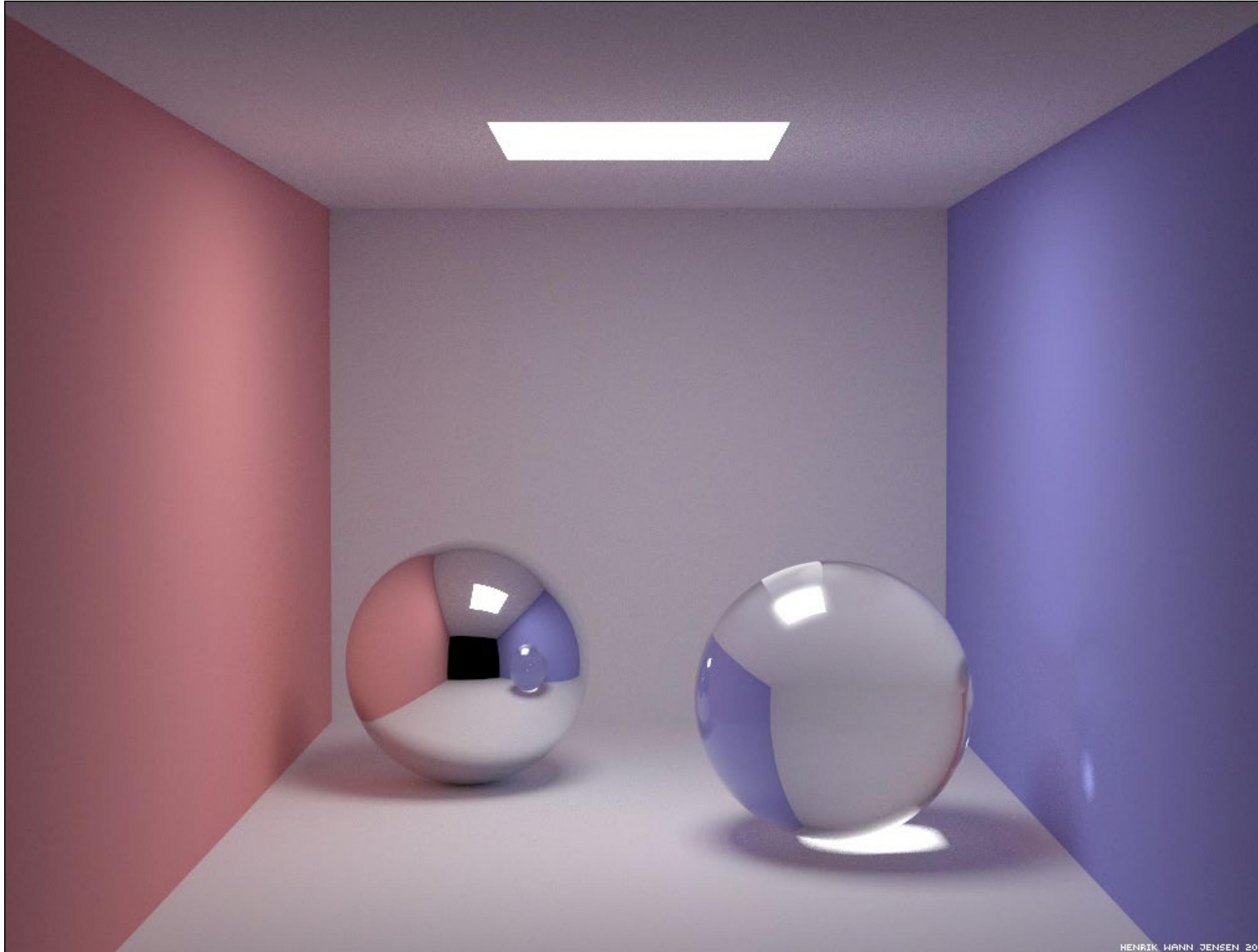
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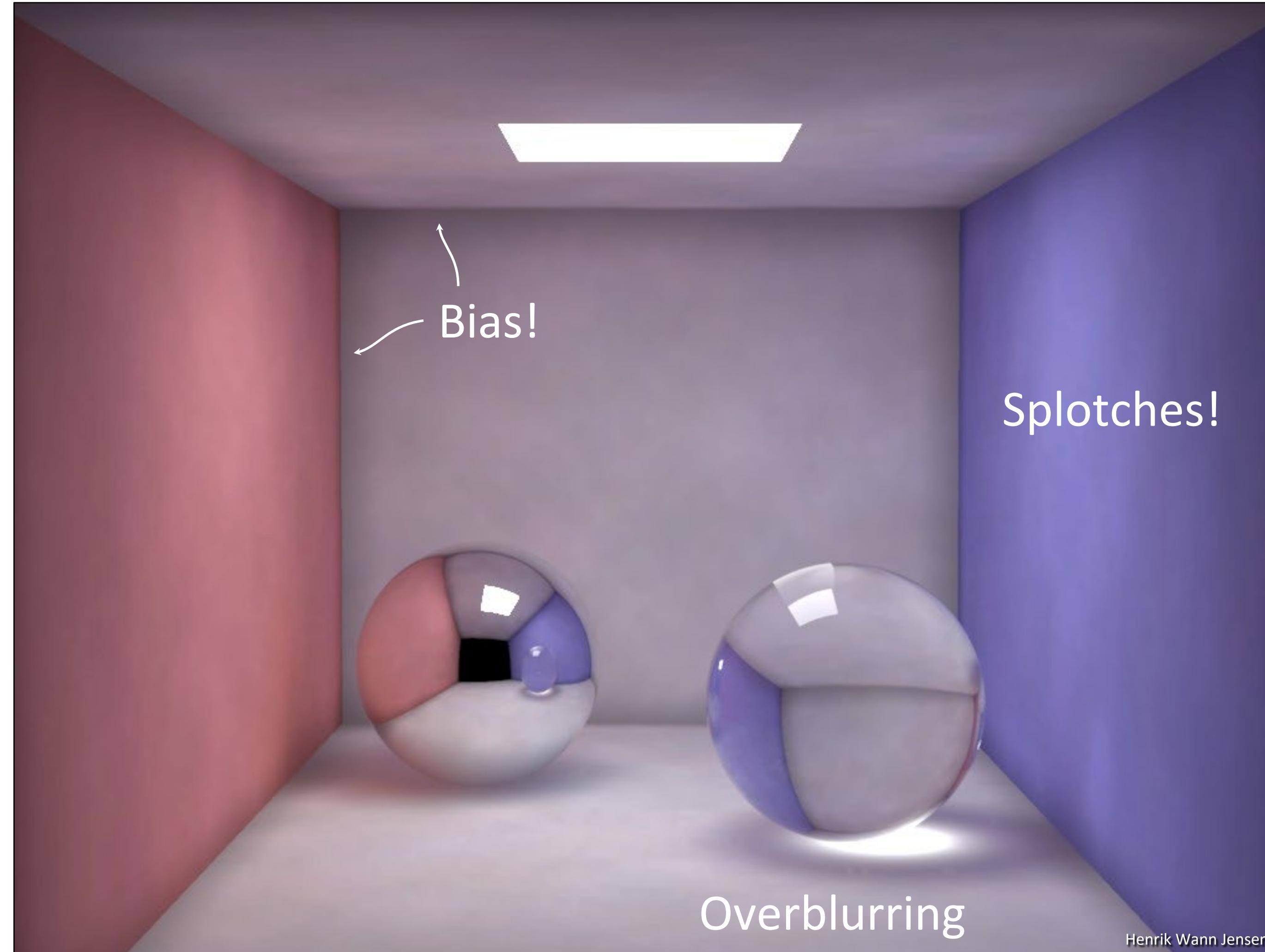
500,000 photons / 500 photons in radiance estimate

# Path Tracing

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# Photon Mapping



500,000 photons / 500 photons in radiance estimate

# Photon Mapping

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Radiance estimate contains error/bias

- Produces darker/brighter, blotchy, blurry appearance
- Requires *many* photons for high quality

Split up lighting computation into components:

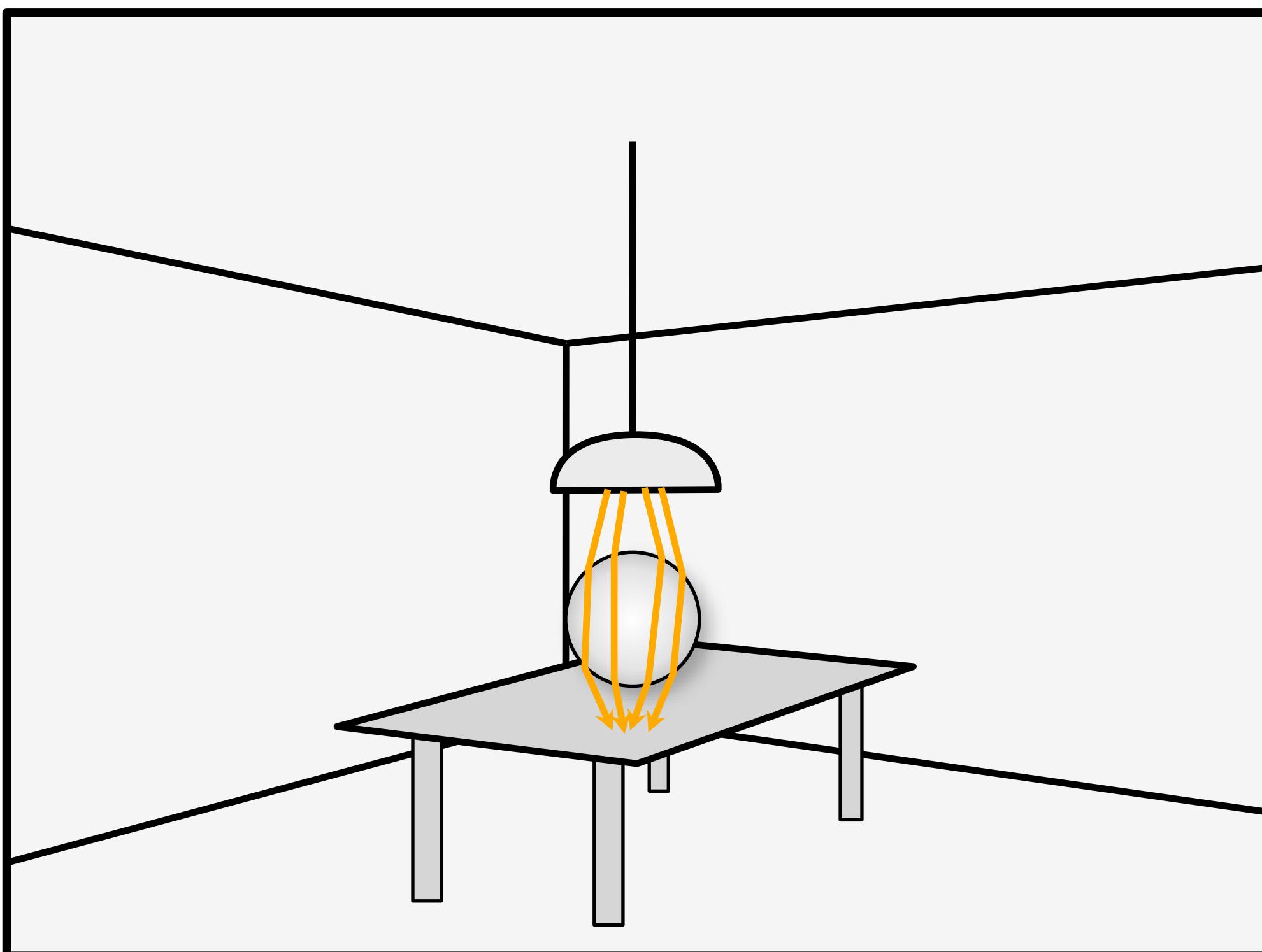
- Direct lighting
- Caustics (caustic photon map)
- Remaining indirect illumination (global photon map)

# Improving Caustics

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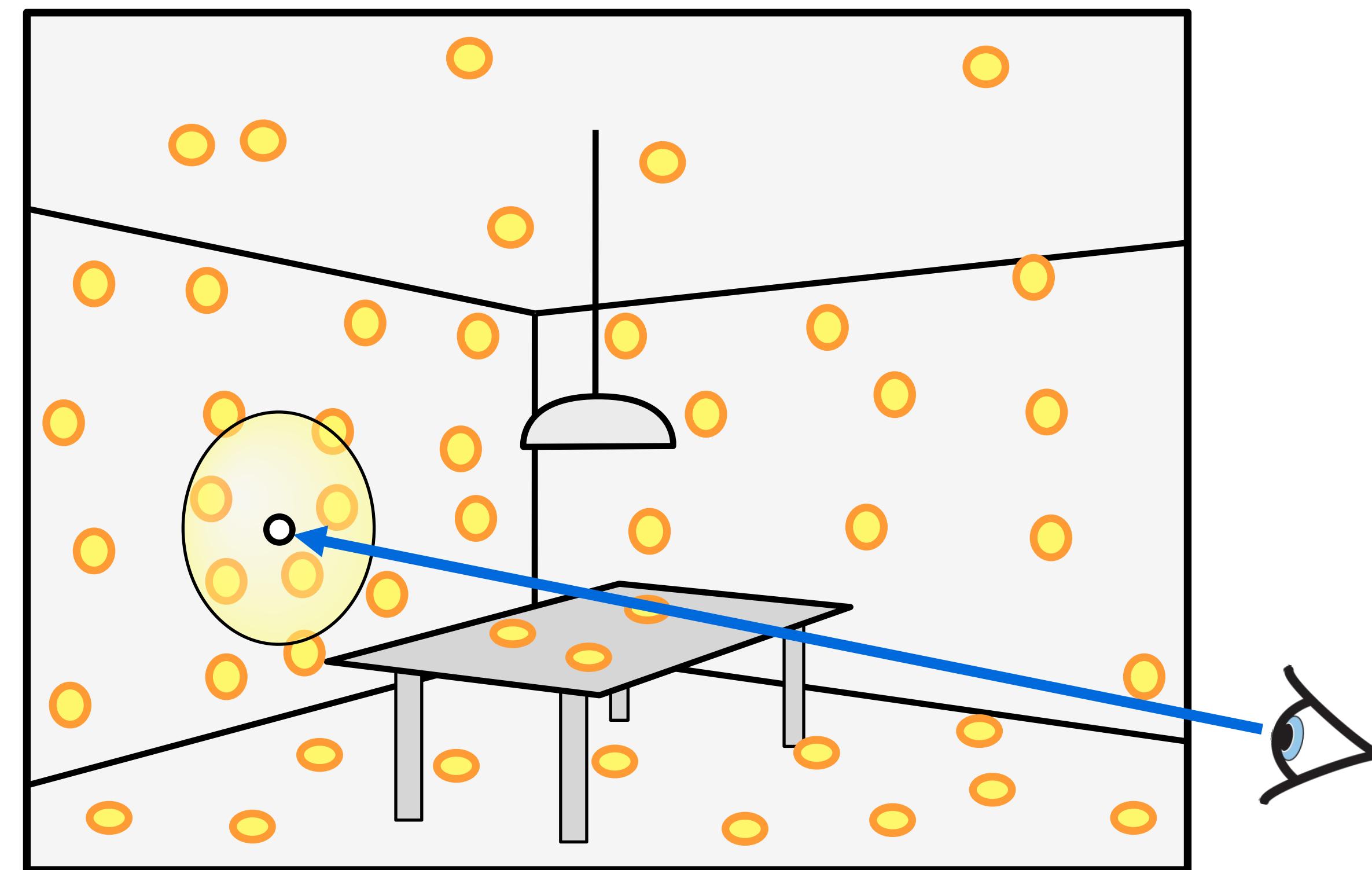
Higher quality photon map for caustics

- Only stores LS+D paths
- Many photons shot directly at specular objects



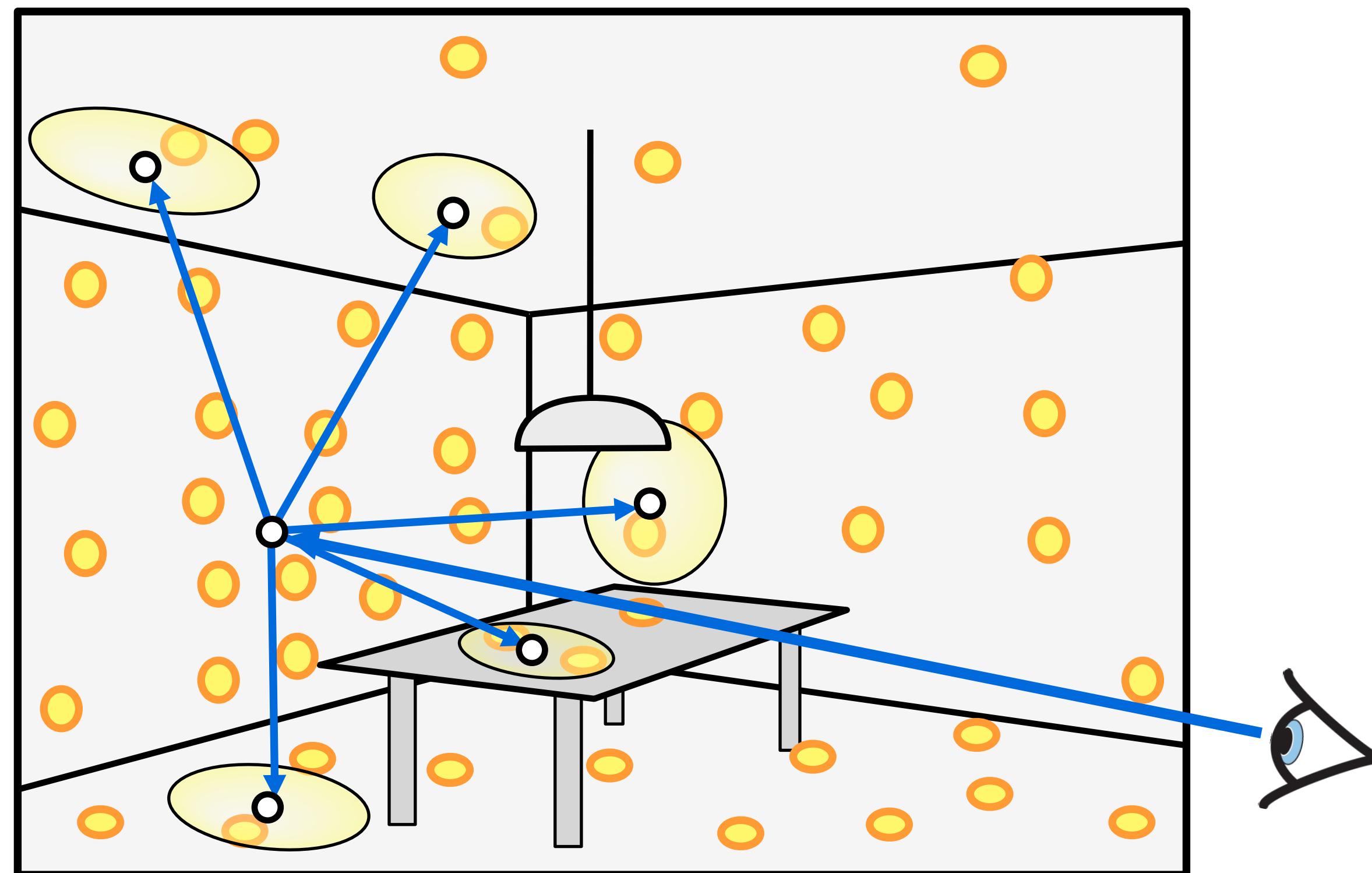
# Improving Remaining Indirect

Original approach: direct density estimation



# Improving Remaining Indirect

Improved approach: using *final gather* (i.e., path trace until second non-specular surface from camera)



# Improved Photon Mapping

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## Camera tracing

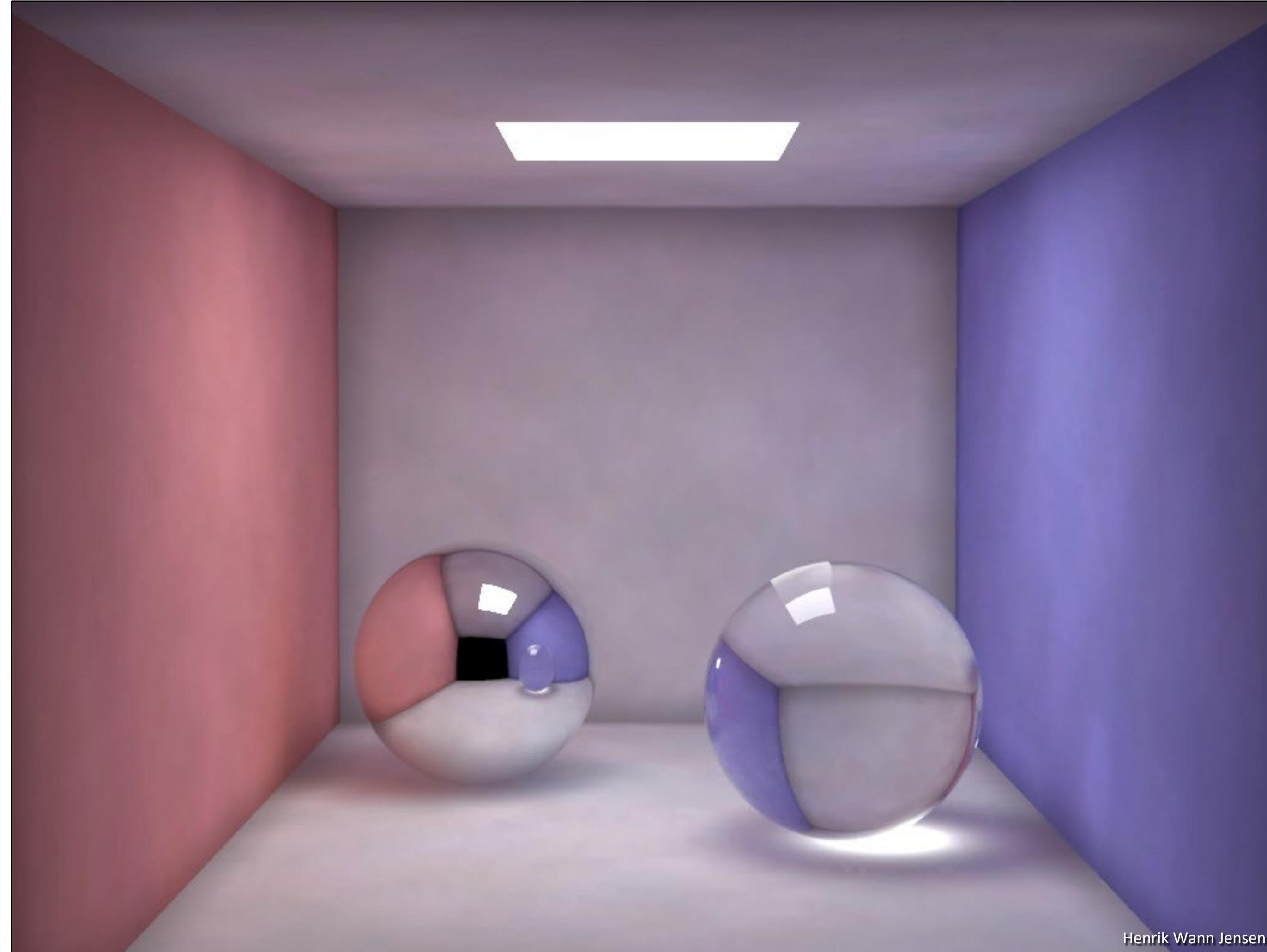
- Trace camera paths until they hit the first non-specular surface point  $x$

