RoseCode

Unofficial Offline Edition

Problem Set From rosecode.neocities.org

Revision **v2024.1**
July 2024

Preface

Time may end, but hope will last forever.

About RoseCode

This document collects all the problems from https://rosecode.neocities.org (RoseCode), which is a website that is along the lines of ProjectEuler. Unfortunately, only the remaining content of the problems can be seen on the website now, and a lot of useful information has been lost.

Due to various reasons, the content of some of the problems are incomplete, and some of them are even unworkable. In addition, the reference answer string is processed by the original answer (see next section), and quite a few answers are misplaced.

Despite all obstacles, the author tries his best to ensure that the content of the problems and the corresponding answers are complete and correct.

Note for the answer

The *answer* to each problem is processed from the original answer string. The following example Python code shows the process of getting the final answer.

```python
from hashlib import md5

f = lambda x: md5(md5(x.encode('ascii')).hexdigest().encode('ascii')).hexdigest()

ans = '123456'
print(f'{ans}\n{f(ans)}')
```

The running result is

```
123456
14e1b600b1fd579f47433b88e8d85291
```

This means that if the original answer string is **`123456`**, after processing the final answer string is **`14e1b600b1fd579f47433b88e8d85291`**.

1 Find a Prime

2009-09-06 00:46:15	by elasolova	5 xp	ProgrammingArchive

Prime numbers are numbers that are only divisible by 1 and themselves. The first prime number is 2. The 10000-th is 104729.

Find the 78200-th prime number.

Answer e30928c4d3d1c530d66198f016081714

2 Fibonacci Run

2009-09-06 02:02:30	**by elasolova**	**5 xp**	**ProgrammingArchive**

Fibonacci terms are generated by adding previous two terms. So by starting with 1, 1 the sequence would be 1, 1, 2, 3, 5, 8, 13, 21, ... and so on.

Find the first Fibonacci term that exceeds 100 in its digital sum (sum of digits).

Answer **5abae30126b49addb6b07816bd0f1e9a**

3 Palindromic Primes

2009-09-06 15:08:15	by elasolova	6 xp	ProgrammingArchive

Palindromic prime is a prime number when reversed is the same prime. For example, 191 is one. Assume all one digit primes are palindromic.

Find sum of all palindromic primes not exceeding one million.

Answer 861b23fd3712b71971f9a803936904b4

4 Distant Twin Primes

2009-09-08 01:46:01	by elasolova	6 xp	ProgrammingArchive

The pair $3, 5$ is special as it is the first prime pair to have difference of two (Twin primes). After this, comes pair $5, 7$. The distance between these two pairs is 2 as it is calculated by subtracting the first member of each pair. Under 100, the pair $59, 61$ has the longest length from such previous pair $41, 43$ with the distance of 18.

Find longest distance of such prime pair, each member below one million, from such previous pair.

Answer 34e2cf40964474f394fc07e94bd32923

5 Fractions to Decimals

2009-09-24 03:26:33	by elasolova	3 xp	ProgrammingArchive

Write a program that will accept a fraction of the form $\frac{N}{D}$, where N is the numerator and D is the denominator, that prints out the decimal representation. If the decimal representation has a repeating sequence of digits, it should be indicated by enclosing it in brackets. For example, $\frac{1}{3} = .33333333\cdots$ is denoted as $.(3)$, and $\frac{41}{333} = .123123123\cdots$ is denoted as $.(123)$.

Typical conversions are:

$\frac{1}{3} = .(3)$
$\frac{22}{5} = 4.4$
$\frac{1}{7} = .(142857)$
$\frac{3}{8} = .375$
$\frac{45}{56} = .803(571428)$

Sample Run

```
ENTER N,D: 1,7
1/7 = .(142857)
```

What is the answer for the fraction $\frac{11}{59}$?

Source: 1993 USACO Qualifying Round

Answer 76a8ed8037d8c6ec28ba93cb5b6e9765

6 Concatable Prime Pairs

2009-09-24 03:57:20	by elasolova	5 xp	ProgrammingArchive

Pair 3, 7 is the first concatable prime pair as 3, 7, 37 and 73 are all prime. When each member is below 100, there are 24 such pairs.

How many concatable prime pairs exist when each member is below one thousand?

Answer 8545e38faf09ff054485fd8208ce3312

7 Palindromic in base 4 and 6

2009-09-08 01:33:51	by elasolova	8 xp	ProgrammingArchive

Number 21 in base 10 is special as it is the first two-digit number to have palindromic base 4 and base 6. We call a number palindromic if the number stays same when reversed. So, 21 in base 4 is 111 which is also palindromic. Similarly, in base 6 it is 33. Find sum of all such numbers in base 10 under ten million.

Note: All one digit numbers are assumed to be palindromic.

Answer 7a88c52c85f9ec23ac975256d2ac42d1

8 Equality in base 8 and base 9

2009-09-08 01:33:51	by elasolova	10 xp	ProgrammingArchive

79 is 117 in base 8. When we reverse 79 we get 97. 97 is also 117 in base 9. Find the maximum number under 10 million that performs this peculiarity. Give your answer in base 8 or in base 9.

Answer 3138765a7685343f87427b42d3341c57

9 Prime Cryptarithm

2009-09-24 03:22:57 **by elasolova** **6 xp** **ProgrammingArchive**

The following cryptarithm is a multiplication problem that can be solved by substituting digits from a specified set of N digits into the positions marked with *. If the set of prime digits $2, 3, 5, 7$ is selected, the cryptarithm is called a PRIME CRYPTARITHM.

```
      * * *
   x    * *
 -----------
    * * * *
  * * * *
 -----------
  * * * * *
```

Write a program that will find all solutions to the cryptarithm above for any subset of digits from the set $1, 2, 3, 4, 5, 6, 7, 8, 9$.

Sample Run

```
ENTER A SET OF DIGITS: 23468

      2 2 2
   x    2 2
 -----------
      4 4 4         <3 more not shown>
    4 4 4
 -----------
    4 8 8 4
```

The number of unique solutions = 4

Test your program with the prime digits $2, 3, 5, 7$. What is the number of unique solutions?

Source: 1993 USACO Qualifying Round 1

Answer 28c8edde3d61a0411511d3b1866f0636

10 6x6 Checker Challenge

2009-09-24 03:57:33	by elasolova	5 xp	ProgrammingArchive

Examine the 6x6 checkerboard below and note that the six checkers are arranged on the board so that one and only one is placed in each row and each column, and there is never more than one in any diagonal. (Diagonals run from southeast to northwest and southwest to northeast and include all diagonals, not just the major two.)

```
     Column
   1 2 3 4 5 6
  -------------
1 | |0| | | | |
  -------------
2 | | | |0| | |
  -------------
3 | | | | | |0|
  -------------
4 |0| | | | | |
  -------------
5 | | |0| | | |
  -------------
6 | | | | |0| |
  -------------
```

The solution shown above is described by the sequence 2 4 6 1 3 5, which gives the column positions of the checkers for each row from 1 to 6.

```
ROW     1 2 3 4 5 6
COLUMN  2 4 6 1 3 5
```

This is one solution to the 6×6 Checker Challenge. Write a program that searches and finds all unique solution sequences to the 6×6 Checker Challenge. What is the total number of solutions found (including reflections and rotations)?

Source: 1993 USACO Qualifying Round 1

Answer 011ecee7d295c066ae68d4396215c3d0

11 All Latin Squares

2009-09-24 03:12:19	by elasolova	7 xp	ProgrammingArchive

A square arrangement of numbers

```
1 2 3 4 5
2 1 4 5 3
3 4 5 1 2
4 5 2 3 1
5 3 1 2 4
```

is a 5×5 Latin Square because each whole number from 1 to 5 appears once and only once in each row and column.

Write a program that will compute the number of $N \times N$ Latin Squares whose first row is:

```
1 2 3 4 5 ... N
```

Sample Run

```
ENTER A WHOLE NUMBER (2-9): 4
THE NUMBER OF 4 x 4 LATIN SQUARES IS 24.
```

What is the number of 5×5 Latin squares?

Source: 1993 USACO Qualifying Round 1

Answer b43e8a9faaf9e1e8334c13d6f20ff797

12 Farm Frenzy

2009-09-24 17:50:21	**by elasolova**	**2 xp**	**MathArchive**

A farmer has 10 acres to plant in wheat and rye. He has to plant at least 7 acres. However, he has only 1200 to spend and each acre of wheat costs 200 to plant and each acre of rye costs 100 to plant. Moreover, the farmer has to get the planting done in 12 hours and it takes an hour to plant an acre of wheat and 2 hours to plant an acre of rye. If the profit is 500 per acre of wheat and 300 per acre of rye, how many acres of each should be planted to maximize profits? Give your answer as a pair in the form (x, y) where x: wheat, y: rye.

Answer 42f226cf66ec4c2b369d100f26a859d0

13 Perfect Pitch

2009-09-24 18:16:43	by elasolova	3 xp	ProgrammingArchive

A perfect number is a positive integer that is equal to the sum of its proper divisors. For example, 6 is a perfect number because $6 = 1 + 2 + 3$.

8128 is also a perfect number. Find the next smallest perfect number that ends with 8128.

Answer 0f38cb3b329691ed5754ed9d5724eb2d

14 The 3n+1 Problem

2009-09-25 03:06:25	by elasolova	6 xp	ProgrammingArchive

Consider the following algorithm:
1. input n
2. print n
3. if $n = 1$ then STOP
4. if n is odd then $n \leftarrow 3n + 1$
5. else $n \leftarrow n/2$
6. GOTO 2

Given the input 22, the following sequence of numbers will be printed
$22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.$

It is conjectured that the algorithm above will terminate (when a 1 is printed) for any integral input value. Despite the simplicity of the algorithm, it is unknown whether this conjecture is true. It has been verified, however, for all integers n such that $0 < n < 1000000$. (and, in fact, for many more numbers than this.)

Given an input n, it is possible to determine the number of numbers printed (including the 1). For a given n this is called the cycle-length of n. In the example above, the cycle length of 22 is 16.

For any two numbers i and j you are to determine the maximum cycle length over all numbers between i and j (inclusive).

What is the answer if $i = 1, j = 1000000$?

Answer 81b9dd0e0451918c42451dbd6ec71366

15 Triplets

2009-09-25 04:23:01	by elasolova	5 xp	ProgrammingArchive

Given:
(1) X, Y and Z are different integers between 0 and 9 (both included)
(2) $\frac{10\times X+Y}{10\times Y+Z}=\frac{X}{Z}$ (with infinite precision)

How many triplets are there?

Source: Prolog Contest 1 Ithaca

Answer 011ecee7d295c066ae68d4396215c3d0

16 Swamp Kangaroo

2009-09-25 20:03:57	by elasolova	10 xp	ProgrammingArchive

Kangaroos are fascinating creatures. For one, the idea of carrying around their offspring in a pouch is very cute, and it reminds us of our own ways of transporting babies. For another, they can jump really far. That can be quite useful, in particular if you are a kangaroo stuck in a swamp with only small islands of land, and crocodiles swimming around. In that case, you would rather not land in the water. Besides mere jumping strength, it is also useful to have some computational power to compute how exactly to use that jumping strength to get where you need to go.

Here is how we model Kangaroo movement. Kangaroos can only move North-South or East-West; no other directions (such as diagonals). Kangaroos can jump any integer distance between 1 and 5 in one hop, which takes them one unit of time. However, after a longer jump, they have to rest before being able to jump again. Specifically, after jumping distance d, the Kangaroo has to rest $(d-1)^2$ time units before being able to jump again. Also, if the next jump is in a different direction from the previous one, the Kangaroo takes an extra one time unit between the jumps to turn around.

The swamp will be described by a two-dimensional grid. Each entry is either water, denoted by a dot (`.`), or land, denoted by `X`. Two locations will be marked with special symbols. `K` denotes the initial position of the Kangaroo, and `G` is the goal the Kangaroo wants to reach (both of these are of course land). You are to find the shortest time in which the Kangaroo could get to the goal (if at all). It does not matter if the kangaroo is tired when it arrives at its destination; it doesn not have to rest.

What is the answer for the following input?

```
.......XXX......XX.....X.X.XX.
XK....XXXXXXXX..XX............
X......XXXXX....XX.X.X.X....X.
.......XXX....XX..............
..............XX...........XX.
...........................XX.
...XX....XX................XX.
..XXXX.....................X..
..XXXX........................
...XX.........XX....X....XXX..
........XX....XX.X..X..X.XGX..
.......XX................XXX..
```

Source: USC Contest Fall 2008

Answer 30ec62934072bd5f3313cd90958318fa

17 Ski Lifts

2009-09-25 20:33:11	by elasolova	20 xp	ProgrammingArchive

Climate changes made the rolling mountains of Alaska a perfect place for all-season skiing. To transform this region into a successful area for skiing, however, ski lifts are needed. People skiing do not like walking upwards. They want to ski downhill; when needed they are willing to ski on the same level for some time.

Optimal use of the area requires that, starting from an arbitrary point, a skier should be able to reach any other point just by skiing downhill or staying at the same level, and occasionally taking a ski lift.

A sufficient amount of ski lifts must be planned and constructed such as to fulfill this condition. On the other hand, building more ski lifts than necessary is a waste of money. What is the minimal number of ski lifts needed?

As ski lifts are built on high poles, we assume that a ski lift can be constructed from any place to any place, regardless of the terrain in between. A ski lift is unidirectional. It is important to know that in Alaska one is not allowed to ski in any other direction than North, South, East or West.

Input format

A line with two positive numbers w and l with $1 \leq w, l \leq 500$: the width and length of the area. w lines, each line containing l positive numbers $h(i, j)$, with $0 \leq h(i, j) \leq 10^9$, representing the height in each point of the area.

Input

ski.txt

Source: Benelux Algorithm Programming Contest 2005

Answer 7b274337ad8441febbe6d566473e7d63

18 Zebra Herd

2009-09-25 21:01:02 **by elasolova** **10 xp** **ProgrammingArchive**

Zebras are very social animals. Like other members of the horse family, they form groups that tend to stick together and hang out fairly regularly, though not exclusively. (Humans also come to mind in this respect.) Lately, researchers have been trying to understand just how the communities of zebras evolve over time, what triggers changes, and so forth. Of course, all they have to go by is observations of where the zebras are over time. From that, we would like to figure out what are the most natural groups. The assumptions are that (a) if a zebra is part of a group, it tends to spend time close to others in that group, (b) if a zebra is not part of a group, it tends to spend time further away from others in that group, and (c) zebras do not change their group membership very often.

Let us make this more precise. You will be given a sequence of observations of zebras. For each observation time, you will have the exact location of each zebra. The distance between two zebras is exactly their Euclidean (straight-line) distance. We assume that there are exactly two groups of zebras in the herd, and will denote them by two colors. What we want to do is color each zebra either red or blue for each time step, expressing membership to one or the other group. To express assumption (c) above, we will assess a penalty of some given number c every time a zebra changes colors. To express assumptions (a) and (b), we look at the distance $d(i, j)$ between every pair i, j of zebras. If i and j are of the same color, then we assess a penalty of $a \cdot d(i, j)$ for this pair. If i and j are of opposite colors, then we assess a penalty of $-b \cdot d(i, j)$ (i.e., we give a bonus).

Thus, if you are given a proposed labeling of all zebras with either red or blue for each time step, you can compute how good an explanation of zebra activity it is. Your goal is to find the best possible labeling, in the sense that it has the smallest possible total penalty. But you will only need to output the total penalty of the labeling, not the labeling itself.

Input Format

The first line of data set contains two integers z, t, the number of zebras $2 \le z \le 10$, and the number of time steps $2 \le t \le 50$. Next comes a line with three floating point numbers $a, b, c \ge 0$, the penalty multipliers. This is followed by t lines, describing zebra positions. Each line contains $2z$ floating point numbers, giving the positions of the zebras in the form `x1 y1 x2 y2 ... xz yz`. The first line contains the positions at time 1, the second line at time 2, and so forth.

Input

zebra.txt

Give your answer rounded to two decimals like `[wholepart].cd`

Source: USC Contest Fall 2008

Answer 715176588d2e2019b5bab7024e9133c0

19 Minimizing a DFA

2010-11-01 17:27:40	by elasolova	5 xp	ProgrammingArchive

Here is a DFA given in the table form

State	a	b
1	2	4
2	1	4
3	1	4
4	3	3

In the minimal representation which states can be represented as just one state? What number is formed if you concatenate the state numbers you have found in increasing order?

Answer d9b1d7db4cd6e70935368a1efb10e377

20 Detrimental

2010-11-01 17:35:43	by elasolova	4 xp	MathArchive

Let

$$A(x) = \begin{vmatrix} x & 2x & x^2 \\ x+1 & 2x+1 & x^2+1 \\ x+2 & 2x+2 & x^2+2 \end{vmatrix}$$

including the determinant operator.

Compute the sum of all $A(x)$ where x varies from 1 to 10^9 inclusive.

Answer dcfcd07e645d245babe887e5e2daa016

21 Bit Sums

2009-09-26 15:35:24	by elasolova	5 xp	ProgrammingArchive

Find the sum of the one hundred thousand smallest positive integers that use only the digits 1 and 0 and are perfectly divisible by 19. We're still dealing with base 10.

Answer 3b00360d0b24dd076621b681048454ea

22 Happy Primes

2009-09-26 19:08:45	by elasolova	6 xp	ProgrammingArchive

A happy number is built by adding the squares of its digits. Doing this permanently, the numbers will end in 1 or 4.

If a positive integer ends with 1, then it is a happy number.

The first few happy prime numbers are:
7, 13, 19, 23, 31, 79, 97, 103, 109, ...

What is the sum of the first 30000 happy prime numbers?

Answer 62a14055f8071f5c5f034c5a1f12ed7c

23 Divisible Hexagons

2009-09-26 19:03:21	by elasolova	2 xp	ProgrammingArchive

A hexagonal number is computed as follows using the formula:
$H(n) = n(2n - 1)$

As shown, the first 5 hexagonal numbers are:
$1, 6, 15, 28, 45$

How many of the first 100000000 hexagonal numbers are divisible by all the numbers from 1 through 15?

Answer 8a615010f0469a53648706843d50122d

24 Grande Facto

2009-10-12 22:50:57	**by elasolova**	**4 xp**	**MathArchive**

What is the sum of the digits of (50000!)?

Answer **574493e0d383d3df8bb6029e5fddce1d**

25 Prime Triangle

2009-10-06 18:56:28	by elasolova	4 xp	ProgrammingArchive

Given the Triangle of primes.

```
2
3 5
7 11 13
17 19 23 29
31 37 41 43 47
53 59 61 67 71 73
...
```

The sum of the 6-th row is 384. Give the sum of the 1000-th row.

Answer 8cab2a505eb34e919e18619c0e7d491c

26 Passion of Pascal

2009-10-12 22:53:02	**by elasolova**	**2 xp**	**ProgrammingArchive**

What is the **digital sum** of **sum of numbers** in the **1000001-th** row of Pascal Triangle?

Answer **62e3dde0ed5f930809b6303fbbbe7493**

27 Surjective Surge

2009-10-13 19:22:55	**by elasolova**	**25 xp**	**MathArchive**

Consider a function $f : A \to B$. What is the digital sum of number of surjective functions if $|A| = 1678, |B| = 1543$?

Answer dd4ef9c7c5b967575806f1d3e1000787

28 Fruit Frenzy

2009-10-14 19:23:56 **by elasolova** **15 xp** **MathArchive**

Assume you have 3 kinds of fruit in a basket: Apples, Bananas, Coconuts. And assume you have an unlimited source of these fruits. Then there are 15 ways to pick 4 fruits

```
4A, 4B, 4C
3A1B, 3A1C, 3B1A, 3B1C, 3C1A, 3C1B
2A2B, 2A2C, 2B2C
1A1B2C, 1A1C2B, 1B1C2A
```

Calculate how many ways to pick up 100000 fruits from 55555 kinds. What is the digital sum of this huge number?

Note: Problem reseted in Nov 26, 2009 as the answer in database was wrong.

Answer **0fd7aec2b5cb03dfd65400989ff1c564**

29 Dividing Cube

2009-10-21 01:20:58	**by elasolova**	**2 xp**	**MathArchive**

How many integers between 1 and 2009 are divisible by a non-trivial cube (p^3 where $p > 1$)?

Note: Please solve by pencil and paper. It is very easy by programming.

Answer 588d0aca3271a7779caf6f0b20ad31b9

30 Madequation

2009-10-30 02:11:33	**by elasolova**	**25 xp**	**MathArchive**

Consider the equation
$a + b + c + \cdots + y + z = 100$
(a to z: There are 26 variables in total)

How many solutions (number of all distinct $[a, b, c, \ldots]$ lists) are there if all a, b, c, ... are odd and positive?

Answer **80ec413696954ca897f01106f841d68f**

31 Game of Preference 1

2009-11-10 23:07:59	**by elasolova**	**10 xp**	**Probability**

In the card game of preference, the deck of 32 cards(4-suit, 7-to-ace) is dealt among three players, so that every player gets 10 cards and the two remaining cards form the widow. How many ways are there to deal hands in such a way that the widow does not consist of two aces?

Answer **06935452afa35c1bf866eb1e886d1582**

32 Game of Preference 2

2009-11-10 23:08:06	by elasolova	10 xp	Probability

Calculate the probability that in the game of preference (32 dealt among three players, 10 cards each, and 2 cards go to the widow) every player gets precisely one ace. Give the ratio in lowest terms.

Answer ad691f5e186253d5bbf99003942a9d1d

33 Relativement Primordiaux

2009-11-10 23:13:34	by elasolova	5 xp	MathArchive

Consider the equation (x, y integers)
$1773\boldsymbol{x} + 1865\boldsymbol{y} = 1$

What is the smallest positive integer value for x?

Answer 4e904ad3a6b3fa22cae1291c3818f31a

34 Attente de Maximum

2009-11-10 23:14:55	**by elasolova**	**5 xp**	**Probability**

You roll a pair of fair dice. Compute the expectation (expected value) of the maximum of the two results. Give the answer in lowest terms.

Info: Expected Value: Wikipedia

Answer **ed703dce39ebb5e07381353b77441f8a**

35 Steps to Hanoi

2009-11-11 16:47:34	**by elasolova**	**3 xp**	**ProgrammingArchive**

What is digital sum of the number of required moves to solve 300000-disk Tower of Hanoi puzzle?

Answer **5a1774856cca211d4ab8eb32c82b603f**

36 Distinct Simple Graphs

2009-11-12 19:42:51	**by elasolova**	**10 xp**	**MathArchive**

What is the number of non-isomorphic (distinct) simple graphs on 28 nodes?

Answer **bf35b33faadfc76e8f16a2ee3f3dca2c**

37 Comparisons in Mergesort

2009-11-13 16:43:48	by elasolova	8 xp	ProgrammingArchive

Mergesort is a recursive sorting algortihm defined as:

- Assume length of input (n) is a power of two
- If $n = 1$, return the element
- If $n > 1$, break list into two
- call mergesort on each half
- merge the sorted lists

How many comparisons are required (in worst case) to sort a list with 2^{150} elements using mergesort?

Answer 814cd7505badaf54f6b874a201b7d4a7

38 Decipher

2010-11-05 20:18:05	**by elasolova**	**14 xp**	**Crypto**

You have this rather mysterious message in decimal ascii form:

```
50 31 22 23 14 29 26 11 16 73 26 8 22 14 26 11 18 13 83 12 4 73 1 0 22 5 93 101
```

You only know that the key to decipher this message is 4 lettered. Find the secret message and type it.

Answer dd8117d36227132688b265677ae47783

39 F*ck World [BF]

2011-04-21 06:00:00	by elasolova	4 xp	Brainf**k

Write “Fuck World” in Brainfuck language.

Answer 4f0522feeda2c4ef50a61e7a476f9b26

40 Lorem Ipsum

2009-11-19 05:02:35	**by elasolova**	**3 xp**	**ProgrammingArchive**

Count the number of unique words in this file: ipsum.txt.

Answer **22ff51a85b792d59648febe87c7e8a89**

41 Output Input [BF]

2011-04-21 06:00:00	**by elasolova**	**10 xp**	**Brainf**k**

Write a Brainfuck code to output the input.

Answer **fabb925607c794f36b8cabafe74d75dc**

42 Mean Square

2009-11-20 20:56:09	**by elasolova**	**10 xp**	**MathArchive**

What is the mean distance between two random points in a unit square? Give your answer to five decimal places.

Answer 8f59484a7283965be4c5e0e3ae6bdf92

43 Input Tuptuo [BF]

2011-04-26 06:00:00	**by elasolova**	**10 xp**	**Brainf**k**

Write a Brainfuck code that outputs the input in reverse.

Answer **1baefe74c697e441fa6013cde77ffa6e**

44 Dicemetrics

2009-11-22 03:22:46	**by elasolova**	**10 xp**	**Probability**

A guy has 8 five-sided dice numbered from 1 to 5. A girl has 6 six-sided dice numbered from 1 to 6. They roll their dice and compare totals. The max total wins and it is a draw if sums are equal. What is the probability that the guy beats the girl? Round your answer to 9 decimal places.

Answer 04973eb7f37f4c9298885ef850e83de2

45 Nonconsecutive Zeros

2009-11-22 23:40:01	**by elasolova**	**6 xp**	**MathArchive**

Derive a formula for the number of strings of length n over the alphabet $\{0, 1, 2\}$ which have no consecutive zeroes. What is the answer for $n = 150$?

Answer **d3c947523bcac173d492689cb227beab**

46 Fibonacci [BF]

2011-04-26 06:00:00	**by elasolova**	**13 xp**	**Brainf**k**

Write a code in Brainfuck that given positive integer input n outputs nth Fibonacci number. **Assume 1st and 2nd are both 1.** Assume that the input will be between 1 and 13.

Answer 749d7d1b0ce950db32aa8b2cfbb9d513

47 State Lottery

2009-11-25 00:10:54	**by elasolova**	**2 xp**	**MathArchive**

In this game of lottery 6 different numbers are chosen among 49. How many different combinations are there? Compute it, namely 49 choose 6.

Contributed by: **Gizmore**

Answer **0684891e5e50dc403248018b565c1996**

48 Lost Password

2009-11-25 08:03:11	**by elasolova**	**12 xp**	**ProgrammingArchive**

I lost my password to a .rar file. It is heavily encrypted and I only get 73 passwords per second. Luckily I remember parts of my password, but I am not sure how long it would take.

The static string `Giz` appears somewhere or maybe does not.

There is definitely a fake 20th century birth year (only the year) somewhere.

There are 1-8 special characters (scattered and no multiple instances) from this charset: `#+%&*!^;:-?/`

Finally there is a word from my dictionary (65535 words) somewhere.

Note that any block can appear anywhere in the password. Could you please tell me how many days it will take to try every password if the speed is at 73 pass/s? Round your answer to 6 decimal places.

Contributed by: **Gizmore**

Answer b5c492635f8821e5da83e6e70ef3f742

49 Binary Decode [BF]

2011-04-26 06:00:00 **by elasolova** **18 xp** **Brainf**k**

Write a Brainfuck code that given a binary string outputs its decimal value. For simplicity only at max 8 -bit binary values will be provided. Note that leading zeroes will be omitted.

Answer **244daf3838d7a55d445df40757cf9934**

50 Digital Sum [BF]

2011-04-26 06:00:00	**by elasolova**	**9 xp**	**Brainf**k**

Write a Brainfuck code that given positive integer n, outputs its digital sum meaning the sum of every digit.

Answer **80ecd7abc0579af356a6ca6fb24ec486**

51 Totient [BF]

2011-04-26 06:00:00	**by elasolova**	**28 xp**	**Brainf**k**

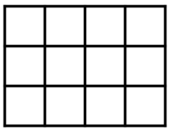
Write a Brainfuck code that given n, outputs n's totient value. n will be random between 1 and 255.

Answer **f8ec53757476417c9350b1f6301bd85d**

52 RectoCount

2009-11-27 04:15:52	by elasolova	4 xp	ProgrammingArchive

A rectangular grid of 4 by 3:



includes a total of 60 rectangles (including squares). How many rectangles are there in a 1453 by 1881 grid?

Answer 9c347a3af808909797b90fb1e9c97242

53 String Decode [BF]

2011-04-26 06:00:00	by elasolova	15 xp	Brainf**k

Write a Brainfuck code that given a string, outputs hex code of given string.

