1 Sum of multiples of 5 or 7

If we list all the natural numbers less than 20 that are multiples of 5 or 7, we get 5, 7, 10, 14 and 15. The sum of these numbers is 51.

Find the sum of all multiples of 5 or 7 less than 2013.

(Inspired by problem 1 of Project Euler)

2 Sum of odd-valued terms in Fibonacci sequence

Each new term in the Fibonacci sequence is generated by adding the two preceding terms. Starting with 1 and 1, the first 10 terms are as follows:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$$

By considering the terms in the Fibonacci sequence whose values do not exceed 4 million, find the sum of the odd-valued terms.

(Inspired by problem 2 of Project Euler)

3 | Largest prime factor of composite number

The prime factors of the number 2013 are 3, 11 and 61, because $3 \times 11 \times 61 = 2013$.

What is the largest prime factor of the number 20130101?

(Inspired by problem 3 of Project Euler)

4 Largest palindrome of multiplying two numbers

A palindromic number reads the same from left to right and from right to left.
The largest palindrome that can be obtained by multiplying two 2-digit numbers is $9009 = 99 \times 91$.

What is the largest palindrome that can be obtained by multiplying a 4-digit number by a 3-digit number?

(Inspired by problem 4 of Project Euler)

5 Sum of digits of 2^{2222}

$2^{15} = 32768$ and the sum of its digits is $3 + 2 + 7 + 6 + 8 = 26$.

What is the sum of the digits that make up the number 2^{2222}?

(Inspired by problem 16 of Project Euler)

6 Sum of digits in number 2013!

$n!$ means $n \times (n-1) \times \cdots \times 3 \times 2 \times 1$.
For example, $10! = 10 \times 9 \times \cdots \times 3 \times 2 \times 1 = 3628800$.
The sum of the digits in the number $10!$ is $3 + 6 + 2 + 8 + 8 + 0 + 0 = 27$.

Find the sum of the digits in the number 2013!.

(Inspired by problem 20 of Project Euler)

7 What is the 23456^{th} prime number?

By listing the first six prime numbers: 2, 3, 5, 7, 11 and 13, we see that the 6^{th} prime number is 13.

What is the 23456^{th} prime number?

(Inspired by problem 7 of Project Euler)

8 | Rectangle-6400

In a rectangle with a length of 4 and a width of 3, we can draw 12 squares with a side length of 1, 6 squares with a side length of 2, and 2 squares with a side length of 3. In total, we can draw 20 squares. We will call this a rectangle-20.

What is the area of the rectangle-6400 whose shape is closest to a square?

Note: The dimensions are whole numbers for both the rectangle and the squares.

(Inspired by the 2012 CMP problem from the mathematics and physics circle of the Société jurassienne d'émulation)

9 Pythagorean triplet

A triplet of non-zero natural numbers (a, b, c) is Pythagorean if $a^2 + b^2 = c^2$. For example, $(3, 4, 5)$ is a Pythagorean triplet.

Among the Pythagorean triples (a, b, c) such that $a + b + c = 3600$, give the largest product $a \times b \times c$.

(Inspired by problem 9 of Project Euler)

10 Sum of prime numbers

The sum of the prime numbers between 1 and 10 is $2 + 3 + 5 + 7 = 17$.

Find the sum of the prime numbers between 1 and 10 000 000.

(Inspired by problem 10 of Project Euler)

11 Mirror of number

The “mirror image of a number n” is the number n written from right to left. For example, $\mathrm{mirror}(7423) = 3247$.

What is the largest number less than 10 million that has the property: $\mathrm{mirror}(n) = 4 \times n$?

(Proposed by Le Coyote)

12 | Triangular numbers with many divisors

The sequence of triangular numbers is generated by adding the natural numbers. Thus, the 7th triangular number is $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$. The first ten triangular numbers are as follows:

$$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \ldots$$

Let us list the divisors of the first seven triangular numbers:

$1 : 1$
$3 : 1, 3$
$6 : 1, 2, 3, 6$
$10 : 1, 2, 5, 10$
$15 : 1, 3, 5, 15$
$21 : 1, 3, 7, 21$
$28 : 1, 2, 4, 7, 14, 28$

We can see that 28 is the first triangular number with more than five divisors.

What is the value of the first triangular number with more than one thousand divisors?