At  $x$  we sum:

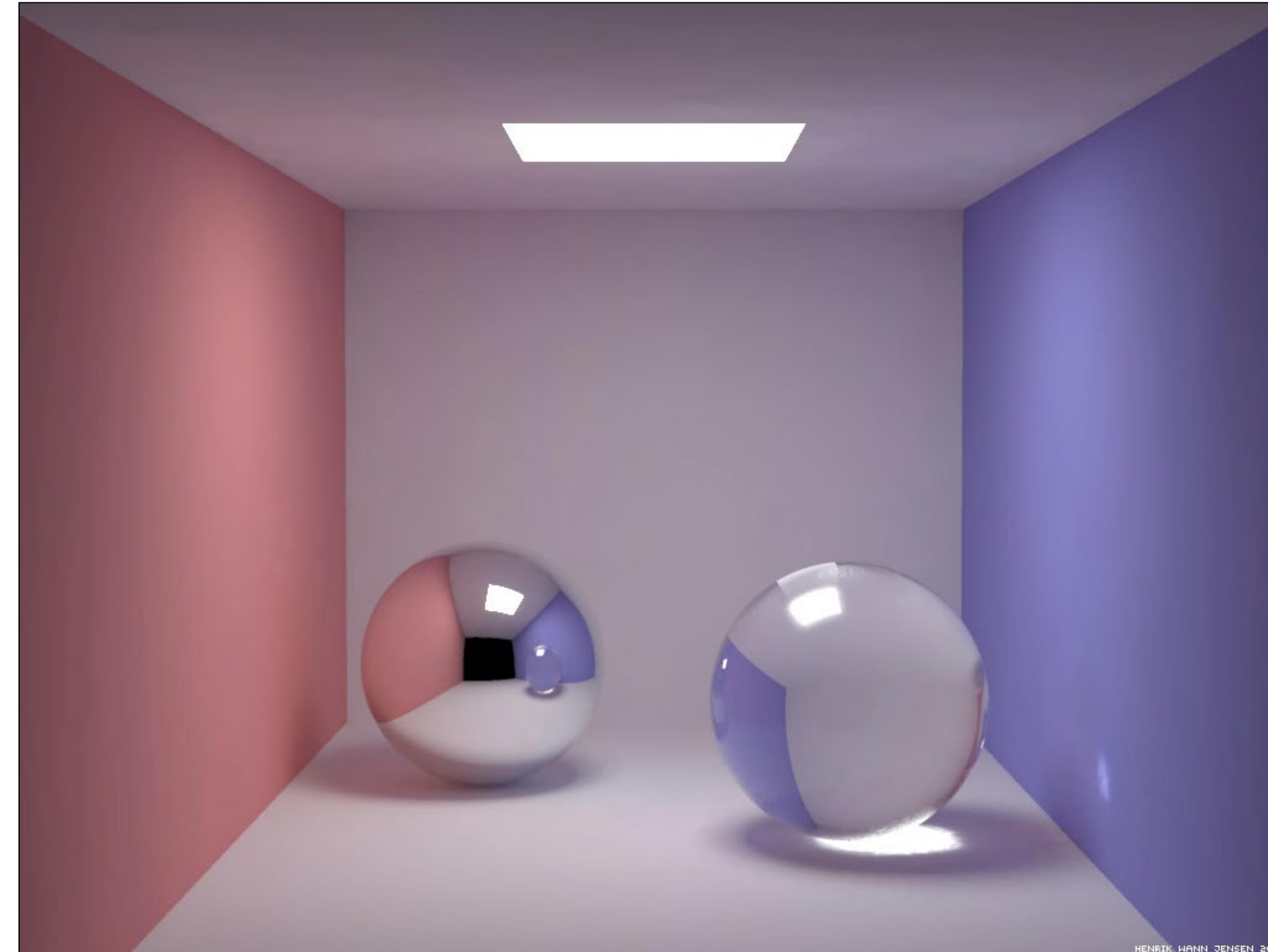
- Emission
- Direct illumination: trace shadow rays to lights
- Caustics: density estimation at  $x$  using only the *caustic* photon map
- Remaining indirect: continue path tracing until next non-specular vertex  $y$ ,  
perform density estimation from global photon map at  $y$

# Photon Mapping

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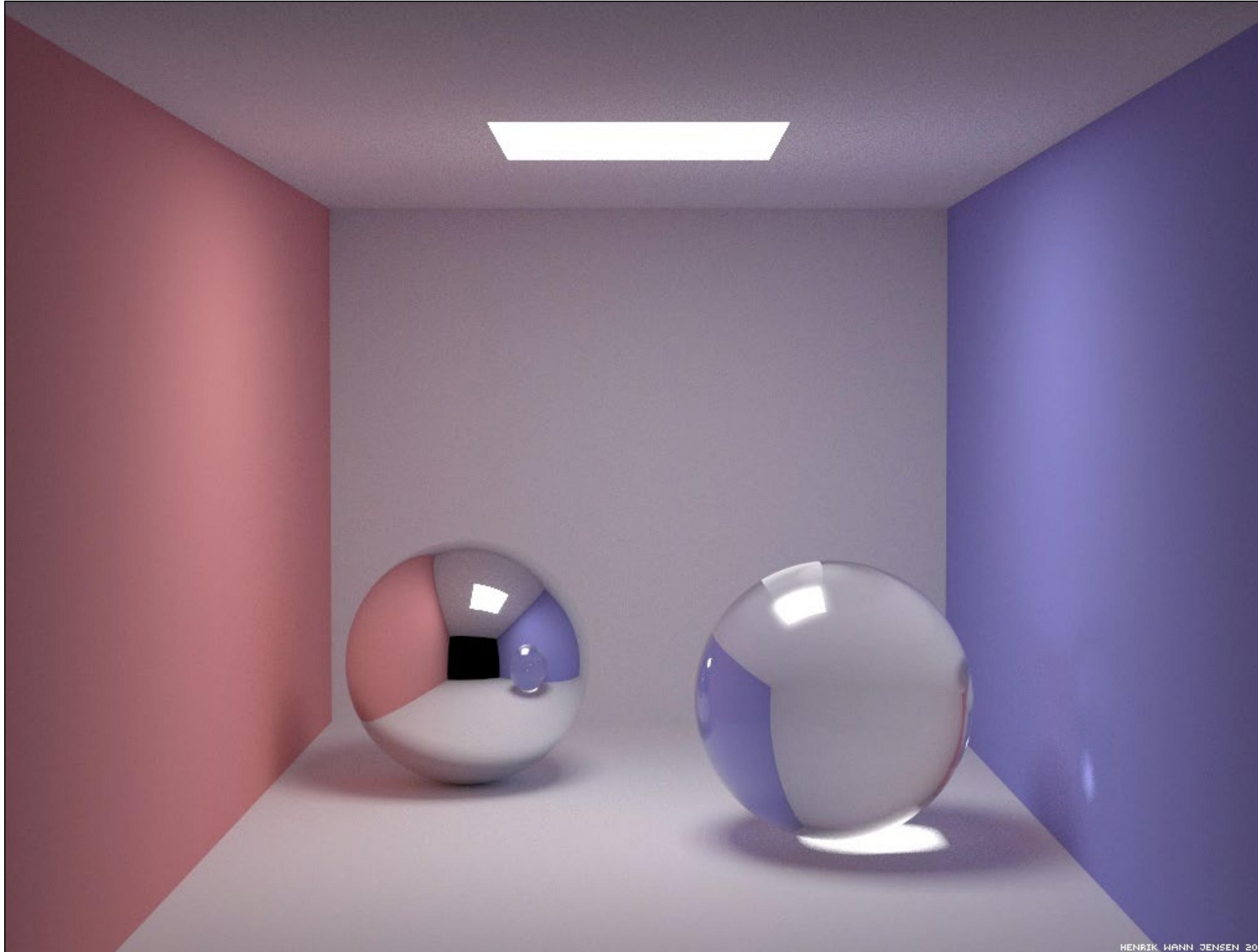
# Photon Mapping (Improved)



final gather + global photon map (200000) + caustic photon map (50000)

# Path Tracing

---



# Validation Tests

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Test idea 1:

- store only direct photons
- visualize photon map directly
- compare to standard direct illumination
- should look identical with many photons

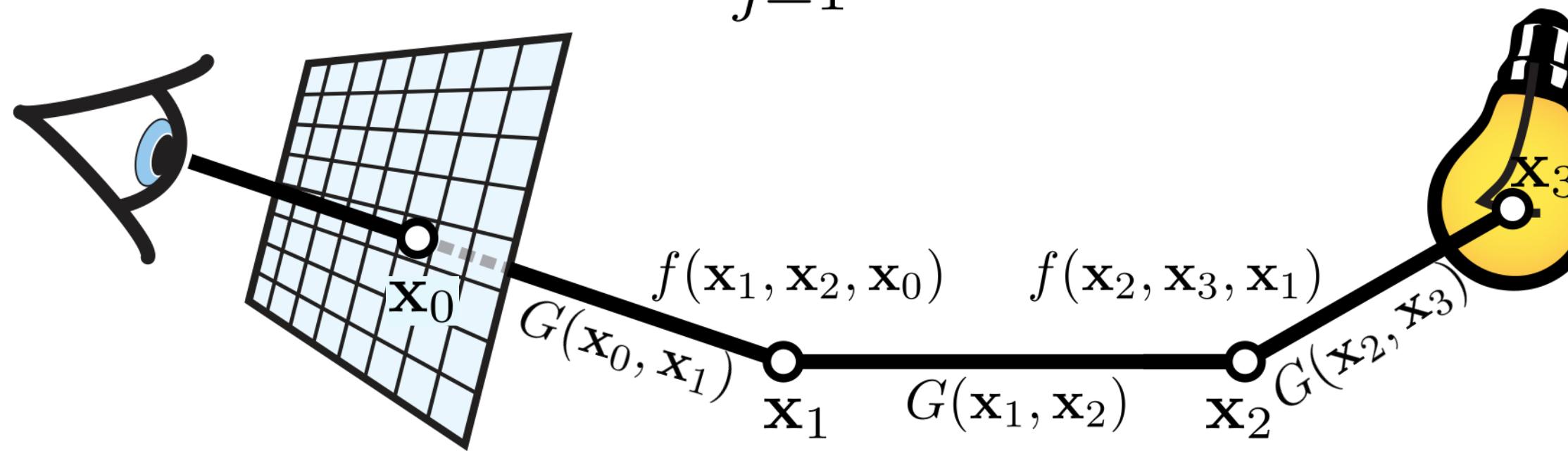
Test idea 2:

- create a perfectly transparent sphere ( $\text{IOR} = 1.0$ )
- store only caustic photons
- render direct illumination + caustics
- shadow should disappear

# Recall: Path Integral Measurement Eq.

$$I_j = \int_{\mathcal{P}} W_e(\mathbf{x}_0, \mathbf{x}_1) L_e(\mathbf{x}_k, \mathbf{x}_{k-1}) T(\bar{\mathbf{x}}) d\bar{\mathbf{x}}$$

path throughput  $T(\bar{\mathbf{x}}) = G(\mathbf{x}_0, \mathbf{x}_1) \prod_{j=1}^{k-1} f(\mathbf{x}_j, \mathbf{x}_{j+1}, \mathbf{x}_{j-1}) G(\mathbf{x}_j, \mathbf{x}_{j+1})$



- Monte Carlo estimator:

$$I \approx \frac{W_e(\mathbf{x}_0, \mathbf{x}_1) T(\mathbf{x}_0, \dots, \mathbf{x}_k) L_e(\mathbf{x}_k, \mathbf{x}_{k-1})}{\underbrace{p(\mathbf{x}_0, \dots, \mathbf{x}_k)}_{\text{joint PDF of path vertices}}}$$

# Photon Mapping

$$I \approx \frac{W_e(\mathbf{x}_0, \mathbf{x}_1)T(\mathbf{x}_0, \dots, \mathbf{x}_k)L_e(\mathbf{x}_k, \mathbf{x}_{k-1})}{p(\mathbf{x}_0, \dots, \mathbf{x}_k)}$$

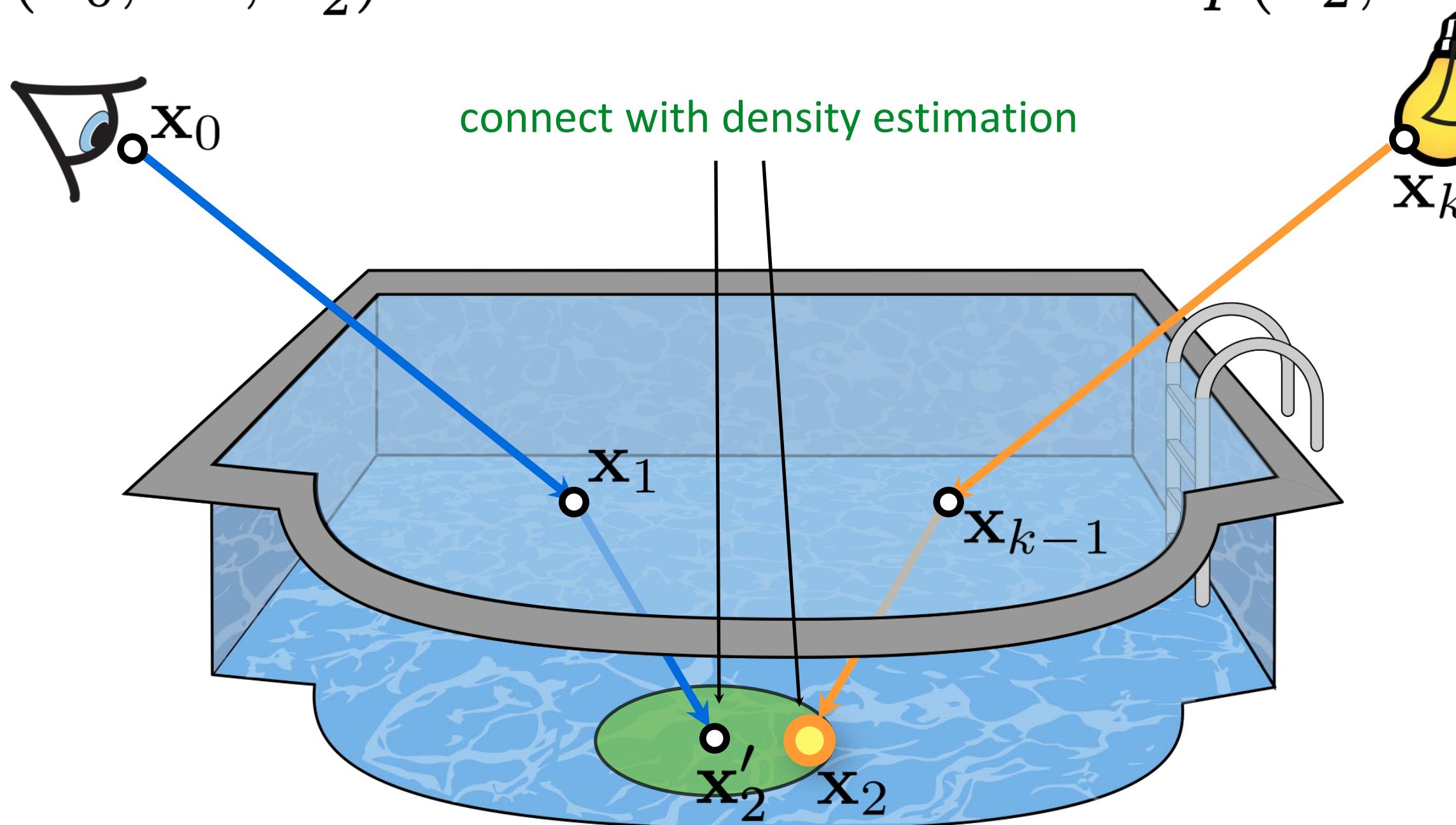
split path contribution into two parts

eye subpath

$$\frac{W_e(\mathbf{x}_0, \mathbf{x}_1)T(\mathbf{x}_0, \dots, \mathbf{x}'_2)}{p(\mathbf{x}_0, \dots, \mathbf{x}'_2)}$$

light subpath/photon “power”

$$\Phi_p = \frac{T(\mathbf{x}_2, \dots, \mathbf{x}_k)L_e(\mathbf{x}_k, \mathbf{x}_{k-1})}{p(\mathbf{x}_2, \dots, \mathbf{x}_k)}$$



# Light Sources in the Real World

---

Complex shape

Covered with transparent materials

Only a small part emits light



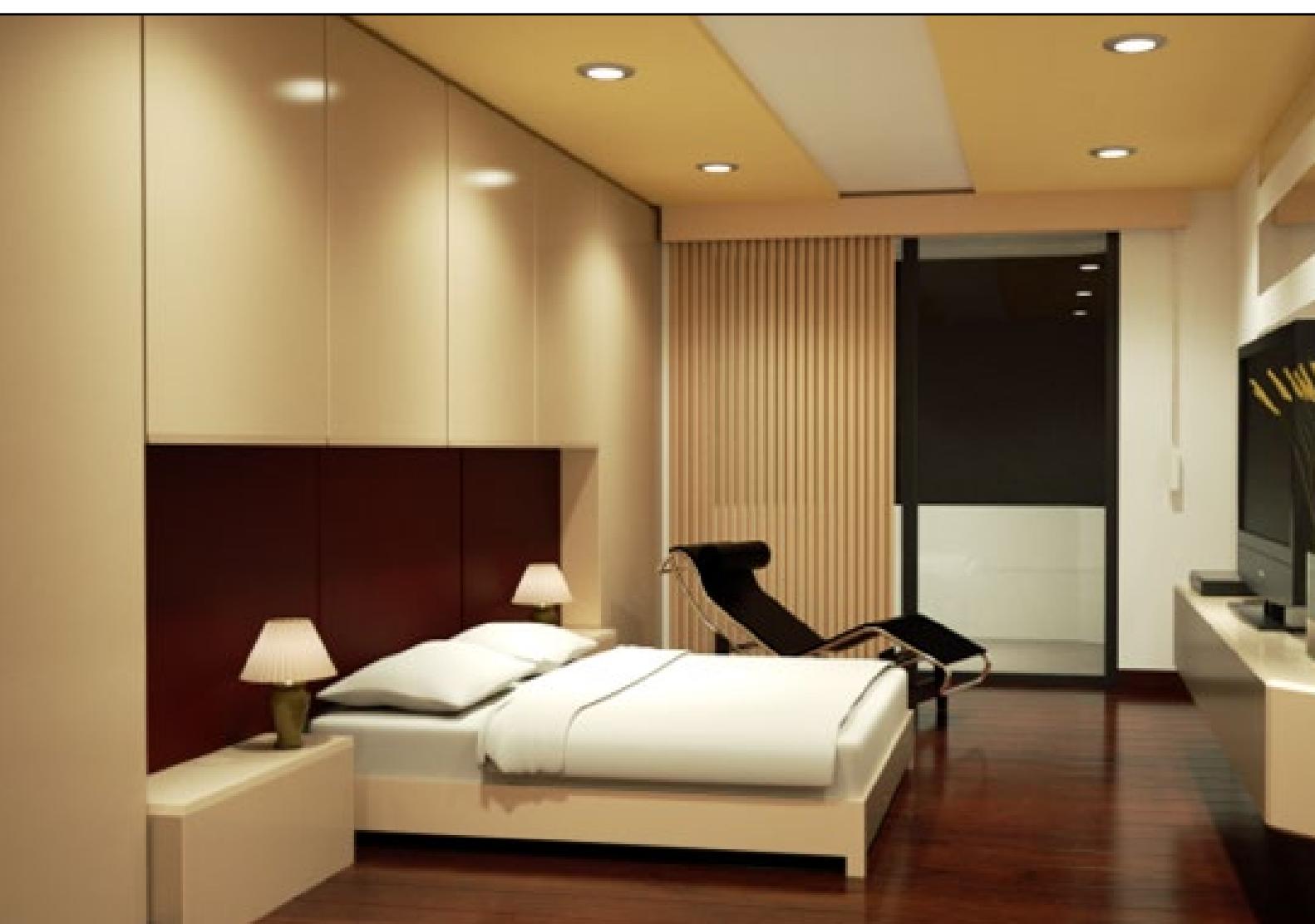
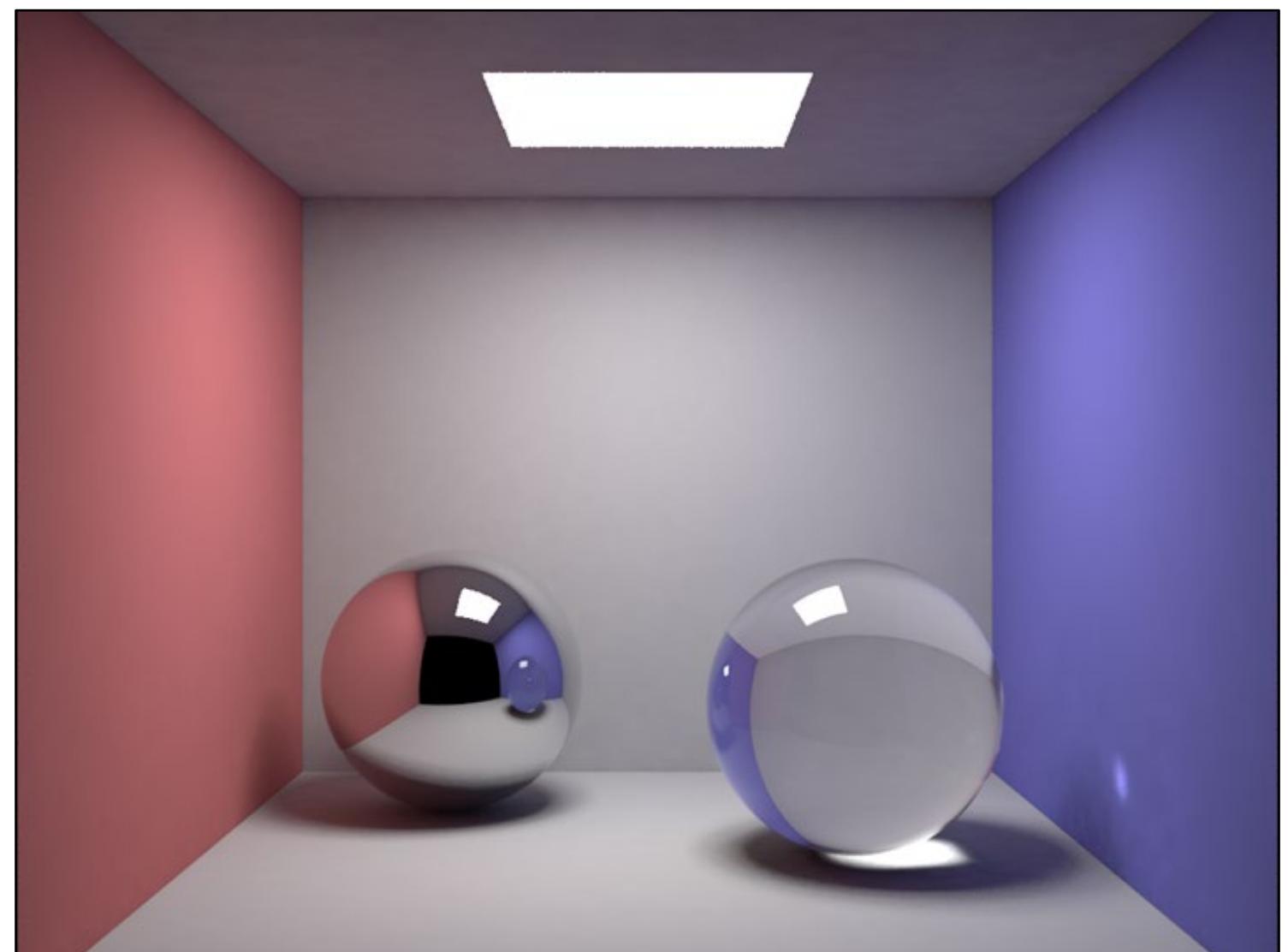
# Light Sources in CG

