A Simple FPTAS for Counting Edge Covers

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Overview

Edge cover

Definition

For an undirected input graph $G = (V, E)$, an **edge cover** of G is a set of edges C covering all vertices.

Example

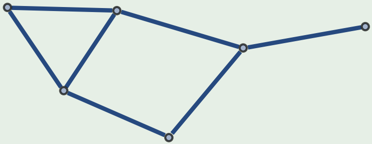

Figure : Example of an edge cover

Edge cover is related to many other problems such as:

- Matching problem.
- Holant problem.
- Rtw-Mon-CNF. (read twice monotone CNF)

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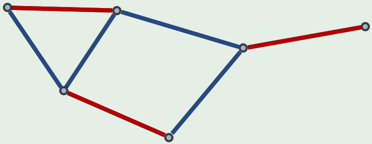

Figure : Example of an edge cover, with edges chosen being highlighted in red.

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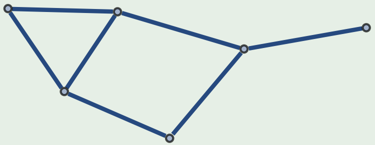
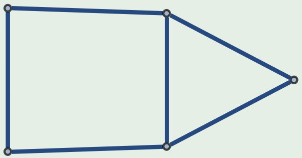
Relation to Matching

Example

Find edge covers by maximal matching?



(a) G has a perfect matching.



(b) G doesn't have a perfect matching.

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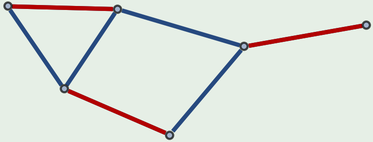

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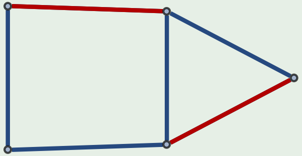

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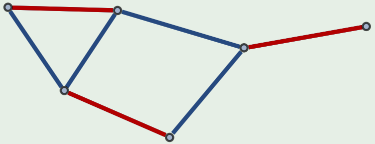

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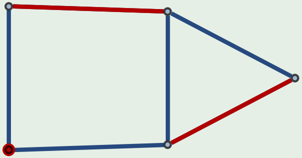

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Approximation Schemes

We are interested primarily in two type of polynomial time approximation scheme:

Definition

(Informally) For given parameter $\varepsilon > 0$ and an instance of a particular problem class, if the algorithm outputs a number $\hat{N}$ such that $(1-\varepsilon)N \leq \hat{N} \leq (1+\varepsilon)N$, where N is the accurate answer of the problem instance, and the running time is bounded by $poly(n, 1/\varepsilon)$ with n being the size of instance, this is called the **FPTAS (fully polynomial time approximation scheme)**.

A randomized relaxation of FPTAS is known as **FPRAS (fully polynomial time randomized approximation scheme)**, which uses random bits and only outputs $\hat{N}$ to the desired precision with high probability.

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