Problem #1

Minimum Value

Let there be a set of N natural numbers 1,2,3,4...N. We are allowed to insert + or − sign in front of each number and add all the resultant numbers. The minimum non-negative value obtained is denoted as $D(N)$.

Find the value of $D(1)+D(2)+...+D(19216812112)$

number theory

Problem #2

Equation

Let x and y be positive real numbers.

Using the equation $y^6=x^6+8x^4-6x^2+8$, find the integral value of $3345x+4321y^2$.

Your answer should not exceed 10^7.

number theory polynomial

Problem #3

Largest Number

Find the positive integer n less than 100000 for which the equation $x^2=ny^2+1$ has the largest fundamental value of x. The fundamental value of (x,y) is the smallest positive non-trivial integral solution to the equation. (1,0) is the trivial solution.

Note: You can ignore those values of n for which there are no fundamental solution to the given equation.

number theory

Problem #4

Palindromes

Find the sum of numbers less than or equal to 1000000000 which are palindromes in binary, octal as well as hexadecimal base.

number theory palindrome

Problem #5

3 Musketeers

There are 3 people A, B and C. Three of them have positive integers written on their hats. One can only see the numbers written on others hats and can not see the number written on his own hat. The number on one of the hats is the sum of the numbers on the other 2 hats. Now the following event occurs

- A was asked about the number on his hat. He replies "Don't know".
- B was asked about the number on his hat. He also replies "Don't know".
- C was asked about the number on his hat. He also replies "Don't know".
- A was asked again the number on his hat. He replies "65".

Find the product of the three numbers on the hats.

number theory implementation

Problem #6

Sum It Up

Let A be the sum of digits of 3334^{3334} when written in decimal notation and B be the sum of digits of A.

What is the value of 2013 times sum of digits of B?

number theory modular

Problem #7

Exponentiation

$$f(x,n) = x^{2^1} + x^{2^2} + x^{2^3} + \ldots + x^{2^n}$$

You are given that *f*(2,10) mod 1000000007 = 180974681

Calculate $\sum_{x=2}^{10^7} f(x, 10^{18}) \bmod 1000000007$

number theory modular

Problem #8

Summation of Primes

Find the sum of digits of all prime numbers below 10^9.

number theory primes

Problem #9

Highly Divisible Number

What is the smallest number to have over 300 divisors?

number theory divisors

Problem #10

Number Spiral Diagonals

Starting with the number 1 and moving to the right in a clockwise direction a 5 x 5 spiral is formed as follows:

```
21 22 23 24 25
20 07 08 09 10
19 06 01 02 11
18 05 04 03 12
17 16 15 14 13
```

It can be verified that the sum of the numbers on the diagonals is 101.

What is the sum of numbers on the diagonals in a 10001 x 10001 spiral formed in the same way?

path traversal matrices

Problem #11

Factorial

Find 1000000008!

Give the answer modulo 1000000009.

number theory modular primes

Problem #12

Prime Gaps

A prime gap is the difference between two successive prime numbers. The n^{th} prime gap is the difference between the $(n+1)^{th}$ and the nth prime numbers.

Using **12.txt**, find the largest prime gap for all the primes in the file.

number theory primes

Problem #13

Pandigital Primes

A number is said to be pandigital if it contains each of the digits from 0 to 9 (and whose leading digit must be nonzero). Find the smallest pandigital prime number.

number theory | primes | pandigital

Problem #14

Count Fibonacci

Find the number of Fibonacci numbers in the file **14.txt**.

number theory | fibonacci

Problem #15

Blind Fold

In a game, played between 2 players there is a circular field and one of the players is blindfolded, who stands in the center of the field. The other player stands at a fixed point on the circumference of the circular field. On the word **GO**, the blind-folded player starts running towards the edge of the field while the second player's aim is to run in and catch him before he moves out of the circle.

If the blind-folded player runs in a random direction with a constant speed v while the second player runs towards the first player with a constant speed m times v, what should be the value of m such that the probability that the second player wins is 0.50?

Report ans as $m*10^6$. If answer is not an integer, round it off to the nearest integer.

probabilities | calculus

Problem #16

GCD

Find the Greatest Common Divisor of $2^{10^{10}} - 1$ and $2^{8^{8}} - 1$.

Since the answer can be very large, enter the answer modulo 1000000007

number theory gcd

Problem #17

Magical Matrix

Let $0 < t < u < v$, where t, u and v are integers. Now, there is a matrix Q with three rows and c columns where $c > 1$. The matrix is special in a way that each of its columns contains the three numbers t, u and v in some order. All the 3 numbers must appear in every column. Sum of all numbers in row 1 is 20, sum in row 2 is 10 and in row 3, the sum is 9. The rows are numbered 1, 2, 3 while columns are numbered 1,2,...,c and each cell in the matrix is denoted by a pair (i,j), where i = row & j = column. Now the cell $(2,c)$ contains v.

Calculate $\sum\left[Q_{i,j} \times (i+j)\right]$ for the whole matrix.

matrices

Problem #18

Game

Sandeep and Varun are playing a series of games. Each of them contributes Rs 1000 to start. Varun's chance of winning an individual game is 3 times that of Sandeep's. They make an agreement that whoever wins 5 games first can take the entire money with him. After first 3 games, Varun has won 1 game and Sandeep has won 2 games. Now they are bored and do not want to play further.

So they want to divide the money on the basis of the result of first 3 games.

Let x be the money Varun gets and y be the money Sandeep gets. Find x^y. If x and y are not integers, round them to nearest integer. Since answer can be very large, enter the answer modulo 10^9+7.

probabilities expectation value

Problem #19

Large Exponentiation

Find the smallest natural number n for which $201413^n \bmod 2097152 = 1$.

Since n can be very large, enter answer as $n \bmod 1000000007$.

number theory   modular

Problem #20

Fives and Sevens

How many 20-digit numbers are there which are formed using only the digits 5 and 7 and divisible by both 5 and 7.

number theory   divisors   combinatorics

Problem #21

Two Squares

You are given 2 squares such that sum of their area is 1. You want to fit these 2 squares inside a rectangle, without overlap, such that sides of the rectangle are parallel to the sides of the squares. Find the area of smallest such rectangle for which we can always fit the 2 squares as per the given constraints.

Enter answer as 10000000 * *area*. In case the answer is not an integer, return the answer rounded off to the nearest integer.

geometry

Problem #22

Totient Function

Let X be the smallest number having $\phi(X)$ equal to 10^8. Find the number of digits in the factorial of X.

ϕ stands for Euler's Totient Function.

number theory totient

Problem #23

Crack The Safe

There is a safe with its 10-digit keypad (as shown below) which is to be cracked.

```
9 8 7
6 5 4
3 2 1
0
```

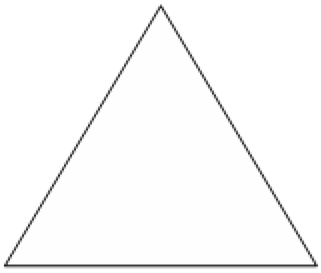
The restriction of the password for that safe is that every pair of neighboring keys in the password is adjacent on the keypad. Adjacent keys are the ones that share a common edge. You know that the password is 109 digits long. Let total number of possible passwords for this safe be X. Since X can be very large, enter X mod 10^9+7.

implementation

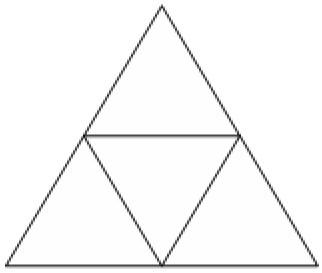
Problem #24

Tangles

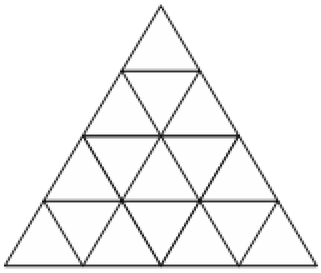
We define the LEVEL of a triangle as in the following image:



LEVEL 1



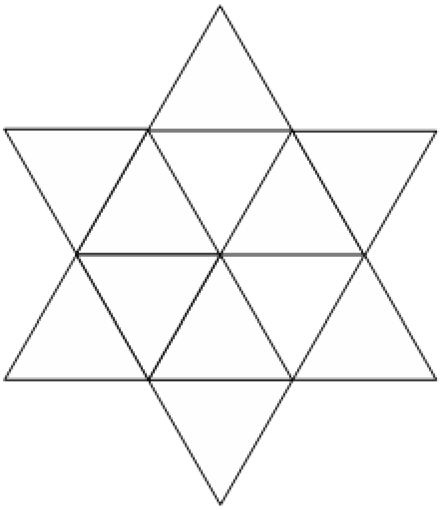
LEVEL 2



LEVEL 4

…………..

Similarly we can define the LEVEL of a hexagram. It's called level n hexagram if it’s joined by 12 triangles, each one is a level n triangle.



Each triangle has level N

Let us define $X(n)$ as the total number of triangles in a level n hexagram. Then $X(n)$ can be written as a polynomial in terms on n.

Let Y be the product of non-zero coefficients of polynomial $X(n)$. Find the factorial of Y. Since answer can be very large, enter the answer modulo 10^9+9

geometry

Problem #25

Butter's Prime Floor

Let there be functions $f(N)$ = no. of primes $x <= N$ and $g(N) = \int_2^N \frac{dx}{\ln(x)}$

Find the value of $\lfloor g(10^{22}) - f(10^{22}) \rfloor$.

Problem #26

Cartman’s Challenge

How many polynomials are there of degree 59 with leading coefficient 1 which cannot be factored in the field of

`411310114921510480003052953791595317048613962353975993313594999488277040407483256849`9?

If your answer is x then input x mod 1000000007

Problem #27

When Kenny Guesses

Let there be an expression $p(x) = \lfloor \theta^{3^n} \rfloor$. The task is to approximate the value of θ so that it gives prime for values from n = 1 to n = 12. Give the answer correct upto 14 decimal places

Problem #28

Mrs Cartman's Desire

Mrs Cartman likes rotational symmetry (everything which is symmetric about the centre) She had a long cherished desire of playing chess. So Randy Marsh challenges her to solve the following puzzle:

She is asked to arrange the non-attacking rooks on the chessboard according to Mrs Cartman's fantasies, only if possible. She is given chessboards of different sizes (n*n) and asked what are the total number of ways of arranging n rooks if n varies from 1 to 61. Give the answer modulo 1000000007.

Problem #29

Mr. Garrison Prepares for the Night

Suppose that Mr Slave wants to buy n sandals $s1,s2,s3,s4...s_n$ and n dresses $d1,d2,d3,d4...d_n$, where s_i is a must to have bought before buying s_{i+1} and the same with d_i. Let the ordered ways to add these $2n$ things to the girl's clothing be $a(n)$, n varies from 1 to 60. So calculate

$$a(1)[(\frac{2207+987\sqrt{5}}{2})^{\frac{1}{16}}+(\frac{24476-10946\sqrt{5}}{2})^{\frac{1}{21}}]+a(2)[(\frac{24476+10946\sqrt{5}}{2})^{\frac{1}{21}}-(\frac{5778-2584\sqrt{5}}{2})^{\frac{1}{18}}]+$$
$$a(3)[(\frac{5778+2584\sqrt{5}}{2})^{\frac{1}{18}}+(\frac{76-34\sqrt{5}}{2})^{\frac{1}{9}}]+a(4)[(\frac{76+34\sqrt{5}}{2})^{\frac{1}{9}}-(\frac{47-21\sqrt{5}}{2})^{\frac{1}{8}}]+$$
$$\ldots a(60)[(\frac{1860498+832040\sqrt{5}}{2})^{\frac{1}{30}}-(\frac{7881196-3524578\sqrt{5}}{2})^{\frac{1}{33}}]$$

Problem #30

Respect My Authoritah!

The wizard Cartman decides to color a torus with colors. Let the minimum number of colors required to color the n-holed torus be $m(n)$. He has a cube of iron. He asks the blacksmith token to cut it (as in a laser plane is passing through the cube) n times. Let the number of different parts of the cube generated be $x(n)$. But he loves to work with prime numbers and so wants $x(n)-k$ and $x(n)+k$ to be prime. Let the smallest k required to accomplish it be $f(n)$. Your task is to find out $\sum_{n=2}^{15}[(m(n)*f(n))]^n$

Problem #31

When Butter Sums It Up

The positive real numbers $x_0, x_1, x_2 ... x_m$ satisfy $x_0 = x_m$ and $x_{i-1} + k/x_{i-1} = k\, x_i + 1/x_i$

Let $x_0(k,m)$ be its maximum possible value of x_0 .

If (k = 2 and m = 59), find the summation of such $x_0(k,m)$ when you increase k by 1 and at the same time decrease m by 2 till both become equal.

Problem #32

Japanese Robo Awesome-O

Professor Chaos controls the robot Awesome-O by pressing the forward or back button. He presses the forward button n times and the back button n times, one at a time. It returns back to him after he presses the buttons $2n$ times.

Let $a(n,i)$ be the number of ways such that the number of moves in the forward direction do not exceed the number of moves in the backward direction and the number of positive moves in the forward direction equals the number of positive moves in backward direction exactly for i times.

Given $a(3,1) = 2$, $a(3,2) = 2$ and $a(3,3) = 1$.

Professor Chaos enters an $n*n$ square board. He can traverse from (0,0) to (n,n) in all possible ways but is restricted to (1,0), (0,1) and (1,1) directions only. Let $b(n)$ be the number of steps on line $x = y$ in all possible paths on which Chaos can travel.

Given $b(2)=6$.

Let $f(n) = \sum_{i=1}^{n} a(n,i)^2$

Find $\sum_{n=1}^{10} f(n) * b(n)$

combinatorics

Problem #33

Angriff der Nazis

In the quiet little mountain town of South Park, there is a square wheat farm ABCD (sides parallel to N-S & E-W directions) with A being towards North-West and A-B-C-D being in clockwise order. The town is attacked by Nazi Bombers and 5 bombs are dropped on the field. Each bomb burns the crop in a radius of 12 km. First bomb is dropped at a point O such that AO is 34 km, BO is 40 km and CO is 26 km. Other bombs are dropped 10 km north, 15 km south, 30 km east and 27 km west, respectively of O.

Find the area of unburnt crop in the farm rounded upto 3 decimal places.

Problem #34

Jewish Factorials

Find in how many ways can we write 100! as a sum of two or more consecutive positive numbers.

Give the answer mod (1000000009)

Problem #35

Cartman’s Large Growing Sequence

Consider the fraction $\frac{AKA}{LOL} = 0.DBUGDBUGDBUG\ldots$

This is a normal fraction which can be written as recurring decimals. Here, the same alphabets stand for the same digits. Find the sum *S* of all such fractions with distinct values of the fraction. *S* = p/q in its most simplified form. Answer *p***q*.

Problem #36

What’s the Answer Makey?

Mrs. Broflovski, Kyle’s mother asked Mr. Makey, "Kyle was visited by three wizards today. If you multiply their ages, you’ll get 2450, and together their age is 2 less than your age. Can you tell me their ages?" After deep thought, the teacher said "No, I can’t." Then Mrs. Broflovski said, "Of course you can’t, but if I tell you that the oldest wizard is older than Kyle, you should be able to work out."

Give the sum of cubes of ages of the three wizards as well as Kyle.

Problem #37

The Allowance Problem

Stan gets his allowance on the 15th day of every month, he gets 1 dollar if its a Monday, 2 dollars if it’s a Tuesday, 3 dollars if its a Wednesday and so on ... 7 dollars if it’s a Sunday. Then on a particular 15th of a random month, find the expected value e of the money Stan will be receiving. Give the integer after ignoring decimal in e*1000000.

Problem #38

When Mr. Garrison Teaches Geometry

A hexagon with area A and consecutive sides of lengths 8, 10, 8, 10, 12 and 12 is inscribed in a circle. Let $N = 100*A$, ignoring the decimal part which may be represented as x^k*y, where $x,y,k > 1$. Take Y as the average of all possible ys in the above expression and find the Y^{th} prime number.

geometry

Problem #39

XOR 'em Up

Say $f(n)$ = bitwise XOR of all natural numbers up to a positive integer n.

Find $f(2^{2013})$ mod 1000000007.

Problem #40

Kyle's Mysterious Sequences

For a given integer n, Consider the following functions $G(n)$ and $F(n)$

$$G(n) = \sqrt{F(n) - \tfrac{(n-1)^2}{n^2} F(n-1) + \tfrac{1}{n^2}}$$
$$F(n) = 8 + \tfrac{(n-2)^2}{n^2} F(n-2)$$
$$F(1) = F(2) = 8$$

Now consider the sequences

$$Y(n) = \sum x \mid x < n, n \bmod x = 0$$
$$Z(n) = Z(n-1) + Y(n)$$
$$Z(0) = Z(1) = 0$$

Find the value of $Z\left(\frac{1}{G(1000000000)-2)}\right)$

number theory

Problem #41

Cartman Loves One Direction

Find the number of occurrences of the digit 1 in all natural numbers between 123456789 to 1234567890.

combinatorics

Problem #42

Medical Marijuana

Men in South Park are suffering from vulva cancer and are getting doctor's reference for medical marijuana. The demand for the same has gone up. A transport company handling the shipments has n identical articles which are to be shipped in k identical trucks.

You simply have to find the total number of ways of distribution of those articles in the trucks.

Compute for $n = k = 10000$.

Answer mod 1000000007.

number theory integer partition dp

Problem #43

Game of Stacks

Drake and Josh play a game to take a break from programming. In this game, there is a stack of N sheets. Drake starts first and alternating turns, they remove an odd number of sheets in each turn and put them in a basket which is initially empty. After every turn they note down the count of the sheets in the basket and thus construct an increasing sequence of numbers. But then, Drake wonders, how many distinct games of this kind can they play such that the stack gets empty at last and there are exactly k numbers in the sequence i.e. the game ends after k turns. Josh answers this when N and k were small. But now Drake who is also good in Mathematics asks you the same question with slight complications:

$N = 999999$ and $k = 5^p$ can be any number where $\frac{(2^{p-1}-1)}{p}$ is a perfect square, i.e. you have to take the sum for all such k!

Give your answer mod 10^9+7.

Note that distinct games generate distinct sequences.

Problem #44

Power Sum

Let $k = 4128^6 + 6^{4128}$.