(Inspired by problem 12 of Project Euler)

13 Palindrome square

The smallest palindromic square with an even number of digits is $698896 = 836^2$.

What is the next palindromic square?

(Source: The Mathematical Gazette)

14 | Longest Syracuse sequence

In mathematics, a “Syracuse sequence” is a sequence of natural numbers defined as follows:

We start with an integer greater than zero; if it is even, we divide it by 2; if it is odd, we multiply it by 3 and add 1.
By repeating the operation, we obtain a sequence of positive integers, each of which depends only on its predecessor.

There is a conjecture that says that the Syracuse sequence of any strictly positive integer reaches 1.

For example, starting from 14, we construct the sequence of numbers: 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. This is called the Syracuse sequence of the number 14. Here, it has a length of 18.

For what starting number less than 1 500 000 do we obtain the longest Syracuse sequence?
There are two solutions, give the smallest one.

(Inspired by problem 14 of Project Euler)

15 Numbers of Mage Hic

Fill in a 3×3 grid with all the numbers from 1 to 9 (green squares). Then calculate the products of the three numbers in each row and each column (grey squares). Finally, add these six products together to obtain the blue number (450 in the example given).

2	9	4	72
7	5	3	105
6	1	8	48
84	45	96	450

Of the 362 880 possible grids, what is the minimum blue number and the maximum blue number? Give the product of these two numbers as the result.

(Inspired by problem 12 from the 2013 FFJM individual quarterfinals: "Le nombre du Mage Hic")

16 | THREE x NINE = TROIS x NEUF

A cryptarithm is a numerical and logical puzzle consisting of a mathematical equation in which letters represent numbers to be found. Each different letter represents a different number, and each number is represented by the same letter.

Solve the cryptarithm below (give the product obtained as the answer):

```
THREE x NINE = TROIS x NEUF
```

Constraints:
`THREE` and `NEUF` are multiples of 9;
`TROIS` and `NINE` are multiples of 3.

(Problem 18 from the quarterfinals of the 2003 mathematical and logical games)

17 Amicable numbers

Let $d(n)$ be the sum of the proper divisors of n (divisors strictly smaller than n). If $d(a) = b$ and $d(b) = a$, with a different from b, then a and b are said to be amicable.
For example, the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110; therefore $d(220) = 284$. The proper divisors of 284 are 1, 2, 4, 71 and 142; therefore $d(284) = 220$. 220 and 284 are two amicable numbers.
The sum of the amicable numbers between 1 and 1000 is $220 + 284 = 504$.

What is the sum of the amicable numbers between 1 and 100 000?

(Inspired by problem 21 of Project Euler)

18 Sum of non-abundant numbers

A perfect number is a number whose sum of proper divisors is exactly equal to the number itself. For example, the sum of the proper divisors of 28 would be $1 + 2 + 4 + 7 + 14 = 28$, which means that 28 is a perfect number.

A number n is called deficient if the sum of its proper divisors is less than n, and it is called abundant if this sum is greater than n.

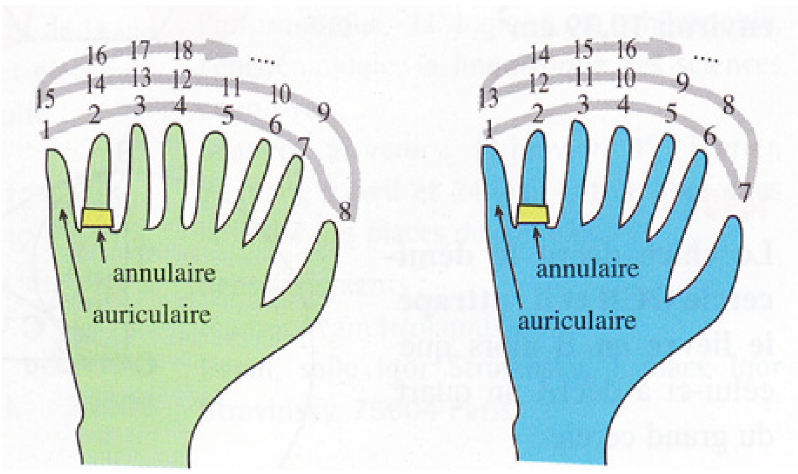
Since 12 is the smallest abundant number $(1 + 2 + 3 + 4 + 6 = 16)$, the smallest number that can be written as the sum of two abundant numbers is 24.