Answer 9cc5e6129c7f78973f8c8bd0eb336032

54 Be Careful!: Feel the Chemistry

2013-07-30 06:48:29	**by elasolova**	**7 xp**	**Sequence**

```
11 21 22 31 32 41 33 42 51 43 52 61 ?
```

Hint: *feel it*

Answer **1cdebc4405e3610c2b0ac08b81789cc9**

55 Path to Palindromes

2009-12-02 07:30:11 | **by elasolova** | **6 xp** | **ProgrammingArchive**

A number can be converted to a palindrome by summing the cube of each digit and repeating this procedure. For example 36 converts to 99 in 2 steps. Likewise 17 converts to 737 in 6 steps. We assume a number is not convertible in this way if it does not reach a palindrome in at most 100 steps. Below one million, which number has to undergo most steps (if there are more than one, assume smallest). Give your answer as `number.steps`

Answer e293d25b2e6f07a9fb373905cb4aae88

56 Convergents of e

2009-12-02 07:29:39	**by elasolova**	**10 xp**	**ProgrammingArchive**

Square root of 2 can be represented as $[1;(2)]$ where (2) means 2 repeats *ad infinitum*. In a similar way, $\sqrt{23} = [4;(1,3,1,8)]$ (Note: Continued Fraction)

Surprisingly,
$e = [2;1,2,1,1,4,1,1,6,1,...,1,2k,1,...]$.

The first ten terms in the sequence of convergents for e are:
$2, 3, 8/3, 11/4, 19/7, 87/32, 106/39, 193/71, 1264/465, 1457/536, ...$

Find the sum of digits in the numerator of the 1000-th convergent of the continued fraction for e.

Note: An extension of Project Euler Problem 65

Answer **0e3a9753e6442ff8e2d954e523898a9c**

57 Non-Repeating Digits

2009-12-02 08:05:02	**by elasolova**	**6 xp**	**ProgrammingArchive**

Write a counter function that counts from 1 to one million (inclusive) but only counts numbers whose digits don't repeat. As an example, 98, (99, 100, 101), 102 is a jump of 4. Submit total count of numbers and biggest jump. Format: `total.jump`

Answer dae0effa2bcebc572983463f92ab797a

58 Meaning of Life

2013-07-31 04:46:51	**by elasolova**	**8 xp**	**Sequence**

Have you ever seen the movie Gattaca?

All capitals.

Hint: *matching strand?*

Answer **0b373dddb5179af216202196b00cba32**

59 Edges of Bipartite

2009-12-03 21:00:06 **by elasolova** **10 xp** **MathArchive**

Let $|V| = 1000$, and assume W a subset of V is selected at random from the collection of all subsets of V. Let us form the complete bipartite graph G with parts W and $V \setminus W$ (in other words, we connect two vertices if and only if one of them is in W while another is not). What is the expected number of edges in G? Round to an integer if needed.

Answer **f04ca46e8ee0bb079788ffaec7ebc5da**

60 Pandigital Ends

2009-12-04 04:20:27	**by elasolova**	**7 xp**	**Programming**

A pandigital number makes use of the digits 1 through 9 at least once but not in any particular order. For example: 146523798 is a 9 digit pandigital.

What is the sum of values of i such that 3^i has it's last 9 digits pandigital from 1 to 9, where $1 \leq i \leq 1000000$?

Answer 76bf00a22f55c046782907a6e4fcd63e

61 Factorial [BF]

2011-04-26 06:00:00	**by elasolova**	**13 xp**	**Brainf**k**

Write a Brainfuck code that given positive integer n, outputs its factorial.

Answer **941d23c1f6df779e74b09c7c965c010f**

62 n-Ovals Problem

2009-12-05 05:55:17	**by elasolova**	**4 xp**	**MathArchive**

Suppose that n ovals (an oval is a closed curve that does not cross over itself) are drawn on the plane such that no three ovals meet in a point and each pair of ovals intersects in exactly two points. How many distinct regions of the plane are create by n ovals? Provide a closed form answer including n.

Note: Use ^ as the power operator. Do not use paranthesis (or spaces) anywhere. Use precedence rules instead. You should write terms decreasingly in terms of power of n.

Answer 19492447d71be6e366b47501a8141a6c

63 Parentheses Preference

2009-12-05 06:34:27	by elasolova	12 xp	ProgrammingArchive

Suppose we want to find the number of ways to parenthesize the expression
$a + b + c + ... + y + z$ (26 terms)
so that a parenthesized form of the expression reflects the process of adding two terms. How many ways are there?

Answer bafd6f63860d3521249c98fbaaaf27e9

64 Sort String [BF]

2011-04-26 06:00:00	by elasolova	10 xp	Brainf**k

Write a Brainfuck code that given string n, it sorts characters in string in increasingly alphabetic order then outputs this sorted string. Thus, given "haeusl" it outputs "aehlsu".

Answer 37ac82dfaf159fcfd623f7e3e894ee53

65 Weird Dice

2009-12-10 01:38:52	**by elasolova**	**3 xp**	**Probability**

Find two six sided dice, such that the probability of each sum from 2 to 12 is the same as two standard dice. Each side must have least one dot. Negative numbers are not allowed. There is another answer besides two standard 123456 dice. Format of the answer should be `111222,113344` (just an example) where first integer is the smaller and integers acquired by concatenation increasingly.

Answer **6f1e421bfc7159d4d48bed4c7fdeed5d**

66 Unique Chars [BF]

2011-04-26 06:00:00	**by elasolova**	**15 xp**	**Brainf**k**

Write a Brainfuck code that given input string x, outputs number of unique characters.

Answer 7bfc85c0d74ff05806e0b5a0fa0c1df1

67 Prime Finder [BF]

2011-04-26 06:00:00	**by elasolova**	**21 xp**	**Brainf**k**

Write a Brainfuck code that given n, outputs n-th prime. Assume n will be between 1 and 54.

Answer **a54c22fe5a0cf0edb9ab5299c8d60d54**

68 Bob and Stephen

2010-01-10 04:00:13	**by elasolova**	**2 xp**	**MathArchive**

Bob and Stephen are brothers. Stephen is twelve years older than Bob. What is the maximum possible age of Bob such that the ratio between ages of two brothers is an integer?

Answer **c8b2f17833a4c73bb20f88876219ddcd**

69 New Year's Equation

2010-01-10 04:02:42	**by elasolova**	**4 xp**	**MathArchive**

How many real solutions are there for the following equation:
$x^{2010} + 2009 = 2008x$

Answer **dcfcd07e645d245babe887e5e2daa016**

70 New Testament

2010-01-10 04:08:01	**by elasolova**	**5 xp**	**MathArchive**

Given $x + y \leq 2009$ and $y + z \leq 2010$. Suppose the number of non-negative integer solutions to these inequalities is in the form $2k(6k + 1)(6k + 2)$ with k positive, find k.

Answer **d8827f2811ee6fbaa9a0fc087a1f8446**

71 Be Careful!: Famous Ensemble

2013-07-31 04:09:52	**by elasolova**	**6 xp**	**Sequence**

```
5 85 987 4482 ?
```

Hint 1: *Think title*
Hint 2: *4 digits*

Answer **45e39d4eb3d1da13702f0b6e763d11f4**

72 8-Puzzle Challenge

2010-01-20 06:24:07	by elasolova	12 xp	ProgrammingArchive

A 9-digit representation of 8-puzzle `abcdefgh0` corresponds to:

```
a b c
d e f
g h
```

where `0` is the place of the empty cell.

Given 100 start states in 8puzzle.txt compute the average of shortest path distance (that takes the least number of moves) to goal `253176408`. You can use any of the suiting heuristic functions you like. Round your answer to 2 decimal places.

Answer a4d6a50b2f0276bda71f0e5f9bb1d190

73 Mouse Escape Time

2010-02-01 04:25:27	**by elasolova**	**4 xp**	**MathArchive**

Consider a mouse in a maze. There are four possible paths such that path 1 takes 5.32 minutes to escape, path 2 takes 3.12 minutes to espace, path 3 takes 8.24 minutes to return to same place and path 4 takes 2.35 minutes to return to same place. Given that the mouse chooses any of the four paths at random what is the expected time of escape in minutes? Round your answer to 4 decimal places.

Answer 2ce54e2f07f3709413249118493 96c28

74 String Shift [BF]

2011-04-26 06:00:00	**by elasolova**	**5 xp**	**Brainf**k**

Write a Brainfuck code that shifts input string chars by one and outputs it. Ex. `javaist` → `kbwbjtu`

Answer **b522a1532d429304912638c9f430b60c**

75 Pragma Series: Question out of the Box

2012-06-15 22:46:32	**by elasolova**	**4 xp**	**Hack**

Here is a simple question:

```
2+2=?
```

The problem here is that there is no usual way to submit the solution. You need to figure that out by yourself!!

Answer 011ecee7d295c066ae68d4396215c3d0

76 Pragma Series: Lucky Enough?

2012-06-15 16:42:30 | **by elasolova** | **11 xp** | **Hack**

Here is a flash dice simulator. It is somewhat biased, however. You need to roll two 6s in order to get the secret code. If you do not hack somehow, you will never be able to get that lucky!! When you get the code submit like you did for problem 75.

Answer b6f20d75395e7531f07ec2e241a40d5d

77 Pragma Series: Unknown Decoder

2012-06-18 23:33:45	**by elasolova**	**8 xp**	**Crypto**

In this problem we provide the answer. It is: javaist. However, the server will first decode the answer you submit. Unfortunately, the encode/decode algorithm used is unknown. However, we provide a basic flash tool to help you. Given an encoded text it provides the original data. Feel free to experiment and then encode the actual answer before submitting.

Note: The returned string of flash tool is only a crude report of actual encoded string. It is not totally dependable. So examine its source code, find the intermediate page and that way your experimentation with encoding will be more dependable.

Answer 0e21e3905108e19bcd1ebe6f27da07f7

78 Pragma Series: RGB Average

2012-06-21 22:41:47	**by elasolova**	**12 xp**	**TimeRace**

p78.php creates a random png image. Your task is to detect the average RGB values for the image and POST it to the same page (p78.php) via `rgb=RAvg:GAvg:BAvg&submit=true`. Round each average to 2 decimal places. Note that you have 1 second to do so after you see the random image.

Answer **5a9e8b445c690639919892e311fd8649**

79 Pragma Series: Original Triangle

2012-06-23 18:51:42	by elasolova	13 xp	TimeRace

In this problem you will be supplied with several thousands of random triangle vertice positions (in cartesian coordinates). Your task is to compute number of triangles that contain the origin.

p79.php supplies the list of triangles in the form `Ax,Ay,Bx,By,Cx,Cy` per line (where ABC are the vertices). When you get the answer POST to same page (p79.php) as `amount=A&submit79=true`. You have 1.86 secs to do so.

Answer 5a9e8b445c690639919892e311fd8649

80 Image Barcode

2012-06-23 21:34:51	**by elasolova**	**5 xp**	**Crypto**

Here is a barcode like image. Can you extract the code?

Answer **2752bf43f7f6efe2008f538796eee63a**

81 Pragma Series: Brute Interrogation Made Easier

2012-09-26 04:21:39	by elasolova	15 xp	Hack

In this problem you need to interrogate an ex-agent prisoner. You can perform 4 actions. The prisoner's state is kept as a session variable on the server side and you will be able to see it once you get to the secret page. You also receive a text regarding the state.

These 4 actions translate to some basic arithmetic operations done on the variable. When the condition is met the prisoner will confess the secret code.

Note: You need to make the variable somehow 1000 to see the password. Brute force is not necessary from now on. It is simple reverse engineering.

Answer 85dc3b233cba0477ea6d82f4fbcc74e7

82 Pragma Series: Sorted Radical Function

2012-06-27 22:42:46 **by elasolova** **12 xp** **TimeRace**

The radical of n, $r(n)$, is the product of distinct prime factors of n. For example, $156 = 2 \times 2 \times 3 \times 13$, so $r(156) = 2 \times 3 \times 13 = 78$. If we calculate $r(n)$ for $1 \leq n \leq 10$, then sort them on $r(n)$, and sorting on n if the radical values are equal, we get:
$1, 2, 4, 8, 3, 9, 5, 6, 7, 10$

let $S(k)$ be the kth element on the sorted list. So $S(3) = 4$, $S(7) = 5$ etc. p82.php will print random M and N values where $r(i)$ is sorted for $1 \leq i \leq M$ and the answer is $S(N)$. POST to p82.php as `ans=S(N)&submit82=true`. You have 1.12 secs to calculate!!.

Note: M will be between 8000 and 10000, whereas N will be between 1000 and 3000.

Fix: Fixed index problem. Now the S array starts from 1 as described above instead of starting from 0.

Answer **5a9e8b445c690639919892e311fd8649**

83 Pragma Series: Binary or Not?

2012-06-28 23:26:45	**by elasolova**	**15 xp**	**TimeRace**

p83.php creates some random binary looking data. Your task is to decode that into english letters (not necessarily forming a meaningful word!). Then POST as `ans=letters&submit83=true`. Time limit is 0.7 secs.

Answer **5a9e8b445c690639919892e311fd8649**

84 BeCareful!: Grasshopper

2012-06-30 22:47:40	**by elasolova**	**7 xp**	**MathArchive**

On the real number axis, Goppy the Grasshopper can hop along (starting from 0) the axis either way, 364 or 715 units length. What can be the minimum distance of any hopped point to point 2010.

Answer **4e44f1ac85cd60e3caa56bfd4afb675e**

85 Find The Next One

2019-01-30 07:01:05	by C_K_Yang	8 xp	Sequence

There is an integer sequence:

```
1, 8, 216, 13824, 4741632, 11119431168, 19421724672, 263128269312, 2241883861124382 72,
391914614322266112, ?, ...
```

Find the number which the question mark represents.

Answer

86 BeCareful!: MultCount

2012-07-01 21:30:48	**by elasolova**	**4 xp**	**MathArchive**

For n bigger than 1, how many multiplications we need to perform (at least) to calculate n to the power 13?

Answer **4e44f1ac85cd60e3caa56bfd4afb675e**

87 BeCareful!: 3-Slot Scales

2012-07-01 21:32:48	**by elasolova**	**8 xp**	**MathArchive**

Looking the same, but having different wieghts, we have 50 balls. We also have a 3-slot scales. This scales can sort 3 balls from least to most in weight with a single measurement and it only accepts 3 balls (no more no less) at a time.

The question then is, how many measurements (at minimum) we need to perform to find the 2nd lightest ball?

Answer **80ecd7abc0579af356a6ca6fb24ec486**

88 BeCareful!: Group Work

2012-07-01 13:38:00	by elasolova	5 xp	MathArchive

One of the professors in Math department gives a lot of homework. We have 3 particular students in this class called Alice, Ben, and Clark. When Alice and Ben works together they finish a day's homework in 5 hours. Clark and Alice together finish it in 3 hours as they kind of fancy each other :) and like to work together. Ben and Clark, together, finish in 4 hours. One day both Ben and Clark are sick and they cannot work. Alice has to do it alone. How long will it take (in hours)? Round to 2 decimal places.

Answer f736b67909f706452fe35fe3f73dd38d

89 BeCareful!: Couple HandShake

2012-07-01 17:36:15	**by elasolova**	**8 xp**	**MathArchive**

You are a reporter of Chicago Tribune Entertainment section. There was a celebrity couple-exclusive party in Chicago. It was a formal and fabulous one :) so everyone in the party had hanshaked others excluding their own partners. Your paparazzo were able to capture 144 handshakes. You know that they are the best in this job. So your director in the newspaper wants you to report how many couples there were in the party?

Answer **4c0d13d3ad6cc317017872e51d01b238**

90 BeCareful!: UnHumDrum

2012-07-01 18:02:48	**by elasolova**	**9 xp**	**MathArchive**

You are the head of a jewelry design firm. One of your clients asked you to design a specific brand of jewelry that will be called UnHumDrum. So here is the challenge. The materials will used are composed of 2 crimson red, 2 metallic blue, and 2 cyanic green beads (there is no difference in these beads other than their color). You have to line these beads on a linear (not circular) silken strand and beads with same color must not be next to each other to avoid humdrum (there comes the brand name :)). So in your catalog for this new brand, how many distinct products can you name?

Answer 4db87140662bd68076ef786f7163cedc

91 Be careful!: Monthly Birthdays

2012-07-22 15:43:17	by elasolova	3 xp	MathArchive

What minimum number of friends do I need to have to be sure that at least 7 of my friends have their birthdays in same month?

Answer 8c9d4c8374b6277fbfddc0fa4e4fbe1f

92 Be careful!: Mega Box

2012-07-07 17:19:05	by elasolova	2 xp	MathArchive

We have a mega box composed of small cubes attached together. The box is a 3d rectangular shape in that each dimension includes $9, 10, 11$ cubes respectively ($9 \times 10 \times 11$ cubes included). We paint the whole surface of this box to red. The question is that how many cubes will not be painted at all?

Answer 40ece03de4fc621a653ad5d565e41701

93 Be careful!: Rook Attack!

2012-07-08 16:43:59	by elasolova	6 xp	MathArchive

On a conventional chess board (8×8), we want to place 3 rooks. How many different placements are there so that they won't have a chance to attack each other?

Answer f1a0ec25e9d0cfad11c7f6f21f072a63

94 BeCareful!: Bombastic Queue

2012-07-16 17:25:26	by elasolova	7 xp	MathArchive

Mr. Bombastic wants to enqueue his students in a row. There are 7 students in the class that are queued according to their heights from shortest to longest currently. Mr. Bombastic has a taste of fashion and does not like orderly stuff. So he decides to disrupt the queue. However, the school directors only permit 2 but only 2 swaps starting from height order. A swap is between **any** two students. So how many different queue options does Mr. Bombastic have by exactly two successive swaps?

Note: There was a confusion about number of swaps available. It is EXACTLY two swaps. No more no less.

Answer 8cdf85a71d67ce91884d391eafb6704a

95 BeCareful!: Prepare for War

2012-07-17 16:36:11	**by elasolova**	**12 xp**	**MathArchive**

You have 10 distinct army units in a strategy game online. One of your allies warns you that you will witness an upcoming attack. This attack will simultaneously target 4 areas of your empire. As a commander, you want to evaluate your options. You decide that each area needs at least one army unit and also you have to deploy all of your 10 units no matter what. How many possible deployment options are there?

Note: There was a spacing error in the answer in the database. Problem fixed on 26 July 2012.

Answer **acb115ad95b90d6aca5460bff66c4cec**

96 Deep Compression

2012-07-24 14:50:53	by elasolova	6 xp	ProgrammingArchive

Some compression has been done on this file. Solution lies deep inside.

Answer 6e86d53e3c80b57efc425c3d81837c52

97 Hitchhiker's Bad Luck

2013-07-31 04:36:17 **by elasolova** **12 xp** **Sequence**

```
10 29 36 bb 61 07 c3 82 90 b1 a6 ?
```

Hint 1: *think baseic*
Hint 2: *think as a whole*

Answer **d15509b48113ffaf089d780645d022ee**

98 Be Careful!: Self Service

2012-09-02 00:58:40	**by elasolova**	**8 xp**	**Sequence**

```
20,23,5,14,20,25,20,23,5,14,20,25,20,8,18,5,5,6,9,22,5,6,15,21,18,20,5,5,14,20,23,...
```

Get the rule for this sequence. Then what term waits longest to make its first appearance, tell me?

Answer 83a5a282f092aa7baf6982b54227bb54

99 Be Careful!: Combination I

2012-09-02 01:17:36	by elasolova	5 xp	Sequence

```
88 op. 63 = 25
25 op. 9 = 16
38 op. 16 = 18
33 op. 18 = 15
32 op. 15 = x
```

Find the rule then find `x`, but think out of the box. `op.` is **not** a conventional operator.

Answer 7bfc85c0d74ff05806e0b5a0fa0c1df1

100 Be Careful!: Combination II

2012-09-02 02:09:44	by elasolova	5 xp	Sequence

```
34-16 = 18
32-18 = 14
36-14 = 22
46-22 = x
64-x  = 40
75-40 = 35
50-35 = 15
35-15 = 20
```

Think out of the box again!! Submit x. "-" is not necessarily subtraction operator.

Answer 29bdbc822df2e6c13dcf4afe6913525f

101 Alertation

2012-09-11 17:37:02	**by elasolova**	**5 xp**	**Hack**

Your task is to inject `<script>alert()</script>` into this page. Note that there are some crude protection and illusions on the way. When you get the pass after injection just submit here.

Answer **ff7c3f88b0ec4cffe5f9e3013659d889**

102 App: Exeintro 1

2012-09-26 03:32:29	**by elasolova**	**5 xp**	**Hack**

This is our starter exe challenge. The solution is obtained through a slight! modification. We are dealing with command line (so expect no UI for now). Feel free to discuss spoiler free thoughts on the forum.

Download the App

Answer **7eec05929e2d0765a82247d358ff16a6**

103 App: Exeintro 2

2012-09-26 03:35:24	by elasolova	6 xp	Hack

Another intro exe challenge. Still we are dealing in command line. This one lacks some arguments obviously.

Download the App

Answer 448d11c0814ddbdbb99a318c423ac141

104 App: ClickMania

2012-09-29 04:47:56	**by elasolova**	**12 xp**	**Hack**

Some clicking involved!

Download the App

Answer **7837471baab8142887d104e02df875f2**

105 Number Cross

2012-10-03 02:15:19	by elasolova	7 xp	ProgrammingArchive

If you place 1 to 8 into this puzzle

```
* * *
  * * *
  * *
```

where no consecutive (+1 / −1) will be adjacent to each other (left, right, up, down, up-right, etc., all directions), you will have 6 solutions. Namely, one is this:

```
7 4 6
  1 8 2
  3 5
```

So the question is, how many solutions are there for 1 to 10 for this puzzle

```
* * *
  * * *
* *
  * *
```

Answer 0c010fbd9548da6b2e31539b3b9968b3

106 Realistic I

2012-12-27 23:42:37	**by elasolova**	**20 xp**	**Hack**

Here you have a link to an insurance website. This insurance company has suspended its services lately due to its underpopularity. Your friend John Milkway is a customer of this website and has already paid for his insurance plan. He wants his insurance back. Aware of your hacking skills he requested help. Help him to get what he wants.

Hint 1: *robots*
Hint 2: *<Limit>*

Select area to reveal hints if desired.

Answer **2223403e559fe371afb33a90cda28201**

107 Realistic II

2012-12-31 23:30:34 **by elasolova** **20 xp** **Hack**

This time javaist.com needs your help. There is a secret hacking team called Munati. They have chosen javaist.com as their newest target. Find exploit and login to their administration before it is too late.

Link to Munati

Hint 1: *schema*

NOTE: Problem is unavailable/unsolvable till a second notice due to database issues. Sorry for inconvenience …

Answer **0096c586ba3b4003636f607f62f4e357**

108 RSA Attack

2013-04-08 06:00:00 | **by sinan** | **10 xp** | **Crypto**

The following message has been encrypted using the RSA algorithm (http://en.wikipedia.org/wiki/RSA).

```
Message:
527052005627025563334979410266214608870058706774369807477440119 2
384735248934549598826525961928648863148331773299424464335735475 1
928762059694003655833574098381027150306494849266342531557648210 8
055974415356757438138959801929670186199325345000688716342081877 3
64884130165379971128518570467092626371748600039
```

For the conversion from text to number, treat a text of N bytes as $N \times 8$-bit big endian number as in the following examples:

```
YOU = 89,79,85 -> hex: 594F55 decimal: 5853013
YOU CAN = 89,79,85,32,67,65,78 -> hex: 594F552043414E decimal: 25138499959341390
```

You are given the following RSA public key:

```
N =
600044820024309699731118027473601013914386694555098820168500217 4
873965886299642285961910921367896805514530168338297622221048013 5
998769586441513634418595494528677385424958835746103287874931239 5
957038728308263132923851840578660779147842182360934548523308654 8
71204913382882966214741283716047066022304286877

E =
515802370774054967302429892313797011792779662844280924717789277 5
110571790951267662711498049902741032484506954046624122090295038 8
339536098555049449293400244538578773117240734726359213644677719 8
260693567057834041124453307095288303167957306144521329947291244 1
18700665840856941580663774265885674204692471913
```

Can you decrypt the message?

Anwser format: The secret phrase

Answer 5c75fb70c67ec3e8f0f73c7a393a6189

109 Repeated Appereance

2013-08-01 06:00:00	**by sinan**	**12 xp**	**Probability**

by **sinan**

How many times must a die with 6 faces be thrown, on average, until one face appears 4 times?

For example, one set of throws might be 1, 2, 1, 4, 3, 5, 2, 4, 6, 1, 3, 1.

Submit your answer as `a/b`, where a and b are two relatively prime, positive integers.

Answer ae5ad70d0cb9f1587d8e028bf57ca717

110 Balls and Boxes

2013-11-27 20:08:42	by elasolova	15 xp	Probability

A box contains r red balls and b blue balls. Balls are removed sequentially from the box (without replacement). What is the expected number of balls left in the box at the first instant at which all the remaining balls are of the same colour?

Give your answer in closed form and simplify as much as you can, use paranthesis where needed. After simplification, there will be a few possible answers depending on ordering of the terms and only one will be accepted so try all.

Answer 62dfff1041b06f07a16160188891fc0d

111 Histheorically?

2014-01-24 07:00:00	**by elasolova**	**15 xp**	**Math**

Let's define a fast-growing sequence
$F_n = 2^{2^n} + 1, n \geq 1.$

Now let's calculate the sum s of all pairwise gcd (namely sum of greatest common divisors of all 2 element combinations). Now the sequence starts with $n = 1$ and goes up to $n = x$. Give the answer as a polynomial in x. Output format is `(a,b,c)` where $ax^2 + bx + c$ is the polynomial. Use reals with one digit after the decimal point.

Answer **029c808e80134d14ccbe18d8e192e94a**

112 Mirror primes 2

2014-05-20 06:19:32	by sinan	5 xp	Math

Mirror (Palindromic) prime is a prime number when reversed is the same prime. Assume all one digit primes are mirror primes.

Find the sum of all mirror primes with all **prime digits** not exceeding 10^{20}.

Answer format: `count,sum`

Example: `8,2227` for all mirror primes with all **prime digits** not exceeding 10^3 (2, 3, 5, 7, 353, 373, 727, 757).

Answer b9e9537ea7e4e8d39d069a7a03f7b68c

113 Odd coefficients

2014-05-20 18:27:02 **by Philippe_57721** **10 xp** **Math**

by **Philippe_57721**

Let $P[X, n] = (1 + X + X^2)^n$

$P[X, 7] = X^{14} + 7X^{13} + 28X^{12} + 77X^{11} + 161X^{10} + 266X^9 + 357X^8 + 393X^7 + 357X^6 + 266X^5 + 161X^4 + 77X^3 + 28X^2 + 7X + 1$

It contains 11 odd coefficients: $1, 7, 77, 161, 357, 393, 357, 161, 77, 7, 1$.

How many odd coefficients are there in $P[X, 6700417]$?

[My timing: < 100 ms]

Answer 26c2e58c3a6556738930d45dff1e687d

114 From End to Beginning

2014-05-21 07:10:08	by sinan	8 xp	Math

Let $a(n)$ be the n-th term of a sequence.

We have the following relation:
$a(n) = 3 \cdot a(n-1) - 2 \cdot a(n-2) + 1 \cdot a(n-3)$

What is $a(1)$ if we have the following values:
$a(10^{100} + 0) = 0$
$a(10^{100} + 1) = 1$
$a(10^{100} + 2) = 2$

Submit your answer as $a(1) \bmod 10^9$.

Problem maintainer: **sinan**

Answer 81ff5274a990a2c94a76cad063e2cca0

115 A special order

2014-05-22 17:48:13	**by sinan**	**6 xp**	**Programming**

Order the numbers 1 to 66 in such a way that sum of any 2 consecutive numbers is a square number. In case there are multiple solutions, find the lexicographically first solution.

Answer format: concatenate every 5th number starting with the 1st. (i.e: 1st, 6th, 11th, …, 66th)

Example: `831116` for 1 to 16 (8, 1, 15, 10, 6, 3, 13, 12, 4, 5, 11, 14, 2, 7, 9, 16)

Answer 908bab3ef7771f48cba7e30af2250628

116 The biggest square (harder)

2016-04-12 08:30:48	by sinan	10 xp	Math

See p319.[1] The same definition and the same answer format. Find those with **100th biggest area** for $R = 10^{10} + 19$.