What is $\sum_{n=1}^{17040384}$(units digit of $k^n + n^k$) (units digit of kn+nk)?

number theory modular

Problem #45

One of Us

Let $a(n)$ be defined as the number of terms in the sequence $2^1, 2^2, ... 2^n$ which begin with digit 1.

Find $\lim_{n\to\infty}(\frac{a(n)}{n})$

Give your answer as the largest integer after multiplying by 10^6.

calculus

Problem #46

When Butter Recurses

Let us define a series M such that $M(n-1) + M(n+1) = 5*F(n)$ where $F(n)$ is the n^{th} term of the Fibonacci sequence with $F(0) = 0$, $F(1) = 1$ and $M(0) = 2$.

Find the value of $\prod_{n=1}^{9} M(2^n)$

Give answer mod1000000007

number theory fibonacci

Problem #47

Stan’s Prime Challenge

What is the number of primes of the form $4k + 3$ between 8589934592 and 17179869184?

number theory primes

Problem #48

Mr Garrison Can't Get a Period

Consider $F(n)$ as the last 8 digits of the n^{th} Fibonacci number.

What is the period of $F(n)$?

number theory fibonacci

Problem #49

Hyper Sum of Fibonacci Numbers

Let $F(n)$ be the n-th Fibonacci number. That is, $F(n)$ satisfies

$F(0) = 0$, $F(1) = 1$, $F(n) = F(n-1) + F(n-2)$ (if $n \geq 2$).

Let $f_k(n)$ be the function such that

$$f_0(n) = F(n),$$

$$f_k(n) = \sum_{i=1}^{n} f_{k-1}(i) \quad (\text{if } k \geq 1).$$

You are given that $f_1(1) = 1$, $f_2(2) = 3$ and $f_{10}(10) = 130965$.

Find $f_{10^8}(10^8) \pmod{1\,000\,000\,007}$.

number theory fibonacci modular

Problem #50

Divisibility

Find the base of a counting system, like decimal(10), binary(2) etc., less than 1000000 in which maximum numbers follow the following property:

"If *x* is divisible by *y*, sum of digits in *x* will also be divisible by *y*."

In case there is more than 1 possible answer, then concatenate the possible answers in ascending order without any spaces in between. For example if the answers are 3, 21 and 54 then your answer should be 32154.

number theory modular

Problem #51

Probability of Squares

Two numbers are chosen at random from 1 to 10^{16}. Let *p* be the probability that their sum is a perfect square. Find integral part of $10^{16}*p$.

probabilities

Problem #52

Modular Roots

Let $y = x*x$. * is multiply modulo N = 4776913109852041418248056622882488319.

You have to find the smallest positive *x* for *y* = 739397.

number theory

Problem #53

Harry and his Magical Boxes

There are 7 magical boxes each containing a label depicting a number. The labels of the boxes are given as 5,2,7,8,7,4,6. Harry Potter goes to each jar once and shouts a spell. When Harry shouts a spell near a box with label x he gets a random integer n from the box such that, 1 <= n <= x. Thus he gets a collection of seven numbers by shouting once near each box. One such collection is 4,2,1,2,6,4,6. Two collections are same if one is a permutation of other. So 1,2,1,1,1,1,6 is same as 2,1,1,1,1,1,6. Now Harry expects you to tell the total number of different collections he can get by repeatedly performing this act.

combinatorics

Problem #54

Find Tuples

Let S denote the set of all 6-tuples $(X_1,X_2,X_3,X_4,X_5,X_6)$ of positive integers such that $X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2 = X_6^2$

Consider the set $T = X_1X_2X_3X_4X_5X_6 : (X_1, X_2, X_3, X_4, X_5, X_6) \in S$.

Find the HCF of all members of T.

number theory gcd

Problem #55

Double Devils

All numbers of the form **6_6_6_0_0_6_6_6_0_0**, where each _ can be any single digit, are called double devil numbers (for example, 6160630507696864040 is a double devil number).

Let x be the unique number whose square is a double devil number. Find x.

Problem #56

Playing with Subgraphs

Find the number of distinct sub-graphs of a complete graph of n labelled vertices, such that the sub-graph is a spanning tree connecting all the vertices and the degree of no vertex is more than 3.

Given n equals 314159, give your answer (mod 1 000 000 007).

Problem #57

Smallest Perimeter

The sides of a triangle have integer lengths k, m and n.

Assume that $k > m > n$ and $\{\frac{3^k}{10^4}\} = \{\frac{3^m}{10^4}\} = \{\frac{3^n}{10^4}\}$.

where {x} denotes the fractional part of a number.

Determine the minimum value of the perimeter of the triangle.

number theory modular

Problem #58

Circle of Death

Let n be 10 times the largest integer which cannot be represented as the sum of five non-zero perfect squares. Now, consider yourself standing in a circle of n people (positioned 1 to n around the circle), where every k^{th} (k = 10) person is killed from the remaining ones standing except the last one. For example, if n = 4 and k = 2, then the order in which people will be killed is 2, 4, 3. Hence person at position 1 will be the survivor.

Let p be the position you will choose so as to survive this game (be the last to be killed). Also, for n = 40, let q be the smallest value of k(>1) for which people are killed in the same order as they are standing (i.e. 1,2,3,...).

What is $(p*q)$ mod 1000000009?

number theory   dp

Problem #59

Ones and Zeros

Let $S = 2^a+2^b+2^c+2^d+2^e$ where a,b,c,d,e are distinct whole numbers.

Let S_n be the n^{th} number such that for all $i < n$, $S_i < S_n$ and for $i > n$, $S_i > S$.

Find $S_{2131646}$ mod 10^9+7.

combinatorics

Problem #60

Rolling Dice

Alex has a 7-sided regular dice. Every side has a distinct value between 1 to 7, each with equal probability of coming up in a roll. Alex rolls the dice once. Let the value obtained be equal to S_1. Now he rolls the dice S_1 number of times. Let the sum of values obtained in these S_1 rolls be equal to S_2. Now he rolls the dice S_2 number of times. Let the sum of the values obtained in these S_2 dice rolls be equal to S_3. This process is continued for an infinite number of times. Find the expected value of $S_{1182014}$. If the answer is x, give your answer as $\lfloor x \rfloor$ mod 10^9+7, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

Problem #61

Derangements

There are n people and n houses, such that every person owns exactly one distinct house. Out of these n people, k people are special ($k <= n$). You have to send every person to exactly one house such that no house has more than one person, and no special person goes to his own house. Let

$S(n,k)$ be the number of ways of doing this. Find the value of $S(654321,123456)$. Give your answer mod 10^9+7.

Problem #62

Magic in Boxland

A magician lives in a mysterious Boxland that comprises of eight cities and all of them contain 6 magic wands. Each wand has been colored with one of n colors, such that no two wands in the same cities are of same color, and no two colors occur together in more than one city. The smallest number n that satisfies this condition is used by the magician to create a ball-box challenge that contains a 2-D array of size $n*n$ of such boxes placed adjacent to each other satisfying following conditions:

- Every box that does not contain a ball shares a side with one which does.
- Given any pair of boxes that contain balls, there is a sequence of boxes containing balls, starting and ending with the given boxes, such that every two consecutive boxes of the sequence share a side.

Find the smallest number of balls that must be there inside those boxes multiplied by 3.

(i.e. Answer = [Total number of balls inside the array of boxes] x 3)

combinatorics graphs greedy

Problem #63

Seat Yourself

Let x be the number of students seated around a round table. Then, let k be the number of ways in which they change their seats resulting in an arrangement in which the students are either on the same seat or one of the seats adjacent to the original one.

Given that x belongs to the set (123345857, 343139869), both not-included, find the possible number of values of k which are congruent to 111 mod 5.

Problem #64

Polynomials

Consider a polynomial of degree $3m$ such that

$P(0) = P(3) = P(6)...P(3m) = 2$

$P(1) = P(4) = P(7)...P(3m-2) = 1$

$P(2) = P(5) = P(8)...P(3m-1) = 0$

and $P(3m+1) = 6562$

Find the value of m?

Problem #65

Identical Boxes

You are given 2500 identical boxes and 5000 identical balls. Find the number of ways in which balls can be distributed such that each box contains at least one ball. Assume the capacity of each box to be infinite.

Give your answer $\bmod 10^9+7$.

Note: All permutations of boxes are counted as a single way as the boxes are identical.

Problem #66

Reaching Point

You have to reach point (x_2,y_2) from (x_1,y_1) by making jumps from one point to another but you have some limitations. You can only move in the direction of the line connecting these two points constantly aiming towards the destination point. Also from one point you can jump to only the next most nearest integral point on the line. For example, (3,3) to (6,6) then path has to be

(3,3)−>(4,4)−>(5,5)−>(6,6) and this required 3 jumps. But now the problem is the a jump comes at a heavy cost.

If the total number of jumps made to reach (x_2,y_2) starting from (x_1,y_1) is n, then the total cost for the journey is said to be the (number of trailing zeroes in $n!$)^n.

Find the total cost for the journey from (−10101099, 127898755387) to (1137947000140424, 1607032990556) mod 10^9+7.

number theory modular

Problem #67

Finding Limits

$$S_0 = 3$$
$$S_1 = 3^3$$
$$S_i = S_0^{S_{i-1}} \text{ for } i > 1$$

Find S_x mod 1073741824 where x tends to infinity.

Problem #68

A Mixture of Ryski's Problems

Ryski was asked compute the sum of the series

$$\sum_{k=0}^{\infty}(1/((4k+1)(4k+2)(4k+3)(4k+4))).$$

Let this summation be p.

Ryski also has 2*n different pens ($n \in N$, $n \geq 2$). Each day, he takes n pens with him to school. After some days the following condition was fulfilled: every two pens were together on at least one day. Let the minimum number of days needed for this to happen be q.

Give the answer as [$p * q * 10^{20}$], where [x] denotes the greatest integer less than or equal to x.

Problem #69

Trio Interchange

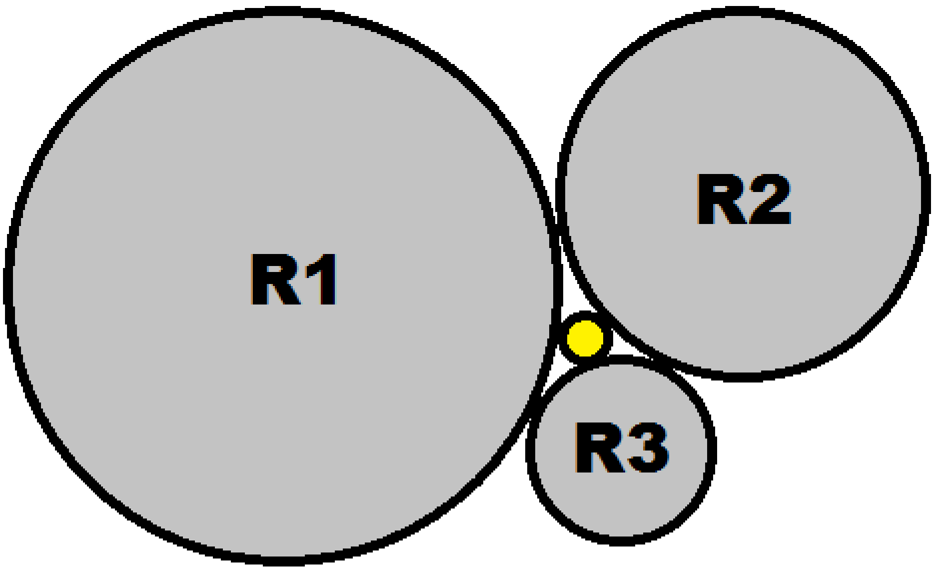
There is a six digit number, which when multiplied with 6, yields another six digit number, with first three digits of original number as the last three digits of the new number, in the same order, and the last three digits of the original number as the first three digits of the new number, in the same order.

Let S be the sum of the digits of that number. Find $S!$ mod 1000000007

number theory

Problem #70

Fibonacci Circle



All circles in the picture above are tangent to each other. If radius $R1 = 2^{12}$, $R2 = 2^{11}$ and $R3 = 2^{10}$, calculate r, radius of the inner yellow circle. Let $x = \lfloor x \rfloor \times 10^{14}$.

We now define a new sequence as

$F(n) = F(n-1) + F(n-2) + (n-1) \quad \forall n \geq 2$

with $F(0) = 0$ & $F(1) = 1$.

Calculate sum $\{F(0)+F(1)+F(2)+...+F(x-1)+F(x)\}$ mod 1000000007.

number theory geometry

Problem #71

Random Selector

A random number selector can only select one of the nine integers 1,2,...,9, and it makes these selections with equal probability. Let p be the probability that after n selections ($n > 1$), the product of the n numbers selected will be divisible by 10. If $n = 5$, find $p \times 59049$.

probabilities

Problem #72

GitHub

You are given six characters `GITHUB` to make a string of length N. However there are some limitations:

- I can not come directly after `G`.
- H can not come directly after `G` or `B`.
- T can not come directly after `T` or `U`.
- The string should be a palindrome.

How many ways are there to form a string of length $N = 7162534$?

Submit the answer modulo 1000000007.

dp

Problem #73

Brute

$x^6 + x^3 + x^3y + y = 147^{157}$

$x^3 + x^3y + y^2 + y + z^9 = 157^{147}$

Let n be the number of solutions for the above system. i.e. if $x = a$, $y = b$, $z = c$ is a solution then the number of solutions is incremented by 1. Give your answer as first six digits of $(n + \mathrm{e})^{134}$.

number theory

Problem #74

Mod It

Let n be the largest positive integer, such that $n!$ can be expressed as the product of $(n-2014^{2015})$ consecutive integers.

Let x be equal to n mod 38980715857. Find x mod 8037517.

number theory

Problem #75

No Primes

Find sum of all 5-digit natural numbers such that they don't contain any subsequence forming prime number. 2, 3, 4 or even 5-digit prime numbers are not allowed.

brute force

Problem #76

Erdős

Given that e,r,d,o,s are real numbers such that

$e + r + d + o + s = 8$

$e^2 + r^2 + d^2 + o^2 + s^2 = 16.$

Let M be the maximum value of s. Find the number of factors of $M \times 30$.

number theory

Problem #77

Prime Fangs

In mathematics, a vampire number is a composite natural number v, with an even number of digits n, that can be factored into two integers x and y each with $n/2$ digits and not both with trailing zeroes, where v contains precisely all the digits from x and from y, in any order, counting multiplicity. x and y are called the fangs, e.g. $1260 = 21 \times 60$.

A prime vampire number is a vampire number whose fangs are its prime factors. $117067 = 167 \times 701$ is the first prime vampire number. Calculate the 27th prime vampire number.

brute force

Problem #78

Golden Path

Let x be a number such that for $n > x$, the nth Fibonacci number has at least one prime divisor that does not divide any kth Fibonacci number for all $k < n$. Find the sum of all primes $p < 10^x$ such that for some integer n, the expression $n^2 - p \times n$ is a prime power.

Note: A prime power is a number of the form q^m where q is a prime and m is a positive integer.

number theory

Problem #79

Strings

Laertes and Roxane play a game together by first drawing a string of Ls and Rs into the sand, and then taking turns removing any one of their respective letters (L for Laertes and R for Roxane) along with the substring to its right. The first player unable to make a move loses the game.

Consider the game played with the string L. If Laertes goes first, he removes the only L and passes to Roxane, who immediately loses, having no valid move. If Roxane goes first, she immediately loses again. So we see that Laertes always has at least one move's worth of an advantage over Roxane. We shall abbreviate this by saying the value of the string L is +1, from Laertes' point of view. Similarly, the string R has value −1. A game played with the two separate strings, L and R (denoted by $L+R$) can be shown to have the property that the first player to play loses the game. We shall call the value of such a string 0, and denote this as $L+R=0$.

Convince yourselves that $LR=0.5$ (try playing the game with $LR+LR+R$). What is the value of LRL?

strings

Problem #80

Piles of Stones

Laertes and Roxane play another game now as they are tired of drawing in the sand. Instead, they play a game with piles of stones that works as follows:

At the beginning of each turn, the player first chooses one of the piles and then removes some number of stones from it (the whole pile may be removed as well, but at least one stone needs to be removed). The turn then passes to the next player. The player that removes the last stone(s) wins.

Laertes always goes first.

Convince yourself that in a game with three piles of 3, 4, and 5 stones respectively, Laertes will always win if he plays optimally, no matter what Roxane does. Let's say that games with this property have value 1.

Consider 6561 separate games, with four piles, and the number of stones per pile varies between 1 and 9. How many of these games have value 1?

games

Problem #81

Hide and Seek

Laertes and Roxane go to the Senate to play a game of Hide-and-Seek. There are 100 rooms in the Senate, and Roxane picks one of them and hides there till the game ends.

Laertes, at the beginning of every turn, picks one room and searches in it. Since he is human and thus fallible, he only has a 60% chance of finding Roxane if she is in fact hiding in the room. If he fails to find her in the room, Roxane's score increases by one, and the next turn starts, whereupon Laertes must pick another room to check (he can also check the same room again). Remember, Roxane remains in the room she initially chose.

Given that Laertes and Roxane play with perfect strategies, what is the expected value of Roxane's score at the end of the game? The answer must be correct to 1 place of decimal.

games

Problem #82

Pebble Trouble

Laertes and Roxane play a game with some black and white pebbles at the beach. They dig twelve small pits in the sand, making a dodecagon. Laertes will place two stones, one black and one white into two adjacent pits in such a way that the white stone is to the left of the black. Laertes cannot then place any stones in the pits next to these. Similarly, Roxane places her stones so that the white one is to the right of the black one. She too cannot use the adjacent pits ever again.

The first person who is unable to place their pair of stones loses. If Laertes goes first, convince yourselves that Roxane can always win with perfect play.

Consider 1000 games where the number of pits in the sand varies from 12 to 1011. How many of these games can Roxane always win, if Laertes goes first each time?

games

Problem #83

Fresh from the Oven

Laertes and Roxane have before them a cake, freshly baked by their mother. It has been lightly scored so as to allow it, when cut, to be divided into $M \times N$ pieces. The two kids decide to play a game to cut the cake. Laertes will cut any rectangle into two pieces along the vertical lines, and Roxane along the horizontal lines. The game ends when the first who is unable to make a move loses.

Laertes always goes first.