Find the sum of all positive integers less than or equal to 2013 that <u>cannot</u> be written as the sum of two abundant numbers.

(Inspired by problem 23 of Project Euler)

19 Close encounters of the fourth kind

Little green men meet little blue men. To their great surprise, they notice that their hands do not have the same number of fingers: 7 for the blue ones and 8 for the green ones. But the scientists of both peoples notice that if you count on your fingers as shown in the figure, moving back and forth from the little finger to the thumb, then from the thumb to the little finger, certain numbers can be counted on both the ring finger of the blue hands and the ring finger of the green hands (2 and 14, for example).
These numbers have been termed “annular” by scholars.



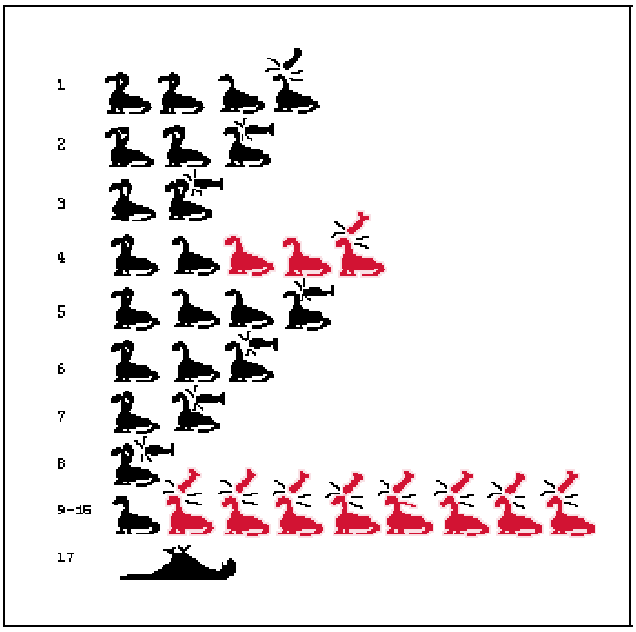
Calculate the sum of the ring numbers between 1 and 2013.

(Inspired by a problem from the Mathematical and Logical Games Championship)

20 Hercules and the hydra

A battle between Hercules and one (or more) hydra(s) can be described by the following steps:

- On the first blow, the Hydra loses a head, and a copy of the wounded Hydra (that is, one with one less head) appears. On the second blow, a Hydra loses a head, and two copies of the wounded Hydra appear. On the tenth blow, a Hydra loses a head, so ten copies appear, and so on.
- When a one-headed Hydra is struck, it disappears.
- To complicate matters, the treacherous Hera forces Hercules to cut off the heads in the worst possible order, so that it takes him as long as possible to destroy the Hydras.



Example: Hercules fights two two-headed hydras and two one-headed hydras.

Assuming Hercules is capable of cutting off a head every second, how many seconds will it take him to defeat a four-headed hydra?

(Jean-Bernard Roux's problem)

21 Happy new year 2013!

2013 has an interesting feature: it is the first year since 1987 to be composed of all different digits. A period of 26 years separates these two dates.

Between year 1 and 2013 (inclusive):

a) How many years have there been with all different digits? (Years 1 through 9 will be counted in this number).
b) What was the length (in years) of the longest period between two dates with all different digits?

Give the product of the results of a) and b).

(Proposed by Le Coyote)

22 | Eightfold anagrams

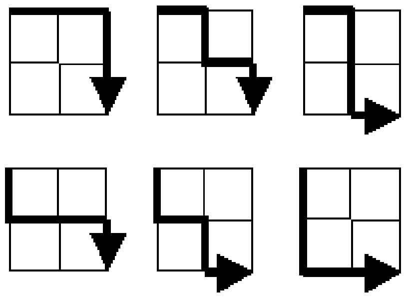
Mathilde found two surprising six-digit numbers. When multiplied by 8, they produce a six-digit number written with the same digits in a different order. Even better: these two numbers are also anagrams!

What are Mathilde's numbers? Give the sum of the two numbers.

