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# Report for HW2

## Part 1 Implementation of algorithm:

**For a)**, I applied the formula showed in Shrev's text book

$$\begin{aligned} V(0) &= S(0)\left[N\left(\delta_+\left(T,\frac{S(0)}{K}\right)\right)-N\left(\delta_+\left(T,\frac{S(0)}{B}\right)\right)\right] \\ &-e^{-rT}K\left[N\left(\delta_-\left(T,\frac{S(0)}{K}\right)\right)-N\left(\delta_-\left(T,\frac{S(0)}{B}\right)\right)\right] \\ &-B\left(\frac{S(0)}{B}\right)^{-\frac{2r}{\sigma^2}}\left[N\left(\delta_+\left(T,\frac{B^2}{KS(0)}\right)\right)-N\left(\delta_+\left(T,\frac{B}{S(0)}\right)\right)\right] \\ &+e^{-rT}K\left(\frac{S(0)}{B}\right)^{-\frac{2r}{\sigma^2}+1}\left[N\left(\delta_-\left(T,\frac{B^2}{KS(0)}\right)\right)-N\left(\delta_-\left(T,\frac{B}{S(0)}\right)\right)\right]. \end{aligned}$$

where

$$\delta_\pm(\tau,s)=\frac{1}{\sigma\sqrt{\tau}}\left[\log s+\left(r\pm\frac{1}{2}\sigma^2\right)\tau\right].$$

The parameters I passed in are Barrier price, Expiry, Interest Rate, Dividend, Spot, and Volatility.

**For b)**, I applied **Euler Scheme code**(the professor said that if I already applied Euler, I am not expected to apply the simple MC generator again. So, I keep it as is.) I wrote last time and created an array of the right size to store the corresponding values out of simulations and wrote functions to calculate the standard error. Since the mean is calculated before standard error. I think it is not necessary to calculate the mean again using for loop inside the standard error function. So, I simply pass it in. All values that I passed in are wrapped up in a struct called EulerScheme besides the extra parameter Barrier price, which is added part of original Euler Scheme. The only amendment to original EulerScheme is the test of whether or not the price path has touched barrier price and if so break out of loop and return payoff 0. The parameters passed in are Number of Iteratals, Interest Rate, Volatility, Dividend rate, Spot price, Strike price, and Expiry (all in the EulerScheme Struct), plus Barrier price.

**x(t)=x(t-1)+a(X(t-1),t)*dt+ b(X(t-1),t)*dW**

(a, b are function input of Euler Scheme. a and b in face take more parameters in implementation, but the mathematical model is the same.)

**For c)**, I write a method called Euler Scheme2. It gets the same input as in part b) but the output is a struct that stores an array of two doubles. The first one is the generated option and the other is the paired option. I tested if the price path touches the barrier price at the end of every iteration. If one of the pair hit the barrier, the loop continues. If both hit the barrier, the loop breaks. The formula is the same as in b).

**For d)**, the first part is a loop to generate first 30% of the paths to estimate the coefficient b. The paired x is chosen to the corresponding European option. The code worked last time is directly applied here. After saving the two array of x and y's, the coefficient is easily calculated using the coefficient formula

**b=Cov(x,y)/σ(x) ∗ σ(x).**

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The coefficient is then used to do the control variant method. Apply the formula

$$\mathbf{y} - \mathbf{b} * (\mathbf{x} - \bar{\mathbf{x}})$$

The resulting variance is used to estimated mean and variance.

## Part 2 Benchmark(Note that this is not for the question about changing Barrier price. There will be another chart showing that.)

| | My Code with standard 10000 simulations | Haug's VBA Code |
|---|---|---|
| Closed-form barrier option price/Standard Error/Time | 0.0517846/NA/0 | **0.514** |
| Monte Carlo barrier option price/Standard Error/Time | 0.0831479/0.653733/0.461 | NA |
| Antithetic variants barrier option price/Standard Error/Time | 0.069555/0.581483/0.25 | NA |
| Control variants barrier option price/Standard Error/Time | 0.343659/0.597526/0.904 | NA |

It seems that the arithmetic method here. It has a relatively close mean and thanks to the perfect negative correlation by construction, antithetic method also has lower standard error and time consumption.

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## Part 3 Control Variance Scheme :

The variance reduction is decided by the correlation between two variables shown in the formula below in the supplementary note of the homework.

$$\frac{\mathrm{Var}[\bar{Y}(b^*)]}{\mathrm{Var}[\bar{Y}]} = 1 - \rho_{XY}^2,$$

| | Estimated b | ρ |
|---|---|---|
| Barrier price=120 | -0.00413149 | -0.00431025 |
| Barrier price=200 | -0.0217959 | -0.0213909 |
| Barrier price=1000 | -0.00413149 | -0.00431025 |

If ρ is close to 1, the variance reduction effect is big. If barrier price is a little bit higher than strike, the effect is not very big. But, significantly higher will yield better result. But if the barrier goes too far away, the variance reduction makes little difference.

| | Estimated b | ρ | Standard error | Price |
|---|---|---|---|---|
| Strike=20 | 0.0119268 | 0.0119258 | 31.3809 | 29.2939 |
| Strike=80 | -0.000630738 | -0.00641176 | 7.66266 | 3.57685 |
| Strike=100 | 0.0108018 | 0.011187 | | |
| Strike=120 | NA | NA | NA | NA |
| Strike=200 | NA | NA | NA | NA |

If ρis closer to 1, the effect is bigger. So, when the option is deep in the money or the strike is close to spot the effect is stronger. Others are smaller. Strike above barrier makes no sense as the option will worth nothing for sure.

For time consumption, the antithetic method greatly reduces the computation time while the control variant method greatly increases the computation time as shown in the graph in part2.