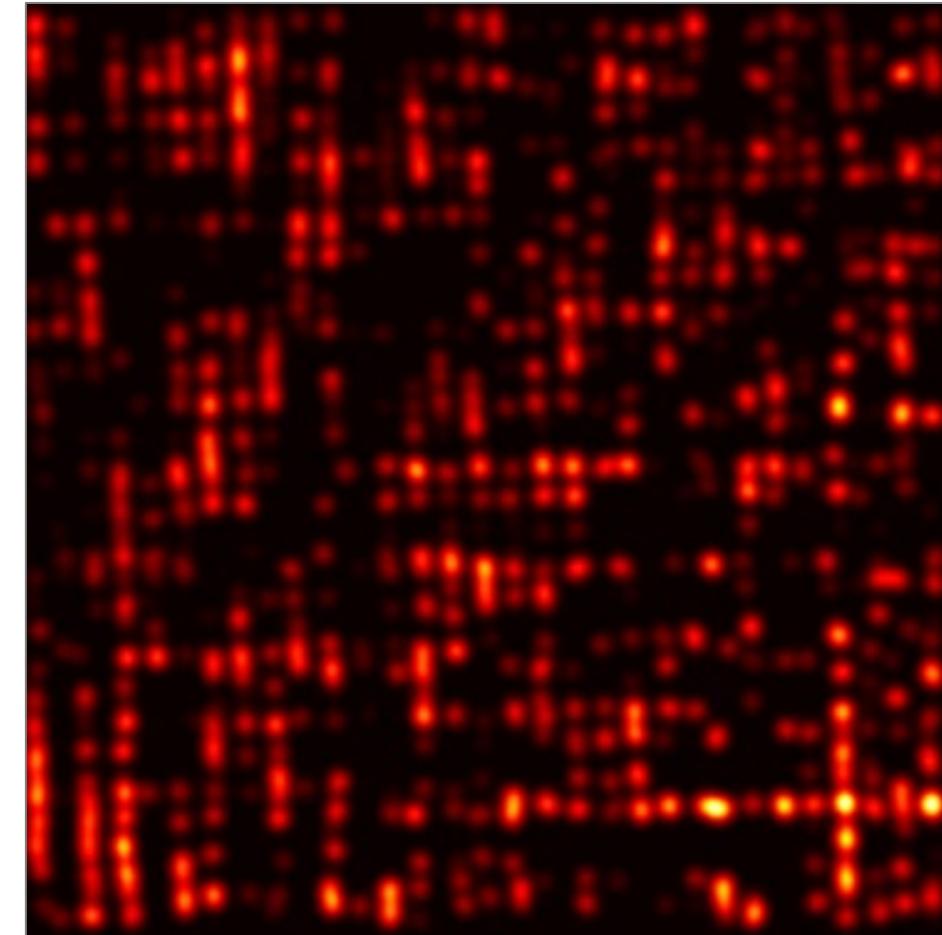
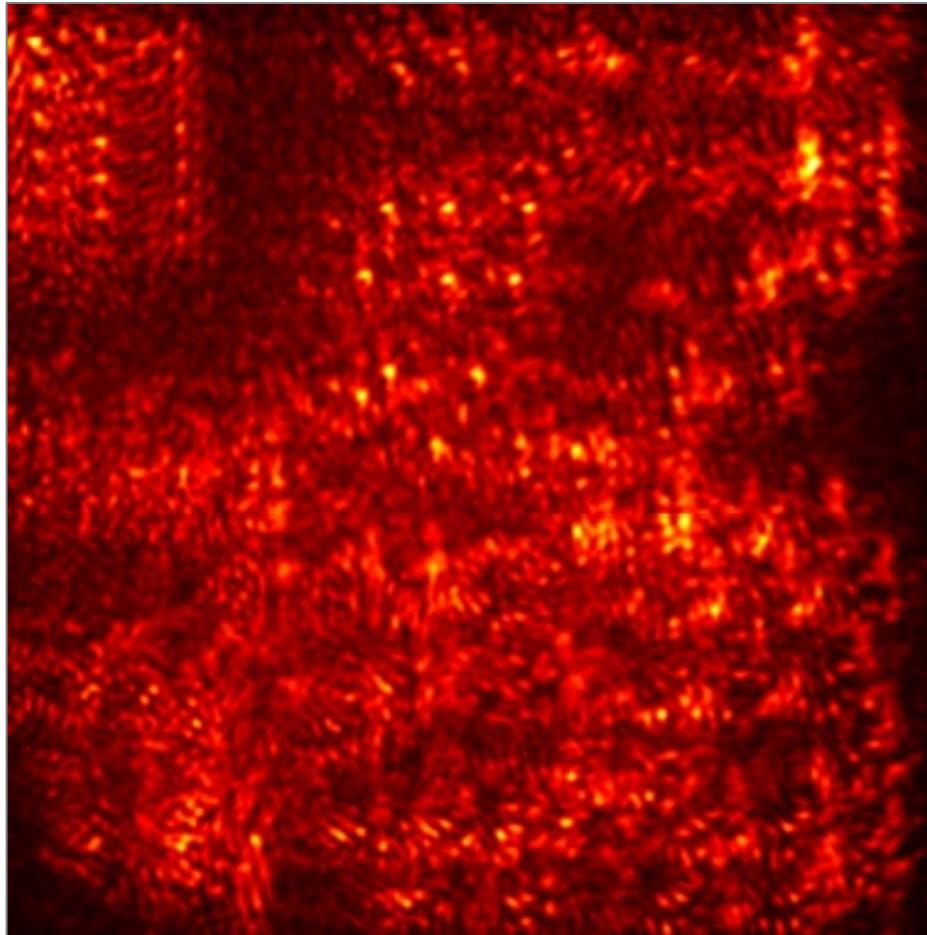


# Rendering wave effects



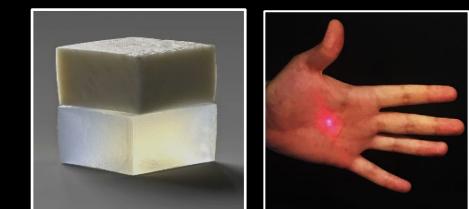
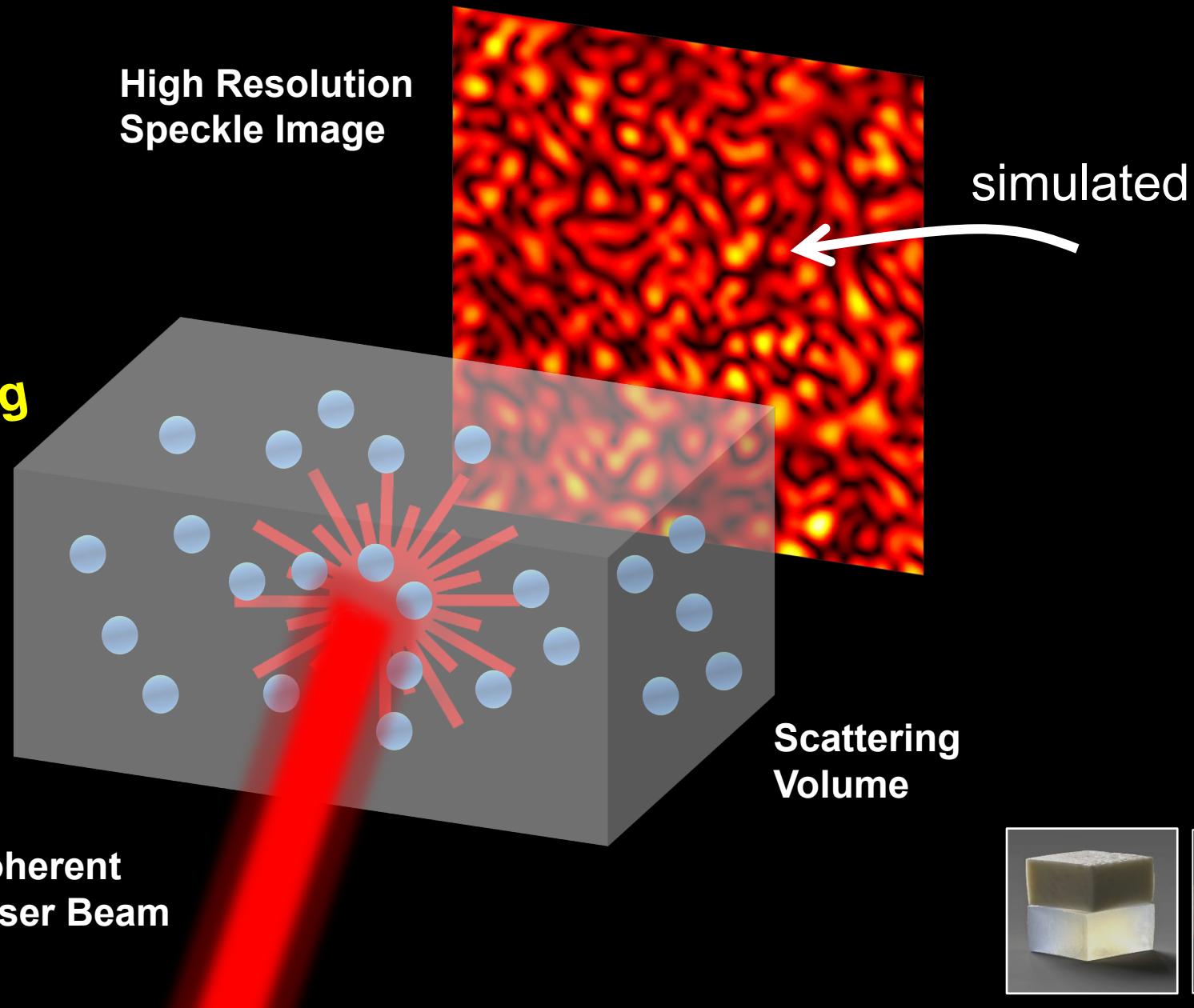
15-468, 15-668, 15-868  
Physics-based Rendering  
Spring 2022, Lecture 16

# Course announcements

- How many of you attended Thomas Mueller's talk?
- Take-home quiz 9-10 posted, due 4/26.
- Nobody was around for yesterday's recitation :-( .
- Will try to go over final project proposals tonight.

# Coherent Scattering and Memory Effect

The Memory Effect is  
very useful in many  
computational imaging  
applications



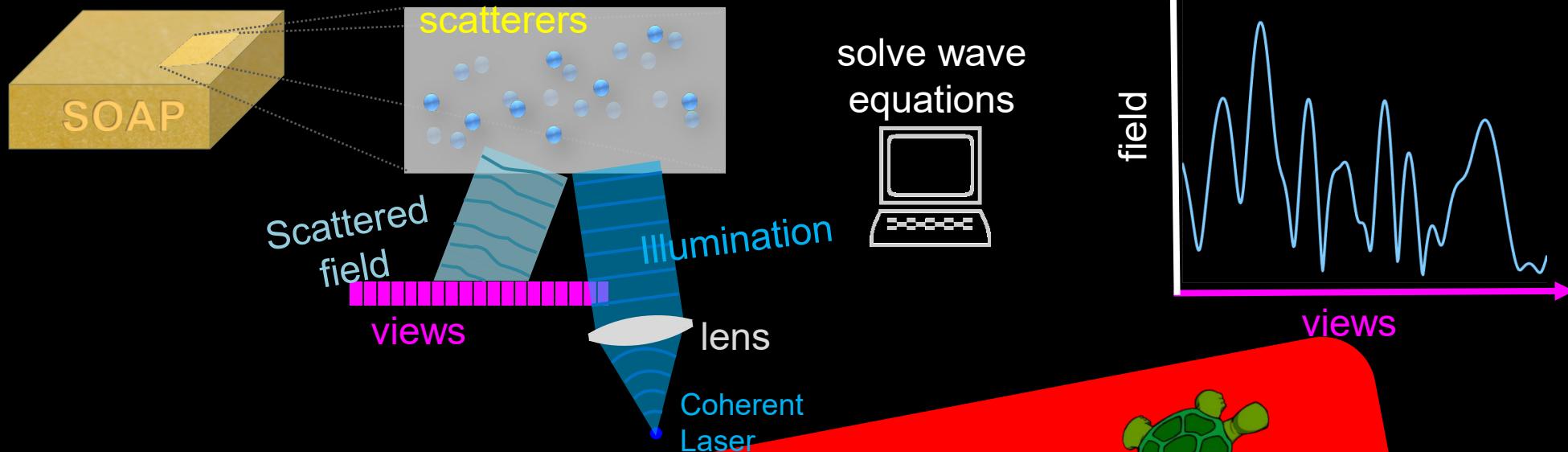


# Simulating speckles

inefficient

Specify exact (sub-wavelength)  
position of scatterers

In graphics we describe  
materials by **statistical**  
bulk parameters, as  
the **density** of scatterers



## Wave equation solvers

- Differential equation F
- Integral equation (e.g.,

Slow

Practical only for tiny  
or optically thin media



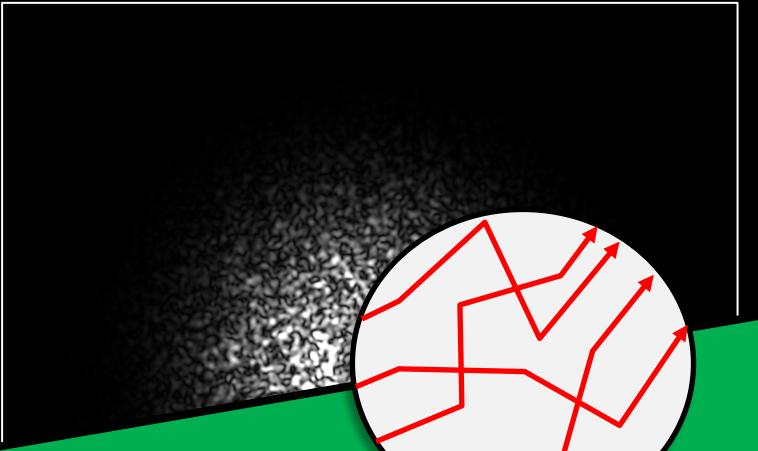
For simplicity: **Flatland**  
Scattering medium is **2D**  
Sensor is **1D**  
Speckle pattern is **1D**

# Monte Carlo (MC) Simulation of Speckles

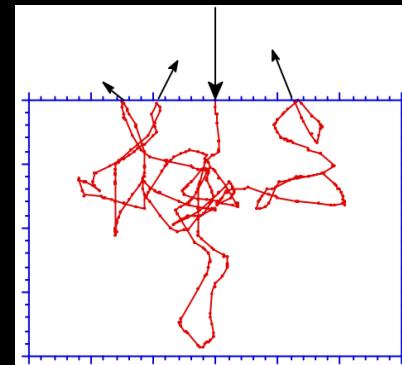
MC Advantage:

1. **Fast**
2. input is scatterer **density** rather than exact scatterer locations

**Our Goal:**  
Extend efficient MC tools  
to evaluate speckles and  
their coherent statistics

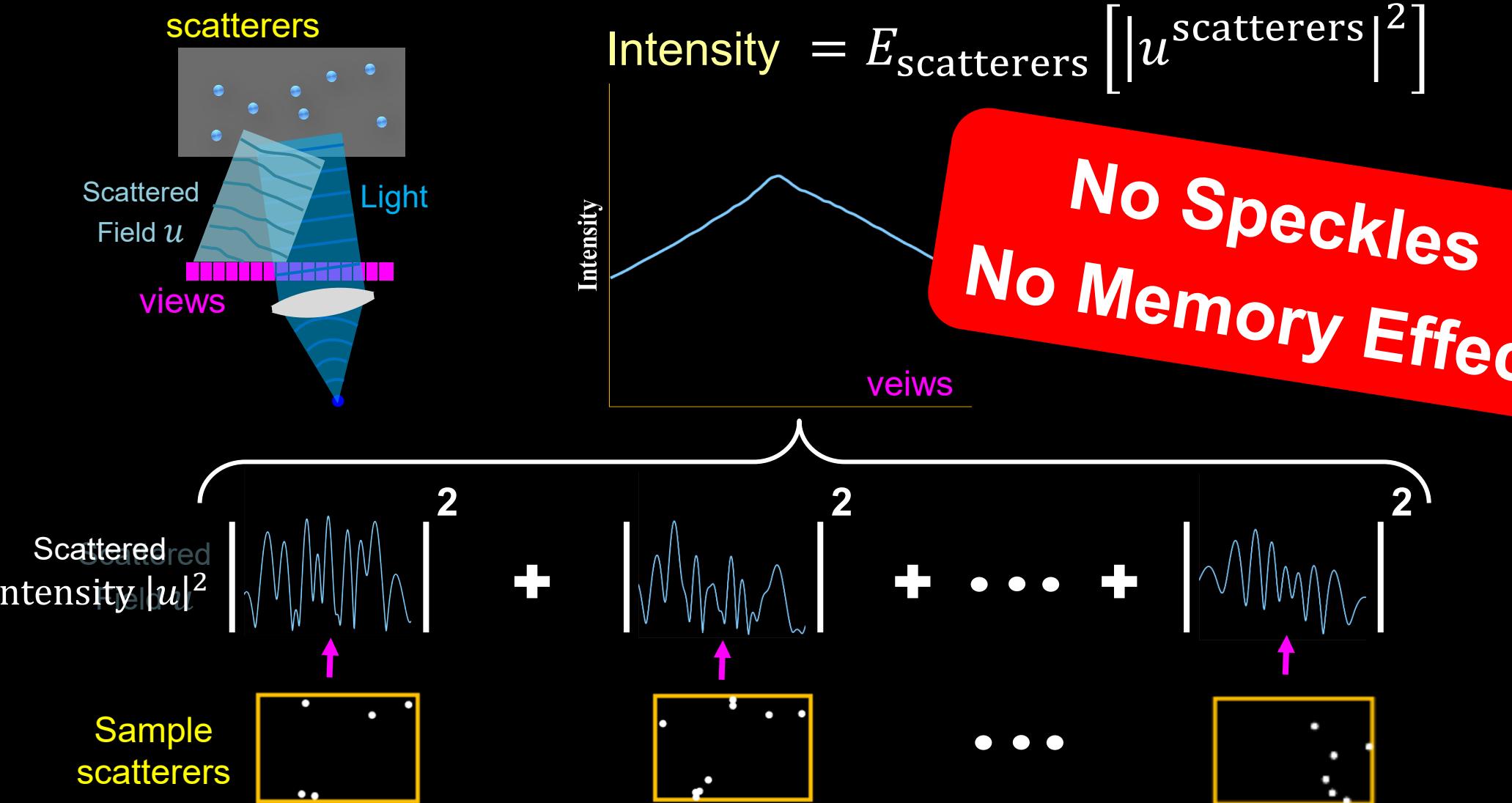


Standard intensity MC



Monte Carlo Modeling of Light Transport in Multi-layered Tissues,  
Wang & Jacques, 1992

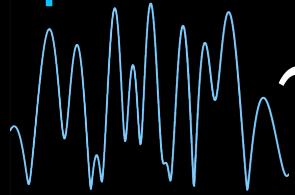
# Wave Solution v.s. Monte Carlo



MC requires the scatterers density – no need for exact positions

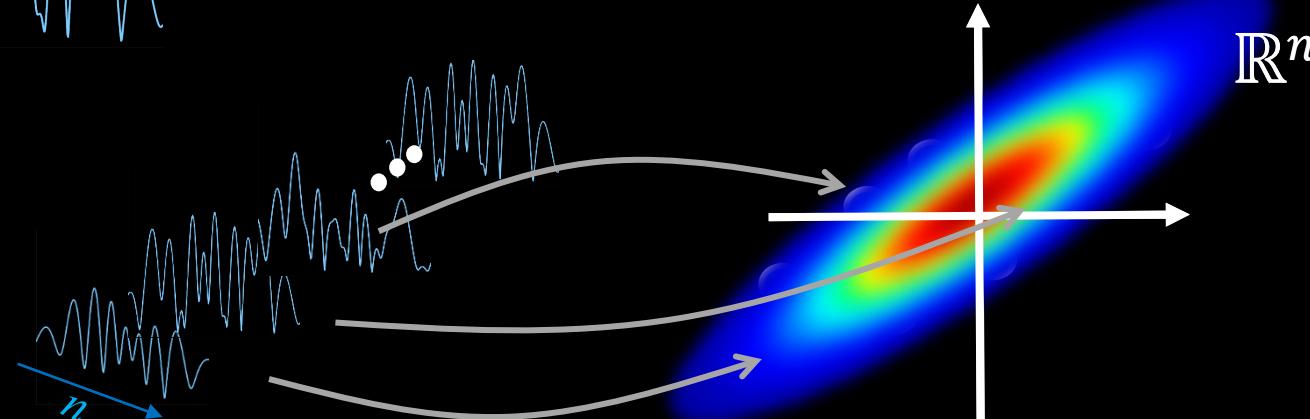
# Speckle Statistics

Speckles



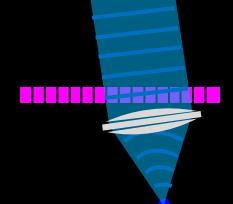
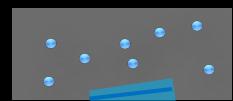
$$\sim \mathcal{N}(\text{Mean}, \text{Covariance})$$

Sufficient Statistics



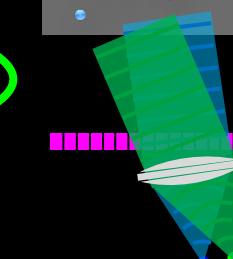
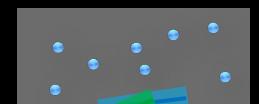
1<sup>st</sup> moment

$$\text{Intensity Mean} = E_{\text{scatterers}} [|\vec{u}_{\text{scatterers}}|^2] \rightarrow \text{Incoherent Summation}$$

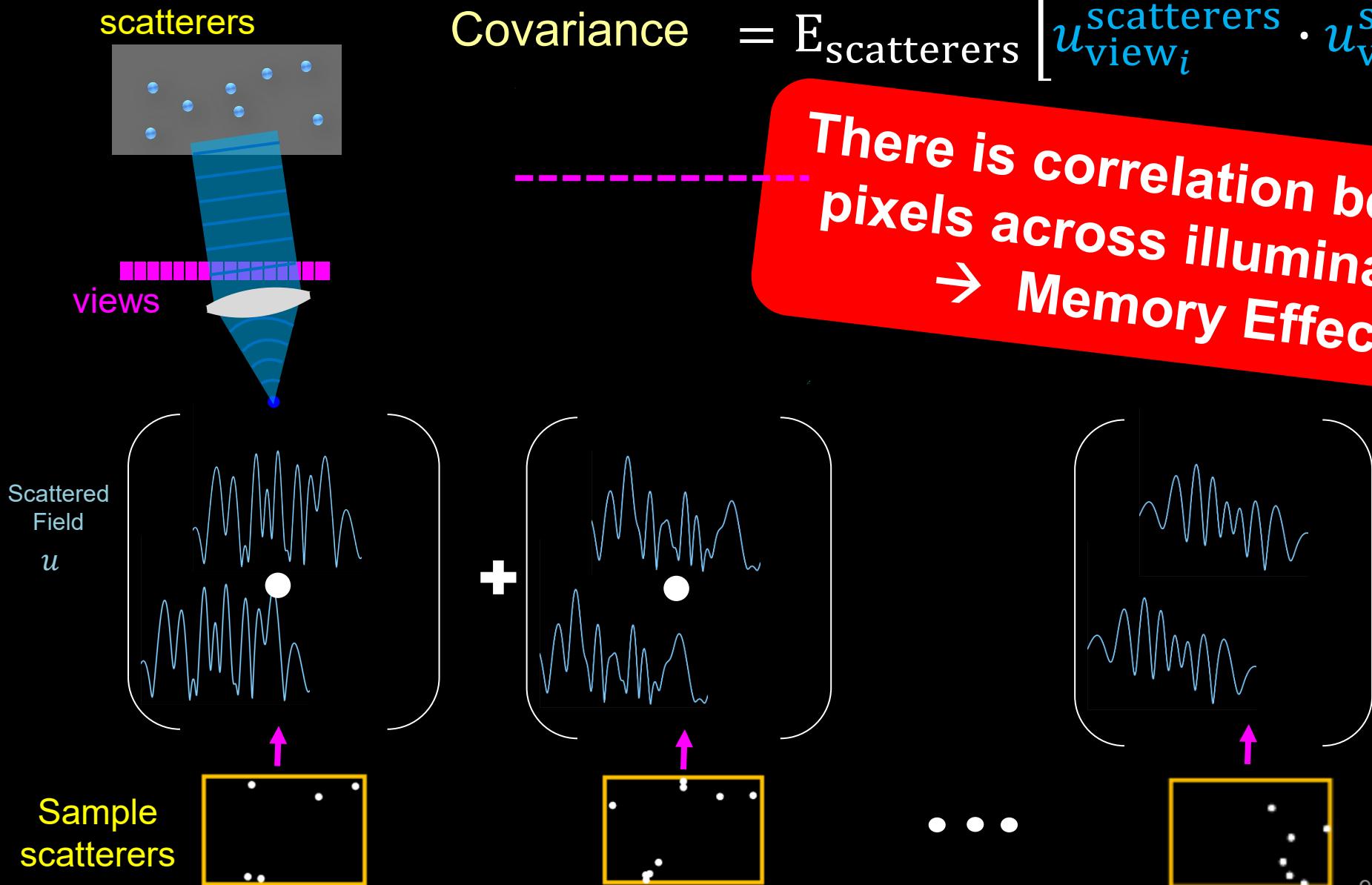


Cross-Illumination

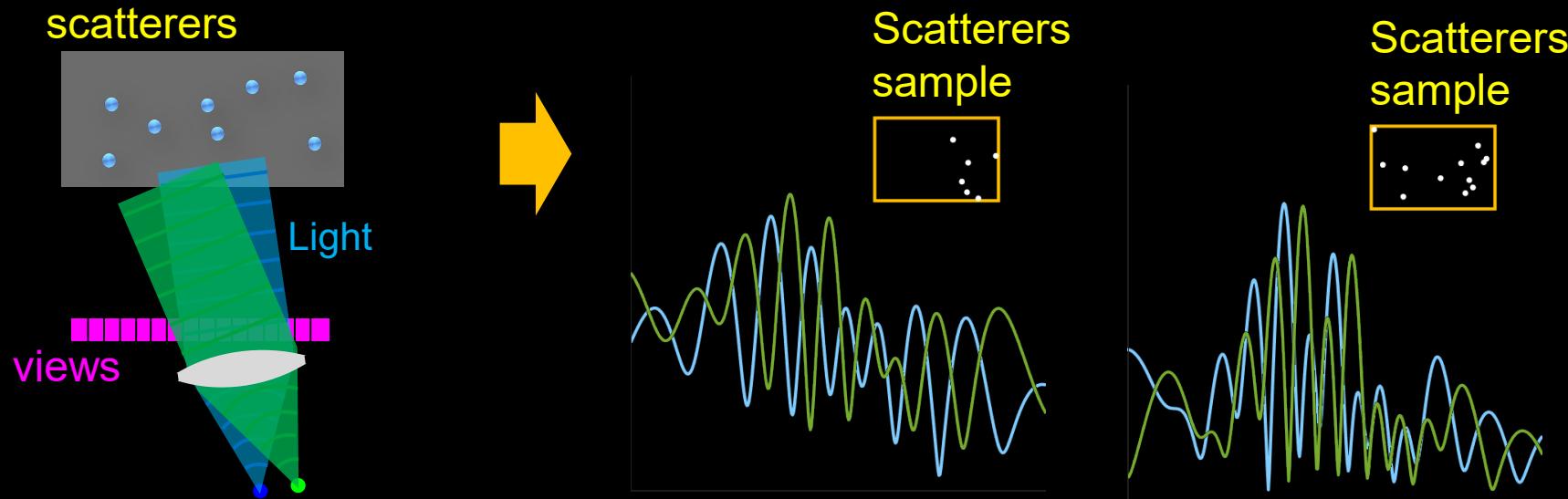
$$\text{Field Covariance} = E_{\text{scatterers}} [u_{\text{view}_i}^{\text{light}_1, \text{scatterers}} \cdot u_{\text{view}_j}^{\text{light}_2, \text{scatterers}*}]$$