Consider 256 games with cakes varying in size from 1×1 to 16×16. How many of these games can Laertes always win, if he plays with perfect strategy?

games

Problem #84

The Gobbler

Laertes and Roxane are tired of cutting cakes. They want to eat some now, and they do so by playing a game as follows:

On his turn, Laertes must choose any square on the grid and eat it, along with the square immediately below it. On her turn, Roxane must choose any square on the grid and eat it, along with the square to its immediate right. The first person who cannot eat two pieces as described above on any turn loses. Let us describe the squares using the conventional matrix coordinate scheme, with the top-left square being (0,0) and the positive x and y axes extend to the right and down respectively. In cases where taking two different squares results in the same cake upto symmetry, then the square that is closest to the origin is preferred. If the square is still ambiguous, then the one that is closer to the x axis is preferred.

Consider the 2×1 cake. It is obvious that no matter who goes first, Laertes can always win by eating the square at (0,0) (and so consequently (1,0)).

The 2×2 cake is different though. Convince yourself that the first person to start will win, no matter who it is.

What are the optimal squares chosen by both players on a 4×2 cake? Assume that Laertes goes first. If you think the sequence is (a,b), (c,d), (e,f), enter *abcdef*.

games

Problem #85

The Stymphalean Birds

The story

The Stymphalean birds were monsters. They were man-eating birds with beaks of bronze and sharp metallic feathers they could launch at their victims. They had destroyed the local crops and fruit trees before Hercules was tasked with killing them. Athena gave Hercules a rattle that he could use to scare the birds off so he could then shoot them down with his bow.

The game

It turns out that shaking the rattle in a specific way would induce absolute terror in the birds. Let L represent shaking the rattle to the left, and R to the right. Then, using the game introduced in **Problem 79**, enter the string that has the value 4.740234375.

strings

Problem #86

The Augean Stables

The story

Hercules' task this time is to clean the Augean stables. They are in a right old mess. Hercules eventually hits upon the idea of diverting a nearby river into the stables in order to clean it. This gets rid of the muck but sadly a lot of vegetation has overgrown on the grounds that Hercules will have to weed out the hard way. In order to pass his time, he invites the local farmhand to play a game with him, using the weeds.

The game

Each weed is divided into a certain number of parts, and players take turns cutting off a part. Any section of the plant that is not connected to the ground after a cut is also removed. Play continues until the last person to make a cut wins.

Hercules always goes first.

Each weed is represented as a graph, with edges representing pieces of the plant that can be cut off in a turn. The node labelled 0 is the one that is connected to the ground.

Now, **here** are the adjacency matrices of 7 such weeds. Consider the 127 games that can be played with some combination of these weeds. For example, Hercules can always win the game played on just weed 1, but he will always lose the game played with both weeds 1 and 3. Find all the combinations of weeds such that Hercules will always lose the game played with them. Enter your input in the following fashion:

The combination (1,3) is considered as the binary number 1010000 (the first and third places are set to 1), and the combination (1,6,7) is 1000011. Convert your combinations into binary numbers and enter their sum in decimal.

games

Problem #87

The Flock of Geryon

The story

Hercules just got another task. This time he has to steal Geryon's golden cow from the far-off Mediterranean island of Erytheia. Hercules knows that Geryon has a large flock of cows among which there is only one golden cow, which Hercules needs to retrieve.

Hercules is tired of fighting and challenges Geryon to a bloodless battle instead. The game Hercules suggests is to be played with the remaining cows.

The game

Each player chooses to eliminate 13 or 50 or 63 cows from the flock at each turn. The player who eliminates the last set of cows gets the golden cow.

Geryon is not very good at games, so he does not tell Hercules how many cows are there in his flock, lest he thinks of a winning strategy, however in his foolishness he lets slip that the number lies anywhere between 0 to 205530. He also forces Hercules to decide who shall begin the game.

Hercules now wants to know what the odds are of him losing the game if he goes first and plays perfectly, given that the number of cows lies anywhere between 0 and 205530.

Can you help Hercules by finding the probability? The answer must be correct to 4 decimal places.

games

Problem #88

The Cretan Bull

The story

Animals seem to wreak a lot of havoc in greek myths. The one under consideration is a bull that used to uproot crops and bring down orchard walls in Crete. Hercules is sent to kill it as usual.

The game

The bull is smart however, and it senses that Hercules is after it. It engages in an impromptu game of hide-and-seek (following the same rules as those in **Problem 81**).

Alas, it only has two choices of hiding places. In the first location, Hercules has a 40% chance to find the bull, if in fact the bull is there. In the second location, Hercules has a 100% chance to find the bull if it is there. The second location is thus a terrible hiding place. But, the bull bets on Hercules dismissing such a location.

If Hercules and the bull play with perfect strategies, what is the expected number of turns it will take for Hercules to capture the bull? The answer is a rational number in lowest terms p/q, and you are to enter p,q.

probabilities

Problem #89

The Horses of Diomedes

The game

Hercules and Abderus play a game with a cake (as described in **Problem 84** of the preliminaries). Hercules eats squares horizontally and Abderus vertically. We define the value of a game as:

+1 if Hercules can win no matter who starts,
−1 if Abderus can win no matter who starts,
+i if the first player can always win, and
−i if the second player can always win.

Consider 30 games, played with cakes of sizes $M \times N$, with $1 \le M \le 3$, and $1 \le N \le 10$.

What is the sum of the values of these games? If your answer is a+ib, enter a,b.

The story

Oh, why is Abderus important? He was eaten alive by the crazed horses of Diomedes, and in revenge Hercules fed Diomedes to his own horses. This seemed to have calmed the crazed horses and they lived happily ever after in a meadow somewhere. Greek myths can be very weird sometimes.

games

Problem #90

Capture of Cerberus

The story

Hercules now has to bring back Cerberus, the three headed dog from the underworld. Cerberus being an intelligent beast, lures Hercules to a game of gates. If Hercules wins, he will come back with him, otherwise, Hercules has to spend the rest of his days in the underworld.

The game

Rules:

- There are a total of N = 18 gates, some of which are open and some which are closed.
- At each turn, the player has to open a closed gate first, and choose to close/open upto 4 gates to the left of the first chosen gate. (Here total number of changed gates c = 5). He can either open a closed gate or close an open one.

The player to open the last closed gate wins the game.

The trial match

Hercules requests for a trial match and Cerberus agrees to play a toned down version of the above game, with N = 5 and c = 3. He sets up the initial configuration as this:

`CCCCO` (`C` = closed, `O` = open)

Cerberus urges Hercules to go first. Now, no matter what Hercules does, it is easy to see that Cerberus will always be the one to open the last closed gate. Such a position will always cause Hercules to lose, that is, if he goes first.

Hercules is wary of losing, and wants to make a list of all initial positions where he will lose if he goes first. How many such configurations are there?

games

Problem #91

The Girdle of Hippolyta

The story

Ares had once gifted Hippolyta, the queen of the amazons, a girdle -- a belt. It is this belt that Hercules is now tasked to steal. But, when word of his task spread to the queen herself, she was very impressed with Hercules' bravery and offered to give him the belt if he won a game of skill against her.

The game

Let $s_i = a_j = \frac{1}{(2-i/g)n-1}$ be a series for each i, $0 \leq i \leq 7$.

And consider $I_n = \int_0^\infty \prod_{j=1}^{n} \frac{\sin(a_j x)}{a_j x} \, \mathrm{d}\, x$ for each s_i.

As an example, let's play the game with $_i$ = 0. Here, $I_1 = I_2 = \cdots I_7$, but $I_7 \neq I_8$. Each I_n is of the form $p/q\ \pi$. We say the value of s_0 is $8+p+q$ (8 being the first integral to break the pattern).

Play the game with all the indicated values of i. Add together the values of each s_i and submit this sum modulo 1000000007 as the answer.

games

Problem #92

The Apples of the Hesperides

The story

Hercules has to steal a hundred apples from the Garden of Hesperides. The apple trees are located at a radius of $\sqrt{200}$ from the centre (0,0), abundantly distributed (read: infinite amounts) along the circumference.

There is a pole at the centre behind which Hercules is hiding from Ladon, the Dragon lord guarding these trees. Hercules can escape from Ladon only if he is hidden from Ladon's line of sight behind the central pole. Ladon is powerful because he can see in all directions. If Ladon sees Hercules at any point of time, he will immediately fly and kill Hercules.

Cutting to the chase

Ladon is travelling to the nearby city of Egypt along a straight line starting from (0,-80), parallel to the x-axis, at a speed of 10 m/s starting at time t=0, nevertheless his all-seeing eye does not tire out from the constant vigilance. Hercules is hiding behind the pole at (0,0) at t=0. Hercules strategizes to pick up the apples located at the circumference, all the while remaining hidden from Ladon's sight, and place them at the centre. But, he can carry only a maximum of 10 apples at a time. Clearly he needs to make 10 such trips. He has a speed of 9 m/s.

What is the shortest time Hercules will need to complete his task, (without getting killed midway, obviously)? If x seconds is your answer, enter $\lfloor x \rfloor$.

games

Problem #93

The Ceryneian Hind

The story

Hercules' next task is to catch the Ceryneian Hind, which is as fast as the wind. Artemis considers this a sacred animal and will not give it up to Hercules easily. In order to capture it, he must play a game against Artemis. Artemis has carefully assembled a stick figure of her favourite animal for the game.

The game

The figure is made out of sticks which can be either yellow, golden, brown or green. The black line in the figure represents the ground.

At each turn, a player plucks out a stick from the figure. Any "hanging" sticks, i.e. those that have no connection to the ground, either directly, or through other sticks, will fall down and are immediately discarded.

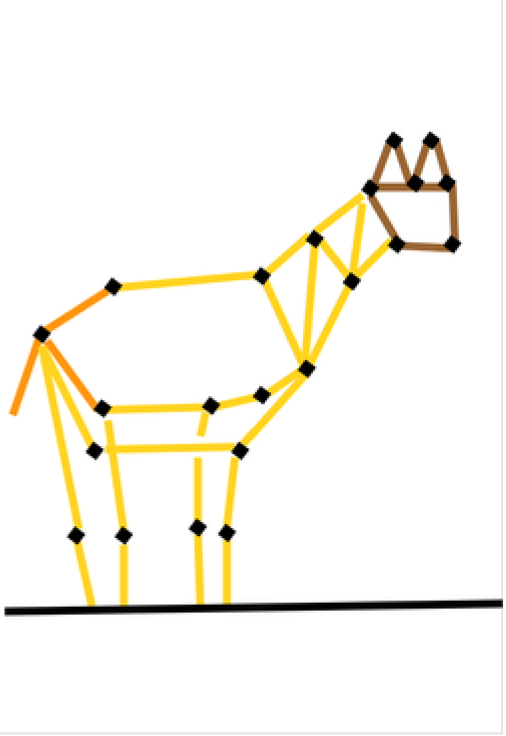
Hercules can only pluck the orange sticks, and Artemis, the brown ones. Both of them can pluck the yellow and green sticks. The one to pluck the last stick wins the game, and gets the Hind.

Hercules also spots a number of other interesting creatures and gambles against Artemis to capture them all, playing the same game for each of them.

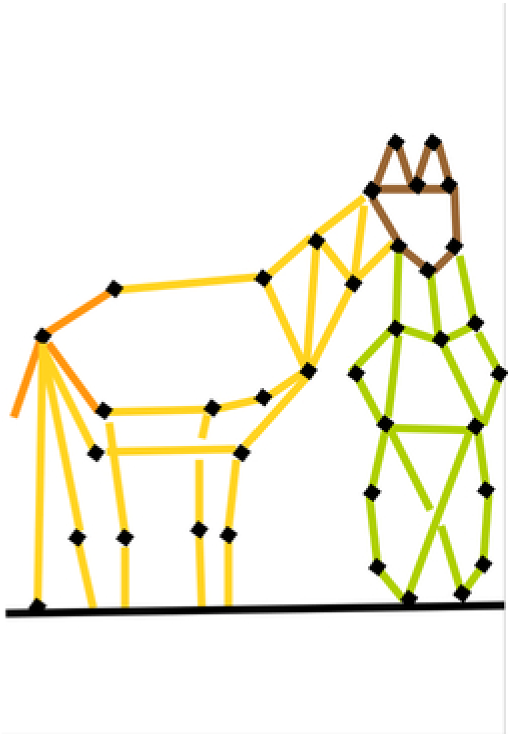
For each game,
A represents Hercules losing no matter who starts, if Artemis plays intelligently.
H represents Hercules wins no matter who starts, if he plays intelligently.
F represents the situation where the first player always loses.
S represents the situation where the second player always loses.

For the 5 figures given below, enter the string of answers in the order of the labels of these figures.

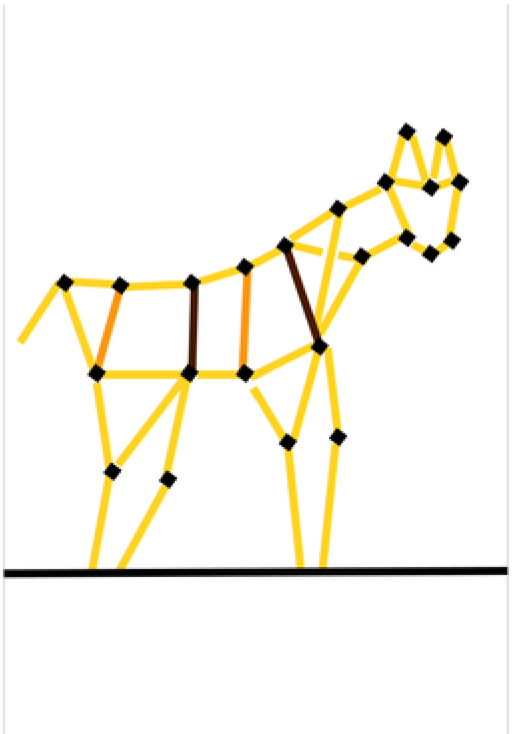
Golden Hind



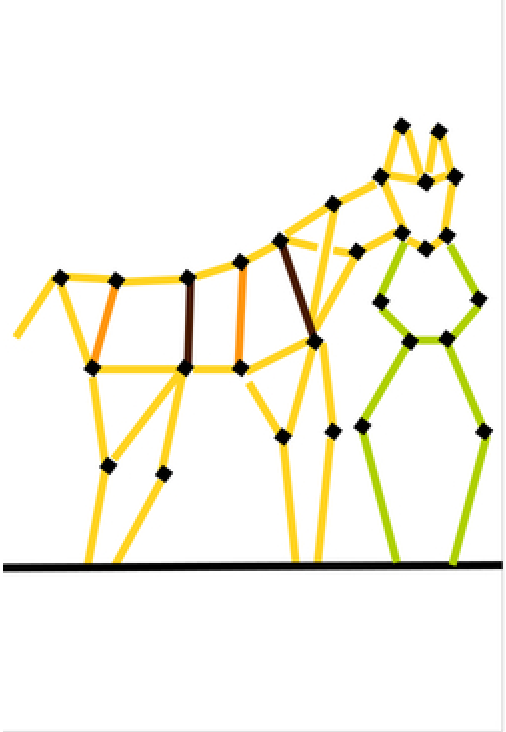
Golden Hind eating grass



Wild Zebra



Wild Zebra eating grass



Poisonous Toad

games

Problem #94

Vacation

Ram and Shyam took a vacation at their grandparents' house. During the vacation, they did all activities together. They either played tennis in the evening or practiced Yoga in the morning, ensuring that they did not undertake both the activities on any single day. There were some days when they did nothing. Out of the days that they stayed at their grandparents' house, they were involved in one of the two activities on 22 days. However, their grandmother while sending an end of vacation report to their parents stated that they did not do anything on 24 mornings and they did nothing on 12 evenings. How long was their vacation?

Give your answer in days.

implementation

Problem #95

Strange Currency

In the strange country Oz, the only official coins are 7-cent coins and 13-cent coins. What is the largest amount that cannot be paid with these coins if a shop has no change at all?

modular   brute force

Problem #96

The Game of Marbles

A box contains two marbles. One black and the other white. In each turn the player takes a marble out at random and notes its colour. After each turn, the marble is replaced in the bag and an extra black marble is added. The player pays Rs. 1 to play and wins if he/she has taken more white marbles than black marbles at the end.

If the game is played for three turns, the probability of the player winning is exactly $7/24$, and so the maximum prize fund that should be allocated for winning in this game should be Rs. 3 before the organizer would expect to incur a loss. The maximum prize is a whole number and includes the Rs. 1 to be given by the player, so the player gets Rs. 2. Find the maximum prize fund to be allocated to a game with sixteen turns.

probability

Problem #97

Triangles

Let us take two triangles $\triangle ABC$ and $\triangle PQR$ as shown in Figure 1.

In $\triangle ABC$, $\angle ADB = \angle BDC = \angle CDA = 120°$.

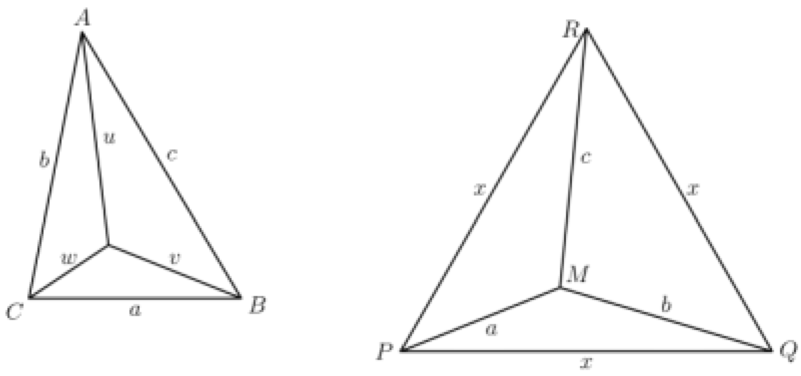


Figure 1

$u = 999$, $v = 899$, $w = 1099$.

D is the internal vertex of $\triangle ABC$

Find the value of x.

geometry

Problem #98

Series Dilemma

If X_n and Y_n denote two sequences of integers defined as follows:

$X_0 = 1$
$X_1 = 1$
$X_{n+1} = X_n + 2\times X_n - 1$

$Y_0 = 1$
$Y_1 = 7$
$Y_n + 1 = 2\times Y_n + 3\times Y_n - 1$

$n = 1,2,3,...$

Thus, the first few terms of the sequences are:

X : 1,1,3,5,11,21,...
Y : 1,7,17,55,161,487,...

