Paper Guide: Bias in apparent dispersion measure due to de-magnification of plasma lensing on background radio sources[1]

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Overview

Two kinds of lensing

- Plasma lensing

  If the intervening object is an inhomogeneous electron density distribution, forming a cloud of cold plasma, the travelling light bundle is distorted due to the interaction between the plasma and the traversing electro-magnetic signal.

- Gravitational lensing

  If an intervening object is an inhomogeneous mass density on top of a background mass density, this inhomogeneity disturbs the homogeneous mass density of the cosmic background spacetime. Consequently, the path of the electro-magnetic signals deviates from their unperturbed path in the background spacetime.

For more information, see [2].

Comparison

Difference between plasma lensing and gravitational lensing.

Gravitational lensing

- Frequency independent
- Projected mass density
- Converge
- Non-dispersive
- Source unobservable
- Magnification

Plasma lensing

- Frequency dependent
- Projected electron density
- Diverge
- Dispersive
- Source observable
- Magnification and **De-magnification**

They are both inhomogeneous and sharing the same mathematical description.

$$\vec{\beta} = \vec{\beta} - \vec{\alpha}(\theta) = \vec{\theta} - \nabla_{\vec{\theta}}\psi(\theta) \tag{1}$$

Where $\vec{\beta}$ and $\vec{\theta}$ are the angular position of the source plane and of the image respectively, $\psi(\theta)$ denotes the effective lens potential, and $\nabla_{\vec{\theta}}$ is the gradient on the image plane. Equation (1) is also called lens equation (for axisymmetric model). Why choose θ as the independent variable, not β ?

Details of lens equation

$$\vec{\beta} = \vec{\theta} - \nabla_{\vec{\theta}}\psi(\theta) \tag{1}$$

The inverse magnification μ^{-1} is the absolute value of the determinant of the Jacobian A, that is $\mu^{-1} = |\det(A)|$.
For axisymmetric model, it is just as simple as the following:

$$\mu^{-1} = \frac{\beta}{\theta}\frac{d\beta}{d\theta} \tag{2}$$

Where $\theta = \sqrt{\theta_x^2 + \theta_y^2}$ and $\beta = \sqrt{\beta_x^2 + \beta_y^2}$.

Details of lens equation

$\vec{\beta}$ and $\vec{\theta}$ are in the vicinity of $\vec{\beta}_0$ and $\vec{\theta}_0$ respectively, which reads:

$$\vec{\beta} - \vec{\beta}_0 = A(\vec{\theta} - \vec{\theta}_0) \tag{3}$$

Apparently, the absolute value of the determinant of transform matrix is the expansion factor of volume element.

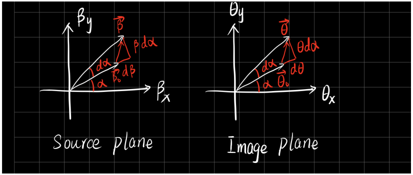
$$\begin{aligned} dV' &= dx'_1 \wedge dx'_2 \wedge \cdots \wedge dx'_n \\ &= (A_{1,j_1} dx_{j_1}) \wedge (A_{2,j_2} dx_{j_1}) \wedge \cdots \wedge (A_{n,j_n} dx_{j_n}) \\ &= \epsilon_{j_1 j_2 \ldots j_n} A_{1,j_1} A_{2,j_2} \ldots A_{n,j_n} dx_1 \wedge dx_2 \wedge \cdots \wedge dx_n \\ &= \mathrm{det}(A) dV \end{aligned} \tag{4}$$

Here, duplicate indexes mean sum.

$$\epsilon_{j_1 j_2 \ldots j_n} = \begin{cases} 1 & j_1 j_2 \ldots j_n \text{ even} \\ -1 & j_1 j_2 \ldots j_n \text{ odd} \\ 0 & \text{If any two number of } j_1 j_2 \ldots j_n \text{ are the same.} \end{cases} \tag{5}$$

