Design and Analysis of Algorithms (2018 Spring)

Assignment 1

Due: April 9, 201

1. (20 points) Prove the follows:

   If $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$, then $f_1(n)f_2(n) = O(g_1(n)g_2(n))$.

2. (25 points) Describe the Master method for solving recurrence relations, and give an asymptotic tight bound for f(n) in the following recurrence relations:

   1、 $f(n) = 9f(\frac{n}{6}) + O(n\log n).$

   2、 $f(n) = 2f(n-3) + O(n).$

   3、 $f(n) = 4f(\frac{n}{2}) + O(n^2).$

3. (25 points) Arrange the following functions in ascending asymptotic order of growth rate:

(a) $f_1(n) = n^{2.5} + 5^{100}n^2$, $f_2(n) = 2^{\log n + \log\log n}$, $f_3(n) = \sqrt{n^{3.5}}$, $f_4(n) = 2^{2n}$, $f_5(n) = 3^n$;

(b) $f_1(n) = n^{\log^2 n}$, $f_2(n) = 2^{\log n + \log\log n}$, $f_3(n) = \log^n \log^2 n$, $f_4(n) = n^{\sqrt{n}\log n}$.

4. (30 points) Use dynamic programming to solve the following knapsack problem: There are n different types of items and each type has two pieces (totally we have 2n items). Item i has weight of $w_i > 0$ kilograms and value of $v_i > 0$ dollars. You have a knapsack that can contain at most W kilograms. How to fill your knapsack with the items to maximum the value?

(a) Please define your subproblem;

(b) Give the recurrence relation based on your subproblems;

(c) Solve the following instance showed in Table 1 by using the bottom-up method. You are required to give the computation steps (the table used to store the solutions to the subporblems).

Items	Weight	Value
1	1	2
2	2	3
3	4	8
4	3	2
5	5	9

Table 1: There are 5 items, each of which has two pieces. The capacity of the knapsack is 10 kilograms.

附加题：一个图如果任何一条边最多包含在一个圈中则称这个图为仙人掌图。设计一个算法在一个边带权重的仙人掌图中找出距离最远的两个顶点。