Let the largest number that occurs in both the sequences be m.

Give the answer as $(m\times 123^{m+1})$

number theory

Problem #99

Divisibility Test

Sum of digits of a 5-digit number is 41. Let the probability that such a number is divisible by 11 be p/q, where $\gcd(p,q) = 1$.

Find $q-p$.

number theory probability

Problem #100

Quadratic Chaos

Let m be an integer such that $1 \leq m \leq 1000$. Find the probability p of selecting at random an integer m such that the quadratic equation $6x^2-5mx+m^2 = 0$ has at least one integer solution.

Give answer as $p\times 1000$.

probability

Problem #101

Fibonacci Twist

Find 67th term of the sequence whose initial terms are as follows:

0th term: power of 3 in $\binom{66}{24}$

1st term: power of 2 in $\binom{73}{27}$

2nd term: power of 3 in $\binom{3280}{1367}$

3rd term: power of 2 in $\binom{3712}{2005}$

4th term: power of 2 in $\binom{14348}{7519}$

5th term: 16
6th term: 19

implementation

Problem #102

Chinese Whispers

Consider a group of 5011 friends and a particular person in the group (say L). L has a message to spread around. Define a "whisper" as an instance where a friend passes the message to another friend. Considering all friends to be distinct, L is interested in finding out the number of sequences of 999983 "whispers" after which he will eventually get the message back. Calculate the answer modulo 1000000007.

Note:

> After a whisper, if A passed the message to B, then only B can whisper to others since only he has the message. Others can't.
>
> During the sequence of whispers, there can be a point in time where A would get the message after less than 999983 whispers. In that case, he again should pass the message to others in order to complete a total of 999983 whispers

dp

Problem #103

Sevens and Threes

Find the sum of all 7 digit numbers which satisfy the following conditions:

1. It contains only digits 3 and 7.

2. The number is divisible by 7.

3. Last digit of number is 3.

number theory modular

Problem #104

Dr. Antlove

Dr. Antlove loves ants (obviously) and he cannot stand anything happening to his ants. Dr. Rage wants to play with him a little. So he challenges him with a trick puzzle. He gives Dr. Antlove a regular polygon with 628318531 sides with Queen ant in the middle and gives him a triangular (ant-trap)net that can be tied at any three random vertices of the polygon(any three vertices can be chosen as the vertices of the net at random). But Dr. Antlove is busy with a seminar this weekend. So he orders you to get the ant for him. Before moving on you believe it would be better to calculate the probability of your success. All choices of vertices being equally likely, let the probability that you would be able to catch the ant inside that net be *p*. (All vertices are distinct.)

Give the answer = $p \times 2 \times (2 \times \lfloor (\pi \times 10^8) \rfloor - 1)$

geometry probability

Problem #105

Vikas and Pizzas

Vikas has currently ordered a pizza from Domino's. Due to many orders, Domino's did not cut the pizza into slices. He was wondering how many maximum pieces can be made of a pizza with n cuts keeping the pizza fixed at its position.

Find the answer when $n = 65536$.

geometry

Problem #106

Back Propagate

Let the initial terms of a sequence be {1,1,1,1,2,11,84,676,5477,44407}.

Propagate this sequence backwards to produce 20 new terms. Find the sum of these 20 terms (written in simplest fraction form as p/q). What is the value of $(p*q) \bmod (10^9+7)$?

dp

Problem #107

Negated Twins

Let $f(x)$ be a polynomial with integer coefficients such that

$f(2) = f(0) = f(1) = f(5) = n$ and

$f(-2) = f(-1) = f(-5) = -n$ for some positive integer n.

Find the smallest possible value of n.

number theory

Problem #108

Divisor Game

Consider a two player game. There are N balls marked 1 to N. A move consists of removing a ball n and all the remaining balls which are divisors of n (including 1). The players alternate the moves. The one who takes the last ball wins the game. Let us assume that both players play optimally. Find the probability that the player who starts the game wins it, given that N will be a random integer between 1 and ∞. Let this probability be x. Give your answer as $x \times 1000000000000$.

games

Problem #109

Extension Conundrum

We have a triangle with side lengths $a = 345678$, $b = 456784$ and $c = 567890$ with vertices named A, B and C, and I denote the incentre of circle. Line segment BI is extended to meet opposite side of triangle i.e. AC at K. Find length of line segment IK.

Note : a is side opposite to vertex A, b is opposite to vertex B and c is opposite to vertex C.

geometry

Problem #110

L's Birthday

L was celebrating his birthday. His mom made a triangular cake as per L's request as he found that round cakes were too mainstream. After the candles were blown, L's friends (who were 9973 in number) decided that they wanted triangular cake pieces. As L's mom is fair towards his friends, she has the task to divide the cake into 9974 triangles of equal volume. Like L, even his mom likes a good challenge. So she decided to cut the cake in the following way:

She would select a point P on the triangular surface and from P she would trace 9974 rays which will intersect the sides of the triangular surface and cut through the rays resulting in 9974 triangular cake pieces of equal volume. But L pointed out that this type of partitioning is not possible for any interior point on the surface. So she is interested in finding out the number of interior points (P) wherein such equal partitioning of the cake is possible.

number theory   geometry

Problem #111

Rectangle has Children

A rectangle 'A' is said to be the child of rectangle 'B' if following 3 conditions are satisfied:

a. 'A' can be completely fit inside 'B' without A's sides touching B's sides.

b. At Least one side of 'A' should be greater (in length) than both the sides of 'B'

c. A’s and B’s sides are positive integers.

A rectangle with sides of length 22543 units and 22541 units has how many children?

(Note : A square is also a rectangle with equal sides)

(Note : A’s (Child) sides need not be parallel to B’s sides when A is fit inside B).

geometry implementation

Problem #112

Rooted Sums

Find the answer to the following summation:

$$\sqrt[5]{\frac{11+5\sqrt{5}}{2}}+\sqrt[8]{\frac{47+21\sqrt{5}}{2}}+\sqrt[6]{\frac{18+8\sqrt{5}}{2}}-\sqrt[16]{\frac{2207-987\sqrt{5}}{2}}-\sqrt[22]{\frac{39603-17711\sqrt{5}}{2}}+\sqrt[11]{\frac{199+89\sqrt{5}}{2}}+\sqrt[30]{\frac{1860498-832040\sqrt{5}}{2}}+\sqrt[13]{\frac{521+233\sqrt{5}}{2}}-\sqrt[12]{\frac{322-144\sqrt{5}}{2}}-\sqrt[10]{\frac{123-55\sqrt{5}}{2}}-\sqrt[26]{\frac{271443-121393\sqrt{5}}{2}}$$

Let the answer be x. Submit the answer as $\lfloor x\times 1000000000 \rfloor$

Problem #113

Monica's Candy Fever

We all know how much Monica loved candies in her childhood. And, her excessive competitiveness is no big secret! There have been various incidents where the Gellars had brought home some candies (say n). Monica always wanted to have more candies than Ross. Now, the Gellars actually gave the candies randomly to both of them. In spite of the randomness, Monica got more candies than Ross every single time! Now, we want to calculate the probability that Monica had more candies than Ross at every instant that the candies were being distributed. Let the probability of that be $F(n)$. Calculate $x = \sum_{i=1}^{1000000000} F(i)$. Submit $\lfloor x \times 1000000 \rfloor$.

combinatorics   probability

Problem #114

Revenge with Derangement

Suppose that there are N persons who are numbered 1,2,...,N. Let there be N hats, also numbered 1,2,...,N. We have to find the number of ways in which no one gets the hat having same number as his/her number.

Let the number of ways to accomplish the above task be $D(N)$. You need to enter the value of $D(N)$ mod P.

Take N = 49770435560715869 and P = 223092870.

combinatorics   chinese remainder theorem

Problem #115

42: The Answer to Everything?

Represent 42 as the sum of N positive real numbers X_1 , X_2 , … X_N where N is an integer, such that the product, $P = X_1X_2\ldots X_N$ is maximised. Find $\lfloor P \rfloor$ where $\lfloor\ \rfloor$ is greatest integer function.

calculus

Problem #116

Funny Factorial

In a right angled triangle *ABC* right angled at *B*, equations of median *AD* and *CF* are $y = x + 1$ and $y = 2x + 4$ respectively . Given *AC* is 60 units find the last five non zero digit of $(X)!$ where *X* is the area of triangle *ABC*.

geometry brute force

Problem #117

Expectation Frenzy

Suppose an M-faced fair dice is tossed *N* times. Whenever the dice is tossed, each face appears with probability $1/M$. After *N* throws of the dice, let the expected value of minimum number obtained on the dice be E. Enter the value of $\lfloor E*10^6 \rfloor$ where $\lfloor \ \rfloor$ is greatest integer function.

Take M=100 and N=20.

probability

Problem #118

Analyzing Complexity

Problems with which we often deals in programming requires us to approximate number of iterations that we are going to perform in loops.

Given below is a simple pseudo code for which you need to tell the number of iteration performed in the given nested list :

```
for ( int x1 = 1 ; x1 <= n ; x1++ )

   for ( int x2 = x1 ; x2 <= n ; x2++ )

      for ( int x3 = x2 ; x3 <= n ; x3++ )
```

```
    ......
      ......
        ......
       for ( int xm = x(m-1) ; xm <= n ; xm++ )
          it++ ;
```

Take initial value of it as 0 and value of m and n to be 25 and 26 respectively.

combinatorics

Problem #119

Minimum Cubes

We define $f(x)$ = minimum number of positive perfect cubes that sum up to x.

For example,

$f(2)=2(2=1^3+1^3)$

$f(9)=2(9=1^3+2^3)$

$f(17)=3(17=1^3+2^3+2^3)$

Find $\sum_{n=1}^{10^6} f(n)$.

dp

Problem #120

Balance the Weight

You are given a beam balance and N objects where i^{th} object weighs $2i-1$, $0 < i <= N$. $F(N)$ denotes the no. of ways of placing these objects(one by one) on balance such that the left side is always heavy (after every placement). Find the value of $F(100)$ and give the answer modulo 10^9+7.

implementation constructive algorithms

Problem #121

The Game of Fibonacci and Powers of Two

You are given a pile of stones. Two people play this game turn by turn. In any turn, a player can remove a number of stones from the pile, say X, such that X can be either a non zero Fibonacci number or a power of 2.

The person that empties the pile wins the game. Both the persons play optimally, i.e. they try to make the best possible move that will help them win. If the number of stones in the pile varies between 1 and 10^6, in how many cases will the first player lose?

dp games

Problem #122

Never Ending Fraction

Given that

$$\frac{P_n}{Q_n} = 0 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\dots\cfrac{1}{(n-1) + \cfrac{1}{n}}}}}$$

for $n > 0$ (n is an integer) where

$P_0 = 0$ and

$Q_0 = 1$.

Calculate $(P_{1000} * Q_{1000}) \bmod (1000000007)$

number theory recurrence

Problem #123

Recurring Roots

Let $\sqrt{x\sqrt{(x+1)\sqrt{(x+2)....\sqrt{(y-1)\sqrt{y}}}}}$ for $x >= 2$ and $x < y$ where x and y are integers.

Also, let $g(x) = T$ where T is the smallest integer such that $x < T$.

Let $h(x) = 2^x \bmod 1000000007$

Find out $h((g(f(g(f(42,111111111111111)),222222222222222)))))$.

number theory induction

Problem #124

Co-prime judgement

We define $\varphi(n)$ as the number of positive integers less than or equal to n that are relatively prime to n.

Lets define $S(n) = \sum\varphi(d)$ over all the divisors of n (i.e. d divides n).

Also define $F(n)=\sum S(x)$ where x varies from 1 to n.

Compute the value of $F(999999)$ + F(888888)

number theory

Problem #125

Happy String

You are given p a's and q b's. You arrange them in the form of a string. If you are able to get a string in which as you move from left to right, the number of a's is strictly greater than b, then it is known as a happy string.

Let the number of such happy strings be denoted by $happy(p,q)$, then find $happy(200,100)$ * 201! * (101!/300!).

probabilities

Problem #126

Playing with constraints

Let x = last 7 non-zero digit of $99^{99}!$

Define $F(n,k)$ = Sum of kth powers of all divisors of n, so for example $F(6,2)=1^2+2^2+3^2+6^2=50$

Define further $G(a,b,k)$ as: Sum of $F(j,k)$ where j varies from a to b both inclusive

You need to enter the value of $G(1,x*x,2)$ modulo (10^9+7)

number theory

Problem #127

Decamping Twice

Let S be a set where $S = \{1,2,3,4,5\ldots\ldots4444444444\}$ and P be a subset of S such that $x \in P$ but $2x \notin P$. Let $|T|$ be the total number of elements in the set T. Find out maximum $|P|$ of all the possible subsets P.

combinatorics

Problem #128

Fibonacci Fun

Given:

$$a_1 * (e^{x_1}) + a_2 * (e^{x_2}) + \ldots + a_{30} * (e^{x_{30}}) = 321123$$

where a_i = ith fibonacci number, e is the Euler's number and x_i is a real variable (i.e. $a_1 = 1$, $a_2 = 1$, $a_3 = 2$ ..)

Let minimum value of: $e^{2x_1} + e^{2x_2} + \ldots + e^{2x_{30}}$ be Z at $x_i = y_i$ where all x_i satisfy the above condition.

Let $k = (y_1 + y_2 + y_3 + \ldots + y_{30})^4 / Z$

Find the greatest integer $\leq 100k$

number theory fibonacci

Problem #129

Fragger and his sequence

Fibonacci sequence is defined as follow: $F_1 = 1$, $F_2 = 2$, $F_i = F_{i-1} + F_{i-2}$ ($i > 2$)

Every natural number X can be represented in Fibonacci system such that X is sum of one or more distinct Fibonacci numbers in such a way that the sum does not include any two consecutive Fibonacci numbers.

Let $X = A_1 * F_1 + A_2 * F_2 + A_3 * F_3 + \ldots\ldots + A_{k-1} * F_{k-1} + A_k * F_k$ where

$0 <= A_p <= 1 \forall p < k$ and $A_k = 1$

We will represent X in Fibonacci system as : $A_k A_{k-1} A_{k-2} \ldots\ldots A_1$ Example $1 = (1)_F$, $2 = (10)_F$, $3 = (100)_F$, $4 = (101)_F$

If we write representation of all natural numbers in Fibonacci system consecutively we will obtain a new sequence (say fragger sequence) which will look like this : 1,1,0,1,0,0,1,0,1…….

You need to count the number of times 1 appears in the first 10^{15} terms of fragger sequence.

fibonacci dp

Problem #130

The Start

Let $S_n = (\sum_{k=1}^{n} a_k)^2$, where $a_k = (k*(k+2))^{-1}$. Let $S_n = (p/q)$, where p and q are positive integers and gcd(p,q) = 1. Find the value of $p*q$ for n = 112358.

series summation

Problem #131

Crazy sum

Let $f(n) = \sum d|n\ \mu(d)h(n/d)$ where $h(x)$ = Sum of divisors of x

$g(n) = \sum d|n\ \mu(d)f(n/d)$

Find $\sum_{n=1}^{10000000000} g(n)$ modulo 2^{64}

Note: here $\mu(n)$ is Mobius Function.

series summation

Problem #132

Functions

For a complex number z, given a function $f(z) = \sum_{i=1}^{n} \frac{1}{i^2}$

Let p_j be the j-th prime number $g(z) = \prod_{j=1}^{m} (1 - p_j^{-z})$

Let $x = \lim_{n,m\to\infty} f(z) * g(z)$

Find the value of $\sum_{i=1,\gcd(i,k)=x}^{i=k} 1$, where k = 354216846978542365.

series functions

Problem #133

The drunk boyfriend

In a single dimension world, a girl is standing at the point $x = 0$. Her boyfriend is standing at some point $x = x_0$. He is drunk. So, he either takes a step towards her girlfriend or a step away, with equal probability. But he can't go beyond $x = n$, because of a wall placed there, So whenever he reaches $x = n$, he has to go back to $x = (n-1)$. Let the expected number of steps for him to reach his girlfriend be $F(x_0,n)$ where x0 is his starting point.

Find the value of $\sum_{i=1}^{N} F(i, N)$ for $N = 10^7$

probability

Problem #134

Triangles Counting

Find the sum of all positive integers up to 10^8 which can be expressed as a hypotenuse of a right angled triangle where other two sides are also positive integer.

number theory

Problem #135

Polynomial Expansion

We wish to generate 1000000 pseudo-random numbers s_k in the range $\pm 2^{19}$, using a type of random number generator (known as a Linear Congruential Generator) as follows:

```
t := 0
```

```
for k = 1 up to k = 1000000:

     t := (615949*t + 797807) modulo 2^20

s_k := t-2^19
```

Thus: $s_1 = 273519$, $s_2 = -153582$, $s_3 = 450905$ etc.

We need to find a polynomial $P(x)$ of lowest possible degree such that $P(i) = s_i$ for all integers $i <=$ 1000000

Enter the value of $(P(1)+....P(1000100))$ modulo (1000000007)

polynomial   binomial

Problem #136

Equate It

Let a, b, c be positive integers such that $a \neq b \neq c$ and a divides b^{69}, b divides c^{69} and c divides a^{69}. Find min m such that that abc divides $(a+b+c)^m$ for any a, b, c satisfying the above condition.

number theory

Problem #137

Harry Potter and Horcrux

In a two dimensional map Harry initially is standing at some point denoted by character 'H'. Also there is a horcrux at some point of map denoted by character 'X'. In order to protect that horcrux You-Know-Who has appointed several Death Eaters at certain points of map denoted by character 'D'.

Harry need to find the horcrux but he is not knowing the exact position of horcrux and hence he will walk in any of the four direction (left, right, top, bottom) with equal probability from his position as long as he stays in map and not go to any place where Death Eaters reside. Points on which Harry can walk will be denoted by 'W' in map.

Harry is determined to find the horcrux and he will not stop until he finds it. You need to find expected number of steps Harry will take to find the horcrux.

There are 20 maps in "maps.txt". For each map first its dimensions are given m,n which indicates map size $m \times n$ .After which map is given in described format. NOTE: There always exist a path between Harry and horcrux for map given in input.