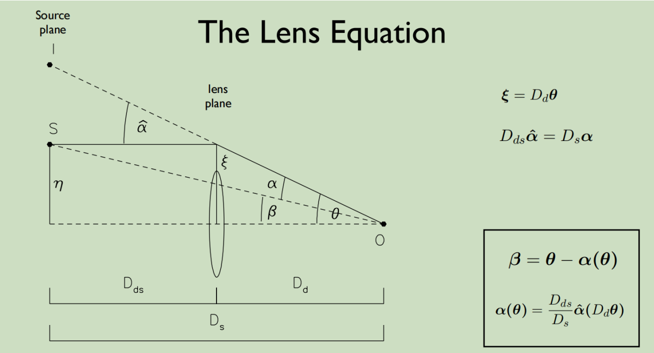
Details of lens equation

However, for axisymmetric model, we do not need to get the μ^{-1} by the labour work of calculating the determinant of Jacobian.



$$\mu^{-1} = \frac{dV_{\text{source}}}{dV_{\text{image}}} = \frac{\beta d\alpha d\beta}{\theta d\alpha d\theta} = \frac{\beta}{\theta}\frac{d\beta}{d\theta} \tag{6}$$

Schematic of lens equation



Caustic

In optics, a caustic or caustic network is the envelope of light rays reflected or refracted by a curved surface or object, or the projection of that envelope of rays on another surface. The caustic is a curve or surface to which each of the light rays is tangent, defining a boundary of an envelope of rays as a curve of concentrated light. Therefore, in the photo to the right, caustics can be seen as patches of light or their bright edges. These shapes often have cusp singularities. More details, see [3].

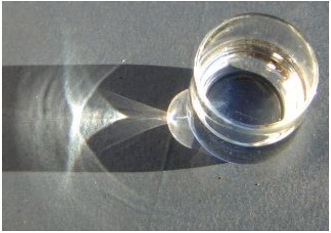


Figure 1: Caustics produced by a glass of water [3]

Outline

- Two density profiles, Gaussian and softened power law (SPL) models.
- The luminosity function of the background sources is modified by plasma lensing.
- Where electron density is high and has a large gradient, there may be no received radio signal (probability up to $\sim 15\%$).
- Several properties that influence the bias.
  - The number of lenses
  - The density model
  - The density parameters

Gaussian model

$$\psi(\theta) \equiv \frac{D_{ds}}{D_d D_s}\frac{\lambda^2}{2\pi} r_e N_\mathrm{e}(\theta) \tag{7}$$

Gaussian model:

$$N_\mathrm{e}(\theta) = N_0 \mathrm{exp}(-\frac{\theta^2}{2\sigma^2}) \qquad (\theta > 0) \tag{8}$$

$$(\frac{dN_\mathrm{e}}{d\theta})_\mathrm{max} = \frac{dN_\mathrm{e}}{d\theta}(\theta = \sigma) = -\frac{N_0}{\sigma}\mathrm{exp}(-0.5) \tag{9}$$

$$\psi(\theta) = \theta_0^2 \mathrm{exp}(-\frac{\theta^2}{2\sigma^2}) \tag{10}$$

$$\mu^{-1}(\theta) = 1 + \frac{2\theta_0^2}{\sigma^2}(1 - \frac{\theta^2}{2\sigma^2})e^{-\frac{\theta^2}{2\sigma^2}} + \frac{\theta_0^4}{\sigma^4}(1 - \frac{\theta^2}{\sigma^2})e^{-\frac{\theta^2}{2\sigma^2}} \tag{11}$$

$$\theta_\mathrm{crit} = \sqrt{e^{3/2}/2}\sigma \tag{12}$$

If $\theta_0 > \theta_\mathrm{cirt}$, more than one one caustic will be generated and this kind of lens is called super critical lens. Since the image suffers extremely de-magnification, the region inside the inner caustic is "forbidden region".

Gaussian model

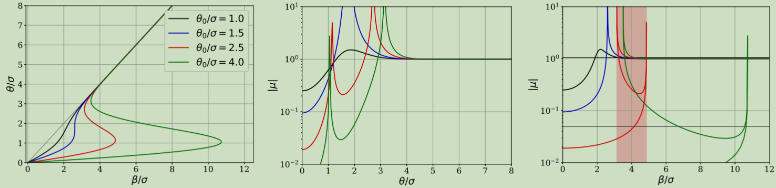


Figure 1. Lensing properties of the Gaussian model:the $\theta - \beta$ relation (left), magnification curve on the image plane (middle), and magnification curve on the source plane (right), respectively. The pink shaded region in the right panel shows the cross section where three images are generated for $\theta_0/\sigma = 2.5$ (red curves). The equivalent parameter ratios for the SPL lens using Eq. 18 are $\theta_0/\theta_c = 1.4, 1.8, 2.5, 3.4$ respectively.