---

Simple shape

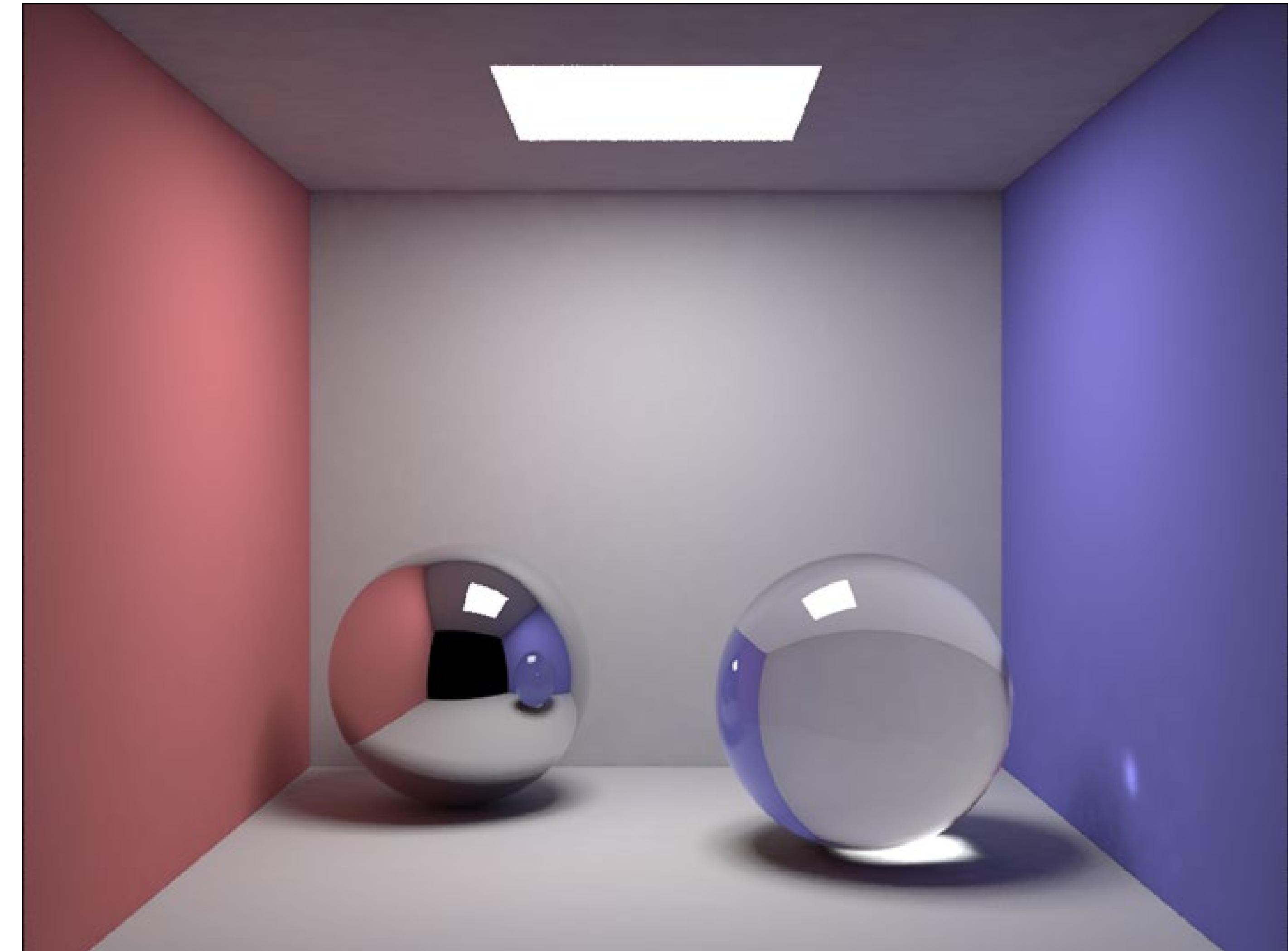
Bare light source

Entire part emits light

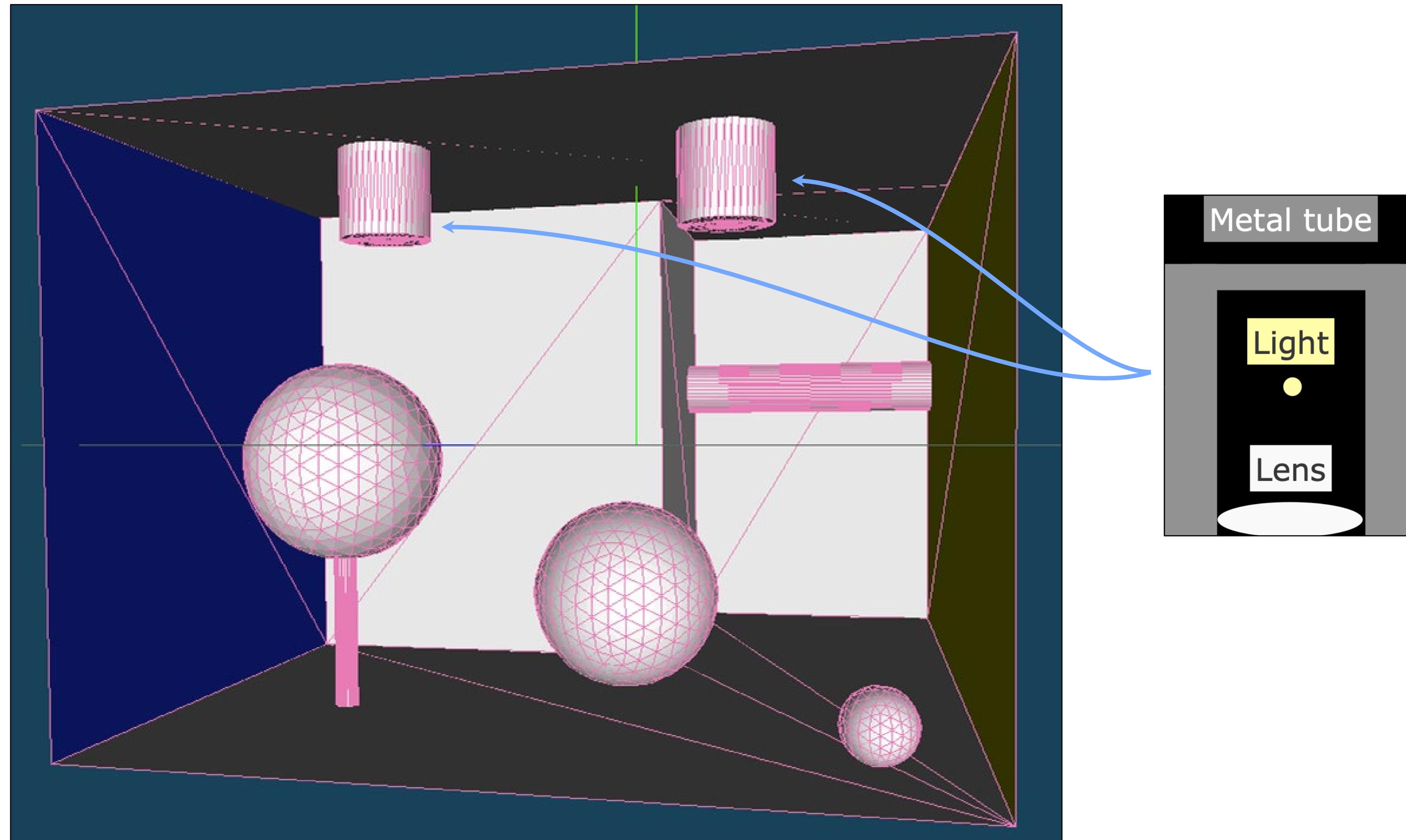


# Why?

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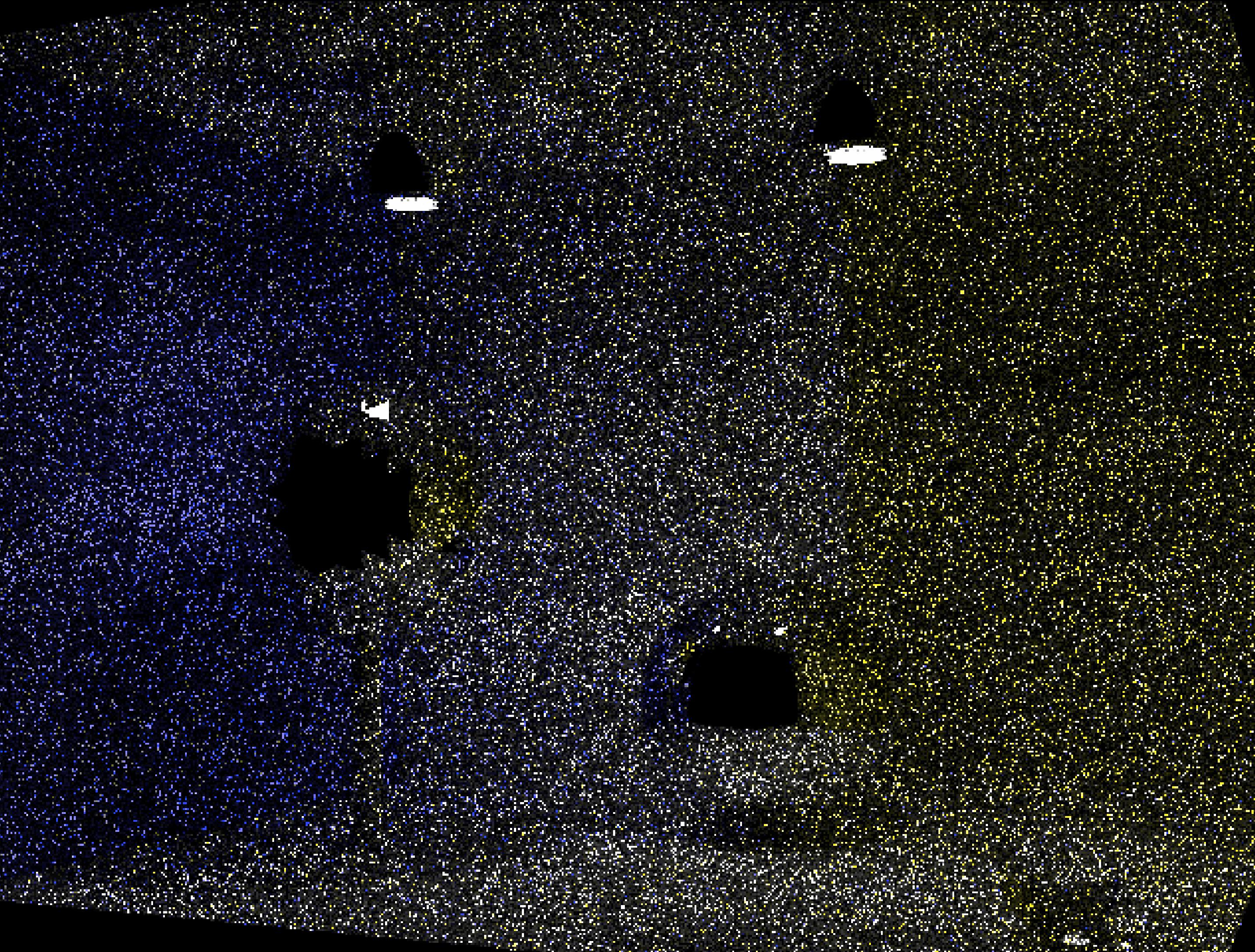
# Scene with “Realistic” Lights



Images courtesy of T. Hachisuka

# Path Tracing

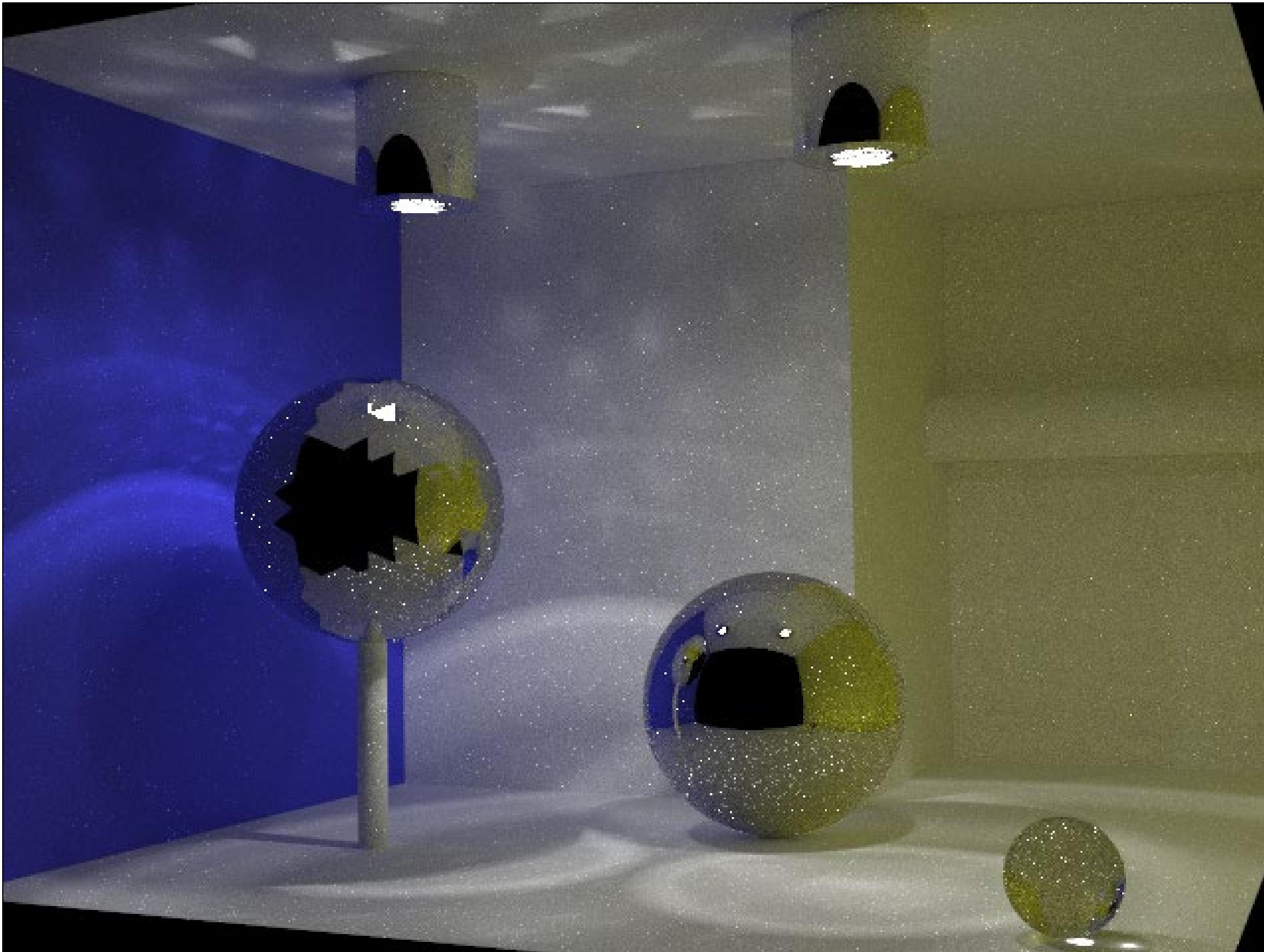
---



Images courtesy of T. Hachisuka

# Bidirectional Path Tracing

---



Images courtesy of T. Hachisuka

# Robustness of Rendering Methods

---

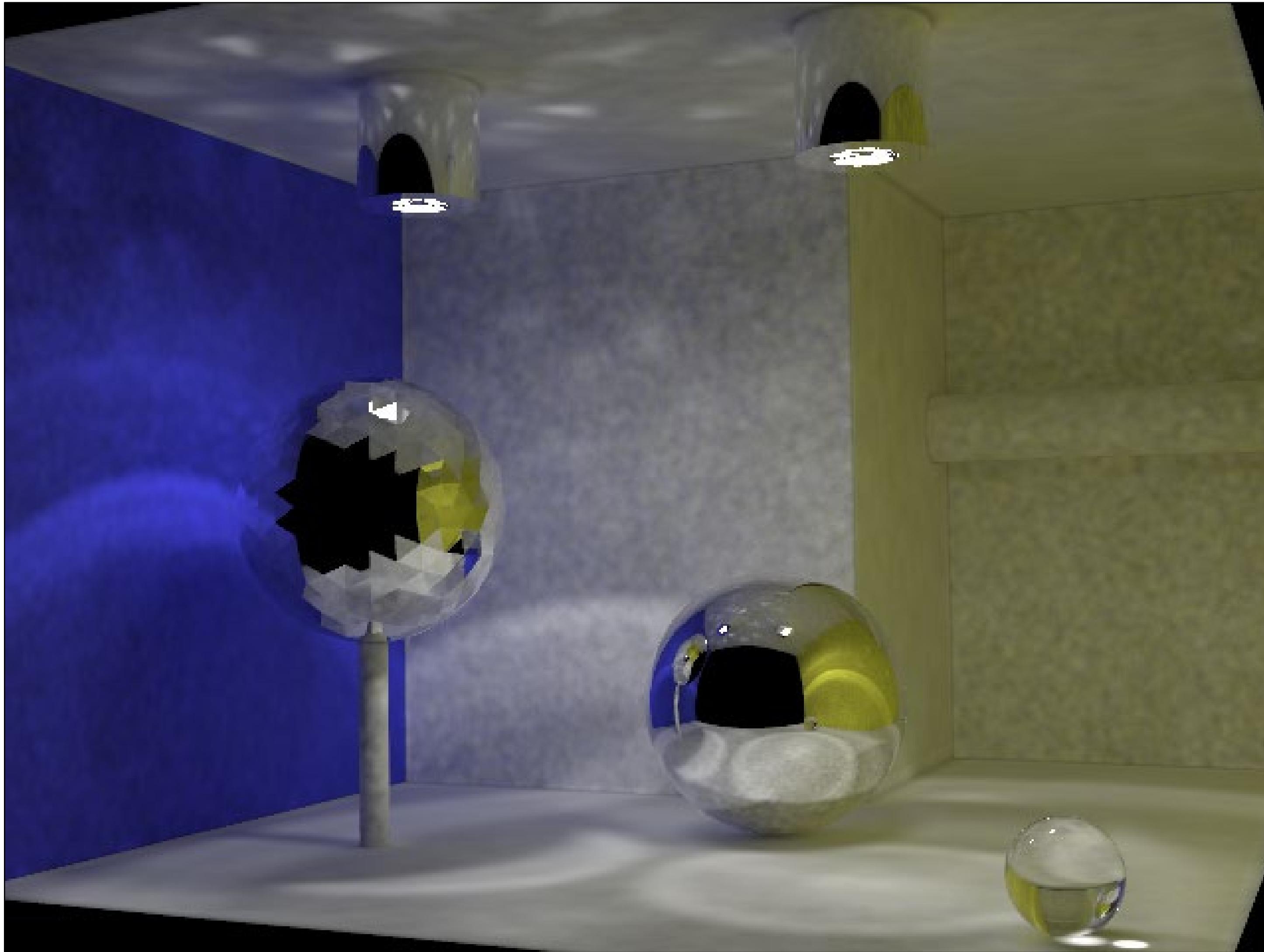
None of these unbiased methods can handle real light sources well:

- Path Tracing
- Bidirectional Path Tracing

Photon Mapping?