[My timing: < 10 s]

Answer c95cc286fc710445c123f141cb186ff9

[1] Actually Problem 323.

117 Pure partitions

2014-05-22 17:55:28 **by Philippe_57721** **10 xp** **Math**

Author **Philippe_57721**

A partition of n is pure if it can express every number between 1 and n in exactly one way.

There are 8 pure partitions of 11:

- $6\ 3\ 1\ 1 = 1\ (1+1)\ 3\ (3+1)\ (3+1+1)\ 6\ (6+1)\ (6+1+1)\ (6+3)\ (6+3+1)\ (6+3+1+1+1)$
- $6\ 2\ 2\ 1 = 1\ 2\ (2+1)\ (2+2)\ (2+2+1)\ 6\ (6+1)\ (6+2)\ (6+2+1)\ (6+2+2)\ (6+2+2+1)$
- $6\ 1\ 1\ 1\ 1\ 1 = 1\ (1+1)\ (1+1+1)\ (1+1+1+1)\ (1+1+1+1+1+1)\ 6\ (6+1)\ (6+1+1)\ (6+1+1+1)\ (6+1+1+1+1)\ (6+1+1+1+1+1)$
- $4\ 4\ 2\ 1 = 1\ 2\ (2+1)\ 4\ (4+1)\ (4+2)\ (4+2+1)\ (4+4)\ (4+4+1)\ (4+4+2)\ (4+4+2+1)$
- $4\ 4\ 1\ 1\ 1 = 1\ (1+1)\ (1+1+1)\ 4\ (4+1)\ (4+1+1)\ (4+1+1+1)\ (4+4)\ (4+4+1)\ (4+4+1+1)\ (4+4+1+1+1)$
- $3\ 3\ 3\ 1\ 1 = 1\ (1+1)\ 3\ (3+1)\ (3+1+1)\ (3+3)\ (3+3+1)\ (3+3+1+1)\ (3+3+3)\ (3+3+3+1)\ (3+3+3+1+1)$
- $2\ 2\ 2\ 2\ 2\ 1 = 1\ 2\ (2+1)\ (2+2)\ (2+2+1)\ (2+2+2)\ (2+2+2+1)\ (2+2+2+2)\ (2+2+2+2+1)\ (2+2+2+2+2)\ (2+2+2+2+2+1)$
- $1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 = 1\ (1+1)\ (1+1+1)\ (1+1+1+1)\ (1+1+1+1+1)\ (1+1+1+1+1+1)\ (1+1+1+1+1+1+1)\ (1+1+1+1+1+1+1+1)\ (1+1+1+1+1+1+1+1+1+)\ (1+1+1+1+1+1+1+1+1+1)\ (1+1+1+1+1+1+1+1+1+1+1)$

How many pure partitions of 46079 are there?

[My timing: < 100 ms]

Answer a6d157db60dca7917186fe31a891a97a

118 A special number

2014-05-25 06:30:15	by sinan	4 xp	Programming

Find the biggest number whose any 4 consecutive digits sum to a square number and whose any decimal digit is not used more than 5 times.

For example 522007 is such a number: $5+2+2+0=9$, $2+2+0+0=4$, $2+0+0+7=9$.

Answer e8dfc63a77ae08b2699743eae1dcbd3b

119 Simplify

2014-05-25 16:12:15 **by Philippe_57721** **8 xp** **Math**

Let's define:
$\mathrm{R2}(x) = x^{\frac{1}{2}}$ (Square root)
$\mathrm{R3}(x) = x^{\frac{1}{3}}$ (Cubic root)

We can simplify the following expression:
`R3(R3(2)-1)` as
`R3(1/9)-R3(2/9)+R3(4/9)`

LaTeX code

$$\sqrt[3]{\sqrt[3]{2}-1} = \sqrt[3]{1/9} - \sqrt[3]{2/9} + \sqrt[3]{4/9}$$

Can you simplify `R2(R3(3645)-R3(2916))`?

LaTeX code

$$\sqrt{\sqrt[3]{3645} - \sqrt[3]{2916}}$$

Format answer: a linear combination of radicands in increasing order using functions `R2`, `R3` and without spaces

Hint: The anwser involves only small integers …

Answer e743f75be0ebebb7a3da7b5697d3154b

120 A secret conversation

2014-05-28 09:29:42 **by sinan** **25 xp** **Crypto**

Alice and Bob have the following RSA keys:

```
N_bob =
2843320233792542251053357394068088686668524975752636080093537940
4832527514764874620628848105181825527933880102457169730719923244
6420151499813375523635289637990354962034693574665424583399133573
0583501632679081431737342001818666796295631297455573102373116440
3029788484704936984714437598885024284453243844454411794042986345
2670573135811710297531056679013108703134713805990185300653031156
4180092399756052224998536204529486287954472923574337261846322017
890592838801890702039

E_bob =
2291100640658304898982037736299578548614658498374369307423059101
7753909793205446710905103866626852874140382358105613997851675753
6616368494790527495119970061488145079661142528335377919451554688
6154257484268335107367291548215957325536731933203947342597628607
0538102530714442994442646163672237217358454654896394943346204459
0665520882478282704144094054610779463057719130141630088734441774
4643471937489057261616554090115047189679240761528659032842549170
461539414559766262563

N_alice =
5686640467585084502106714788136177373337049951505272160187075880
9665055029529749241257696210363651055867760204914339461439846489
2840302999626752018801395151071678016047051400133923004670186013
9988684396623348180130983267413984406693145186191663434333107341
4271159645546495517290635898239652290125489708412903139175746507
8882509062160255488830168051051374738819714402801269575387131769
8468196572196081277369024478340281325985826876817304126462736931
162058746229329458443

E_alice =
5532204639026134156498081092471471626870989697962125080810121970
0937589706597407035452447205974152734582820015569746612727007206
2814426400049844220852036436793883376956881347807180964186086536
6805991700540784345980142718081120280543581590237686483113697701
6509366984984445443725555441703117867313078723749470914317101582
9520122197714627967971392769730377826127068782925537780549266077
9768718083990120844013377666526983516809292705750575751662270100
621679600058468454699
```

Alice sends Bob the following message:

```
M_AtoB =
1396364112413623642374178953014019904568061507434558505408267555
8231810369233960158591102037668709444252754006802542256490072017
0291609574950964015823622596731476468470628411816615719782348534
7273167553704712490006234283555813449834048635758312335972910209
5849958053264948388306938543259511664955402250394909679552507339
```

```
2828805017197137725781299915845926173013085987451784686823149814
4313379547441762202636503884816587950694638762568426358881638049
169947376779771961136
```

What should Bob send back to Alice?

Input format: The first 100 digits of the message encrypted using Alice's public key

[My timing: < 1 m]

Answer a5acdbbbb14723ff5a0a81b6763e02c1

121 In a distance

2014-05-29 12:01:53	by Philippe_57721	6 xp	Programming

Author **Philippe_57721**

We are reading the decimal part of PI by slices of 10 digits:
1415926535, 8979323846, 2643383279, 5028841971, …

In the first 20000000 digits, how many slices are the closest of 1415926535 (not included) in the sense of the Damerau-Levenshtein distance?

Answer format: `Count,(Slice with the closest euclidian distance)`

Example: `3,1145726635` for the first 2000000 digits.

[My timing: 35 sec]

PS: As there are two slightly different algorithms for this distance, the problem is using the restricted edit distance version.

Answer 247298a7f5fa1474d5e5961150f87106

122 Smallest difference

2014-06-01 16:48:34 **by Philippe_57721** **5 xp** **Math**

For $1 \leq x, y \leq 1000$, the smallest possible value of $|x \cdot \mathrm{PI} - y \cdot \mathrm{E}|$ is reached for $(x, y) = (777, 898)$ ($\mathrm{PI} = 3.141592\cdots$, $\mathrm{E} = 2.71828\cdots$)

Which pair $(x, y) \leq 10^{20}$ does yield the smallest value?

Answer format: `x,y`

Answer eb7779a6bc6f3bebff839f0bac9ff4de

123 Creatures

2014-06-04 18:22:30	**by sinan**	**12 xp**	**Programming**

Let there be creatures with different N colors $(1, ..., N)$ for $N > 7$. Let $\mathrm{num}(i)$ be the number of creatures with i-th color on a specific day.

The following events occur within an N-day period.

On the i-th day of the period:

Number of cretures with i-th color increase by 1000 more than the sum of existing creatures with $(i+1)$-th, $(i+2)$-th and twice the $(i+3)$-th colors.
$\mathrm{num}(i) = \mathrm{num}(i) + \mathrm{num}(i+1) + \mathrm{num}(i+2) + 2 \times \mathrm{num}(i+3) + 1000$

Number of ceatures with $(i+4)$-th color becomes 17 more than twice the existing creatures of that color.
$\mathrm{num}(i+4) = 2 \times \mathrm{num}(i+4) + 17$

All the $(i+5)$-th colored creatures but one change their color to i-th color (i.e. one of them doesn't change its color).

Number of creatures with $(i+6)$-th color quadruples.
$\mathrm{num}(i+6) = 4 \times \mathrm{num}(i+6)$

Find the total number of creatures after 10^{19} days if $N = 11$ and initially there is 1 creature of each color.

Assume that the first day is the first day of the period.

Note: $(i+n)$-th color should mean $((i+n) - N)$-th color if $i + n > N$. For example if $i = 9$ then $(i+4)$-th should mean 2nd $(9 + 4 - 11 = 2)$.

Input format: total mod 10^9

Example: `619784765` for 100-th day when $N = 10$.

[My timing: < 1 s]

Answer 02ddab418694e6c02bd6f5ec23fbd71d

124 Slices of PI E - Part 1

2014-06-05 18:03:06 by Philippe_57721 **7 xp** Programming

Author **Philippe_57721**

We are reading the decimal parts of PI and E by slices of 10 digits:
$\mathrm{PI} = 1415926535, 8979323846, 2643383279, 5028841971, \dots$
$\mathrm{E} = 7182818284, 5904523536, 0287471352, 6624977572, \dots$

What is the “first” slice appearing in both constants?

“first” means it requires the minimum numbers of digits in the decimal parts.

For slices of 5 digits, the answer would be: 32823.

[My timing: < 3 sec]

Answer 08299d7d2db9272ad0d7271947bd3b32

125 A Game of Risk

2014-06-07 07:59:33 **by elasolova** **12 xp** **Probability**

Two friends, Arya and Bran are being held captive in separate places. They request a trial by game from their captivators.

Here is the deal: Arya has to choose a random number between 1 and 101010101 (inclusive). Bran has to choose also but between 1 and 303030303 (inclusive). There is no possible communication between them and they will only be freed if the sum of their numbers is odd. Otherwise they are doomed forever.

What is the probability of them being freed? Give the answer as an irreducible fraction.

Answer **ce05b49bc7beea9ccfe3e3ac43d75c7c**

126 Bombastic Queue - revisited

2014-06-08 14:03:25	**by sinan**	**15 xp**	**Math**

Mr. Bombastic wants to enqueue his students in a row. There are 50 students in the class that are queued according to their heights from shortest to longest currently.

Mr. Bombastic has a taste of fashion and does not like orderly stuff. So he decides to disrupt the queue. However, the school directors only permit 20 but only 20 swaps.

A swap is between any two students. So how many different queue options does Mr. Bombastic have such that at least 30 or more students are not at their original places after exactly twenty successive swaps?

[My timing: < 1 s]

Answer 97ca9584f8ecc7df6e786176701c3ac6

127 Factorial Fine Factorisations

2014-06-09 15:40:34 **by Philippe_57721** **8 xp** **Math**

A factorisation of N in factors is fine if each factor contains only 1 prime number.

$6! = 720$ possesses 10 fine factorisations:

- 5 9 16
- 3 3 5 16
- 2 5 8 9
- 2 3 3 5 8
- 4 4 5 9
- 3 3 4 4 5
- 2 2 4 5 9
- 2 2 3 3 4 5
- 2 2 2 2 5 9
- 2 2 2 2 3 3 5

How many fine factorisations are there for 10000!.

Give the digital sum of this large number.

[My timing: 1 sec]

Answer 3e2b8b6ab4cf4d52ceb2158dd118ad68

128 Flip-flop Bit Sets

2014-06-14 06:32:42 **by Philippe_57721** **10 xp** **Math**

Let B be the binary digits of an integer N. $B = \{b_1, b_2, b_3, ..., b_n\}$, b_1 is the least significant bit and $b_n = 1$.

Consider the subsets of B whose elements are alternately 0 and 1.

Example: $N = 40$, $B = \{0, 0, 0, 1, 0, 1\}$.

There are 11 such flip-flop subsets (including the empty set):

Index	Values
$[]$	$\{\}$
$[1, 4]$	$\{0, 1\}$
$[1, 6]$	$\{0, 1\}$
$[2, 4]$	$\{0, 1\}$
$[2, 6]$	$\{0, 1\}$
$[3, 4]$	$\{0, 1\}$
$[3, 6]$	$\{0, 1\}$
$[5, 6]$	$\{0, 1\}$
$[1, 4, 5, 6]$	$\{0, 1, 0, 1\}$
$[2, 4, 5, 6]$	$\{0, 1, 0, 1\}$
$[3, 4, 5, 6]$	$\{0, 1, 0, 1\}$

How many flip-flop subsets are there for $N = 1234567891011121314$?

[My timing: < 100 ms]

Answer 2c1904be55334a7dc8159abfa3dd06b5

129 Crazy World Cup

2014-06-15 23:26:23 **by elasolova** **15 xp** **Math**

World cup 2014 got crazier then ever.

Each new day a new team registers to world cup with an insane match schedule. Team registered at day i dictates that they will only be able to play i days and then they will take rest for i days after and this pattern will continue on.

In an example team registered at day 1 will have an availability schedule like:
$10101010\cdots$ (where 1 denotes available)

Availability for Team registered in day 2:
$011001100\cdots$

Availability for Team registered in day 3:
$00111000111\cdots$

So on and so forth …

The administration wants to match up all teams at i-th day. So, number of available teams at day i must be even to do that.

The question is just for mathematical enthusiasm and it may make no sense. So, how many days will **NOT** be valid for such a matching if the world cup lasted for 10^{18} days?

Answer ca283dd1b30f520848c50620d8be43b8

130 Mills-like Constant

2014-06-19 17:55:56 **by Philippe_57721** **10 xp** **Math**

In 1947, William Mills proved the following theorem:

There exists a real number M such as $\left[M^{3^k}\right]$ is a prime number for all $k \geq 1$ ($[x]$ is the integer part of x).

Find the smallest real P such as $\left[P^{2^k}\right]$ is prime for k in $\{1, 2, 3, 4, 5, 6, 7\}$.

Answer format: round your answer to 30 digits after the decimal point

[My timing: < 1 sec]

Answer **8fd9723df80b95e1edcab5716f1399b0**

131 Exploring the Champernowne Constant

2014-06-22 16:40:59 **by Philippe_57721** **8 xp** **Programming**

The Champernowne constant is the real number whose decimal part is the concatenation of all positive integers:
$C = 0.1234567891011121314151617181920\cdots$

At which index does appear the first occurence of the substring 803396454804396?

Hint: The substring 91 appears at index 9.

[My timing: < 100 ms]

Answer 475d2de6c4c9996aac8e2336192106da

132 Perfectly protected pillars

2014-06-27 18:04:46 **by Philippe_57721** **10 xp** **Programming**

In the main hall of the King's palace there are pillars disposed in a rectangular grid $N \times M$.

The king wants to dispose 4 guards at the 4 directions of each pillar. A guard at the east of a pillar is at the west of the next pillar, etc …

Being mathematically inclined, the king wants that for each pillar, there are exactly 2 guards looking at it and 2 guards turning their back.

Here is an example with 2×3 pillars (and 17 guards):

$$\begin{array}{ccccccc}
 & \downarrow & & \uparrow & & \uparrow & \\
\leftarrow & \bigcirc & \rightarrow & \bigcirc & \rightarrow & \bigcirc & \leftarrow \\
 & \uparrow & & \uparrow & & \downarrow & \\
\rightarrow & \bigcirc & \rightarrow & \bigcirc & \rightarrow & \bigcirc & \rightarrow \\
 & \uparrow & & \uparrow & & \downarrow &
\end{array}$$

How many possible configurations are there for 4×6 pillars?

You are given: 374 configurations for 2×3 pillars.

[My timing: 25 sec]

Answer d3ae2cb07ed2d42845c786b144fd88c4

133 Find a recurrence

2014-07-01 14:10:31	**by sinan**	**20 xp**	**Programming**

See the problem 130.[1] Find a recurrence relation for $M \times 8$ grid for $M > 0$.

Then how many configurations are there for $8^{88} \times 8$ grid?

Input format: enter the rightmost 20 digits

Hint: Solve the p130 first.[2] Once solved, you can access to a reasonably fast code in the problem forum. If you opt to use it, you need to improve it to solve this one. Then you need to calculate the solutions for $M = 1, 2, \ldots, 72$ to find a recurrence equation.

[My timing: 45 s]

Answer 2ba729c4a035e52e521f1b1780d01140

[1] Actually Problem 132.
[2] Actually Problem 132.

134 Fibonacci Partitions

2014-07-03 18:28:26 **by Philippe_57721** **8 xp** **Programming**

105 is the smallest integer which can be decomposed in 10 ways as sum of distinct Fibonacci numbers:

- 1 2 5 8 13 21 55
- 1 2 5 8 34 55
- 1 2 5 8 89
- 1 2 13 34 55
- 1 2 13 89
- 3 5 8 13 21 55
- 3 5 8 34 55
- 3 5 8 89
- 3 13 34 55
- 3 13 89

What is the smallest integer with 5000 decompositions?

[My timing: 80 sec]

Answer 130a700b5b21f11c778dbd0ed6b7e127

135 PPT PPH

2014-07-04 10:29:53 **by sinan** **8 xp** **Programming**

Consider the primitive Phytagorean triangles with an hypotenuse equal to a Pythagorean prime (i.e. a prime of the form $4n + 1$).

How many such triangles are there if their hypotenuses are between 10^{19} and $10^{19} + 10^6$?

Input format: `count,sum of the smallest sides`

Example: `11,248` for such triangles with hypotenuses in the range $[1, 100]$.

[My timing: < 8 s]

Answer 25ec37758f913b9dbe38e4af367d5e76

136 Prime factorization

2014-07-07 13:34:05	**by gerrob**	**6 xp**	**Math**

Let $n = \text{product}(p = 2, 2 \times 10^{13}, \text{GCD}(((p-1)! + 1)/p, p))$, where p runs on primes in the $[2, 2 \times 10^{13}]$ interval. Find the prime factorization of n.

Answer format: `p1,p2,...`
(the prime factors (in increasing order) separated by commas)

Answer ff52fc43f71e5c9512fb27dc89b77520

137 Prime factorization II

2014-07-07 17:17:58	by gerrob	10 xp	Math

Let $n = a^3 + b^3 + c^3 - 3 \cdot a \cdot b \cdot c$ for $a = 6^{58}$, $b = 9^{77}$ and $c = 10^{107}$. Find the largest prime factor of n.

Answer format: the first 15 digits of p
(where p is the largest prime factor of n)

Answer 30a875a3e60b92c8d3d60f084b379e82

138 Prime factorization III

2014-07-08 17:41:52 **by gerrob** **20 xp** **Math**

Factorize

```
n=6072763559395513956949829660448533632513422921778233734175 87
8471387717282140650810510890458076264309972245103807179057762 9
```

if you know that $n = p \cdot q \cdot r \cdot s \cdot t$ (where $p < q < r < s < t$ are primes) and $a(k) = p^k + q^k + r^k + s^k + t^k$ is also given for $k = 1, 2, 3, 4$.

```
a(1)=45330606091644572644695584271000126664541588217136917454 0
5459244544616227925211 97
```

Answer format: `L(p),L(q),L(r),L(s),L(t)`
(where $L(x)$ is the first five digits of x)

Answer 880efa4b35b2beed4d9ea492c5efd358

139 Towers of Hanoi

2014-07-10 11:03:51 **by sinan** **10 xp** **Programming**

There is a 30 disk layout as shown below (from bottom to top) — **not necessarily reached by way of always making the optimal moves**:

```
1| 21 13 8 5 3 2 1
2| 25 16 9 4
3| 30 29 28 27 26 24 23 22 20 19 18 17 15 14 12 11 10 7 6
```

Target is to move all the disks to the second peg. To reach that target, what would be the layout after 10^9 **optimal** moves and what would be total of the moves if we assign 12 for a move from first to second, 13 for a move from first to third, etc. (possible moves would be 12, 13, 21, 23, 31, 32)?

Input format: `1st peg,2nd peg,3rd peg,total`

Example: `10965,321,874,2191` for the following

First layout for a 10-disk configuration:

```
1| 10 9 8 5 4
2| 7 6 3
3| 2 1
```

The layout after 100 moves targeting to move all the disks to the 3rd peg:

```
1| 10 9 6 5
2| 3 2 1
3| 8 7 4
```

total = 2191

Answer 158c546adea42d68fc2a75f2a4566c2b

140 Exploring the Champernowne Constant - Ultimate

2014-07-10 18:01:47 by Philippe_57721 **12 xp** Programming

C is the Champernowne constant (See problem 131).

Define $K(n) = [C \times 10^{10^n}]$ where $[x]$ is the integer part of x (the first 10^n digits of C).

Example: $K(1) = 1234567891$.

What is the digital sum of the digital sum of $K(100)$?

You are given:
$K(10) = 44567901235$
$K(15) = 4441049382716054$
$K(20) = 441486267257142575580$

[My timing: < 100 ms]

Answer e6d29d6863a7820ae06871d24dd09913

141 Prime factorization IV

2014-07-11 17:57:36 **by gerrob** **20 xp** **Math**

$p = 798889237986664049$ divides $n = a^b + c^d$ for some $0 < a, b, c, d < 18000$ (integers) where $a < c$. Find n, so the a, b, c, d quadruplet. The solution is unique.

Answer format: `a,b,c,d`
(where $a < c$)

[My timing: 61 seconds]

Answer c522cf2facbf8732e867b3d7405c76f5

142 Arbitrage

2014-07-11 18:14:13 **by gerrob** **8 xp** **Math**

On an event there are three possible outcomes. The provided odds on these are: 2, 3 and 7. Suppose that you have $m = 614889782588491410$ (money), and you can place bets on all of these 3 outcomes. What is the maximal profit that you can reach (indepedently from the occuring outcome)? The answer is an integer.

Answer format: `profit`

Answer **0aff4e37c4438dee4c18c592ccd28bca**

143 Lattice points

2014-07-14 13:37:43 **by sinan** **8 xp** **Math**

For a, b, c, d all being positive integers, we have the following:
$L_1 : y = (b/a) \cdot x$
$L_2 : y = -(a/b) \cdot x + a$
L_1 and L_2 cross at a $P(c, d)$ point.
The distances from P to origin $(0, 0)$ and to $(0, a)$ and to $(b, 0)$ points are all integers.

Let $f(R)$ denote the number of (a, b, c, d) solutions if P is inside $x^2 + y^2 = R^2$ circle.

You are given: $f(10^5) = 3782$.

What is $f(10^{19})$?

[My timing: < 100 ms]

Answer 4201ed1ab8e63702376c1c1d41894748

144 Find a recurrence 2

2014-07-15 11:34:04	**by sinan**	**5 xp**	**Math**

Find a recurrence relation for the following sequence:
$1, 2, 3, 5, 8, 3, 11, 14, 25, 39, 64, 25, 89, 114, 203, 317, 520, 203, 723, 926, \ldots$

Then what is the 10^{100}-th number modulo 10^9?

Answer **782c636f4af8f269b023f4e616a0ceed**

145 Equilateral triangles

2014-07-15 15:02:39 **by sinan** **10 xp** **Math**

Assume a D point inside an ABC equilateral triangle with $AD = x$, $BD = y$, $CD = z$, $0 < x \le y \le z < 100$ and x, y, z are all integers.

Let the length of the sides (s) of the triangle be a function of x, y, z: $s = f(x, y, z)$.

Find all such triangles.

Input format: `cnt,sum of s's (3 digits after decimal point)`

Example: `13,59.831` for $0 < x \le y \le z < 5$.

Answer b256f46666c9d0e16b2e7e9b6caa3649

146 Equilateral triangles 2

2014-07-18 11:59:34	by sinan	15 xp	Math

Assume a D point on the AB side of an ABC equilateral triangle with $AD = x, BD = y, CD = z, 0 < x < y < z$ and x, y, z are all integers.

Let the sides of the ABC triangle be equal to $s = x + y$.

Find all such triangles for $s < 10^8$.

Input format: `cnt,sum of z's,sum of s's`

Example: `35,1853,2081` for $s < 100$.

[**My timing: < 10 s**]

Answer 9ae3990a735c5d057ce0a48914f94314

147 What else

2014-07-20 15:14:55 | **by Philippe_57721** | **8 xp** | **Programming**

The function `T` is defined as:

```
T(x,y,z) =
    if x <= y
        return y
    else
        return T(T(x-1,y,z),T(y-1,z,x),T(z-1,x,y))
```

The function `U` is defined as:

```
U(n) = T(n,0,n+1)
```

We consider the number of times the **else** clause in the `T` function is invoked when computing `U(n)` (no memoization). Let $E(n)$ this number.

You are given $E(5) = 223$.

What is $E(100)$?

Answer format: give the digital sum of this huge number

[My timing: < 100 ms]

Answer 9a93bc40e53f31e10ff47d97bd93c4a2

148 Highly composite numbers decomposition

2014-07-26 06:27:05 **by Philippe_57721** **8 xp** **Programming**

36 can be written in 9 ways as product of factors > 1:

- $\{36\}$
- $\{3, 12\}$
- $\{4, 9\}$
- $\{6, 6\}$
- $\{2, 18\}$
- $\{3, 3, 4\}$
- $\{2, 3, 6\}$
- $\{2, 2, 9\}$
- $\{2, 2, 3, 3\}$

A number is highly composite if it has more divisors than all smaller numbers.

What is the first highly composite number which can be written in more than 10^8 ways as product of factors > 1?

Answer format: `n,(number of products)`

Example: `20160,1261` for a threshold of 1000.

[My timing: 45 sec]

Answer 1fc82c619c401a0544708b1945b802b0

149 Number decomposition again

2014-07-28 09:20:44	by sinan	10 xp	Programming

Let P be the product of first 20 primes.
$P = \text{prod}(i = 1, 20, \text{prime}(i)) = 557940830126698960967415390$

In how many ways can $200 \times P$ be written as product of factors > 1?

[My timing: < 10 s]

Answer fa5770f838653162e4bd3cb64ca35a67

150 Towers of Hanoi revisited

2014-08-02 13:04:50	by Philippe_57721	10 xp	Programming

In a game of Hanoi with n disks, we can consider the layout after m moves as a permutation of $\{1, \ldots, n\}$.

For instance, with 15 disks, after 12345 moves the layout is:

- `Peg-1 : 7 8 9 10 11 12 15`
- `Peg-2 : 1 4 5 6 13 14`
- `Peg-3 : 2 3`

corresponding to permutation: `7 8 9 10 11 12 15 1 4 5 6 13 14 2 3`

The index of this permutation in lexicographic order is 563569656784.

With 70 disks, what is the index of the permutation corresponding to the layout after 123456789101112131415 moves?

[My timing: < 100 ms]

Answer c60eafe2663ec705c125f4eaf967f0e5

151 Similar triangles

2014-08-03 11:02:55 **by sinan** **8 xp** **Math**

For a, b, c, d all being positive integers, consider 2 similar triangles with the following lengths:
$T_1 : a, b, c \ (a < b < c)$
$T_2 : b, c, d \ (b < c < d)$

What is the smallest pair of such triangles for which the ratio $(a + b + c)/(a + b - c)$ first exceeds 10^{10}?

Input format: `perimeter of T1 (a+b+c)`

Answer a1742ea582aebc415eda487e97c7a5bc

152 A lattice points puzzle

2014-08-06 10:54:52 by sinan **10 xp** Math

Condider $A(x_a, y_a)$, $B(x_b, y_b)$, $C(x_c, y_c)$, $D(x_d, y_d)$ lattice points in the first quadrant (i.e. coordinates with integer x,y: $x \geq 0$ and $y \geq 0$).

AB segment is parallel to DC and AD parallel to BC.

