Photometric Stereo

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1 Definitions and Notations

Suppose we have M images that are indexed by i, and each image has N pixels that are indexed by j. The intensity of j-th pixel in i-th image is therefore denoted as $I_{i,j}$.

At j-th pixel, denote $\mathbf{n}_j$ the surface normal, ρ_j the albedo, and $\mathbf{b}_j = \rho_j \mathbf{n}_j$ the scaled normal. Note that these quantities are the same across all images and therefore independent of i.

Each image comes from a different lighting environment, which has a different rendering function $\mathcal{R}_i(\cdot)$, written as

$$I_{i,j} = \rho_j \mathcal{R}_i(\mathbf{n}_j), \tag{1}$$

and we consider following rendering models in this note.

1. Directional lighting (distant point light source)

$$\mathcal{R}_i(\mathbf{n}) = \max\{\boldsymbol{\ell}_i^\top \mathbf{n},\ 0\}. \tag{2}$$

2. Directional lighting plus an ambient component (first order spherical harmonics[1])

$$\mathcal{R}_i(\mathbf{n}) = \max\{\boldsymbol{\ell}_i^\top \mathbf{n} + \alpha_i,\ 0\}. \tag{3}$$

We also define

$$\lambda_i = \|\boldsymbol{\ell}_i\|, \quad \widehat{\boldsymbol{\ell}}_i = \frac{\boldsymbol{\ell}_i}{\lambda_i}, \tag{4}$$

where λ_i is the strength of i-th lighing, and $\widehat{\boldsymbol{\ell}}_i$ is a unit vector for direction of i-th lighting.

The goal of photometric stereo is to recover the scene property $\{\rho_j, \mathbf{n}_j\}$ from the intensity measurements $I_{i,j}$, given (fully or partially) calibrated lighting environment $\mathcal{R}_i(\cdot)$.

2 Fully Calibrated Lighting

2.1 Directional lighting

For notational simplicity, we first only consider pixels that are not in shadow, and drop the $\max\{\cdot,\ 0\}$ part of the rendering function

$$I_{i,j} = \rho_j \boldsymbol{\ell}_i^\top \mathbf{n}_j = \boldsymbol{\ell}_i^\top \mathbf{b}_j. \tag{5}$$

[1] The first order spherical harmonics model sometimes does not have the $\max\{\cdot,\ 0\}$ part.

Define $\mathbf{I}_j \in \mathbb{R}^{M\times 1}$ the (vertical) concatenation of all $I_{i,j}$ for $1 \le i \le M$, and $\boldsymbol{L} \in \mathbb{R}^{3\times M}$ the (horizontal) concatenation of all $\boldsymbol{\ell}_i$. Then we have

$$\mathbf{I}_j = \boldsymbol{L}^\top \mathbf{b}_j, \tag{6}$$

where $\mathbf{I}_j$ are measured intensities, $\boldsymbol{L}^\top$ are calibrated lighting parameters, and $\mathbf{b}_j$ is the unknown scene property, which can be easily solved in the least squares manner

$$\mathbf{b}_j = (\boldsymbol{L}^\top)^\dagger\, \mathbf{I}_j = (\boldsymbol{L}\boldsymbol{L}^\top)^{-1}\, \boldsymbol{L}\, \mathbf{I}_j. \tag{7}$$

And finally, we can recover ρ_j and $\mathbf{n}_j$ as

$$\rho_j = \|\mathbf{b}_j\|, \quad \mathbf{n}_j = \frac{\mathbf{b}_j}{\rho_j}. \tag{8}$$

2.2 Directional lighting with an ambient component

The rendering equation in this model can be written as

$$I_{i,j} = \rho_j \left(\boldsymbol{\ell}_i^\top \mathbf{n}_j + \alpha_i\right) = \boldsymbol{\ell}_i^\top \mathbf{b}_j + \alpha_i \|\mathbf{b}_j\|. \tag{9}$$

Then we can formulate estimation of scene property as the following optimization problem

$$\min_{\mathbf{b}_j} \quad \sum_{i=1}^{M} \left(I_{i,j} - \boldsymbol{\ell}_i^\top \mathbf{b}_j - \alpha_i \|\mathbf{b}_j\|\right)^2. \tag{10}$$

Unfortunately, we do not have a fast (non-iterative) algorithm for solving this problem yet.

3 Partially Calibrated Lighting

In reality, we can usually calibrate part of the lighting property accurately. For example, using a chrome sphere, we are able to calibrate the lighting directions $\widehat{\boldsymbol{\ell}}_i$, but not their strengths λ_i. Therefore, we also need to estimate the unknown lighting parameters λ_i from the measurements $I_{i,j}$.

Since the number of images M is much smaller than the number of pixels N, the number of unknown lighting parameters is also much smaller than number of scene property parameters. We define a cost function on lighting parameters

$$\mathrm{cost}(\{\lambda_i\}) = \min_{\{\rho_j, \mathbf{n}_j\}} \sum_{i=1}^{M} \sum_{j=1}^{N} \left(I_{i,j} - \rho_j \mathcal{R}_i\left(\mathbf{n}_j\right)\right)^2, \tag{11}$$

where the minimization can be solved as a fully-calibrated problem described in previous section. We can then perform a optimization on this cost to find the optimal lighting parameters $\{\lambda_i\}$.