(Inspired by problem 14 from the quarterfinals of the 2010 Mathematical and Logical Games)

23 | Paths in a grid

Starting from the top left corner of a 2×2 grid, there are 6 paths to the bottom right corner, moving only to the right or down.



How many such paths are there in a 30×20 grid?

(Inspired by problem 15 of Project Euler)

24 Lexicographic permutations

A permutation is an ordered arrangement of objects. For example, 3124 is a possible permutation of the digits 1, 2, 3 and 4.
The list of permutations in lexicographical order of 0, 1 and 2 is as follows:

$$012, 021, 102, 120, 201, 210$$

In lexicographical order, what is the two-millionth permutation of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9?

(Inspired by problem 24 of Project Euler)

25 First Fibonacci number with 2013 digits

The Fibonacci sequence is defined by the recurrence relation:

$$F_n = F_{n-1} + F_{n-2}, \text{where } F_1 = 1 \text{ and } F_2 = 1.$$

Thus, the first 12 terms are: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...
The 12th term, F_{12}, is the first term that contains 3 digits.

What is the index of the first term in the Fibonacci sequence that contains 2013 digits?

(Inspired by problem 25 of Project Euler)

26 Number divisible by pieces

Find an integer $c_1 c_2 c_3 \cdots c_9$ composed of all digits from 1 to 9, such that $c_1 \cdots c_k$ is divisible by k, for $k = 1, \ldots, 9$.

Let's take a 3-digit example: 321.

3 is divisible by 1.
32 is divisible by 2.
321 is divisible by 3.

(Source unknown)

27 | Euler's polynomial for prime numbers

Euler published the remarkable quadratic polynomial:

$$n^2 + n + 41$$

It turns out that this formula provides 40 prime numbers for successive values of n ranging from 0 to 39. However, when $n = 40$, $40^2 + 40 + 41 = 40(40 + 1) + 41$ is divisible by 41.

Using a computer, the incredible formula $n^2 - 79n + 1601$ was discovered. It provides 80 prime numbers for successive values of n ranging from 0 to 79 (each prime number appears twice):

1601, 1523, 1447, 1373, 1301, 1231, 1163, 1097, 1033, 971, 911, 853, 797, 743, 691, 641, 593, 547, 503, 461, 421, 383, 347, 313, 281, 251, 223, 197, 173, 151, 131, 113, 97, 83, 71, 61, 53, 47, 43, 41, 41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151, 173, 197, 223, 251, 281, 313, 347, 383, 421, 461, 503, 547, 593, 641, 691, 743, 797, 853, 911, 971, 1033, 1097, 1163, 1231, 1301, 1373, 1447, 1523, 1601.

The product of the coefficients, -79 and 1601, gives -126479.

Consider quadratic polynomials of the form:
$n^2 + an + b$, where $|a| < 1500$ and $|b| < 1500$
where $|n|$ is the absolute value of n (for example, $|11| = 11$ and $|-4| = 4$).

Give the product of the coefficients a and b of the quadratic polynomial that generates the longest sequence of prime numbers for successive values of n, starting from $n = 0$.

(Inspired by problem 27 of Project Euler)

28 | Spiral numbers

Starting with the number 1, and turning clockwise, a 5×5 spiral is constructed as follows:

21	22	23	24	**25**
20	**7**	8	**9**	10
19	6	**1**	2	11
18	**5**	4	**3**	12
17	16	15	14	**13**

We can verify that the sum of the numbers on the diagonals is 101.

What is the sum of the numbers on the diagonals of a 2013×2013 spiral constructed in the same way?

(Inspired by problem 28 of Project Euler)

29 Distinct powers

Consider a^b for $2 \le a \le 5$ and $2 \le b \le 5$:

$$2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32$$
$$3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243$$
$$4^2 = 16, 4^3 = 64, 4^4 = 256, 4^5 = 1024$$
$$5^2 = 25, 5^3 = 125, 5^4 = 625, 5^5 = 3125$$

If we sort these numbers in ascending order, removing any duplicates, we obtain a sequence of 15 distinct terms:

$$4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125$$

How many distinct terms are there in the sequence obtained as above for $2 \le a \le 1000$ and $2 \le b \le 1000$?