Consider N points $M_1, M_2, ..., M_N$ on BC segment that split BC into N equal segments:
For $x_c > x_b$ and $y_c > y_b$:
$M_1(x_b + 1 \cdot (x_c - x_b)/N, y_b + 1 \cdot (y_c - y_b)/N)$,
$M_2(x_b + 2 \cdot (x_c - x_b)/N, y_b + 2 \cdot (y_c - y_b)/N)$,
...
$M_N(x_b + N \cdot (x_c - x_b)/N, y_b + N \cdot (y_c - y_b)/N) = C(x_c, y_c)$.

$M_1, M_2, ..., M_N$ are **not necessarily lattice points** but all the line segments drawn from A to $M_1, M_2, ..., M_N$ cross BD segment at lattice points.

What is the smallest area of $ABCD$ you can get for $N = 1000$?

Input format: $\text{Area}(ABCD) \bmod 10^9$

Answer d8079566f3a5c22981ec25a6f432e31e

153 A best approximation

2014-08-06 11:01:16	**by sinan**	**10 xp**	**Math**

For a, b, c being positive **coprime** integers and $a < b < c$, consider the following 2 integer sided squares:
$S_1 : A(0,0); B(c,0); C(c,c); D(0,c)$
$S_2 : A'(a,0); B'(a+b,a); C'(b,a+b); D'(0,b)$

What is the best approximation for $\text{Area}(S_1)/\text{Area}(S_2) = 2$ if $\text{Area}(S_2) < 10^{24}$? In other words find the (a,b,c) triples that minimize $|\text{Area}(S_1)/\text{Area}(S_2) - 2|$.

Input format: enter the list of "a" values separated by commas (possibly more than one)

Example: `a1,a2,...,an` $(a1 < a2 < \cdots < an)$

Answer d86a7941ba160e7a5c7d9db75e3b665c

154 A boring sequence

2014-08-07 17:31:46 **by Philippe_57721** **12 xp** **Math**

Let $P(x) = x^{19} + 6$
Let $G(n) = \text{GCD}(P(n), P(n+1))$

The sequence $\{G(1), G(2), G(3), ...\}$ seems constant and equal to 1 for an awful long time …

What is the first n for which $G(n) > 1$?

Answer format: n

[My timing: < 5 sec]

Answer aa0405a3757e36455aadd556c35da575

155 Hint for 152

2014-08-10 15:33:10 **by Philippe_57721** **5 xp** **Programming**

If we apply the permutation P with index I (in lexicographic order) to the string S for the following values:

```
I = 366899275
S = "-EHILNOOSTTU"
```

we get $P = \{10, 3, 2, 1, 9, 7, 5, 12, 11, 4, 8, 6\}$
$\Rightarrow$ `"THE-SOLUTION"` (Take the 10th char, then the 3rd, … in 1-based origin)

Apply the permutation P with index I to the string S for the following values:

```
I =
6044912527595411749579227181311120239399976240987501298038504481
584636103259545029612786266672153273978138480777064 2238
S =
"-------------AAABBBDEEEEEEEEFHHIILLLLLLMMNNNNOOOOOOOOOOPPPRRRSSSSSSTTTTTTTUUVWYY"
```

and you'll get a good lead to solve problem 152.[1]

Answer format: the string

Answer 858c57b45de337f451b135866ff7c36c

[1] Actually Problem 154.

156 Brute interrogation - revisited

2014-08-17 09:09:44 **by sinan** **10 xp** **Programming**

See the following code:

```
function(N)
begin
    UP=101
    DOWN=211
    FACTOR=-2
    temp=0
    loop for each decimal digit of N from left to right
    begin // loop
        if digit is equal to 1; add UP to temp
        else if digit is equal to 2; subtract DOWN from temp
        else if digit is equal to 3; multiply temp by FACTOR
    end // loop
    return temp
end
```

What is the smallest N for which function(N) returns 100000001?

[My timing: < 1 m]

Answer 6f0b32018ad04bb380409912ce1e89c8

157 Morley's triangles - Warming up

2014-08-17 12:46:19	by Philippe_57721	7 xp	Math

A beautiful theorem of elementary geometry only discovered in 1898 by Frank Morley states:
In any triangle, the three points of intersection of the adjacent angle trisectors form an equilateral triangle.

What is the side of the Morley's triangle for the integer sided triangle: $(20424, 24167, 29791)$. It's a rational number.

Answer format: `P/Q`
(in irreductible form)

Answer 4ac8fad9b1873593c3956aa24e4cc609

158 Morley's triangles - Practise

2014-08-17 12:50:19 | **by Philippe_57721** | **12 xp** | **Programming**

Find an integer sided isoceles triangle

- whose perimeter is between 94000 and 95000
- whose Morley's triangle has a rational side

Answer format: `a,b,p/q`

- $a < b$ are the 2 sides of the isoceles triangle (a and b are coprime)
- p/q is the side of the Morley's triangle in irreductible form

[My timing: 15 min ...]

Answer 3ea0a97e5080c5e7d76875812cc18d48

159 End of Fibonacci

2014-08-19 02:58:11	by elasolova	6 xp	Math

Find the smallest $n > 0$ such that Fibonacci(n) ends in nine nines.
$F(1) = 1, F(2) = 1, F(3) = 2$

by **Buri**

Answer be8e77c6e27147cd5bf9b9c071ebed08

160 Programming exercise - Counting

2014-08-24 06:09:36	by Philippe_57721	8 xp	Programming

Let the following function

```
function(n)
begin
    count = 0
    for i in 1 to n
        for j in (i+1) to n
            if 1 = GCD(i,j)
                count = count+1
    return count
end
```

What is $\text{function}(123456789)$.

You are given: $\text{function}(10^5) = 3039650753$.

[My timing: 80 sec]

PS: I slightly changed the parameter (and the answer) as the original solution might be easily found on the web.

Answer d8abcc59bbe09d9031523f0d5d68b682

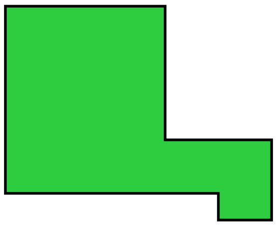
161 Hunting the golygons

2014-08-28 05:14:04 by Philippe_57721 **12 xp** Programming

A golygon is a polygon:

- whose all angles are right
- whose lengths are the consecutive integers 1, 2, 3, ... in that order

There is 1 non-crossing golygon with 8 sides:



It's area is 52.

Find the minimum and maximum area amongst all non-crossing 24 sides golygons.

Answer format: `min,max`

Example: `384,664` for 16 sides golygons.

[My timing: 30 sec]

Answer 2c055674fd21f210928e25c312a4ef97

162 Quasi-perfect numbers

2014-09-05 08:28:05	by Philippe_57721	7 xp	Programming

An integer is perfect if it is equal to the sum of its proper divisors.

Let say an integer is quasi-perfect if it can be written as a sum of some distinct of its proper divisors.

For instance, $N = 150$ as the following proper divisors:
$1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75$
and can be written as:

- $1 + 3 + 5 + 6 + 10 + 50 + 75$
- $1 + 3 + 6 + 10 + 25 + 30 + 75$
- $1 + 3 + 6 + 15 + 50 + 75$
- $2 + 3 + 5 + 10 + 25 + 30 + 75$
- $2 + 3 + 5 + 15 + 50 + 75$
- $2 + 3 + 15 + 25 + 30 + 75$
- $5 + 15 + 25 + 30 + 75$
- $10 + 15 + 50 + 75$
- $25 + 50 + 75$

Find the first such decomposition for 1000768 (in lexicographic order).

Answer format: comma separated list of divisors in ascending order

Example: `1,3,5,6,10,50,75`

[My timing: 50 sec]

Answer 1c7c4710a5cddfcc911c41803fc0a90d

163 Quasi-perfect numbers (revisited)

2014-09-06 07:22:22	**by gerrob**	**15 xp**	**Programming**

See Quasi-perfect numbers problem from Philippe. This time find the first decomposition in lexicographic order for $N = 30380400$.

Answer format: $P \bmod 98961651023$
(where P is the product of all terms in the decomposition)

Example: `3375000` (for $N = 150$).

[My timing: 24 seconds]

Answer **fa3a0795556d4c2922dbb4d1998e6e30**

164 A sum with constraints

2014-09-14 16:28:01 **by Philippe_57721** **7 xp** **Programming**

Find the sum of all integers with at most 12 digits and containing exactly one '1', two '2', three '3' and four '4' in their digital representation (e.g.: 23313447424).

You are given: 121970799987802920 for all integers with at most 10 digits and containing exactly one '1', two '2' and three '3'.

[My timing: 15 sec]

Answer ea24d9d11dc6d7772c3d6dbf6a5d34ff

165 Consecutive numbers

2014-09-15 12:24:01	**by sinan**	**15 xp**	**Programming**

For $N > 0$, consider the ways to write N as the sum of consecutive integers from a to b where $b \geq a > 0$.
$N = a + (a+1) + (a+2) + ... + (b-1) + b$

For example:
$N = 9 = 4 + 5 = 2 + 3 + 4$
$N = 10 = 1 + 2 + 3 + 4$

Let N be the product of first 42 primes:
$N = \mathrm{prod}(i = 1, 42, \mathrm{prime}(i))$

$N = 5397346292805549782720214077673687806275517530364350655459511599582614290$

How can we write N in such a way that $(b - a)$ is **maximized**?

Input format: `a,b`

Example:
`1,20` for the product of first 4 primes:
$N = 2 \times 3 \times 5 \times 7 = 210 = 1 + 2 + 3 + \cdots + 20$

Answer **4e4502a6fd2ce6db3233f573803bea66**

166 GCD Arithmetic Mean

2014-09-20 14:04:32	**by Philippe_57721**	**7 xp**	**Math**

Let's define $A(n)$ = Arithmetic mean of $\text{GCD}(n, k)$ for k in $\{1, ..., n\}$.

Find $A(30!)$.

You are given $A(1000) = 17/2$.

Answer format: `P/Q`

[My timing: 36 sec]

Answer 5578e1d2a1278c5b517fb12a98fbd123

167 GCD Harmonic Mean

2014-09-20 14:05:28 **by Philippe_57721** **8 xp** **Math**

Let's define $H(n) =$ Harmonic mean of $\mathrm{GCD}(n, k)$ for k in $\{1, \ldots, n\}$.

For the record, if $V = \{V_1, V_2, \ldots, V_n\}$ the harmonic mean of $V = n/(1/V_1 + 1/V_2 + \cdots + 1/V_n)$.

Find $H(25!)$.

You are given $H(1050) = 52500/22403$.

Answer format: `P/Q`

[My timing: 5 sec]

Answer d6674a5c27ed26d2c266d4a843076fea

168 Hendeca divisibility

2014-09-27 17:32:56 **by Philippe_57721** **10 xp** **Programming**

It can be proved that there is a smallest integer $\boldsymbol{h}$ such as for any sequence of k (greater or equal to h) consecutive integers there is always one whose digital sum is divisible by 11.

It is not true for less than h consecutive integers.

What is h?
What is the first exception, i.e. the smallest $\boldsymbol{e}$ such as the $(h-1)$ consecutive integers starting with $\boldsymbol{e}$ have their digital sum not divisible by 11?

Answer format: `h,e`

Answer **b0eaa924044418df29602b7de3d84852**

169 Circle Festival

2014-09-28 10:58:49	by elasolova	8 xp	Math

A huge crowd is celebrating the new season.

There are N people (numbered from 1 to N) in the crowd and they gather in a **circle** in sequence.

They decide to play a game like this:
It starts with 1st person. He eliminates person 2 with a wooden sword and gives the sword to 3. Then 3 eliminates 4 and sword is passed to 5. So each person eliminates the one next to himself and passes the sword to the other next. The winner of the game is the last person standing.

Which number is the winner if $N = 9 \times 10^{29}$?

Hint: Deduce a formula from smaller cases.

Answer 23e2455505f38cd37e060afe37c87ebd

170 A modular equation

2014-10-05 16:19:06 | **by Philippe_57721** | **12 xp** | **Programming**

Solve the modular equation

$$314159265358979323846^x = 271828182845904523536$$
$$\text{modulo } 3506457161893013907641318390893283649218125103$$

Answer format: x

[My timing: 45 sec]

Answer **f6aa44213e9d8471c3a216fa8be34c96**

171 Too long...

2014-10-09 08:58:47 **by sinan** **10 xp** **Hack**

Just get the correct value and enter as the answer! Try it

The JavaScript code for this problem is as follows:

```
function f(N)
{
    var a,b,c,d,e,f,g,h,s=0,tb,tc,td,te,tf,tg,th;
    for(a=0;a<=N;a+=1){
        for(b=0;b<=N;b+=2){
            tb=a+b;if(tb>N)break;
            for(c=0;c<=N;c+=3){
                tc=tb+c;if(tc>N)break;
                for(d=0;d<=N;d+=4){
                    td=tc+d;if(td>N)break;
                    for(e=0;e<=N;e+=5){
                        te=td+e;if(te>N)break;
                        for(f=0;f<=N;f+=6){
                            tf=te+f;if(tf>N)break;
                            for(g=0;g<=N;g+=7){
                                tg=tf+g;if(tg>N)break;
                                for(h=0;h<=N;h+=8){
                                    th=tg+h;if(th>N)break;
                                    if(th==N)
                                    {
                                        s++;
                                        if((s&0x3FFFF)==0)
                                        {
                                            var ans=prompt('Taking too long...
Continue?','Yes');
                                                if(ans!='Yes')return (0-
document.myform.myinput.value);
                                        }
                                    }
                                }
                            }
                        }
                    }
                }
            }
        }
    }
    return s;
}

function try_it()
{
    var N=document.myform.myinput.value;
    if (N==f(1234)){
```

```
        alert("Right! Just submit it below.");
    }else{
        alert("Wrong");
    }
}
```

Answer **b73ff51339812bbabaa1de71e25cd4a2**

172 Programming exercise - Fixed points

2014-10-11 08:41:23 **by Philippe_57721** **10 xp** **Programming**

The function F is defined by
$F(1) = 1$
$F(3) = 3$
$F(2 \times n) = F(n)$
$F(4 \times n + 1) = 2 \times F(2 \times n + 1) - F(n)$
$F(4 \times n + 3) = 3 \times F(2 \times n + 1) - 2 \times F(n)$

A fixed point is n such as $F(n) = n$.

How many fixed points less or equal to 10^{100} have F?

Answer format: the count

[My timing: < 100 ms]

Answer ea7e1472196dd23a82602b2b9b2c51fa

173 Cube painting

2014-10-14 10:06:03	**by sinan**	**10 xp**	**Math**

Consider the number of ways to paint the faces of a cube using n distinct colors.

Let $d(i)$ be i-th decimal digit of $41152/333333 = 0.123456123456\cdots$ i.e. $d(1) = 1, d(5) = 5, d(9) = 3$, etc.

Let $F(n)$ be the number of different cubes that can be obtained if it is allowed that upto $d(i)$ faces can be painted with the i-th color. $i = 1, 2, ..., n$.

What is $F(1001)$?

Example: 415 for $F(5)$.

Answer 30736c4a411ce324908fad408b72e1c8

174 Next 2

2014-10-14 11:22:05	by sinan	5 xp	Sequence

```
23800-728-5852-36176-23800-3360-3312-78624-25536-5408-988-192192-?-?
```

Input format: `number1-number2`

Hint: This sequence is a tweaked form of a sequence that otherwise can be found easily (even in OEIS).

Another hint: It has been tweaked in such a way that
OEIS: $f(0,1) - f(1,1) - f(2,1) - \cdots$
This: $f(0,0) - f(1,0) - f(2,0) - \cdots$

Yet another hint: solve p188[1]

Answer a2d643035def3800e946f56a7b139a3d

[1] Actually Problem 190.

175 Pandigital primes

2014-10-16 18:29:04 **by Philippe_57721** **4 xp** **Math**

We have $19 = 201$ in base 3.

Can you find all the prime numbers p such as there is a base b in which p can be written with all digits $\{0, \ldots, b-1\}$ used exactly once?

Digit 0 on the left is allowed.

Answer format: `p1,p2,...`
(in ascending order)

Answer 6cf8c44959f790ea9b0f088b94a72f2b

176 GCDs

2014-10-16 18:30:02	by Philippe_57721	4 xp	Math

What is the GCD of all numbers $(n^{37} - n)$ when $n \geq 1$?

Answer format: the GCD

Answer c0cf95f0dcdc1bafed14499e05bd4e07

177 Find it

2014-10-17 13:48:28 **by sinan** **8 xp** **Hack**

Just get the correct value and enter as the answer! [ ] Test it

The JavaScript code for this problem is as follows:

```
var M=13;

function func(X,N)
{
    if(((X-1)%M)==0){
        return (func(((X-1)/M)*(M-2),N-1));
    } else if(N>0){
        return (-1);
    } else {
        return (X);
    }
}

function test()
{
    var t=document.myform.myinput.value;
    t>>=30;
    t>>=17; // answer < 2^47
    if (t == 0)
    {
        t=func(document.myform.myinput.value,M);
    }
    if(t>0)
        alert("Right! Just submit it below.");
    else
        alert("Wrong");
}
```

Answer 6d7c5709003d40c36a11b0b16cc7e561

178 Find the maximum

2014-10-24 17:24:48	by Philippe_57721	8 xp	Math

The Diophantine equation $n^2 - n \cdot m - m^2 = 1$ $(n, m > 0)$ has infinitely many solutions.

If $n, m < 10^{15}$, what is the maximum of $n^2 + m^2$?

Answer b3602fe156a5dacb82b7c14c70f517d9

179 How many solutions

2014-11-02 17:17:47 **by Philippe_57721** **8 xp** **Math**

Consider the diophantine equation $x^2 + 3 \cdot y^2 = z^2$ ($0 < x, y, z$ and coprime).

How many solutions are there with $z \leq 10^9$?

You are given:
- 276 solution for $z < 1000$
- 2767 solution for $z < 10000$
- 27574 solution for $z < 100000$

Answer format: the count

[My timing: 120 sec]

Answer 7999e24efa33e6ea8d9a25e15f894d16

180 How many points

2014-11-03 12:03:58	**by sinan**	**12 xp**	**Math**

For real numbers $a, k > 0.0$, let $F(a, k)$ denotes the number of lattice points inside the OAB triangle with the following vertices:
$O(0.0, 0.0)$
$A(a, 0.0)$
$B(0.0, k \cdot a)$

For example:
$F(5.0, 5.0) = 81$ for $O(0.0, 0.0)$, $A(5.0, 0.0)$, $B(0.0, 25.0)$.

What is $F\left(10^{16} + 1, 2/\left(\sqrt{5} + 1\right)\right)$?

[My timing: 1 s]

Answer 9cbe139c05aeb03b1f3597fe3715d4d9

181 How many points 2

2014-11-04 10:35:03 **by sinan** **8 xp** **Math**

For real numbers $a, m > 0.0$, let $F(a, m)$ denote the number of lattice points inside the OAB tringle with the following vertices:
$O(0.0, 0.0)$
$A(a, 0.0)$
$B(x_b, y_b)$ which is the intersection point of the following lines:
$L_1 : y = x/m$
$L_2 : y = m \cdot (x - a)$

For example:
$F(8.0, 3.0) = 19$ for $O(0.0, 0.0)$, $A(8.0, 0.0)$, $B(9.0, 3.0)$.

What is $F\left(123456789101112, 2/\left(\sqrt{5} - 1\right)\right)$?

[My timing: 1 s]

Note: You are advised to solve p178 first.[1]

Answer **903f453f0844edb8b9bf416854d407b4**

[1] Actually Problem 180.

182 Tubes

2014-11-06 07:26:05 **by sinan** **20 xp** **Programming**

Consider 6 tubes named A, B, C, D, E, F with capacities $10, 10, 7, 7, 7, 7$ respectively.

In the first state they contain the following unit amounts of a certain liquid (A to F):
$9, 7, 6, 5, 0, 0$

And the goal state is as follows (A to F):
$9, 2, 2, 2, 5, 7$

Treat each state as a hex number (`FEDCBA`) and assign a 6-digit number for them as in the following example:
The first state = `005679`
The last state = `752229`

We need to reach to the goal state with the mimimum number of steps.

Let A be the eventual array of size k with the states (from first to last) in it:
$A = [$`005679`$, \ldots,$`752229`$]$
where $A[i]$ is the state at the i-th phase, $i = 1, \ldots, k$.

Let N be the hex number obtained by concatenating the hex digits of the above array:
$N =$ `005679...752229` or
$N = A[1] \cdot B^{k-1} + A[2] \cdot B^{k-2} + \cdots + A[k] \cdot B^0$ where $B = 16^6$.

Find the smallest N. (i.e. Find the lexicographically first, shortest path.)

Answer format: $N \bmod 1000000007$

Example:
Capacities: $7, 6, 5, 4, 4, 4$
First: $7, 5, 0, 0, 0, 0$
Last: $3, 2, 3, 4, 0, 0$

$1 : 7, 5, 0, 0, 0, 0 \quad s =$ `000057`
$2 : 3, 5, 0, 4, 0, 0 \quad s =$ `004053`
$3 : 3, 5, 4, 0, 0, 0 \quad s =$ `000453`
$4 : 3, 6, 3, 0, 0, 0 \quad s =$ `000363`
$5 : 3, 2, 3, 4, 0, 0 \quad s =$ `004323`

$N =$ `000057004053000453000363004323` (hex)
$N \bmod 1000000007 = 10356901$ (decimal)

[My timing: 25 s]

Answer 45a1acb875e00bb1d45c6b96ccb0be9a

183 Run it (javascript)

2014-11-08 10:39:29 **by sinan** **10 xp** **Programming**

Here's a javascript code in action. Experiment with it and submit the value for $M = 1000000000$.

[My timing: 42 s]

M = 20 Run it

```
M=20

a=2 b=2 k=1 nt=1
a=4 b=4 k=2 nt=2
a=6 b=3 k=2 nt=3
a=6 b=6 k=3 nt=4
a=8 b=8 k=4 nt=5
a=12 b=4 k=3 nt=6
a=12 b=6 k=4 nt=7

found 7 solutions
```

The JavaScript code for this problem is as follows:

```
function func()
{
    var M=myform.myinput.value;
    var nt=0;
    var outputDiv = document.getElementById("outputDiv");
    outputDiv.innerHTML = "<pre\\>";
    if (M<=0 || M>99) {
        alert("Should be >0 and <100");
    } else {
         outputDiv.innerHTML += "<font size=+2 \\><b\\>M="+M+"<\/b\\><\/font\\><br\\><br\\>";
        for (var a=1; a<M; a++){
            for (var b=1; b<=a; b++){
                var ab=a+b;
                if (ab>=M) break;
                var k=Math.floor(0.5+(a*b)/ab);
                var diff=Math.abs(a*b-k*ab);
                if (diff==0){
                    nt++;
                    outputDiv.innerHTML += "a="+a+" b="+b+" k="+k+" nt="+nt+"<br\\>";
                }
            }
        }
        outputDiv.innerHTML += "<br\\><b\\>found "+nt+" solutions<\/b\\>";
    }
    outputDiv.innerHTML += "<\/pre\\>";
}
```

Answer **ba6b80a01d4472dbbc29516d571fb2d5**

184 ones and twos

2014-11-11 17:03:50 **by Philippe_57721** **7 xp** **Programming**

There is one integer N such as $N \times 2^{100}$ is a 100-digits number containing only 1's and 2's.

Can you find it.

[My timing: < 100 ms]

Answer 7c7cbf2dc04676bc626a8d8649fc8852

185 Dividing triples

2014-11-16 09:27:27	by Philippe_57721	7 xp	Math

Find all the triples (a, b, c) with $0 < a < b < c$ such as:
$(a-1) \cdot (b-1) \cdot (c-1)$ divides $a \cdot b \cdot c - 1$.

Answer format: `(a1,b1,c1)(a2,b2,c2)...`
$(a1 < a2\ ...)$

Answer cfc3a56fdcfd56276be2b74019ee73b7

186 Tubes 2

2014-11-17 14:33:18	by gerrob	24 xp	Programming

See p180 from Sinan.[1] Now there are 10 tubes named with $A, B, C, D, E, F, G, H, I, J$ with capacities $14, 13, 12, 12, 10, 10, 7, 7, 7, 7$ respectively. The first state is (A to J) $10, 9, 11, 10, 9, 8, 5, 5, 4, 2$. The goal state is ($A$ to J) $13, 12, 11, 11, 4, 6, 6, 0, 6, 4$. Encode each state as a hex number `JIHGFEDCBA`, so the starting state is `245589AB9A`, the ending state is `460664BBCD`.

We need to reach to the goal state with the mimimum number of steps.

Let A be the eventual array of size k with the states (from first to last) in it:
$A =$ [`245589AB9A`, ...,`460664BBCD`]
where $A[i]$ is the state at the i-th phase, $i = 1, \dots, k$.

Let N be the hex number obtained by concatenating the hex digits of the above array:
$N =$ `245589AB9A...460664BBCD` or
$N = A[1] \cdot B^{k-1} + A[2] \cdot B^{k-2} + \cdots + A[k] \cdot B^0$ where $B = 16^{10} = 2^{40}$.

Find the smallest N. (i.e. Find the lexicographically first, shortest path.)

Answer format: $N \bmod 1000000007$

[My timing: 60 sec] (using less than 1GB of Ram)

Answer e7a0b25841cb0e7799f438f5ca332ea9

[1] Actually Problem 182.

187 Dividing sextuplets

2014-11-20 10:20:18	**by gerrob**	**10 xp**	**Programming**

There is an obvious generalization of p183 from Philippe.[1] Find all $1 < a < b < c < d < e < f$ (integer) sextuplets for that $(a-1)\cdot(b-1)\cdot(c-1)\cdot(d-1)\cdot(e-1)\cdot(f-1)$ divides $a\cdot b\cdot c\cdot d\cdot e\cdot f-1$. You can easily check that this is a valid sextuplet:
$[3, 5, 17, 257, 65555, 226112997]$.

Answer format: `cnt,s`
where `cnt` is the number of solutions and `s` is the sum of $a+b+c+d+e+f$ values over all solutions.

[My timing: < 1 sec]

Answer 5715d4489daa7c557f18c3ee33b3f268

[1] Actually Problem 185.

188 Dividing septuplets

2014-11-20 10:28:55	by gerrob	12 xp	Programming

Find all $1 < a < b < c < d < e < f < g$ (integer) septuplets for that $(a-1)\cdot(b-1)\cdot(c-1)\cdot(d-1)\cdot(e-1)\cdot(f-1)\cdot(g-1)$ divides $a\cdot b\cdot c\cdot d\cdot e\cdot f\cdot g-1$.

Answer format: `cnt,s`
where `cnt` is the number of solutions and `s` is the sum of $a+b+c+d+e+f+g$ values over all solutions.

[My timing: 2 min]

Answer 0e7d27f0743a50572e7a2867ebfc7033

189 Smallest partition

2014-11-23 10:23:32	by Philippe_57721	10 xp	Math

Proposition:

For any partition of $\{3, ..., n\} = A \cup B$ the equation $x \cdot y = z$ where x, y, z (not necessarily distinct) are all in A or all in B has always a solution.

Find the least integer n such as this proposition is true.

[My timing: pencil and paper]

Answer a345ba5a87227571c02d216368d858e6

190 Bruteforce it

2014-11-24 08:51:05 | **by sinan** | **20 xp** | **Programming**

Here is an hex dump of a password protected data:

```
50 4b 03 04 0a 00 01 00 00 00 ae 60 6b 45 68 81
7f 1e 18 00 00 00 0c 00 00 00 01 00 00 00 01 3e
24 52 bf a8 02 52 f9 7a 98 64 65 da e2 4e 27 a7
f4 22 2c 77 6b 7b fd
```

I believe you can easily figure out what it is and brute force the password using the following hints:

- Static string “sinan” is a part of the password (any uppercase lowercase combination is possible)
- It has a 3-digit decimal number in it [000-999]
- 4 distinct characters are also present in it from the set {‘(’, ‘)’, ‘:’, ‘;’, ‘_’, ‘.’, ‘,’}
- The above mentioned parts may appear in any order (i.e. 6! possibilities)

Answer format: enter the decrypted data

Note: The decrypted data is an hint for the problem 172.[1] So this problem is more rewarding than its points.

[My timing: 460 s]

Answer 03b4aea6bfbf3faa105f27ae6fb0e8fa

[1] Actually Problem 174.