Let the sum of expected value for all map be P. Enter the value of floor($P*10^3$). Link to **maps.txt**

probability graph

Problem #138

Mul Me

Let the number of ordered quadruples satisfying this equation be $f(n)$

$a*b*c*d <= n$

$f(1) = 1$

$f(2) = 5$

Find $f(10^{12})$

number theory

Problem #139

Sum Me

Let $F(n)$ = sum of all divisors of n.

Find the sum of all n such that $F(n) = 2*n$ and $1 <= n <= 5*10^{35}$

Report your answer modulo 10^9+7.

number theory

Problem #140

Bored with GCD

Define $P(m)=\sum_{k=1,\gcd(k.m)=1}^{m} k$

Also $F(n)=\sum_{i=1}^{n} P(i) \bmod i$

Find the value of $\sum_{i=1}^{12} F(10^i) \bmod 1000000007$

number theory gcd

Problem #141

Complex Summation

An nth root of unity, where n is a positive integer (i.e. n = 1, 2, 3, …), is a number z satisfying the equation:

$z^n = 1$

An nth root of unity is primitive if it is not a kth root of unity for some smaller k:

$z_k \neq 1$ (k = 1,2,3,...,n−1)

Let $S(n)$ be the sum of all the primitive nth roots of unity.

Define $F(n,k)=\sum_{i=1}^{n} i^k S(i)$

Enter the value of $F(10^{11},10^3)$ mod 1000000007.

number theory algebra complex numbers

Problem #142

Inequality Marathon

Let $S=X_1,X_2,X_3\ldots\ldots X_k$ where $0<=X_i<=1$ for $1<=i<=k$ and $\sum_{i=1}^{k} X_i = 1$.

Also $0<=P(x)<=1$ for all $x \in S$. Given that $\sum_{i=1}^{k} P(X_i) = 1$.

Let us say T = Max $(1/(\sum_{i=1}^{k} (X_i)^2))$

Also, $F(x) = \sum_{i=1}^{k} P(X_i)\log(1/P(X_i))$ (log with base 2).

Given that Max($F(x)$)=13.

Find (T!) mod 1000000007

probability inequality

Problem #143

Exponential Fibonacci

Let F_n be the nth fibonacci number where $F_0=0$ and $F_1=1$.

Also let $a = 3^{3^{43}}$ and $b = 3^{3^{42}}$.

If $F_x=\gcd(F_a,F_b)$ where x can be represented as 27^y.

If $(F_m)^3 - (F_n)^3 = F_y - (F_{y/3})^3$

Find the value of (m*n) mod 1000000007.

fibonacci matrix exponentiation

Problem #144

Circular paradigm

Let the number of arrangements of $3n$ identical Type A balls and 3 identical Type B balls in a circle be $F(n)$. Find $F(100000)*F(100001)$.

combinatorics

Problem #145

Vasu And Houses

We have a total of n-1 persons (P_1,P_2,...P_{n-1}) and n houses (H_1,H_2,H_n).

Find the number of ways for n=6 such that each person visits only 1 house and the following conditions are satisfied:

1. Each person visits a different house

2. Person P_i can't visit House H_i

3. P_1 can't visit House H_n

derangements

Problem #146

A Very Easy Sum

A number is called special if when prime factorized as $p1^{a1} * p2^{a2} * \cdots * pk^{ak}$, then a_i<=2 for all 1<=i<=k.

Let $S(N)$ be the sum of all special numbers from 1 to N.

Given $S(10)$=47 and $S(10^4)$=41586160

Enter $S(123456789123456789)$ modulo 10^9+7.

number theory summations

Problem #147

Divisor Sum

Define $F(n,0)=1$

$F(n,k)=\sum_{d|n} F(d,k-1)$, where $d|n$ means d is a divisor of n.

Let $S(n,k)=\sum_{n=1}^{N} F(n,k)$

Given $S(9876,234) \bmod 1000000007 = 208863976$

Enter the value of $S(98765432,2345678) \bmod 1000000007$

number theory　divisors　summations

Problem #148

Levels in Tree

Consider a tree with n nodes rooted at node 1. We define parent[i] as the parent of ith node. For each $2<=i<=n$, parent[i] can assume any value from 1 to $i-1$ with equal probability. Let $F(n)$ the expected value of the sum of levels of all nodes. 1st node is at level 1 and for each $2<=i<=n$, if node i has level x, parent[i] has level $x-1$.

Given $F(10)=29.28968$

Enter your answer as the greatest integer less than or equal to $F(12345678)$.

probabilities

Problem #149

Fibonacci Sum

Define: $F(0)=0$

$F(1)=1$

$F(n)=F(n-1)+F(n-2)$, $n>=2$

Define $S(N,K)=\sum_{n=0}^{N} F(1+n*K) \bmod 1000000009$

Given $S(10^{12},100)=878943097$

Enter the value of $S(221^{221^{10^{18}}}, 55^{55^{10^{18}}})$

fibonacci modular roots

Problem #150

Fun with Expectation

Consider an array of n numbers where each element can be any non negative x bit number (0 to 2^x-1) with equal probability. Let $F(n,x)$ be the expected value of the sum of bitwise xor of all possible subsequences of the array.

Eg. Consider the array 1,2,3.

Sum of xor of all possible subsequences of this array = 1 + 2 + 3 + 1^2 + 1^3 + 2^3 + 1^2^3 = 1 + 2 + 3 + 2 + 1 + 3 + 0 = 12.

Note: ^ is the bitwise xor operation here.

$F(2,2)=4.5$

Enter your answer as floor($F(29,30)+F(30,29)$).

probabilities bits

Problem #151

The Pebble Game

Given a Directed Acyclic Graph (DAG) , at a given instant of time we can do one of the following two operations:

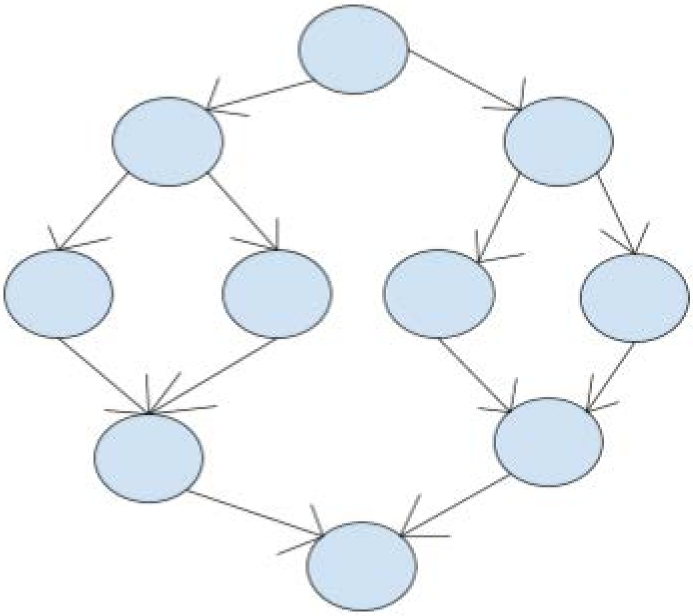
1) Place pebble on a node,if all its immediate ancestors have a pebble

2) Remove pebble from a node.

Initially, there are no pebbles on any node of the DAG.

Our goal is to minimize the maximum number of pebbles over time such that finally, pebbles are on all leaf nodes (nodes with no outgoing edges) . Assume all DAGs of the form: 2 equal height full binary trees, one kept inverted. The leaves of each binary tree are at the same level.

Eg.: This is a DAG made to resemble 2 binary trees of height 2.



Say $F(n)$= Answer for a DAG similar to 2 binary trees of height n.

Find $\sum_{i=0}^{20} F(i)$

graphs

Problem #152

Modular Test

Let $F_p(x)$ = maximum k such that p^k divides $x!$

Let $G_p(x)=p^{Fp(x)}$

Let $Q(N) = \left(N!/\left(G_2(N)\cdot \mathrm{G}_3(N)\cdot G_7(N)\cdot G_{101}(N)\right)\right) \bmod 7711956$

Let $S(N) = \sum_{n=1}^{N} Q(10^{16} + n\cdot 10^5)$

Enter the value of $S(10^5)$

chinese remainder theorem factorial

Problem #153

Costly Coins

Akhil has 100000000 coins in his FlipCart. He spreads them on a table. Initially all of them are facing head. Daga selects any 34117 coins at random and inverts all of them. Daga repeats this process 1000 times.

What is the expected number of head after the process?

Submit the integer part of the expected number (ignoring the part after decimal point).

expectation

Problem #154

Vikas and Cows

Vikas has a rectangular farm of dimensions a*b in 'Amazon' forest. He also has 4 cows. The 4 cows are very aggressive. So he ties his cows to the 4 corners of the farm through ropes. The length of ropes are r1, r2, r3, r4. He wants to maximise the grazing area. But, as the cows are aggressive, he does not want the cows to fight.

$F(a,b)$= Maximum area of the farm that can be grazed by the cows

$$S(a,x)=\sum_{b=a}^{x} F(a,b)$$

Output $\lfloor S(4,1000) \rfloor$

geometry

Problem #155

Irreconcilable Differences

Saurv4u and Daenerys were a loving couple. But due to 'Irreconcilable Differences', the couple is thinking of 'breaking' up.

To measure the depth of the differences D, Saurv4u has shaded the inner region of the rectangles (The sides of the rectangle are parallel to the Cartesian axes and the center of rectangle is origin) such that the perimeter of rectangle is less than 4*a , while Daenerys has shaded (on the same plane) the inner region of all the rectangles (The sides of the rectangle are parallel to the Cartesian axes and the center of rectangle is origin) such that the area of rectangle is less than 3*a*a/4.

The measure of the depth of differences D is the common area bounded by the two.

Given the value of a = 1000, find the value of $\lfloor D \rfloor$

$\lfloor x \rfloor$ means the greatest integer <= x

geometry

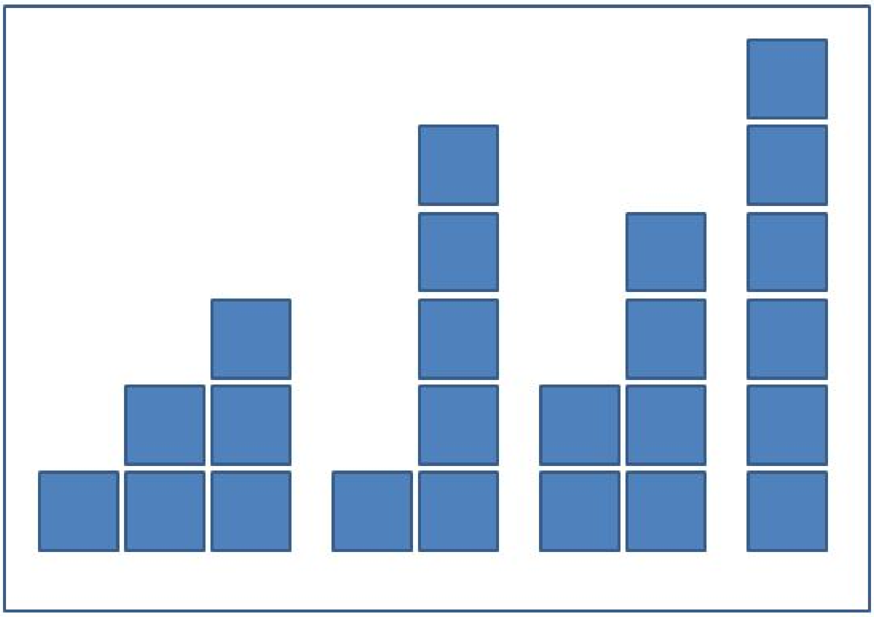
Problem #156

The Next Big Tower

Princi is playing 'Make Towers with Blocks'. She wants to make towers and then jump from one tower to the next.

The rules of the game are: She has N blocks. She can make any number of towers with them but the heights of the towers must be in a strictly increasing order i.e. height of any two towers must not be equal (so she can jump higher and higher).

Now she wants to find the number of ways in which she can make towers like this with $N = 196$ blocks (the different ways for $N = 6$ are shown in the image). Since, she is busy doing her work, you have to find the answer for her.



For n = 6

dp

Problem #157

For the love of tomatoes

Mishra loves to eat tomatoes. He likes tomatoes so much that he shares them with no one (not even, well, no one). Mishra likes to store his tomatoes safely. He puts all his tomatoes in boxes and puts all boxes in a 3D Cartesian space on the coordinates (i,j,k) such that $1<=i<=N$,$1<=j<=N$ and $1<=k<=N$ So there are a total of N^3 boxes.

Now, adkroxx wants to steal his tomatoes. He has the following information:

- Number of tomatoes in each box is different.
- Number of tomatoes in any box A_{ijk} is $1<=A_{ijk}<=N^3$

- The number of tomatoes in each row along positive x axis is strictly increasing.
- The number of tomatoes in each row along positive y axis is strictly increasing.
- The number of tomatoes in each row along positive z axis is strictly increasing. Now, before stealing the tomatoes, adkroxx defines ADK_i as the product of all possible values of A_{iii}

$P(N)$ is given by:

$$P(n) = \sum_{i=1}^{N} ADK_i$$

Find $P(N)$ for $N = 216$. Give the answer modulo 10^9+7 .

brute force

Problem #158

Best Hacker in the World

Foxtrot and Vampire decided to play a game to decide who is a better hacker. They have n tiles in a line out of which m are white rest all are black. Their task is to change all the black tiles with white by following a simple procedure. They can change a black tile with white only if there is atleast one white tile adjacent to that black tile. Now they have $n = 74$ and $m = 13$ and the 13 positions (indexed 1) of white tiles are 6,14,19,20,21,24,30,43,58,61,69,70,73

They want to find the total number of ways in which it can be done. Help Vampire find the answer as he cannot come out in daytime.

Report your answer modulo 10^9+7.

combinatorics

Problem #159

Revenge of The Fox

After losing to Vampire, Foxtrot wants to take revenge.

Vampire has 30 piles of coins, each contains N number of coins. Every gold coin weighs 10 grams, while counterfeit coin weighs 9 grams. Some of the piles contain all counterfeit coins. He has a digital scale capable of accurately weighing any number of coins. He knows that at most 3 of the 30 piles are counterfeit. He needs to find the minimum value of N so that, with just one measurement, he can guarantee identifying which (if any) piles are counterfeit. Vampire is busy in some work and says if Foxtrot can solve this for him, he will declare Foxtrot as the best hacker.

Find the answer for Foxtrot.

Problem #160

Such Sum Wow

Vaibhav is participating in MCA PCIC - largest international Math competition where he gets this problem:

Given N equations of the form:

$a_i \cdot x_i^2 + b_i \cdot x_i + c_i = 0$, i varies from 1 to N,

where $a_i = b_i = i$th Fibonacci number and $c_i = (i+2)$th Fibonacci number.

Let $g_i = (-1) \cdot (2 \cdot a \cdot x_i + b_i)^2$ for all the equations.

$$h(N) = \sum_{i=1}^{N} g_i$$

Submit $h(N) \bmod (10^9+7)$ for $N = 987654321342198766$.

Note: Fibonacci series:

Fib[0] = 0, Fib[1] = 1, Fib[i] = Fib[i−1] + Fib[i−2]

Now Vaibhav doesn't like maths, so you have to solve it for him.

fibonacci

Problem #161

Binary Play

Consider a set $S(N)$ such that $S(N)$ contains all sequences of 0 and 1 of length N in which no two $1'$s are adjacent. For example:

$S(3)$=(000,001,010,100,101)

Let A = No. of elements in $S(30)$

B = Sum of digits of A

You have to output no. of 0 appearing in Bth sequence of $S(30)$ when arranged lexographically.

fibonacci dp

Problem #162

One Last Game

During a game of Mafia, after Verma is killed by Mafia in the first round, he starts getting bored and keeps disturbing others in the game. This irritates everyone else and to keep the game interesting, Akhil asks Adarsh to keep Verma occupied in something else.

So Adarsh gives him a $1*N$ matrix which has N cells. Also, he gives him M colours numbered from 1 to M . Now, Verma can choose any submatrix with even number of cells from the matrix and colour it with one of the M different colours. He can colour as many times as he wants . If Verma tries to recolour any cell which is already coloured, the cell gets coloured in the last colour.

Adarsh asks Verma to make as many distinct matrices that he can make such that every cell is coloured. Each matrix is different from other if atleast one of the cell has a different colour.

Find the number of matrices Verma can make and submit the answer modulo 1000000007

Take N = 99999999999999 and M = 999999999

combinatorics

Problem #163

Bored with Shinigami

Ryuk is bored with the Shinigami world and goes to the human world to find something interesting. He is fascinated with the GCD function and fibonacci numbers and starts playing with them.

Fibonacci numbers are defined as :

$F(n) = F(n-1) + F(n-2) \quad \forall n \geq 2$

with $F(0) = 0$ & $F(1) = 1$.

Ryuk wants you to find the value of $\gcd(F(10^{12185} - 17^{2185}), F(10^{14807} - 17^{4807})) \bmod 1000000007$.

Problem #164

Kill the remaining Chinese

Kira has killed all criminals in Japan and has now started killing Chinese criminals. He kills Z criminals every week Where $X = 100!$ and $X^{9449771607341027425} \cong Y \bmod 9449771616229914661$

$Z = Y \bmod 1000000008$.

As a supporter of Kira, help him by finding Z.

Problem #165

L's Successor

L is fighting Kira which can be dangerous as Kira has the Death Note. Watari has to decide a successor for L in case L dies. Watari gives a point (x,y,z) in cartesian plane to Mello and Near and they have to play a game such that they take turns. In one turn, the player can change one of the coordinates (say w) to$[w/2]$,$[w/3]$,$[w/5]$.($w > 0$) So (8,9,10) can be reduced to (4,9,10) or (2,9,10) or (1,9,10) or (8,9,3) and so on. The person who changes the point to origin wins the game. Watari wants Near to win but he forgot the z coordinate so he randomly selects z between 0 and 12345 (both inclusive). So the point is L(2^{60}, 2^{61}, z), where z is between 0 and 12345 (both inclusive). Given that Near starts play calculate the probability that Near will win the game if both players play optimally.

If the answer is of the form u/v where u and v are coprime you have to input $u+v$ as the answer.

Problem #166

Apples for Ryuk

Light Yagami has to buy apples for Ryuk without attracting any unwanted attention towards himself. Interestingly, the number of apples he buys everyday is always a Fermat Number. A Fermat number is a number of the form $2^{2^n}+1$. $G(N)$ = sum of digits of Nth Fermat Number modulo 9 Find the sum of $G(i)$ for all i between 1 and 999999999999 (both inclusive).