(Inspired by problem 29 of Project Euler)

30 Sum of fifth powers

Surprisingly, there are only three numbers that can be written as the sum of the fourth powers of their digits:

$$1634 = 1^4 + 6^4 + 3^4 + 4^4$$
$$8208 = 8^4 + 2^4 + 0^4 + 8^4$$
$$9474 = 9^4 + 4^4 + 7^4 + 4^4$$

We do not count $1 = 1^4$ because it is not a sum.

The sum of these numbers is $1634 + 8208 + 9474 = 19316$.

Find the sum of all numbers that can be written as the sum of the fifth powers of their digits.

(Inspired by problem 30 of Project Euler)

31 Swiss coins

In Switzerland, there are seven coins: 5 cents, 10 cents, 20 cents, 50 cents, 1 franc, 2 francs, and 5 francs.

You can make 10 francs like this: 1×5 francs $+ 2 \times 2$ francs $+ 10 \times 10$ cents.

How many ways can you make 10 francs using any number of Swiss coins?

(Inspired by problem 31 of Project Euler)

32 Pandigital identities

The product 7254 is interesting because the identity $39 \times 186 = 7254$, composed in the order of multiplicand, multiplier, and product, uses exactly once all the digits from 1 to 9.

Find the sum of all products having this property.

Warning! The same product can be obtained in several ways. It should only be counted once in the total.

(Inspired by problem 32 of Project Euler)

33 Easter in April

During the years 2001 to 9999 (inclusive), how many times will the date of Easter fall in April, in the Gregorian calendar?

(Proposed by Le Coyote)

34 | $1! + 4! + 5! = 145$

145 is a curious number. Indeed, $1! + 4! + 5! = 1 + 24 + 120 = 145$.

Find the product of all numbers that are equal to the sum of the factorial of their digits.

Notes:
1! = 1 and 2! = 2 are not sums and will not be included in the product.
Remember that 0! = 1.

(Inspired by problem 34 of Project Euler)

35 Circular prime numbers

The prime number 197 is called “circular” because all circular permutations of its digits: 197, 971 and 719 are also prime numbers.

There are 13 circular prime numbers less than 100: 2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79, and 97.

How many circular prime numbers are there less than 100 000?

(Inspired by problem 35 of Project Euler)

36 Palindromic numbers in two bases

The decimal number 585 (1001001001 in binary) is a palindrome in both bases.

Find the sum of all numbers less than 10 million that are palindromes in base 10 and base 2.

Note: Whether in base 10 or base 2, the numbers must not start with a 0.

(Inspired by problem 36 of Project Euler)

37 | Truncable prime numbers

The number 3797 has an interesting property. It is prime, and it remains prime when we eliminate its digits one by one from left to right: 3797, 797, 97 and 7 are all prime numbers. The same is true when eliminating the digits from right to left: 3797, 379, 37, and 3 are also all prime numbers.

Find the sum of the eleven prime numbers that can be truncated from left to right and from right to left.

Note: 2, 3, 5 and 7 are not considered truncable prime numbers.

(Inspired by problem 37 of Project Euler)

38 | Conway sequence

The Conway sequence is a mathematical sequence invented in 1986 by mathematician John Horton Conway, initially under the name “audioactive sequence”. In this sequence, a term is determined by announcing the digits that form the previous term.

$T_1 = 1$
$T_2 = 11$
$T_3 = 21$
$T_4 = 1211$
$T_5 = 111221$
$T_6 = 312211$
$T_7 = 13112221$
$T_8 = 1113213211$
$T_9 = 31131211131221$
…

If $T_1 = 2$, how many 1s will there be in T_{50}?

(Proposed by Le Coyote)

39 Right triangles with given perimeter

If p is the perimeter of a right triangle with integer side lengths, there are exactly three solutions for $p = 120$: $\{20, 48, 52\}$, $\{24, 45, 51\}$, $\{30, 40, 50\}$.

For what value of $p < 10\,000$ is the number of solutions maximum?

(Inspired by problem 39 of Project Euler)

40 Champernowne's constant

The Champernowne constant is an irrational real number created by concatenating positive integers:

$$0.12345678910\mathbf{1}112131415161718192021\cdots$$

We can see that the 12th digit of the fractional part is 1.