# 2<sup>nd</sup> Moment - Covariance



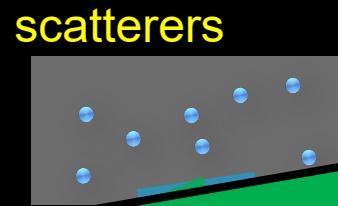
# Cross –illumination statistics



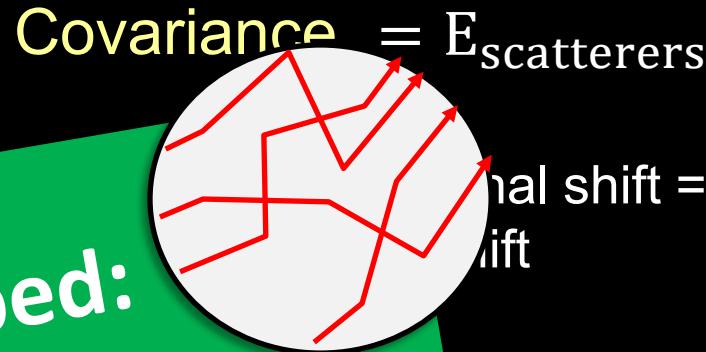
**Memory Effect:**  
tilting illumination results in highly correlated shifted speckles

Next: Cross Illumination Covariance

# Cross -illumination statistics



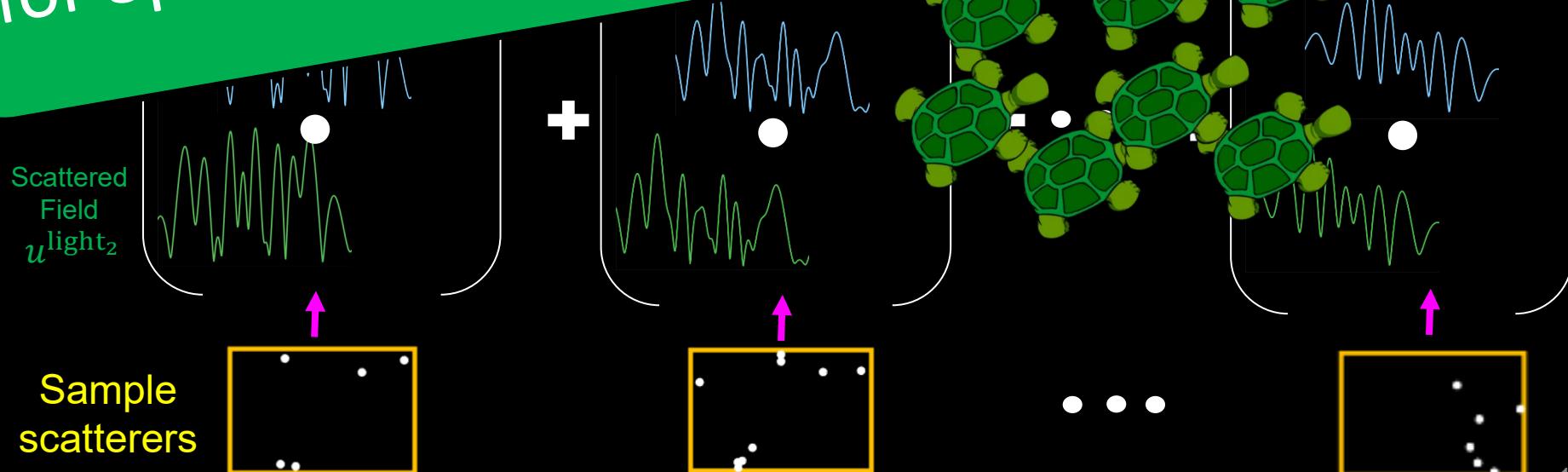
We developed:  
Efficient MC  
for Speckle Covariance



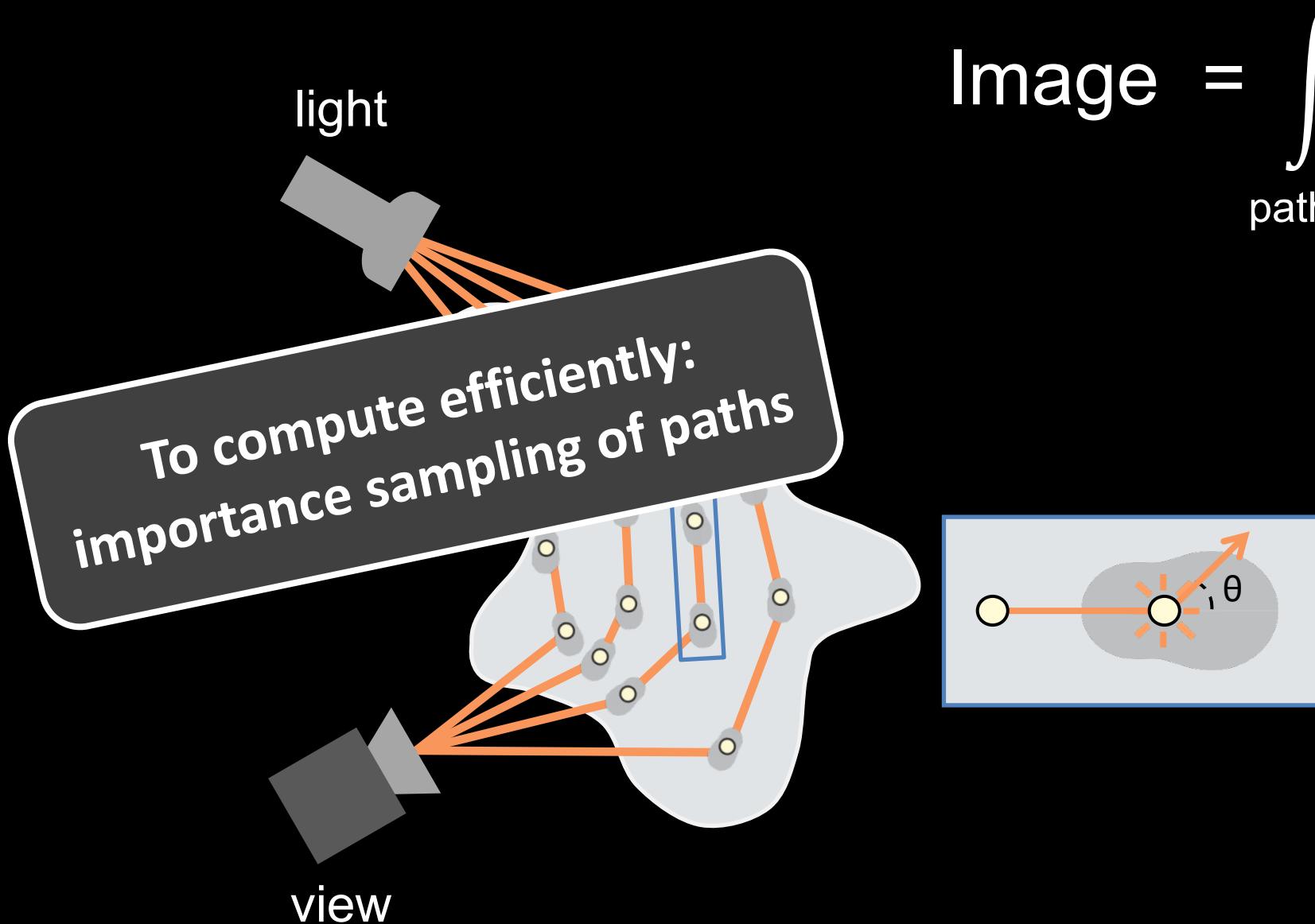
$$\text{Covariance} = \mathbb{E}_{\text{scatterers}} [u_{\text{view}_i}^{\text{light}_1, \text{scatterers}} \cdot u_{\text{view}_j}^{\text{light}_2, \text{scatterers}^*}]$$

Speckles are Gaussian:  
Mean + Covariance  
are sufficient statistics

This is all we need to describe speckles



# Monte Carlo Rendering 101



$$\text{Image} = \int_{\text{paths}} f(\text{path})$$

Throughput that acts on each path, depends on the scattering material

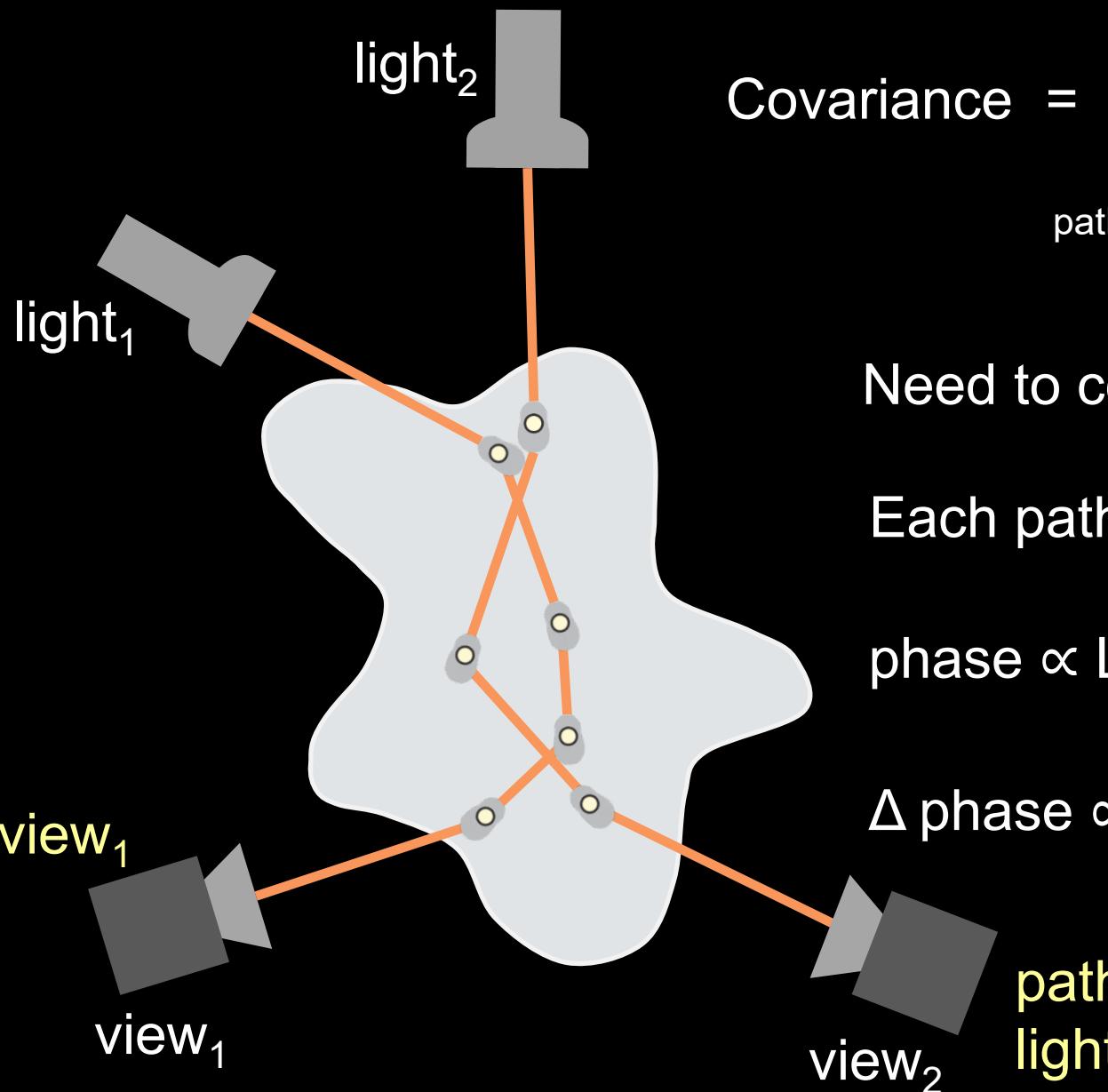
volumetric density  
(extinction coefficient)

material  
scatter albedo

phase function

$$\begin{bmatrix} \sigma \\ a \\ p_\theta \end{bmatrix}$$

# Covariance Rendering



$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$

$\mathbf{u} = |\mathbf{u}| e^{i \cdot \text{phase}}$

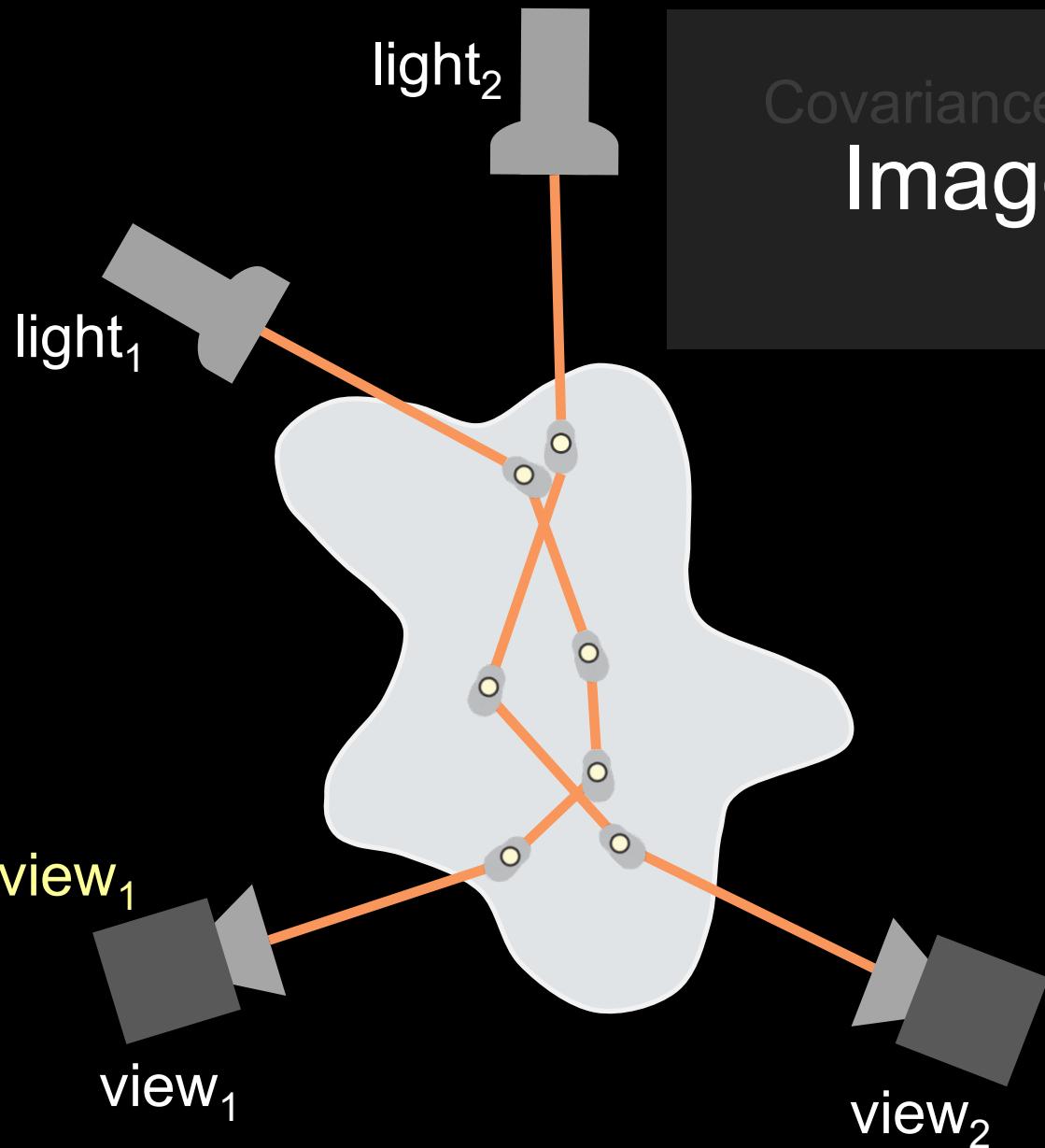
Need to consider products of **pairs** of paths

Each path contributes a complex number  $\mathbf{u}$

phase  $\propto$  Length ( path )

$\Delta$  phase  $\propto$  Length ( path<sub>1</sub> ) - Length ( path<sub>2</sub> )

# Covariance Rendering



$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} u(\text{path}_1) \cdot u^*(\text{path}_2)$$

**Image** =  $\int_{\text{paths}} f(\text{path})$

Real throughput  $u = |u| e^{i \cdot \text{phase}}$

$$\text{path}_1 = \text{path}_2$$



Same complex contribution

$$u(\text{path}_1) = u(\text{path}_2)$$



$$\Delta \text{phase} = 0$$

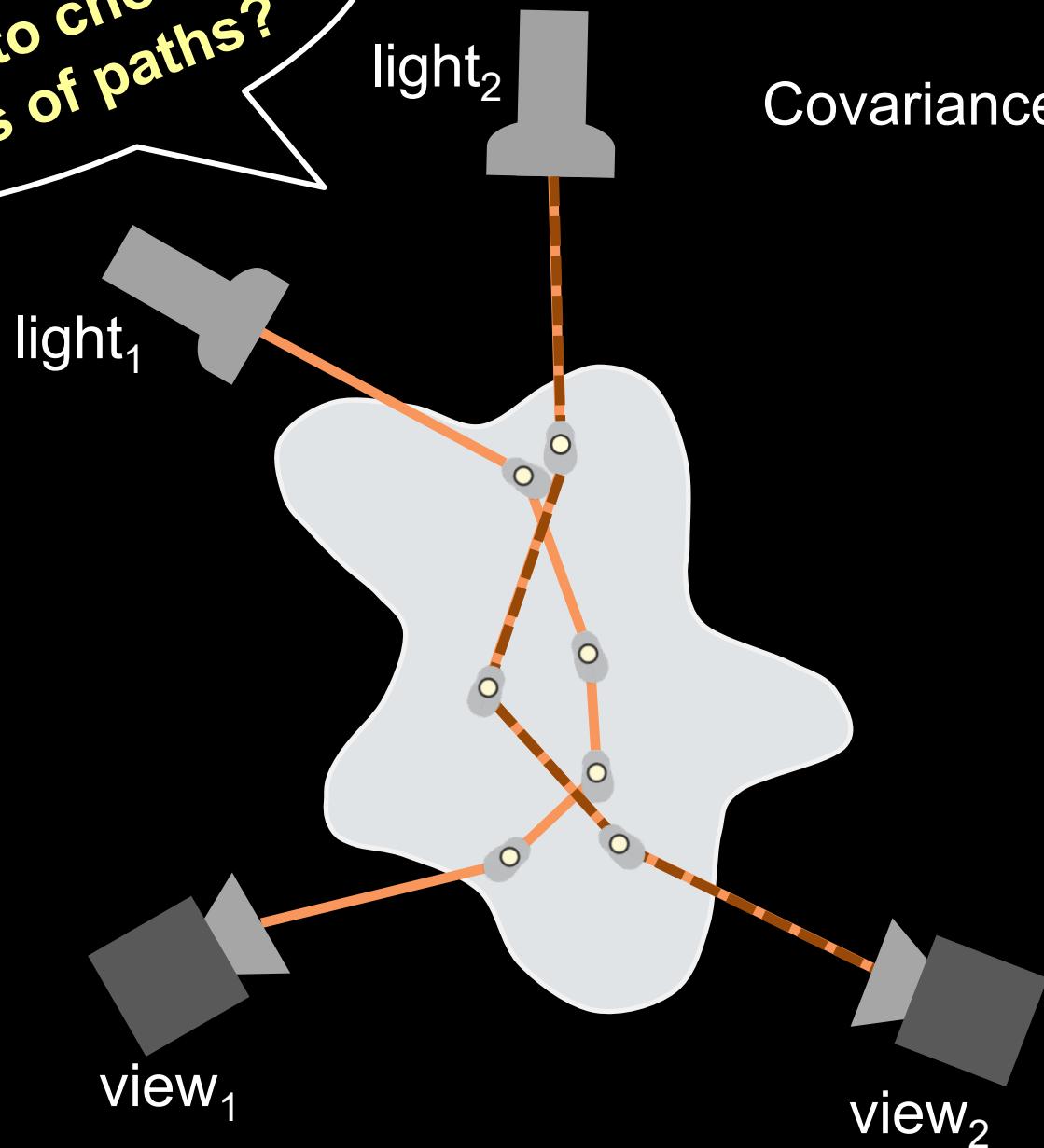
Real

$$\text{path}_2 : u(\text{path}) \cdot u^*(\text{path}) = f(\text{path})$$

$\text{light}_2 \rightarrow \text{view}_2$

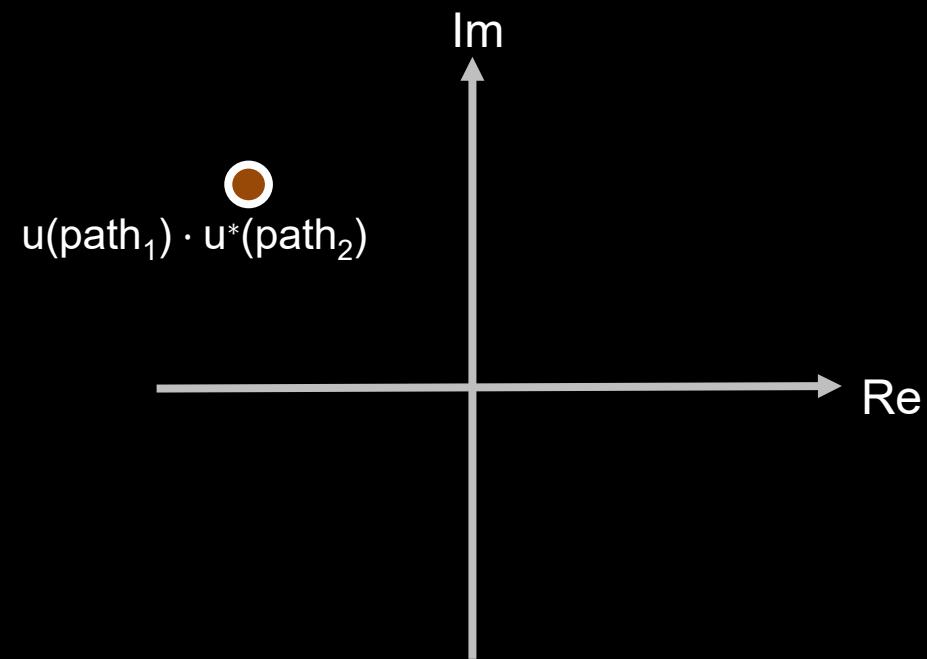
# Covariance rendering

How to choose  
pairs of paths?

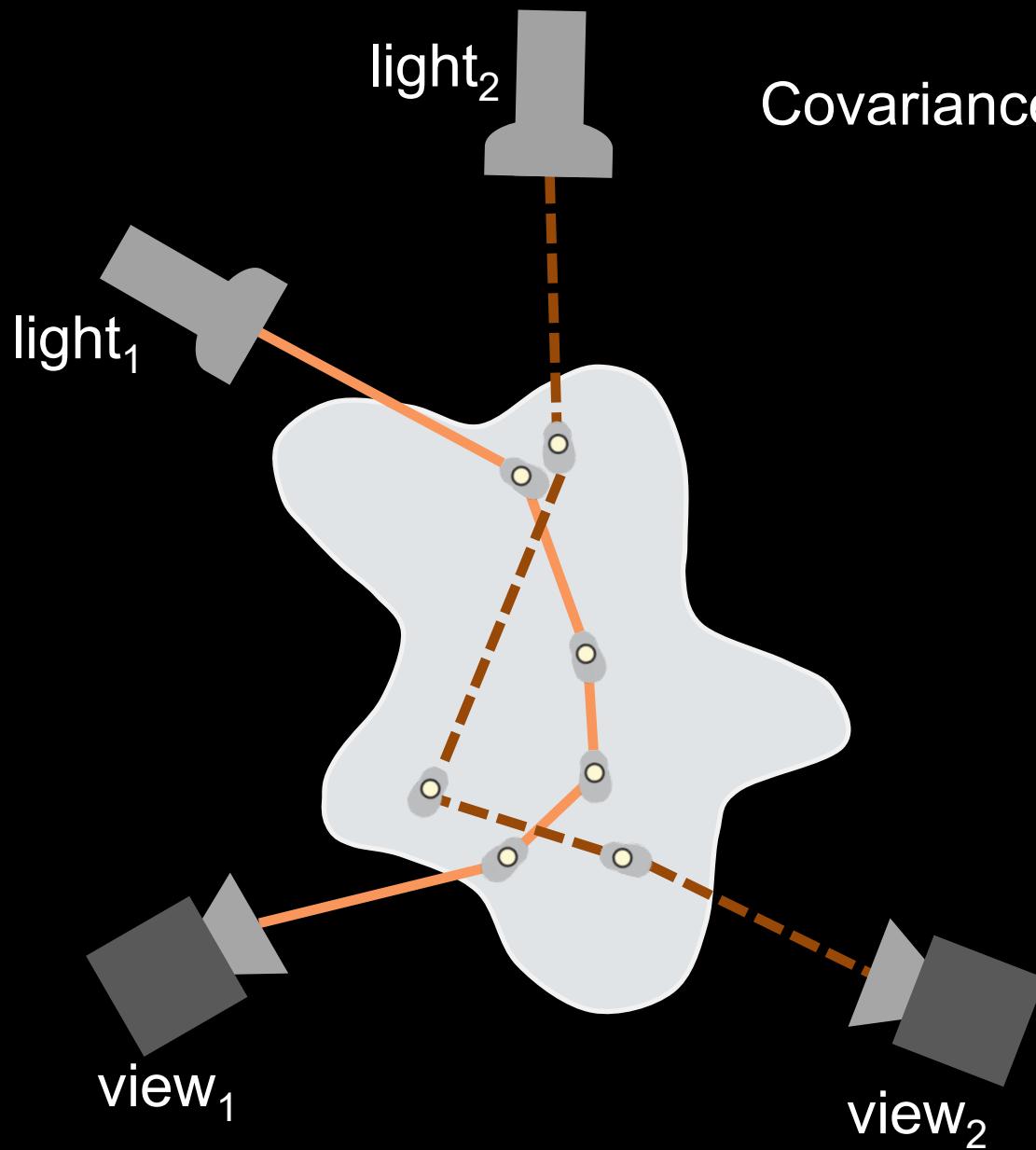


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$

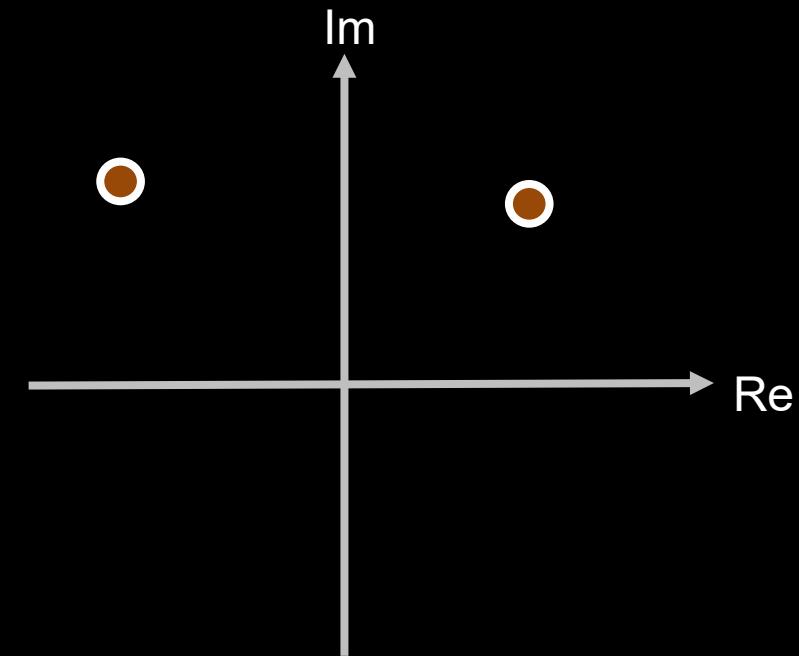
$\mathbf{u} = |\mathbf{u}| e^{i \cdot \text{phase}}$



