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Report for HW4

Part 1 Implementation of algorithm:

For a), the closed formula is exactly the same as in HW1. The Black-Scholes formula is as below:

$$C(S,t) = N(d_1)\ S - N(d_2)\ Ke^{-r(T-t)},$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}.$$

The function is from Joshi code. MC vanilla call and Park-Miller uniforms are from HW3. QMC are using Sobol sequence and is presented in HW3 as well.

For b), I wrote a brand new class for GeometricAsianCF. It is one instance of PathDependent Class. The difference between PathDependentAsian and this new class is that the new class instead of getting the arithmetic mean of the discounted spot values obtains the geometric mean of the discounted spot values. One thing to note here is that I didn't divide the discount rate back to get the original spot values because the discount rate is the same in both arithmetic and geometric Asian option cases. Therefore, just for coherence, I still put the value there in order to have a fair comparison between arithmetic Asian option and geometric Asian option. I also wrote the closed formula class function for the geometric Asian option.

Please be noted that for the MC simulations, I used Antithetic variance reduction method in the HW3 and for the QMC simulations I just applied the Brownian Bridge and terminal stratification algorithm as in the supplementary note.

I quote it here for your convenience.

for i = 1…..K,
    generate U~Unif[0; 1];
V ←(i -1 + U)/K;
$W(t_m)$←sqrt(t_m)*InverseCumulativeNormal(V);
for j = 1…. M- 1;
    generate Z ~ N(0, 1);
    a =(t_m-t_j)/(t_m-t_{j-1})
    b=(t_j-t_{j-1})/(t_m-t_{j-1})
    c=sqrt((t_m-t_j)* (t_j-t_{j-1})/(t_m-t_{j-1}))
    W(tj) ←a*W(tj1) + b*W(tm) + c*Z;
return (W(t1)….W(tm));
end;

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For c), Arithmetic Asian option is given by the Joshi code. I added BrownianBridgePath to the QMC one and got pretty good result.

Part 2 Benchmark(Note that this is not for the question about changing Barrier price. There will be another chart showing that.)

	My Code with standard 10000 simulations (mean/std-dev)	Haug's VBA Code
Closed-form Vanilla call option price	10.0201	**10.0201**
MC vanilla call price with Park-Miller uniforms and antithetics	9.82118/20.2609	**9.9861**
QMC vanilla call price with Sobol sequence	9.99624/19.4387	**10.0151**
Closed-form geometric Asian call price	4.09575	**N/A**
MC geometric Asian call, Park-Miller uniforms	4.21109/9.0616	**N/A**
QMC geometric Asian call, Sobol sequence	4.28687/8.00289	**N/A**
MC Arithmetic Asian call, Park-Miller uniforms	4.54773/9.69062	**NA**

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QMC Arithmetic Asian call, Sobol sequence	4.50227/8.52529	**5.1126**

Part 3 Comparison of Algorithms:

It seems that the algorithms to apply Sobol sequence as well as stratification and Brownian Bridge are giving nicer and closer results than Park Miller sequence and antithetic combined. To see if it persists.

I tested more values with Strike 70. The results are listed below:

	My Code with standard 10000 simulations (mean/std-dev)	Haug's VBA Code
Closed-form Vanilla call option price	34.3953	**34.3953**
MC vanilla call price with Park-Miller uniforms and antithetics	34.3237/29.7753	**N/A**
QMC vanilla call price with Sobol sequence	34.3675/29.3042	**34.3892**
Closed-form geometric Asian call price	30.526	**N/A**
MC geometric Asian call, Park-Miller uniforms	30.5696/17.9111	**N/A**
QMC geometric Asian call, Sobol sequence	32.192/16.3855	**N/A**

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MC Arithmetic Asian call, Park-Miller uniforms	31.249/18.2791	**NA**
QMC Arithmetic Asian call, Sobol sequence	32.8669/16.5802	**31.5184**

If I run similar simulation with regard to Geometric Asian option using the ExoticBSEngine offered by Joshi. The result will be a little bite closer to the exact solution- 4.19916 and stddev=9.10843. It seems that the reduction in variance is more significant in the case where I have BrownianBridgePath instead of ExoticBSEngine. In the Arithmetic case, the formula given by Joshi seems to be a little off from Excel formula, but I test it again when strike is 70, in which case the Joshi code gives 31.2549 for MC arithmetic Asian and 32.8669 for QMC. They are very close to **31.5184** as give in the Excel result. In fact, I directly copied Joshi's code and his code is proved to be "right". This also safeguards it.

In this case, Sobol sequence combined with BrownianBridgePath also gives a more accurate value is European case and persistently smaller variance even if antithetic method is used alone with Park Miller generator. It seems to me that our Columbia Alumni did discover something useful!