If d_n represents the nth digit of the fractional part, what is the value of the following expression:

$$\boldsymbol{d_1 \times d_{10} \times d_{100} \times d_{1000} \times d_{10000} \times d_{100000} \times d_{1000000} \times d_{10000000} \times d_{100000000}}$$

(Inspired by problem 40 of Project Euler)

41 Pandigital prime

We say that a number with n digits is pandigital if it contains all digits from 1 to n exactly once. For example, 2143 is pandigital and is also prime.

What is the largest pandigital prime number?

(Inspired by problem 41 of Project Euler)

42 | UN x UN + UN = DEUX

A cryptarithm is a numerical and logical puzzle consisting of a mathematical equation where letters represent numbers to be found.

Solve the cryptarithm below (give the answer as the number `DEUX`):

```
UN x UN + UN = DEUX
```

(Proposed by Le Coyote)

43 | Palindrome squares

A palindromic number reads the same backward as forward. A single-digit number is a palindrome.

Give the sum of the numbers whose square is a palindrome of at most 13 digits.

(Proposed by Le Coyote)

44 Pentagon numbers

Pentagonal numbers follow the formula $P_n = n(3n-1)/2$. The first ten pentagonal numbers are:

$$1, 5, 12, 22, 35, 51, 70, 92, 117, 145, \ldots$$

We can see that $P_4 + P_7 = 22 + 70 = 92 = P_8$. However, their difference, $70 - 22 = 48$, is not pentagonal.

Find the two pentagonal numbers P_j and P_k, whose sum and difference are pentagonal, and whose $D = |P_k - P_j|$ is minimized. Give the value of D.

(Inspired by problem 44 of Project Euler)

45 Triangular, pentagonal, and hexagonal

Triangular, pentagonal, and hexagonal numbers follow these formulas:

Triangular $T_n = n(n+1)/2 : 1, 3, 6, 10, 15, \dots$
Pentagonal $P_n = n(3n-1)/2 : 1, 5, 12, 22, 35, \dots$
Hexagonal $H_n = n(2n-1) : 1, 6, 15, 28, 45, \dots$

We can verify that $T_{285} = P_{165} = H_{143} = 40755$.

Find the next triangular number that is also pentagonal and hexagonal.

(Inspired by problem 45 of Project Euler)

46 Another conjecture of Goldbach

Christian Goldbach conjectured that every odd composite number could be written as the sum of a prime number and twice a square number.

$$9 = 7 + 2 \times 1^2$$
$$15 = 7 + 2 \times 2^2$$
$$21 = 3 + 2 \times 3^2$$
$$25 = 7 + 2 \times 3^2$$
$$27 = 19 + 2 \times 2^2$$
$$33 = 31 + 2 \times 1^2$$

It turns out that this conjecture is false.

What is the smallest odd composite number that contradicts this conjecture?

(Inspired by problem 46 of Project Euler)

47 Distinct prime factors

The first two consecutive numbers with two distinct prime factors are:

$$14 = 2 \times 7$$
$$15 = 3 \times 5$$

The first three consecutive numbers with three distinct prime factors are:

$$644 = 2^2 \times 7 \times 23$$
$$645 = 3 \times 5 \times 43$$
$$646 = 2 \times 17 \times 19.$$

Find the first four consecutive numbers with four distinct prime factors. What is the smallest of these numbers?

(Inspired by problem 47 of Project Euler)

48 | $1^1 + 2^2 + 3^3 + \cdots$

$1^1 + 2^2 + 3^3 + \cdots + 10^{10} = 10405071317$.

Give the first 10 digits of the series $1^1 + 2^2 + 3^3 + \cdots + 2013^{2013}$.

(Inspired by problem 48 of Project Euler)

49 A very special arithmetic progression…

The arithmetic progression 1487, 4817, 8147, with a common difference of 3330, is special for two reasons:

1. all 3 terms are prime;
2. all 3 terms are composed of the same digits

There is only one other arithmetic progression of 4-digit numbers with the same properties. Which one?

Give the first term multiplied by the common difference as your answer.

(Inspired by problem 49 of Project Euler)

50 | Hamming sequence

Consider the integers whose only prime divisors are 2, 3, and 5. Arrange them in ascending order. This is the Hamming sequence:

$$2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 27, 30, 32, 36, 40, 45, 48, 50, \ldots$$

What is the 2013th term in this sequence?