Problem #167

The Perfectionist Yagami

L gives Light the following challenge and claims that if he is unable to solve it, he is Kira. Help Light deceive L by solving the challenge. A 3-perfect number is defined as a number whose sum of divisors equals thrice that number.you have to fiind the sum of all 3-perfect numbers of the form $n = 2^k 3p$ that are less than 10^9, where p is an odd prime number and k is positive Integer. Give the answer modulo 10^9+7.

Problem #168

L's Ideology

L is sure that Light is kira and Misa is second Kira . He randomly selects two permutation of 1 to N and store it in $A[0],A[1],...A[N-1]$ and $B[0],B[1],...B[N-1]$. All permutations are equally probable . Now he randomly selects a integer X, $0 <= X < N$, and shifts permutation B to left by X.

For Example - [2,3,1,4,5] be a permutation , shifting it left by 2 will result [1,4,5,2,3].

He defines another sequence

$g[i] = (A[i]*1) \bmod B[i] + (A[i]*2) \bmod B[i] + (A[i]*3) \bmod B[i] ...... (A[i]*B[i]) \bmod B[i]$.

Find expected value of $g[0]+g[1]+........+g[N-1]$ for $N = 1111$.

Answer greatest integer of expected value.

Problem #169

Attendance and Probability

As we all know adkroxx is a great programmer of our institute. But due to programming he started missing a lot of classes. Now the institute decides to offer him cash reward for good attendance. If he is absent for 4 consecutive days or late on more than one occasion they will forfeit his prize. During an n-day period a ternary string is formed for each day consisting of L's (late), O's (on time), and A's (absent).

Adk being a smart kid tries to find out what is the total number of prize string that exists. Although there are eighty-one ternary strings for a 4-day period that can be formed, exactly forty-seven strings would lead to a prize.

Now you have to help adkroxx to find out number of prize strings for $n = 10000007$ and report the answer modulo 10^9+7.

Problem #170

Magical Distance

Consider all points with integral positive coordinate (x,y) which lies on the curve

$|x^2 - 4xy - y^2| = 1$

Let P_i be the coordinate of ith point when all the points satisfying above equation are sorted in ascending order according to their distance from the origin.

Let D_i = distance between (0,0) and P_i

If $S(n)=\sum_{i=1}^{n} D_i^2$

Enter the value of $S(10^{15})$ mod 1000000009.

Problem #171

Random Name

$$g(x)=2^{2x^3}-1$$
$$f(x)=\sum_{i=1}^{x} g\left(\left\lfloor \frac{x}{i} \right\rfloor\right)$$
$$F(x)=\sum_{i=1}^{x} f(i)$$

Find $F(12345678)$ mod 10^9+7

Problem #172

Polynomial Expansion 2

Consider $f(x)=1+5x+6x^2+3x^3+10x^4+11x^5-19x^6-2x^7$

Consider $g(x)=x^5-1$

Let $R(k,x)$ be the remainder obtained when $f(x)^k$ is divided by $g(x)$.

Example:

$$R(1,x)=12-14x+4x^2+3x^3+10x^4$$
$$R(2,x)=-112-247x+352x^2+60x^3+172x^4$$

Let $S(k,x)$ be the sum of coefficient of $R(k,x)$.

You need to enter the value of $S(18^{10^{18}}, x)$ modulo 100086841.

Problem #173

Permutation mania

Permutation p is an ordered set of integers $p1,p2,...,pn$ consisting of n distinct positive integers, each of them doesn’t exceed n. We will denote the ith element of permutation p as pi. We will call number n the size or the length of permutation $p1,p2,...,pn$.

We will call position i ($1 \le i \le n$) in permutation $p1,p2,...,pn$ good, if $|p[i]-i| = 1$.

Your task is to count the number of permutations of size n with exactly k good positions.

Give the answer for $n = 1000$ and $k = 700$ modulo 1000000007.

Example: for $n = 3$ and $k = 2$ answer will be 4.

(1,3,2),(3,1,2),(2,1,3),(2,3,1) all have exactly 2 good position.

Problem #174

Dabba and Dhakan

Dabba loves Maths and his girlfriend Dhakan, so he either spends his day with dhakan or does maths all day. But he hates spending two consecutive days with dhakan (because she is a dhakan). He wants to make his schedule for his summer vacation as a sequence of doing maths or spending day with dhakan.

No of days of his summer vacation can vary from l to r. You need to count in how many ways Dabba can select k different schedule of the same number of days for his summer vacations, whose days can vary from l to r.

For example if $k = 2$, $l = 1$ and $r = 2$, if we define Maths day as {1} and Dhakan's day as {0}, here are all possible combination: {0}, {1}, {00}, {01}, {10}, {11}. But '00' can not be selected because it has 2 consecutive Dhakan's day in a row. Now, we need to count in how many ways we can select $k = 2$ schedule of the same length in range [1,2]. Here they are: for no. of days = 1 {0,1}; no. of days = 2 {01,10}; {01,11}; {10,11}. so answer for $k = 2$, $l = 1$ and $r = 2$ will be 4.

He wants you to tell him in how many ways can do that, modulo 1000000007 for $k = 200$, $l = 1$ and $r = 10^{18}$.

Problem #175

The Rocking Problem

Bunty is trying to solve a problem and he is unable to do it so he asks you to solve his problem.

Let $F(n)$ is the number of subsets of $(1,2,3....n)$ that contain no consecutive integers modulo 1000000007.

Now you have to report $G(F(100000007))$;

$G(n)$ = number of binary sequences of length n that have no consecutive 0′s modulo 1000000007.

Problem #184

Color problem

Mr. Pink likes summation series and Mr. White likes to take big powers of numbers. In order to satisfy both of them Mr. Brown want you to evaluate the following sum

$$\sum_{i=1}^{n} i^k = 1^k + 2^k + \cdots + n^k$$

modulo 10^9+7 for $n = 10^9$ and $k = 10^6$

Problem #186

Euler and Fibo

$$S(a,b) = \sum_{k|a,k|b} \Phi(k)$$

where $k|x$ means k divides x, $\Phi(n)$ is Euler totient

$$Z0(n) = \sum_{a=1}^{n} \sum_{b \geq a}^{n} S(a,b)$$

$F(n) = F(n-1) + F(n-2)$ for $n > 1$

$F(0) = 0$ and $F(1) = 1$

$$Z1(n) = \sum_{a=1}^{n} \sum_{b \geq a}^{n} S(F(a), F(b))$$

Find $Z0(n)^{Z1(n)}$ mod10^9+7 for $n = 10^8$

Problem #189

Night King and The Army of the Dead

Night king is marching with an army of the dead. The Night King has N deads in his army and they are labelled from 1 to N. Jon Snow is trying hard to stop Night King to cross the wall and destroy Westeros. Jon figures out that killing a walker leads to the disintegration of some of his associated wights.

If Jon kills the kth dead all the dead with a label which divides k will be disintegrated. For N = 123456789, what is the minimum number of kills required to clear the army?

Problem #190

Semi Prime Sum

What is the sum of semi-primes up to 10^7.

Semi Prime is a number of the form $a*b$ where a and b are both prime numbers.

Problem #191

Save Jon Snow

Jon Snow hands Gendry a message, tells him to go to Eastwatch and send a raven to Daenerys to rescue them. Currently, Gendry is at (0,0) and the castle is situated at (N,N). In order to escape the battlefield, he can only move from (i,j) to $(i,j+1)$ or $(i+1,j)$ or $(i+1,j+1)$.

Find the number of paths that can be taken by Gendry to reach the castle modulo 10^9+7 for $N = 10^7$.

Problem #192

Colorful Matrix

Consider a 2000*2000 matrix which is to be filled with 15 colors. Find the number of ways to color the matrix. Two colorings are same if rotating one of them along the axis perpendicular to the plane (in multiples of 90 degrees) gives the other. Give the answers modulo 10^9+7.

Problem #193

Bend the knee or Play

Jon has a well-shuffled deck of $M = 4999$ white cards and $N = 4999$ green cards. Each permutation of cards in the deck is equally likely. Dany has set some rules for Jon. Every time Jon flips a white card he gets one coin, otherwise he loses one coin.At any moment (even at the beginning), Jon is allowed to stop playing the game and keep the number of coins that he has.During the game-play the balance of coins that Jon has may be negative. Let k be the expected amount of coins Jon will have if Jon plays optimally. Evaluate $[k*10000]*123456$.

[x] represents the greatest integer <=x.

Problem #194

Integers on a roll

Let $F(a)$ = Number of integral points on the equation is $x^2 + y^2 = a$.

Find the sum of $(a*d*k)^{(F(17455420182 15^a * 2017^d * 408^k))}$ for all values 1<=a,d,k<=100 modulo 10^9+7.

Problem #195

Flip 'em all

Consider a row of N cards all facing up. Now a player flips all the multiples of 1, then 2 and so on till N. Find the number of cards facing up if N = 12345678987654321.

Problem #196

Game of Thrones

In game of thrones you either win or you die. Each one of Jon, Dany, Cersei and Euron are ruling one of the four kingdoms. Ruler who gets to rule two kingdoms sits on the iron throne. Dany decides to attack one of the other kingdoms so that she can sit on the iron throne. But there is a catch. There are three dragons distributed among the 4 rulers in any possible way. (Dany herself can have all the three dragons or maybe she won't have any!) All the dragons are alike and there is no difference between them.

If Dany attacks on a kingdom whose ruler has more dragons than her, she loses. The probability that she sits on the throne is of the form p/q where p and q are coprimes and q is not equal to zero. Submit the ans p^q modulo 10^9+7.

Problem #197

Kill the King

Daenerys has 3 dragons and 9 XORgons (a creature that can breathe any number from the range [0,123456789]). If multiple XORgons throw numbers to a target at a time then the resulting number will be the XOR of the individual numbers thrown by each XORgon. Surprisingly the night king can be killed by the number 123456789. Help Tyrion to find out how many ways all 9 XORgons can throw number to the night king such that night king can be killed.

To cut the long story short how many unordered 9−tuples (a,b,c,d,e,f,g,h,i) are there such that $a \wedge b \wedge c \wedge d \wedge e \wedge f \wedge g \wedge h \wedge i$ = 123456789 and a,b,c,d,e,f,g,h,i are numbers from the range [0,123456789]. Submit the answer modulo 1000000016000000063.

$\wedge$ represents the bitwise XOR operation.

Problem #198

XOR Again

Find the sum of $i \wedge j$ modulo 10^9+7 for all integers i and j in [0,1073742824].

$\wedge$ represents bitwise XOR operation.

Problem #199

Zero and Big Bits

Zero is a very powerful person. He has N countries (out of total 197) under his control all of which are identical to him. Once he counted the number of ways of arranging all the countries in a list of size 197 (modulo M) and stored the number in a computer (in binary format) as Zero likes bits. Zero also likes big numbers: More the number of set bits, the happier Zero will be. His happiness is equal to number of set bits in the number. The answer is his happiness.

The world knows only about modular arithmetic with $M = 10^9+7$. Calculate all the values modulo M. Here $N = 191$.

Problem #200

Stick to the Bases

$F(i,j)$ = sum of positive integers (in base 10) having an identical representation in both base i and base j.

$F(2,3) = 1$

Let $S(N)$ be the sum of all values of $F(i,j)$ for $2 \le i < j \le N$

Find $S(N)$ mod 22011663 for $N = 10^{18}$

Problem #201

Tricky Game

TNBT likes playing with sticks, so once he broke a stick into 3 pieces and calculated the probability of the pieces forming a triangle with much difficulty. He now wishes to calculate the probability of the pieces NOT forming an N-sided polygon given the stick was broken into N pieces. Since he found it difficult for 3 pieces, tell him the answer for $N = 733$.

If the probability is p/q where $\gcd(p,q) = 1$, the answer is $[q/p]$ modulo 10^9+9.

$[x]$ represents the greatest integer $\le x$.

Problem #202

Power and Modulo

$F(k) = 5^k \bmod (10^{k'})$ where $k' = k \bmod 6$

Let $S(N)$ be the sum of $F(k)$ for all $1 \le k \le N$.

Find $S(10^{18})$ mod 10^9+7.

Problem #203

Couples

Swegwan is getting more naughty day by day. Recently he has come to know about the new variety of Sal trees that can share emotions. So he is highly excited about making their couples. He is God so he knows the tree's gender. Because he has enough free time he counts the number of male and female Sal trees now and then. Since it's very hard to know the exact positioning help him find the expected number of probable couples p. Every adjacent male and a female tree is a probable couple for Swegwan. According to him, there are 6367 male trees and 6571 female trees (all planted in a row).

For example, MT FT MT FT has 3 probable couples.

Give your answer as $[p*10^4]$.

$[x]$ is the greatest integer $\le x$.

Problem #204

Combination Addition

$$S(N) = \sum_{n=0,k=0}^{2k\le n\le N} \binom{n-k}{k}$$

Find $S(10^{18})$ modulo 10^9+7.

$\binom{n}{k}$ represents the binomial coefficient.

Problem #205

Prime Sum

$P(N)$ represents the largest prime factor of N.

Find the sum of $P(k^3 + 1)$ for $1 \le k \le 10^7$ modulo 10^9+7.

Problem #206

Finding Primes

Find the smallest prime number p such that there exist an integer x which satisfies, $(x+8) * (x+10) * (x+12) = p$

Problem #207

Unlucky base

Given a number a, consider the set $S(a) = \{a^i : i >= 0\}$. You create a sorted sequence G formed by taking any number of distinct elements of the set $S(a)$ and adding them. Let the k^{th} smallest number which can be constructed using this for a given a be denoted as (a,k).

Find the answer for (a,k) = (13,423732713), submit your answer modulo 4222236787

The elements of the sequence G go as follows 1,13,14,169,170,182,183...

Problem #208

Just Mod it!

For all the integers in range 1 to 10^{18}. Find out the number of integers n such that $n^n \bmod (100) = n$
Note: Here, mod refers to the remainder operator and not the relation.

Problem #209

Harry and Magic Squares

Harry and Hermione find an ancient square in the forest. Its vertices V1, V2, V3, V4 are each labeled with a number.

Hermione hates asymmetry, and she wants all numbers to be equal. Fortunately, Harry carries a magic wand with him. In one spell, he can choose any two adjacent vertices and increase their labels by one (each). He is allowed to use any amount of spells until the numbers become equal. Unfortunately, Harry is not so good at math, and he does not know if it is possible to make them equal.

Harry tells you that the initial values of the four numbers can be randomly distributed between 1 and 1000. Find the probability that he can use these spells to make all four labels of the vertices equal. Note that there is no restriction on the range of the final values of the magic numbers. If this probability can be expressed as p/q, where p and q are coprime numbers, enter the value of p.

Examples:

Case 1:
8 6
8 6
Here, Harry can apply the spell twice to make the four numbers equal to 8. Both spells act on the top-right and bottom-right vertices.

Case 2:
1 3
4 5
Here, any amount of spells will fail to achieve the end-goal of making all four labels equal.

Problem #210

Circle and his friend tangent

Consider two points A0 and A1 which are on a unit circle C centered at origin. Where point A0 is (1,0) and A1 is such that angle between positive direction of X-axis and radius vector through A1 is 1°.

The points Ai ($i >= 2$) are obtained in the following way :

Consider vector Vi=A(i−1)−A(i−2), if A(i−1) and A(i−2) coincide take V as the tangent at A(i−1). Ai is obtained by intersection of circle with line which passes through Point A0(1,0) and is parallel to Vi.

Find the angle between X-axis and radius vector through An for n = 123456789987654321. Give answer in degrees (Only output the integral part of the answer, if your answer is 19.23 enter 19 (19.0 will be wrong)).

Problem #212

Functional Permutations

Given a permutation p of length n, we define the following functions

$S(p) = \{i : p(i) > p(i+1), 1 \leq i \leq n-1\}$

$f(p)$ = Sum of elements of $S(p)$.

Let $F(n,k)$ = number of permutations p of length n with $f(p) = k$.

Find $F(123,321)$ modulo 10^9+7

Problem #213

Lonely Expansion

Let $f(r,n)$ be the number of co-efficient which appears only once in expansion of $(x_1+x_2+...+x_r)^n$

then, $f(3,2) = 0$ and $f(3,3) = 1$

Let $A(i) = \sum_{j=1}^{n} f(i, j)$

and $B = \sum_{i=1}^{n} A(i)$

Find the value of B modulo m if n = 1000000000000 (10^{12}) and m = 998244353.

Problem #214

Easy Expressions

Find the value of the following summation

$$\sum_{x=1}^{10^{12}} (x^2 + x + 1) \cdot x!$$

$x!$ denotes factorial of x.

Give your answer modulo 3^{20}

Problem #215

Go for Gold

Sparky is yet again being accused of being a robot. He needs to solve this question to prove his innocence.

Given a number x and has defined the following functions:

$p(x) = x!$

$s(x) = 1 + 2 + 3 + ... + x$

Find the value of $p(x)$ mod $s(x)$ for $x = 10^9+6$

Hint: 10^9+7 is a prime number

Problem #216

Simple Alphabet

Manas is the smallest Panda on this planet. So he loves writing on the wall. Today Manas is presented with a special wall which is infinitely long and which resembles a notebook with horizontal lines at a distance 10 units apart. Since he knows only 3 letters of the English alphabet, “O”, “N”, “I”, he starts writing these characters on this wall in any angle at any position. He writes “O” 677 times, “N” 733 times and “I” 779 times. We define a cut as the intersection of character with the horizontal lines on the wall. Find the expected number of cuts Manas will make.

The dimensions of the characters:
“I”: Line segment of length 1 unit.
“O”: Circle of radius 0.5 unit.
“N”: Smaller line segments have length 1 unit and longer line segment has a length 2 units.

If the answer is p, give the answer as $\lfloor p \times 10^5 \rfloor$

Note: A single character can make multiple cuts, for example, “O” can never make a single cut.