# Photon Mapping

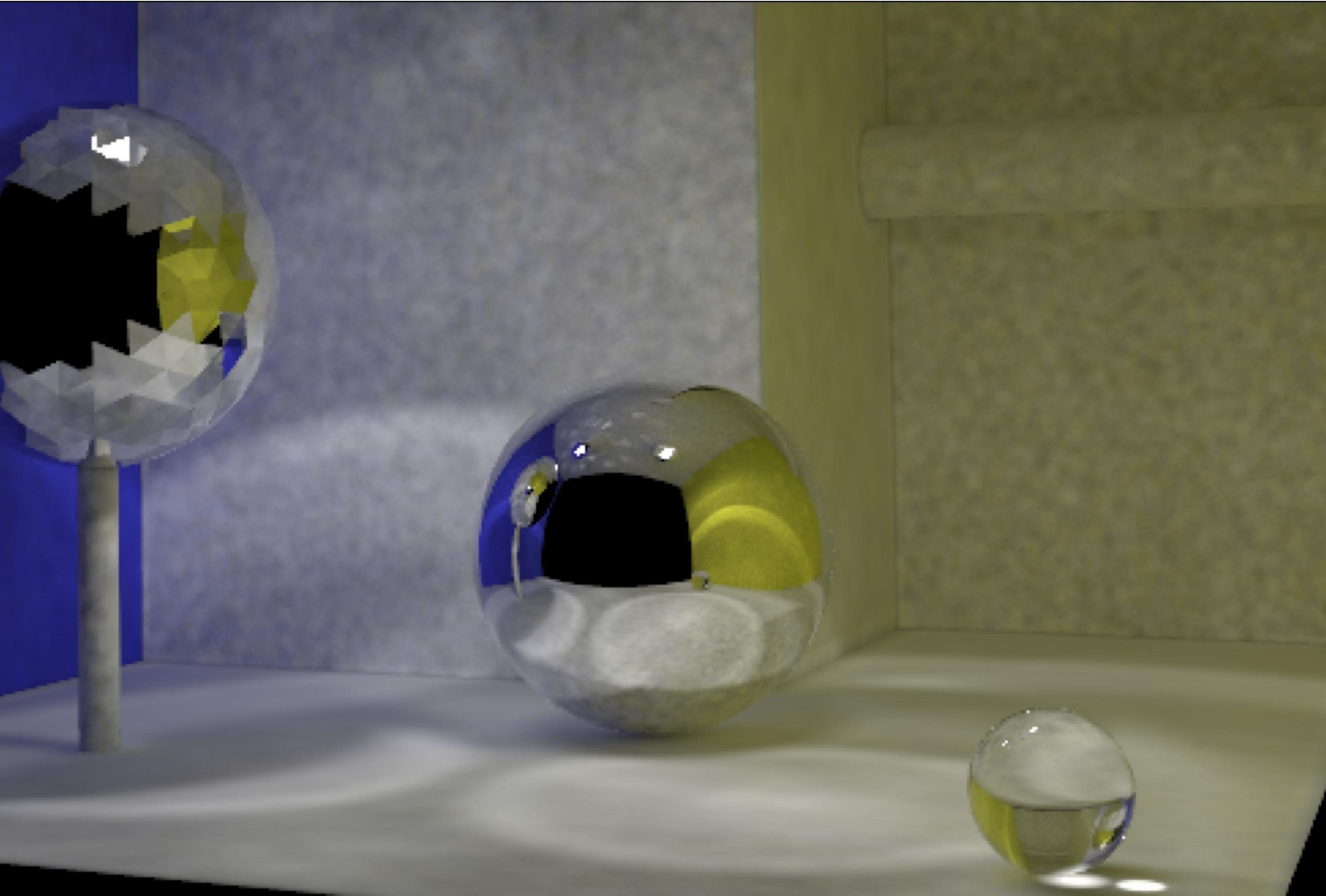
---



Images courtesy of T. Hachisuka

# Photon Mapping

---



Images courtesy of T. Hachisuka

# Photon Mapping - Summary

---

## Advantages

- Handles difficult paths more robustly than unbiased algorithms
- Consistent estimator
- Reuse of computation (photons)

## Disadvantages

- Bias shows up in many different forms
- Requires additional data structure (KD-tree)
- No progressive rendering
- Large memory footprint  
Non-intuitive hyperparameter fine-tuning

# Characteristics of Estimators

---

## *Unbiased estimator*

- expected value equals the true value being estimated

$$E[F] = \int f(x) dx$$

- variance (noise) is the only error
- averaging infinitely many estimates (each with finite number of samples) also yields the correct answer

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \langle F^k \rangle = \int f(x) dx$$

# Characteristics of Estimators

---

## *Bias* of an estimator

- difference between the expected value of the estimator and the true value being estimated

$$\beta = E[F] - \int f(x) dx$$

- expected average difference
- averaging infinitely many estimates yields the correct answer plus the bias

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \langle F^k \rangle = \int f(x) dx + \beta$$

# Characteristics of Estimators

---

*Consistent estimator*

- bias disappears in the limit

$$\lim_{N \rightarrow \infty} E[F] = \int f(x) dx$$

*Consistent estimators and unbiased estimators are asymptotically equivalent*

- both need an infinite number of samples to reduce the error to zero

# Characteristics of Estimators

---

*Mean Squared Error (MSE)* of an estimator

- combines variance and squared bias

$$\text{MSE}[F] = \text{Var}[F] + \text{Bias}[F]^2$$

*Root Mean Squared Error (RMSE)*

- has the same units as the quantity being estimated
- for unbiased estimators equal to std. deviation

$$\text{RMSE}[F] = \sqrt{\text{MSE}[F]}$$

# Rendering Techniques

---

Examples of unbiased methods

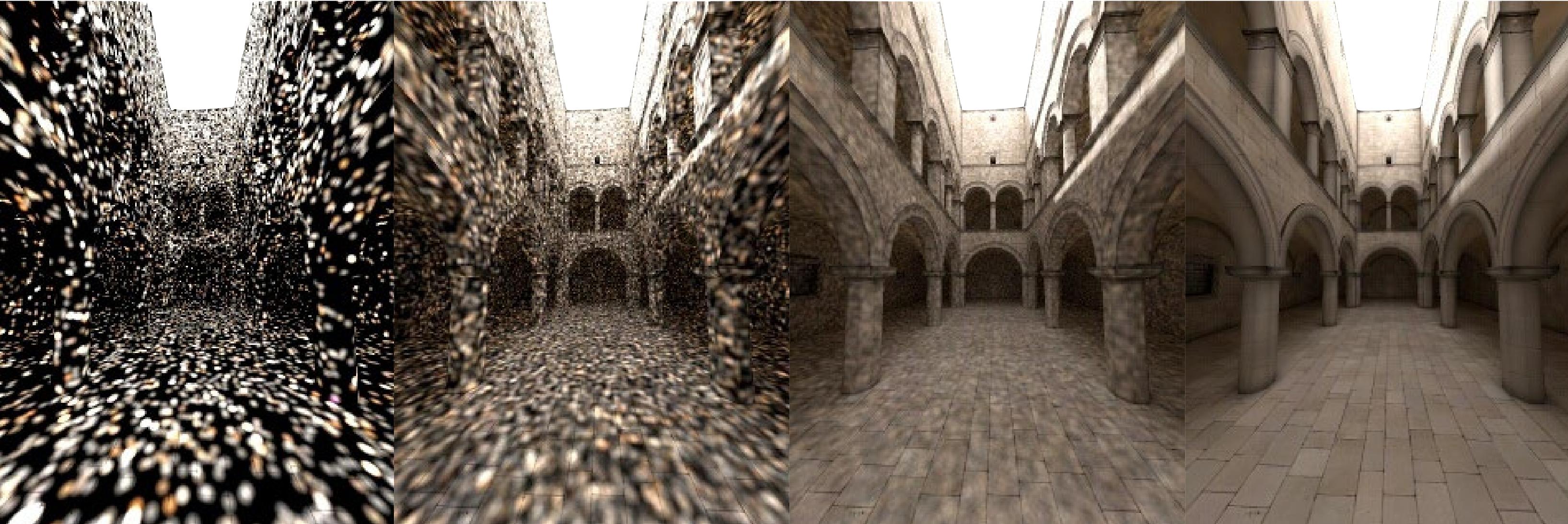
- Path tracing
- Light tracing
- Bidirectional path tracing

Examples of biased/consistent methods

- (Progressive) photon mapping
- Many-light methods

# Consistency of Photon Mapping

Result converges to the correct solution



Conditions for convergence:

- Infinitesimally small radius
- Infinite number of nearby photons
  - **Infinite storage requirement!**

# Progressive Photon Mapping

# Key Idea

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Progressively shrink the density estimation kernel

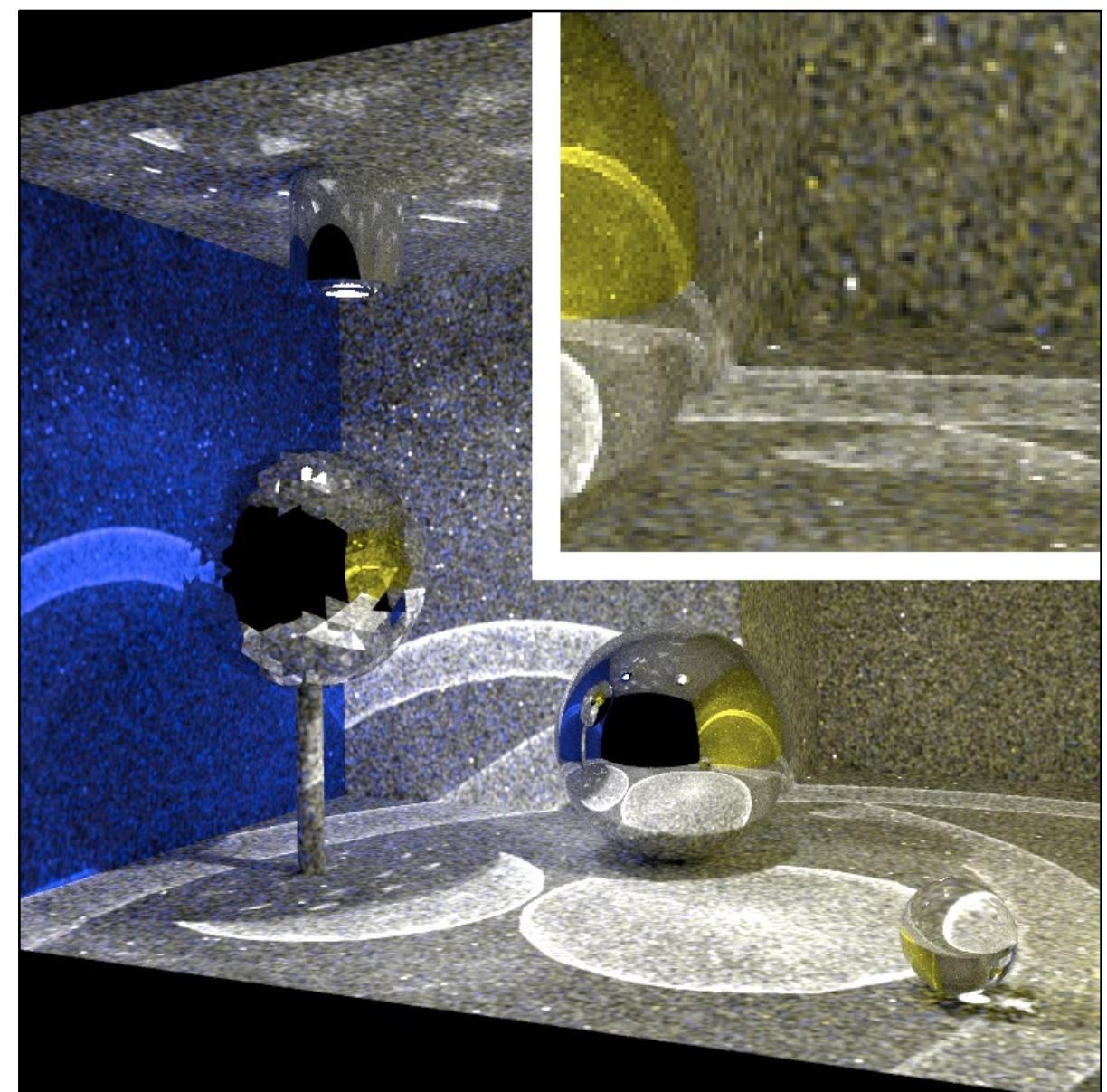
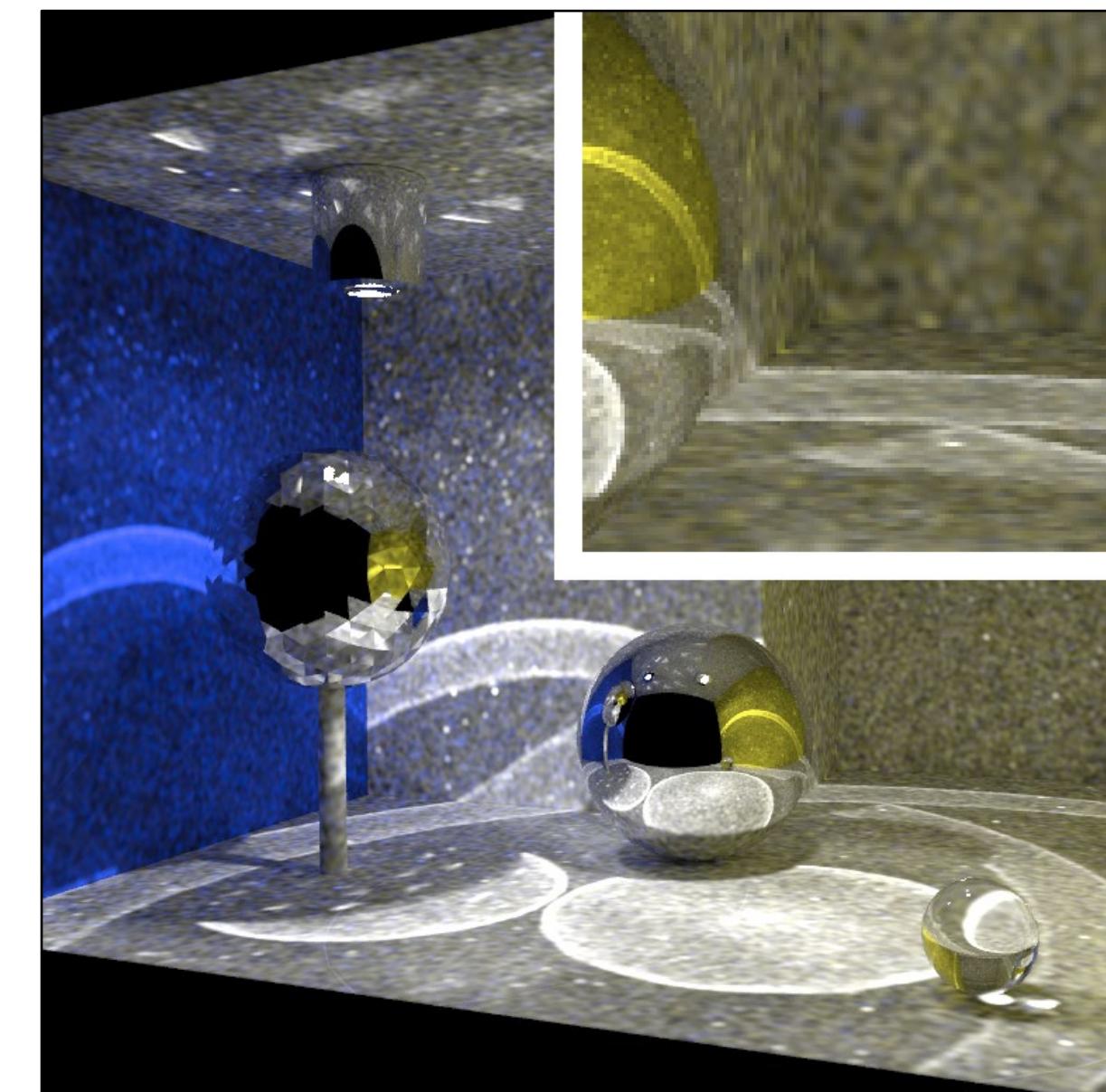
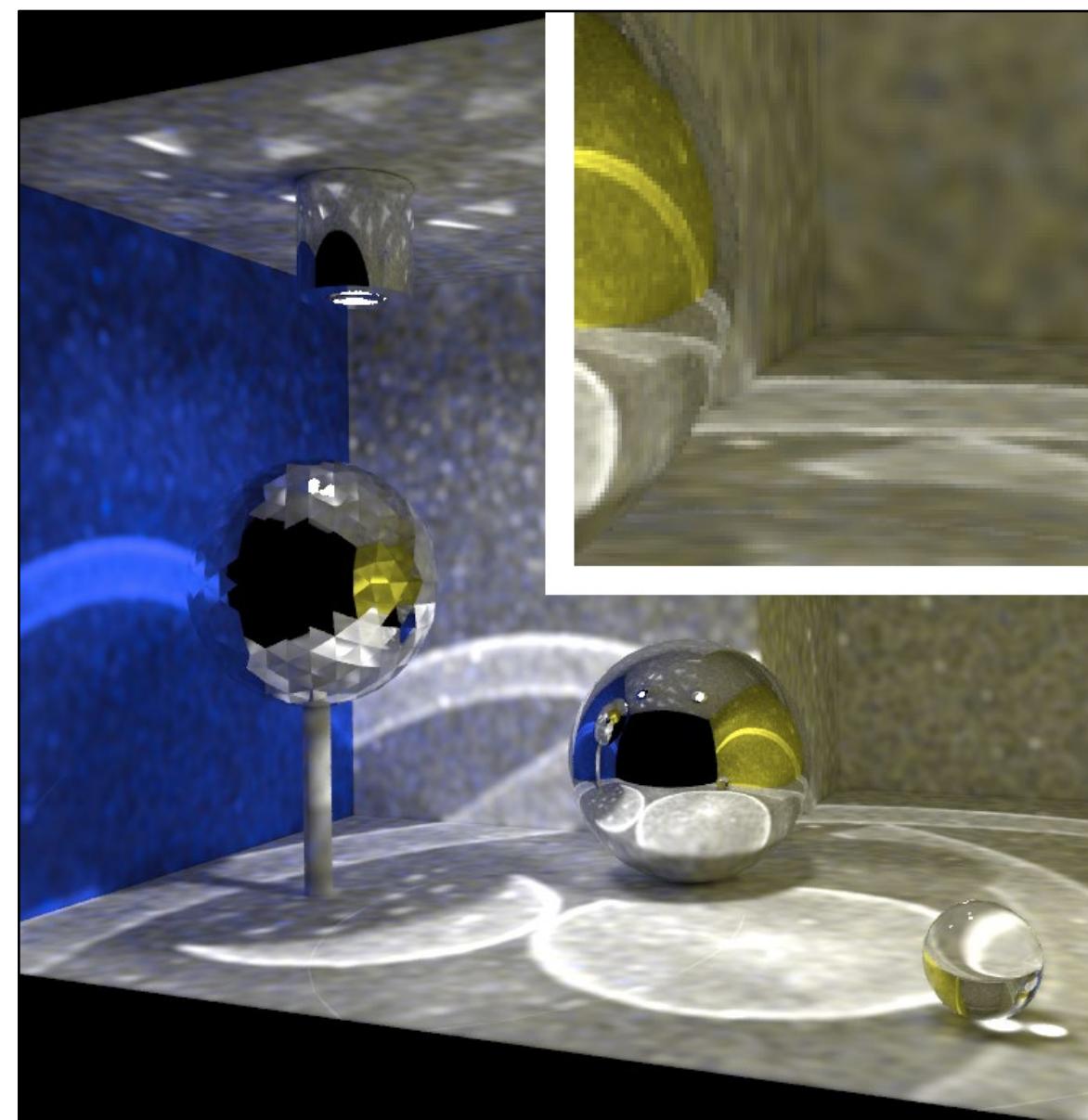
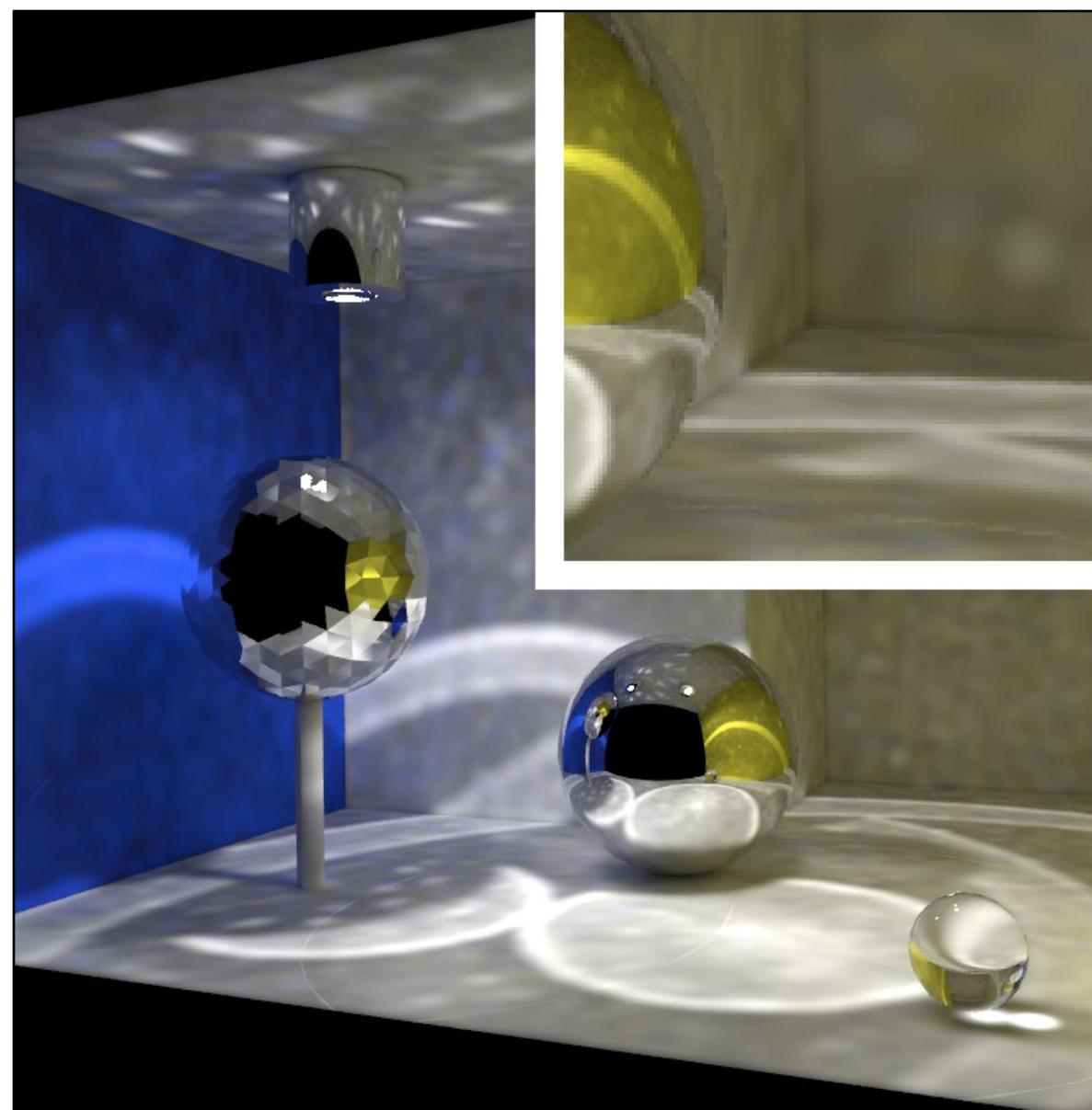
Hachisuka et al. 2008, 2009, ...