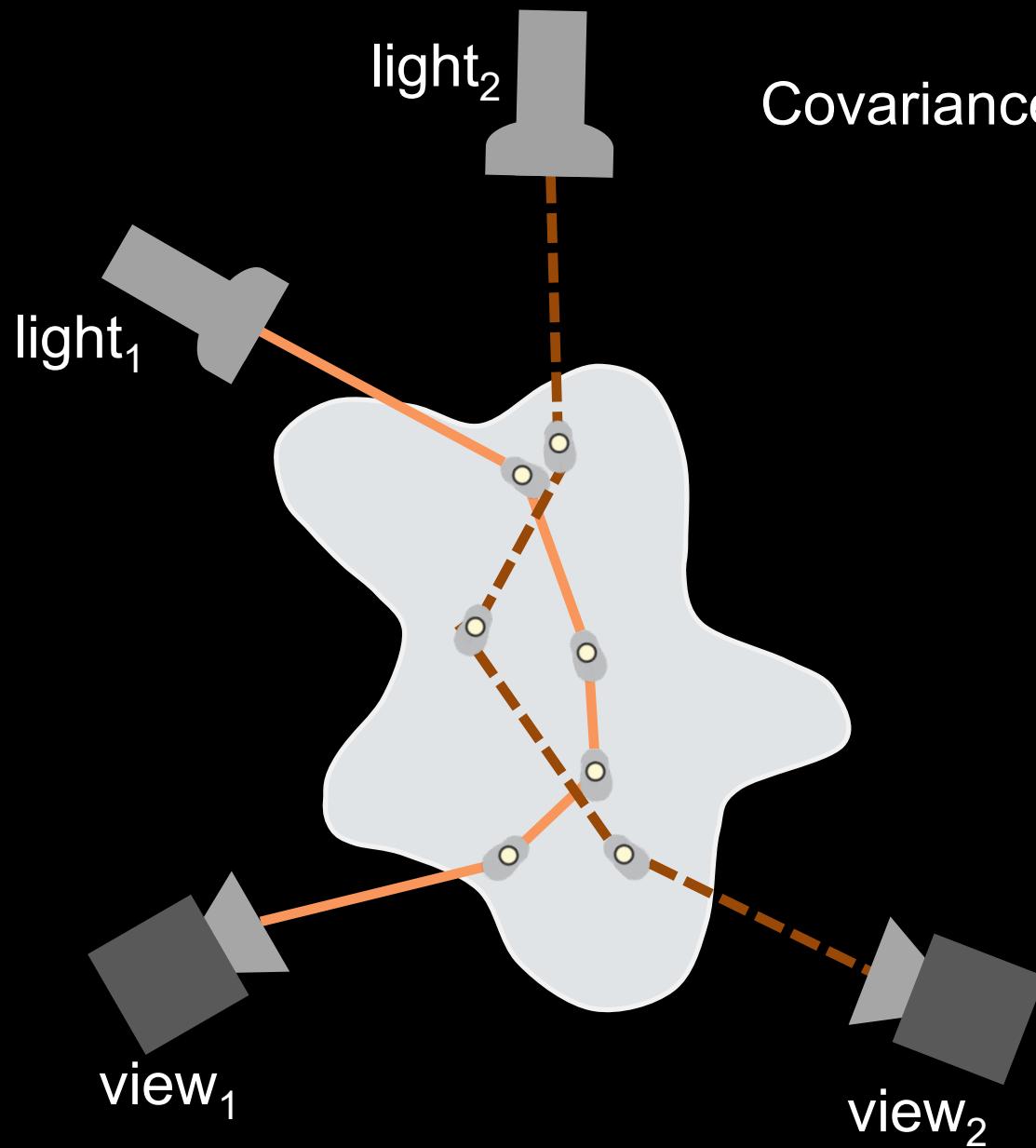
# Covariance rendering



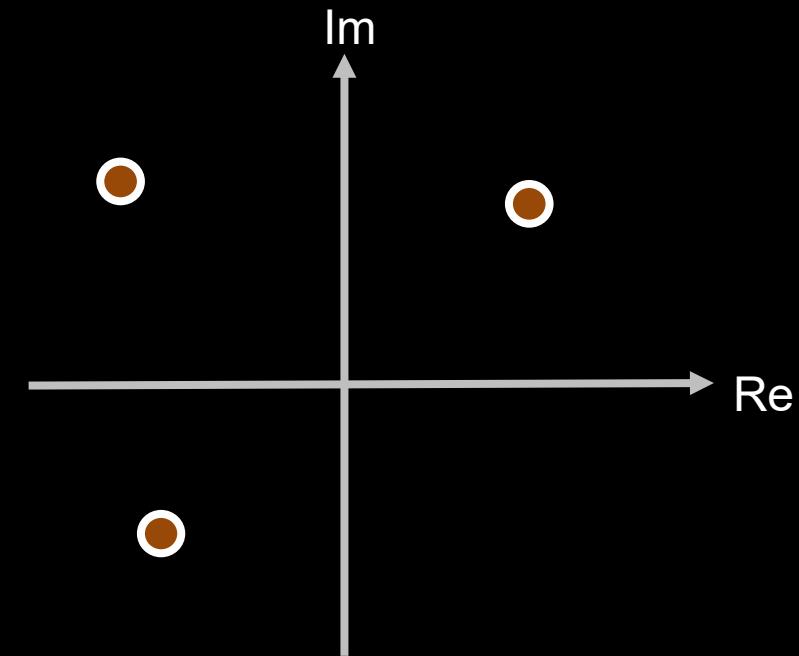
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$



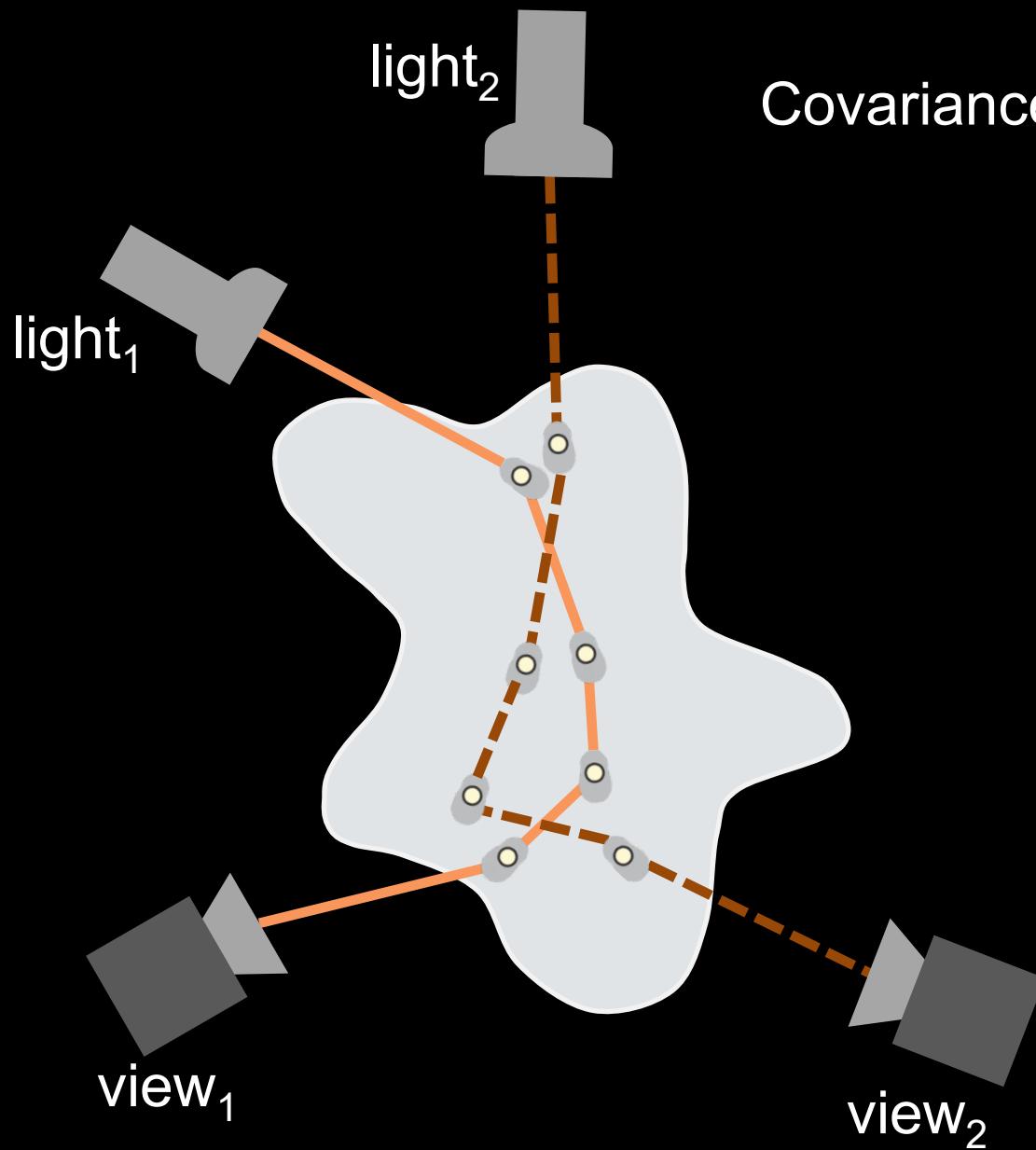
# Covariance rendering



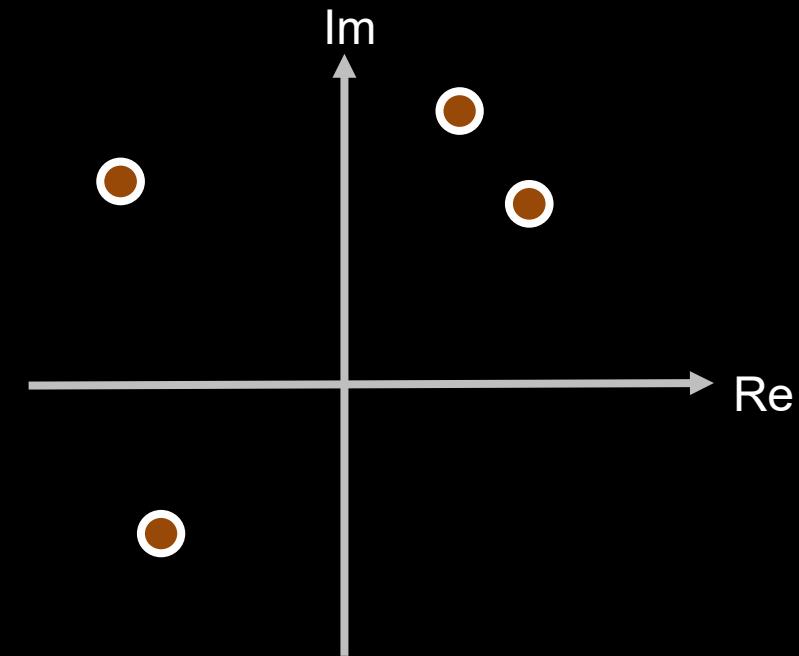
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$



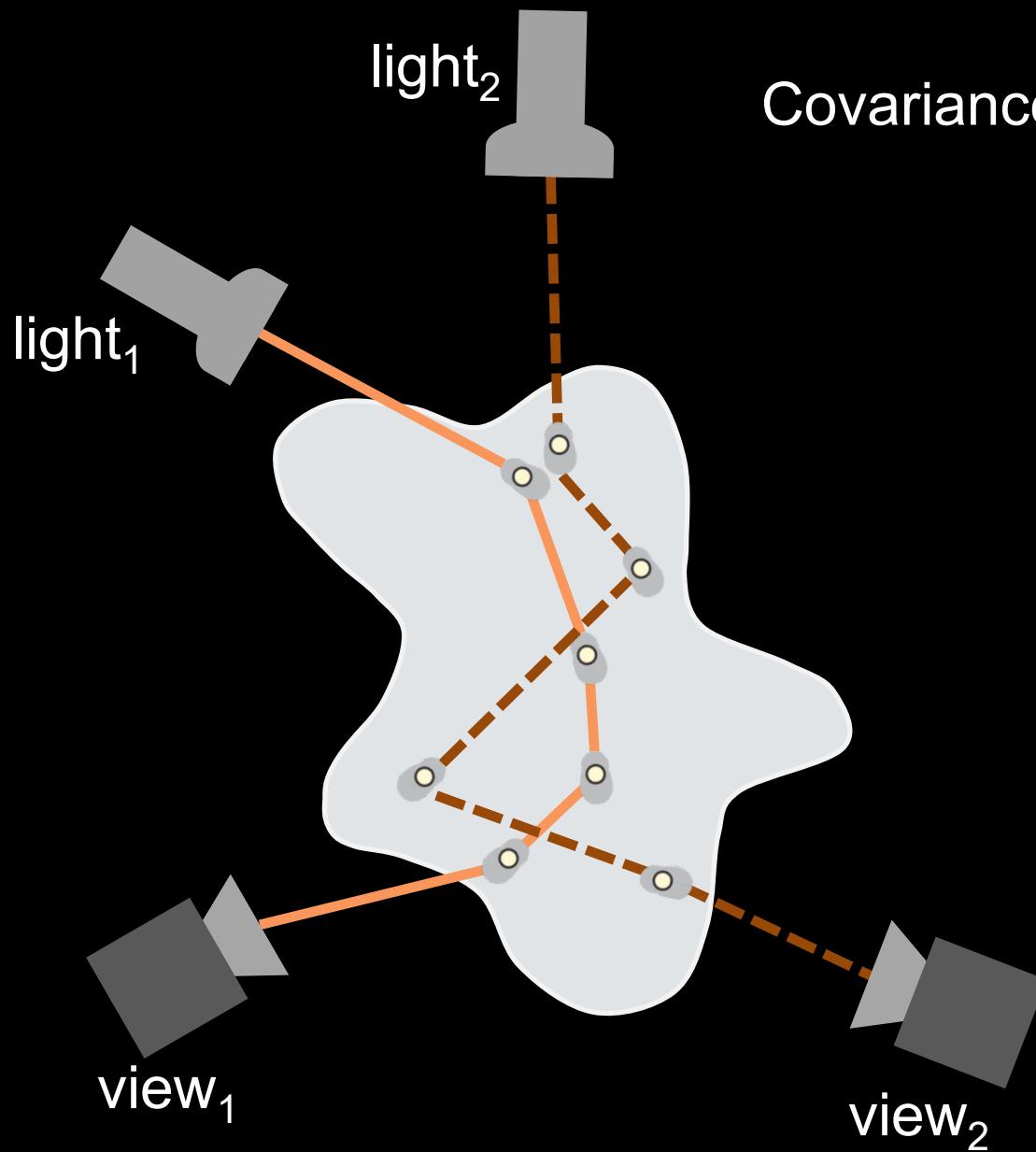
# Covariance rendering



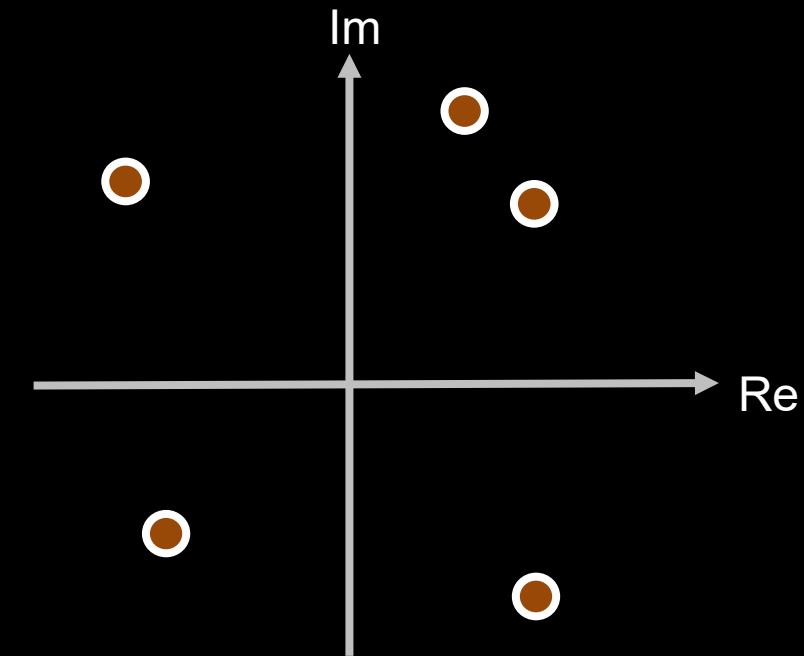
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$



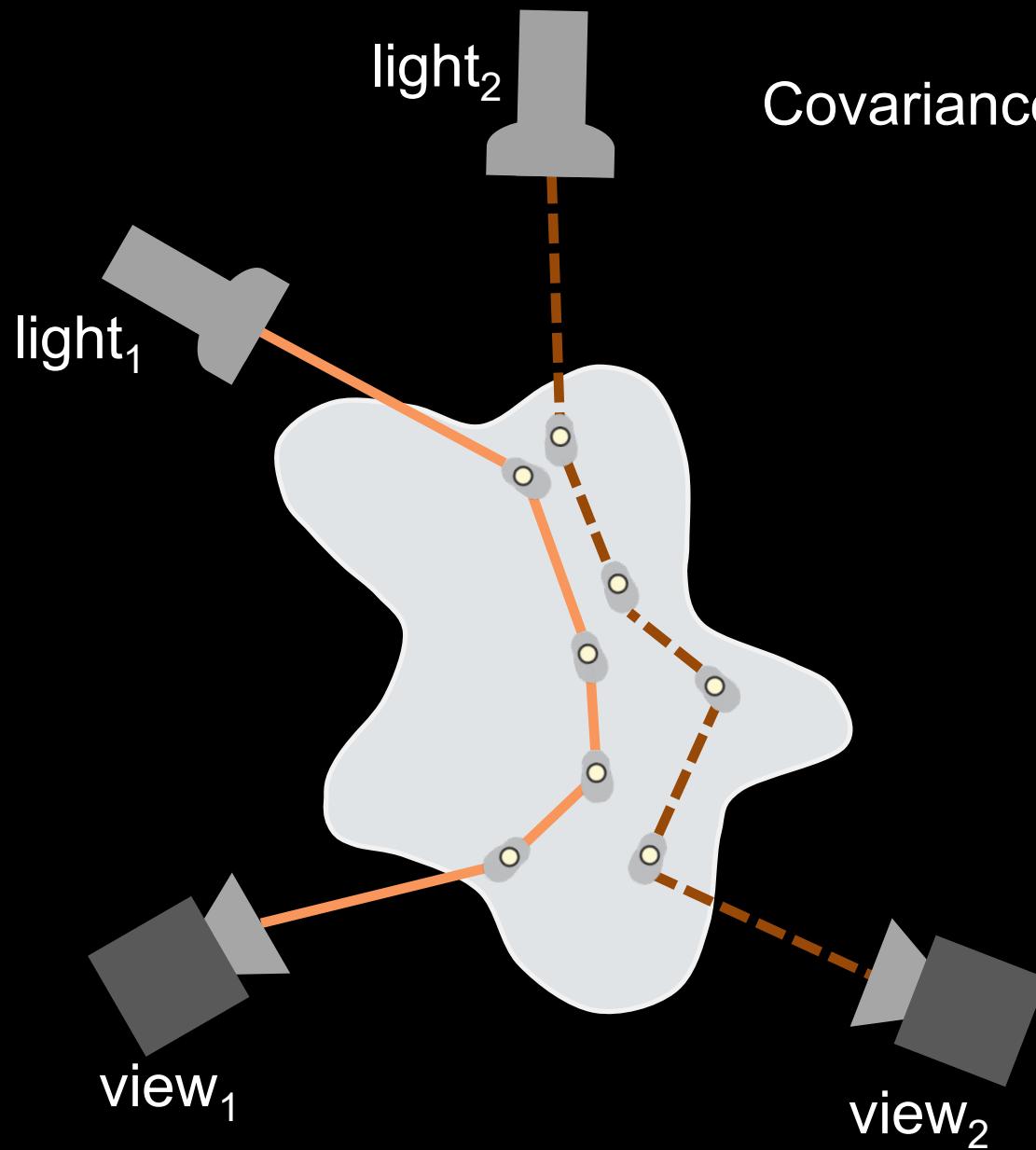
# Covariance rendering



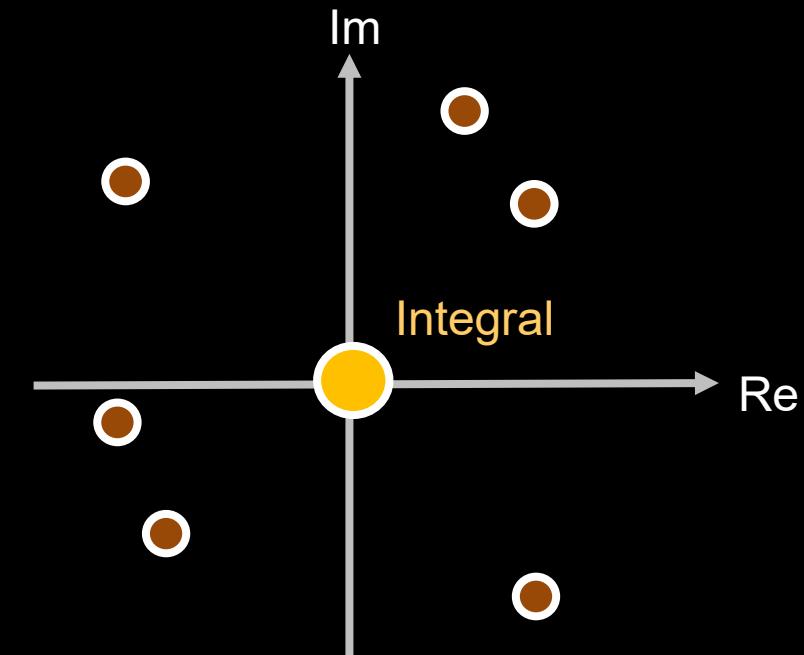
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$