Hint - https://brilliant.org/wiki/linearity-of-expectation/

Problem #217

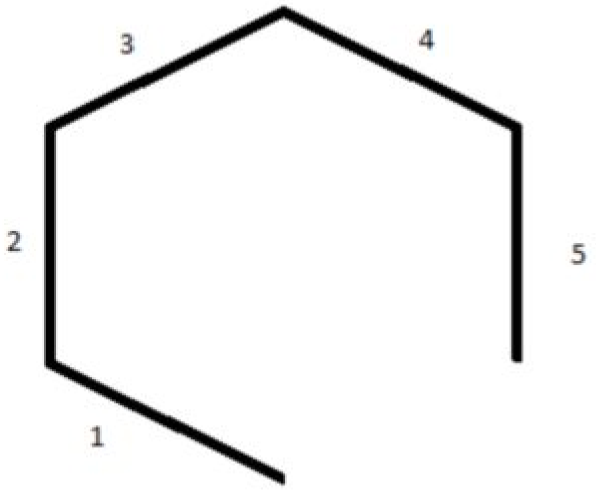
Stick Game

There are N sticks ($N \geq 3$) of same size kept such that they make a regular polygon. Now one of the sticks from the polygon is removed to make it an open polygon.

Alex and Bob are playing a game (starting with Alex) in which they perform the following moves. A player chooses a stick and removes it. Now we have two sets of connected sticks that were originally in contact with the removed stick. The one with the smaller length is also removed with along with the selected stick. If both of the sets are of same size any one is removed. The person who can not make a move in their turn loses.

Example for N = 6:

One of the sticks is removed and the figure looks like this.



Now, if the chosen stick is terminal one like 1 or 5 then only that particular stick is removed. If some other stick is chosen (say stick numbered 2) then on removing there will be 2 sets of connected sticks. The set with less number of sticks gets removed too, and thus the stick with number 1 will also be removed.

You are required to find how many starting positions are there below 10^{10^6} such that Alex loses, assuming that both Alex and Bob play optimally.

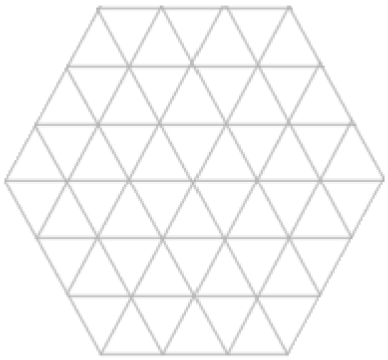
A tutorial on Game Theory

Problem #218

Counting Hexagons

A regular hexagon with integer length n units is divided in $6n^2$ equilateral triangles with sides of length 1 unit and this hexagonal lattice formed has a total of $3n^2+3n+1$ lattice points.

For n = 3, refer to the image below



Let $H(n)$ denote the number of *regular hexagons* that can be formed by connecting any 6 points. Let $S(n)$ denote the summation of $H(k)$ for $1 \leq k \leq n$.

Given $H(3) = 36$ and $S(10) = 7942$, Calculate $S(10^{18})$.

Give your answer modulo 10^9+7.

Hint: Can you observe any pattern?

Problem #219

Divide and Destroy

Given n and k, find the number of a sequences of integers $a_1, a_2, \ldots, a_{k+1}$ such that a_{i+1} divides a_i for $1 \leq i \leq k$ and $a_1 = n$, $a_{k+1} = 1$.

Find the answer modulo 10^9+7 for $(n,k) = ((1101^{17})\cdot(2019^{29}), 3\cdot 367\cdot 673)$.

Problem #220

Fibonazi

We define,

$f(i) = f(i-1) + f(i-2)$ and,

$F(i) = (i^2)f(i)$

Find $\left(\sum_{i=1}^{N} F(i)\right) \bmod (10^9 + 7)$

$N = 10^{18}$, $f(1) = 1$, $f(2) = 1$

Hint - https://www.geeksforgeeks.org/matrix-exponentiation/

Problem #221

Harvey's Shot

Harvey is set on a mission to test his luck. To test a person's luck there is a circular room which is marked with 180 points with numbers 1, 2, 3, …, 180 written on them on the circumference equidistant from the adjacent points. Harvey stands at the point 1 on the boundary with a gun in his hand. He shoots the gun aiming inside the circle at any of the marked points from 2 to 90 (90 included). Assuming an elastic collision between the bullet and the wall the bullet reflects as soon as it touches any point on the circumference. If the bullet reaches the point 1 it hits Harvey and he dies. The bullet stops when it reaches a point where the number written on it is smaller than that of the number written on the point from which it was reflected.

Given that Harvey chooses the point where he shoots randomly, let the probability that Harvey dies in the game be m/n (m and n are coprime). Enter your answer as $m \cdot n$.

Problem #222

Houdini and Picking Cards

Houdini has a deck consisting of 10^6 cards, where the i-th card (starting with 1) is labeled with the number i. He randomly picks two cards from the deck, and wants to find the probability that one of the card labels is divisible by the other. The probability can be written in the form m/n where m and n are coprime integers. Enter your answer as $m + n$.

Problem #223

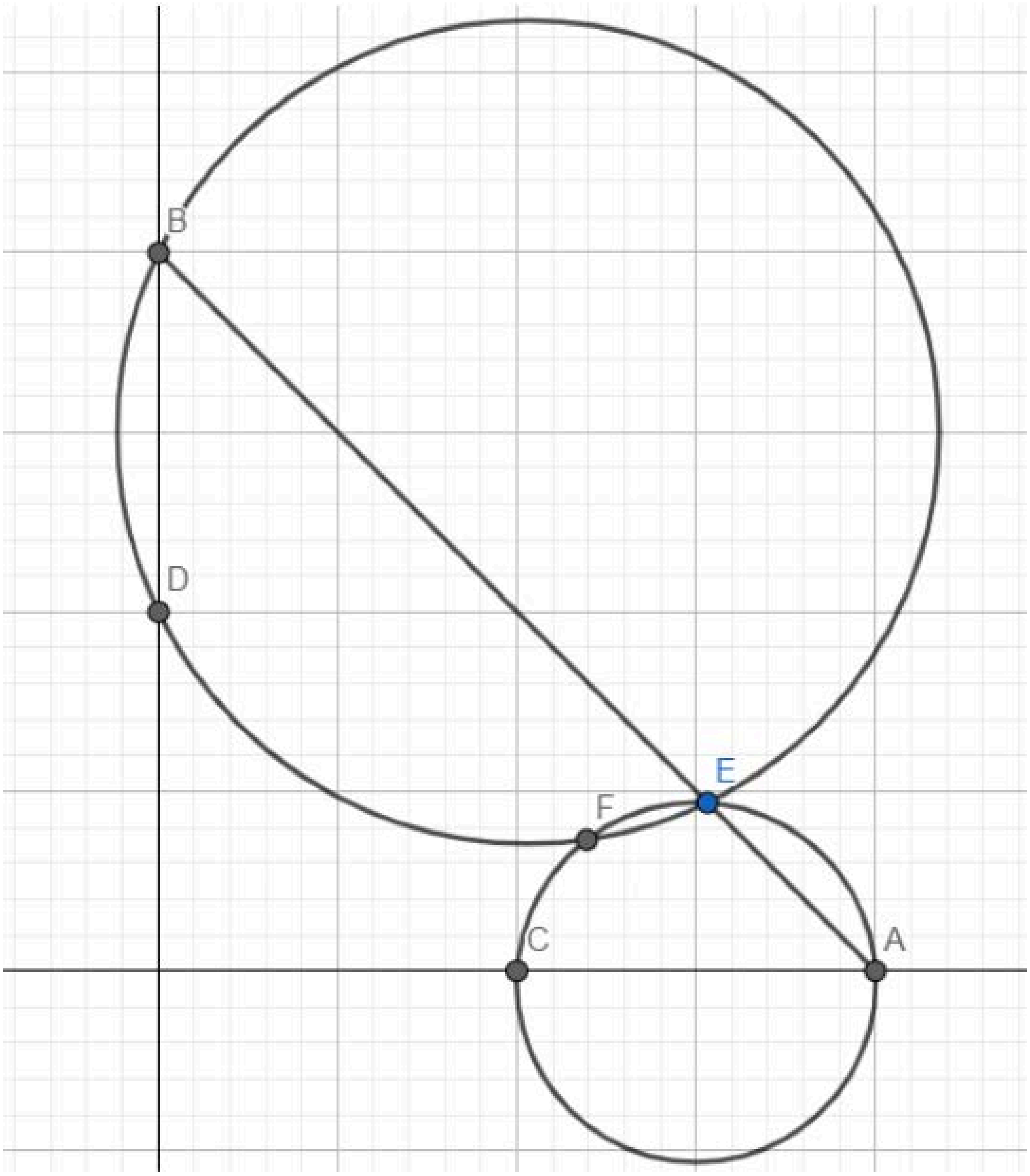
Lines and a Point

You are given a set of lines $S := x(6a+b)-y(11a+2b)=3a-183b$ for all real values of (a,b) and a point $(713,732)$. A closed curve is formed by taking the reflection of this point from all the lines in set S.

Let A be the area enclosed by this curve, find A/π.

Problem #224

Circles and a Point



Given integer a = 100.

Consider points A($2a$,0), B(0,$2a$), C(a,0), D(0,a) and a line AB.

Point E is on the line at point (x,y) and the circles intersect again at point F.

As Point E moves from A to B let the distance traveled by point F be d.

Find [d]. Where [.] is the greatest integer function.

Note: A new pair of circles is formed for every location of E using the points ACE and BDE. The second point of intersection is labeled F

Problem #225

Totient Fun

Find the first integer n such that $\varphi(n) = \varphi(n+1) = \varphi(n+2)$.

Enter the answer modulo 1729.

$\varphi(n)$ is totient of n.

You can read about the totient function here: Euler Totient Function

Problem #226

The Pass

The King of Zugzwangtria is dead in a war. Since the king had no successors, his existing marvellous brothers Dratox and Keraze are claiming for the throne. Dratox being the elder has the right on the throne but his younger brother Keraze refuses and threatens to war.

The wise ministress Freixola suggests both the players to play a game, whoever wins would get the throne. Both agree to this! The game is as follows:

First Dratox picks up a natural number from 1 to N, call it '*a*'.

Then Keraze choses a real number '*b*' of the form $\frac{\mathrm{a}^{1+\sqrt{i}}}{i^{\sqrt{a}}}$ (where i is some integer from 1 to N).

Then Freixola tries to pass the golden stone through the magical ring. If the stone can possibly pass through the ring then Dratox wins, else Keraze wins.

The golden stone is a rigid regular tetrahedron of side length '*a*' and the magical ring is a rigid circle of radius '*b*' (negligible width).

Let X be the total number of *a*'s for which Dratox has a guaranteed win.

Enter X^X mod 10^9+7, $N = 10^{123456789}$

Problem #228

Generating Sets

For given numbers n, a and a set S, define the set $a^S = \{a^t \bmod n : t \text{ is in } S\}$.

For n = 998244353 and S = Set of natural numbers, find

$$\sum_{i=1}^{n} \left| i^S \right|$$

Give ans modulo 10^9+7.

Note: $|X|$ denote the size of set X.

Problem #229

Powerful Roots

Let $f(i) = \begin{cases} f(i-1) + 2f(i-2) + 4f(i-3), & i > 3 \\ i, & i \le 3 \end{cases}$

We define a polynomial $p(x) = x^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$ where $a_i = f(2^i) \bmod M$

Let the roots of $P(x)$ be $b_1, b_2, ..., b_n$

Let

$$S(k) = \sum_{i=1}^{n} b_i^k$$

Find $S(k) \bmod M$. for n = 60, k = 123456789987654321, M = 998244353

Problem #230

Points

There are K points and M lines such that

- Every line contains 8 points
- Every point lies on 8 lines
- Any two distinct lines intersect in a unique point
- Any two distinct points lie on a unique line.

Lines may be straight or curved. What is $K*M$?

Problem #231

Maximal Set Reduction

Let $S = \{1, 2, 3, ..., 2019\}$. Find the maximum value of x such that when any of the x elements are removed from the set then there exist two distinct elements in the remaining set having their sum equal to 2019.

Problem #232

A Long Walk

You have a set S that stores distinct points.

Let the points consisted in a walk with 30 teleportations be

$(x_0, y_0), (x_1, y_1), ..., (x_{30}, y_{30})$

$(x_0, y_0) = (0, 0)$

$(x_{30}, y_{30}) = (X, Y)$

For $1 \le I \le 30$ $(x_i, y_i) = (x_i - 1, y_{i-1} + 2^{i-1})$ or $(x_i, y_i) = (x_i - 1 + 2^{i-1}, y^{i-1})$

A walk is called a valid walk when Y lies in the range [123456789, 987654321].

In a valid walk for $0 \le i \le 29$, insert point $(X - x_i, Y - y_i)$ in set S.

Find the maximum size of set S by taking 28 valid walks.

Problem #233

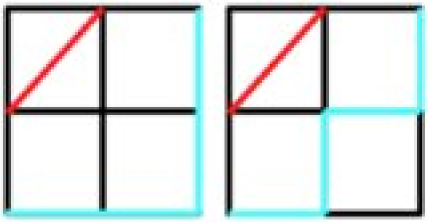
Manhattan Count

Let $f(n, x)$ be a function that denotes the number of ways to travel from (0, 0) to (n, n) in a 2-dimensional grid without touching/crossing the diagonal formed by the points $(0, x)$ and $(n - x, n)$. From a given point (a, b) you can either travel to $(a + 1, b)$ or $(a, b + 1)$ in one step.

Define $g(n) = \sum_{i=1}^{n} f(n, \mathrm{i})$

Calculate $g(100000) \bmod 10^9+7$

Example: $f(2, 1) = 2$, The two valid paths being –



Problem #234

Expected GCD

You have 2019 cards in a deck numbered as 1, 2, 3, …, 2019. You draw two cards from the deck without replacement. Let x be the expected value of the gcd of the two numbers. Report your answer as [1000*x], where [x] denotes Greatest Integer Function of x.

Problem #235

Divisor Sum

Let $d(n)$ be the smallest prime divisor of n.

Calculate

$$\sum_{k=1}^{15120} d(15120! + k)$$

Problem #236

Frequency Restriction

Let $f(n, m)$ count the number of sequences $(a_1, a_2, \ldots, a_m)$ of length m and $1 \le a_i \le n$, such that for each $1 \le i \le n$, $\mathrm{cnt}(i) \le i$, where $\mathrm{cnt}(i)$ is the number of times i occurs in the sequence.

Find $f(50, 1000) \bmod 10^9+7$.

Problem #237

Another Permutation Statistics

Let $a(n)$ be the number of permutations π of length n satisfying $\pi(y) = x$ if and only if $\pi(n - x + 1) = y$ for all x.

Find $\sum_{k=1}^{10^6} a(k)$ mod 10^9+7

Example: $a(4) = 2$ and two permutations are : 2413, 3142

Problem #238

Power GCD Sum

Evaluate :

$$X = \sum_{i=1}^{n} \left[\sum_{d|i} \frac{\left(\sum\limits_{\substack{0 \le k \le i \\ \gcd(k,i)=d}} a^{k} \right)}{\left(\sum\limits_{\substack{0 \le k \le i \\ \gcd(k,i)=d}} a^{-k} \right)} \right]$$

for n = 7777777, a = 788788.

Since X may be large. Give your answer as X mod 10^9+7.

Problem #239

Divisor Count

The function $d(n)$ denotes the number of positive divisors of an integer n. For example, $d(6) = 4$, because there are 4 divisors of 6 and they are 1,2,3, and 6.

We create a function $f(n)$ which denotes “The summation of the number of divisors of the divisors” of an integer n.

For example, $f(6) = d(1) + d(2) + d(3) + d(6) = 1 + 2 + 2 + 4 = 9$.

Find $\sum_{i=1}^{2019} f(i!)$ mod 10^9+7, where $n!$ means factorial of n.

Problem #240

Sieve of Pi-thagoras

Everybody knows the Sieve of Eratosthenes, but does anyone know the Sieve of Pi-thagoras? In the following pseudocode, assume that “real” represents a real number with infinite precision.

```
real sieve_of_pithagoras (integer n) {
    real ans = 0;
    for (i = 1; i <= n; i += 1) {
        for (j = i; j <= n; j += i) {
                ans += 1 / j^4;
        }
    }
    return ans;
}
```

Bunty is a kid living in the year 3019. He has access to computation power so immense that he can run this code for as large an integer as possible. You challenge Bunty to find a finite integer input n so that it returns at least r. Deep down, based on your math skills, you know that this isn't possible for any $r \geq r_0$. You are required to find $[10^{18} \times r_0]$ modulo 10^9+7, where $[x]$ denotes the integral part of x.

"^" in the code implies the power operator.

Problem #241

Bunty's Algorithm

Bunty recently learned about the significance of $d(n)$, the smallest prime divisor of n, through which he can quickly extract prime factorisations of numbers!

He sits down and drafts an interesting sequence of integers where the integers are bound between the values 2 to 1,000,000. To him, a sequence a_i is interesting if a_i is strictly increasing and the corresponding sequence $d(a_i)$ is also strictly increasing.

Bunty attempts to write all interesting sequences but soon realises that there are too many of them. Can you, the algorithmist, count the number of interesting sequences modulo 10^9+7?

Problem #242

Unlucky Seven

Find the remainder obtained when 77…7 (10^{16} times) is divided by 12345678.

Problem #243

Determinant

Let $A_{n\times n}$ be a square matrix of order n = 12345.

The elements of the matrix are defined as $A_{ij} = |i - j|$.

Find $|A|$ mod 10^9+7, where $|A|$ denotes the determinant of the matrix A.

Problem #244

Fun with Divisors Sum

Let $f(a,b)$ denote the sum of divisors of $ab+1$ which lies in the range (a,b).

For example, $f(3,23) = 5 + 7 + 10 + 14 = 36$.

Calculate $\sum_{i=1}^{p-1} f(i,p)$ for p = 1000000007.

Problem #245

Sequence Triples

Let f_m denote the number of sequence triples $(\{x_0, x_1, \ldots, x_m\}, \{y_0, y_1, \ldots, y_m\}, \{z_0, z_1, \ldots, z_m\})$ such that:

1. $x_0 = y_0 = z_0 = 0$

2. Sequences x_i, y_i, z_i are all strictly increasing

3. $x_m + y_m + z_m = 4040$

Then calculate $\sum_{i=1}^{\infty} f_i$ mod 10^9+7.

Problem #246

Bella Ciao!

El Professor has a solid plan to heist Royal Mint. Rio needs to hack into its digital security system. Rio chooses a random number between 5 and 10^{10} (both inclusive). Given two numbers $A = 94069$ and $B = 50549$, the system can be hacked if the number is chosen can be expressed as $Ax+By$, where x and y are non-negative integers. What is the probability Rio hacks into the system?