191 Non attacking queens

2014-12-07 19:20:08 **by Philippe_57721** **8 xp** **Programming**

On a 5×5 chessboard, there are 8 ways to place 3 white queens and 5 black queens so that no queen of a given color can attack a queen of another color.

One position is

```
+---+---+---+---+---+
| W | W |   |   |   |
+---+---+---+---+---+
|   |   |   | B | B |
+---+---+---+---+---+
|   |   |   |   | B |
+---+---+---+---+---+
| W |   |   |   |   |
+---+---+---+---+---+
|   |   | B | B |   |
+---+---+---+---+---+
```

How many ways are there to place 5 white queens and 6 black queens on a 6×6 chessboard the same way?

Answer format: `Count/White queens positions/Black queens positions`
for the 1st solution in lexicographic order.

A position is given as: (i, j) where i, j are the row and column index starting from the upper left cell, index starting at 1.

Example: `8/(1,1)(1,2)(4,1)/(2,4)(2,5)(3,5)(5,3)(5,4)` for 3 white and 5 black queens on a 5×5 chessboard.

[My timing: 40 sec]

Answer 2be117d6f72ae808edab7d7b9fa5e81e

192 The Best Sub-sequence

2014-12-21 17:43:15 **by Philippe_57721** **8 xp** **Programming**

Consider the decimal part of PI by groups of 2 digits.

We get the following sequence for the 20 first elements:
$14, 15, 92, 65, 35, 89, 79, 32, 38, 46, 26, 43, 38, 32, 79, 50, 28, 84, 19, 71$

Subtract from each element of this sequence 50:
$-36, -35, 42, 15, -15, 39, 29, -18, -12, -4, -24, -7, -12, -18, 29, 0, -22, 34, -31, 21$

You can verify that the sub-sequence starting at index 3 and ending at index 7 $(42, 15, -15, 39, 29)$ has the largest sum: 110.

Now group the decimal part of PI by groups of 10 digits and subtract $10^{10}/2$ from each element.

We get the sequence:
$S = -3584073465, 3979323846, -2356616721, 28841971, 1939937510,$
$820974944, 923078164, -4371379101, 3628034825, -1578829321, \ldots$

What is the sub-sequence with the largest sum in the first 5000000 elements of S?

Answer format: `Sum,Starting index,Ending index`

Example: `110,3,7`

[My timing: 2 sec] (Computation of PI not included!)

Answer 90e000007b4e78fac882ac2d3051c819

193 No remainder

2014-12-25 16:45:07	by sinan	10 xp	Math

Let $F(n) = 11 + 105^n$.

Let S be the serie of $F(n)$ values for $n = 1, 2, ...$:
$116, 11036, 1157636, 121550636, 12762815636, 1340095640636, ...$

What is the 10^4-th term **that can be divided by 27126986671934931497 with no remainder**.

Answer format: the term mod 10^9

[My timing: 3 s]

Answer 6907587da01f0ffaee64f29c55e1c8c2

194 Trains

2014-12-27 09:58:34 **by Philippe_57721** **7 xp** **Programming**

On a circular track, three trains travel at the same speed: one unit per second.

The track has a length of 641 units.

- Train no. 1 has a length of 11 units and occupies the position 0 to 10. It travels clockwise
- Train no. 2 has a length of 13 units and occupies the position 100 to 112. It travels counter-clockwise
- Train no. 3 has a length of 17 units and occupies the position 200 to 216. It travels clockwise

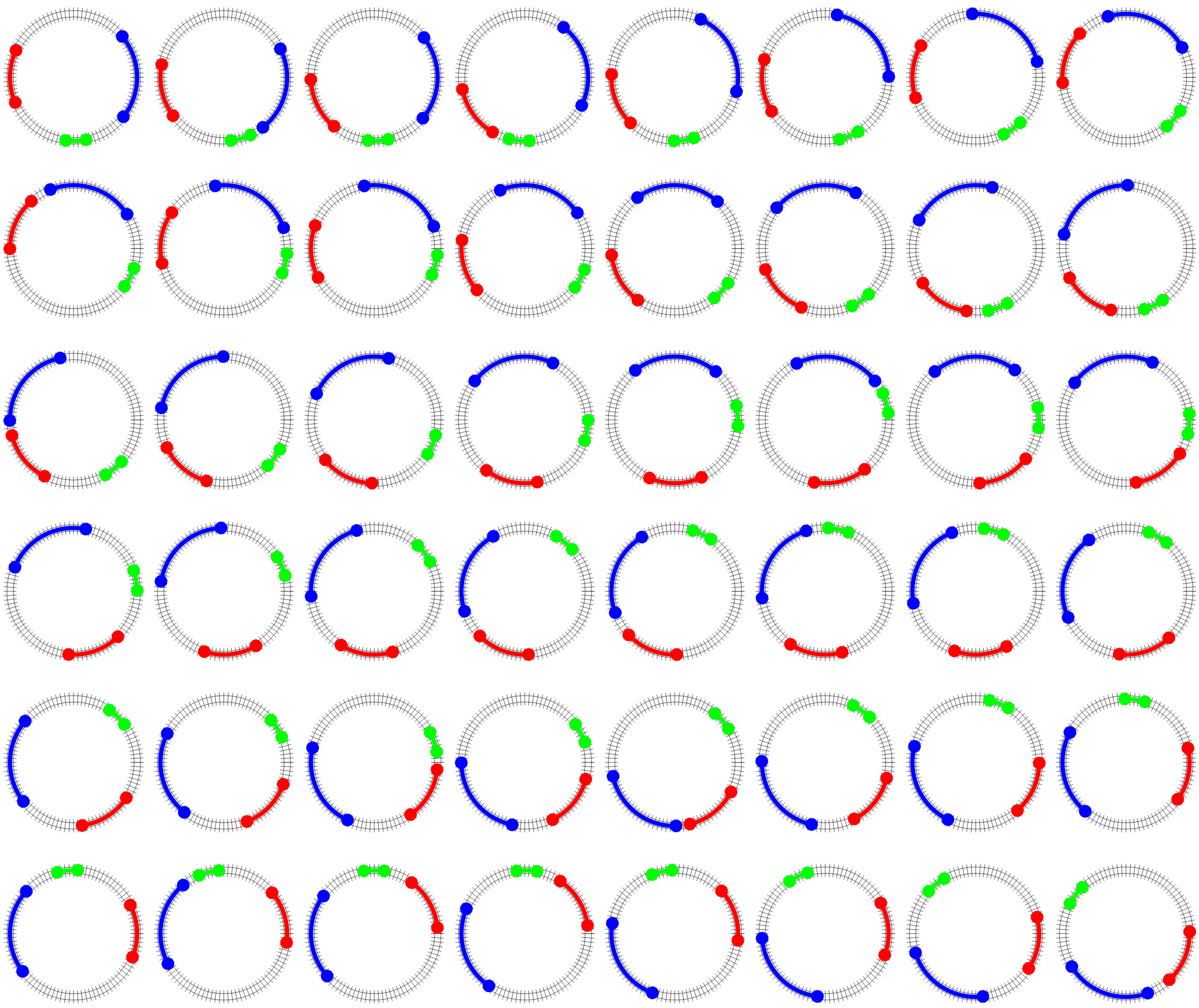
When two trains collide, they collide elastically, and go in the opposite direction at the same speed.

The trains start to move at $T = 0$.

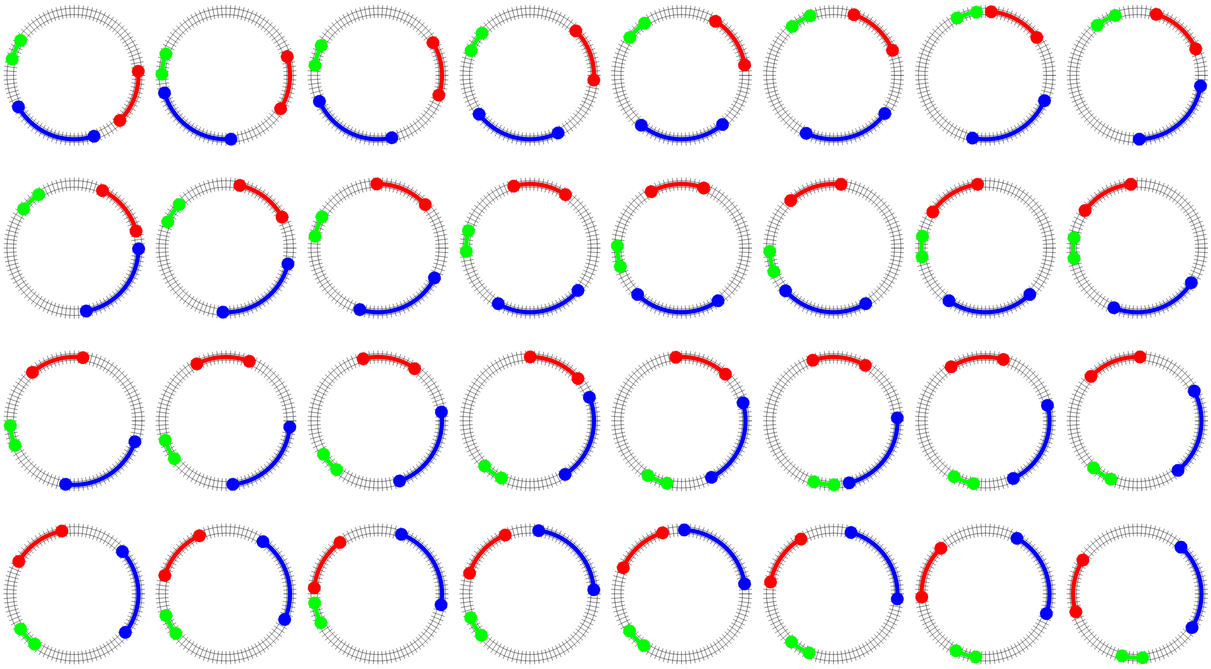
At which T do the trains occupy the same position again?

[My timing: 5 sec]

Here is an example:[1]



[1] Selected key frames from the original GIF.



(Image credit: apgoucher. I will remove it if there is a copyright issue.)

Answer e3138a4b634d2999696df71280c942c2

195 Antipodean numbers

2015-01-04 18:53:35 **by Philippe_57721** **7 xp** **Programming**

A number is antipodean if it is equal to is rotation by 180°: you type such a number on a calculator and turn it upside down, you read the same value.

- 1961 is such a number

How many antipodean numbers with a number of digits less or equal to 17 are there and what is there sum?

Answer format: `Count,Sum`

You are given: `14404,5947658589706` for 9 digits or less.

[My timing: 50 sec]

Answer **3795546a18e4350d1760f72a011f5fbe**

196 Meeting time

2015-01-06 11:39:57	**by sinan**	**8 xp**	**Math**

There are 2 points (A and B) 100 kms apart. A vehicle (V_A) starts to travel from point A to point B and another (V_B) from B to A at the same time (00:00:00).

V_A travels at a speed (km/h) equal to the distance-to-B $+13$ at any time while V_B travels at the speed of 61 km/h at any time.

So for example when V_A is 91.1 kms away from B, its speed would be 104.1 km/h at that time.

At what time do the two vehicles meet?

Answer format: `HH:MM:SS:ms`
(rounded to milliseconds)

Example: `01:11:17:691`

Answer **13ec87ce779e984a43eccf42266b3c7e**

197 A Fractal Sequence

2015-01-10 07:15:05	by Philippe_57721	7 xp	Programming

The application T operates on a sequence S as:
$T(S) = \{1, S[1], 2, S[2], ..., S[n], n + 1\}$

For instance:
$T(\{10, 20, 30\}) = \{1, 10, 2, 20, 3, 30, 4\}$

Consider the infinite sequence $S = T(T(T(T(T(\cdots(\{1\})\cdots)))))$. T is applied infinitly.

What is $S[12345678910111213]$?

[My timing: < 100 ms]

Answer eaf6173aad3ee5d6eddec5b384c2386f

198 A special grid

2015-01-14 09:58:28	by sinan	6 xp	Programming

Consider an $N \times N$ grid ($N < 16$) where hex digits 1 to N are used only once in each row or column and treat the whole grid as a number starting from first row to the last.

What is the smallest number for $N = 13$?

Answer format: last 2 rows as uppercase hex

Example for $N = 5$:

```
1, 2, 3, 4, 5
2, 1, 4, 5, 3
3, 4, 5, 1, 2
4, 5, 2, 3, 1
5, 3, 1, 2, 4
```

Number = 1234521453345124523153124
Answer = 4523153124

Answer 7712803abea4d16c2598c4db2aa11526

199 A special sequence

2015-01-14 10:46:41	**by sinan**	**4 xp**	**Math**

```
1,17,283,48405761,21491955953121731720119,?
```

Hint: solve p196 first.[1]

Answer 1822e0f42054d5aa316026798312a8cd

[1] Actually Problem 198.

200 2-derangements

2015-01-16 16:45:50 **by Philippe_57721** **10 xp** **Programming**

A derangement is a well known kind of permutation with no fixed point.

Define a 2-derangement as a pair P_1, P_2 of derangements where: $P_1[i] \neq P_2[i]$ for all i.

Here are 2 examples of 2-derangements for 5 elements:

```
2 4 5 3 1
4 5 1 2 3

4 5 1 2 3
2 4 5 3 1
```

How many 2-derangements are there for 208 elements?

As the answer is a BIG number, use the following condensed representation:
`First 10 digits[number of remaining digits]Last 10 digits`

For instance, for 2^{127}, the representation is:
`1701411834[19]5884105728`

[**My timing: 40 sec**]

Answer 27c1456beebd0bd571af0311091e4737

201 Divisibility

2015-01-17 17:10:10 | **by sinan** | **12 xp** | **Programming**

How many numbers below 10^{15} can be divided by sum of their digits?

For example, 54 is such a number, the sum of its digits is $5 + 4 = 9$, 9 can divide 54.

Answer format: number

Example: 32 if below 10^2 or 11871 if below 10^5.

[My timing: 6 s]

Answer 2ee2dfffa650b06cdd86f7d726e54acf

202 Run it (javascript)

2015-01-22 09:45:50 **by sinan** **15 xp** **Math**

Here's a javascript code in action. Experiment with it and submit the value for $M = 100$ and $N = 200$.

Use the following condensed representation:
`First 10 digits[number of remaining digits]Last 10 digits`

For instance, for 2^{127}, the representation is:
`1701411834[19]5884105728`

M = 5 N = 5 Run it

```
M=5, N=5

COUNT=771
```

The JavaScript code for this problem is as follows:

```
/*
See the following pseudo code:

   define global variables ROWS, COLS and COUNT
   define GRID as a ROWS by COLS matrix and initialize it to zero

recursive_function(row=1, col=1, row_max=1)
begin
   set GRID[row, col] to 1
   if col+1 is less than or equal to COLS
   begin // if
       if GRID[row, col+1] is equal to zero; call recursive_function(row, col+1,
row_max)
   end // if
   else if row_max is equal to ROWS; add 1 to COUNT

   if row+1 is less than or equal to ROWS and GRID[row+1, col] is equal to zero;
      then call recursive_function(row+1, col, row_max+1)

   if row-1 is greater than zero and GRID[row-1, col] is equal to zero;
      then call recursive_function(row-1, col, row_max)
   set GRID[row, col] to 0
end

main(M,N)
begin
   ROWS=M
   COLS=N
   COUNT=0
   call recursive_function(row=1, col=1, row_max=1)
   return COUNT
end
*/
```

```
var ROWS;
var COLS;
var GRID = new Array();
var COUNT;

function recursive_function(row,col,row_max)
{
    GRID[row-1][col-1]=1;
    if(col+1<=COLS){
        if(GRID[row-1][col+1-1]==0) recursive_function(row,col+1,row_max);
    } else if (row_max==ROWS) COUNT++;
                        if      (row+1<=ROWS      &&      GRID[row+1-1][col-1]==0)
recursive_function(row+1,col,row_max+1);
    if (row>1 && GRID[row-1-1][col-1]==0) recursive_function(row-1,col,row_max);
    GRID[row-1][col-1]=0
}

function main()
{
    var M=myform.myM.value;
    var N=myform.myN.value;
    if (M<=0 || M>=10 || N<=0 || N>=10) {
        alert("Should be >0 and <10");
    } else {
        ROWS=M;
        COLS=N;
        COUNT=0;
        for (i=0;i<ROWS;i++){
            GRID[i]=new Array();
            for (j=0;j<COLS;j++){
                GRID[i][j]=0;
            }
        }
        recursive_function(1,1,1);
        outputDiv.innerHTML = "<pre><font size=+2 \\><b\\>M="+M+", N="+N+"</b\\></
font\\>";
        outputDiv.innerHTML += "<b\\>COUNT="+COUNT+"</b\\></pre>";
    }
}
```

Answer d8451dd59c2fda41729a367f13fcd1bd

203 Balanced triangles

2015-01-23 14:19:32 **by Philippe_57721** **7 xp** **Programming**

Starting with a sequence of signs + and -, we form a “pascal-like” triangle applying the following rules:

- the 1st row is the initial sequence
- each next row is built by combining each 2 consecutive signs from the previous row, with the classical rules (+ ⊗ + ⇒ +, + ⊗ - ⇒ -, etc.)

For instance, starting with ‘-+--’, we obtain the following triangle:

```
- + - -
 - - +
  + -
   -
```

Such a triangle is said balanced if it contains the same number of + and -. The following triangle is balanced:

```
- + - +
 - - -
  + +
   +
```

Find the number of sequences with 23 signs containing the largest number of + which produce a balanced triangle and give the last one (lexicographic order).

Answer format : `count,sequence of + and - without spaces`

PS: `count` is the number of sequences with that largest number of +.

For 7 signs, the anwser would be: `5,+++-++-`.

[My timing: 100 sec]

Answer f3aab82c97287883eeab217f9751f175

204 Happy numbers

2015-01-24 21:07:19	by gerrob	5 xp	Math

Let $f(n)$ be the sum of the squares of the digits of n. Iterate this process, if we can reach 1 in this sequence then we say n is a happy number. $H(n)$ denotes the number of (positive) happy numbers up to n. For example $n = 736$ is a happy number as the iterated sequence is $736, 94, 97, 130, 10, 1$.

Find $H(12345678)$.

Example: $H(1000) = 143$.

Answer cda22fae4535a2af0b8cc838359eabcb

205 Happy numbers II

2015-01-24 21:09:40	**by gerrob**	**8 xp**	**Math**

Find $H(2^{300})$ so the number of happy numbers up to 2^{300}.

Answer **c43fb08a56292827c2c6933b11e323a5**

206 Balanced triangles (revisited)

2015-01-24 22:37:18	by gerrob	10 xp	Programming

See p201 from Philippe.[1] Find the answer for 47 signs!

Answer format: `count,sequence of + and - without spaces`
where `count` is the number of sequences with that largest number of `+`

[My timing: 2 min]

Answer 8654c39acfb3e3cc9533888c6c57b118

[1] Actually Problem 203.

207 Laver Tables

2015-01-30 07:28:02 **by Philippe_57721** **8 xp** **Programming**

Laver tables, discovered by Richard Laver, are very intriguing mathematical objects.

Although their definition is elementary, some of their properties cannot be proved in the classical set theory (ZFC), but require (so far) some hypotheses about large cardinals.

Let's define the operation $\star$ for integers in range $[1..n]$ (n is a power of 2) by the following axioms:
$x \star 1 = (1 + x) \bmod n$
$x \star (y \star z) = (x \star y) \star (x \star z)$

Here is the Laver table for $n = 8$:

```
    1   2   3   4   5   6   7   8
  + - + - + - + - + - + - + - + - +
1 | 2 | 4 | 6 | 8 | 2 | 4 | 6 | 8 |
  + - + - + - + - + - + - + - + - +
2 | 3 | 4 | 7 | 8 | 3 | 4 | 7 | 8 |
  + - + - + - + - + - + - + - + - +
3 | 4 | 8 | 4 | 8 | 4 | 8 | 4 | 8 |
  + - + - + - + - + - + - + - + - +
4 | 5 | 6 | 7 | 8 | 5 | 6 | 7 | 8 |
  + - + - + - + - + - + - + - + - +
5 | 6 | 8 | 6 | 8 | 6 | 8 | 6 | 8 |
  + - + - + - + - + - + - + - + - +
6 | 7 | 8 | 7 | 8 | 7 | 8 | 7 | 8 |
  + - + - + - + - + - + - + - + - +
7 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
  + - + - + - + - + - + - + - + - +
8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
  + - + - + - + - + - + - + - + - +
```

You'll notice that, except for the last one, all rows are periodic.

Find the periodic part of the 1st row for $n = 2^{16}$.

Answer format: comma delimited list of values

Example: for $n = 8$, the anwser would be: `2,4,6,8`.

N.B.: It can be proved that the periodicity of the 1st row is unbounded. But the first n for which the period is greater that the one you'll find for 2^{16} is greater than $A(9, A(8, A(8, 255)))$. A is the Ackerman function …

[My timing: < 1 s]

Answer 8c7d5b4304bc480b373d3febe5b19a98

208 Laver Tables II

2015-02-02 18:56:06 **by Philippe_57721** **15 xp** **Programming**

See problem 205.[1]

Find the periodic part of the 1st row for $n = 2^{23}$.

Answer format: to keep the answer short, give the first difference of the vector of values divided by 2

For instance, for 2^{10}, the first row is:
2, 12, 14, 240, 242, 252, 254, 768, 770, 780, 782, 1008, 1010, 1020, 1022, 1024

So the answer would be:
`5,1,113,1,5,1,257,1,5,1,113,1,5,1,1`

[My timing: 32 sec]

Answer 885382d1ebc7fcff661f2f3af8ba4250

[1] Actually Problem 207.

209 Huge Fibonacci

2015-01-31 16:15:31 **by sinan** **5 xp** **Math**

Let $\mathrm{Fib}(N)$ be N-th Fibonacci number with $\mathrm{Fib}(1) = 1$ and $\mathrm{Fib}(2) = 1$.

What is $\mathrm{Fib}(\mathrm{Fib}(1000000)) \bmod 1000000007$?

[My timing: 2 s]

Answer **3938bb50dc7ba66da0db5120836de83b**

210 Trigonometric equations

2015-02-01 13:37:04 **by gerrob** **5 xp** **Math**

Solve in real variables:
$(\sin x)^2 + (\cos y)^2 = y^2$
$(\sin y)^2 + (\cos x)^2 = x^2$

Submit all solution in lexicographic increasing way.

Answer format: `(x1,y1),(x2,y2),...`
Here lex. sorting means that $(a, b) < (c, d)$ if $a < c$ or $\{a = c$ and $b < d\}$. You can assume that all solution is in fact rational, so submit them as $x = p/q$, where $q > 0$, $\gcd(p, q) = 1$ (and not use q if x is integer).

Example: `(-1/2,0),(3/4,7)`

Answer 06469901718bc2eae5427ad21bd9a6b1

211 Enumerate the rationals

2015-02-06 12:47:10 **by Philippe_57721** **7 xp** **Programming**

Consider the function $F(X) = 1/(2 \times [X] - X + 1)$ where $[X]$ is the integer part of X.

It is quite remarkable that the sequence $\{0, F(0), F(F(0)), F(F(F(0))), \ldots\}$ contains every non-negative rational exactly once. $\{0, F(0), F(F(0)), F(F(F(0))), \ldots\} = \mathbb{Q}^+$.

We note $F(0)^n = F(F(\cdots(F(0))\cdots))$, function F is applied n times.

What is $F(0)^{14131211110987654321}$.
Find n such as $F(0)^n = 1415926535/5772156649$.

Answer format: `N/D,Index`

[My timing: < 100 ms]

Answer e2929fff9a1d65b51a35b2ce94a4a43b

212 Maximising a Ratio

2015-02-08 09:27:23 **by sinan** **10 xp** **Programming**

Consider a Phytagorean right-angled triangle with lengths a, b and c ($a < b < c$).

Its Perimeter: $P = a + b + c$.
Its Area: $A = a \times b/2$.

Find the (a, b, c) triple with maximum A/P ratio if $P \leq 10^{17}$.

Answer format: `a,b,c`

Example: `280,294,406` if $P \leq 10^3$
$A = 41160$, $P = 980$, $A/P = 42$

[My timing: 40 s]

Answer 0e949fcafb999ff1ff848e21149a5def

213 Subsets

2015-02-08 11:37:04	by sinan	6 xp	Programming

Let S be the set of numbers up to 40. $S = \{1, 2, ..., 40\}$.

Note the following subsets:
S_1 is the prime numbers in S.
S_2 is the semiprime numbers in S.
S_3 is the all other numbers of S.

Now consider 4 subsets all with 4 distinct numbers.
A is a subset of S_1.
B is a subset of S_2.
C is a subset of S_3.
D is also a subset of S_3 but C and D are disjoint.

Find the number of A, B, C and D subsets such that
- they all sum to the same number
- the smallest element of $A <$ that of $B <$ that of $C <$ that of D

Example:
$A = \{5, 19, 23, 37\}$
$B = \{6, 14, 26, 38\}$
$C = \{8, 12, 28, 36\}$
$D = \{16, 18, 20, 30\}$
$5 + 19 + 23 + 37 = 6 + 14 + 26 + 38 = 8 + 12 + 28 + 36 = 16 + 18 + 20 + 30 = 84$
$5 < 6 < 8 < 16$
This counts as one solution.

[My timing: 21 s]

Answer 671b7c6d831d522c204b0a453579f832

214 Subsets II

2015-02-08 20:45:53 | **by gerrob** | **10 xp** | **Programming**

See p211 from Sinan,[1] same problem, but for the first 256 positive integers, so for $S = \{1, 2, ..., 256\}$.

[My timing: 3 seconds]
(without any trick using roughly 210 MB of Ram, the answer fits in 64 bits.)

Answer a5d11669f58f0ed29b12a598682489b5

[1] Actually Problem 213.

215 Compositorial plus one

2015-02-08 21:39:47	by gerrob	6 xp	Math

Let $c(n) = n!/n\#$ be the n-th compositorial number, where $n\#$ is the product of primes up to n. For example $c(7) = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7/(2 \times 3 \times 5 \times 7) = 24$.

$S(p) = \{n : c(n) + 1 \text{ is divisible by } p\}$.

$H(p) = |S(p)|$, so the size of $S(p)$.

Example: $H(5) = 4$ as $S(5) = \{4, 5, 6, 7\}$.

Find several values of $H(p)$ (where p is prime).

Answer format: `H(6031001),H(6873371),H(9103859),H(6133003),H(7834447)`

Answer e816ca2c258bf2a7af780d67713ba73a

216 Shift Me Down

2015-02-13 15:59:39	**by elasolova**	**4 xp**	**Sequence**

3 4 6 8 12 ?

Answer f93b8bbbac89ea22bac0bf188ba49a61

218 Prime 11-tuplets

2015-02-13 16:11:01	by sinan	4 xp	Programming

S is a set of 11 prime numbers such that

- the average is also a prime number
- the biggest element < 75

How many such S are there and what is the last S?

Answer format: `cnt;p1,p2,...,p11`
$(p1 < p2 < \cdots < p11)$

Answer 59b52cca72cc5aca7eaee289af3247c5

219 Girls and boys

2015-02-13 16:47:22 **by Philippe_57721** **5 xp** **Math**

At school, little boys are mischievous: they pull girls' hair, they kick. Girls don't like boys and prefer to be with girls.

In a classromm a teacher decides to place children in row in such a way that each girl gives the hand to at least one other girl.
`B G G B` — good
`B G B G` — not good

How many possibilities are there in a classroom with 40 children.

Answer **f21fd32134cef2996463d1cde5ba3aa7**

220 Minimising a Ratio

2015-02-21 10:18:53 **by sinan** **5 xp** **Math**

Let $N = 3^{4^5} + 4^{5^6} = F1 \times F2\ (F1 < F2)$.

Find $F1$ and $F2$ proper divisors of N that minimise $(F2 - F1)/N$.

Answer format: `F1,F2`
where `F1` and `F2` is entered using the following condensed representation:
`Leading 10 digits[number of digits not shown]trailing 10 digits`

For example, 2^{127} in this representation would be shown as
`1701411834[19]5884105728`

Answer **31779bda880b78154467cddd7f2dffc5**

221 Prime change

2015-02-21 10:19:21	**by sinan**	**6 xp**	**Programming**

There are some coins worth 1, 2, 5, 11, 23, 47, 101, 211 and bills worth 503, 1009, 2003, 5003, 10007.