# Covariance rendering

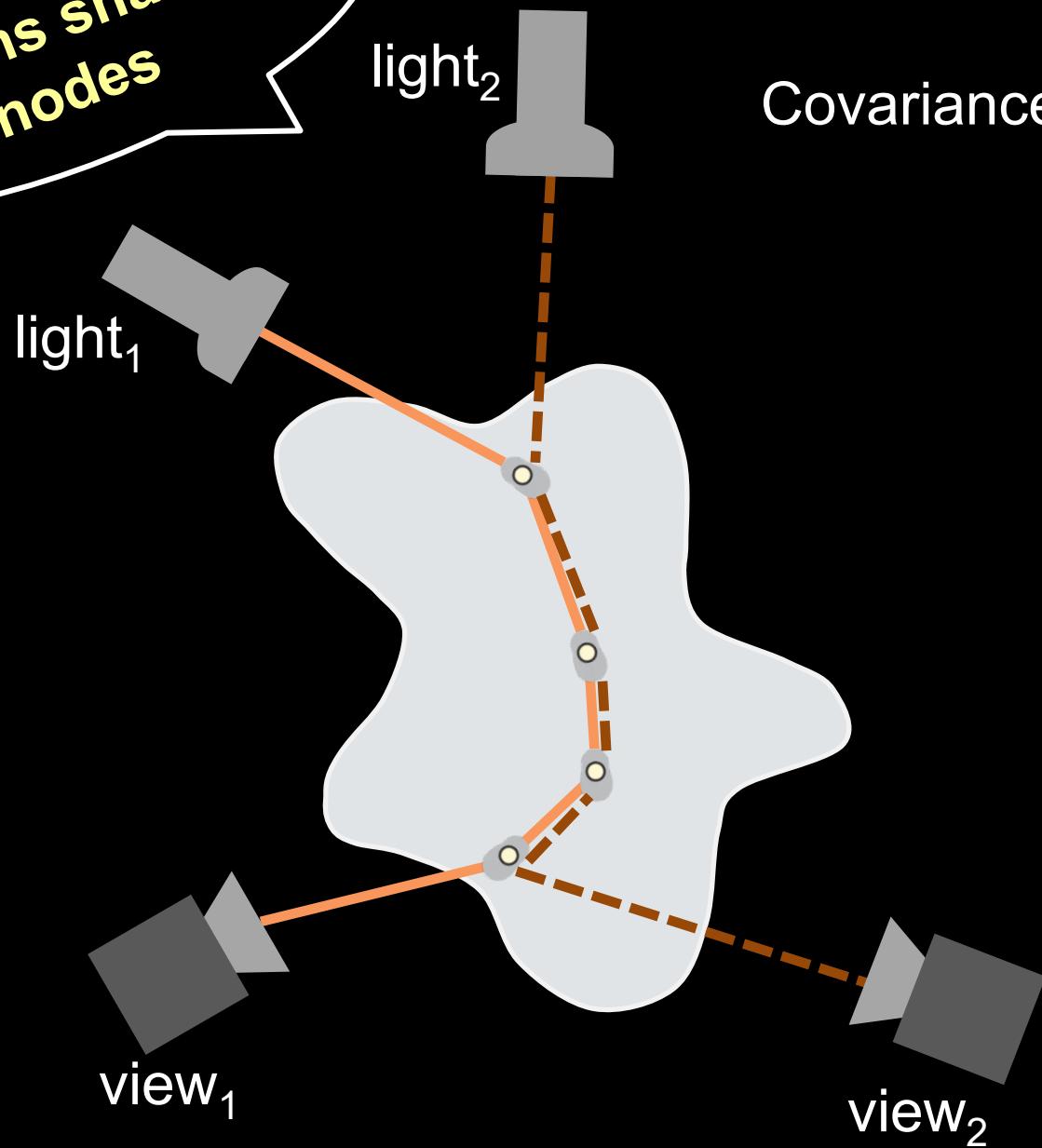


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$

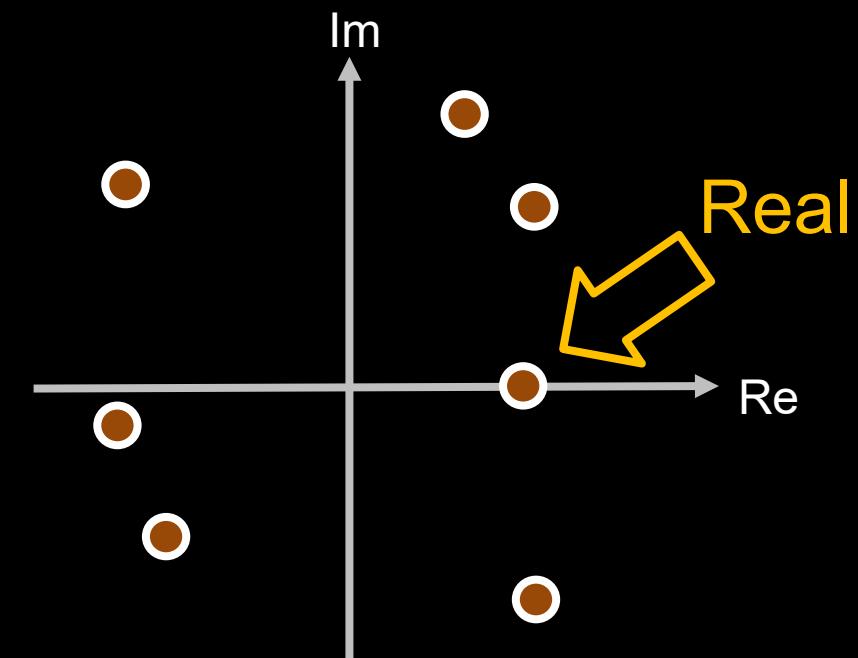


# Covariance rendering

Paths share  
nodes

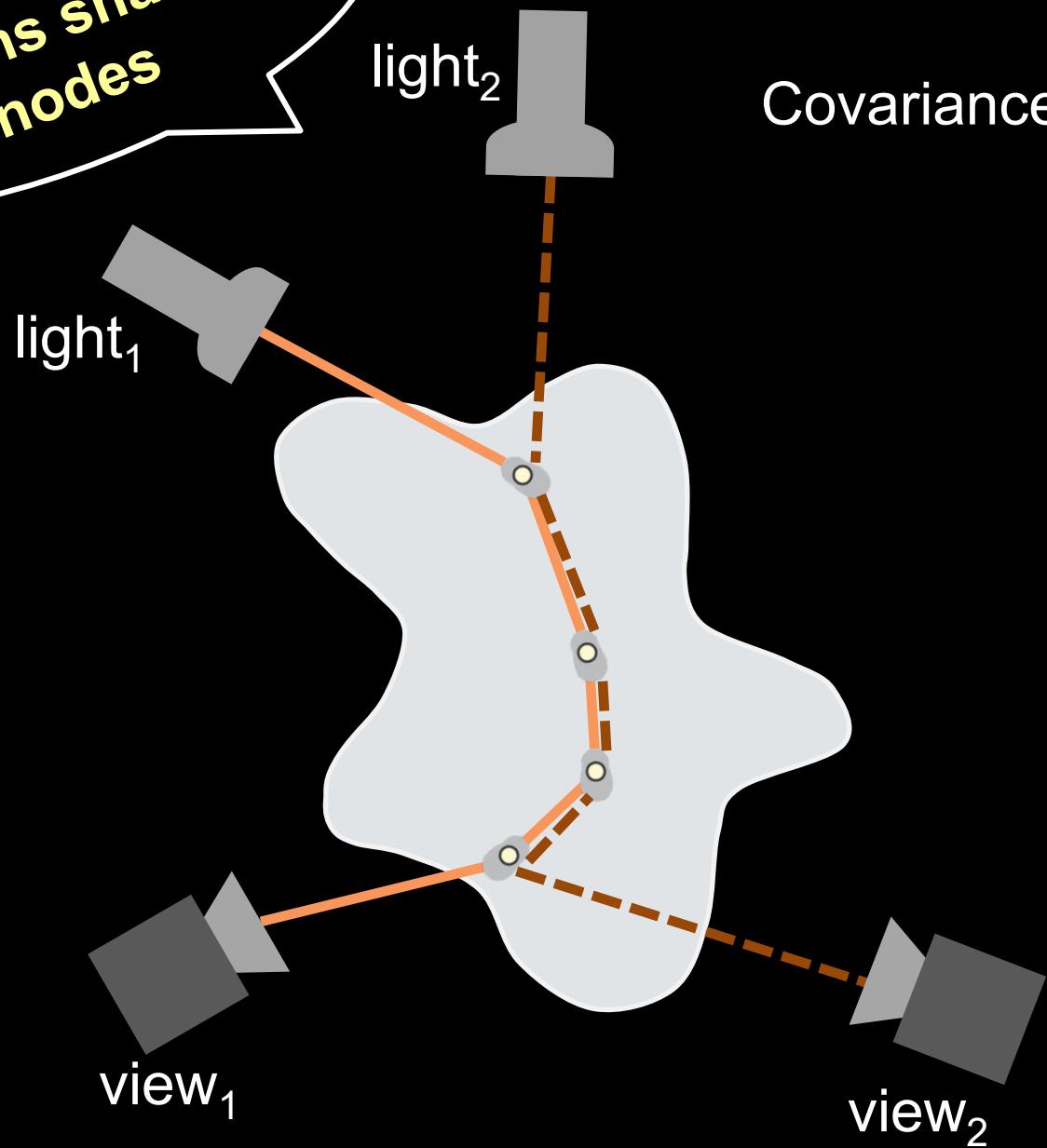


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$

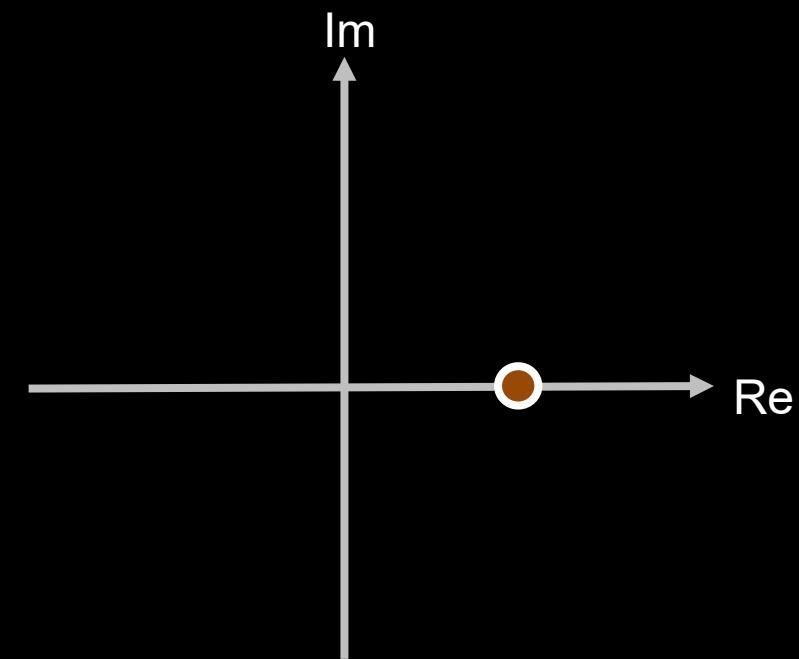


# Covariance rendering

Paths share  
nodes

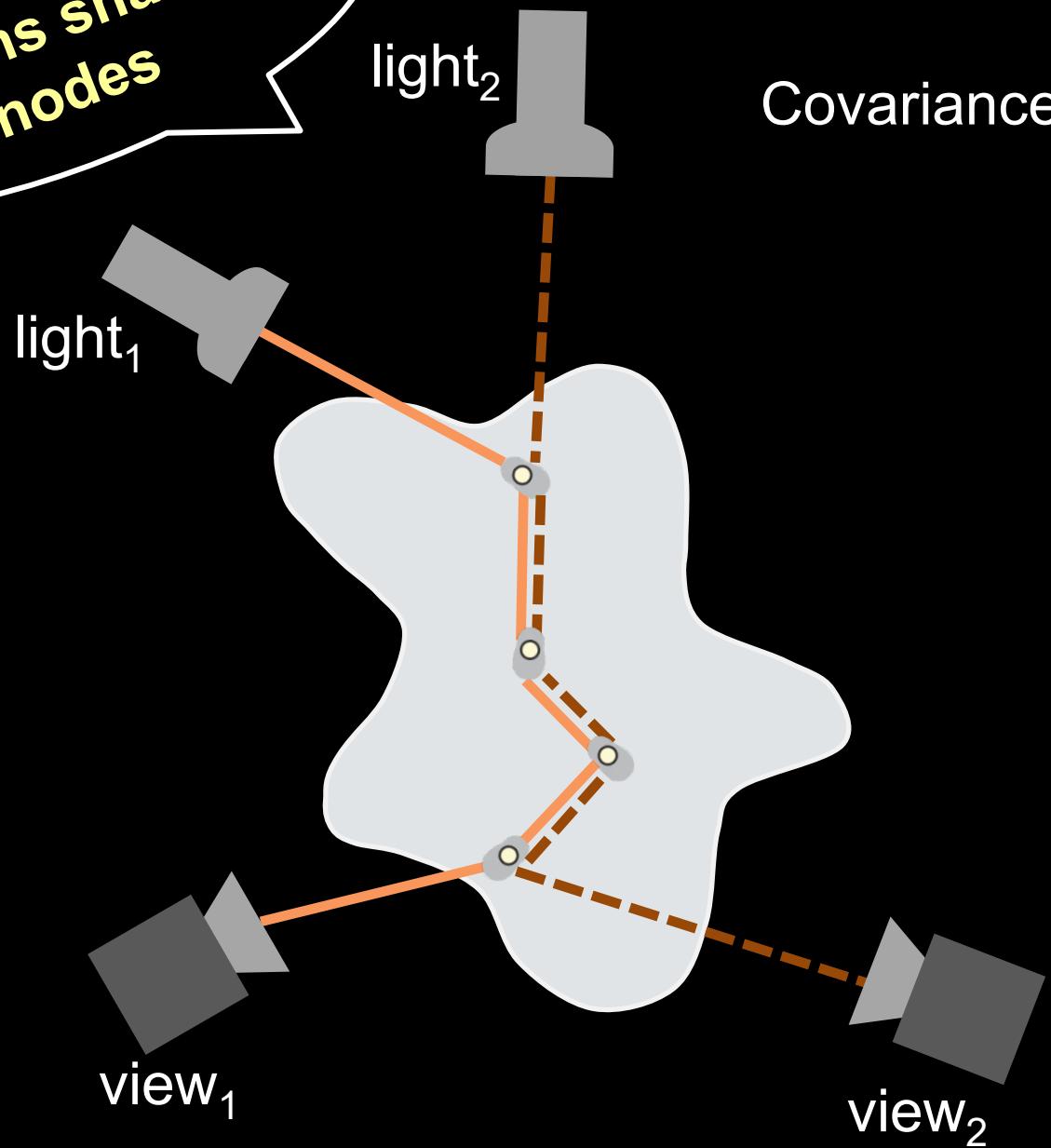


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$

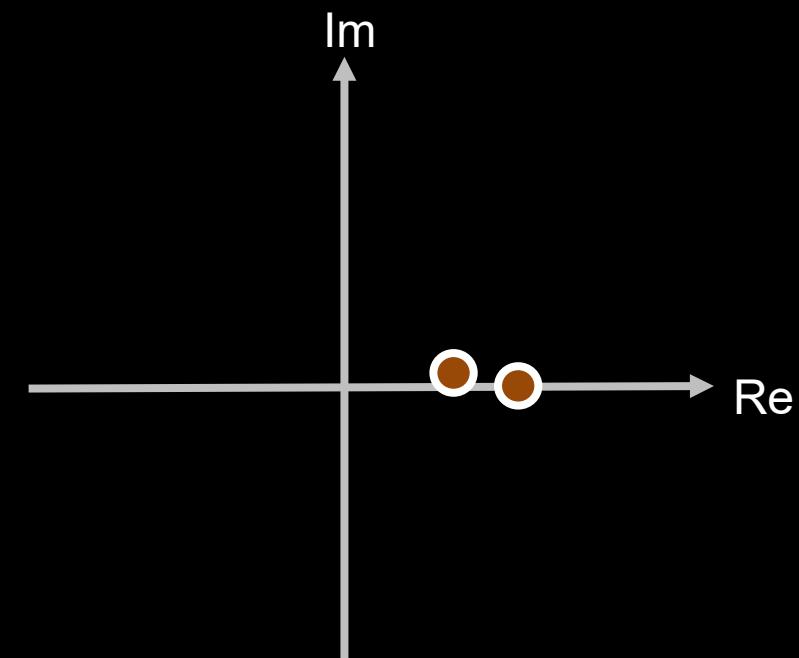


# Covariance rendering

Paths share  
nodes

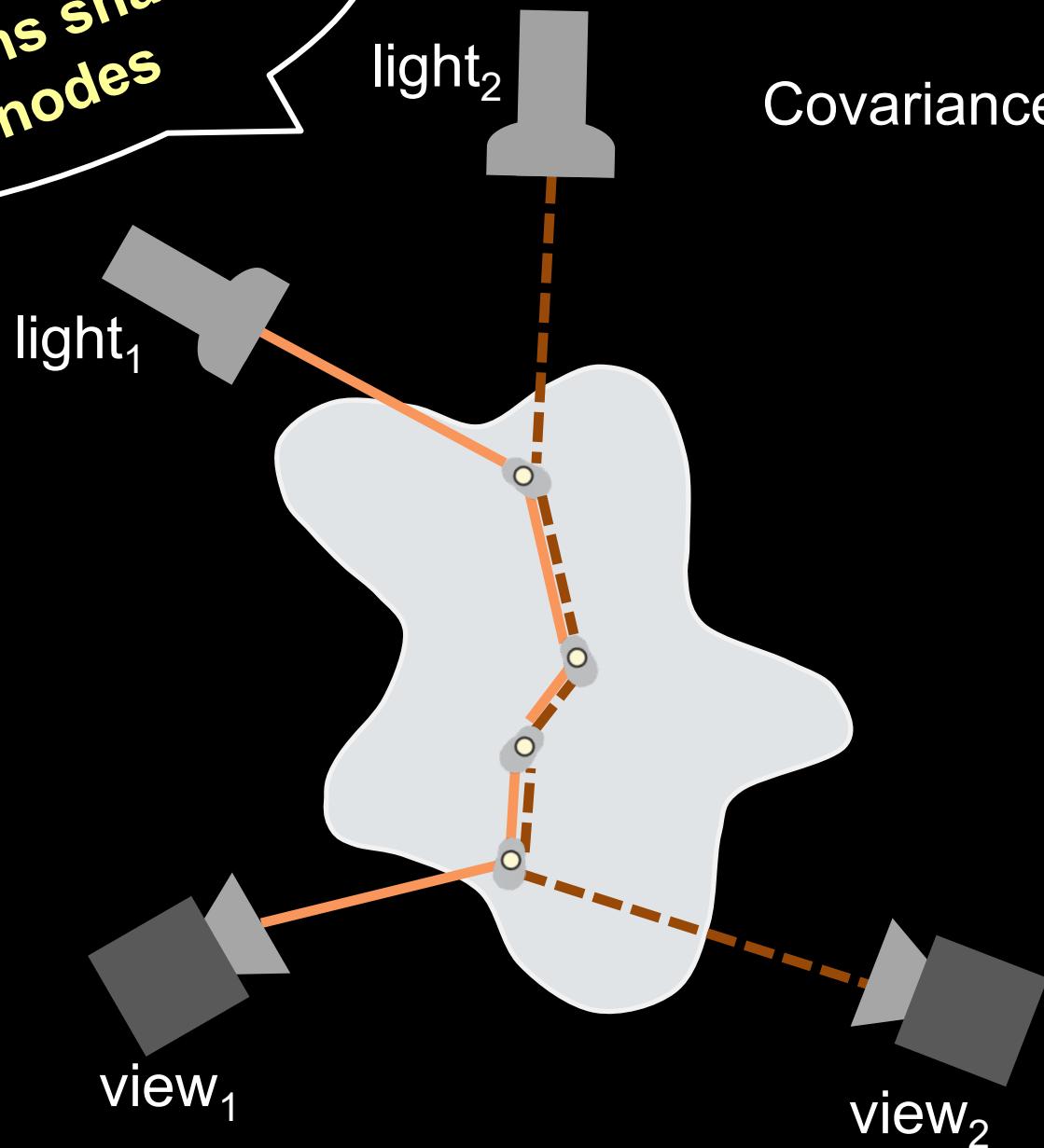


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$

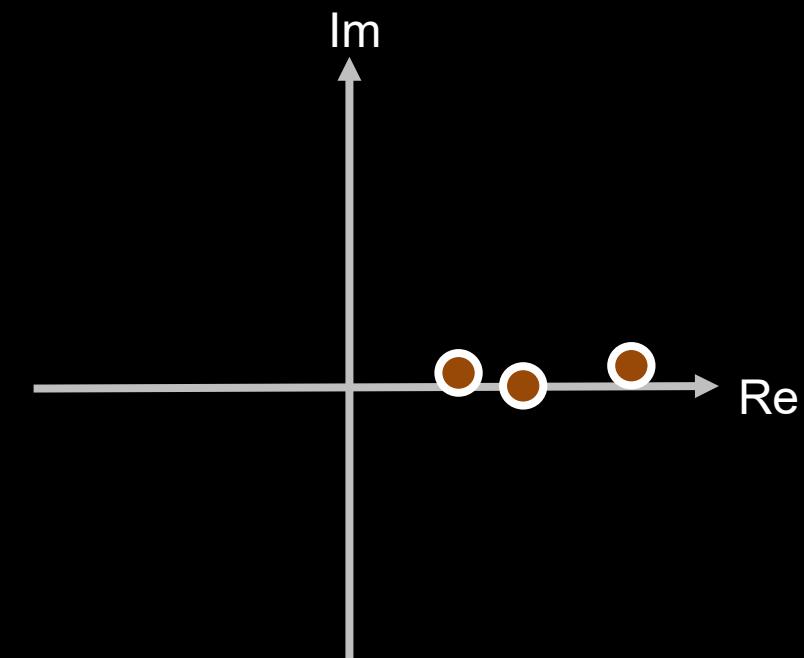


# Covariance rendering

Paths share  
nodes

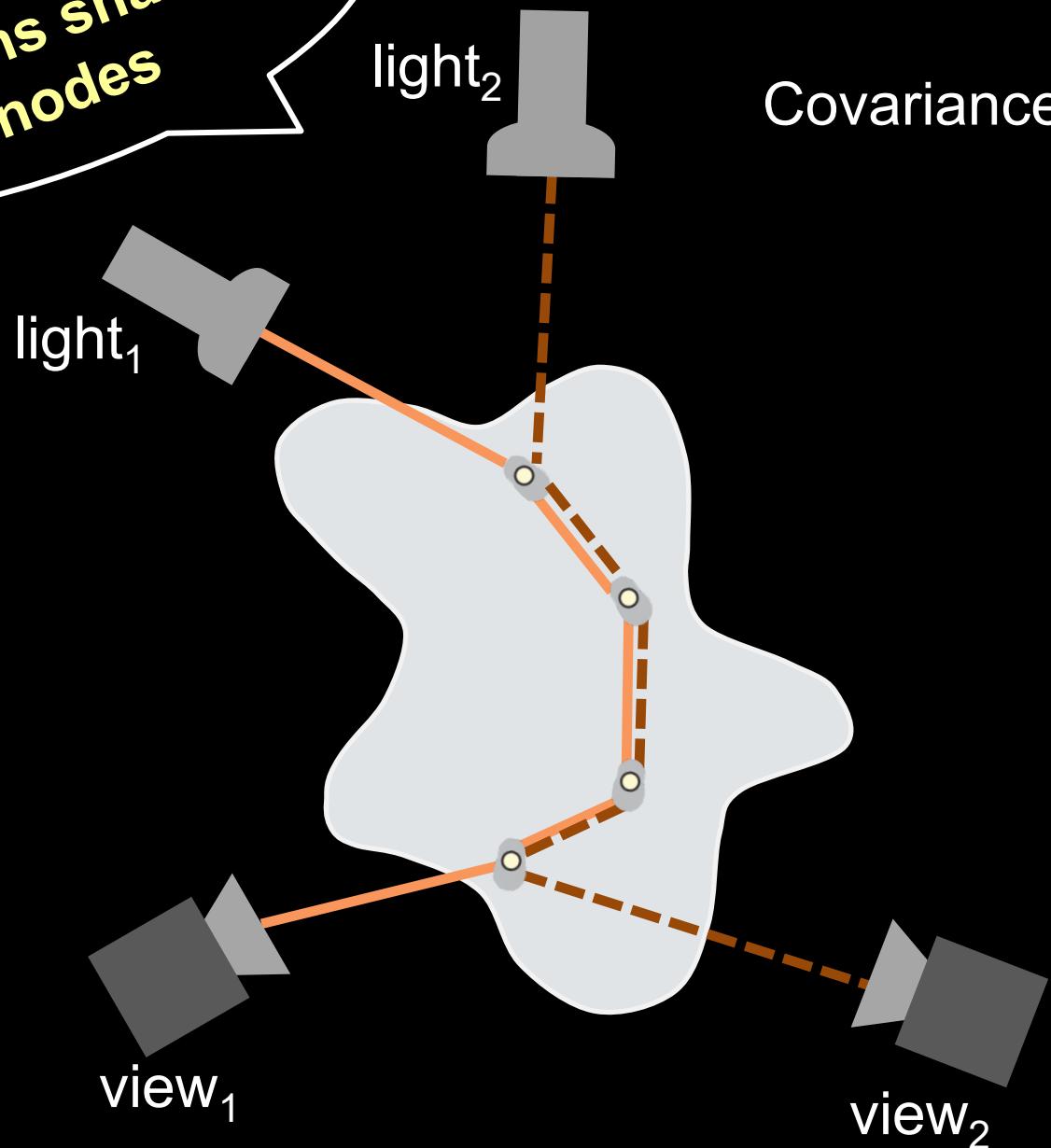


$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$

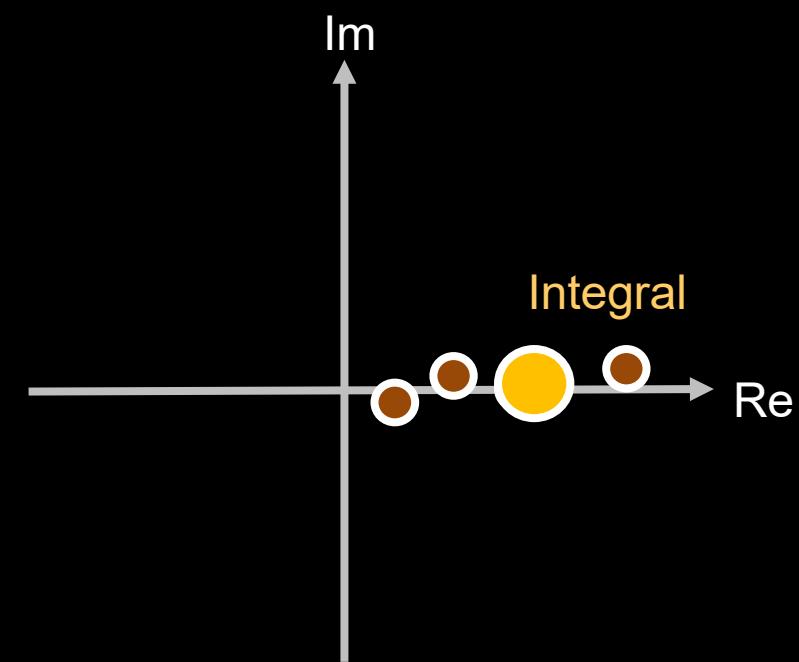


# Covariance rendering

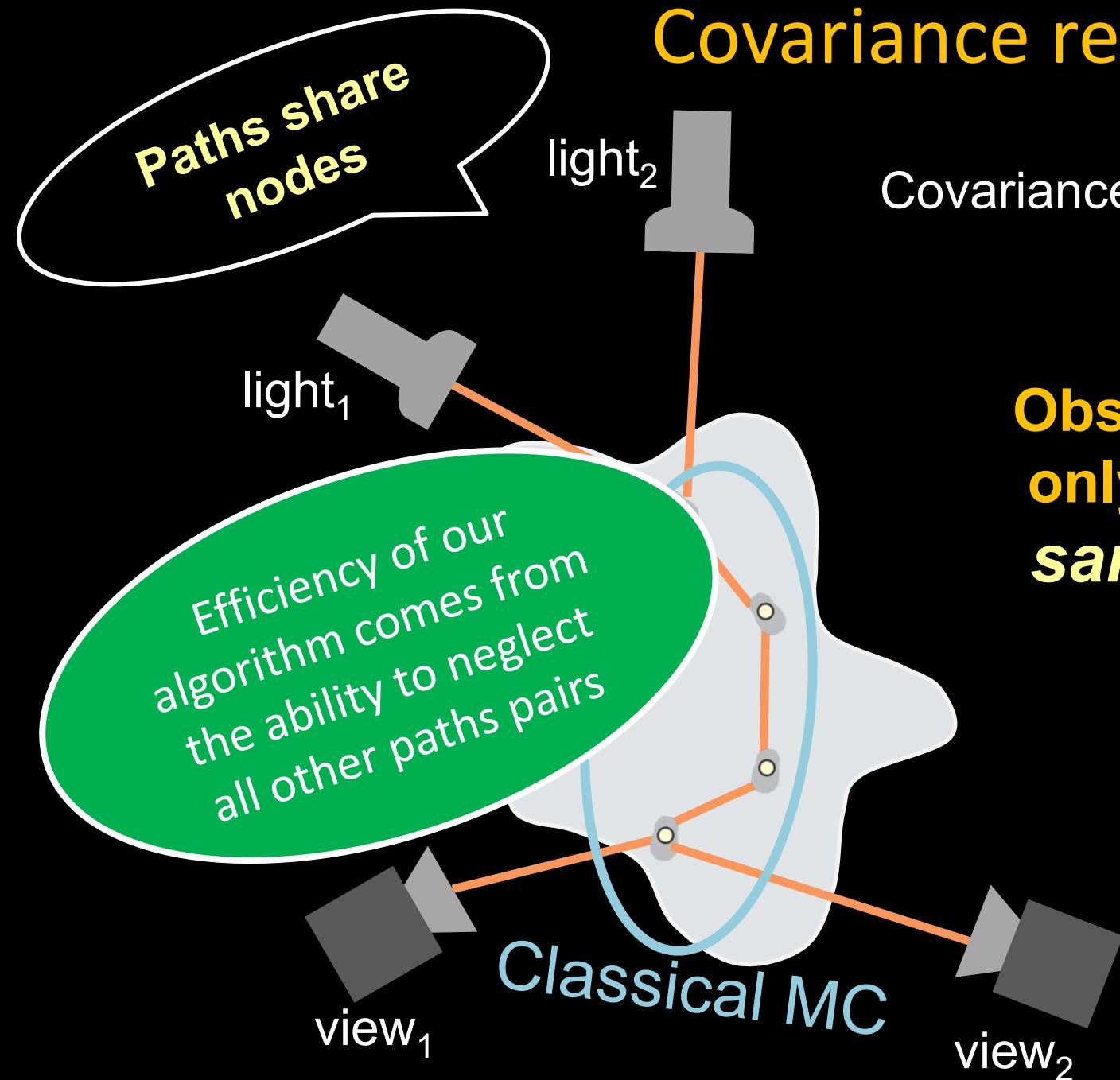
Paths share  
nodes



$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$

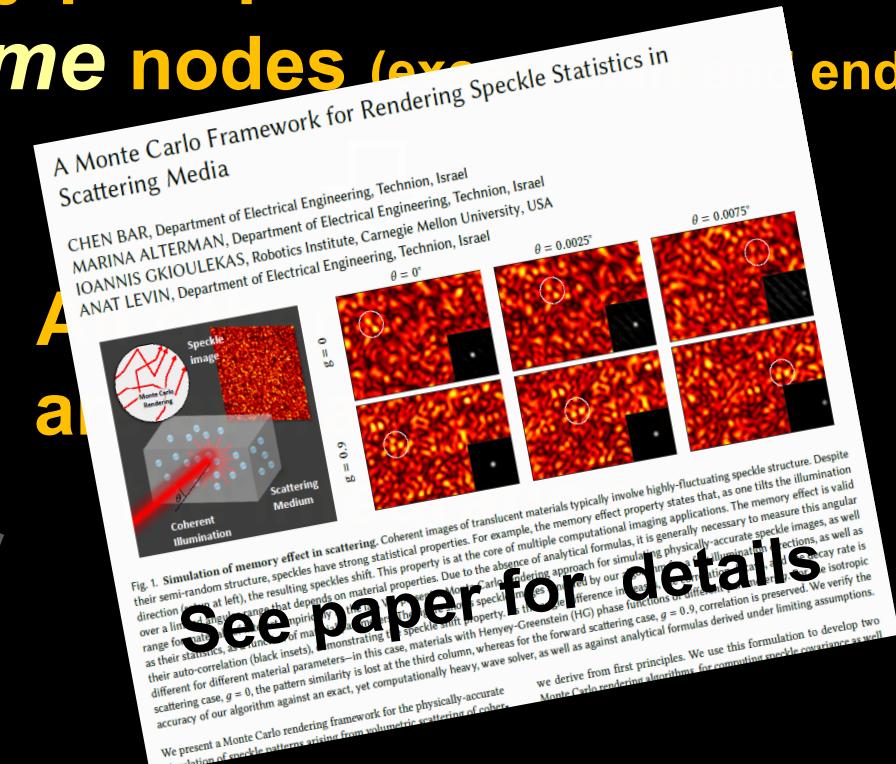


# Covariance rendering



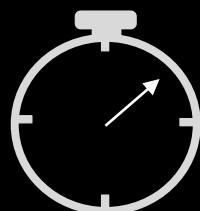
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$