Report your answer as $\lfloor \text{probability} \times 10^{12} \rfloor$, where $\lfloor x \rfloor$ denotes the integer part of x.

Problem #247

Too Difficult

Given a sequence a consisting of n integers, an inversion is defined as a pair of indices (i,j) such that $i < j$ and $a[i] > a[j]$. Let inv(a) denote the total number of inversions in a. A permutation p of order n is a sequence p of size n such that for any $1 \leq i \leq n$, there exists a valid index j such that $p_j = i$. For example, the sequence $p = (1,3,2,4)$ is a permutation of order 4, and it has 1 inversion, namely (2,3).

Calculate the sum of inv(p) over all distinct permutations of order $n = 15$. (Two permutations p and q are different if there is an index i for which $p[i] \neq q[i]$.)

Problem #248

Largest Angle

Given three points $A(10^{17},10^{17})$, $B(3\times10^{17},3\times10^{17})$ and $C(X,0)$.

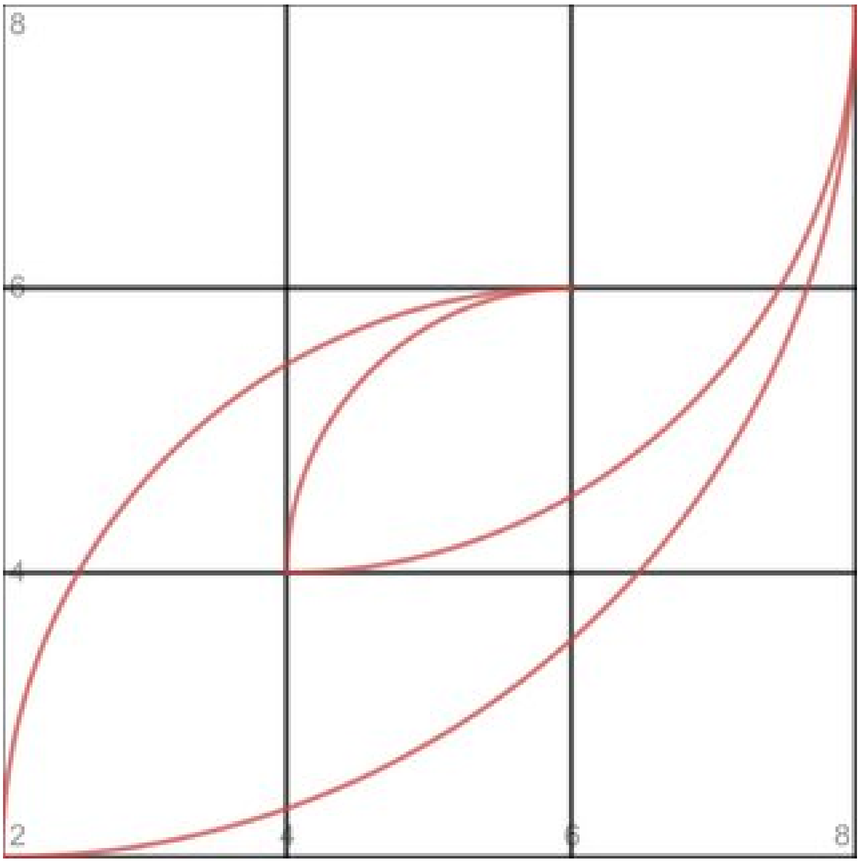
If $\angle ACB$ is the maximum possible, calculate $\left\lfloor \sqrt{X} \right\rfloor$

where $\lfloor x \rfloor$ denotes the integer part of x.

Problem #249

Mysterious Figure

One mysterious night a random shape and a box appeared on a square piece of land. Upon close inspection, it was realized that the shape was in fact not at all random and held the passcode for the box. You being the crypto expert were called to open the box. The box read as follows: “The enclosed red area holds the answer”. You are provided with the length of the inner squares, which is 98765. Please find out the passcode, floored down to the nearest integer.



Problem #250

Random Expectations

Adarsh decided to attend CEN-105 lecture feeling he will learn something new. It turns out the class is extremely boring. He begins writing a random number starting with "0.". After the decimal point, he writes a long string of numbers, formed by randomly choosing any prime between 1 and 9 repeatedly (Ex. 0.7223557333…).

Let the expected number formed be represented as p/q, where p and q are coprime integers, find $p + q$.

You can learn about expected value.

Problem #251

Ricket Numbers

Rick is working on an important mission but Beth has asked him to look after Morty. Rick doesn't want to be disturbed so he wants to keep Morty busy. He knows how much Morty loves challenges and how much he hates numbers, so he asks Morty to "help" him by counting all the rickety numbers upto 12345678.

A number N is said to be Ricket if and only if it is composite and there exists a way to permute all its divisors (except 1) onto a circle such that the GCD of any two adjacent elements is greater than 1.

Problem #252

Pegs on Board

We define $f(R,C)$ as follows:

Consider a $2R \times 2C$ chessboard. $f(R,C)$ is the number of ways in which you can place $R \times C$ pegs on the white coloured squares such that no two pegs are diagonally adjacent to each other.

Calculate $f(12345678,87654321)$ modulo 1000000007.

Problem #253

Another Board Game

Annu wanted to invent a board game to play with his girlfriend Chunnu on their next date. He came across the following problem while trying to look for optimal coloring for his board game.

Consider an $N \times N$ board where each cell can be coloured black or white. Let x_i denote the number of white cells in the ith row and y_j denote the number of white cells in the jth column. Define $f(N)$ as the maximum possible value of the sum:

$$\sum_{i=1}^{N} x_i \cdot (N - y_i)$$

Calculate $f(13579)$.

Problem #254

Small permutations

For a permutation p of size n, an index r is called 'small' if $p(r) = \min(p(1), p(2) \ldots p(r))$. Calculate the number of permutations of size 4040 with exactly 2000 small indices modulo 1000000007.

Problem #255

Infinite graph

Let $G = (V,E)$ be an infinite undirected graph whose vertices are ordered pairs of integers, that is $V = \mathrm{Z}^2$. Given two natural numbers a and b, two vertices (x_1,y_1) and (x_2,y_2) share an edge if and only if either one of these conditions hold: 1. $|x_1 - x_2| = a$ and $|y_1 - y_2| = b$, or 2. $|x_1 - x_2| = b$ and $|y_1 - y_2| = a$

Let's define the following functions

$f(a,b)$ be 1 if the graph is connected (there exists a path between any two vertices), and 0 otherwise, and $g(i) = \sum_{j=1}^{10^6} f(i, j)$

Evaluate $\sum_{i=1}^{10^6} g(i)$

Problem #256

Many jumps

Anya is currently at $x_0 = 100$. She will do 10 jumps as follows:

If after i jumps, she is at coordinate x_i, then she will jump to some coordinate in the range $[0,x_i]$ chosen uniformly.

Let $\mathrm{P}([a,b])$ denote the probability that after 10 jumps Anya lies in the range $[a,b]$. Also, let

$$L = \lim_{x \to 0} \frac{P([1, 1+x])}{x}$$

Find the value of $\lfloor L \cdot 10^6 \rfloor$, where $\lfloor x \rfloor$ denotes the integer part of x.

Problem #257

Subset union

X is a set of size 202020. Let $S_1, S_2 \ldots S_{100}$ be subsets of X, such that:

For any distinct indices a, b, c, the union of S_a, S_b, S_c is not equal to X.

For any distinct indices a, b, c, d, the union of S S_a, S_b, S_c, S_d is equal to X.

Find the maximum value of $|S_1| + |S_2| \ldots + |S_{100}|$.

Problem #258

Centroids

Rishak was getting bored during the lockdown. He wanted to text someone special, but his internet was not working, so he started drawing instead. He drew a rectangle $ABCD$ and then labelled G_1 as the centroid of $\triangle ABC$ and G_2 as the centroid of $\triangle ACD$. The length of G_1G_2 was 377427. Also, the perimeter of the rectangle $ABCD$ was measured as 3184398. Can you find out the area of the rectangle?

Problem #259

Teleportation

Nik wants to meet Anya, and so he fires up his teleporter. The teleporter works by independently choosing a random real number between 0 and 1 thrice. Suppose the three numbers it chooses are X,Y and Z. The teleporter then transports Nik to $x = \max(X,Y,Z)$.

As Anya is smart, she will wait for Nik at the coordinate where the expected distance from her to where Nik lands will be the minimum. If she waits at $x = L$, find $\lfloor L \cdot 10^9 \rfloor$, where $\lfloor x \rfloor$ denotes the integer part of x.

Problem #260

Totient sum

Let

$$L = \sum_{n=1}^{\infty} \frac{\varphi(n)}{2020^n - 1}$$

Here $\varphi(n)$ is the totient function. It can be shown that L can be represented as P/Q, where P and Q are coprime integers, and $Q \not\equiv 0 \pmod{1000000007}$. Find the value of $P \cdot Q^{-1}$ modulo 1000000007.

Problem #261

Occupying Houses

There are 65 houses arranged along a street and numbered from 0 to 64. Initially, only the 0^{th} and 64^{th} houses are occupied and the rest are empty. Then, one by one 63 buyers come to purchase the remaining houses. Each buyer can choose to occupy a house lying in the center of two consecutive occupied houses. In other words, if the a^{th} and b^{th} houses are occupied and no houses between them are occupied, then the buyer can occupy the $(a+b)/2^{th}$ house.

Calculate the number of ways all the remaining 63 houses can be occupied modulo 1000000007. Two ways are different if the order of occupying the houses is different in both ways. (Eg. the 2^{nd} person can either occupy the 16^{th} house or 48^{th} house, both of which are different ways.)

combinatorics recursion

Problem #262

Jumping Frogs

A circular array consists of 7 points such that the distance between any two neighboring points along the circumference is 1 unit distance. Two frogs are currently lying on the array.

Every second, each frog jumps to one of the adjacent points, with equal probability. Let $f(x)$ be the expected time after which both the frogs occupy the same position in the array if initially, they were x unit distance apart.

You are required to find the value of $f(1)\cdot f(2)\cdot f(3)$.

probability expected value

Problem #263

Subset Intersection

Let S be a set of size n. Let T_k be the set of all ordered k-tuples $(A_1, A_2, A_3, A_4, \ldots, A_k)$ where A_i is a subset of S, such that $A_1 \cup A_2 \cup A_3 \cup A_4 \ldots, A_k = S$.

Given $n = 2718281828$ and $k = 3141592653$, find the sum of $|A_1 \cap A_2 \cap A_3 \cap A_4 \ldots \cap A_k|^2$ for all k-tuples $(A_1, A_2, A_3, A_4, \ldots, A_k)$ belonging to T_k. The answer can be huge, so compute it modulo 10^9+7.

combinatorics

Problem #266

The Set 128

Let T be an array of size 127 with $T[i] = i$ (T is 0 indexed), In other words $T = \{0, 1, 2, \ldots, 126\}$. Let M be a set of numbers such that:

$$M = \left\{ \frac{a_1}{127} + \frac{a_2}{127^2} + \frac{a_3}{127^3} + \frac{a_4}{127^4} : a_i \in T,\ i = 1, 2, 3, 4 \right\}$$

All the numbers in M are arranged in ascending order. Let x be the 20212021st term. Answer the nearest integer of $x \cdot 10^7$.

Problem #267

Sum it up Rationally!

Let $F(p)$ be the number of different rational numbers X in the range (0,1) such that when X is written as an irreducible fraction, the numerator and denominator sum to 10^p. In other words, if $X = a/b$, such that a and b are coprime, and $a + b = 10^p$

Find summation $F(i)$ from $i = 1$ to $i = 10^{10}$, modulo 10^9+7.

Problem #268

Hypothetical Dice

Jaideep is playing a game with 1234567-sided dice, its sides are numbered from 1 to 1234567 and each side has an equal probability of appearing. First, he rolls a die and if the value obtained on the die was k, he takes k dice and rolls all of them. Then find the expected value of the sum of values on all $k + 1$ dice.

probability expected value

Problem #269

Escaping the Elf

You are trapped at the center of a big circle of radius 10^9 units by a mischievous magical elf. Every minute he asks you to choose a direction in which you want to move. In the ith minute, you will be allowed to move exactly 'i' units in that direction.

However, the elf wants to make it hard for you to escape, so every time you choose any direction he decides whether he wants to allow you to move in the chosen direction or in the direction exactly opposite to your choice. So, if you choose to move towards the left, then the elf can force you to move towards the right, or may allow you to move left.

You must take exactly '*i*' steps in the *i*th minute. Find the time in minutes required to escape if both you and the elf play optimally.

Note: 'Escaping' refers to reaching a point that is outside of the circle.

geometry

Problem #270

Yet another sum problem

Let $S = \sum_{j=0}^{p} {}^{p}C_j \cdot {}^{p+j}C_j \cdot 3^j$.

Calculate S mod p^2 for $p = 10^9+7$

combinatorics

Problem #271

GCD Fun

Let $f(x_1, x_2, x_3, \ldots, x_n) = x_1 \cdot x_2 \cdot \ldots x_n \cdot 2^{\gcd(x1,\, x2,\, \ldots,\, xn)}$. Let S_n be the sum of $f(x_1, x_2, x_3, \ldots, x_n)$ over all sequences $(x_1, x_2, x_3, \ldots, x_n)$ where $1 <= x_i <= 10^6$. Find S_{31415} modulo 10^9+7.

gcd

Problem #272

The Quirky Binomial

$$f(n)=\sum_{i=0}^{\lfloor n/2 \rfloor} {}^{n-2i}C_{2i}$$

Find:

$$\left(\sum_{i=0}^{10^6} f(n)\right) \bmod \left(10^9+7\right)$$

binomial coefficient observation

Problem #280

777 Strikes Again

Given N = 777...777 (19 digits). Find the number of values $r (0 \le r \le N)$ such that ${}^{N}C_{r}$ is divisible by 31. Since the number of such values can be huge, compute the answer modulo (10^9+7)

Problem #281

Open Sets

Let n=192837. For each i from 1 to n, let x_i and y_i be two independently chosen random numbers from (0, 1). Let $a_i = \min(x_i, y_i)$, $b_i = \max(x_i, y_i)$, and S be the intersection of open intervals (a_i, b_i). In other words,

$$S=\bigcap_{i=1}^{n}(a_i, b_i)$$

It is easy to see that S is itself an open interval. Find the expected length of S.

It can be shown that the answer can be expressed as p/q, where $\gcd(p, q) = 1$ and $q \neq 0 \bmod 998244353$. Output $pq^{-1} \bmod 998244353$.

Problem #282

Game Time

Let $a_{i+1} = (c * a_i + d) \bmod m$ where $c = 5447196$, $d = 1953840$ and $m = 998244353$ be a pseudo random number generator for $0 \leqslant i \leqslant N-3$ where $N = 100000$ and $a_0 = a_{N-1} = a_N = 0$

One day, Taniya created a game in which there were N blocks arranged sequentially. A player starts at the 1st block. To go from the ith block to the $(i+1)$th block, the player has to take a jump. The jump will succeed with probability p, meaning that the player will go to the $(i+1)$th block with probability p, otherwise he will have to start from the beginning and will be landed at the 1st block. The game ends when the player reaches the Nth block. Further, there is a penalty of a_i coins each time a player lands on block i. Taniya was not sure what value to pick for p and so she randomly selected a number from [0.5, 1]. Calculate the Expected number of coins a player has to pay before the game ends.

It can be shown that the answer can be expressed as p/q, where $\gcd(p, q) = 1$ and $q \neq 0 \bmod m$. Output $pq^{-1} \bmod m$.

Problem #283

Count the Assignments

Mark is working on a battleground game where he needs to develop a lobby-making system which assigns players to the teams. Since this game requires extreme teamwork, there needs to be at least 2 players in each team. However, there is no restriction on the maximum number of players that can be assigned to a particular team.

Mark has decided to have 100 players and 25 teams in the lobby. He wants to know the number of ways to assign these players to the teams. Since the answer can be large, you are required to find the answer modulo (10^9+7)

Two assignments are considered different if it is possible to find a pair of players, who belong to the same team in one assignment and to different teams in another assignment.

Problem #284

Beautiful Grids

Let there be a 2718281×2718281 grid of squares with some squares coloured black and others coloured white. It is not possible to have unicoloured grids, i.e there must be at least one square of each colour in this grid.

A grid is called beautiful if it looks the same even when the entire square is rotated by 90∘ anticlockwise around its center any number of times. A beautiful grid also looks the same when it is reflected across a line joining mid points of opposite sides or a line joining opposite corners. Find the number of possible beautiful grids. Since the answer can be large, compute the answer modulo (10^9+7)

Problem #286

Tiling Game

Alice and Bob are playing a game on an $n \times m$ grid. Initially, the grid is empty. On each move, a player can place either a 1×1 domino, or a 1×2 domino (either vertically or horizontally), such that the new domino does not partially or completely overlap with any previously placed domino. Alice moves first. The player who cannot make a move loses the game.

For an $i \times j$ board, define $f(i, j)$ =

1, if Alice wins with optimal play.

2, if Bob wins with optimal play.

Calculate $\sum_{i=1}^{989898} \sum_{j=1}^{989898} f(i, j) \cdot (i + j)$

Problem #287

Heptagonal Distances

Let there be a regular Heptagon inscribed in a circle of radius 357 units. Find the sum of the square of distances between all pairs of vertices of the Heptagon. In other words, let v_1, v_2, v_3...v_7 be the vertices of the Heptagon, and D_{ij} be the distance between vertex v_i and v_j. Find $S = \sum_{1 \le i < j \le 7} D_{ij}^2$

Problem #288

Stonks

The stocks of a company were valued at N = 123456 on day 0. Every day for the following M = 217127 days, the value either rose by 1 or decreased by 1. Note that the value never remained the same. After M days, the final value was observed to be 172213.

Determine the number of distinct ways in which the stock value could have changed over the M days, such that it never exceeded L = 200000.

Since the number of ways can be huge, print them modulo 10^9+7.

Problem #289

The Cubone Problem

A cube of side 12345 units is inscribed in a right circular cone having ratio of Height to Radius as 9876:1. It is inscribed such that one face of the cube is contained in the base of the cone. Calculate the Radius R of the Cone.

You are required to find $[10^5 \times R]$ where $[x]$ denotes the integral part of x