In how many ways can you pay 12345, if you are allowed to use **upto 47 coins**?

[My timing: 15 s]

Answer 1ad527b8e44ab56ef2e8b3b9c59d219e

222 Conspicuous buildings

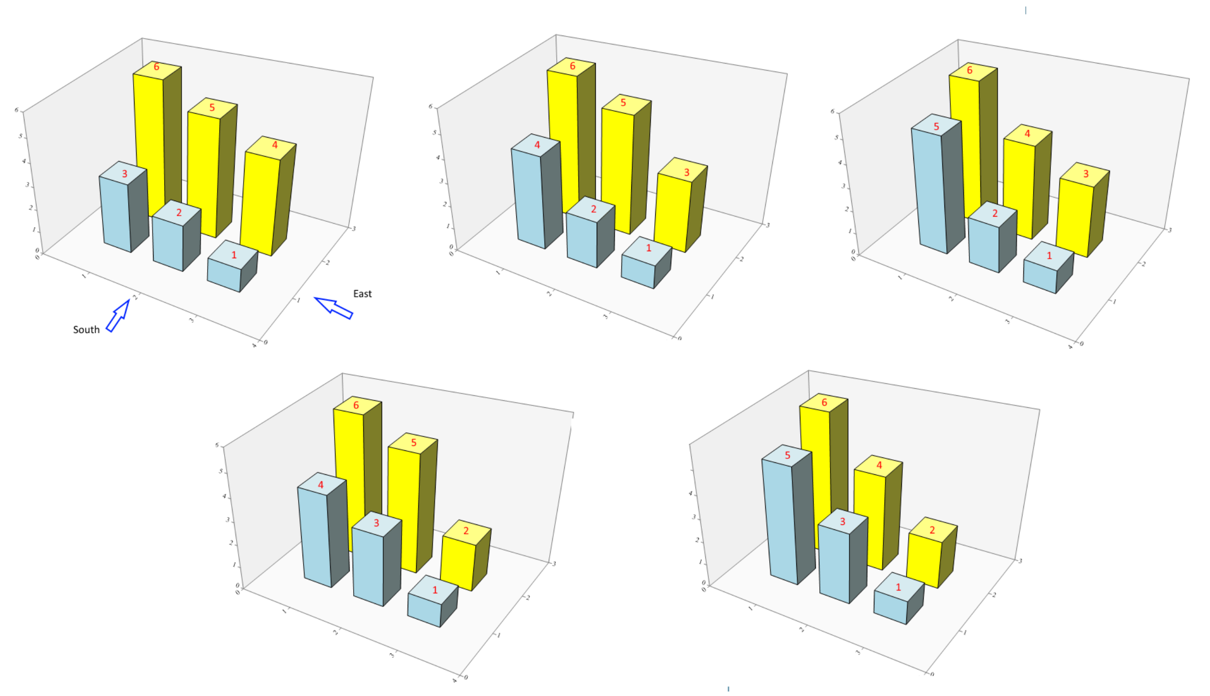
2015-02-20 11:35:12 **by Philippe_57721** **9 xp** **Programming**

In a city, all the buildings are located in the same district.

This district is a rectangle of size W by H, its sides are labelled South, East, North and West.

- Each building occupies a 1-unit square, there are $W \times H$ buildings.
- There height are all integers from 1 to $W \times H$.
- When we look at the buildings facing South face or East face, we can see each building, partly or totally. No building is hidden behind a higher one.

For a 2×3 rectangle, there are five possible arrangements for the buildings.



How many possible arrangements are there for a 15×20 rectangle?

Answer format: the count in compressed format as in problem 198[1]

[My timing: < 100 ms]

Answer **a57abb9af4a3c8dd29c2284b2148dad3**

[1] Actually Problem 200.

223 Fusible Numbers I

2015-02-27 14:06:15 **by Philippe_57721** **6 xp** **Programming**

Fusible numbers are an extraordinary set of rational numbers. You can learn more here.

There are defined by the following rules:

- We define the operation ~ between 2 reals as $a{\sim}b = (a + b + 1)/2$
- 0 is a fusible number
- c is a fusible number is there exist fusible numbers a and b such as $c = a{\sim}b$ AND $|a - b| < 1$

Here are some examples:
$1/2 = 0{\sim}0$
$1 = (0{\sim}0){\sim}(0{\sim}0)$
$5/4 = (0{\sim}0){\sim}((0{\sim}0){\sim}(0{\sim}0))$

It is easy to see that all fusible numbers other than 0 can be written $p/2^q$ for some integers p and q where p is odd.

Find all fusible numbers in the interval $]1/2, 5/4[$ (bounds excluded) with a denominator ≤ 128.

Answer format: comma separated list in ascending order
(We represent a fusible number $p/2^q$ as $p.q$, e.g.: $3/4 = 3.2$)

Example: `15.4,63.6,1.0,5.1`

[My timing: < 1 sec]

Answer 39cfedb769400f229f0d367595ae7a60

224 Fusible Numbers II

2015-02-27 14:06:29 **by Philippe_57721** **10 xp** **Programming**

We define a canonical representation of a fusible number as a representation with only 0.

For instance, the shortest canonical representations of 21/16 are:

- (((0~0)~0)~(0~0))~(0~0)
- (((0~0)~0)~0)~((0~0)~0)
- (((0~0)~0)~0)~(0~(0~0))
- ((0~(0~0))~(0~0))~(0~0)
- ((0~(0~0))~0)~((0~0)~0)
- ((0~(0~0))~0)~(0~(0~0))
- ((0~0)~((0~0)~0))~(0~0)
- ((0~0)~(0~(0~0)))~(0~0)
- ((0~0)~0)~(((0~0)~0)~0)
- ((0~0)~0)~((0~(0~0))~0)
- ((0~0)~0)~(0~((0~0)~0))
- ((0~0)~0)~(0~(0~(0~0)))
- (0~((0~0)~0))~((0~0)~0)
- (0~((0~0)~0))~(0~(0~0))
- (0~(0~(0~0)))~((0~0)~0)
- (0~(0~(0~0)))~(0~(0~0))
- (0~(0~0))~(((0~0)~0)~0)
- (0~(0~0))~((0~(0~0))~0)
- (0~(0~0))~(0~((0~0)~0))
- (0~(0~0))~(0~(0~(0~0)))
- (0~0)~(((0~0)~0)~(0~0))
- (0~0)~((0~(0~0))~(0~0))
- (0~0)~((0~0)~((0~0)~0))
- (0~0)~((0~0)~(0~(0~0)))

and the last in alphabetic order is (0~(0~(0~0)))~(0~(0~0)).

What is the last (in alphabetic order) shortest canonical representation of 111/64?

Hint: The number of 0s in a representation is 13.

[My timing: 4 sec]

Answer 3da9a492d7de6699c611846d69a45798

225 Fusible Numbers III

2015-02-27 14:06:42	by Philippe_57721	8 xp	Programming

A fusible number can be written in several ways as $a{\sim}b$ for some fusible numbers a and b ($a \leq b$).

For instance $101/64 = (3/4{\sim}45/32)(15/16{\sim}39/32)(31/32{\sim}19/16)$.

Find all such representations for $111/64$.

Answer format: comma separated list of $a{\sim}b$ where $a \leq b$ and first terms in ascending order
(if $(ai{\sim}bi)$ and $(aj{\sim}bj)$ are 2 representations with $i < j$, then $ai < aj$)

A fusible $a = p/2^q$ is represented as $p.q$ (see problem 220).[1]

Example: `3.2~45.5,15.4~39.5,31.5~19.4` for $101/64$.

[My timing: 1 sec]

Answer 184ba57d97d750bb365565ba1f27b7d4

[1] Actually Problem 223.

226 P-automorphic numbers

2015-03-06 15:04:39 by Philippe_57721 **6 xp** Programming

A number is *p-automorphic* if the last digits of its *p*-th power are the number itself.

For instance 68751 is a 3-automorphic number:
`68751^3 = 324965351`**`68751`**

There are 10 10-digits 6-automorphic numbers:

- `1787109376^6 = 325766783646508283749948868949208315940937996`**`1787109376`**
- `2000000001^6 = 640000001920000002400000001600000000600000000`**`2000000001`**
- `3787109376^6 = 2950170318650991258032575341918572870335539112113`**`3787109376`**
- `4000000001^6 = 40960000061440000038400000012800000000240000000002`**`4000000001`**
- `5787109376^6 = 375638539633456705990030343693166780341969844246`**`5787109376`**
- `6000000001^6 = 466560000466560000194400000043200000005400000000036`**`6000000001`**
- `7787109376^6 = 222975757734371332908041996970049236323178429737137`**`7787109376`**
- `8000000001^6 = 262144000196608000061440000010240000000960000000048`**`8000000001`**
- `8212890625^6 = 306885405287085842075678332463439801358617842197418`**`8212890625`**
- `9787109376^6 = 878874068949224372247665088145076247737279875049649`**`9787109376`**

How many 50-digits 73-automorphic numbers are there?

Answer format: `count,sum`

Example: `10,57148437509` for 10-digits 6-automorphic numbers.

[My timing: 3 sec]

Answer 111167427cd8295481e964148140d50b

227 A long period

2015-03-02 13:37:40	**by gerrob**	**8 xp**	**Math**

Define the following sequence: $a(0) = (\sin(2\pi/n))^2$ and for $k > 0, a(k) = 4 \times a(k-1) \times (1 - a(k-1))$. For odd n value this sequence is periodical!

Find the period for $n = 31415926535897932384626433832795028841971693993751058 2097$.

Answer **113c04a6709a703859b2d6276488345c**

228 Hyperbolic prime triples

2015-03-13 15:35:23 **by Philippe_57721** **6 xp** **Programming**

There are (probably infinitely) many triples of prime numbers $P_1 < P_2 < P_3$ such as $P_i + P_j \cdot P_k$ is a square number for all combinations (i, j, k) with $i \neq j \neq k$.

Here is an example: $4049, 126001, 130051$.

Find the triple with the largest $P_3 < 180000$.

Answer format: comma separated list

Example: `4049,126001,130051`

[My timing: 95 sec]

Answer f05e9dd8eddf49feb70afae21c746515

229 Meanderings

2015-03-20 09:27:42 **by Philippe_57721** **4 xp** **Programming**

A ant walks exploring each lattice point in the first quadrant of the plane with the following pattern:

37	→	38	→	39	→	40	→	41	→	42	→	43
↑												↓
36	←	35	←	34	←	33	←	32	←	31		44
										↑		↓
17	→	18	→	19	→	20	→	21		30		45
↑								↓		↑		↓
16	←	15	←	14	←	13		22		29		46
						↑		↓		↑		↓
5	→	6	→	7		12		23		28		47
↑				↓		↑		↓		↑		↓
4	←	3		8		11		24		27		48
		↑		↓		↑		↓		↑		↓
1	→	2		9	→	10		25	→	26		49

The 1-st point visited (index = 1) is $(0, 0)$
The 10-th point is $(3, 0)$
The 100000-th point is $(143, 316)$

What is the 10^{17}-th point?
What is the index of the point $(718281828, 141592653)$?

Answer format: `x,y/index`

Example: `143,316/51594074` for 10^5 and $(7182, 1415)$.

[My timing: < 100 ms]

Answer 691be549e68a17df7aac35d46c884caa

230 Anti constitutionnal amendment

2015-04-03 09:57:33	by Philippe_57721	4 xp	Programming

The word **anticonstitutionnellement**, meaning unconstitutionally is the longest word in French.

How many subsequences can you find for this word?

For example, the word ‘nine’ has 14 subsequences: (empty string included)
‘’, ‘e’, ‘i’, ‘n’, ‘ie’, ‘in’, ‘ne’, ‘ni’, ‘nn’, ‘ine’, ‘nie’, ‘nin’, ‘nne’, ‘nine’

[My timing: 90 sec]

Answer f6ceccccb8ee326bf73fc64d3f73a216

231 F-sequences

2015-03-27 10:38:59 **by Philippe_57721** **10 xp** **Programming**

Given two sequences $X[1]\cdots X[n]$ and $Y[1]\cdots Y[m]$ ($n \leq m$), we say that the former is a subsequence of the latter if there if a sequence $1 \leq i[1] < i[2] < \cdots < i[n] \leq m$ such as $X[j] = Y[i[j]]$ for all $1 \leq j \leq n$.

E.g. 'PAIN' is a subsequence of 'DIOPHANTINE'.

We say a sequence $S[1]\cdots S[n]$ is a F-sequence if no sequence $S[i]\cdots S[2 \times i]$ ($2 \times i \leq n$) is a subsequence of any sequence $S[j]\cdots S[2 \times j]$ ($i < j$ and $2 \times j \leq n$).

What is the length of longest F-sequence over the alphabet {`A`, `B`}?
Find the 1st F-sequence in lexicographic order over the alphabet {`A`, `B`, `C`} with length 80.

Hint: Here an example of a F-sequence of length 40:
`AACBABBBBBABBCCCCCCCCCBBBBBBBBBBBBBBBBBA`

Answer format: `count,sequence`

[My timing: < 100 ms]

Answer c0f662afe30b93fb1c89d5044e226261

232 Congruent numbers

2015-04-10 07:35:36 | **by Philippe_57721** | **12 xp** | **Programming**

How many squarefree numbers are congruent in the range $[10^7, 10^7 + 1000]$?

Hint: Assume the BSD conjecture.

Answer format: count,sum modulo 10^7

[My timing: 45 sec]

Answer 1ee2af07c54c9acf0b0d8fb5844edf3b

233 Integer Averages

2015-04-17 09:54:33 **by Philippe_57721** **6 xp** **Programming**

Let define a sequence S as follow:

- $S[1] = 1$
- $S[n+1] =$ the first integer not in $S[1..n]$ such as the average of $S[1..n+1]$ is an integer.

Find $S[2357111317192329]$.

[My timing: < 100 ms]

Answer 5072ee14c435036270d679ab189bb5a0

234 Meeting point

2015-04-02 09:02:39 **by sinan** **10 xp** **Math**

There are 2 particles moving on the unit circle ($x^2 + y^2 = R^2$ and $R = 1$).

Particle A moves clockwise at a speed equal to $2 + x$ while particle B moves counter-clockwise at a speed equal to $2 + x$, where x and y are the coordinate values of the moving particle.

At $T = 0$ time, A is located at $(-1, 0)$ and B at $(0, -1)$, so the speeds would be 1 and 2 for A and B respectively.

Where do they first meet and when?

Answer format: `X,Y,elapsed time`
X, Y rounded to 3 digits after decimal point, elapsed time is a decimal number (rounded to 12 digits after decimal point)

Answer 433a9ce8cbae494af44f05925e2bb9b9

235 Totient Proportionality

2015-04-05 15:02:36 **by Min_25** **15 xp** **Math**

Let $C(N)$ be the number of positive integer $n \leq N$ such that

$$\frac{n}{\phi(n)} = \frac{\phi(n)}{\phi(\phi(n))},$$

where $\phi(n)$ is the totient function.

You are given that $C(10^5) = 342$.

Find $C(10^{15})$.

[My timing: 10.9 sec]

Answer c0794ebc58f691d538260f93e4f37c11

236 Inverse Sigma Function

2015-04-05 00:59:29	**by Min_25**	**8 xp**	**Math**

Find the sum of n such that $\sigma(n) = 13!$, where $\sigma(n)$ is the sum of divisors function.

Example: 104530 for 7!.

[My timing: 4.6 s]

Answer e0bcad5f84744d8a11a9f60b183b506a

237 Programming exercise - Fractions madness

2015-04-24 10:07:03 **by Philippe_57721** **10 xp** **Programming**

Consider the sequence of fractions:
$F = \{\frac{23}{95}, \frac{57}{23}, \frac{17}{39}, \frac{130}{17}, \frac{11}{14}, \frac{35}{11}, \frac{19}{13}, \frac{1}{19}, \frac{35}{2}, \frac{13}{7}, \frac{7}{1}\}$

We define the following algorithm (`n` is an integer):

```
func(n) {
  i = 0;
  Start:
    j = 0;
    while (F[j]*n not integer) {
      j++;
    }
    i++;
    n = n*F[j]
    if (n has only factors 2 and 3) {
      print(i);
    }
    goto Start;
}
```

The first values `func(6)` yield are:

- 10, 24, 46, ...

Find the 100-th value.

You are given the 20-th element: 185500.

[My timing: < 100 ms]

Answer 8f670dd1464d9f034a81426095c9f838

238 Phinary representation

2015-04-30 15:43:17 **by Philippe_57721** **10 xp** **Math**

$\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.

Any positive integer can be represented as a sum of powers of ϕ with integer exponents:

- $1 = \phi^0$
- $2 = \phi^1 + \phi^{-2}$
- $3 = \phi^2 + \phi^{-2}$
- $4 = \phi^2 + \phi^0 + \phi^{-2}$
- $5 = \phi^3 + \phi^-1 + \phi^{-4}$
- $6 = \phi^3 + \phi^1 + \phi^{-4}$
- $7 = \phi^4 + \phi^{-4}$
- $8 = \phi^4 + \phi^0 + \phi^{-4}$
- $9 = \phi^4 + \phi^1 + \phi^{-2} + \phi^{-4}$
- $10 = \phi^4 + \phi^2 + \phi^{-2} + \phi^{-4}$

If we add the constraint that no exponent are consecutive, this representation is unique. This representation is called the phinary representation of an integer.

For an integer n, we define $M(n)$ as the sum of all exponents in its phinary representation:

$$M(641) = \sum 641_{\text{base-}\phi} = \sum\{13, 9, 7, 5, 2, 0, -3, -11, -14\} = 8$$

In the range $[1, 10000000]$, find the minimun for M, the first integer which reaches the minimum, the maximum, and the first integer which reaches the maximum.

Answer format: $M_{\text{min}}, n_{\text{min}}, M_{\text{max}}, n_{\text{max}}$
(`-17,77,8,46` for the range $[1, 100]$)

[My timing: 10 sec]

P.S: Thanks to **sinan** for his suggestions which made this problem more interesting.

Answer 0c7b49c9371501d8a089298098f52cd1

239 Find the period

2015-05-14 21:37:28	**by Philippe_57721**	**8 xp**	**Programming**

The period of a prime p is the period of the fraction $1/p$.

For example,

$$\text{Period}(79) = 13 \quad \left(\frac{1}{79} = 0.\underbrace{0126582278481}_{13}\cdots\right)$$

Find the smallest prime with a period of 798.

Answer c20dd77c6da3e3c3794b7f539e008edb

240 Large factorization

2015-05-07 13:25:50 **by Philippe_57721** **10 xp** **Math**

We have:

$$\begin{aligned} 2^{10000000002}+1 &= f_1 \times f_2 \\ &= \underbrace{41776\cdots\cdots01}_{1505149979\text{ digits}} \times \underbrace{41776\cdots\cdots05}_{1505149979\text{ digits}} \end{aligned}$$

Find the last 10 digits of f_1 and f_2.

Answer format: $f_1 \bmod 10^{10}, f_2 \bmod 10^{10}$

Leading zeros are ignored

Answer 4373f0631fbffb19dd89938596d20cb2

241 Smooth factorisation

2015-05-22 06:51:56 **by Philippe_57721** **8 xp** **Programming**

Let's define a smooth factorization of $n!$ as $n! = f_1 \times f_2 \times \cdots \times f_n$ with:
$f_i \leq f_{i+1}$
$f_n = \text{the largest prime factor of } n!$

Examples:

$$\begin{aligned} 13! &= 2 \times 3 \times 3 \times 5 \times 5 \times 6 \times 6 \times 6 \times 7 \times 8 \times 8 \times 11 \times 13 \\ &= 3 \times 4 \times 4 \times 4 \times 5 \times 5 \times 6 \times 6 \times 6 \times 6 \times 7 \times 11 \times 13 \\ 14! &= 4 \times 4 \times 4 \times 4 \times 4 \times 5 \times 5 \times 6 \times 7 \times 7 \times 9 \times 9 \times 11 \times 13 \end{aligned}$$

Find the last (in numeric lexicographic order) smooth factorization of 28!.

Answer format: comma separated list of factors

Example: `5,6,6,6,6,6,6,6,6,7,7,8,8,10,10,11,13,17,19` for 19!.

[My timing: 70 sec]

Answer 84b6c2c4c5e758c8957e9df95fedaa96

242 Generalized Pisano Period 1

2015-05-07 04:56:35 **by Min_25** **8 xp** **Math**

Let $F_k(n)$ be a linear recurrence sequence such that

$$F_k(n) = 1 \ (1 \le n \le k),$$
$$F_k(n) = \sum_{i=1}^{k} F_k(n-i) \ (n > k).$$

For example, $F_2(1) = 1$, $F_2(2) = 1$ and $F_2(3) = 2$.

Let $P(k, m)$ be the (minimum) period of $F_k(n)$ modulo m.

You are given that $P(2, 10) = 60$ and $P(2, 10^9) = 1\,500\,000\,000$.

Find $P(10, 10^9)$.

[My timing: instant]

Answer 4ef3d650778b312c9f165d97216eb1a5

243 Generalized Pisano Period 2

2015-05-07 04:56:58 **by Min_25** **15 xp** **Math**

Let $F_k(n)$ be a linear recurrence sequence such that

$$F_k(n) = 1 \; (1 \le n \le k),$$
$$F_k(n) = \sum_{i=1}^{k} F_k(n-i) \; (n > k).$$

For example, $F_2(1) = 1$, $F_2(2) = 1$ and $F_2(3) = 2$.

Let $P(k, m)$ be the (minimum) period of $F_k(n)$ modulo m.

You are given that $P(2, 10) = 60$ and $P(2, 10^9) = 1\,500\,000\,000$.

Find $P(50, 10^9)$.

Note: The answer is a 29 digit number.

[My timing: 42 sec (PyPy)]

Answer 1ecd8de7460356ed39e0e4133252b61d

244 Brazilian numbers

2015-05-29 09:59:20 **by Philippe_57721** **8 xp** **Programming**

A number is said brazilian if it can be written with the same 'digit' for some base ($0 < \text{digit} < \text{base}$).

For instance, the following numbers are brazilian:
$2222 = (2, 2, 2, 2)_{10}$
$1281 = (7, 7, 7)_{13}$
$33500852988 = (127, 127, 127, 127)_{641}$

How many brazilian numbers $\leq 100\,000\,000\,000$ are there? (We consider only numbers with at least 3 'digits'.)

You are given:
114 numbers $\leq 1\,000$
617 numbers $\leq 10\,000$
3016 numbers $\leq 100\,000$
14387 numbers $\leq 1\,000\,000$

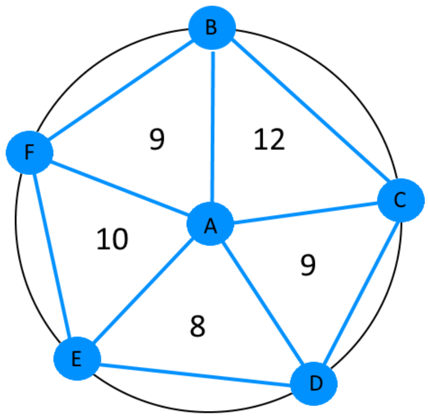
[My timing: 4.7 sec]

Answer 3aa7c660b1f5591170a837c69c60bfb6

245 Weighted triangles

2015-06-05 03:48:11	by Philippe_57721	5 xp	Math

5 points are placed on a circle. Consecutive integers 1 to 6 are placed on the center of the circle and on each point. Joining each consecutive pair of points with the center, we define 5 triangles. The sums of integers for each triangle are: 12, 9, 8, 10, 9. You can guess that the integers are respectively (center first, points clock-wise): 2, 4, 6, 1, 5, 3.



Now, we are using 11 points.
The sums inside the triangles are respectively: 26, 22, 26, 22, 14, 16, 22, 26, 22, 24.

Find the values for each point (same order as in the example). There are more than one solution.

Answer format: comma separated lists between slashs (in lexicographic order)

Example: `1,2,3/3,2,1`

Answer f898a16ad01044fead55081c20408009

246 A special set

2015-05-25 04:42:34 **by sinan** **10 xp** **Programming**

Let S be a set like $\{1, ..., M\}$ with cardinality N.

Any element but the first can be written as a sum of some two lesser and not necessarily distinct elements.

How many such sets can you find if $M = 1881$ and $N = 15$?

Input format: `cnt,S_1001`
(`S_1001` means 1001-st such set)

Example: `cnt,{1,2,3,6,7,13,26,52,104,208,209,418,836,1045,1881}` if asked 101-st such set.

Sets are ordered comparing each element from 1 to M:
100: $\{1, 2, 3, 6, 7, 13, 26, 52, 104, 208, 209, 418, 627, 1254, 1881\}$
101: $\{1, 2, 3, 6, 7, 13, 26, 52, 104, 208, 209, 418, 836, 1045, 1881\}$
Here, there is a difference at 13-th elements.

[My timing: 2 s]

Answer fb0b5b31ac3afb49b24217cfc2d3aaff

247 Crossings and nestings

2015-06-12 09:03:07 **by Philippe_57721** **12 xp** **Math**

Consider the permutation $\{2, 10, 1, 8, 9, 4, 6, 3, 5, 7\}$.

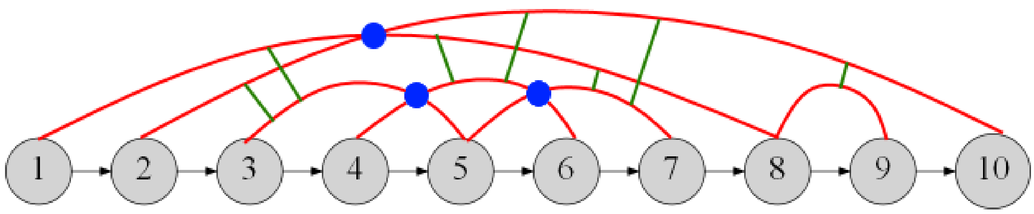
It can be decomposed in 4 “runs” of ascending values:

- $2, 10$
- $1, 8, 9$
- $4, 6$
- $3, 5, 7$

Let’s draw the following graph:

- 10 nodes (1 to 10)
- we join by an edge each consecutive node in the same run

We get the following graph:



We count how many times edges are crossing: 3 (blue dots)

- $[1, 8]$ crosses $[2, 10]$
- $[3, 5]$ crosses $[4, 6]$
- $[4, 6]$ crosses $[5, 7]$

We count how many egdes are nested below another one: 7 (green lines)

- $[3, 5]$ nested below $[2, 10]$
- $[3, 5]$ nested below $[1, 8]$
- $[4, 6]$ nested below $[2, 10]$
- $[4, 6]$ nested below $[1, 8]$
- $[5, 7]$ nested below $[2, 10]$
- $[5, 7]$ nested below $[1, 8]$
- $[8, 9]$ nested below $[2, 10]$

Among all the permutations of $\{1, 2, ..., 10\}$ how many have exactly 4 crossings and 8 nestings?
What is the latest in lexicographic order?

Answer format: `count/permutation` (as comma separated list)

Example: `99/1,2,3`

[My timing: 20 sec]

Answer 2336c357e289f2f5256e1c32c48909a6

248 Smooth factorisation (hard)

2015-05-22 23:15:58 **by gerrob** **20 xp** **Math**

See the problem r237 from Philippe![1] Find the answer for $n = 450$, as the answer would be quite long, give only $\sum_{i=1}^{n} f_i^6$.

Example: `80930279` for $n = 19$.

[My timing: 58 sec.]

Answer 51e4c7ccba556721e4970b6746e683f6

[1] Actually Problem 241.

249 Friend Exponent

2015-06-02 16:18:49 **by Min_25** **15 xp** **Math**

For given two real numbers α and β, a positive integer n is called k-friend exponent of α for β if α^n and β shares the first k-significant digits.

For example, $n = 9$ is a 1-friend exponent of π for e because $\pi^9 = 29809.099333446\cdots$ and $e = 2.71828\cdots$ shares "2".

Let $E(\alpha, \beta, k)$ denote the minimum k-friend exponent of α for β. It can be verified that $E(\pi, e, 1) = 9$, $E(\pi, e, 2) = 19$, $E(\pi, e, 5) = 430881$ and $E(\pi, e, 10) = 2448322414$.

Find $E(\pi, e, 50)$.