- store/update statistics at each camera ray hitpoint

Knaus & Zwicker 2011

- no statistics, just render independent images with smaller and smaller radius, and average

# Different kernel radii



Images courtesy of C. Knaus and M. Zwicker

# Progressive Radius Reduction

---

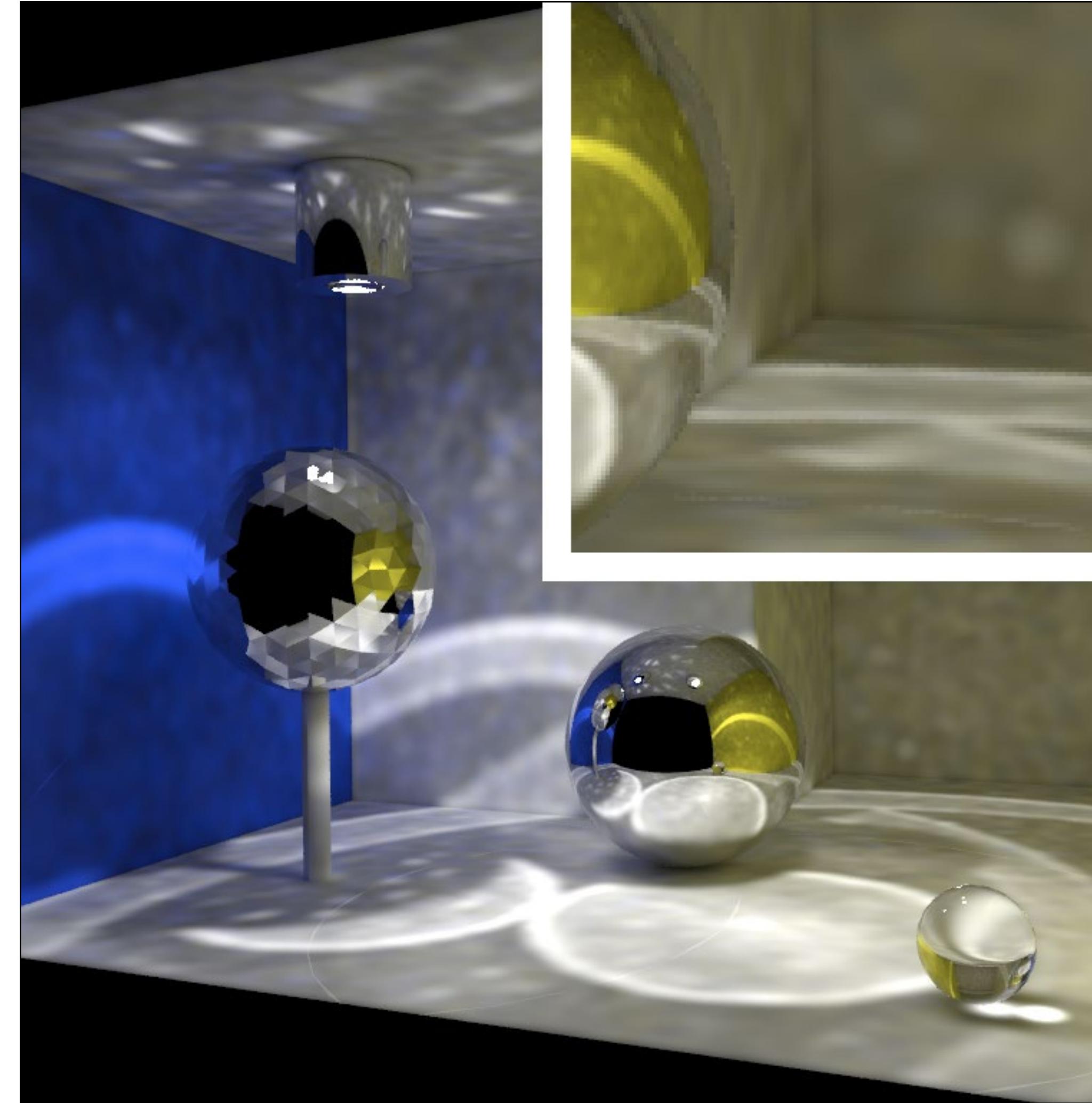


Image 1,  $r = 20$

Images courtesy of C. Knaus and M. Zwicker

# Progressive Radius Reduction

---

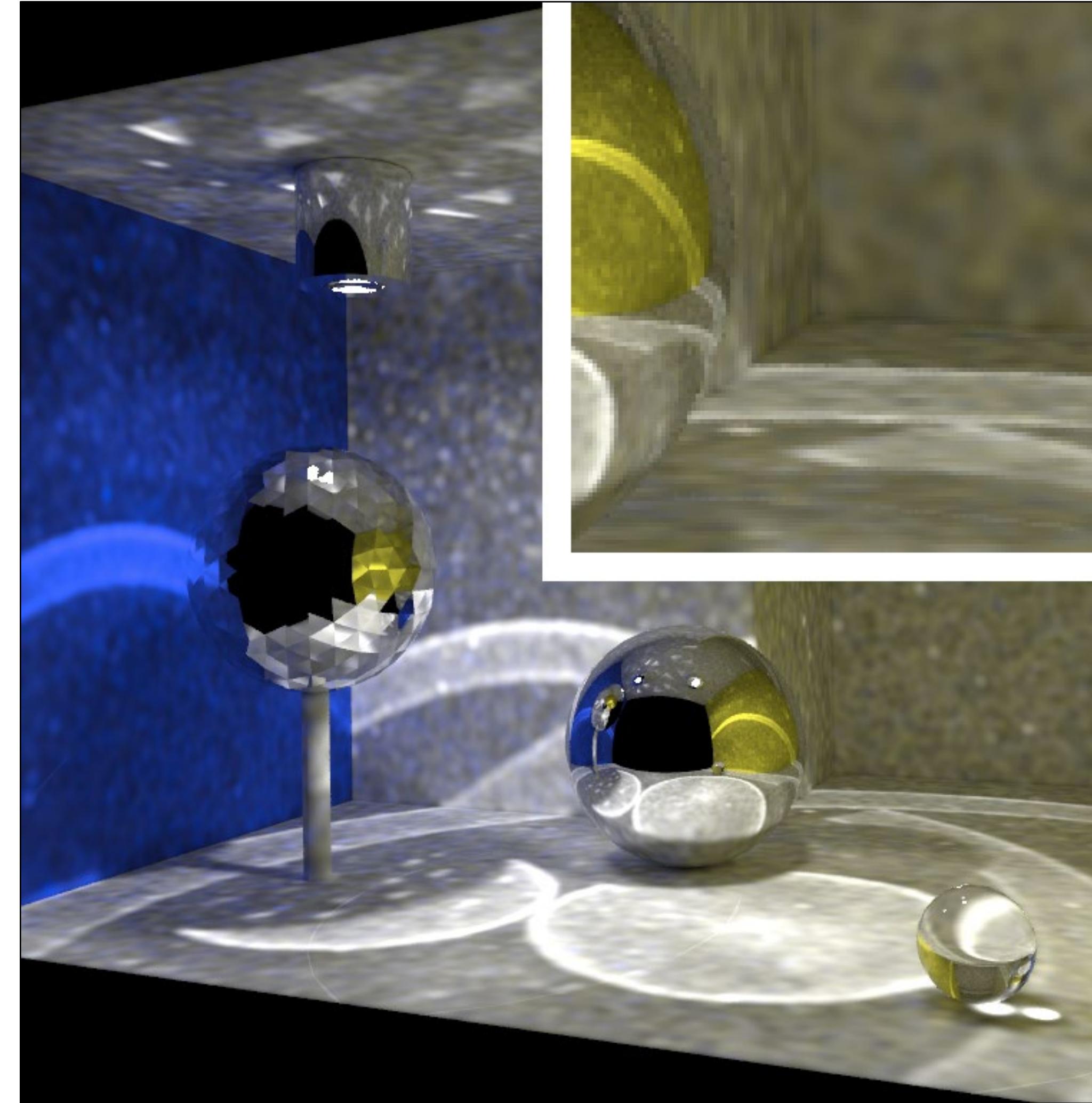


Image 10,  $r = 11.87$

Images courtesy of C. Knaus and M. Zwicker

# Progressive Radius Reduction

---

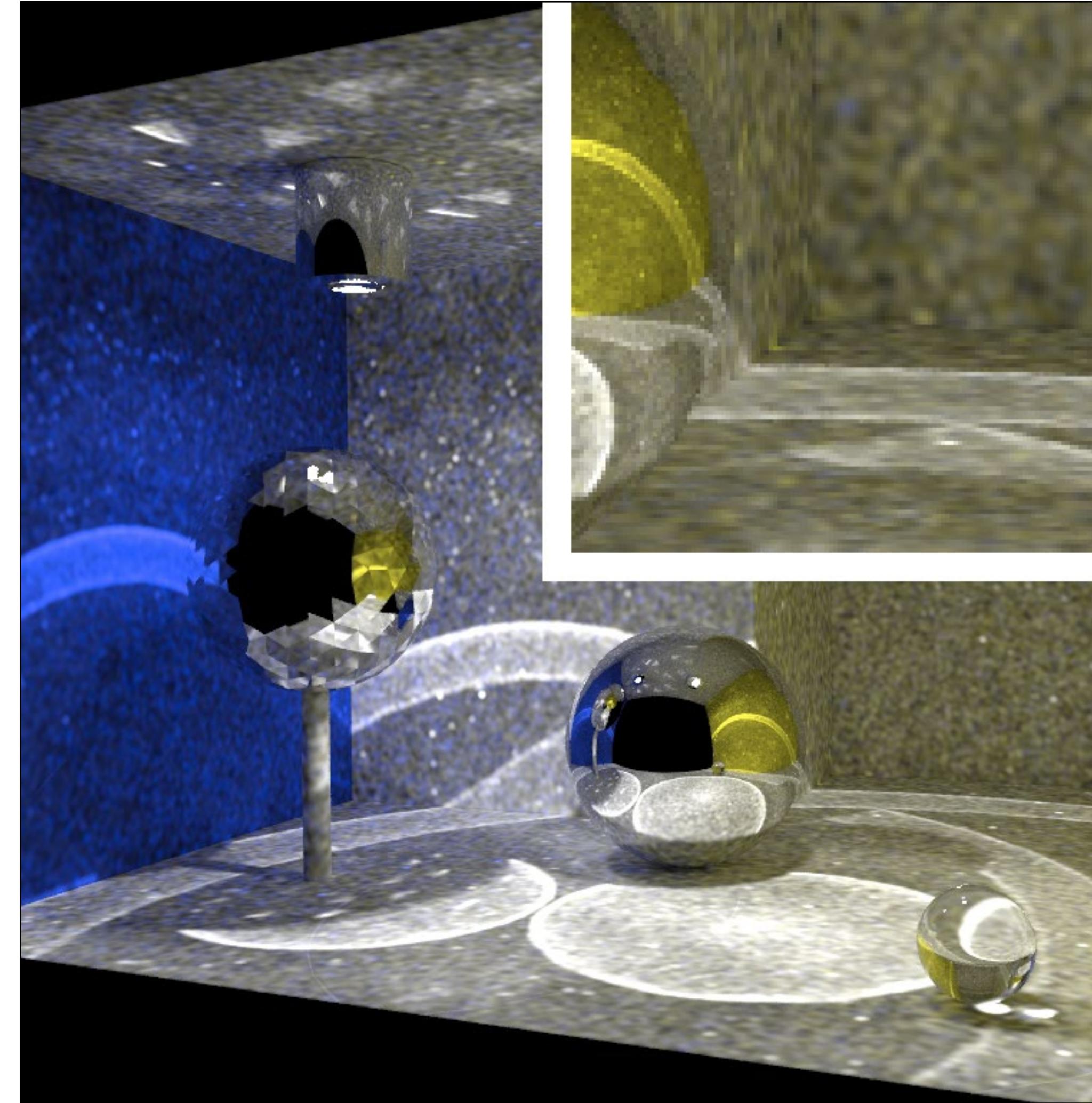
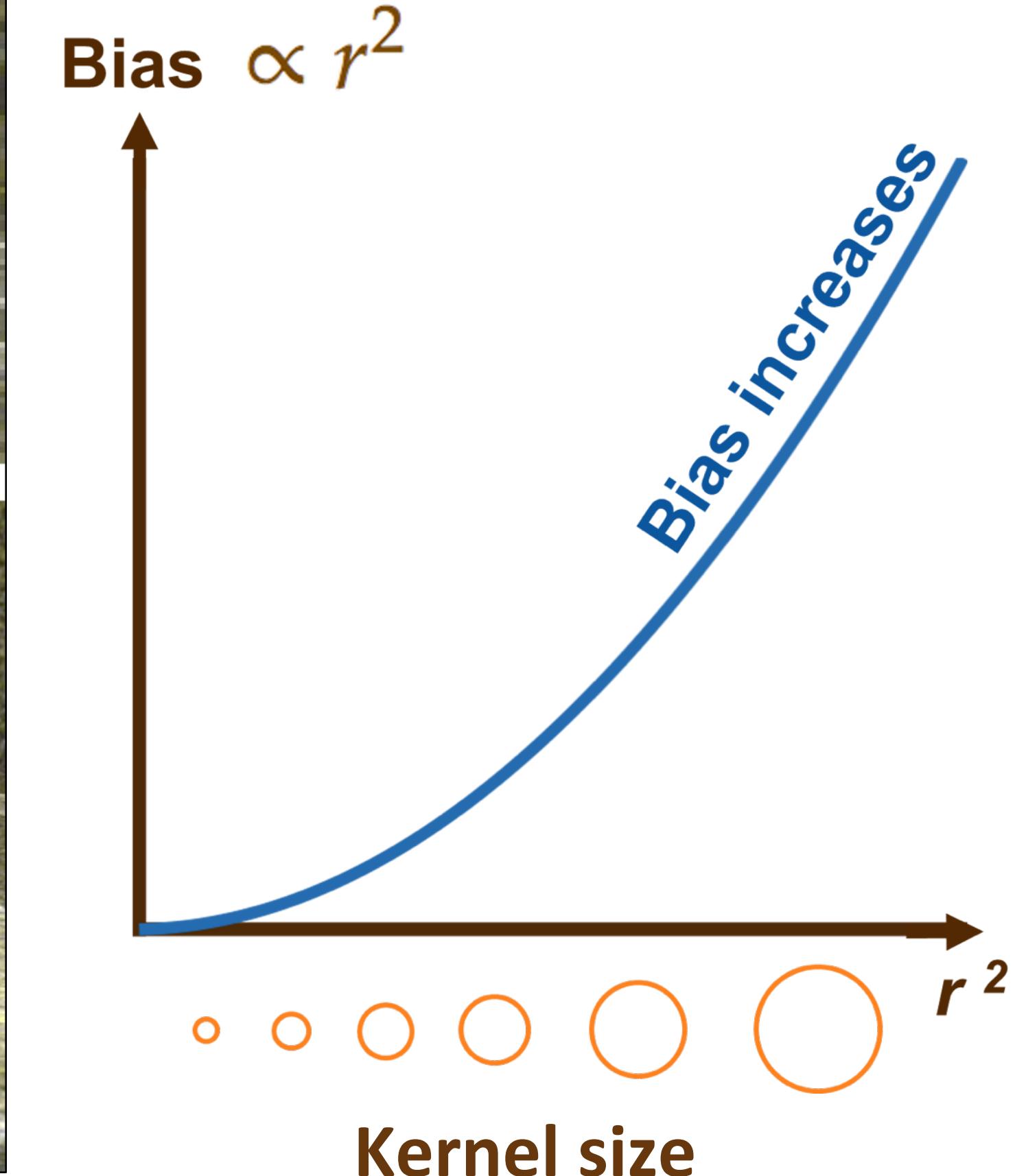
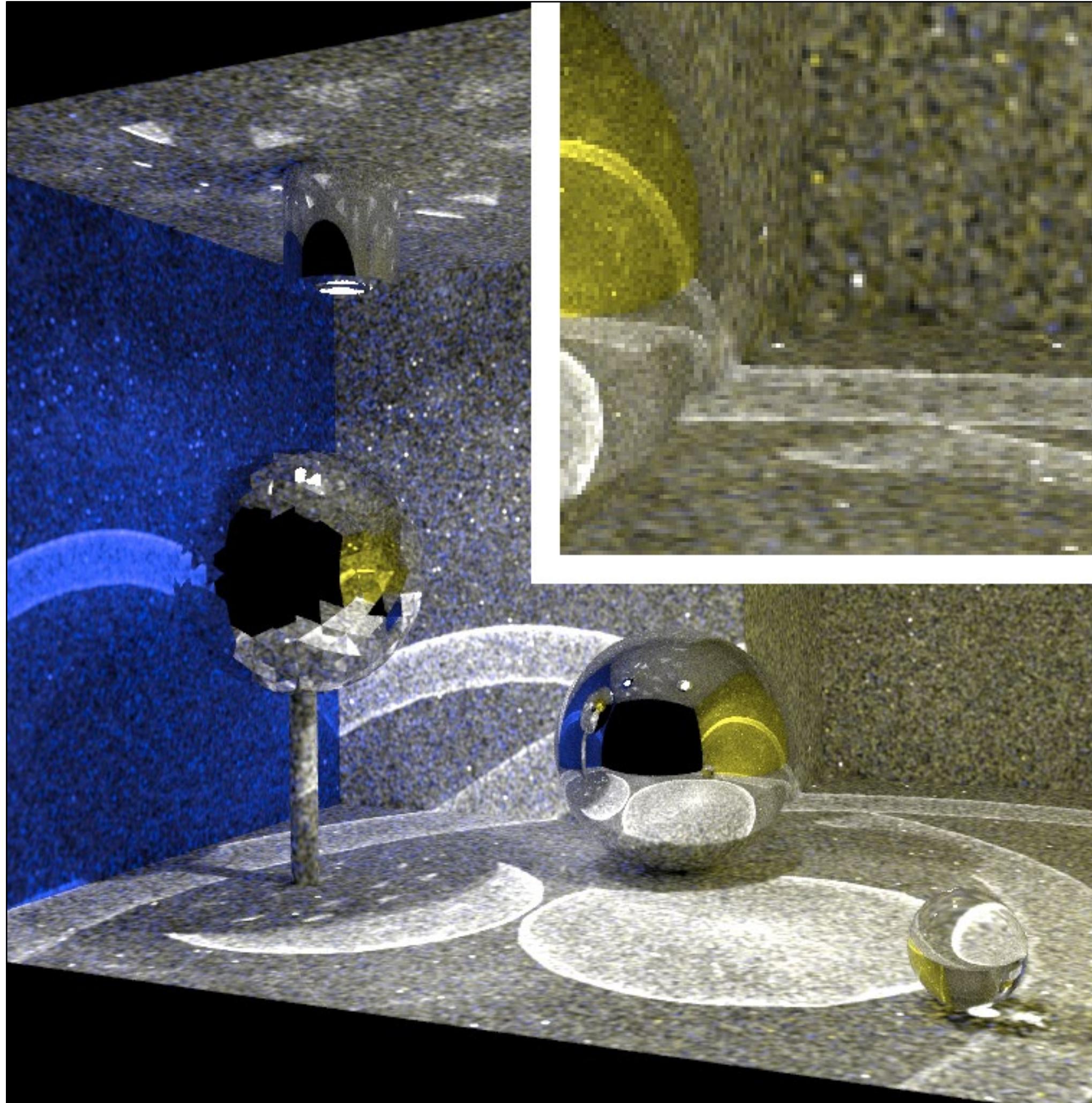
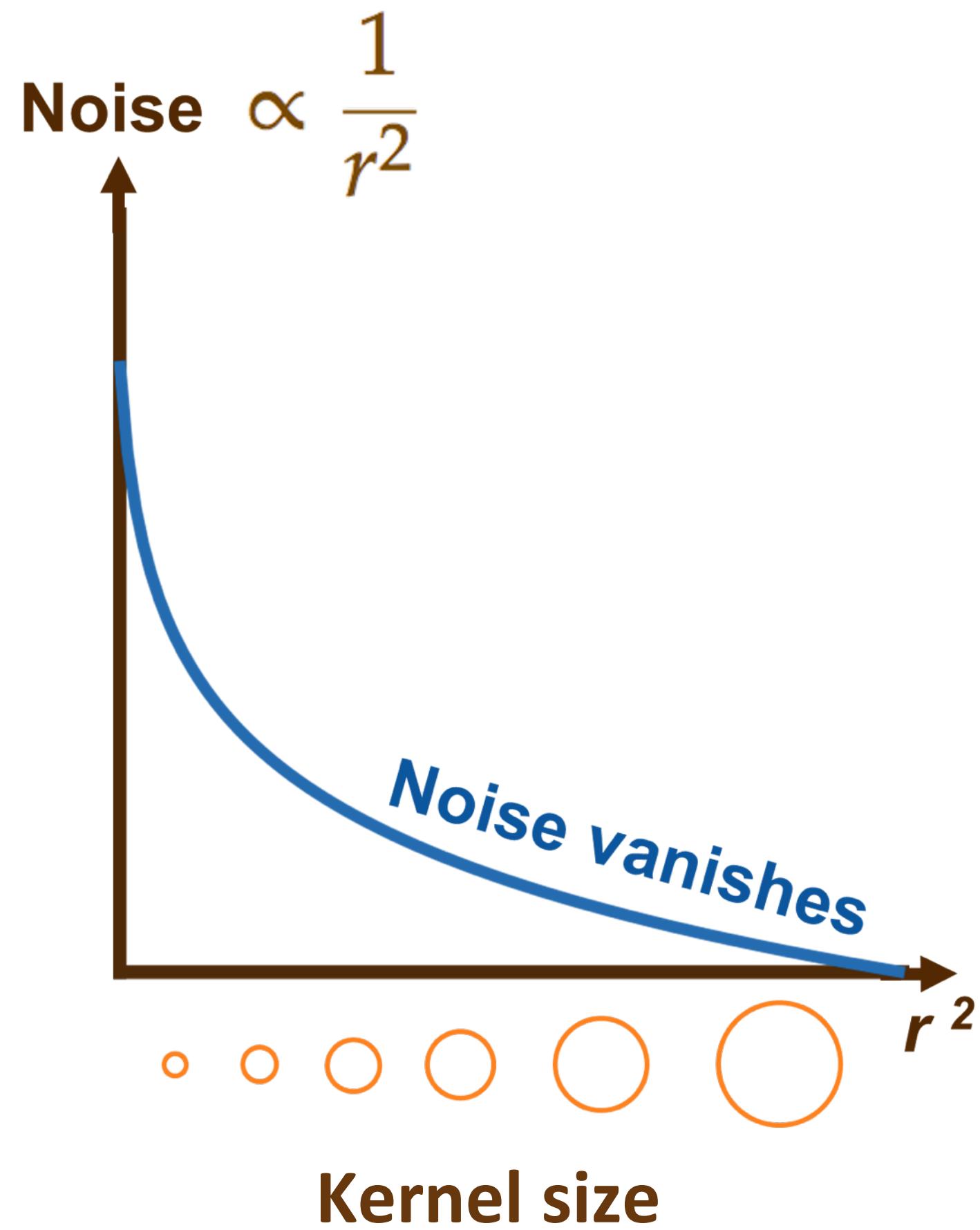


