

2012-2013第一学期 高数E考试题 A 卷（生命学院）试题 答案

第一题: 填空题

1. -1 2. $f(x) = 2^x - \frac{x}{\ln 2}$ 3. $\ln(1 + \sqrt{2})$ 4. $\frac{1}{2}$ 5. $y - \frac{4}{3} = \frac{5}{3}(x - \ln 3)$ 6. $3^{97}(3x + 98)e^{3x}$

第二题: 选择题

1.C 2.C 3.D 4.D 5.B 6.A

第三题: 计算题

1.

$$\int \frac{7x-5}{(x-2)(x-3)} dx = \int \left(\frac{9}{x-2} - \frac{16}{x-3} \right) dx \dots\dots\dots (4\text{分})$$

$$= 9 \ln |x-2| - 16 \ln |x-3| + C \dots\dots\dots (4\text{分})$$

2.

$$\int_3^8 \frac{1}{(x+1)[(x+1)^{5/2}-1]} dx$$

$$t = \sqrt{x+1}, x = t^2 - 1, dx = 2t dt$$

$$x = 3, t = 2; x = 8, t = 3$$

$$\int_2^3 \frac{2t dt}{t^2(t^5-1)} = \frac{2}{5} \int_2^3 \frac{dt^5}{t^5(t^5-1)} \dots\dots\dots (4\text{分})$$

$$= \frac{2}{5} [\ln |\frac{t^5-1}{t^5}|]_2^3 = \frac{2}{5} (\ln(242/243) - \ln(31/32)) \dots\dots\dots (4\text{分})$$

3.

$$\sqrt{x^2 + y^2} - \ln y = 3$$

$$\frac{x+yy'}{\sqrt{x^2+y^2}} - \frac{y'}{y} = 0 \dots\dots\dots (4\text{分})$$

$$y' = \frac{-xy}{y^2 - \sqrt{x^2+y^2}} \dots\dots\dots (4\text{分})$$

$$4. \begin{cases} x = 2 \cos^3 \theta \\ y = 3 \sin^2 \theta \end{cases}$$

$$\frac{dy}{dx} = \frac{6 \sin \theta \cos \theta}{-6 \sin \theta \cos^2 \theta} = \frac{-1}{\cos \theta} \dots\dots\dots (4\text{分})$$

$$\frac{d^2y}{dx^2} = -\left(\frac{1}{\cos \theta}\right)' \frac{1}{-6 \sin \theta \cos^2 \theta} = \frac{1}{6 \cos^4 \theta} \dots\dots\dots (4\text{分})$$

$$5. \int_1^x f'(t^2) dt = x \ln x - x + 1$$

$$f'(x^2) = \ln x$$

$$f(t) = 1/2(t \ln t - t) + C \dots\dots\dots (4\text{分})$$

$$f(x) = 1/2(x \ln x - x) + 3/2 \dots\dots\dots (4\text{分})$$

第四题: 证明题

$$\begin{aligned}
& 1. \quad y = 1/x \\
& \quad y' = -1/x^2 \\
& \quad y - y_0 = -1/x_0^2(x - x_0) \dots\dots\dots (4\text{分}) \\
& \quad x_1 = 2x_0, y_1 = 2/x_0 \\
& \quad x_1y_1 = 4 \dots\dots\dots (4\text{分}) \\
& 2. \quad \arcsin x - 0 = \frac{1}{\sqrt{1-\xi^2}}x \\
& \quad \frac{1}{\sqrt{1-\xi^2}}x < \frac{\sqrt[3]{1+x^3}}{\sqrt{1-x^2}} \dots\dots\dots (4\text{分}) \\
& \quad x > 4/5, \frac{1}{\sqrt{1-\xi^2}}x > x/(3/5) > 3x/2 \dots\dots\dots (4\text{分}) \\
& 3. \\
& \quad f(0) = e, f(1) = 1 \\
& \quad F(x) = e^x f(x) \\
& \quad F(0) = e, F(1) = e \dots\dots\dots (4\text{分}) \\
& \quad F'(\xi) = 0 \\
& \quad e^\xi f(\xi) + e^\xi f'(\xi) = 0 \dots\dots\dots (4 \text{ 分}) \\
& \quad f(\xi) + f'(\xi) = 0
\end{aligned}$$

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