**Observation: need to consider only path pairs that share the same nodes (except end)**



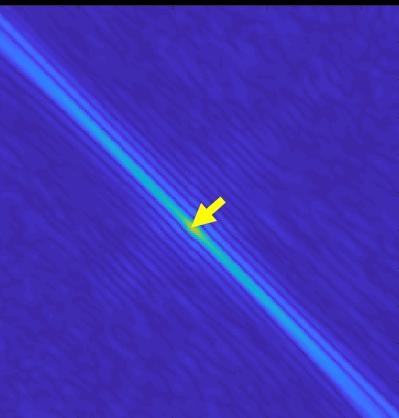
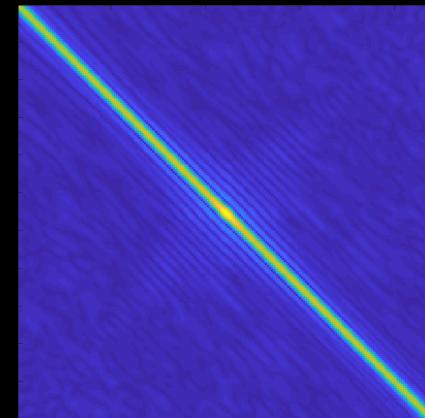
# Validation: Wave Equation Covariances v.s. MC

Computation takes days



Monte Carlo

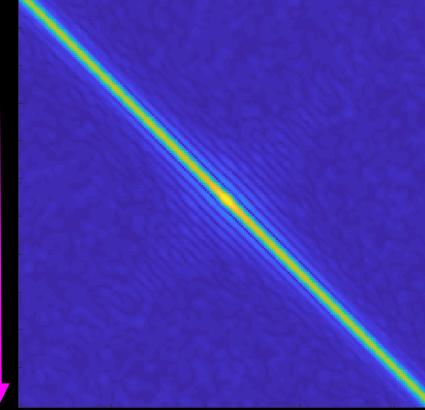
view  $i$



Reveal new types of unexplored correlations

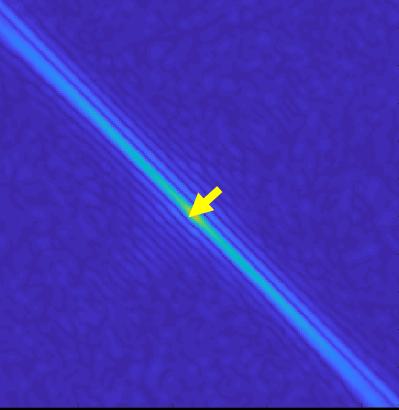
view  $i$

light<sub>1</sub> = 0°  
light<sub>2</sub> = 0°

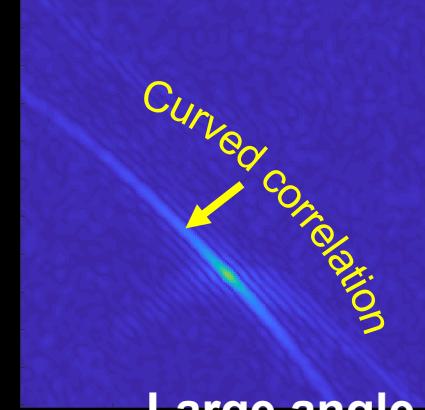


view  $j$

light<sub>1</sub> = 0°  
light<sub>2</sub> = 4°

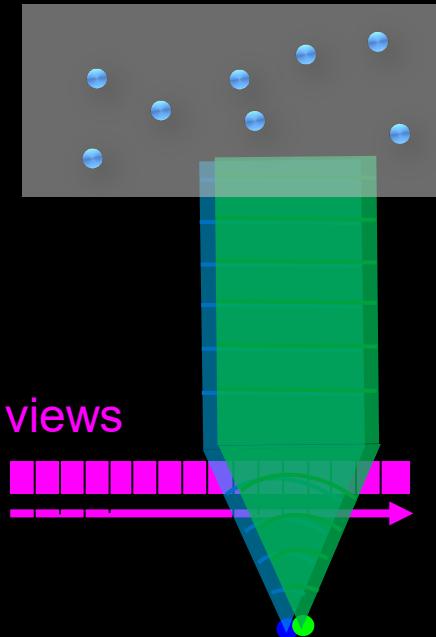


light<sub>1</sub> = 0°  
light<sub>2</sub> = 20°



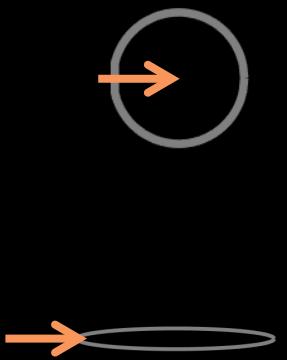
Classical ME holds for relatively small angles

Setup

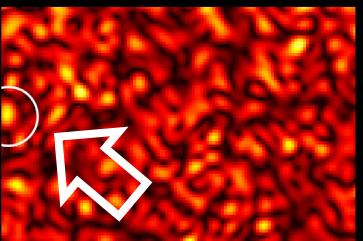


# Rendering Speckles

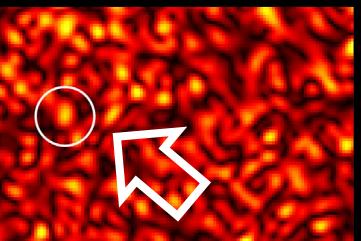
Phase Function



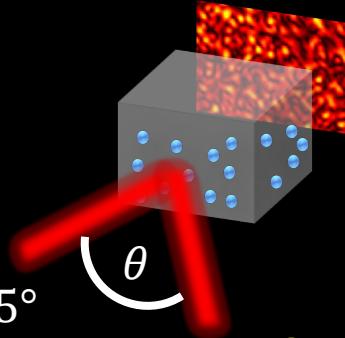
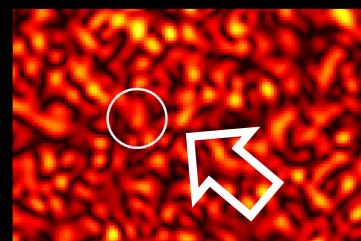
$\theta = 0^\circ$



$\theta = 0.0025^\circ$

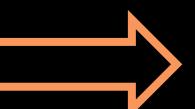
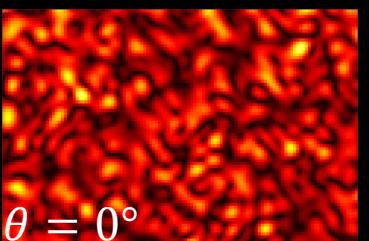


$\theta = 0.005^\circ$

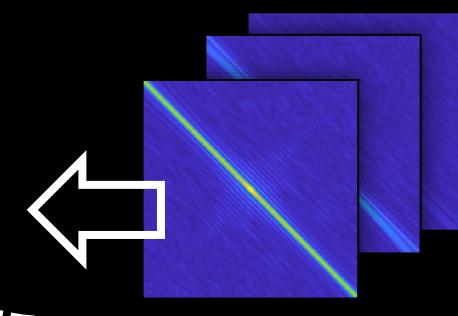


Classical ME holds for relatively small angles

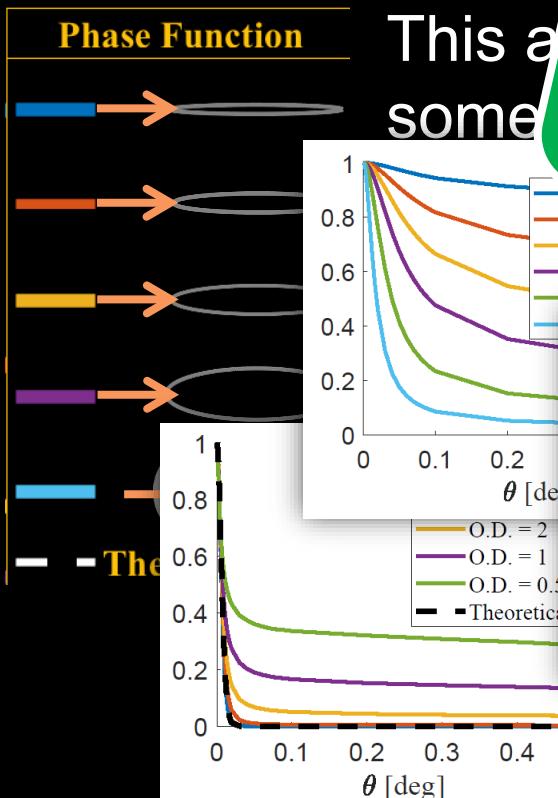
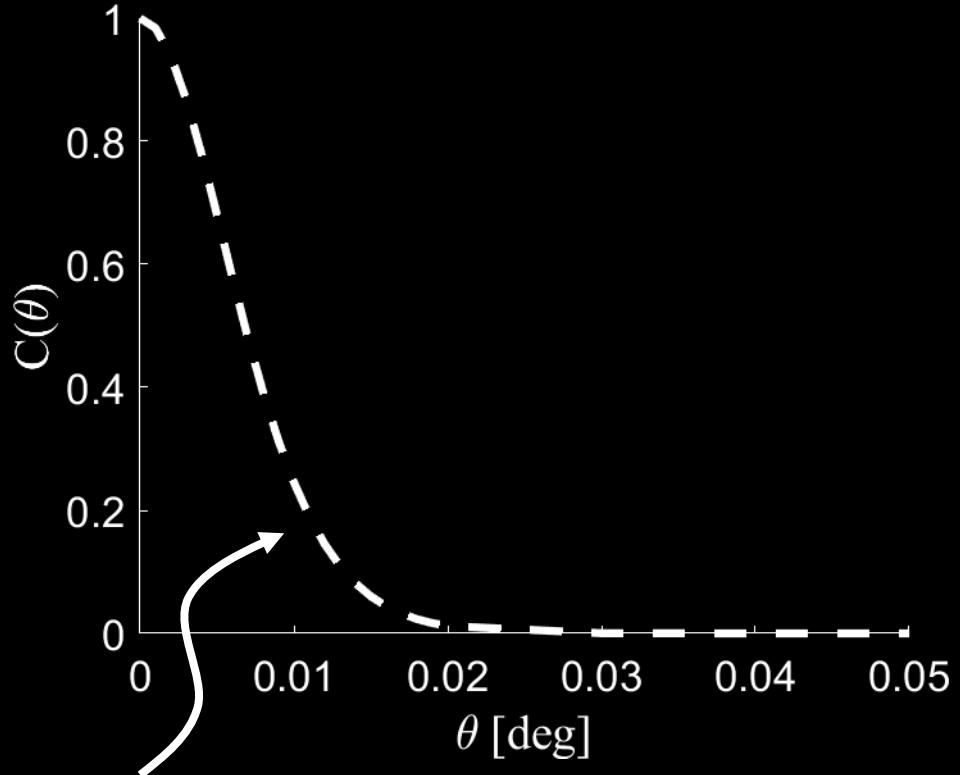
Computing ME extent as a function of  $\theta$  :



$\theta$



# Evaluating the Memory Effect

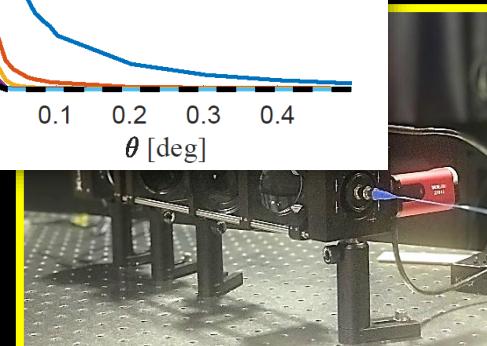
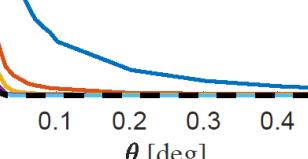
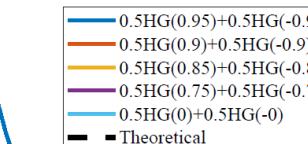


This allows us to

compute ME curves  
for all scattering parameters

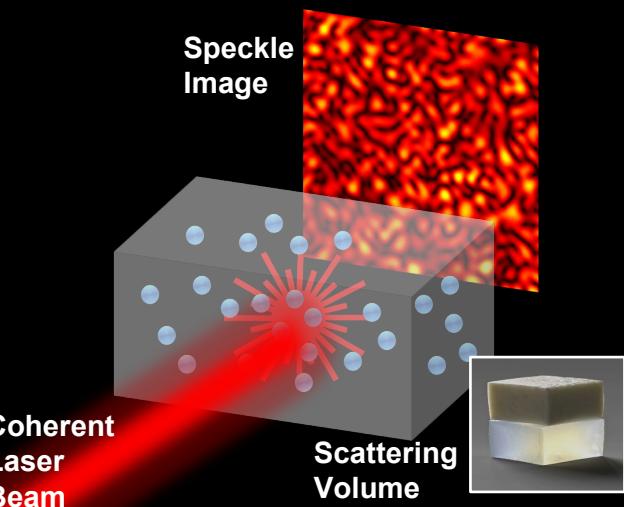
extremely

efficiently



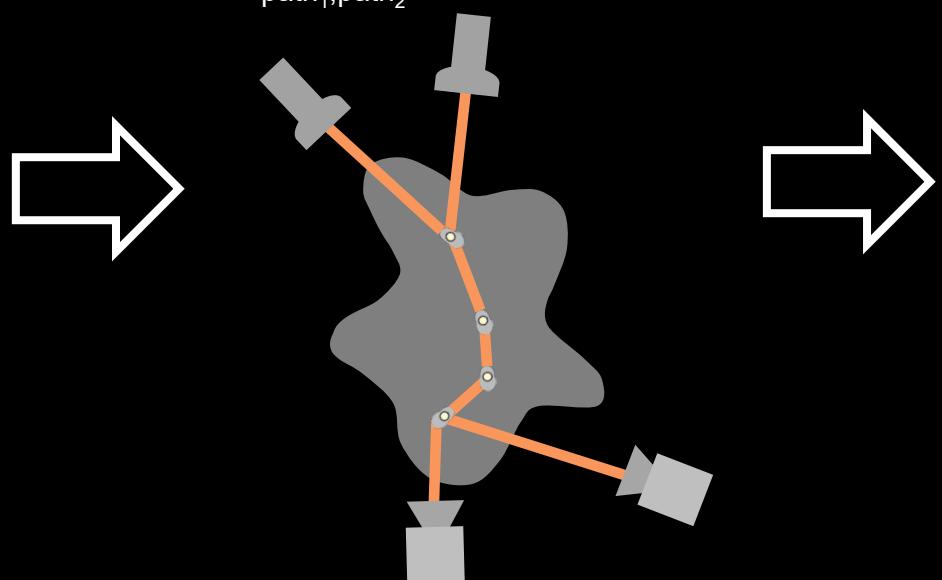
# Summary

Problem:  
Coherent Scattering

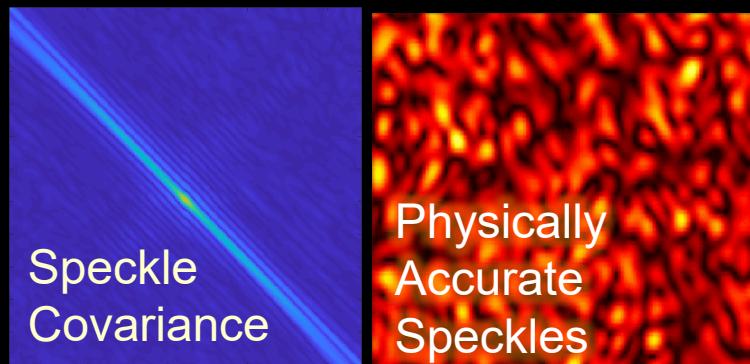


Path-integral formulation  
for speckle covariance

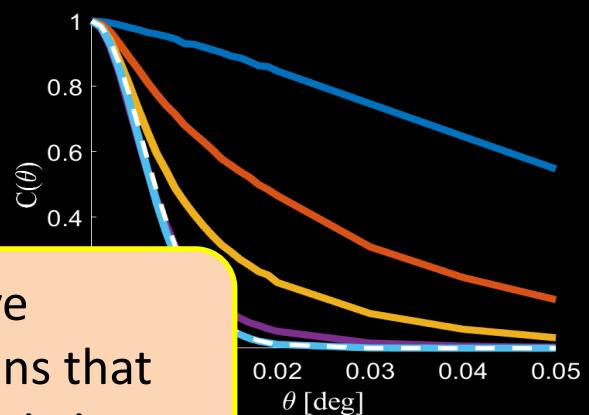
$$\text{Covariance} = \int_{\text{path}_1, \text{path}_2} \mathbf{u}(\text{path}_1) \cdot \mathbf{u}^*(\text{path}_2)$$



Efficient MC Rendering



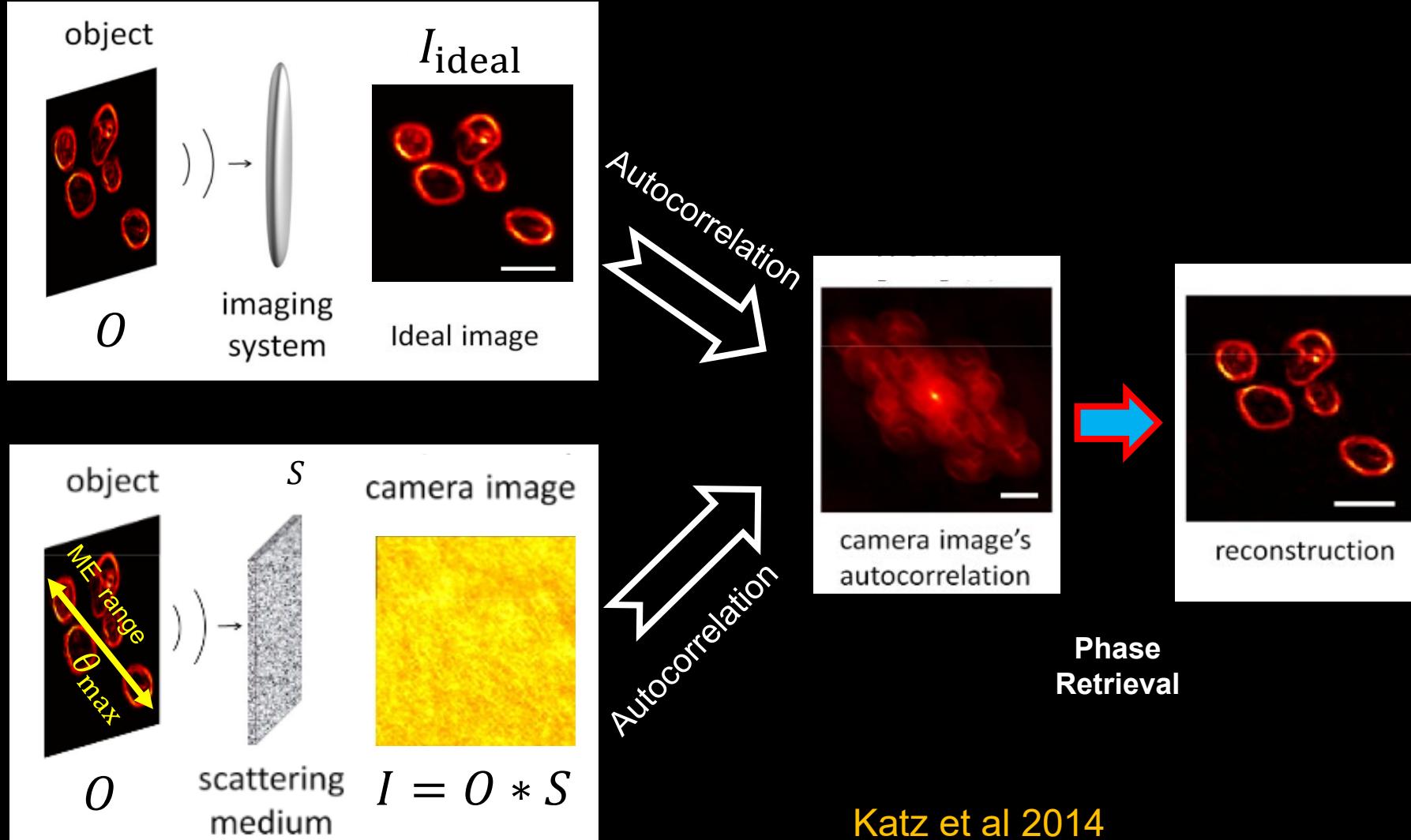
Memory Effect Evaluation



Potentially improve  
imaging applications that  
rely on speckle statistics

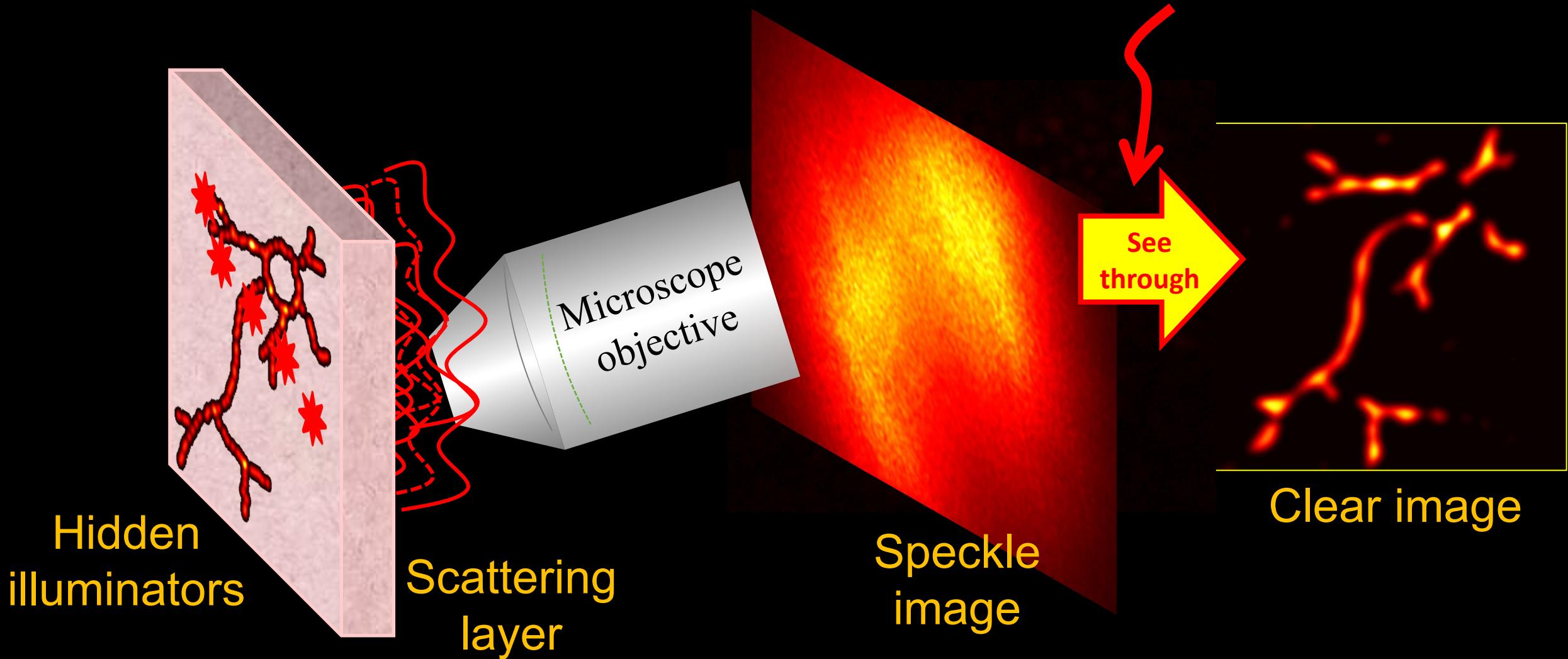
# Seeing Through Scattering Layers

Cool Application by Ori Katz et al 2014

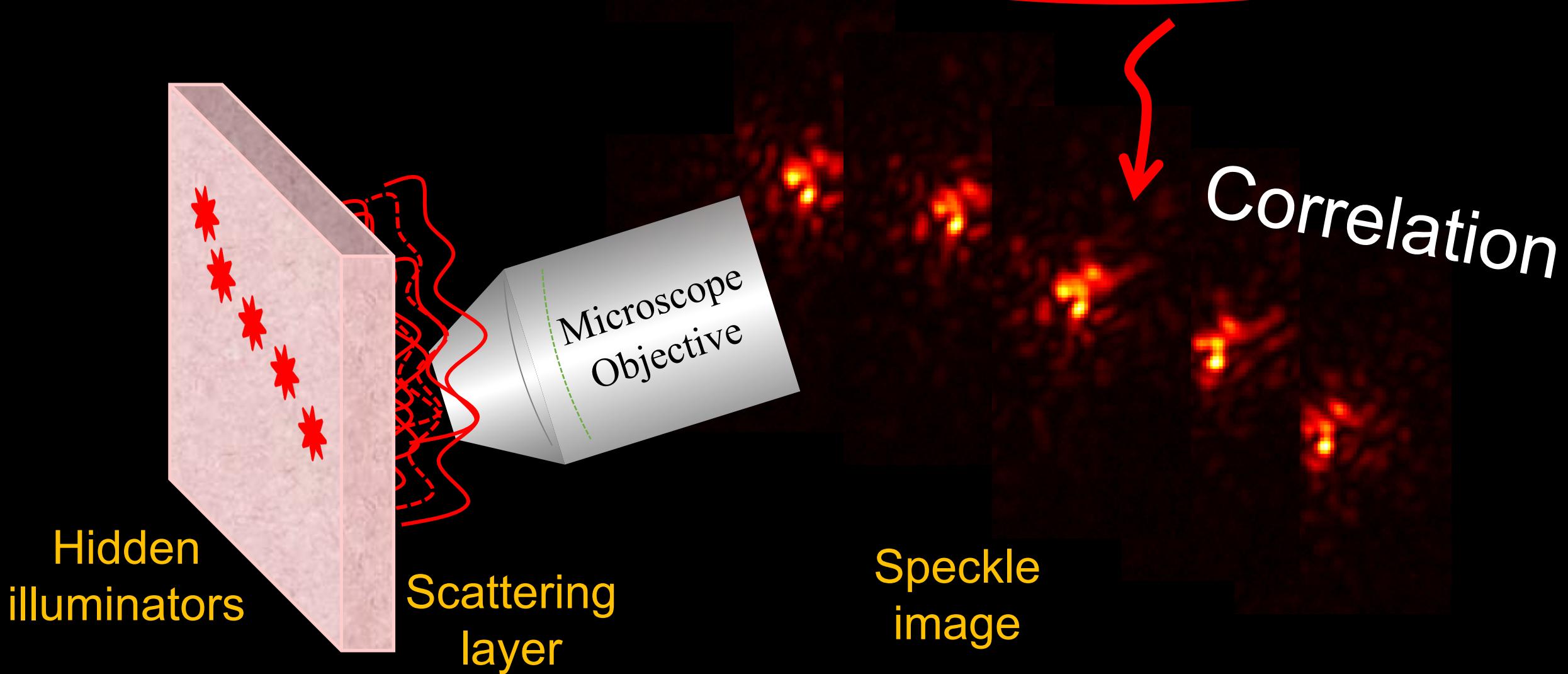


Katz et al 2014

# Coherent scattering and memory effect (ME)

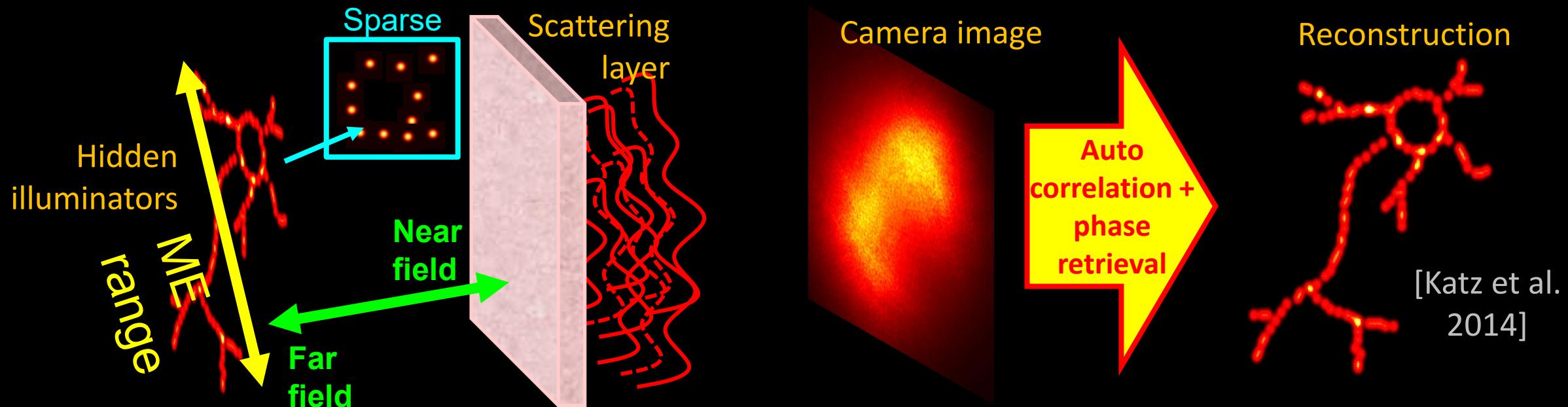


# Coherent scattering and memory effect (ME)



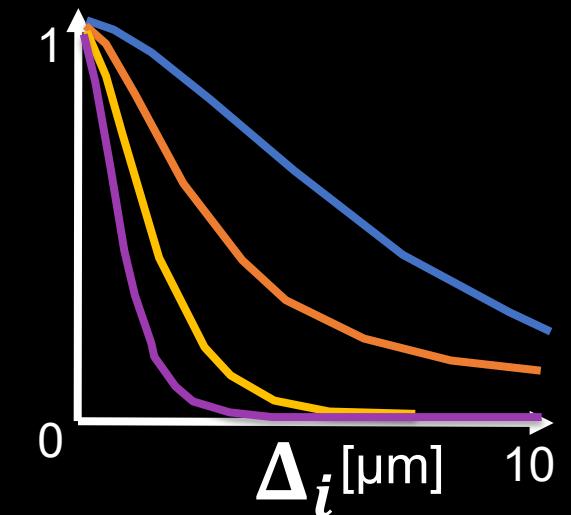
# Problems with classical see-through approach