Image 100,  $r = 6.71$

Images courtesy of C. Knaus and M. Zwicker

# Progressive Radius Reduction



# Running Average

---

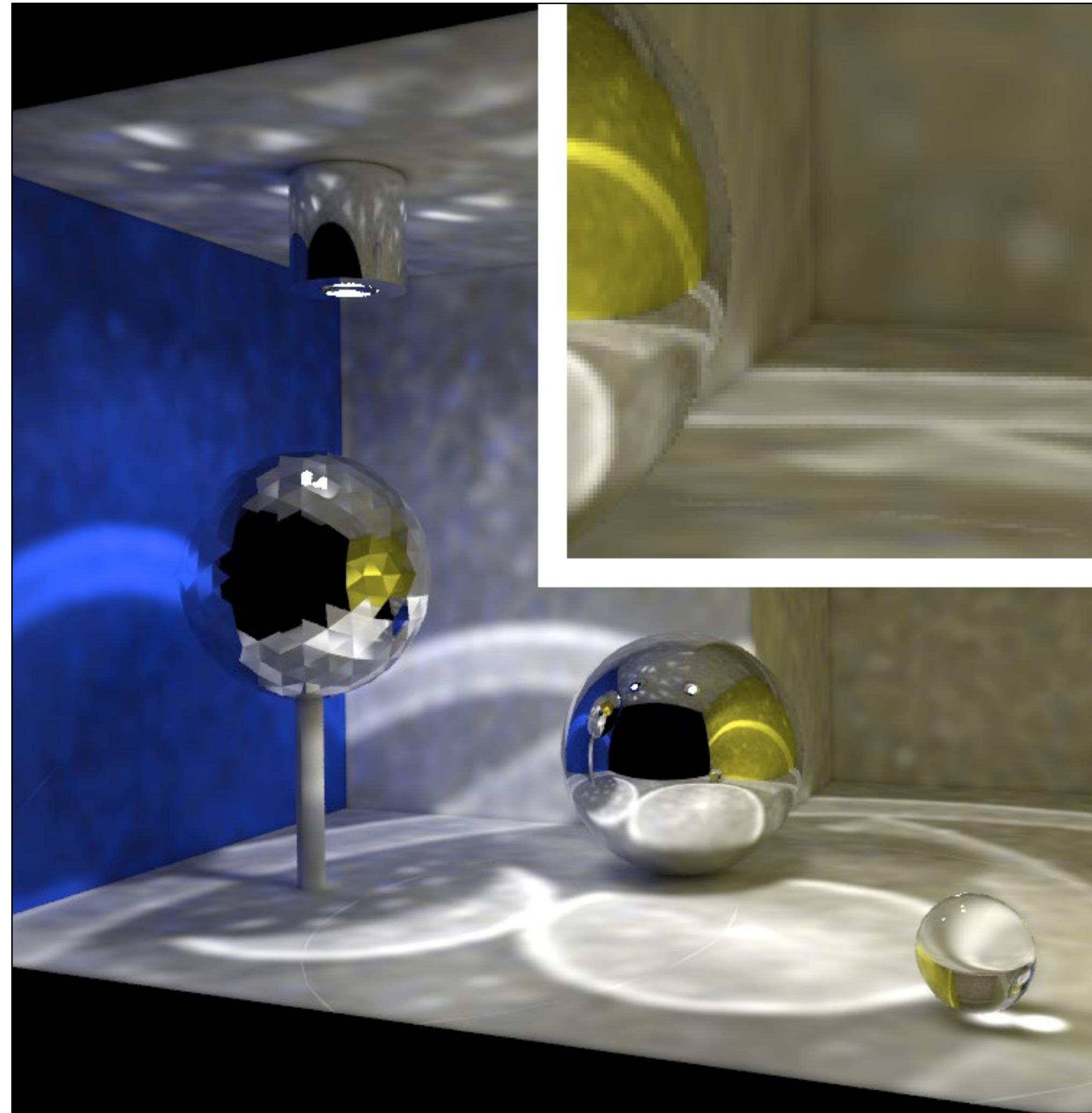
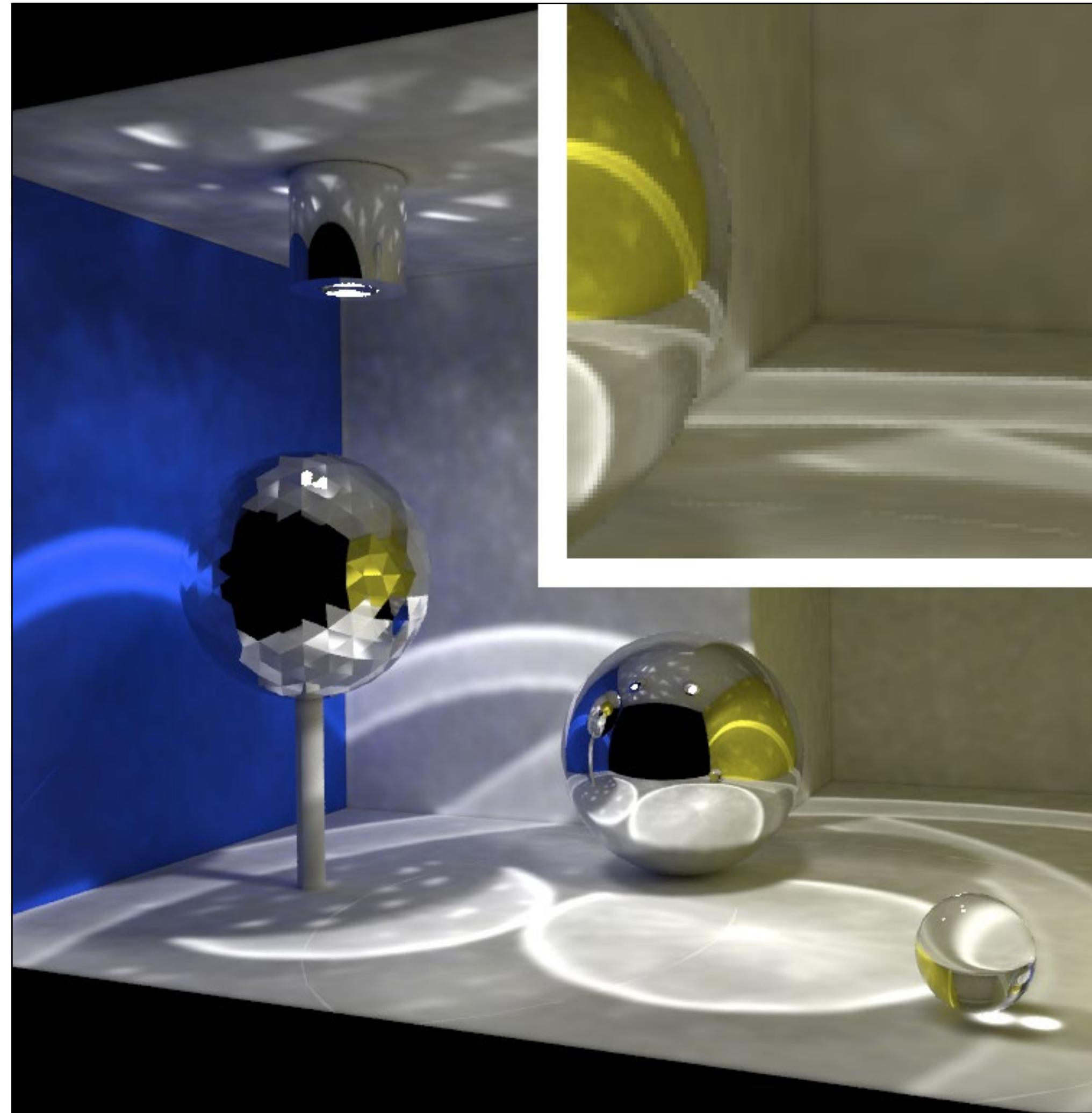