SPL model

SPL model:

$$N_{\mathrm{e}}(\theta)=\frac{n_0R_0^2}{D_d}\frac{1}{\sqrt{\theta^2+\theta_c^2}} \qquad (\theta>0) \tag{13}$$

$$(\frac{dN_{\mathrm{e}}}{d\theta})_{\max}=\frac{dN_{\mathrm{e}}}{d\theta}(\theta=\theta_c/\sqrt{2})=-\frac{2}{3^{\frac{3}{2}}}\frac{n_0R_0^2}{D_d}\theta_c^{-2} \tag{14}$$

$$\psi(\theta)=\frac{\theta_0^3}{\sqrt{\theta^2+\theta_c^2}} \tag{15}$$

$$\mu^{-1}=1+\frac{\theta_0^3\left(2\theta_c^2-\theta^2\right)}{\left(\theta^2+\theta_c^2\right)^{5/2}}+\frac{\left(\theta_c^2-2\theta^2\right)\theta_0^6}{\left(\theta^2+\theta_c^2\right)^4} \tag{16}$$

$$\theta_{\mathrm{crit}}=\theta_c/[2(2/5)^{5/2}]^{1/3}=1.7\theta_c \tag{17}$$

The forbidden region increases with θ_0/θ_c as well. The most important difference is that the forbidden region on the source plane can increase to a large area. The compare this two models, the $N_{\mathrm{e}}(\theta)_{\max}$, $(\frac{dN_{\mathrm{e}}}{d\theta})_{\max}$, and $\psi_{\max}$ are constrained to the same.

SPL model

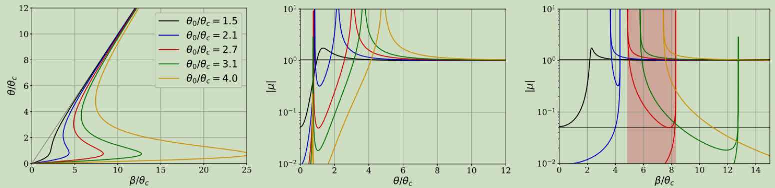


Figure 2. Lensing properties of the softened power-law model: the $\theta - \beta$ relation (left), and the magnifications curves on the image (middle) and source planes (right), respectively. The equivalent parameter ratios for the Gaussian lens using Eq. 18 are $\theta_0/\sigma = 1.2, 1.9, 2.8, 3.5, 5.1$ respectively.

Relation between DM and electron density

From now on, we just adopt the Gaussian model. It has been proposed to use radio pulsing sources to trace the ionised baryons throughout the universe. The relation of the Dispersion Measure (DM) to electron density, however, relies on several assumptions and approximations.

$$\mathrm{DM} = \int_0^L n_e dl \tag{18}$$

Where L is the distance to the source, with n_e in cm^{-3}, L in pc. A significant difference of plasma lensing is that it can cause **strong de-magnification** to the source (even for the brightest primary image), which therefore cannot be observed at all. Thus, the information we extract from observations of radio sources will be in principle incomplete, a distinct conclusion compared to gravitational lensing. Such a "selection effect" can lead to a systematic bias in the estimate of the electron density.

Strong de-magnification $\Longrightarrow$Bias in the estimate of the electron density

$\Longrightarrow$Bias in the estimate of the apparent dispersion measure

The underestimate description

A probability of a background source falling in the exclusion region is defined as

$$\tau = \frac{A_{\rm ex}}{A_{\rm tot}} \tag{19}$$

The true value with the estimated one by the relative ratio

$$r = \equiv (\bar{N}_{\rm e,est} - \bar{N}_{\rm e,true})/\bar{N}_{\rm e,true} \tag{20}$$

$$r = -a\bar{N}_{\rm e} - b \tag{21}$$

Equation (21) is just a linear fit, for more details, see the **Figure 8** of the paper.

Simulations

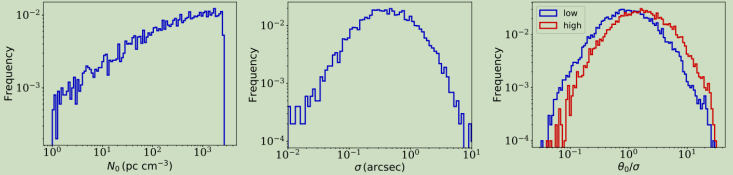


Figure 5. The distribution of lens parameters (N_0, σ, θ_0/σ) that we used in our simulations. The red curve shows the distribution of the high density gradient case.