- Limited range: Only illuminators within the ME range can be recovered.
- Limited density: Only a small number of illuminators can be recovered.
- Unrealistic setup: Far-field imaging conditions do not apply to tissue imaging.



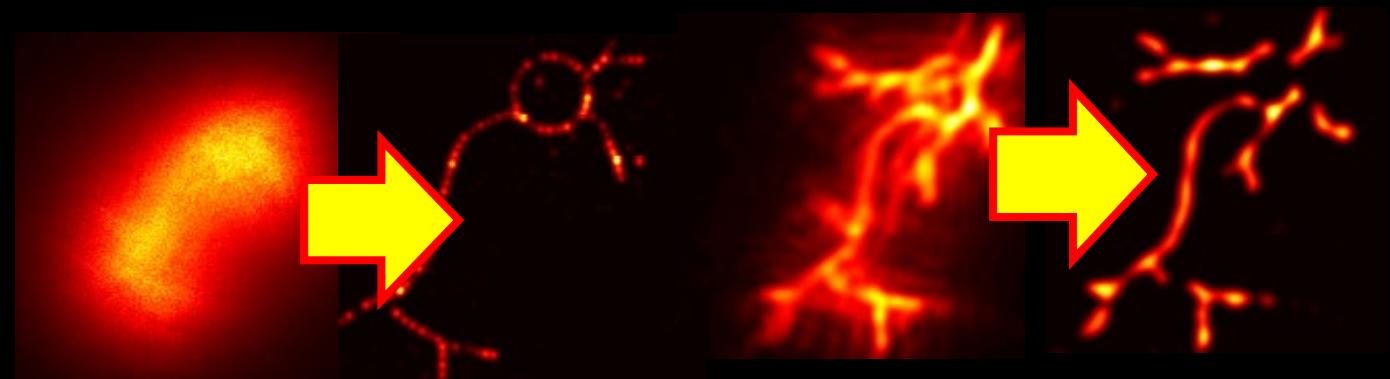
# Our contributions

( i ) Theoretical and rendering based analysis of ME in **near** and **far**-field settings



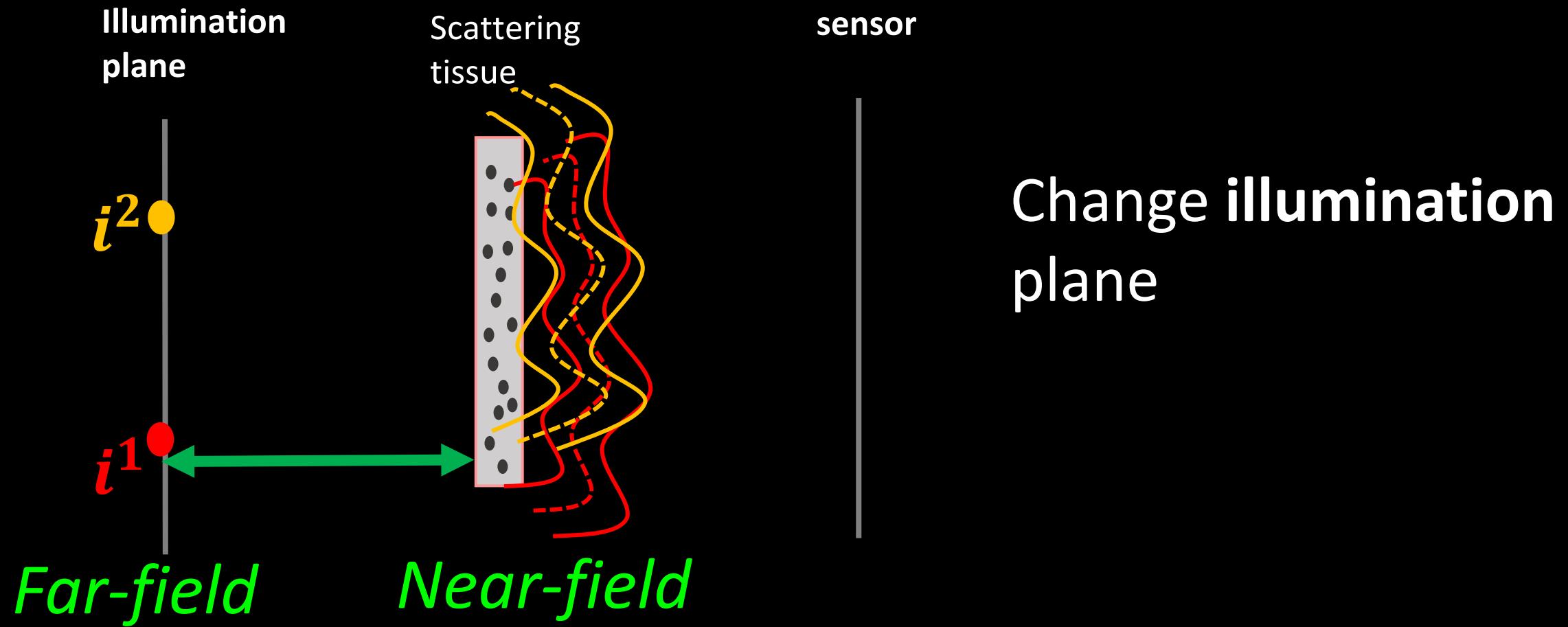
( ii ) Develop a better algorithm for imaging through scattering: 1. higher density 2. wider range 3. near sources

( iii ) Real Lab experiments in **near** and **far** fields

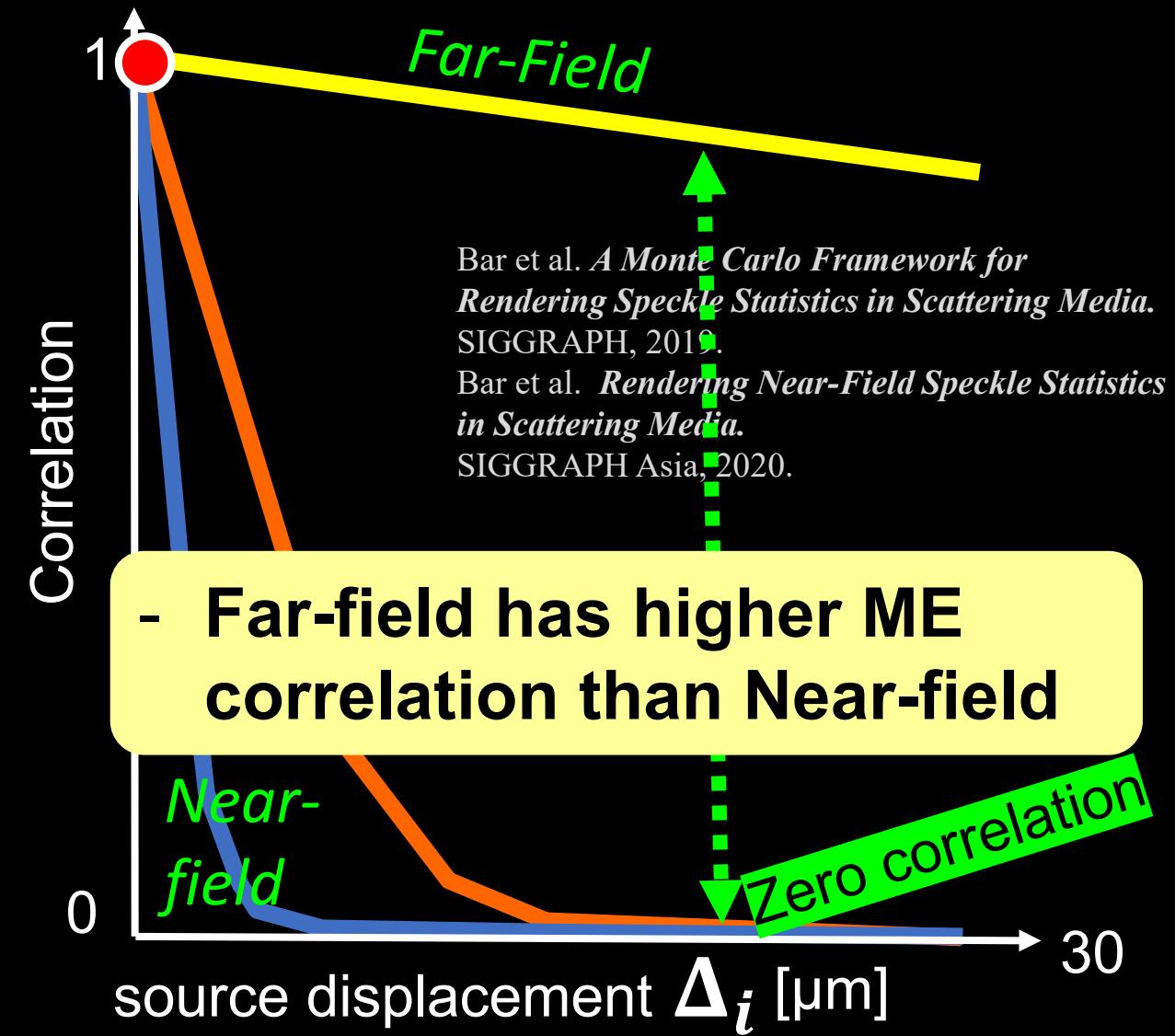
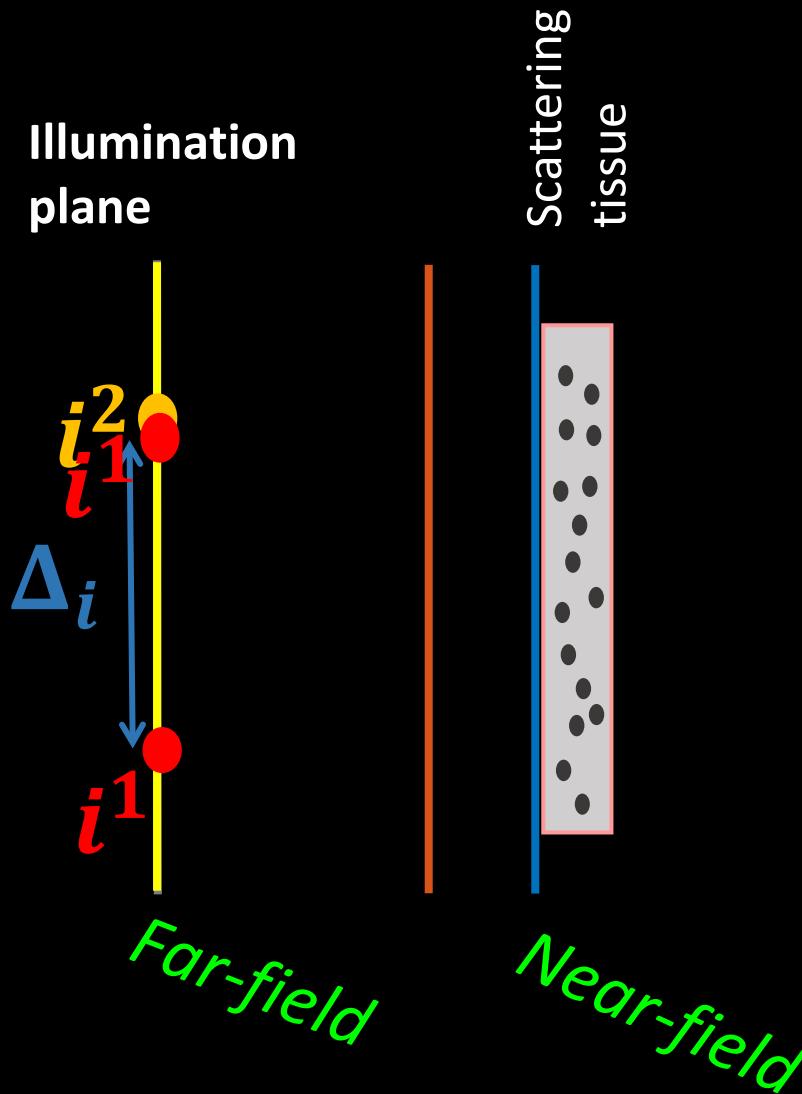


# Theory and simulation analysis

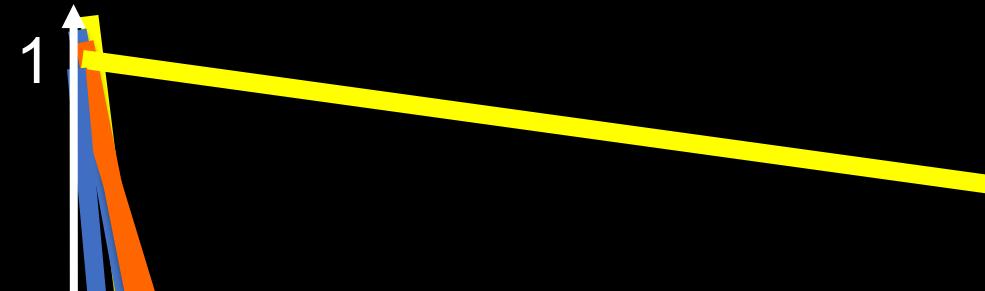
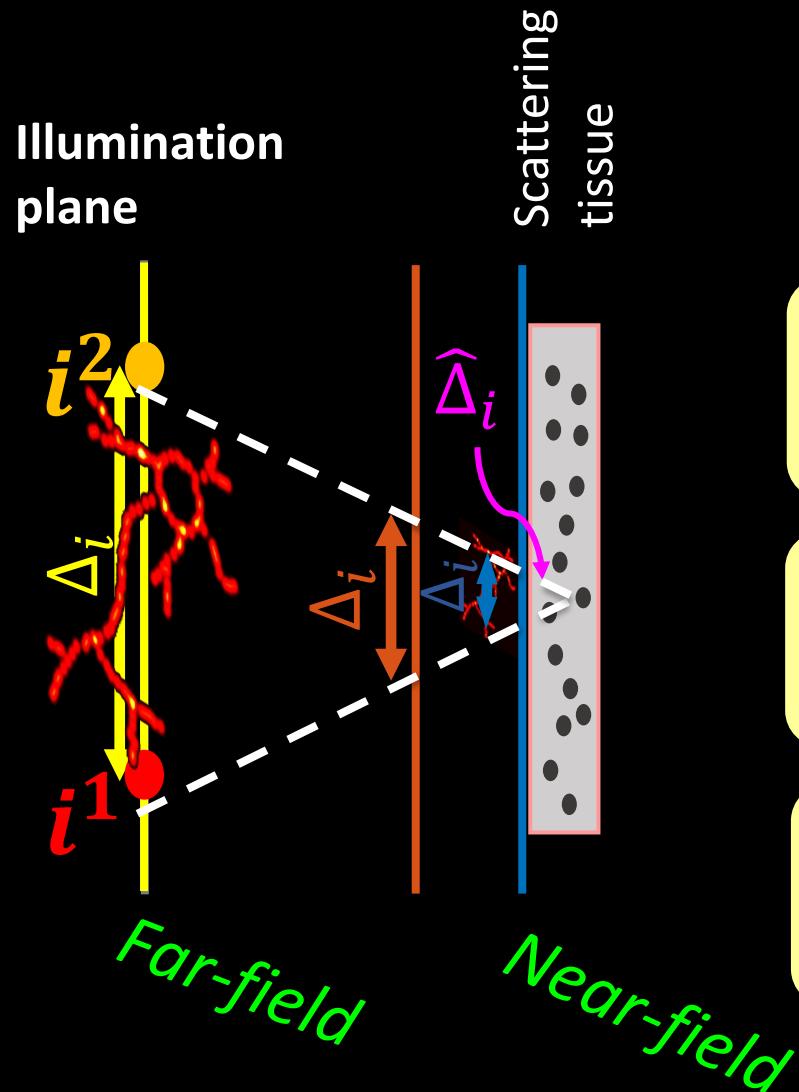
# Problem setting and memory effect



# ME correlation in near-field vs. far-field



# Aligning ME correlation in the near and far fields

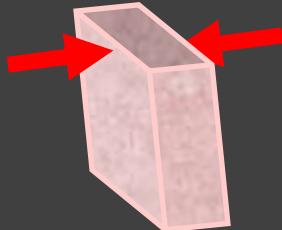


- Theorem: ME range depends only on angular displacement.
- Moving the illumination plane farther away, scales the ME range to cover larger patterns.
- Imaging-through-scattering is easier in far-field than near-field

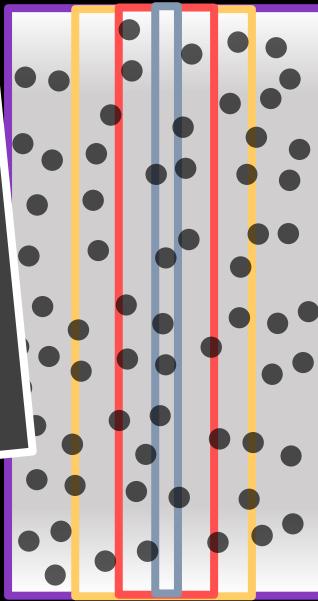
$\Delta_i$  [rad]

# ME correlation in the near-field

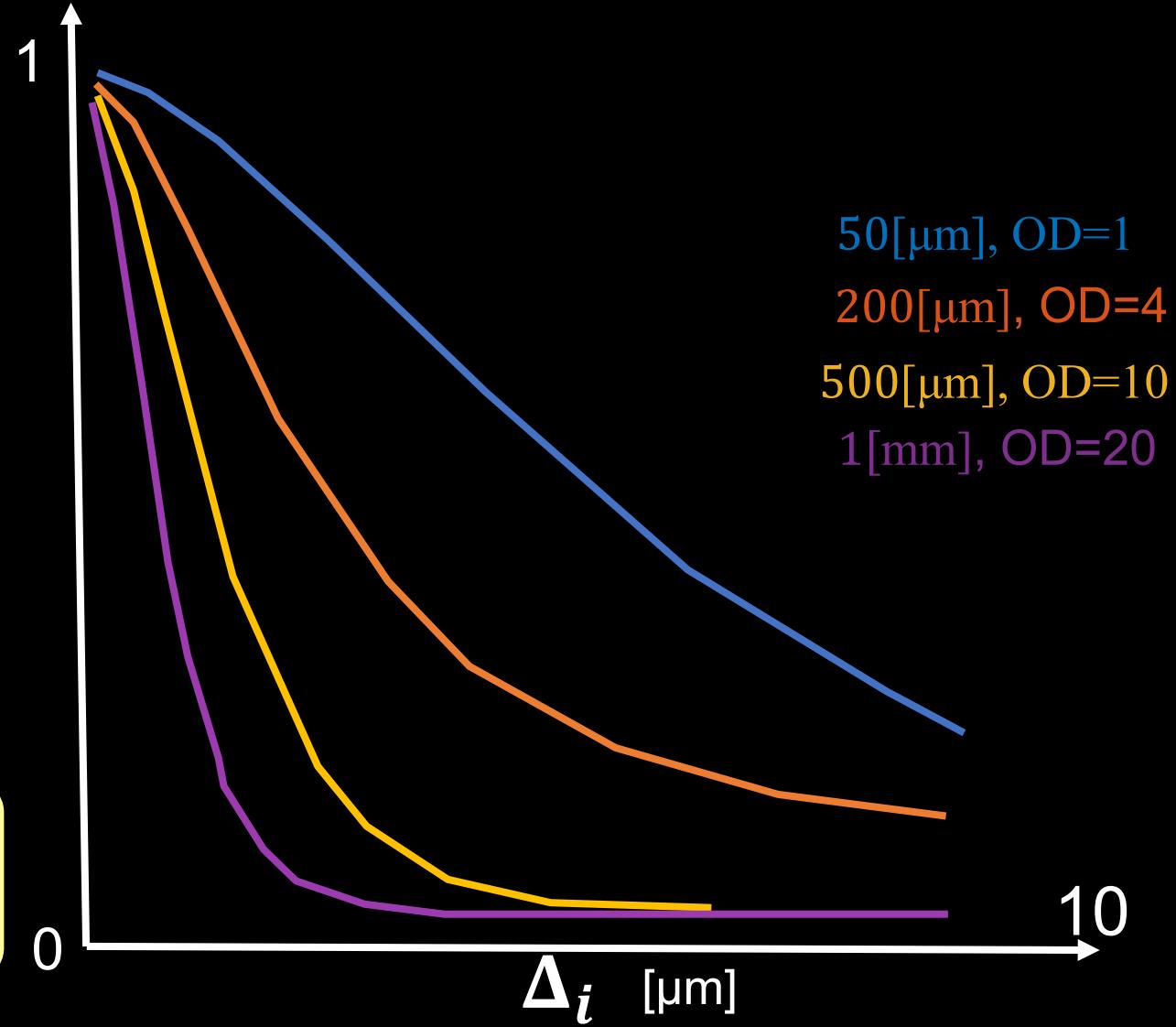
Practical only for modest thickness



Scattering tissue

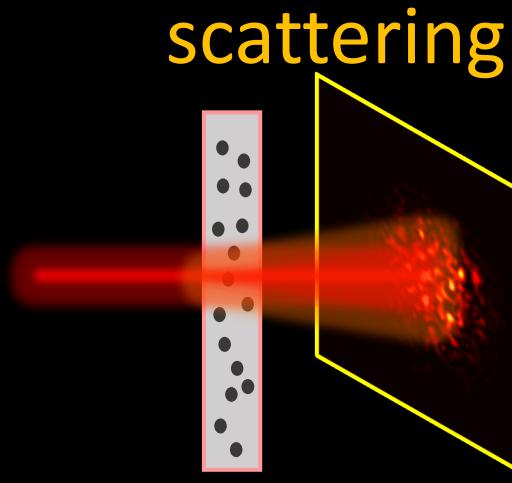


- As the thickness increases, ME correlation decreases.