[My timing: 0.5 sec. (PyPy)]

Answer c5f69d009f1ce0587d773f8db076c9bb

250 Polyominoes 1

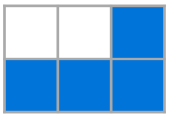
2015-07-03 12:24:49 **by Philippe_57721** **11 xp** **Programming**

A polyomino is a geometric figure obtained by joining equal squares edge by edge. We consider that one side of each square is painted: when comparing two polyominoes for symetry, the painted sides must always stay up. Two polyominoes are equal if they differ only by a rotation of a multiple of $\frac{\pi}{2}$.

To sort the polyominoes, we define the "canonical value" of a polyomino as follow:

- A polyomino is represented by a matrix of 0 and 1.
- We consider all 4 rotations and keep those where the matrix is such as the number of rows is $\leq$ the number of columns.
- We add a column of 0 at the end, flatten the values and consider them as a binary representation of some integer. The canonical value is **the smallest of these integers**.

For example, consider the polyomino



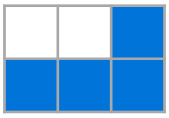
It can be represented either by $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ or by $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

The first representation yields the values $\{0, 0, 1, 0, 1, 1, 1, 0\} = 46$. The second representation yields the values $\{1, 1, 1, 0, 1, 0, 0, 0\} = 232$. Thus, the canonical value for this polyomino is 46.

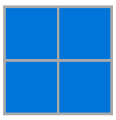
With 4 squares, there are 7 possible polyominoes (sorted by their canonical values):

- 30

- 46



- 54



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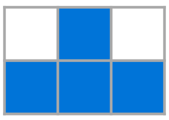
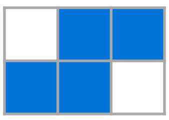
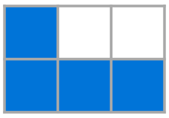


- 108



- 142



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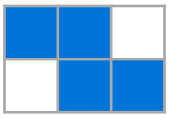


How many polyominoes with 8 squares are there?
What is the canonical value of the 666-th poylyomino?

Format answer: `Count,Value`

[**My timing: 40 sec**]

Answer 74029fadf52b6bd2e1db689c9d8c7506

251 Polyominoes 2

2015-07-17 13:44:51	by Philippe_57721	13 xp	Programming

For the definitions, see problem 247.[1] Among all the 8-squares polyominoes, which ones can cover perfectly a 4×8 rectangle? (A polyomino covers perfectly a rectangle if we can tile without hole or overlap, the rectangle with multiple copies of that poylomino.)

Answer format : comma separated list of canonical values (ascending order)

Example: `126,190,222,238,798,2110` for covering a 2×6 rectangle with 6-squares polyominoes.

[My timing: 6 sec]

Answer ec0fb152d0168b4f876e3342ab6e2dd2

[1] Actually Problem 250.

252 Sort

2015-06-19 11:02:12 **by Philippe_57721** **6 xp** **Math**

Consider the following HUGE numbers:

$$a = \text{googol!}$$
$$b = \text{Skewes' number} = e^{e^{e^{79}}}$$
$$c = 2^{3^{4^{5^{6}}}}$$
$$d = 6^{5^{4^{3^{2}}}}$$
$$e = 4!!!!$$
$$f = p_{\text{googol}}\# \quad (p_n\# \text{ is the primorial function})$$

Sort these numbers in ascending order.

Answer format: comma separated list

Example: `f,e,d,c,b,a`

For the record, the primorial function $p_n\#$ is the product of the first n prime numbers.

$$p_5\# = 2 \times 3 \times 5 \times 7 \times 11 = 2310$$

Answer db361498f85de7014be5d4aeaf909afe

253 Colored coins

2015-06-26 12:51:33	**by Philippe_57721**	**7 xp**	**Programming**

$T(n)$ is the n-th triangular number.

$T(n)$ coins are arranged in a triangle. We color each coin with 4 different colors such as no two touching coins have the same color.

For $n = 2$, there are 24 possible configurations:

```
+---+ +---+ +---+ +---+ +---+ +---+
| 1 | | 1 | | 1 | | 1 | | 1 | | 1 |
|2 3| |2 4| |3 2| |3 4| |4 2| |4 3|
+---+ +---+ +---+ +---+ +---+ +---+

+---+ +---+ +---+ +---+ +---+ +---+
| 2 | | 2 | | 2 | | 2 | | 2 | | 2 |
|1 3| |1 4| |3 1| |3 4| |4 1| |4 3|
+---+ +---+ +---+ +---+ +---+ +---+

+---+ +---+ +---+ +---+ +---+ +---+
| 3 | | 3 | | 3 | | 3 | | 3 | | 3 |
|1 2| |1 4| |2 1| |2 4| |4 1| |4 2|
+---+ +---+ +---+ +---+ +---+ +---+

+---+ +---+ +---+ +---+ +---+ +---+
| 4 | | 4 | | 4 | | 4 | | 4 | | 4 |
|1 2| |1 3| |2 1| |2 3| |3 1| |3 2|
+---+ +---+ +---+ +---+ +---+ +---+
```

We represent a configuration:

```
+-----+
|  A  |
| B C |
|D E F|
+-----+
```

as `A,BC,DEF`.

How many configurations are there for $n = 10$?
What is the 10000000-th (in lexicographic order)?

Example: `192/1,32,214` if we ask the 20-th configuration for $n = 3$.

[My timing: 7 sec]

Answer e5b0cb04de243669196cf8426dcae991

254 Semiprimes

2015-07-10 10:57:10	by Philippe_57721	12 xp	Programming

An integer is semiprime if it is the product of 2 (not necessarily distinct) prime numbers.

There are 34 semiprimes below 100:
4, 6, 9, 10, 14, 15, 21, 22, 25, 26, 33, 34, 35, 38, 39, 46, 49, 51, 55, 57, 58, 62, 65, 69, 74, 77, 82, 85, 86, 87, 91, 93, 94, 95

How many semiprimes are there below 30 000 000 000?

[My timing: 60 sec]

Answer d6f3ef9fc98c27fc08ed9c0fbc2ab78d

255 Complex Base 1

2015-08-01 08:13:27 **by Philippe_57721** **15 xp** **Math**

A complex integer is a number $p + q \times i$ ($i^2 = -1$) where p and q are integers.

It can be proved that every complex integer has a unique representation in base $i - 1$ with 'digits' 0 or 1.

For instance, we have:

$$2 = 0 \times (-1+i)^0 + 0 \times (-1+i)^1 + 1 \times (-1+i)^2 + 1 \times (-1+i)^3 \quad (1100)$$

$$6 = 0 \times (-1+i)^0 + 0 \times (-1+i)^1 + 1 \times (-1+i)^2 + 1 \times (-1+i)^3 + 1 \times (-1+i)^4 + \cdots$$
$$+ 0 \times (-1+i)^5 + 1 \times (-1+i)^6 + 1 \times (-1+i)^7 + 1 \times (-1+i)^8 \quad (111011100)$$

Actually, 6 is the smallest natural integer whose representation contains 6 digits to '1'.

Let's define the function $R(n)$ for an natural integer n as follow: consider the digits of n in base $i - 1$ as the binary representation of an integer.
Thus:
$R(2) = 12$
$R(6) = 476$

Find the smallest integer a, whose representation contains 60 digits.
Find the smallest integer b, whose representation contains 93 digits to '1'.

Answer format: `a,R(a),b,R(b)`

[My timing: instant]

Answer 565879355f5ec90aa60c99a7ba22a45f

256 Complex Base 2

2015-08-01 08:14:10	by Philippe_57721	15 xp	Programming

The definitions are the same as in problem 252.[1]

Consider the following pseudo-random generator:

```
static long seed = 641;
static long FastRandom()
{
    seed = (214013 * seed + 2531011) % 43097251273073;
    return seed;
}
```

Let V the sequence of the first 100000 values of FastRandom.

You are given the following values:

1	139713344
10	34286987424384
100	2133420965406
1000	31159015095342
10000	29256942849607
100000	42800860506246

Find the sum of $R(n)$ for all n in V.

[My timing: 12 sec]

Answer b96cc8fdab20499d7ffcb06d03260653

[1] Actually Problem 255.

257 Pattern Locks

2015-07-17 13:45:07	by Philippe_57721	7 xp	Programming

Pattern locks are a popular mechanism to secure our cell phones.

Given a grid 3×3, the user define a path by joining some points on that grid:

1	2	3
4	5	6
7	8	9

We add the following constraints:
- the path starts with point 1
- we visit every point exactly once

One possible path is: $1, 4, 3, 5, 6, 2, 7, 8, 9$.

Actually, there are exactly 5040 possible paths with the above constraints. There are 2 paths which reach the maximal length:
- $1, 6, 7, 2, 9, 4, 3, 8, 5$
- $1, 8, 3, 4, 9, 2, 7, 6, 5$

The maximal length is $16.652\cdots$

A path starting with $1, 3, 2$ would not be valid, for when joining point 1 and 3 we have visited point 2.

How many possible paths are there on a 3×4 grid?
Which paths reach the maximal length?

Answer format: `count/(comma separated list of points)`

Example: `5040/1,6,7,2,9,4,3,8,5/1,8,3,4,9,2,7,6,5` for a 3×3 grid.

[My timing: 40 sec]

Answer ac4c1be08f1d7ba33327f51183d93ff7

258 P-sets

2015-08-07 08:48:24	by Philippe_57721	8 xp	Programming

Let P a finite set of prime numbers.

A P-set is the set of all numbers whose prime factors are in P.

For instance, if $P = \{2, 3, 5\}$, the corresponding P-set is
$\{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 27, 30, 32, 36, ...\}$.

The 100000-th number of this P-set is 290142196707511001929482240000000000000.

What is the 100000000-th number in the $\{2, 3, 5, 7, 11\}$-set?

[My timing: 40 sec]

Answer 6fab782829208d2d114c46e306d2ea4f

259 Alphabet permutations

2015-07-24 11:50:16	**by Philippe_57721**	**6 xp**	**Probability**

The classical latin alphabet {**a**, b, c, d, **e**, f, g, h, **i**, j, k, l, m, n, **o**, p, q, r, s, t, **u**, v, w, x, **y**, z} contains 20 consonants and 6 **vowels**.

What is the probability that in a permutation of this alphabet, there are no consecutive vowels?

Answer format: `P/Q`
with `P` and `Q` coprime

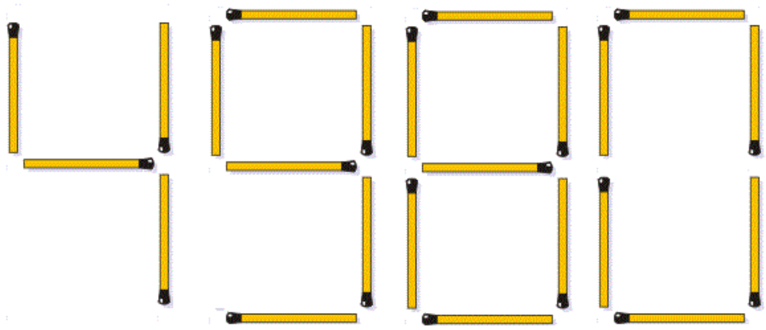
[My timing: < 1 sec]

Answer ee7c1d30528d58345a78d06331a3a9c3

260 Matchstick math

2015-07-31 08:56:32 **by sinan** **5 xp** **Math**

See the following number:



What is the biggest number you can get if you are allowed to move 1, 2, 3 or 4 matchsticks?

Answer format: `n1,n2,n3,n4`

Notes:
- You should look for the ways to increase the number of digits. For example out of "8", you can make "51" by moving 2 sticks.
- Ignore any spacing issues after the move, think all the digits as a part of the one new number.

Answer ed3c36764ecf0e17dd4ed04e3de66db8

261 A slow converging serie

2015-09-27 15:49:28 **by Philippe_57721** **12 xp** **Math**

P is the set of prime numbers.

Let $S = \sum_{p \in P} \frac{1}{1+p^2} = 0.3890595553\cdots$.

Find the value of S with a precision of 50 digits after the decimal point (truncated not rounded).

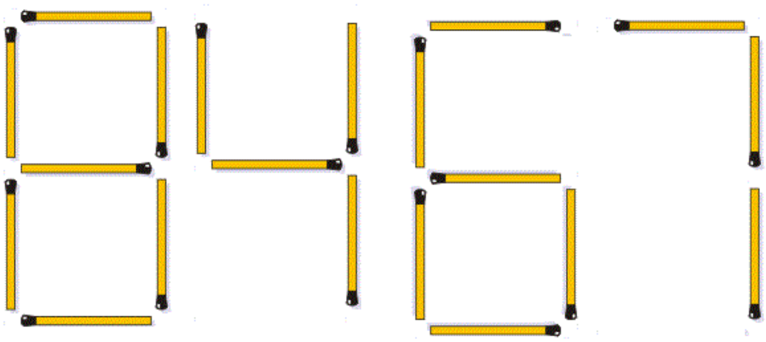
[My timing: 15 sec]

Answer 55ff5d2af96d514bde27a073bca5f6b3

262 Matchsticks prime

2015-08-22 07:07:32 **by Philippe_57721** **6 xp** **Programming**

Using 20 matchsticks, we can write the following prime number:



How may different prime numbers can you write using exactly 20 matchsticks?

[My timing: < 200 ms]

N.B.: Non significative 0s are not allowed, thus: 0019 is not valid.

Answer bedc8b5da097e4f6a7a4e0a0681b9771

263 Digit Sum Sequence

2015-08-02 02:41:12 **by Min_25** **12 xp** **Math**

Let $x_1 = 1$ and $x_i = x_{i-1} + \text{sum\_of\_digits}(x_{i-1})$ for $i > 1$.

You are given that $x_{10} = 62$, $x_{100} = 1205$ and $x_{100000} = 2609882$.

Find $x_{10^{17}}$.

[My timing: 0.2 sec.]

Answer **bbd61f255581afe1721496ab570c43f5**

264 Anti-Pythagorean equation

2015-08-14 04:46:41	by Philippe_57721	8 xp	Math

We have

$$1040^{-2} + 4095^{-2} = 1008^{-2}$$

How many solutions are there to the "anti-pythagorean" equation $x^{-2} + y^{-2} = z^{-2}$ with x, y, z coprime and $x \leq y$ and $x \leq 10^{12}$?

Answer format: `count,`$\sum z$

[My timing: 9 sec]

Answer 36b2088bd7a9db01d82fe1cc39eeaa10

265 Anti-Pythagorean equation 2

2015-08-14 04:54:21	**by Min_25**	**12 xp**	**Math**

See the problem 261 from Philippe for the description.[1]

In this problem, find the answer for $x \leq 10^{18}$.

[My timing: 0.8 sec.]

Answer **fe300e68829e970bdc5a4cb56a83f575**

[1] Actually Problem 264.

266 Parallelepipedic Box

2015-08-28 09:48:22	by Philippe_57721	7 xp	Math

The parallelepipedic box of dimensions $2 \times 3 \times 6$ has a diagonal of 7.

A parallelepipedic box with integer sides has a diagonal of 100000.

Let x, y, z its dimensions with $x \leq y \leq z$.

Find all the possible triples x, y, z.

Answer format: `count,`$\sum z$

[My timing: 10 sec]

Answer 45d20c53c61d79e58387c9f17e297689

267 The Schinzel-Sierpinski conjecture

2015-09-04 11:37:30 **by Philippe_57721** **4 xp** **Math**

This conjecture states:

$$\textit{Every rational } \frac{n}{d} \textit{ can be written } \frac{p+1}{q+1} \textit{ where } p, q \textit{ are prime numbers.}$$

For instance: $\frac{22}{7} = \frac{43+1}{13+1}$, $\frac{355}{113} = \frac{2129+1}{677+1}$.

Find the representation for $\frac{2^{19}-1}{2^{19}}$.

Answer format: `p,q`
give the smallest pair p, q.

[My timing: < 1 sec]

Answer c76e5252fc2160fd4afccf02f291d3c6

268 Meeting probability

2015-08-25 05:41:10 **by sinan** **10 xp** **Math**

There is a grid of roads ($M \times N$). Two people, one at $A(0,0)$ and the other at $B(N,M)$, start to walk from A to B and B to A respectively at the same time. They both walk the shortest path to their destination. For instance, if you come from B to A, you must always move from East to West (left) or North to South (down).

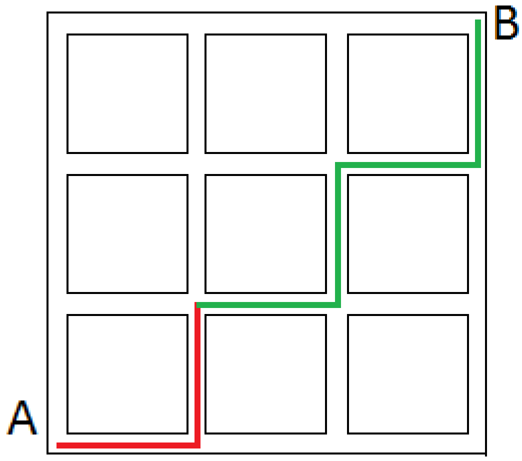
The man at B walks faster than the one at A by a factor of k ($v_B = k \cdot v_A$ where $k > 1$).

If they walk in any **possible** direction equally likely on a 19×23 grid, what is the probability that they meet each other on their way if k is equal to $4/3$ (1) or $6/5$ (2)?

Answer Format: $P_1/Q_1, P_2/Q_2$

Example: Answer=`5/14,5/28` for $k = 2$ and $k = 3/2$ on 3×3 grid.

The following shows a possible meeting for $k = 2$:



Answer 48377a799d0df282b4cb35a4652fc420

269 Counting the zeros

2015-09-11 11:05:04 **by Philippe_57721** **9 xp** **Programming**

Consider the integers in range [111, 122] in binary form:

```
1 1 0 1 1 1 1
1 1 1 0 0 0 0
1 1 1 0 0 0 1
1 1 1 0 0 1 0
1 1 1 0 0 1 1
1 1 1 0 1 0 0
1 1 1 0 1 0 1
1 1 1 0 1 1 0
1 1 1 0 1 1 1
1 1 1 1 0 0 0
1 1 1 1 0 0 1
1 1 1 1 0 1 0
```

and count how many times the digit 0 appears. There are 28 zeros.

How many times does the digit 0 appear in the range [12345678910111213, 31211101987654321]?

[My timing: < 100 ms]

Answer 34fe1bd23852aa4af8268fb79f98fedd

270 Vectors Sums

2015-11-01 06:06:58 **by Philippe_57721** **8 xp** **Math**

V is a vector $\in \{-1, 1\}^n$.

We define the following n sums:

$$S_j = \sum_{i=1}^{j} V_i - \sum_{i=j+1}^{n} V_i \quad \text{for } 1 \leq j \leq n$$

For example if $V = (-1, -1, 1, 1, 1)$ then $S = (-3, -5, -3, -1, 1)$.

We define then $T_j = 1$ if $S_j > 0$ and 0 otherwise. Thus $T = (0, 0, 0, 0, 1)$.

There are 2^n possible vectors V.

How many different vectors T are there?

Give your answer for $n = 50$.

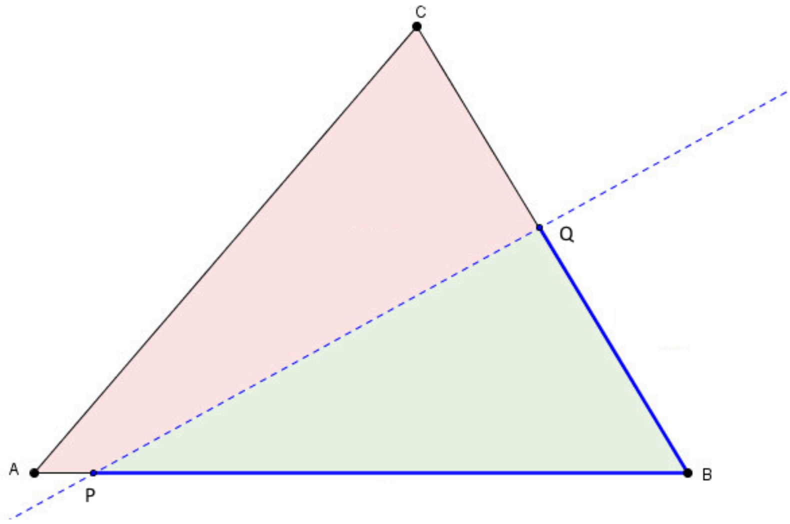
Hint: there are 2728 possibilities for $n = 15$.

Answer 39c05f8b98eb919ef0f22c215b101f5b

271 Triangle Fair Split

2015-09-19 03:41:24 **by Philippe_57721** **10 xp** **Math**

For a triangle, there is always at least one line that splits simultaneously the perimeter and the area in two equal parts.



You are given a triangle of sides 7182, 8182, 8459.

Find how many lines split fairly this triangle and give there length (line PQ on the picture) in ascending order (comma delimited, rounded to 5 digits after the decimal point).

Caveat: If 2 distinct lines have the same length, they will be counted as 2 solutions.

Answer 913cc08eceafb0bffc1c870d87df811d

272 Functional Equation

2015-10-10 12:21:37 **by Philippe_57721** **7 xp** **Math**

f is a function such as

- $f : [0, \infty[\to \mathbb{R}$
- f is strictly increasing
- $f(x) > -\frac{1}{x}$
- $f(x) \cdot f\left(f(x) + \frac{1}{x}\right) = 1$ for all $x > 0$

What is $f(1)$?

Give your answer rounded to 10 digits after the decimal point.

Answer 44f73401c571b0e00ca741f9b9e4d3cc

273 Diophantine Equation

2015-10-03 13:04:46	by Philippe_57721	8 xp	Math

Solve the diophantine equation (Euler)

$$2^{200} = 7 \cdot x^2 + y^2 \text{ with } x, y \text{ odd} > 0$$

Answer format: `x,y`

[My timing: < 100 ms]

Answer 00bfa1db27c71433b903fbbf8534684f

274 Beautiful permutations

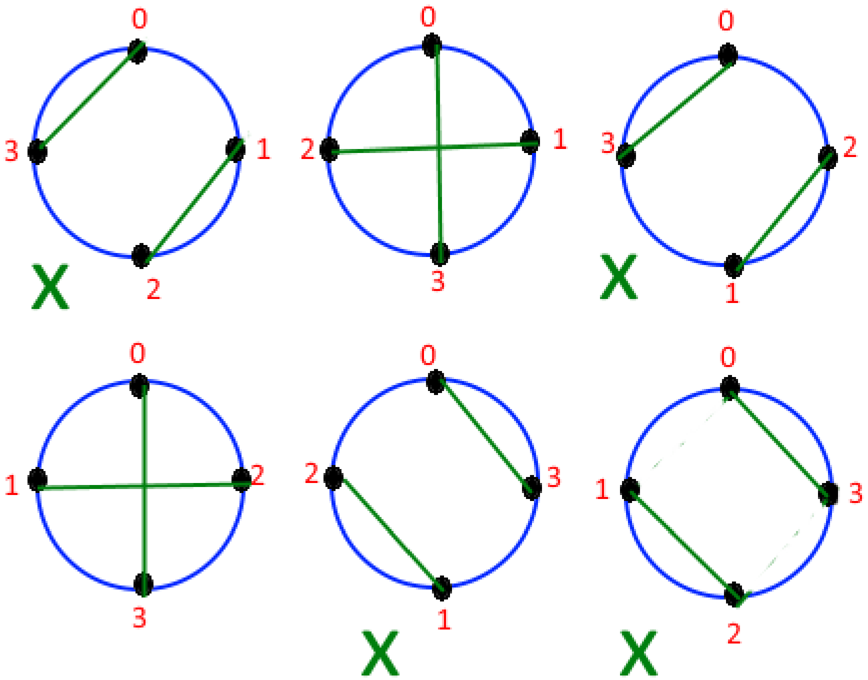
2015-10-25 05:32:05 **by Philippe_57721** **15 xp** **Math**

n is an integer > 2.

The $n+1$ point of a permutation of $0, 1, \ldots, n$ are placed on a circle.

The permutation is said beautiful if for any 4 numbers $0 \le a, b, c, d \le n$ with $a + c = b + d$, the chord joining a and c does not intersect the chord joining b and d.

For $n = 3$, there are 4 beautiful permutations:



How many beautiful permutations are there for $n = 1000$?

You are given:
$C = 6$ for $n = 4$
$C = 10$ for $n = 5$

[My timing: < 1 s]

Answer 037a6f5a4cfcbdf7d5a6d1c4e465a686

275 Counting the solutions

2015-10-17 19:15:36 **by Philippe_57721** **7 xp** **Math**

How many pairs of positive integers $(a, b) \leq 10^9$ are there such as

$$\frac{a^2}{2ab^2 - b^3 + 1}$$

is an integer.

[My timing: instant]

Answer 458e90eac17694d1b240d3d172512405

276 Merge

2015-10-22 17:05:23 **by sinan** **10 xp** **Hack**

I'm sure you can make something out of this. **Pay attention to the title!**

The background image code for this problem is as follows:

```
background-image:url('
data:image/bmp;base64,
Qk1+AQAAAAAAAD4AAAAoAAAAIAAAAAoAAAABAAEAAAAAAEABAAAAAAAAAAAAAAAAAAAAAAAAAOwEAADsB
AABQSwMEFAABAAAAOZBWR9XAp3AwAAAAJAAAAAEAAAAypRjqizuI0Jynv8UeROBStDrYee20F7as2tqC
uitUVXHmECq1ycLTsyC6fu8IUP/gUEsDBBQAAQAAADmQVkccxXIuQgAAADYAAAABAAAAM1AP8NKjrG/W
ycRbRewGuIGx5gdK/hgWxaYtlKMKKwy2kXEkS6Koqo/+hUbEDWKtWzzD/Y2nQKkQlYXb/7lZkhPY/VBL
AwQUAAEAAAA5kFZHktMJ32YAAABaAAAAQAAADUwmjixRkWjMCRwKdPomcAjXe6rPet0Hdz+le67Mh+J
llBPk/xFhOO5j+W2U49yPHeGhmY7aIofM4zfhcBn+1gRwYm3WX6KTnluvWuFDce9qChAxXKHgidEux+r
mhO8H/oul3ZFww9QSwECHgAFBAMCAQ==
')
```

Answer c9d8de3dadb95a84f050e2ab42a8817e

277 Lenstra's Numbers

2015-11-07 17:30:43	**by Philippe_57721**	**8 xp**	**Math**

We have $1233 = 12^2 + 33^2$.

In homage to Hendrik Lenstra who first noticed this property let's call such integers Lenstra's numbers.

Find the first 36-digits Lenstra's number i.e. n such as:
$n = d_1 d_2 \cdots d_{36} = (d_1 d_2 \cdots d_{18})^2 + (d_{19} d_{20} \cdots d_{36})^2$

[My timing: < 1 sec]

Answer 643d4465aecff239163bb90a5cef181c

278 "...THISISTHEEND"

2016-01-01 08:25:43 **by sinan** **15 xp** **Math**

A random generator emits letters based on the English letter frequencies as given below: Out of 10000 letters, it would emit 1203 E's, 910 T's, 812 A's, 768 O's, 731 I's 695 N's, 628 S's 602 R's, 592 H's, 432 D's, 398 L's, 288 U's, 271 C's, 261 M's, 230 F's, 211 Y's, 209 W's, 203 G's, 182 P's, 149 B's, 111 V's, 69 K's, 17 X's, 11 Q's, 10 J's, and 7 Z's.

The generated letters are concatenated to a string starting from an empty string.

What is the probability that we would see a **"…THISISTHEEND"** before seeing any **"…THIS…"**, **"…IS…"**, **"…THE…"** or **"…END…"**?

Answer format: `N/D`
(irreducible fraction)

Check point:
Answer=`1/245` to see a **"…THEEND"** before seeing any **"…THE…"** or **"…END…"** if it were to emit E,T,N,H,D with equal probabilities.

Answer **9778e6bd33c1fee59b921d0b97865210**

279 Equations system 1

2015-11-14 22:51:59	by Philippe_57721	5 xp	Math

Solve the system:

$$\frac{4x^2}{1+4x^2} = y$$
$$\frac{4y^2}{1+4y^2} = z$$
$$\frac{4z^2}{1+4z^2} = x$$

with $x, y, z \neq 0$.