(Inspired by puzzle 5 from "Jeux et casse-tête à programmer" by J. Arsac)

51 Swiss Cryptarithm

A cryptarithm is a numerical and logical puzzle consisting of a mathematical equation in which letters represent numbers to be found.

Solve the cryptarithm below (give the sum obtained as the answer):

```
  ARGAND
+ EULER
-------
  GULDIN
```

Additional information:

- the number 2 does not appear;
- the number 0 appears.

(Proposed by Marie-Pierre Falissard)

52 Multiples contain the same digits

We can see that the number 125874 and its double 251748 contain the same digits, but in a different order.

Find the smallest positive integer n such that n, $2n$, $3n$, $4n$, $5n$ and $6n$ contain the same digits.

(Inspired by problem 52 of Project Euler)

53 | Grid of sums

Consider a square grid with 3 rows and 3 columns. We want to fill this grid with whole numbers. To start with, we place the number 20 in one cell and the number 13 in another. The other cells are filled in one after the other, in an order to be defined. The number placed in a square must be the sum of the numbers in the surrounding squares (i.e., the squares that touch the square to be filled on one side or corner). The largest number placed is called the grid's filling number.

Where should the two starting numbers be placed, and in what order should the grid boxes be filled to obtain the largest possible filling number?

What is the largest filling number?

(Les défis mathématiques du Monde, épisode 3)

54 | Many 2s

Consider the multiplication:

```
_ 2 _ _ _ x _ _ = 2222 _ _
```

In this operation, we have written both 2s (there are no others).

Find the largest possible multiplicand.

(Lewis Carroll Trophy 2013 for high school students)

55 | Lychrel numbers

If we take 47, reverse it, and add the two numbers together ($47 + 74 = 121$), we get a palindrome, i.e., a natural number that is equal to itself when read from left to right or right to left. The same applies to 20: $20 + 02 = 22$.
Not all numbers produce palindromes so quickly. For example, it takes three iterations starting from 349: $349 + 943 = 1292$, $1292 + 2921 = 4213$, $4213 + 3124 = 7337$.
There are also numbers, such as 196, that will never produce a palindrome using this process. These numbers are called “Lychrel numbers”.

Given that 10 677 is the first number requiring more than 50 iterations to produce a palindrome, how many Lychrel numbers are there below 10 000?

(Inspired by problem 55 of Project Euler)

56 Maximum sum of digits of a power

A gogol (10^{100}) is an enormous number: 1 followed by 100 zeros. However, the sum of its digits is only 1.

Considering natural numbers of the form ab, where $a < 250$ and $b < 250$, what is the maximum sum of digits that can be obtained?

(Inspired by problem 56 of Project Euler)

57 Continued fraction

The square root of two can be represented by a continued fraction:

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}}$$

The first four iterations give:

$1 + 1/2 = 3/2 = 1.5$
$1 + 1/(2 + 1/2) = 7/5 = 1.4$
$1 + 1/(2 + 1/(2 + 1/2)) = 17/12 = 1.41666\cdots$
$1 + 1/(2 + 1/(2 + 1/(2 + 1/2))) = 41/29 = 1.41379\cdots$

The next three fractions are $99/70$, $239/169$, and $577/408$. The eighth fraction, 1393/985, is the first one where the numerator has more digits than the denominator.

Among the first 10 000 fractions, how many have a numerator with more digits than the denominator?

(Inspired by problem 57 of Project Euler)

58 Prime numbers on a spiral

Starting from 1 and forming a spiral by turning counterclockwise, we obtain a square with a width of 7:

```
37 36 35 34 33 32 31
38 17 16 15 14 13 30
39 18  5  4  3 12 29
40 19  6  1  2 11 28
41 20  7  8  9 10 27
42 21 22 23 24 25 26
43 44 45 46 47 48 49
```

It is interesting to note that the odd squares are located on the diagonal line starting from 1 towards the lower right corner. It is even more interesting to note that 8 of the 13 numbers on the diagonals are prime numbers, representing approximately 62% of the prime numbers.
If we add another layer to the spiral, we will obtain a square with a width of 9.
Continuing this process, for what width of the spiral will there be less than 13% prime numbers on the diagonals for the first time?

(Inspired by problem 58 of Project Euler)

59 Untouchable numbers

An untouchable number is a natural number that cannot be expressed as the sum of the proper divisors of an integer (divisors other than the integer itself).