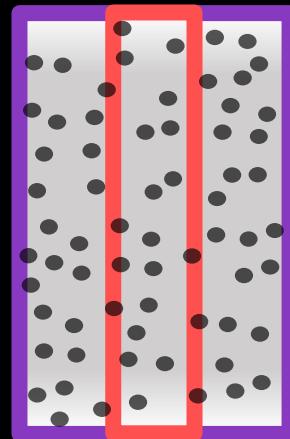


# Speckle Local Support

Tissue is forward

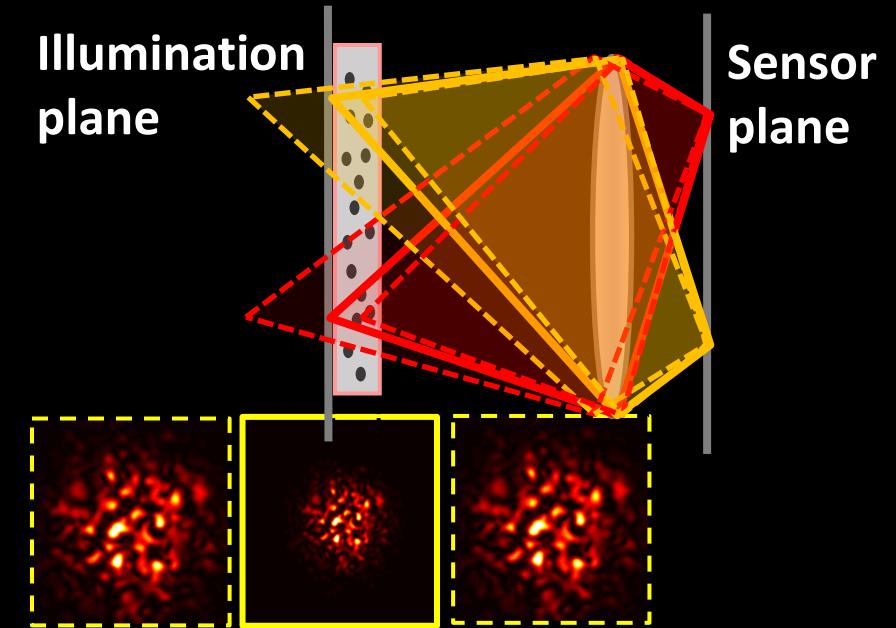


Modest thickness



Modest  
Thick

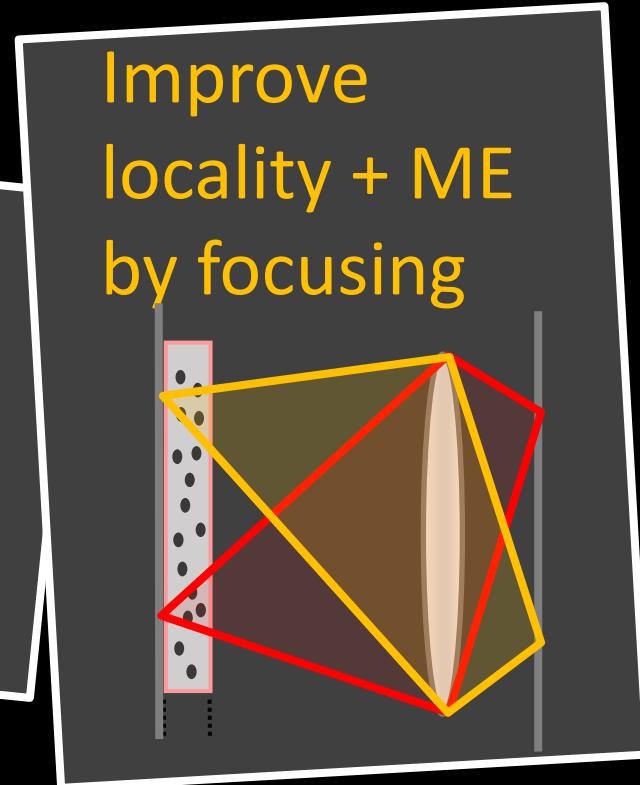
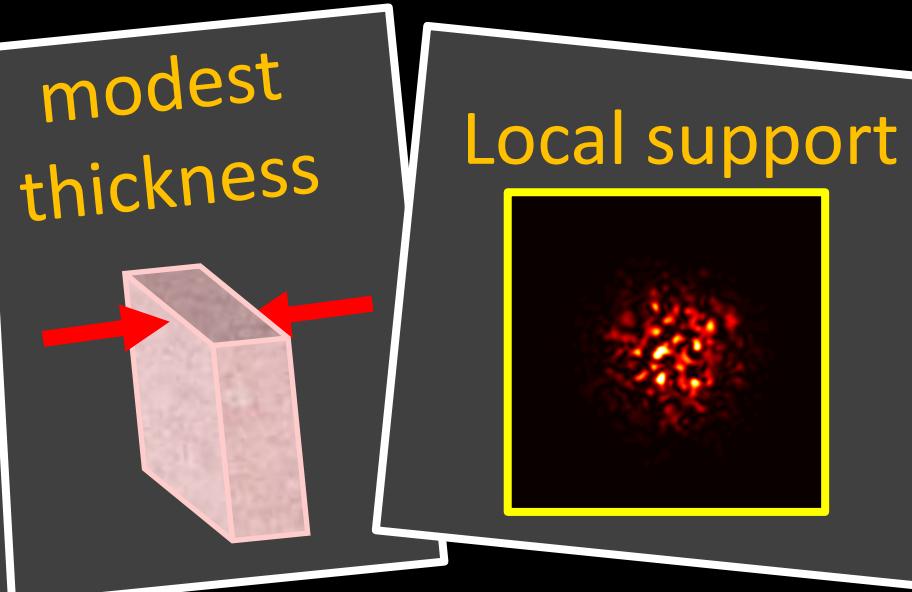
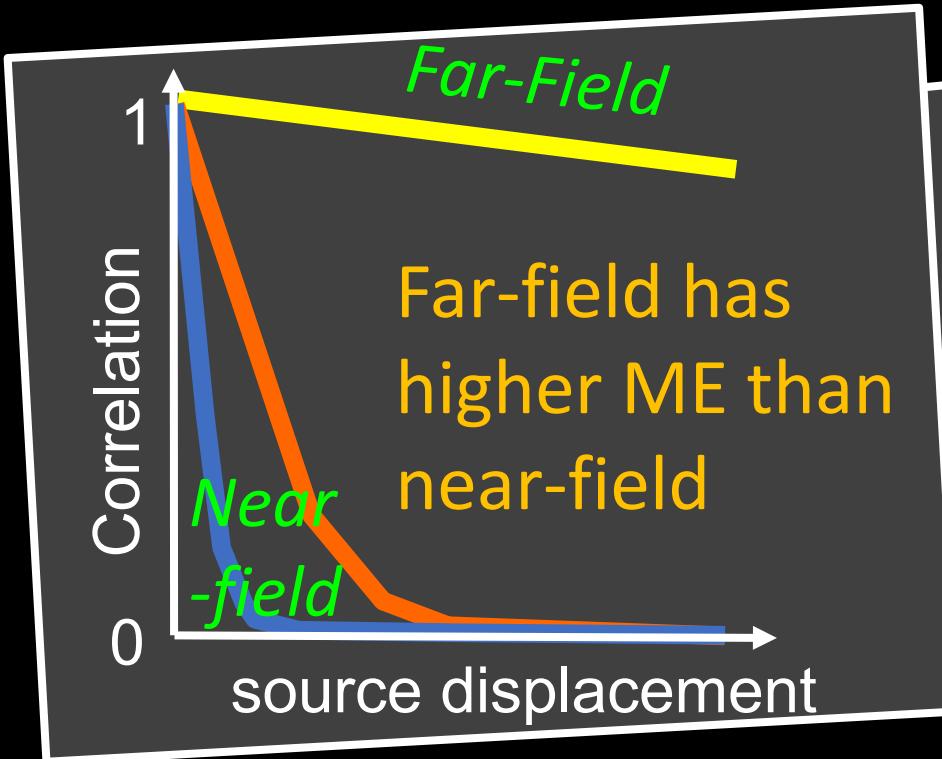
Focusing on illumination



Local support is key for enhancement of seeing-through-scattering algorithms.

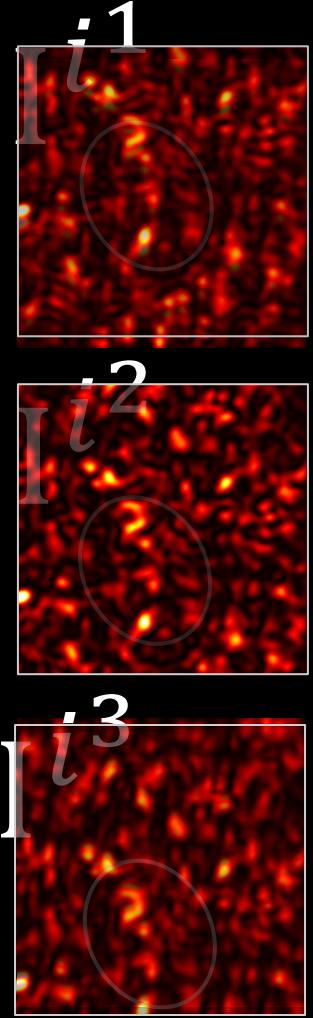
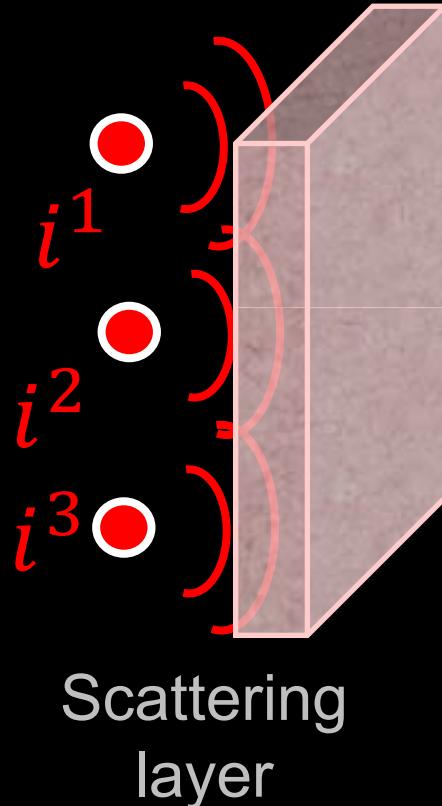
# Summary of theoretical contributions

( i ) Theoretical and rendering based analysis of ME in **near** and **far**-field settings

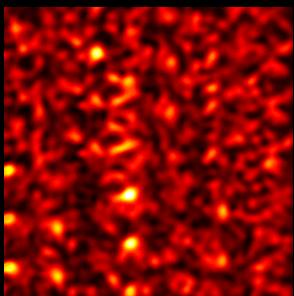


# Our algorithm

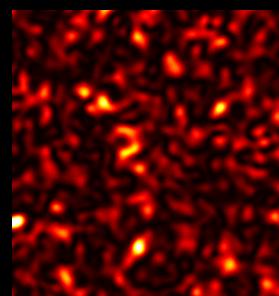
# Seeing through scattering using ME



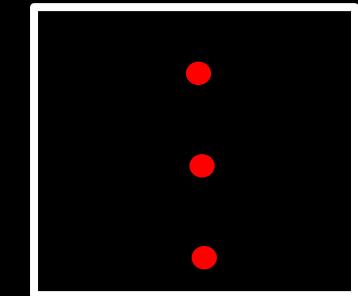
$$I = \sum I^{t^k}$$



$$S$$



Scatter free  
image  
 $O$



$$I * I^{\text{overlap}} * S * S \approx O * O \approx O * O$$

$$S * S \approx S$$

**Full-frame**

$$O = \text{Phase Retrieval}(I * I)$$

# Reconstruction using local speckle correlation

What illuminators produce this image?

Previously:  
sum over full frame

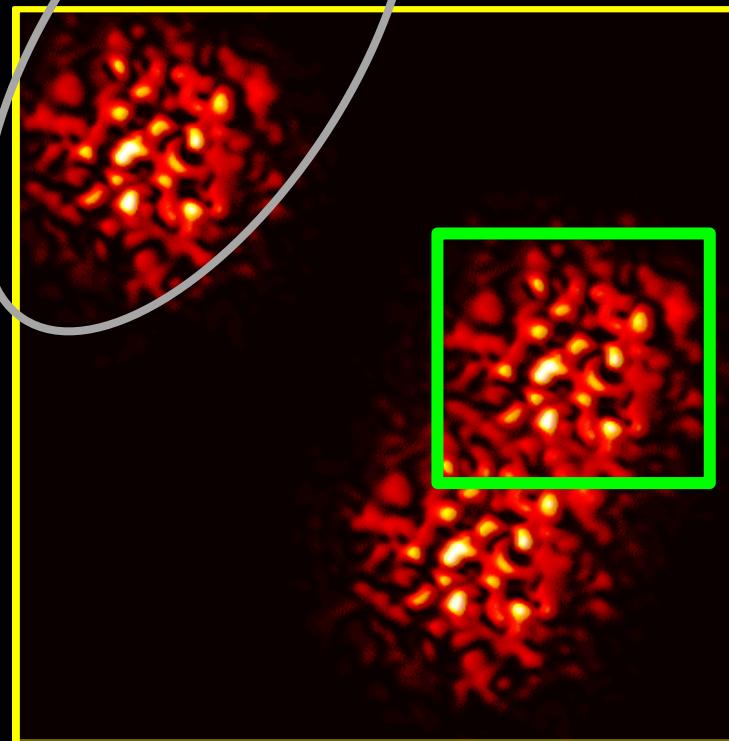
Ours:  
sum over local window

$$\text{Correlation}(\Delta) = \sum_x I_x \cdot I_{x+\Delta}$$

Noisy

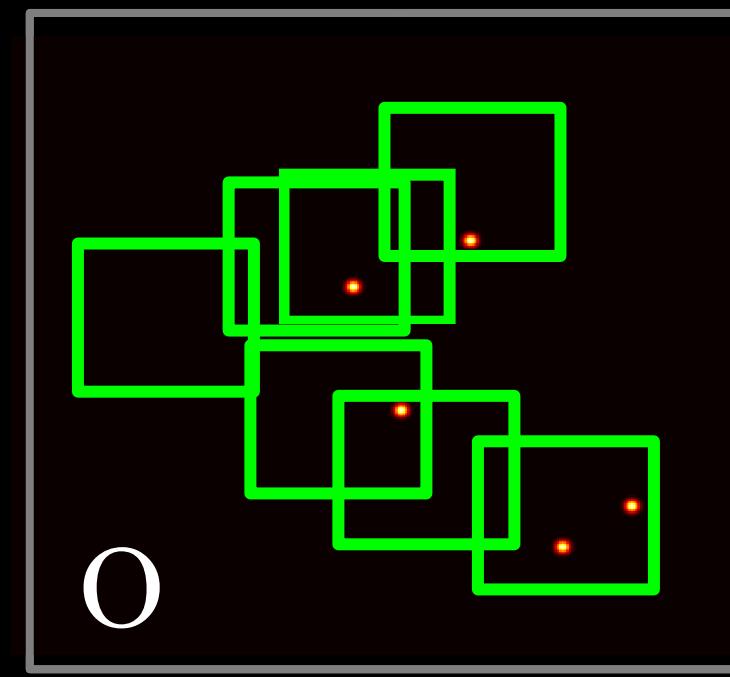
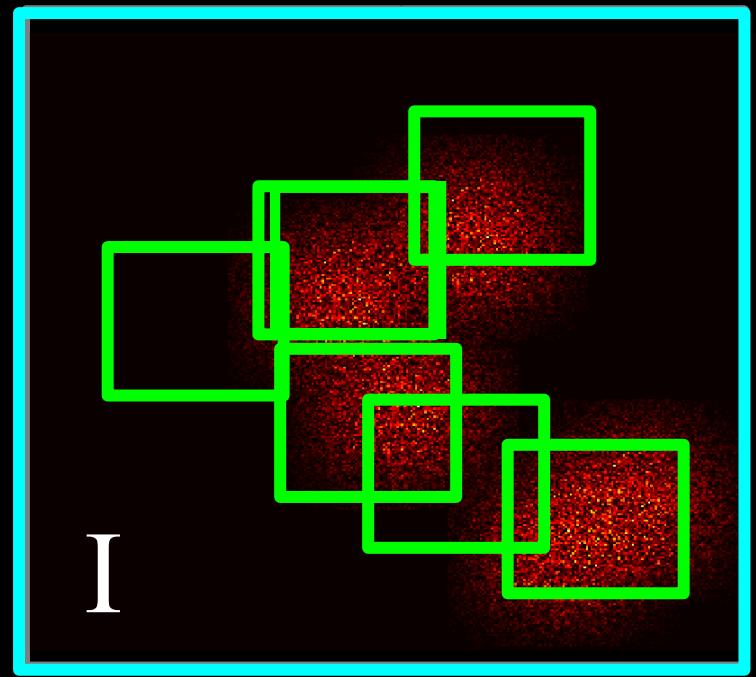
Theorem:  
order of magnitude increase in SNR

Just noise



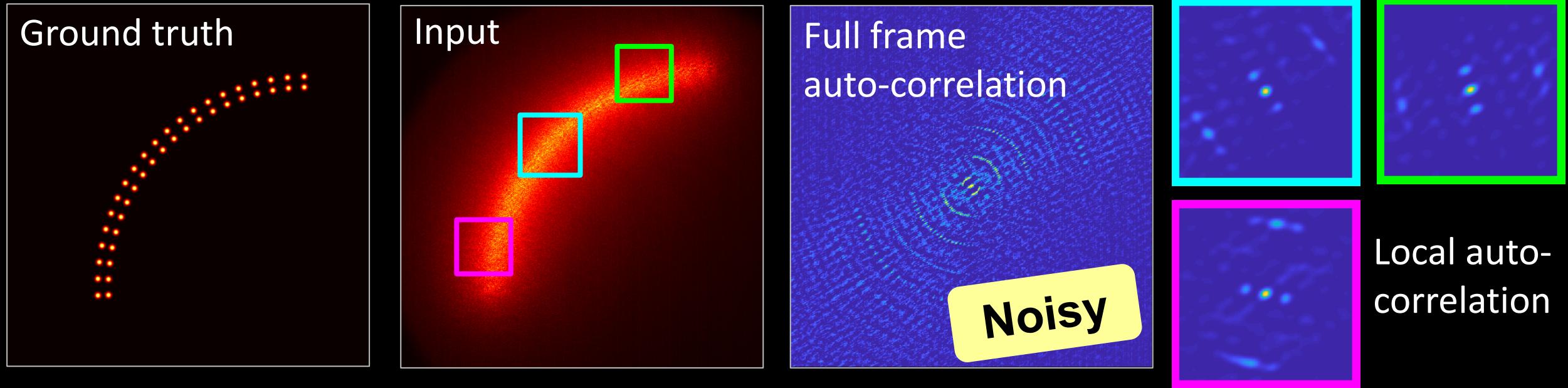
# Optimization with local support

Full-frame  
algorithm



$$\min_{\mathbf{O}} \sum_{\text{All local windows}} \left( \left\| \mathbf{I} - \mathbf{O} \right\|^2 \right)$$

# Local vs. global correlation

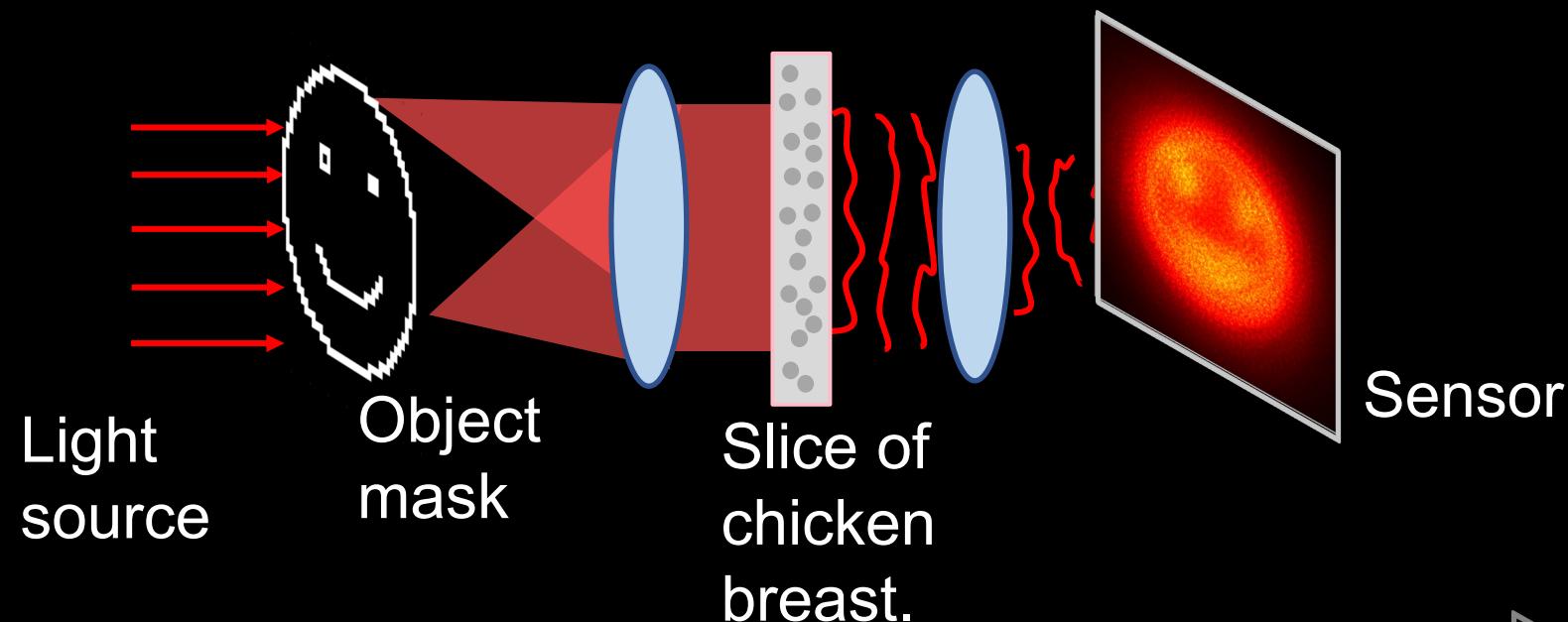


Local correlation: **less noisy and reveals some information about the desired shape**

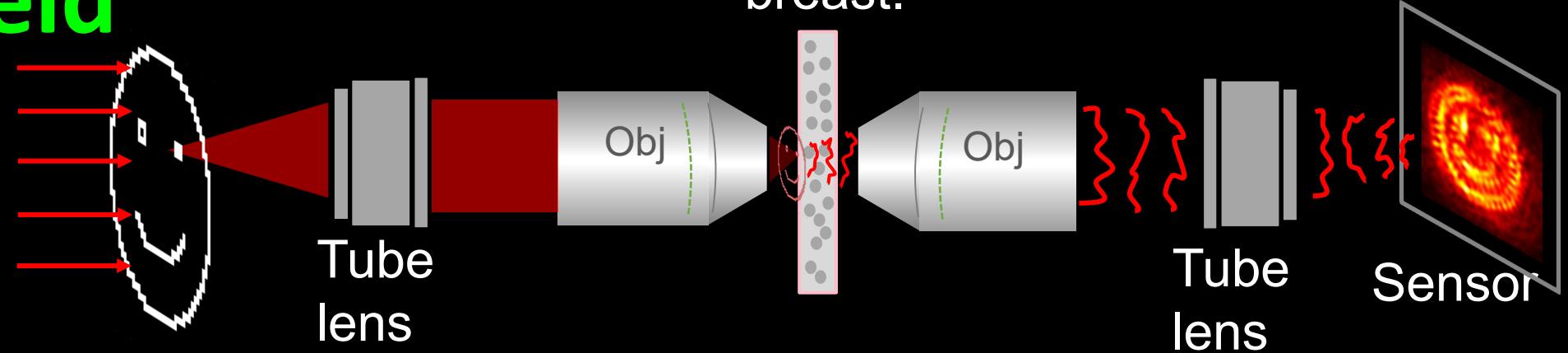
# Lab Results

# Lab setup

Far-field



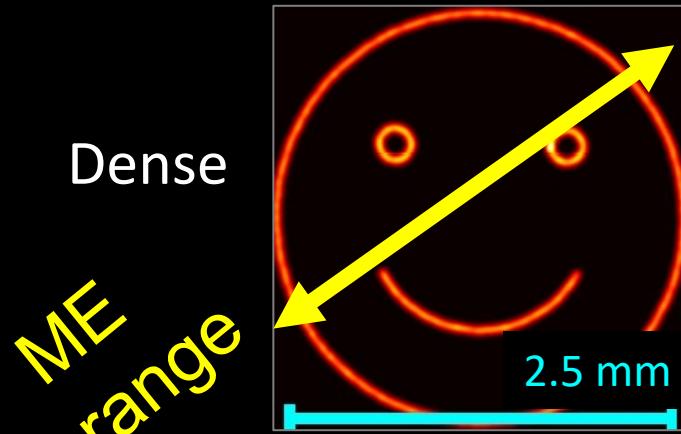
Near-field



Chicken breast  
thickness 200  $\mu\text{m}$

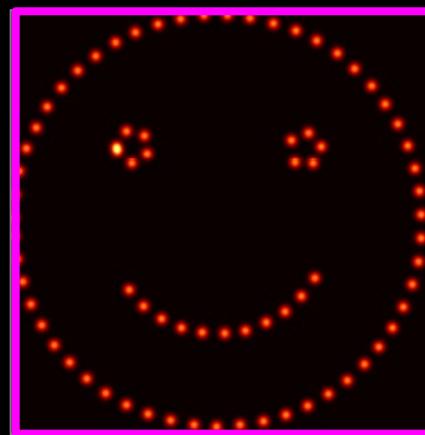
## Lab results: far-field

Ground truth

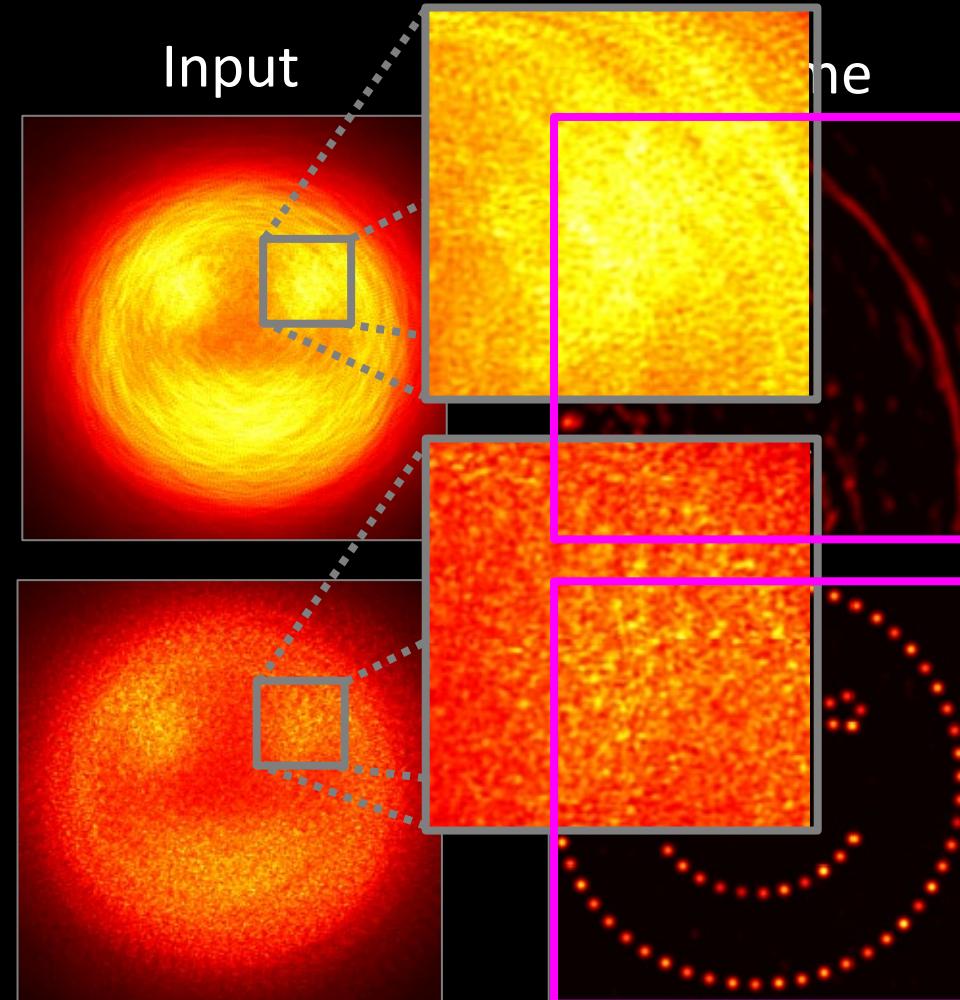


Dense  
ME  
range

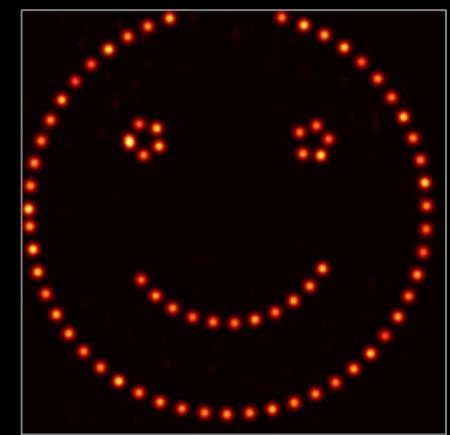
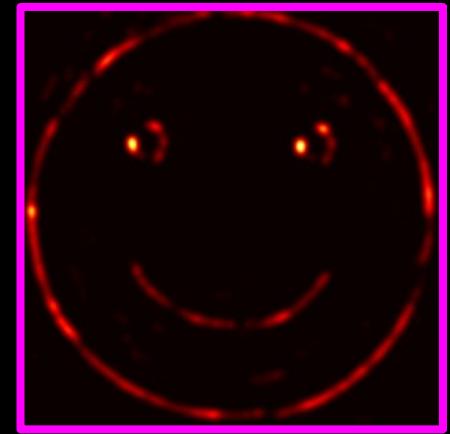
Sparse  
1:4