Simulations

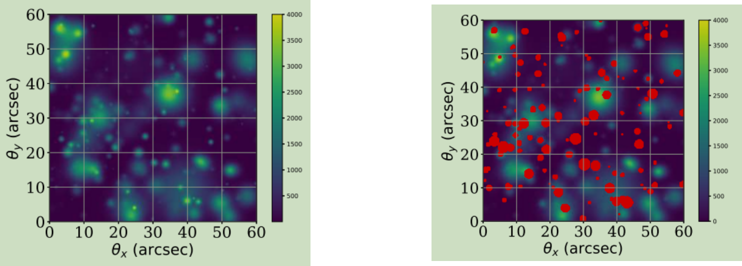


Figure 6. One realisation of maps of the electron density over the field in our test: the left is the initial true map, while the right is the map excluding the de-magnification regions (red). The unit of the colour code is pccm^{-3}. 700 plasma clumps are uniformly distributed on the field. There is 5% probability of a background source falling in the exclusion region.

Bias

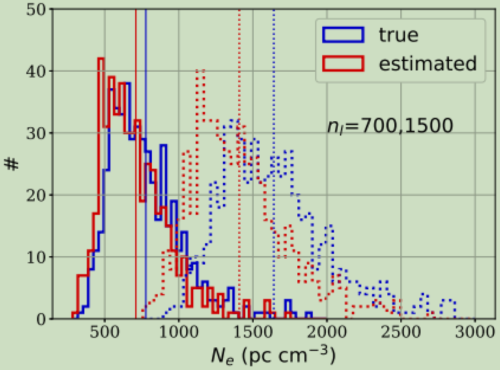


Figure 7. The histogram of the average Dispersion Measure (DM) over the field for 500 realisations: the blue curve presents that over the whole field, the red curve excludes the de-magnification region ($\mu_T < 0.25$). The solid (dotted) lines present that 700 (1500) plasma clumps are placed in the field. The vertical lines indicate the mean electron density $\bar{N}_e$ over the 500 realisations.

Bias

n_l	$\bar{N}_{\text{e, true}}$	$\bar{N}_{\text{e, est}}$	r	τ
100	109.6	105.1	−0.060	0.0079
700	780.1	712.8	−0.086	0.054
1500	1641.1	1408.2	−0.14	0.11
100	114.8	109.2	−0.070	0.097
700	786.5	706.6	−0.11	0.063
1500	1691.0	1409.8	−0.17	0.13

Table 1: Comparison of simulated true electron density ($\bar{N}_{\text{e, true}}$ in unit of pc cm^{-3}) and estimated one $\left(\bar{N}_{\mathrm{e,est}}\right)$ for different number of lens n_l. Gaussian lens model is adopt in all the tests. In the bottom three rows, higher density gradient is used in the simulation.

Luminosity function

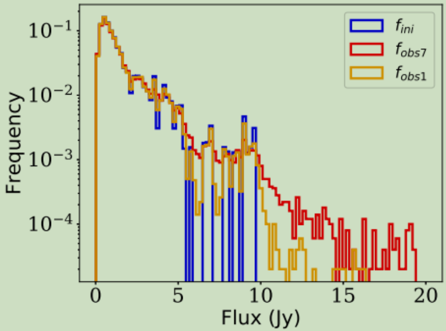


Figure 9. The input luminosity function (blue) and the observed luminosity function (red, orange). To generate this figure, 700 (red) and 100 (orange) plasma lenses are placed in the field.

The observed luminosity will have a **wider** range of minimum and maximum luminosity. One can see that most of the high flux sources are lensed

THANKS

References

[1] Xinzhong Er, Jiangchuan Yu, Adam Rogers, Shihang Liu, and Shude Mao. Bias in apparent dispersion measure due to de-magnification of plasma lensing on background radio sources. 8(November):1–8, 2021.

[2] Jenny Wagner and Xinzhong Er. Plasma lensing in comparison to gravitational lensing – Formalism and degeneracies. (2012):1–16, 2020.

[3] Wikipedia contributors. Caustic (optics) — Wikipedia, the free encyclopedia, 2021. [Online; accessed 30-November-2021].