For example, 9 is not untouchable, because 15 has proper divisors 1, 3, 5, and $1 + 3 + 5 = 9$.

On the other hand, 52 is untouchable because no integer has 52 as the sum of its proper divisors.

The first untouchable numbers are: 2, 5, 52, 88, 96, 120, 124, 146, 162, 188, ...

If k is not untouchable, let $p(k)$ be the smallest integer whose sum of proper divisors is k. **What is the maximum value of $p(k)$ for k less than 666?**

(Proposed by Nicolas Quinodoz)

60 | Mass suicide

The 2013 members of a cult decided to commit suicide. To perform the funeral ritual, they formed a circle and numbered themselves from 1 to 2013.
They began counting, starting with number 1. Every 7th person would have to die. Thus, the first to die will be number 7, the second number 14, the third number 21, and so on.
You are a member of this cult, but you have no desire to die! Your task is to find the position in the circle that will allow you to be designated last, and thus escape death.

Which position will save you?

(Proposed by "problème de Josèphe")

61 No to isolation!

A digit is isolated if its left and right neighbors are different from itself. For example, in 776444, 6 is isolated, but the other digits are not.

On the other hand, the first three numbers that are not multiples of 10 whose squares contain no isolated digits are:

$88^2 = 7744$
$74162^2 = 5500002244$
$105462^2 = 11122233444$

What is the fourth?

(Proposed by Le Coyote)

62 Cubic permutations

The digits of the cube 41063625 (345^3) can be permuted to produce two other cubes: 56623104 (384^3) and 66430125 (405^3). In fact, 41063625 is the smallest cube with this property.

Find the smallest cube for which exactly four permutations of its digits are cubes.

Note: the cube itself + 3 permutations of its digits.

(Inspired by problem 62 of Project Euler)

63 | x^n gives an n-digit number

The 5-digit number $16807 = 7^5$ is also a number raised to the power of 5. Similarly, the 9-digit number $134217728 = 8^9$ is a number raised to the power of 9.

How many positive integers with n digits are also numbers raised to the power of n?

(Inspired by problem 63 of Project Euler)

64 Multiplicative persistence

Consider a positive integer, for example, 377. Multiply its digits: $3 \times 7 \times 7 = 147$. Do the same with the result 147: $1 \times 4 \times 7 = 28$. Repeat: $2 \times 8 = 16$. Again: $1 \times 6 = 6$. Once we reach a single-digit number, we stop.

377, 147, 28, 16, 6 is the "multiplicative sequence" of 377, and the "multiplicative persistence" p of 377 is the number of times the digits had to be multiplied before arriving at a single-digit number; here, $p = 4$.

We conjecture that p cannot exceed 11...

What is the smallest integer less than 1 000 000 with the largest multiplicative persistence p?

(Proposed by Le Coyote)

65 | The numbers of the year 2014

If we add the product of its four digits to the number 2014: $2014 + 2 \times 0 \times 1 \times 4$, we get 2014.

Find the sum of the other positive integers that add up to 2014 when you add the product of their digits.

(Individual quarterfinals of the 28th Mathematical and Logic Games Championship)

66 Period in the decimal expansion

A unit fraction is a rational number written as a fraction where the numerator is 1 and the denominator is a positive integer.

$1/2 = 0.5$
$1/3 = 0.(3)$
$1/4 = 0.25$
$1/5 = 0.2$
$1/6 = 0.1(6)$
$1/7 = 0.(142857)$
$1/8 = 0.125$
$1/9 = 0.(1)$
$1/10 = 0.1$

$0.1(6)$ means $0.166666\cdots$ $1/6$ has a period of 1 digit in its decimal expansion. We can see that $1/7$ has a period of 6 digits, while $1/2$ has no period.

Find the value of $d < 5000$, where $1/d$ has the longest period in its decimal expansion.

(Inspired by problem 26 of Project Euler)

67 Stern's diatomic sequence

Stern's diatomic sequence is the result of the following small equations:

$s_0 = 0$
$s_1 = 1$
$s_{2n} = s_n$
$s_{2n+1} = s_n + s_{n+1}$

What is the value of $s_{10\,000\,001}$?

(Proposed by Le Coyote)

68 | SIX^2 = TROIS

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