Input

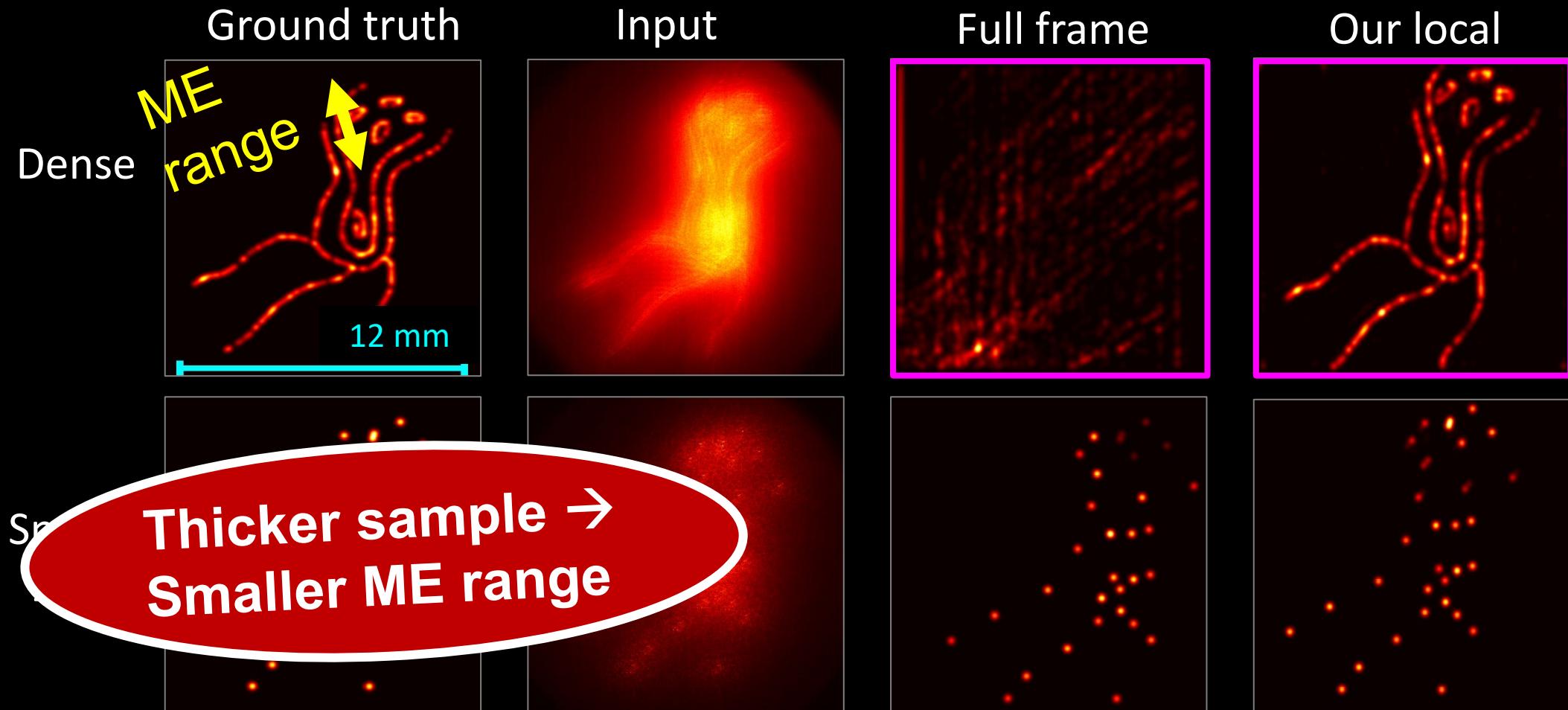


Our local

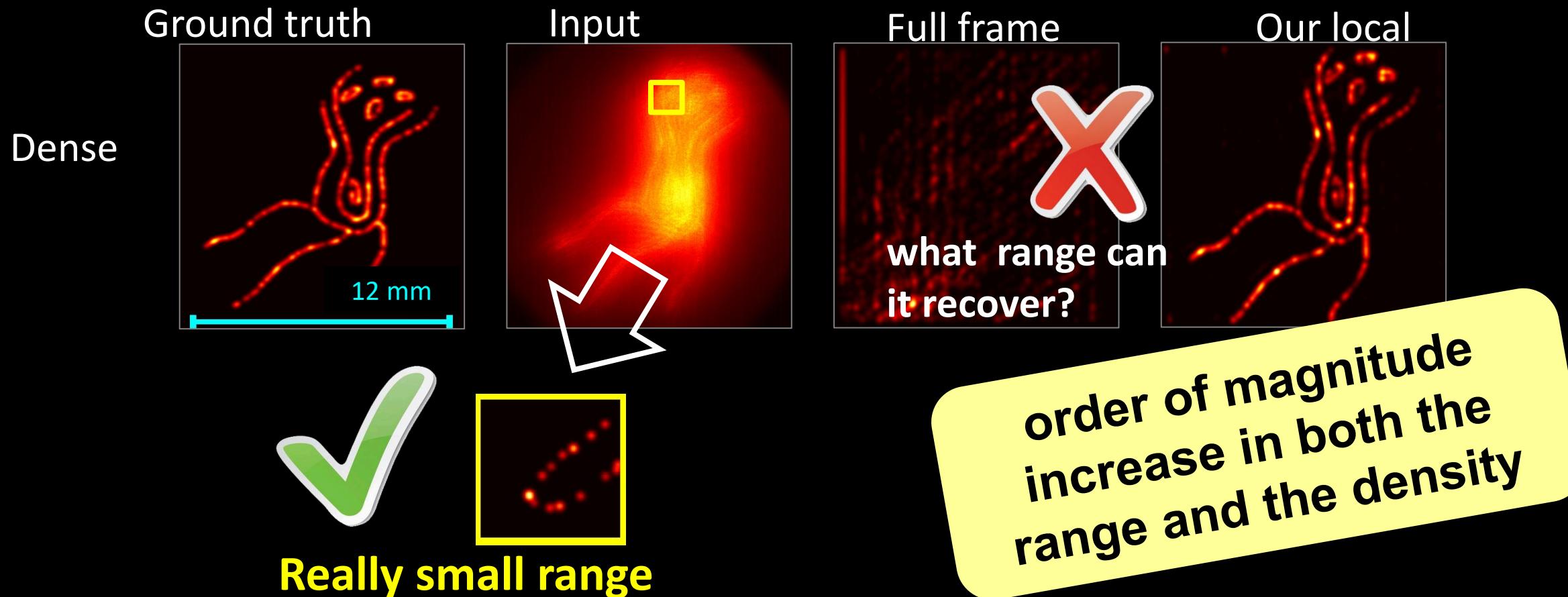


Chicken breast  
thickness 340  $\mu\text{m}$

## Lab results: far-field

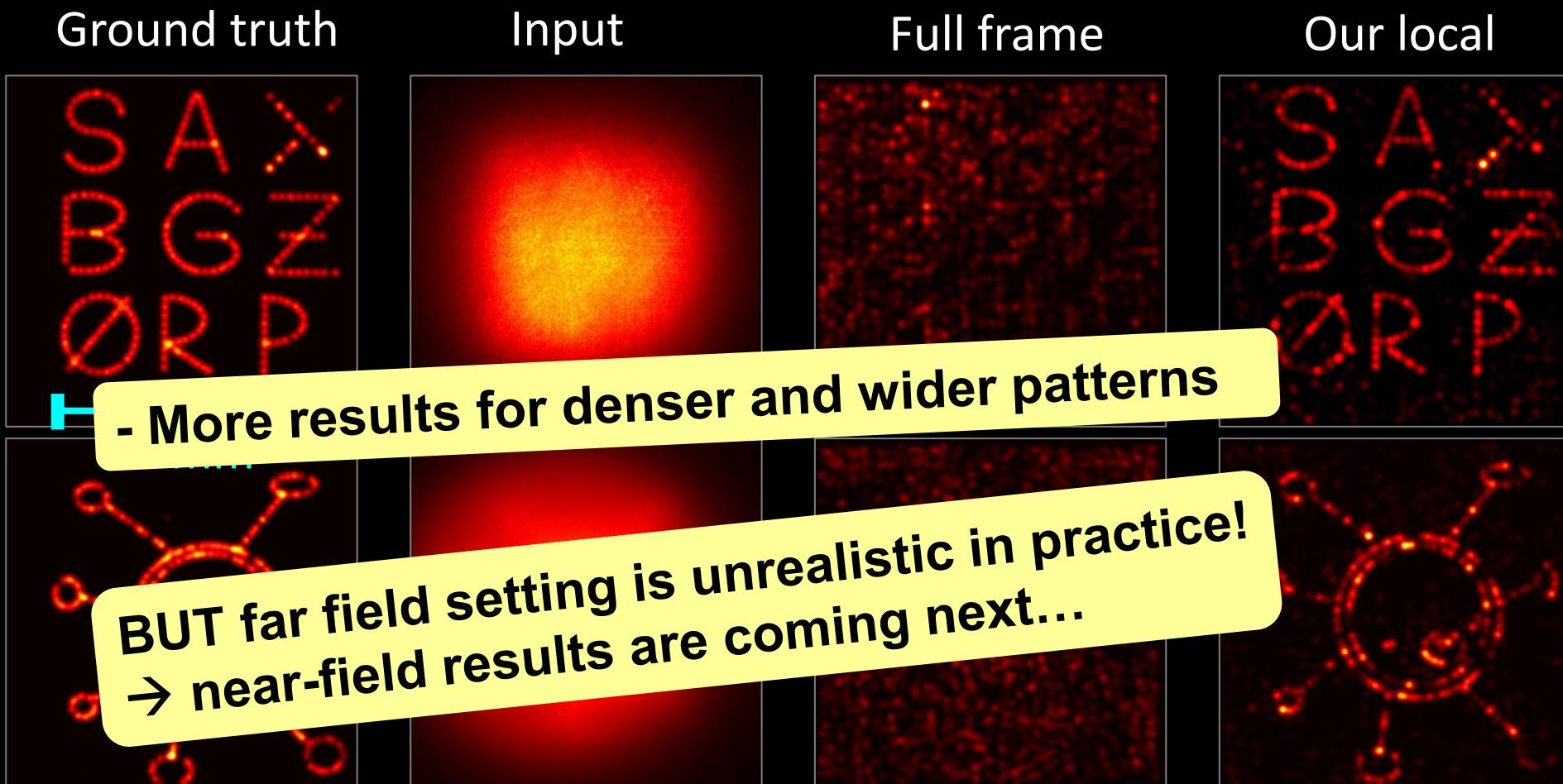


# Lab results: far-field



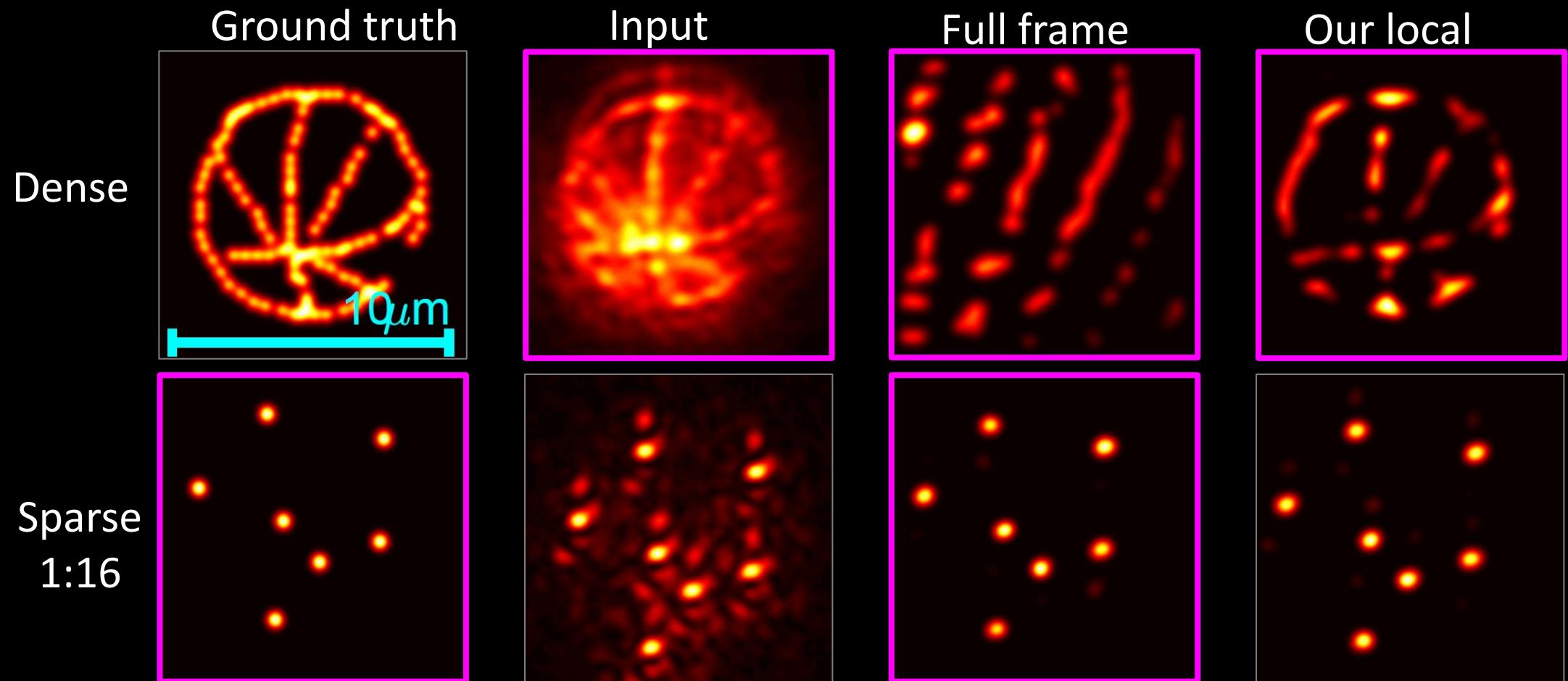
Chicken breast  
thickness 170 µm

## Lab results: far-field



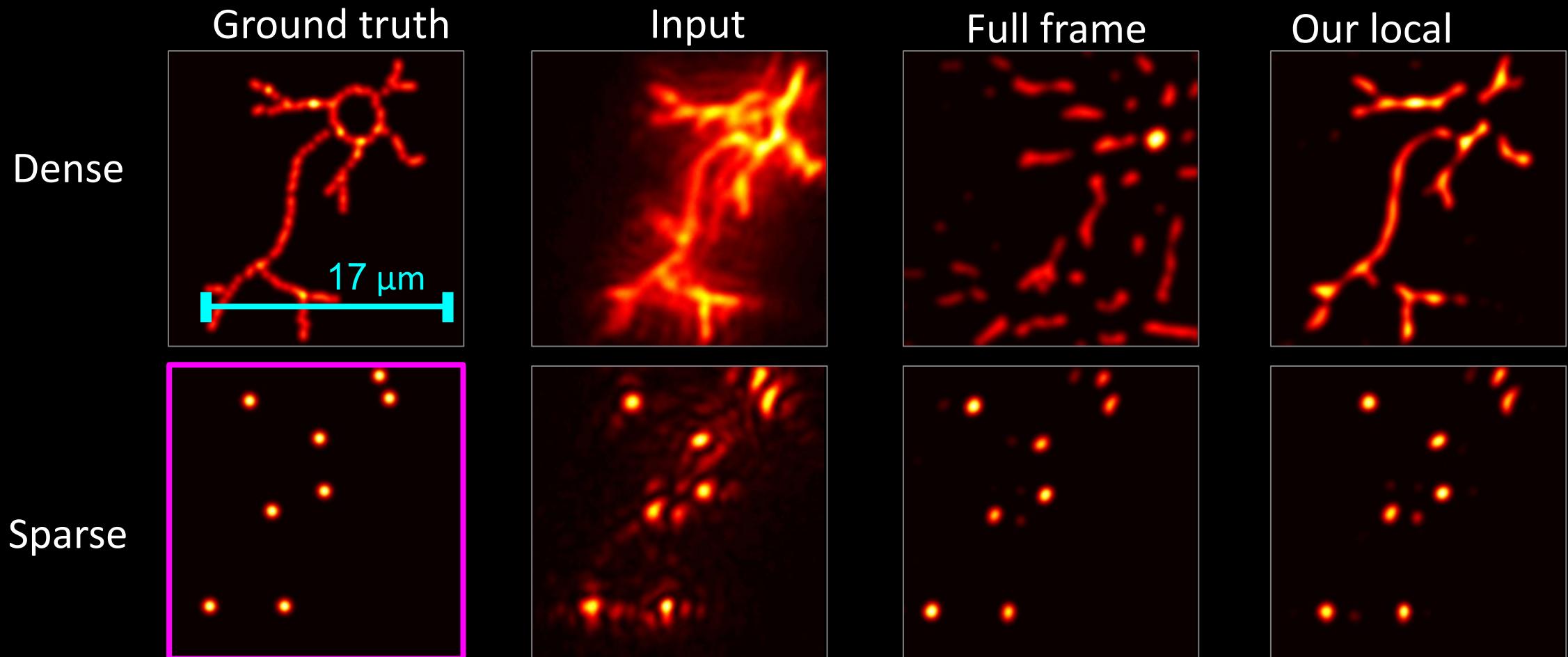
Chicken breast  
thickness 140  $\mu\text{m}$

## Lab results: near-field



Chicken breast  
thickness 140  $\mu\text{m}$

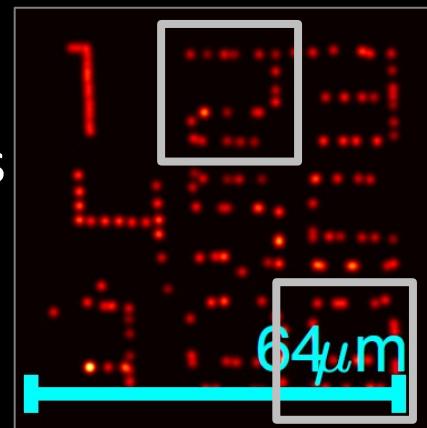
## Lab results: near-field



Chicken breast  
thickness 200  $\mu\text{m}$

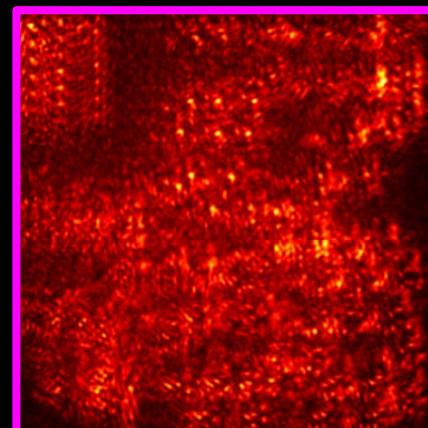
## Lab results: near-field

Ground truth

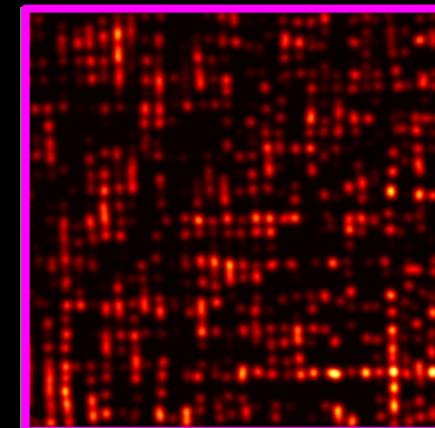


Numbers

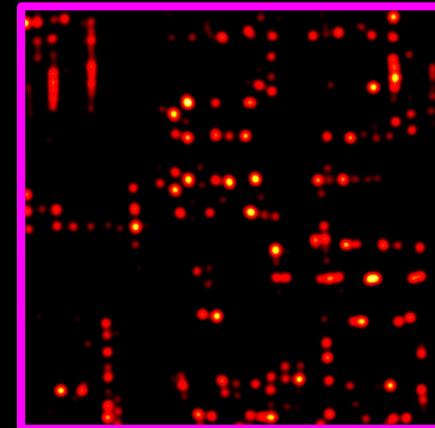
Input



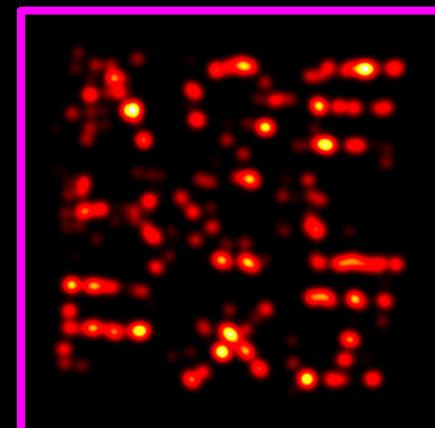
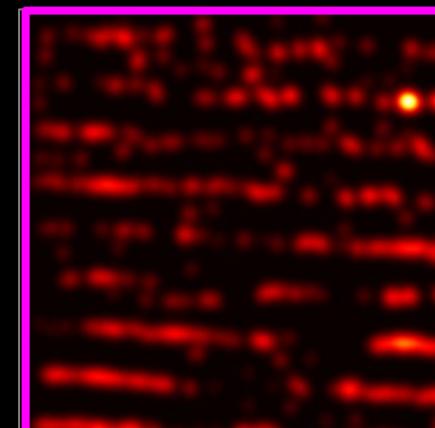
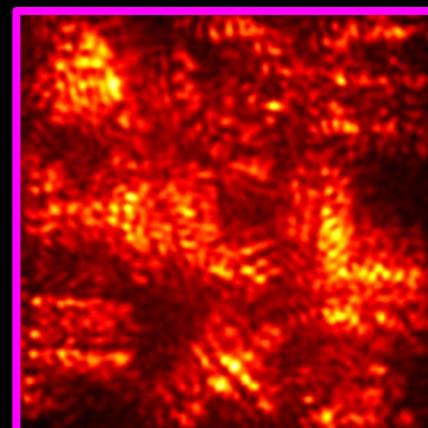
Full frame



Our local



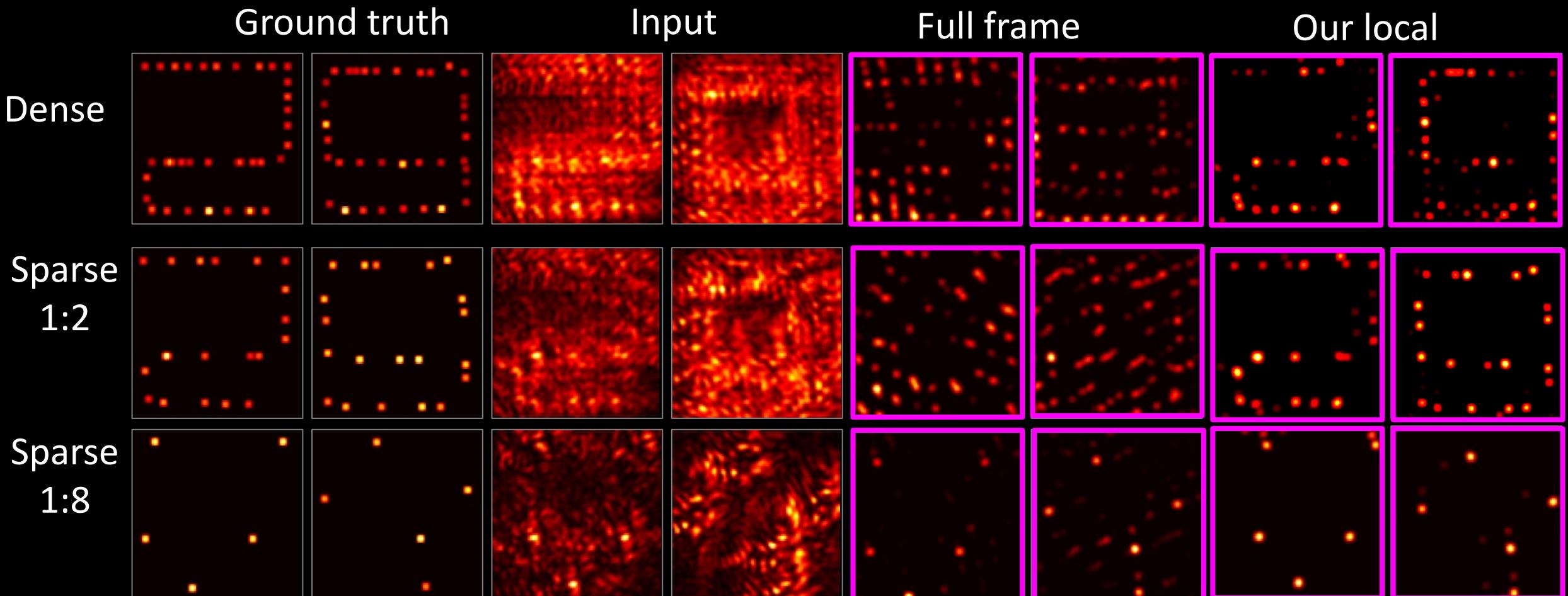
Letters



Larger  
degradation

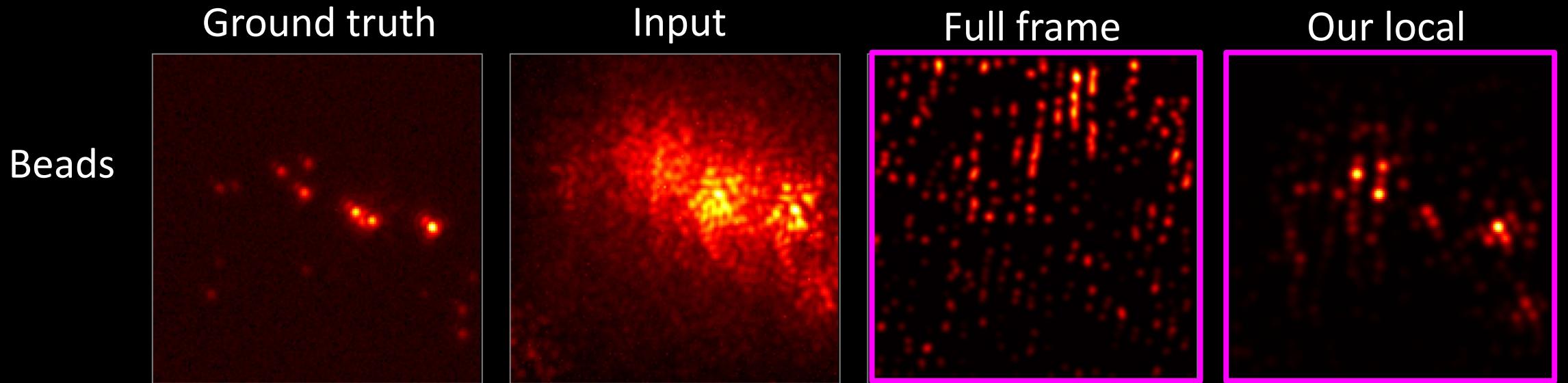
Chicken breast  
thickness 200  $\mu\text{m}$

## Lab results: near-field



Chicken breast  
thickness 100  $\mu\text{m}$

## Lab results: near-field



Reconstructing  
fluorescent beads

Thanks to **Dr. Lucien Weiss** from Schechtman lab  
for preparing the fluorescent samples