Image 1

Images courtesy of C. Knaus and M. Zwicker

# Running Average

---

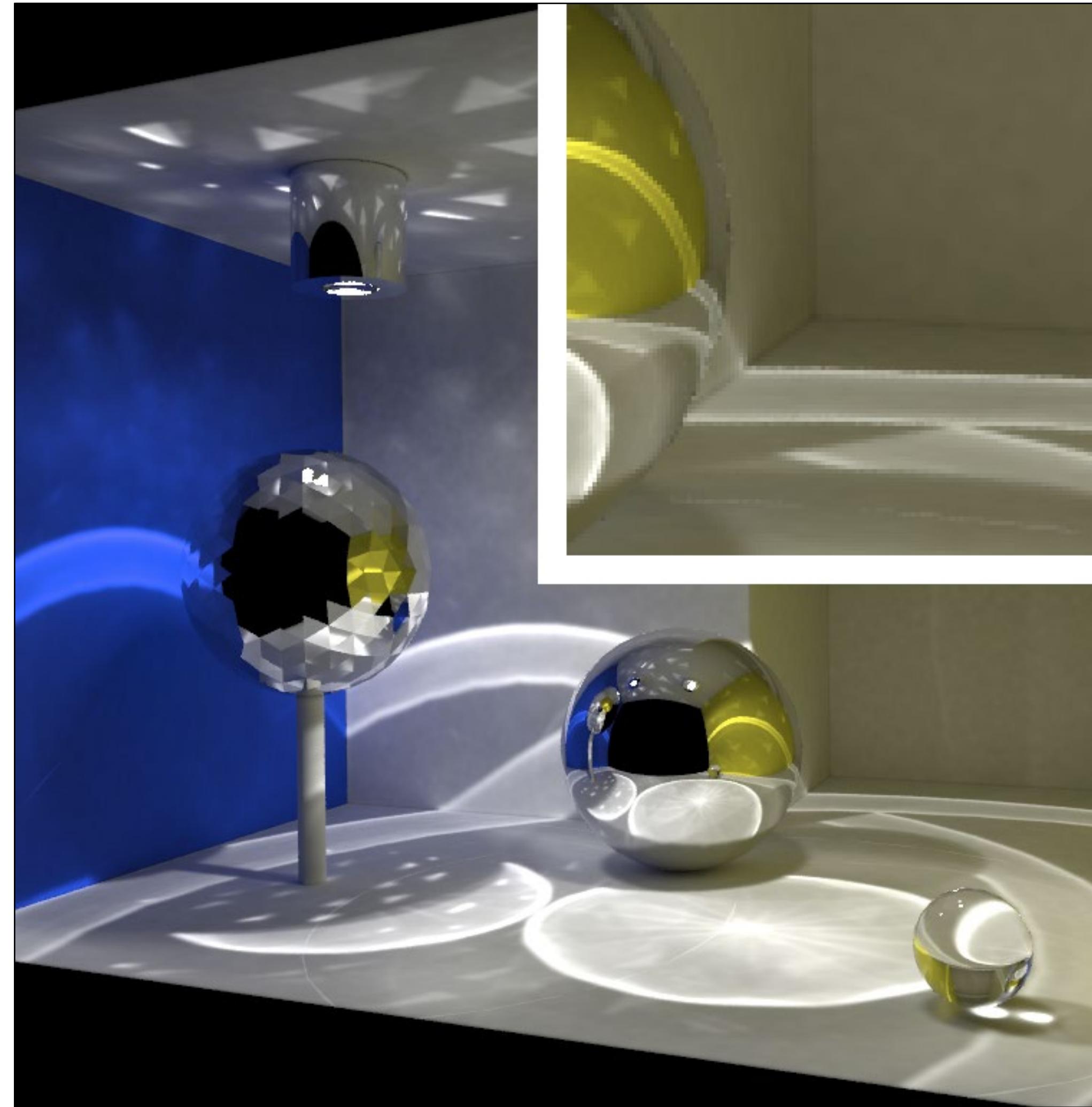


Average of Images 1-10

Images courtesy of C. Knaus and M. Zwicker

# Running Average

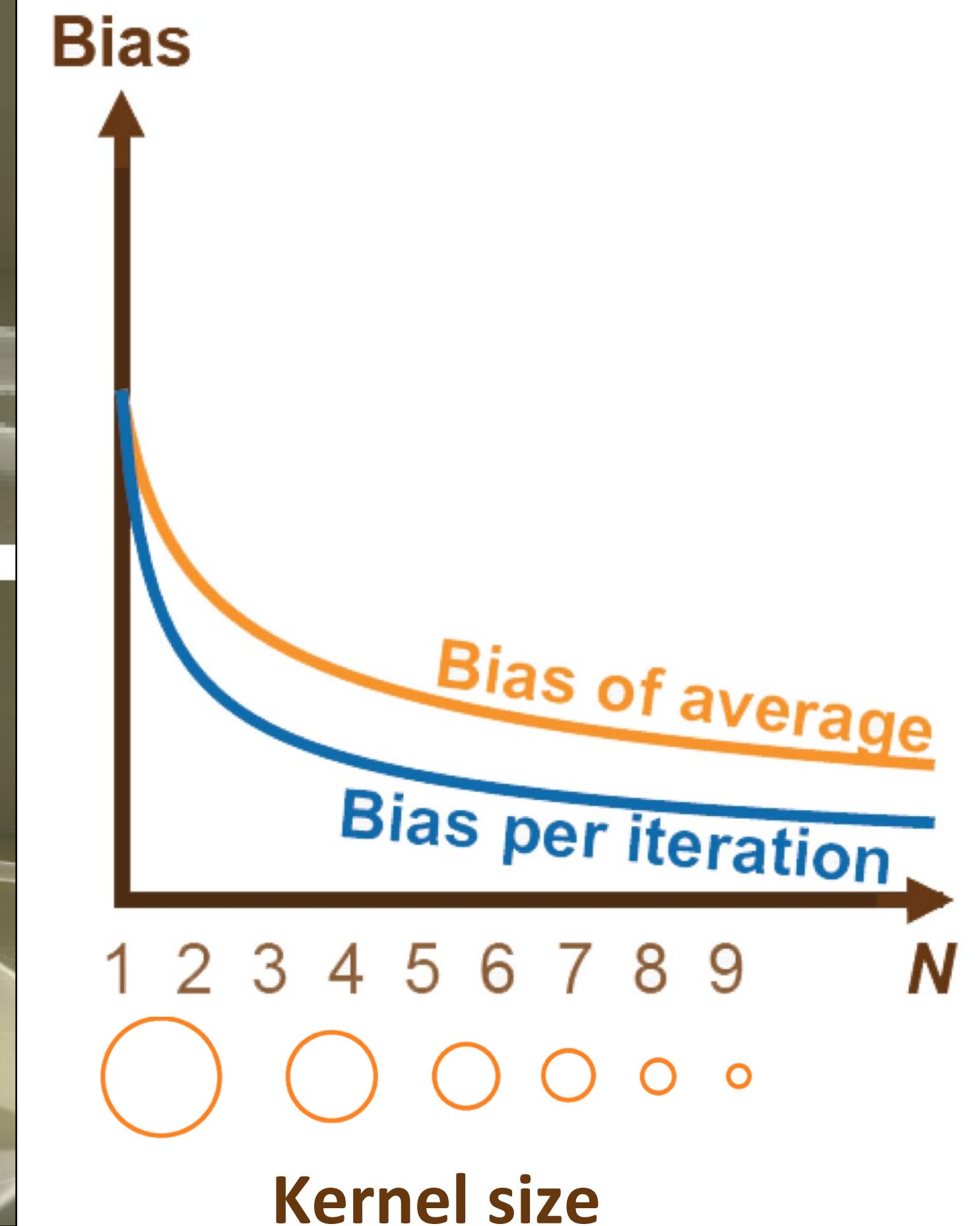
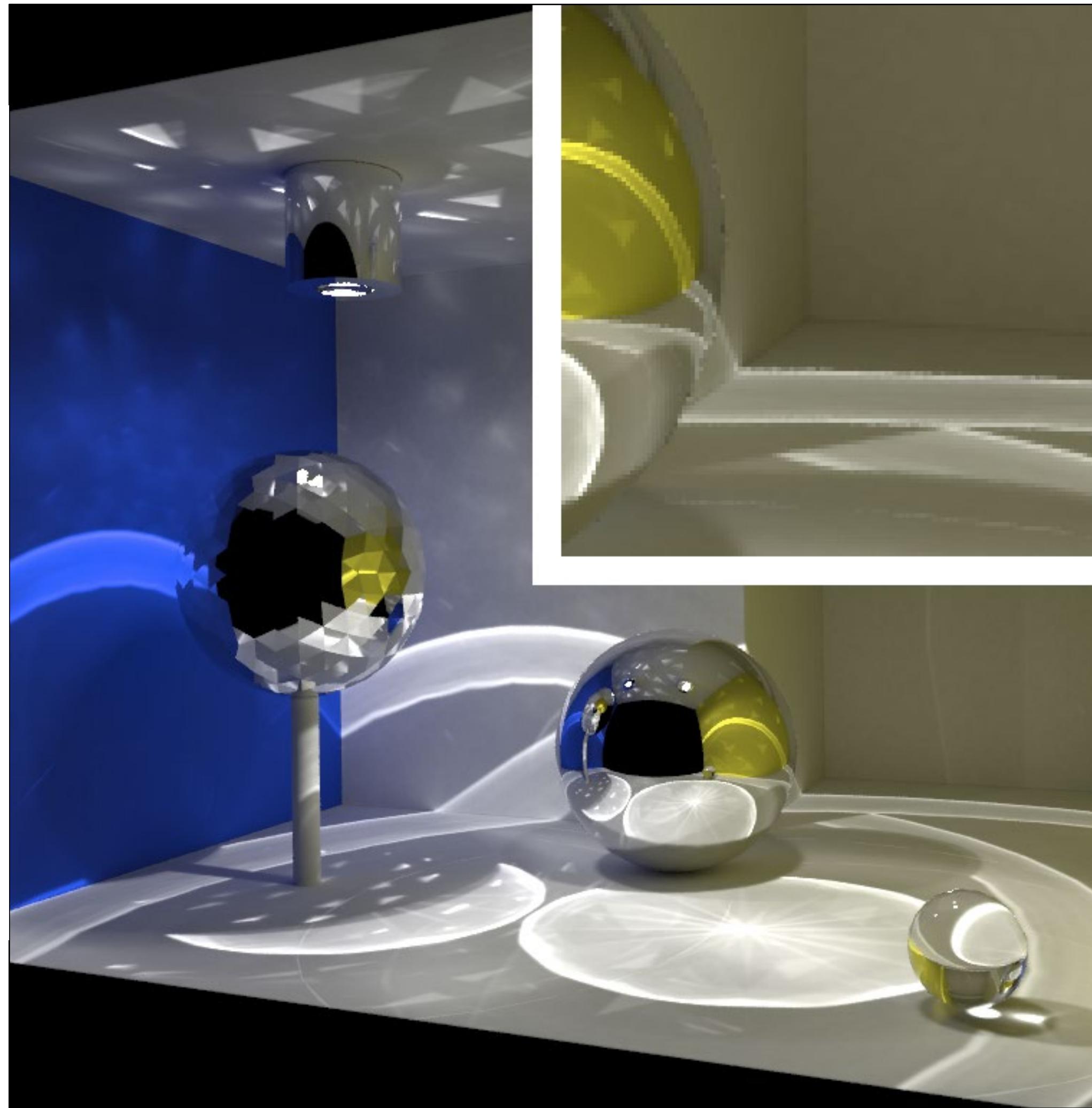
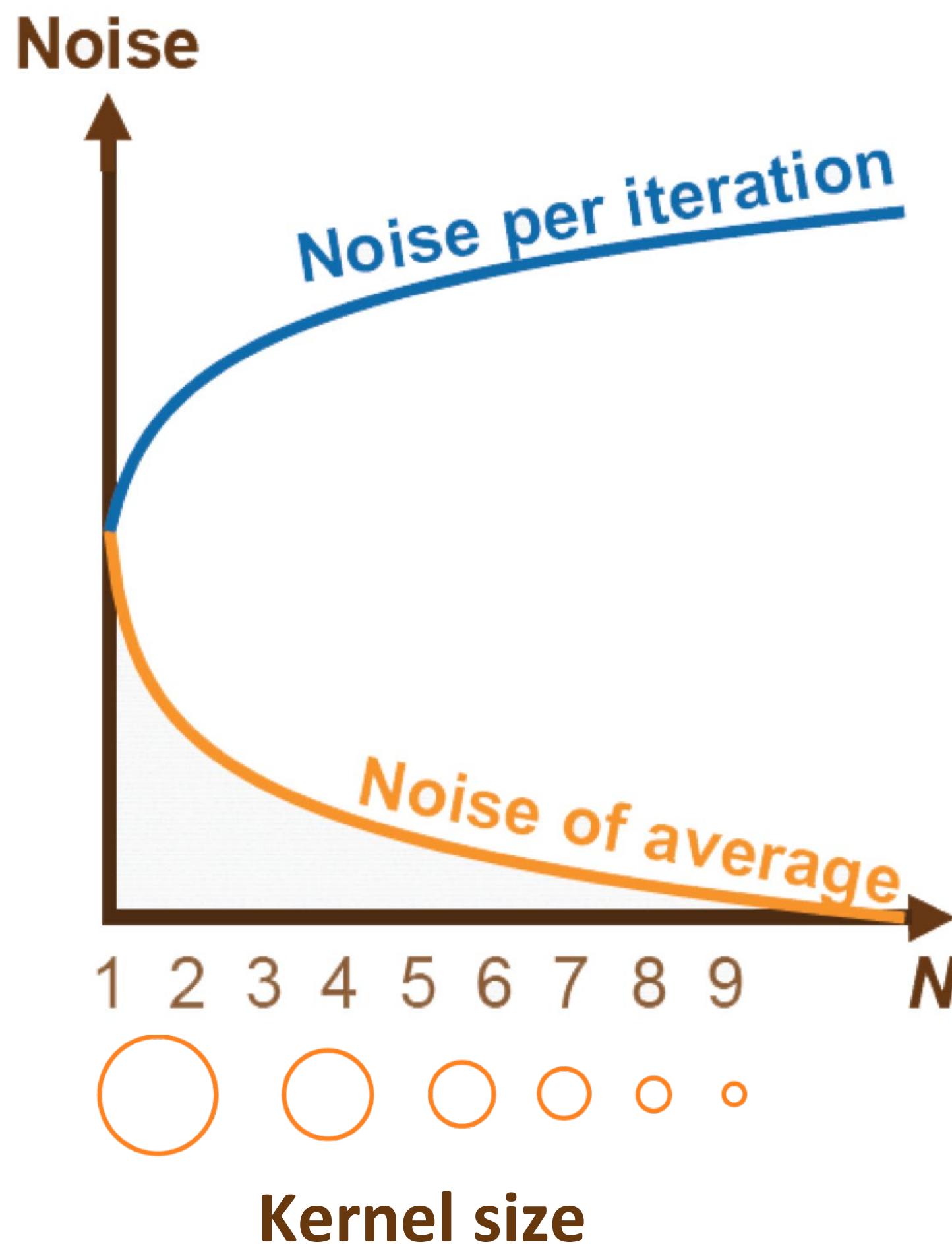
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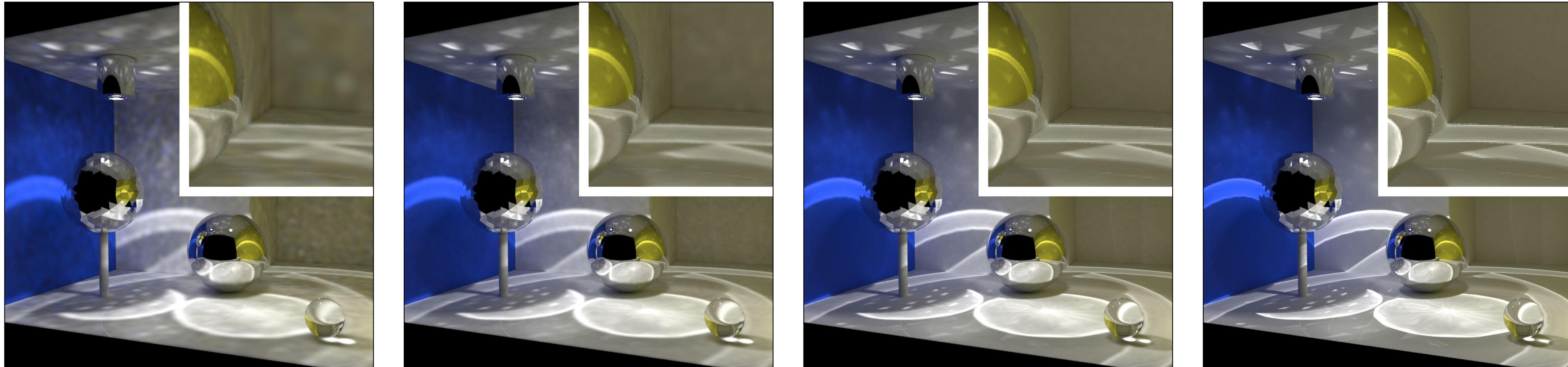
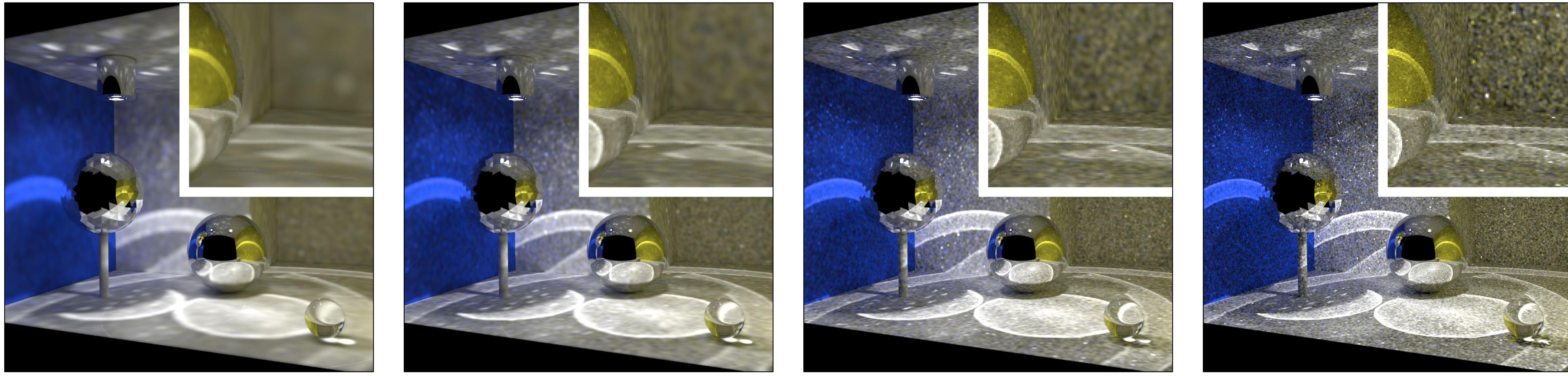
Average of Images 1-100

Images courtesy of C. Knaus and M. Zwicker

# Running Average



# Individual iterations



Running average

Images courtesy of C. Knaus and M. Zwicker

# Radius Reduction

---

Given:

- Iteration  $i$
- Kernel radius  $r_i$
- Parameter  $\alpha \in (0, 1)$  for controlling the shrinking

The radius for the next iteration is:  $r_{i+1}^2 = \frac{i + \alpha}{i + 1} r_i^2$

See [Knaus & Zwicker 2011] for derivation

# Algorithm

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Step 1:

- Photon tracing: emit, scatter, store photons

Step 2:

- Trace camera paths
- Evaluate radiance estimate using radius  $r_i$

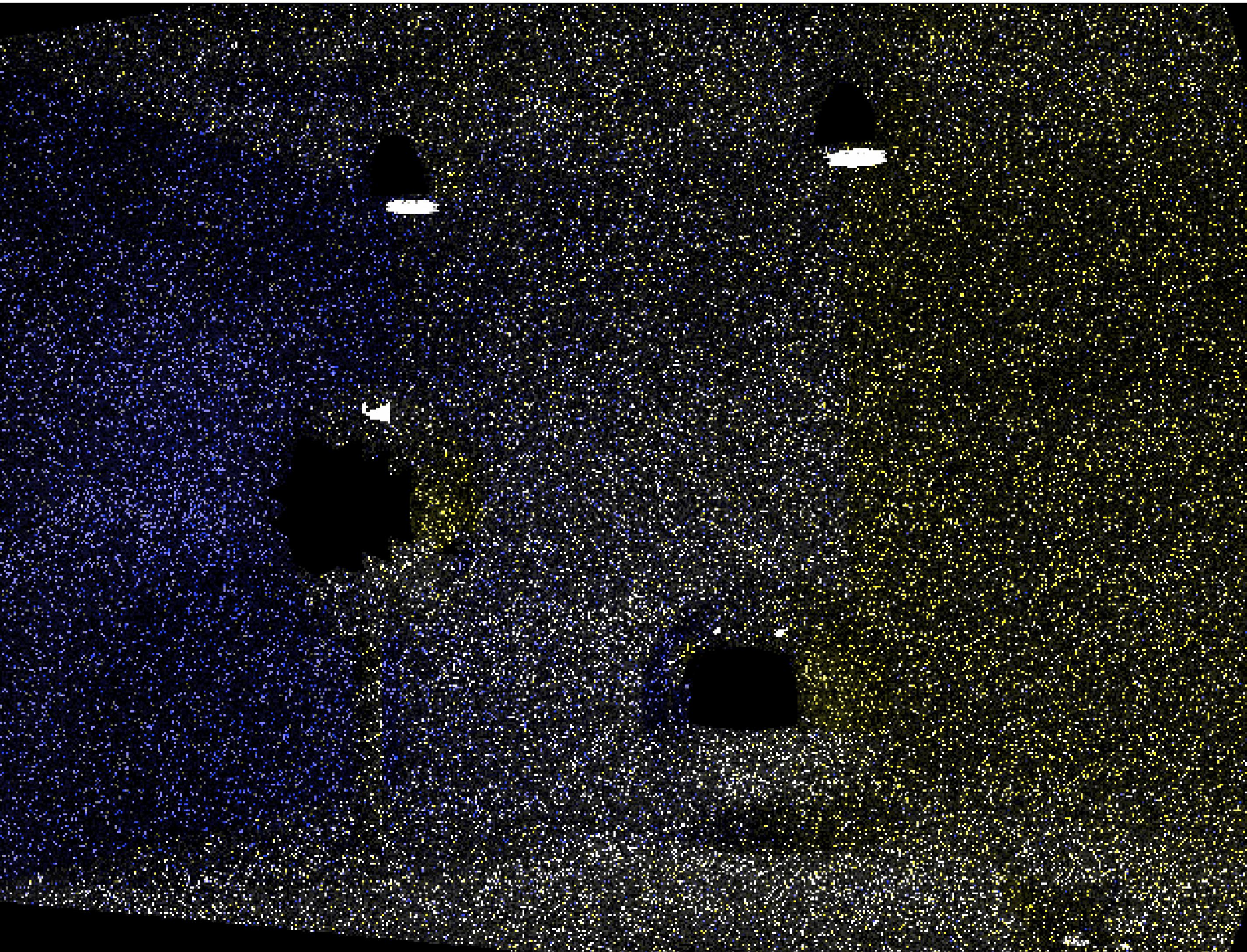
Display running average

Compute new radius  $r_{i+1}^2 = \frac{i + \alpha}{i + 1} r_i^2$  and repeat...

