

Homework #3

Due date: 18:00, October 24th, Monday, 2016

Problem statement

Given an unsigned integer $n \geq 2$, factor it into primes and use the factorization to determine the number, sum, and product of divisors of n .

Let $n = p_1^{e_1} p_2^{e_2} \dots p_k^{e_k}$, where $p_1 < p_2 < \dots < p_k$ are primes and $e_i > 0$, be the prime factorization of n . Then,

the number of divisors of $n = \prod_{i=1}^k (1 + e_i) \quad \dots (1)$

and

the sum of divisors of $n = \prod_{i=1}^k \sum_{j=0}^{e_i} p_i^j \quad \dots (2)$

and

the product of divisors of $n \quad \dots (3)$

For example, $20 = 2^2 * 5$ has 6 divisors, namely, 1, 2, 4, 5, 10 and 20, that sum up to 42, and product is equal to 8000, agreeing with formula

(1): $(1 + 2)(1 + 1) = 6$

(2): $(2^0 + 2^1 + 2^2)(5^0 + 5^1) = 42$

(3): $1 * 2 * 4 * 5 * 10 * 20 = 8000$.

Requirements

1. The easiest factorization method is the trial division algorithm that consists of the following loop:

```
While (not finish yet) {  
    Find the next prime  $p$   
    Find the largest integer  $e$  such that  $p^e$  divides  $n$   
  
    Reduce  $n$  to  $\frac{n}{p^e}$   
}
```

For example, let $n = 20 = 2^2 * 5$, the values of p , e , and n at the end of each iteration are shown below:

1 st iteration	$p = 2$	$e = 2$	$n = 5$
2 st iteration	$p = 3$	$e = 0$	$n = 5$
3 st iteration	$p = 5$	$e = 1$	$n = 1$

2. The product of divisors of n is also an unsigned integer.
3. Be careful of integer overflow especially the product of divisors of n .
(you should check whether the “calculation” will overflow instead of checking overflow after calculation!)
4. Write a C program that is capable of handling input.
5. Properly comment your program.
6. Plagiarism is not allowed!
7. See the sample run below for the required output format.

Submission

Be sure to upload your source code to E3 by the due date and name your file as “xxxxxxx_hw3.c”, where xxxxxxx is your student ID.

Sample run

```
Enter an unsigned integer >= 2: 20  
Prime factorization of 20 = 2^2x5^1  
Number of divisors = 6  
Sum of divisors = 42
```

Product of divisors = 8000

Enter an unsigned integer ≥ 2 : 81

Prime factorization of 81 = 3^4

Number of divisors = 5

Sum of divisors = 121

Product of divisors = 59049

Enter an unsigned integer ≥ 2 : 1421

Prime factorization of 1421 = $7^2 \times 29^1$

Number of divisors = 6

Sum of divisors = 1710

Product of divisors = 2869341461

Enter an unsigned integer ≥ 2 : 84

Prime factorization of 84 = $2^2 \times 3^1 \times 7^1$

Number of divisors = 12

Sum of divisors = 224

Product of divisors overflow!

Enter an unsigned integer ≥ 2 : 3287037600

Prime factorization of 3287037600 = $2^5 \times 3^2 \times 5^2 \times 7^3 \times 11^3$

Number of divisors = 864

Sum of divisors overflow!

Product of divisors overflow!

Enter an unsigned integer ≥ 2 : 4198216889

Prime factorization of 4198216889 = $60917^1 \times 68917^1$

Number of divisors = 4

Sum of divisors = 4198346724

Product of divisors overflow!

Enter an unsigned integer ≥ 2 : 4294967279

Prime factorization of 4294967279 = 4294967279^1

Number of divisors = 2

Sum of divisors = 4294967280

Product of divisors = 4294967279

Enter an unsigned integer ≥ 2 : ^Z