Trivially parallelizable by iteration

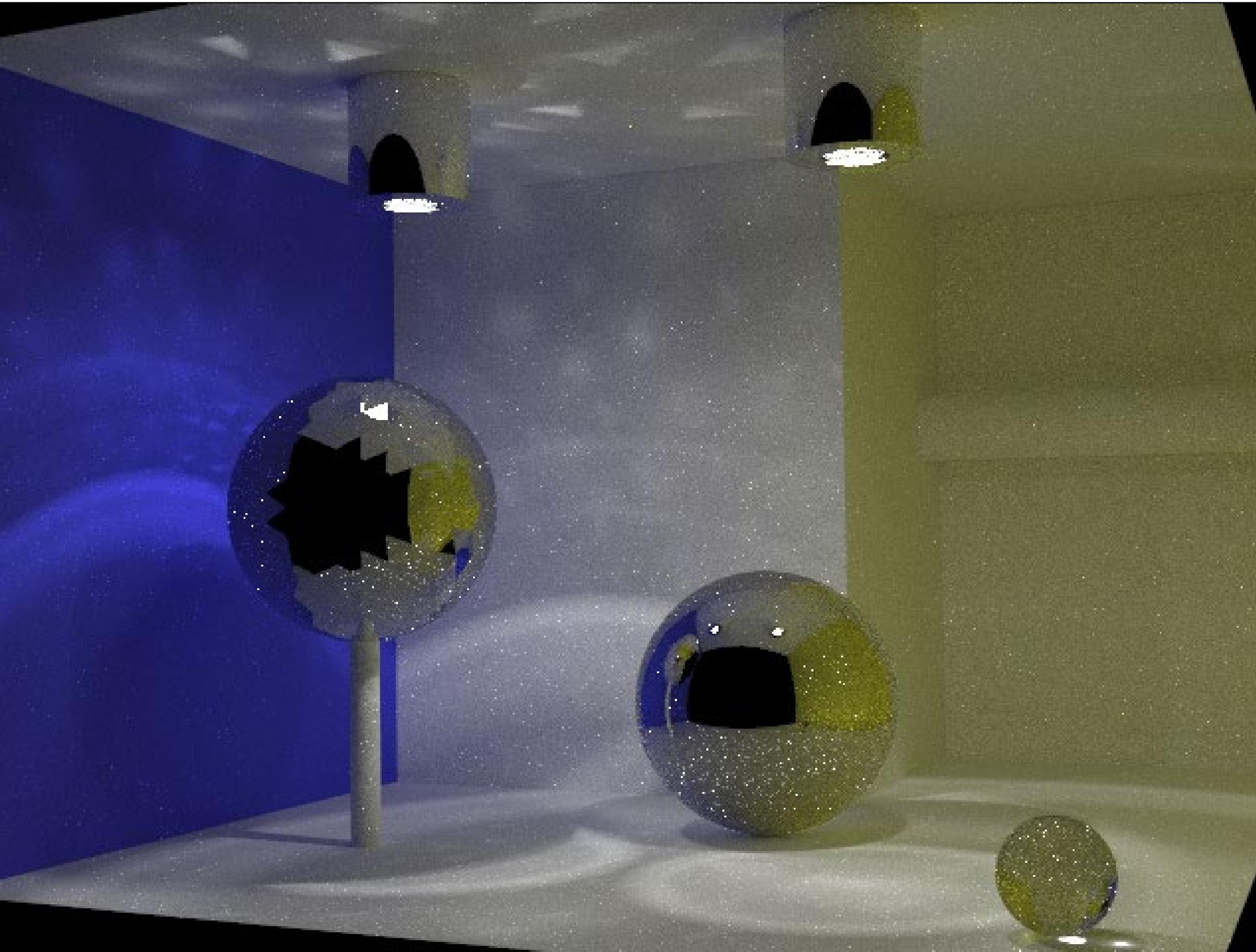
# Path Tracing

---



# Bidirectional Path Tracing

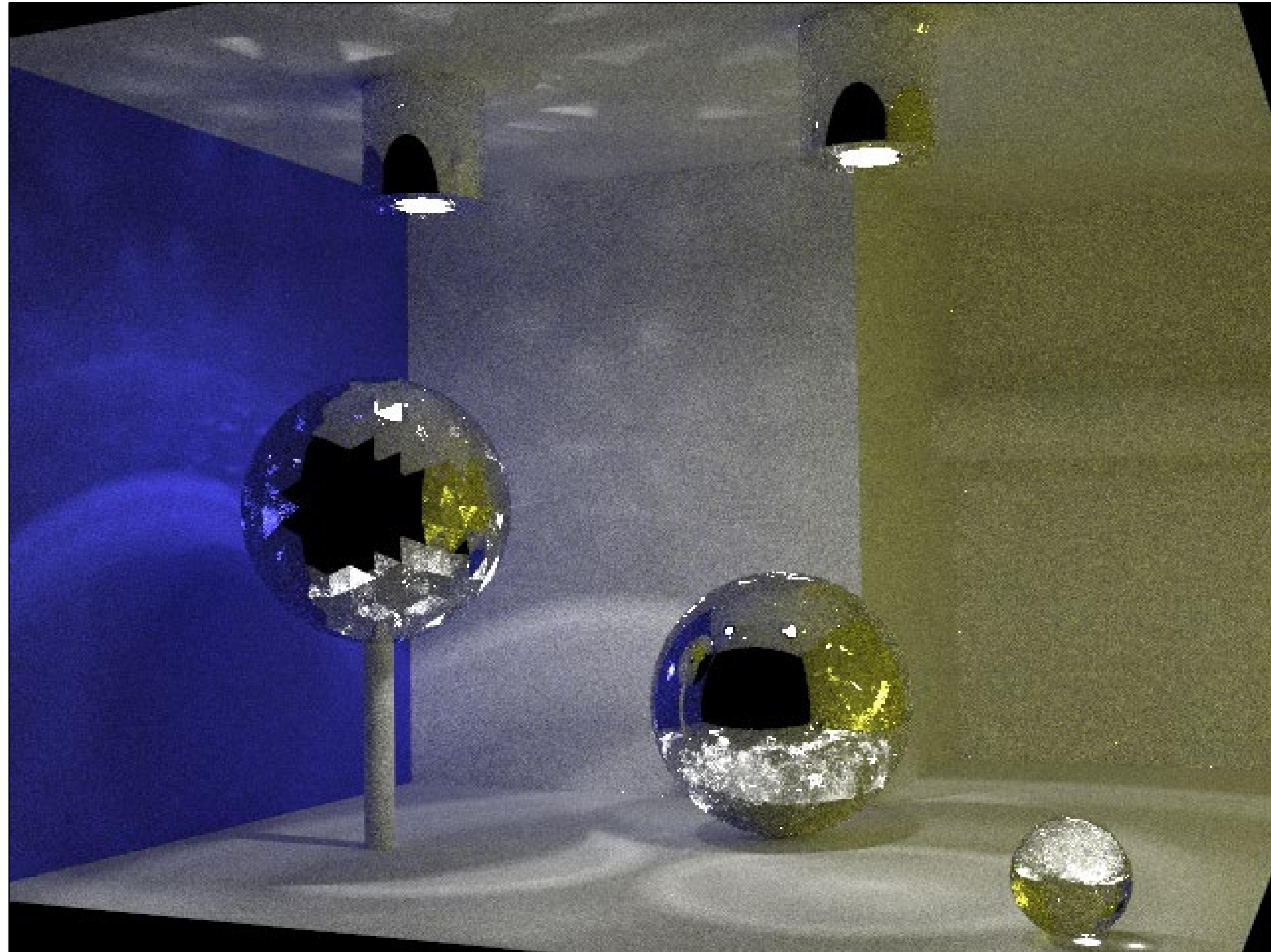
---



Images courtesy of T. Hachisuka

# Metropolis Light Transport

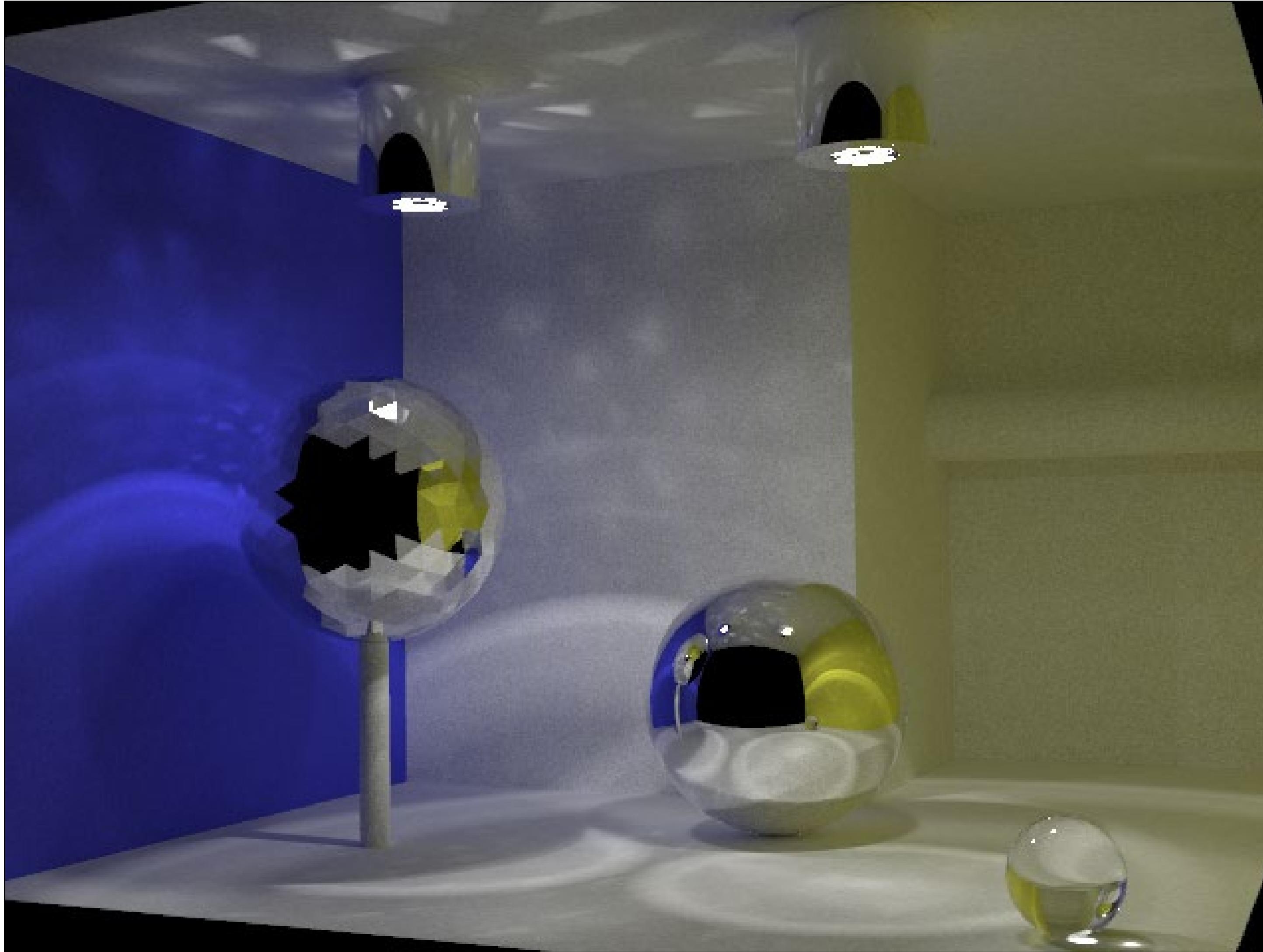
---



Images courtesy of T. Hachisuka

# Progressive Photon Mapping

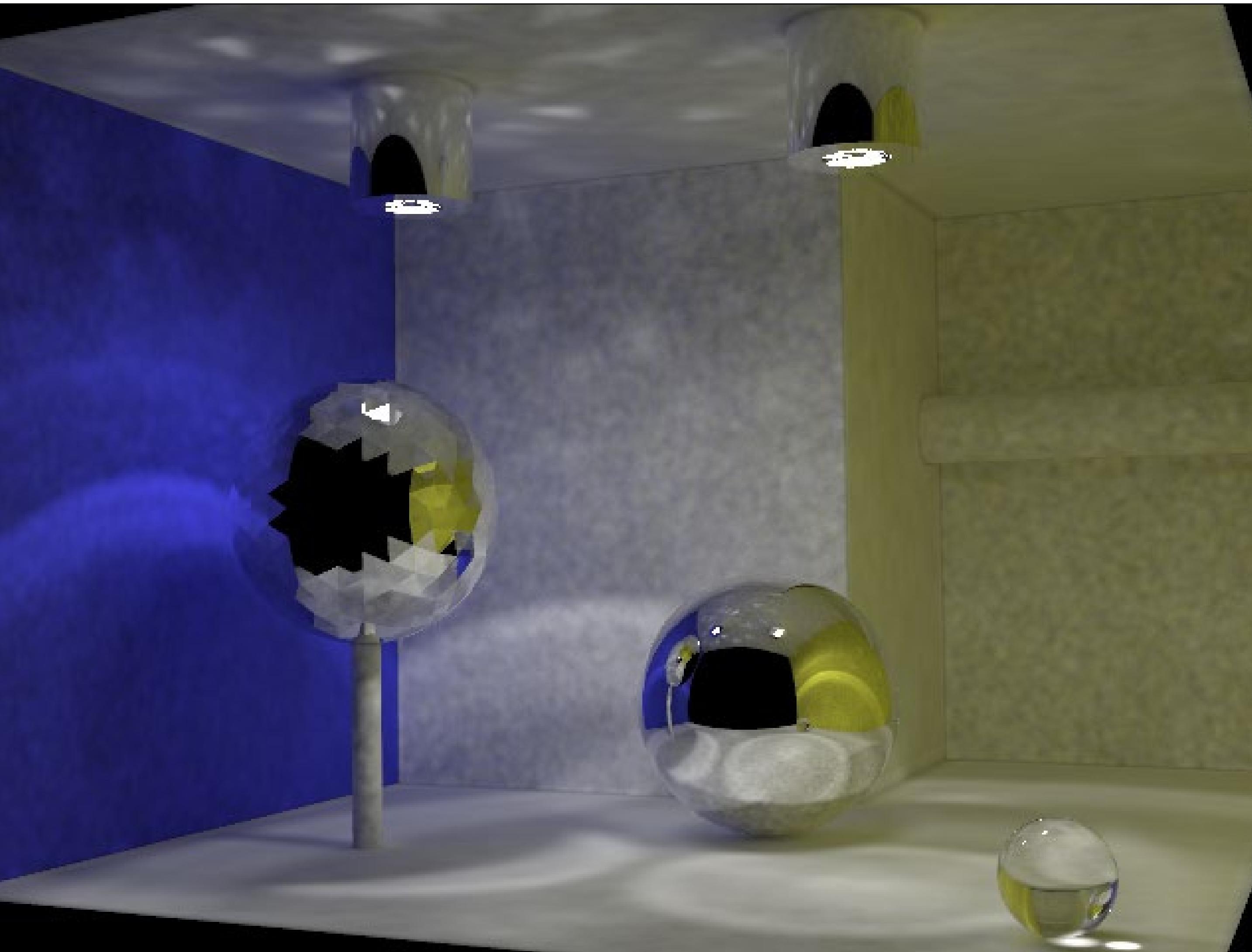
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Images courtesy of T. Hachisuka

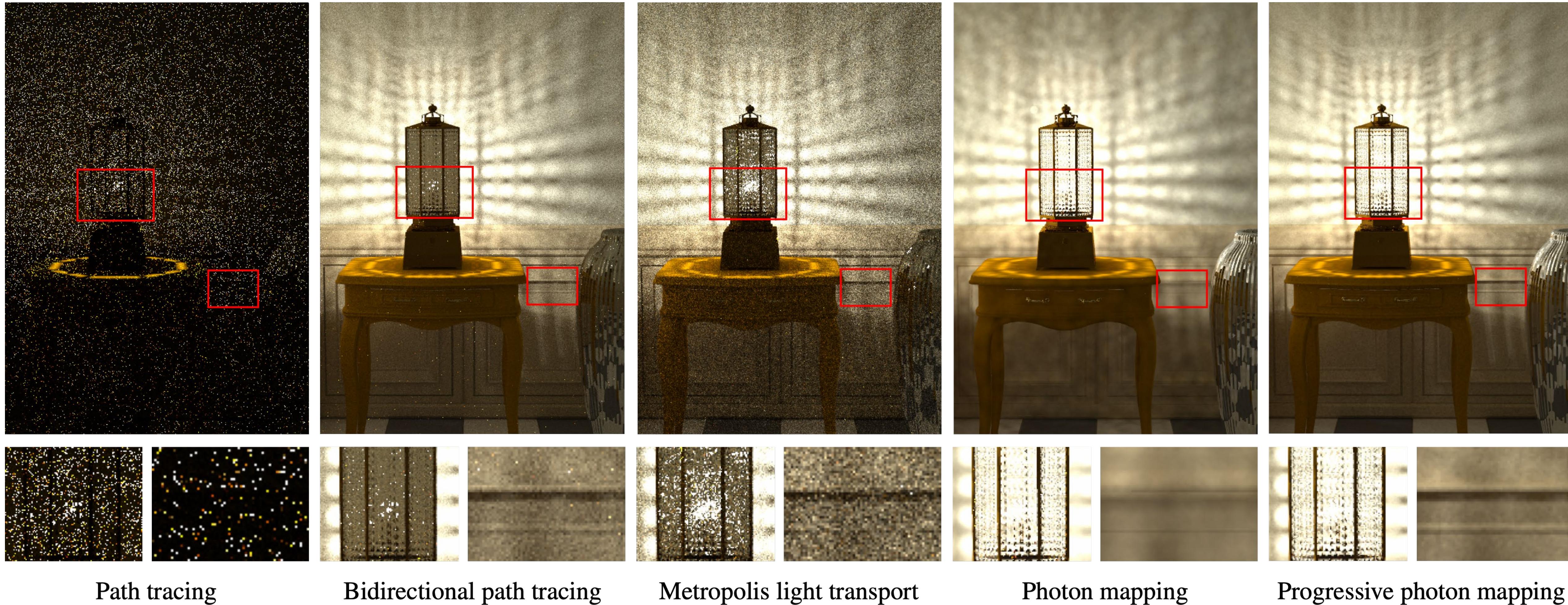
# Photon Mapping

---



Images courtesy of T. Hachisuka

# Glass Lantern



Path tracing

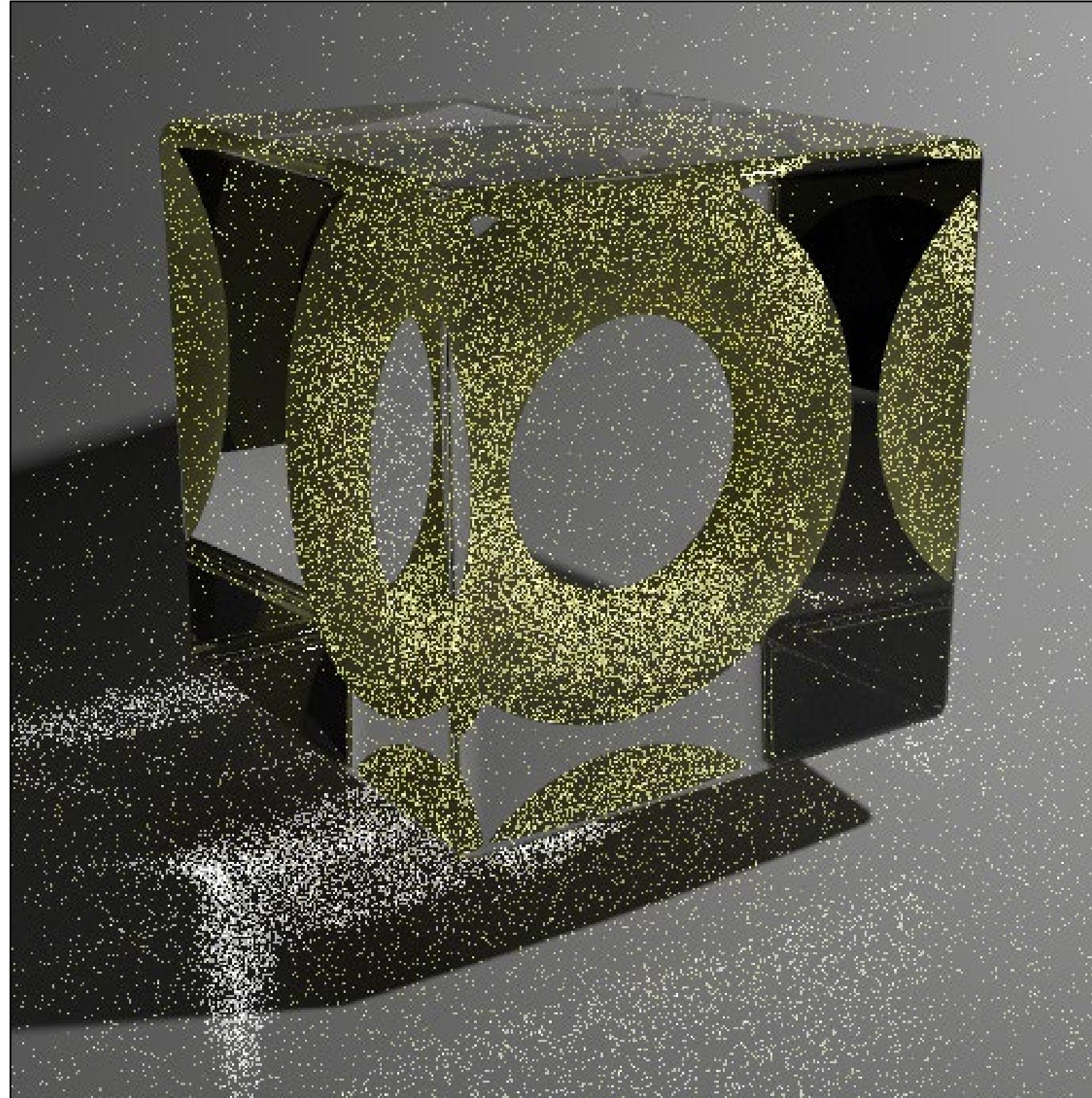
Bidirectional path tracing

Metropolis light transport

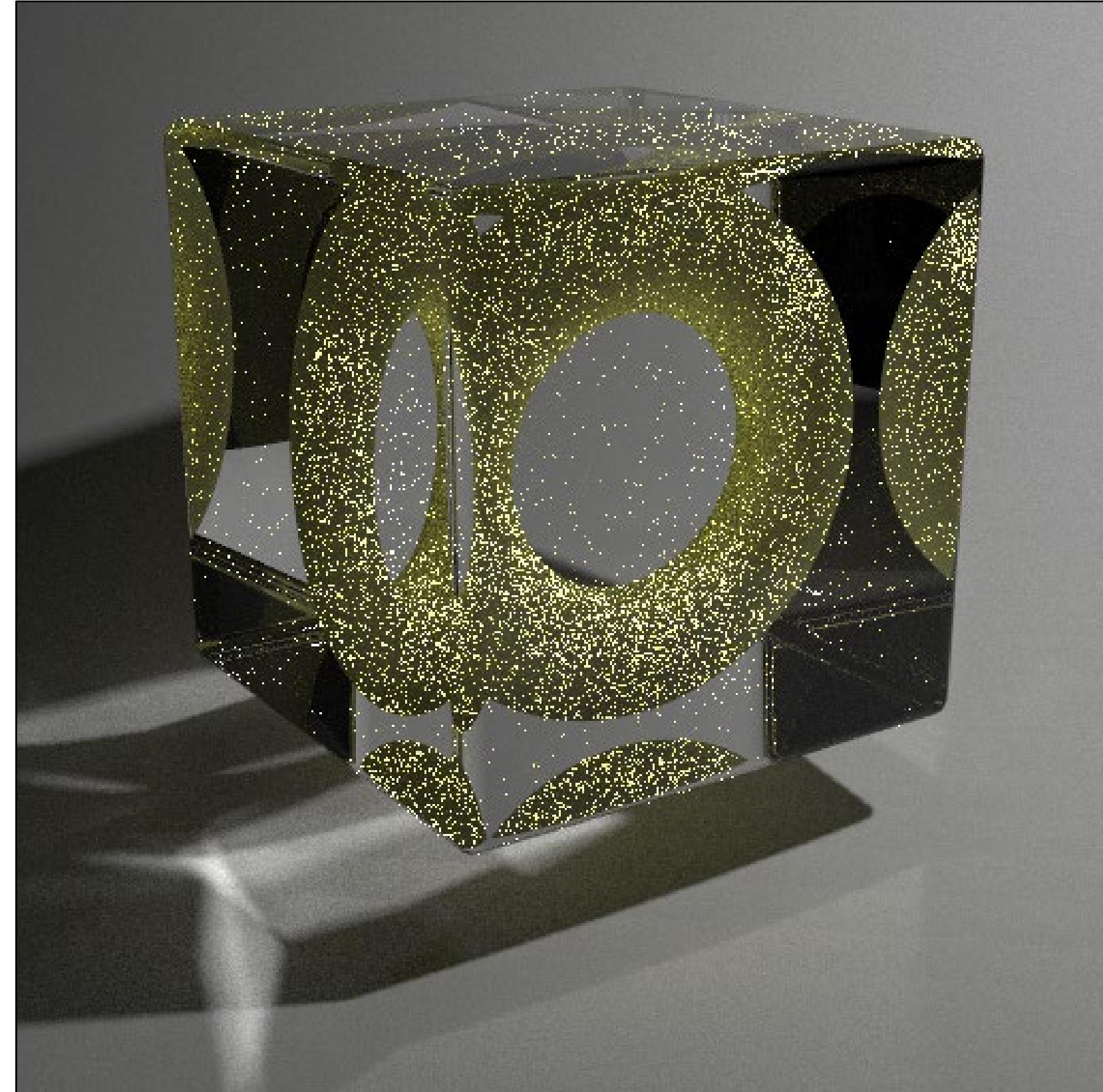
Photon mapping

Progressive photon mapping

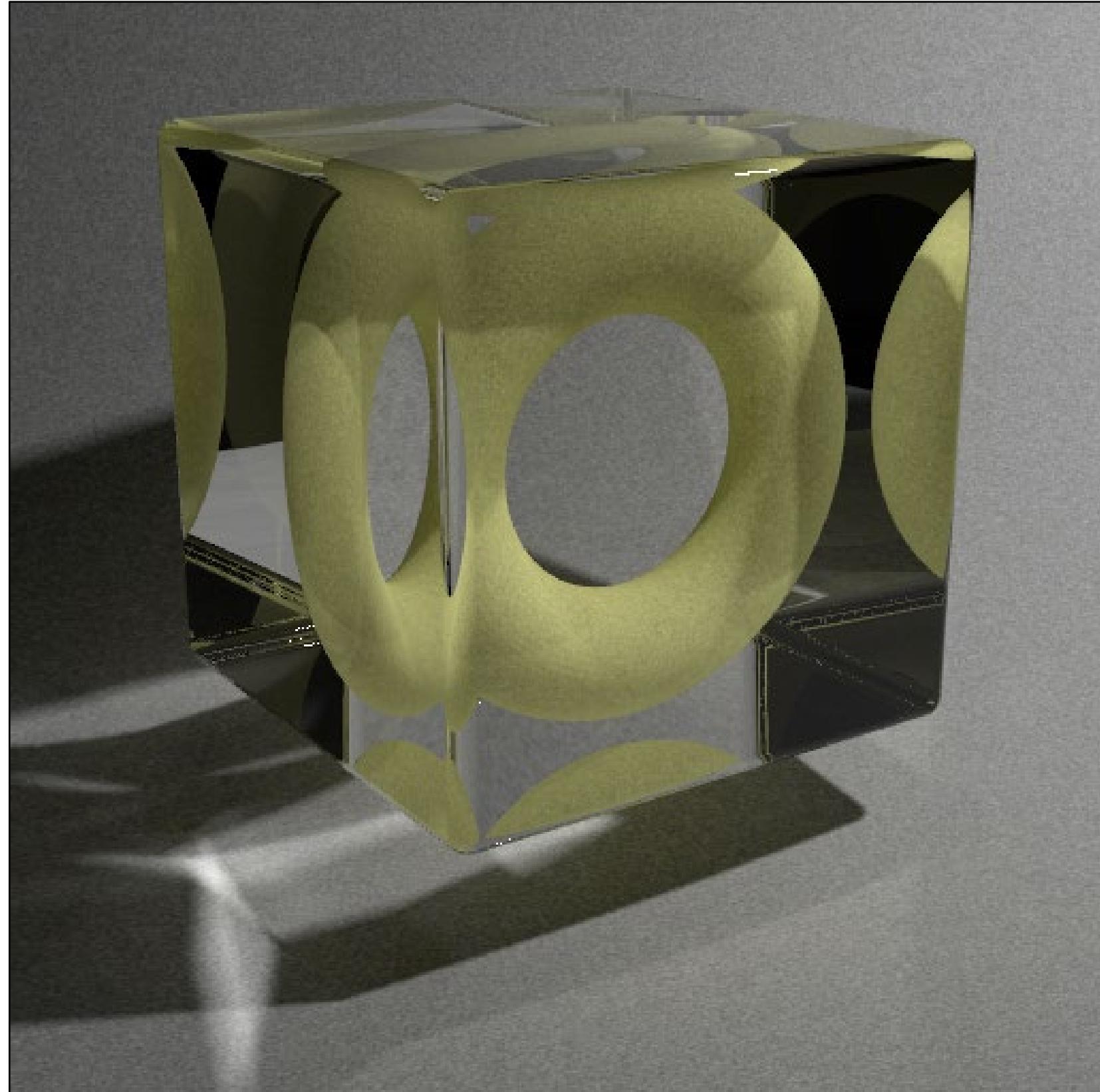
# Torus in Cube (LS+D\*+E)



Path Tracing



Bidirectional Path Tracing



Progressive Photon Mapping

# Progressive PM - Summary

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Reduces memory footprint

- Converges without requiring infinite memory

Renders progressively (user-friendly)

Data structure does not need to be as sophisticated

No need to bother using a caustic map, just use a single photon map for everything

# More On Photon Mapping

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