Answer format: `x,y,z`
(in form `p/q` as they are rational numbers)

Answer 45a9579813ee6b5bffa8136876389297

280 Equations system 2

2015-11-14 22:52:34 **by Philippe_57721** **7 xp** **Math**

Given n nonnegative real numbers $a_1 \leq a_2 \leq \cdots \leq a_n$ we have:

$$\sum_{i=1}^{n} a_i = 96$$

$$\sum_{i=1}^{n} a_i^2 = 144$$

$$\sum_{i=1}^{n} a_i^3 = 216$$

Find n, a_1 and a_n.

Answer format: n, a_1, a_n
(in form `p/q` as a_i are rational numbers)

Answer a75c59831a952477eb099b4103e4c3b0

281 Big Numbers

2015-11-09 08:47:07 **by sinan** **8 xp** **Crypto**

```
2669036874315126775104877548238488463891833998332712460583098453881000,
```

You need to make something out of these numbers.

Answer d223c9a68135220dfec7027f3ca53efa

282 Wilson quotient

2015-11-22 14:32:39 **by Philippe_57721** **9 xp** **Programming**

Given a prime number p, we define the Wilson quotient as:
$W_p = \frac{(p-1)!+1}{p}$
and its residue modulo p as:
$w_p = W_p$ modulo p in the interval $-\frac{p}{2} \leq w_p < \frac{p}{2}$.

Find $w_{200000039}$.

[My timing: 45 sec]

Answer 72e323a4d237ed4785a0ccd9f04ae874

283 Champions

2016-01-29 08:55:31 **by Philippe_57721** **13 xp** **Programming**

Consider the sequence $S(n)$ defined by:

$$S(1) = 2$$
$$S(n) = S(n-1) + \mathrm{GCD}(S(n-1), n-1+(-1)^n) \quad (n > 1)$$

The first values are:
$2, 4, 5, 6, 9, 12, 13, 14, 21, 22, 23, 24, 25, 26, 39, 40, 45, 54, 55, 60, \ldots$

We calculate the vector of first differences $D(n) = S(n+1) - S(n)$:
$2, 1, 1, 3, 3, 1, 1, 7, 1, 1, 1, 1, 1, 13, 1, 5, 9, 1, 5, 1, 1, 1, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 43, \ldots$

Then, we select in this vector each value greater than all the previous ones:
$2, 3, 7, 13, 43, \ldots$

Find the 32-nd element of this sequence.

You are given:
- $M(1) = 2$
- $M(2) = 3$
- $M(10) = 2659$

[My timing: 7 min]
Some solvers found it under 1 min.
There is a fascinating conjecture about this sequence of champions:
Each champion is the larger prime of a twin pair of primes.

Answer 59f2a193f7f8a5245452f76b6361e738

284 Awesome algorithm

2015-11-28 17:12:59 **by Philippe_57721** **7 xp** **Math**

Given a vector A of rational numbers, we define the following transformation:
$B = \{1 \times (A_1 - A_2), 2 \times (A_2 - A_3), 3 \times (A_3 - A_4), ...\}$

We repeat the process:
$C = \{1 \times (B_1 - B_2), 2 \times (B_2 - B_3), 3 \times (B_3 - B_4), ...\}$

And so on …

Consider now the vector
$Z = \{B_1, C_1, D_1, ...\}$

If $A = \left\{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\right\}$, find the first index such as the denominator of Z_i is $> 10^{50}$.

Answer format: `index,denominator`

[My timing: < 1 sec]

Answer ec7cbf506c9cbcdee1aa599be064ebce

285 Caesarian shift

2015-12-01 07:11:39	by sinan	8 xp	Crypto

You need to decode the following string:
`yparjcfwnixigfwzt`

Here's the pseudo code to get this encoded string:

```
function encode(string, n=29101923, offset=10^9)
begin
  len=length of string
  let array be an array with offset+len elements
  for i=1 to n
  begin
    array[i]=1
  end
  for i=n+1 to offset+len
  begin
    array[i]=sum of previous n elements
  end
  for i=1 to len
  begin
    encoded_string[i]=string[i] shifted by (array[offset+i] mod 26)
  end
  return encoded_string
end
```

Example: `encode("solution", 101, 10^5)` returns `"bvktehbs"`.

[My timing: < 1 m]

Answer 800f2ef92f2bb49ba2d5b2d19414a3e5

286 Twelve faces of different colors

2016-02-08 07:33:02 **by sinan** **15 xp** **Math**

Let there be N distinct colors.

Let $a_n = (a_{n-1} + a_{n-2}) \bmod 9$ with $a_1 = a_2 = 1$.

Consider the ways to paint all the faces of dodecahedron using these N distinct colors. **If selected, you can use up to** $a_i + 2$ **but no less than 2** of i-th color where $i = 1$ to N.

What is the total number of ways to paint a dodecahedron if $N = 1000$?

Answer format: `number`

Example: `652706` if $N = 5$ or `130992447` if $N = 10$.

[My timing: 2 s]

Answer 31b1023f835b61bfe65763095b21bb0f

287 Shifted digit

2015-12-15 11:31:32	**by sinan**	**6 xp**	**Math**

Consider the numbers in base-12 that can be written with the decimal digits only (0 to 9).

Of those, find **the smallest** number which, after a ROR 1 (**RO**tate to **R**ight) operation, gives a number equal to 8 times the original number.

Example: 2497 becomes 7249 which is 3 times the original
$(7249)_{12}/(2497)_{12} = 3$

Answer format: the number in base-12

See also: Circular shift - Wikipedia, the free encyclopedia

Answer dee3651b30302aa78a762487b9064d02

288 Which one is the best, e or Pi ?

2016-01-11 09:29:54	**by sinan**	**8 xp**	**Math**

e and **Pi** are transcendental numbers meaning they cannot be the roots of an algebraic equation with rational coefficients.

So we look for some approximations such that the root of an quadratic equation is as close as possible.

Let the equation be given with the A, B, C triples: $A \cdot x^2 + B \cdot x + C = 0$ where A, $|B|$ and $|C|$ are integers $< 10^{10}$ and > 0.

Find the $(A_{\mathrm{e}}, B_{\mathrm{e}}, C_{\mathrm{e}})$ and $(A_{\mathrm{Pi}}, B_{\mathrm{Pi}}, C_{\mathrm{Pi}})$ triples and corresponding roots ($\mathrm{root}_{\mathrm{e}}$ and $\mathrm{root}_{\mathrm{Pi}}$), and calculate the following ratios:
$R_{\mathrm{e}} = |\mathrm{e} - \mathrm{root}_{\mathrm{e}}|/\mathrm{e}$
$R_{\mathrm{Pi}} = |\mathrm{Pi} - \mathrm{root}_{\mathrm{Pi}}|/\mathrm{Pi}$

What are the best triples for e and Pi?

Answer format: $A_{\mathrm{e}}, B_{\mathrm{e}}, C_{\mathrm{e}}, A_{\mathrm{Pi}}, B_{\mathrm{Pi}}, C_{\mathrm{Pi}}$
(the triples with the smallest ratio)

See also:
Transcendental number - Wikipedia, the free encyclopedia
Integer relation algorithm - Wikipedia, the free encyclopedia

Answer cc866b17490238d00d82d60c2d493edd

289 A Diophantine Equation

2015-12-08 07:17:56 **by sinan** **12 xp** **Math**

$x^2 + y^2 - k \cdot x \cdot y = 1 \quad$ (1)
where k is an integer > 1 and $0 < x < y$.

Let $\mathrm{pol}(n)$ be a monic polynomial of degree n:
$k^n + a_{n-1} \cdot k^{n-1} + \cdots + a_1 \cdot k + a_0$

If the first solution to the above equation (1) is $(1, k)$ pair then the solution to n-th would be $(\mathrm{pol}(n-1), \mathrm{pol}(n))$.

What is the 100-th solution?

Answer format: $sum_1 + sum_2$
where $sum_1 = \sum a_i^3$ for $\mathrm{pol}(99)$ and $sum_2 = \sum a_i^3$ for $\mathrm{pol}(100)$.

Example: `-17402` (875 − 18277) for 10-th solution.

[My timing: < 1 s]

See also:
Monic polynomial - Wikipedia, the free encyclopedia
Diophantine equation - Wikipedia, the free encyclopedia

Answer 2e71305a0d7bb9d59b0af232ab2e6864

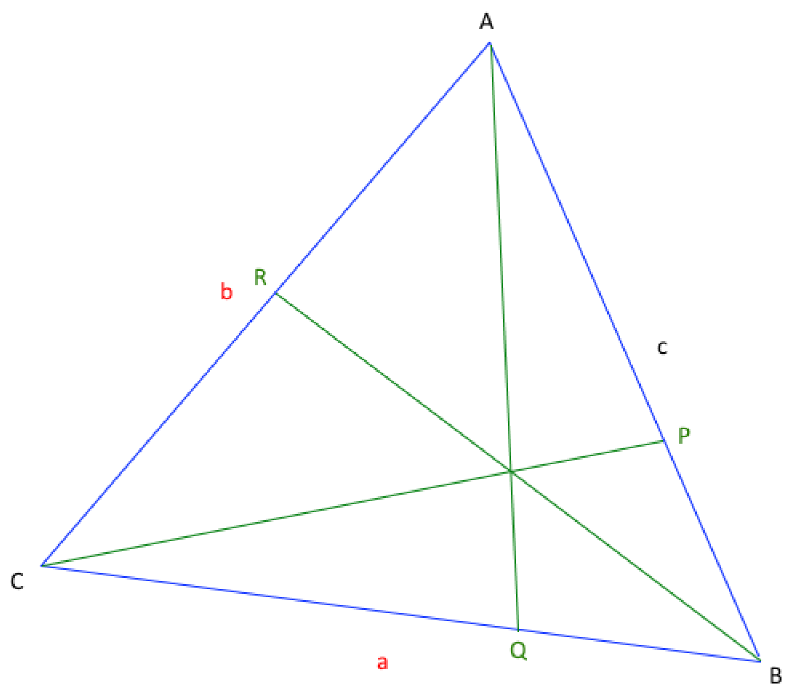
290 Concurrent lines

2015-12-11 15:01:33 **by Philippe_57721** **8 xp** **Math**

A triangle ABC has the following dimensions:
- $BC = a = 1033$
- $AC = b = 1553$
- $AB = c = 1973$

We choose 3 points:
- P on AB at a distance p of A $(0 < p < c)$
- Q on BC at a distance q of B $(0 < q < a)$
- R on AC at a distance r of C $(0 < r < b)$

$(p, q, r$ are integers)



How many triples (p, q, r) are there such as the lines AQ, BR and CP are concurrent?

Answer format: Count, $\sum p, \sum q, \sum r$

You are given: `4,214,202,206` for the triangle $101, 103, 107$.

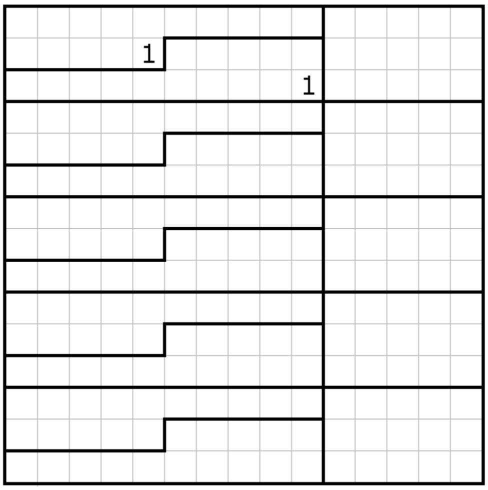
[My timing: < 20 sec]

Answer 6d12fe7551e438131d7c41b7bbaf84a0

291 First sudoku grid

2015-12-22 10:19:11 **by sinan** **8 xp** **Programming**

Using lowercase hex digits `1` to `f`, fill the grid given in the picture in such a way to get the lowest possible hex number when you concatenate the lines starting from top to bottom.



Answer format: last 2 lines
(last 30 digits of the number)

[My timing: < 10 s]

Answer 19a410bc0a9b3c37ecc2cad39ae8d1a8

292 Bonus for p286

2015-12-15 13:49:09	**by sinan**	**6 xp**	**Math**

[1] Consider the following Diophantine equation:
$x^2 + y^2 - k \cdot x \cdot y = 1$ where $k > 1$ and $0 \leq x < y$.

If the first solution is $(0, 1)$, what is the k-th solution for $k = 3^{4^5}$?

Answer format: $x\%M, y\%M$
where $M = 1000000007$

[My timing: < 1 ms]

Answer 5a2c36c95d3fa650c0eac1568b532f17

[1] Actually Problem 289.

293 Decode it (javascript)

2015-12-21 09:15:47 **by sinan** **8 xp** **Crypto**


```
X5dzkDFkg5BnNUEchPVkYb87f7X1QNu0MDR6U1pzQLjCPHUlNPT0KJRsRHZu
iJd0eYTohf5kbLA/bcLmgeWmMGmomEpC
```


Enter the key to decode it.

Key = 1 Decode it

```
Key=1


Not a key!
```

The JavaScript code for this problem is as follows:

```
function func()
{
     var keyStr = "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz0123456789+/
=";
    var A=parseInt(myform.myinput.value);
    var K=404;
    var KK=K*K;
    var AA=A*A;
    var encodedText = document.getElementById('encoded_text').innerHTML;
    var outputDiv = document.getElementById("outputDiv");
    outputDiv.innerHTML = "<pre\>";
    if (A<=0) {
        alert("Should be >0");
    } else {
        outputDiv.innerHTML += "<font size=+2 \><b\>Key="+A+"</b\></font\><br\><br\>";
        // K*A^2+K*x-A-x^2=0 : x^2-Kx+A-KA^2=0
        var delta=Math.sqrt(KK+4*(AA-1)*K);
        var x=Math.floor((K+delta)/2);
        for(var ii=-1; ii<2; ++ii){
            var t=x+ii;
            var tt=t*t;
            var z1=K*(AA+t), z2=(tt+A);
            if (z1==z2){
                var arr=new Array();
                var i=0, keylen=0, s=t, cd=0;
                while (s>0) {
                    arr[keylen++]=s%10;
                    s=Math.floor(s/10);
                }
                outputDiv.innerHTML += "Decoded: ";
                var j=keylen, k=0;
                for(i=0; i<encodedText.length; ++i) {
                    var inx=keyStr.indexOf(encodedText.charAt(i));
```

```
                    if(inx<0) continue;
                    if(--j<0)
                        j+=keylen;
                    inx-=arr[j];
                    if (inx<0)
                        inx+=64;
                    outputDiv.innerHTML += keyStr.charAt(inx);
                    if(i>0 && (i&63)==0)
                        outputDiv.innerHTML += "<br\>";
                    cd+=k++*keyStr.charCodeAt(inx);
                }
                if (cd==360117)
                    outputDiv.innerHTML += "<br\><br\><font color=\"green\"/>Decoded
successfully.</font/><br\>";
                else
                     outputDiv.innerHTML += "<br\><br\><font color=\"red\"/>Decoded
something but it doesn't seem to be OK.</font/><br\>";
                break;
            }
        }
        outputDiv.innerHTML += "<br\>";
        if (ii==2)
                outputDiv.innerHTML += "<br\><font color=\"red\"/>Not a key!</font/
><br\>";
    }
    outputDiv.innerHTML += "</pre\>";
}
```

Answer **aee3adb3a618b282b6a43547e1e2441b**

294 Ratio of triangles areas

2015-12-24 07:57:48 **by Philippe_57721** **9 xp** **Math**

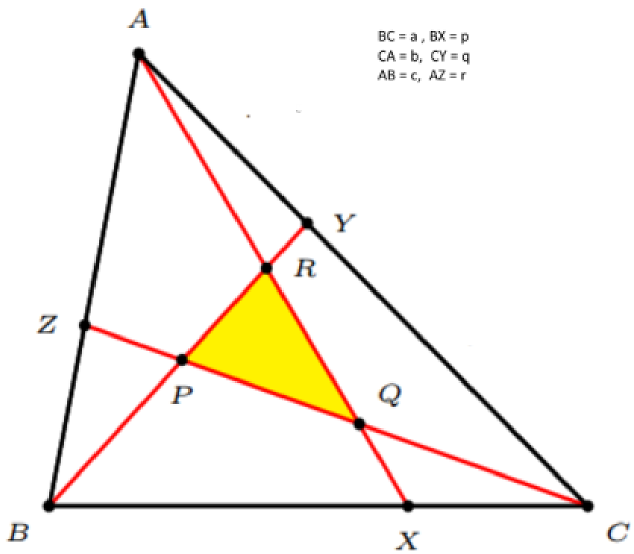
A Heronian triangle ABC has the following dimensions:

- $BC = a = 203$
- $CA = b = 265$
- $AB = c = 372$

We choose 3 points:

- point X on BC at a distance p of B
- point Y on AC at a distance q of A
- point Z on AB at a distance r of C

(p, q, r are integers)



For how many triples (p, q, r) the ratio $\text{Area}(ABC)/\text{Area}(PQR)$ is an integer?

Answer format: `Count/p,q,r`
for the smallest ratio

You are given: `6/2,3,2` for the triangle $(3, 4, 5)$.

[My timing: 10 sec]

Answer f8c39b511d329baba9514342cbc3de17

295 The Hurwitz equation

2016-01-01 17:36:37 **by Philippe_57721** **8 xp** **Math**

The Hurwitz equation is the following diophantine equation. For fixed $n, k > 0$:

$$x_1^2 + x_2^2 + \cdots + x_n^2 = kx_1x_2\cdots x_n \quad \text{for } k, n \in \mathbb{N}^*$$

It can be proved that when this equation has solutions, then $k \leq n$.

Can you find all the k such as this equation has solution for $n = 61$?

Answer format: comma delimited list of k_s

You are given: `1,4,5,13,23` for $n = 23$.

[My timing: 90 sec]

PS: There are much faster solutions than mine …

Answer 5415465a1e6996d5d66aea6041a390a0

296 Farey polygons I

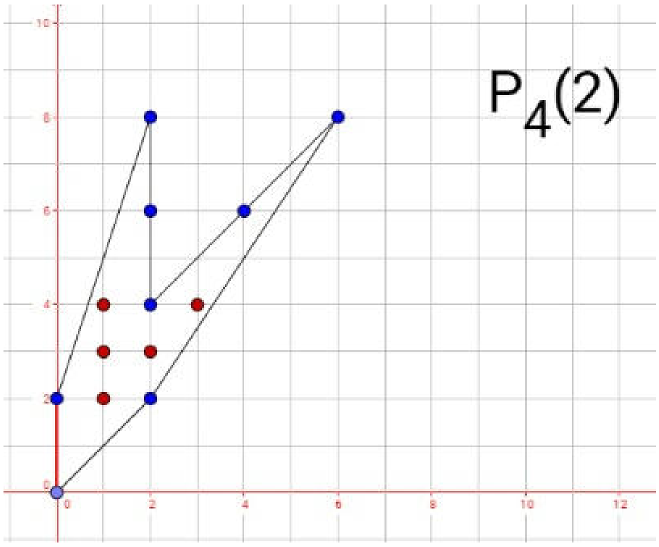
2016-01-08 16:15:50 | **by Philippe_57721** | **7 xp** | **Programming**

Consider the Farey sequence of order n.

For instance $F_4 = \{\frac{0}{0}, \frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1}\}$.

We define $F_n(z)$ by applying a zoom z i.e. multiplying each term by z.

We consider each term of the sequence $F_n(z)$ as a point and join these points respecting the order. We obtain the Farey polygon $P_n(z)$ of order n and zoom z



$P_4(2)$ contains 5 points in its interior.

Find the first Farey polygon $P_n(2)$ whose inside contains (strict) more than 2000 lattice points.

Answer format: `n,count`

[My timing: 40 sec]

Answer 387a2f6e02eef5869aa07d3ac19dae8f

297 Farey polygons II

2016-01-15 12:14:34 **by Philippe_57721** **6 xp** **Math**

What is the area of the Farey polygon (See problem 296) of order 10000000 and zoom 1?

You are given: 1522 for the Farey polygon of order 100.

[My timing: 40 sec]

Answer 57282e01a0afb8ce024e0777bb34b8d4

298 Decode it 2 (javascript)

2015-12-28 07:42:40 **by sinan** **8 xp** **Crypto**

```
4pxLgKsp+pVENasG+apWefzl9Ko/RbD7vGgUTbz7wm8UTb3lt5xNiOA3r5hS
NOc0ApxRd+w48FdHefLu`
```

Enter the key to decode it.

Key = 12345678 Decode it

```
Key=12345678



Not a key!
```

The JavaScript code for this problem is as follows:

```
function func()
{
     var keyStr = "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz0123456789+/=";
    var Key=parseInt(myform.myinput.value);
    var encodedText = document.getElementById('encoded_text').innerHTML;
    var outputDiv = document.getElementById("outputDiv");
    outputDiv.innerHTML = "<pre\>";
    if (Key<=0) {
        alert("Should be >0");
    } else {
        var arr=new Array();
        var i=0, keylen=0, s=Key, cd=0;
        while (s>0) {
            arr[keylen++]=s%10;
            s=Math.floor(s/10);
        }
        if(keylen!=8) {
            alert("key should be 8 digits long");
        } else {
                   outputDiv.innerHTML += "<font size=+2 \><b\>Key="+Key+"</b\></font\><br\><br\>";
            var inx=keylen;
            while (1) {
                for (i=s=0; i<keylen; ++i) s+=arr[i];
                if (--inx<0) inx+=keylen;
                arr[inx]=s;
                if (s==Key) {
                    outputDiv.innerHTML += "Decoded: ";
                    var j=keylen, k=0;
                    for(i=0; i<encodedText.length; ++i) {
                        var inx=keyStr.indexOf(encodedText.charAt(i));
                        if (inx<0) continue;
```

```
                         if(--j<0)
                             j+=keylen;
                         inx+=arr[j]&63;
                         if (inx>=64)
                             inx-=64;
                         outputDiv.innerHTML += keyStr.charAt(inx);
                         if(i>0 && (i&63)==0)
                             outputDiv.innerHTML += "<br\>";
                         cd+=++k*keyStr.charCodeAt(inx);
                     }
                     if (cd==281077)
                       outputDiv.innerHTML += "<br\><br\><font color=\"green\">Decoded
successfully.</font><br\>";
                     else
                        outputDiv.innerHTML += "<br\><br\><font color=\"red\">Decoded
something but it doesn't seem to be OK.</font><br\>";
                     break;
                 } else if (s>Key) {
                     outputDiv.innerHTML += "<br\><br\><font color=\"red\">Not a key!
</font><br\>";
                     break;
                 }
             }
         }
         outputDiv.innerHTML += "<br\>";
     }
     outputDiv.innerHTML += "</pre\>";
}
```

Answer c5a4591951353e248873e43f6ee020a6

299 a*b=c*d where a>b and c<d

2015-12-24 08:48:05	by sinan	10 xp	Sequence

```
1,3,6,1,6,21,82,9,81,....,?
```

Note: **This is partly a stegano challenge. And think of the title as a hint.**

Answer e068e1c49217cecc3644bebc48900512

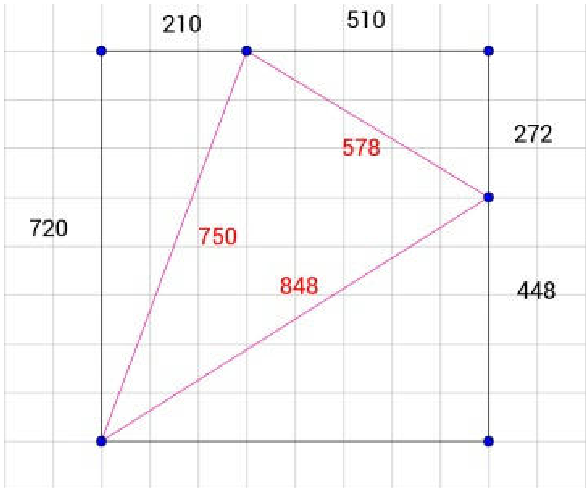
300 Squares dissection

2016-01-22 10:58:37 **by Philippe_57721** **8 xp** **Math**

Consider the square with side $= 720$.

We can dissect it in 3 Pythagorean triangles:

- $(720, 210, 750)$
- $(510, 272, 578)$
- $(448, 720, 848)$



We represent this dissection as: $c_1, c_2, c_3 = 720, 210, 272$ ($c_2 \leq c_1/2$).

How many squares with integer sides ≤ 20000 can be dissected this way?

Anser format: Count, $\sum c_1, \sum c_2, \sum c_3$

You are given: `3,2040,1035,569` for a threshold of 1000.

[My timing: 24 sec]

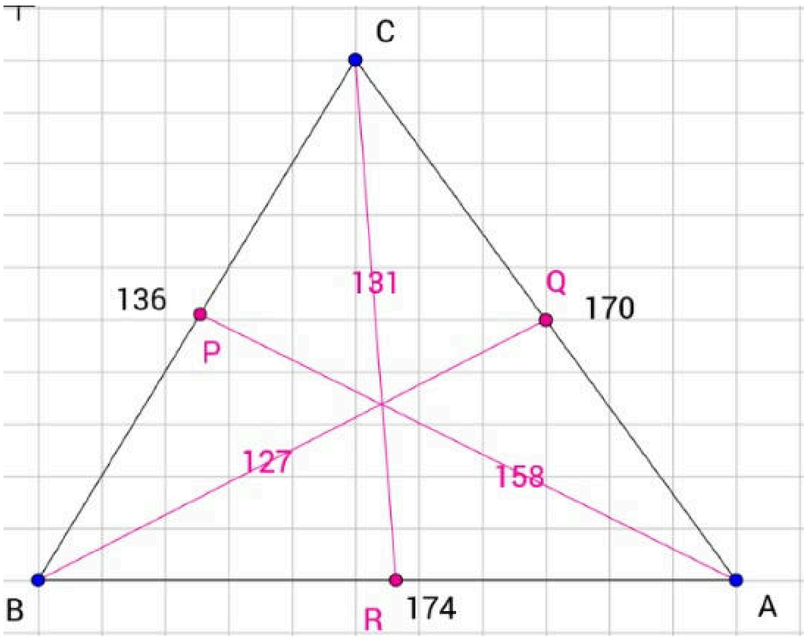
Answer 64917581018c373c22db086f1e1df652

301 Integer medians

2016-02-05 11:59:29 **by Philippe_57721** **7 xp** **Programming**

The following integer triangle with sides 136, 170, 174 has medians with integer length:

- $AP = 158$
- $BQ = 131$
- $CR = 127$



Find all the integer triangles with perimeter ≤ 10000 having integer medians. (We consider only primitive triangles: if (a, b, c) is a solution so is $(k \cdot a, k \cdot b, k \cdot c)$. We count only (a, b, c).)

Anser format: Count, $\sum a, \sum b, \sum c$
$(a, b, c$ being the sides of the triangles)

You are given: `2,390,432,490` for a threshold of 1000.

[My timing: 80 sec]

Answer 9c270d2df827a3f6bf5acb9d32682b1d

302 A 5-way recurrence

2015-12-30 07:20:29 **by sinan** **6 xp** **Math**

$$\begin{aligned}
a_{n+1} &= 1 \cdot a_n - 1 \cdot b_n + 2 \cdot c_n - 3 \cdot d_n + 5 \cdot e_n - 8 \\
b_{n+1} &= 1 \cdot b_n - 2 \cdot c_n + 3 \cdot d_n - 5 \cdot e_n + 8 \cdot a_n - 13 \\
c_{n+1} &= 2 \cdot c_n - 3 \cdot d_n + 5 \cdot e_n - 8 \cdot a_n + 13 \cdot b_n - 21 \\
d_{n+1} &= 3 \cdot d_n - 5 \cdot e_n + 8 \cdot a_n - 13 \cdot b_n + 21 \cdot c_n - 34 \\
e_{n+1} &= 5 \cdot e_n - 8 \cdot a_n + 13 \cdot b_n - 21 \cdot c_n + 34 \cdot d_n - 55 \\
a_0 &= b_0 = c_0 = d_0 = e_0 = 1
\end{aligned}$$

What is n-th solution for $n = 3^{4^5}$?

Answer format: $a_n\%M, b_n\%M, c_n\%M, d_n\%M, e_n\%M$
where $M = 10000019$

Note: **The answer should consist of positive numbers.**

Answer 76fe6137c46092422a7a2d45b3116ede

303 The Pell-Fermat equation

2016-02-19 11:01:53 **by Philippe_57721** **5 xp** **Math**

It is well known that the Pell-Fermat equation $x^2 - n \cdot y^2 = 1$ (**1**) has always solutions when n is not square. But for the symetrical equation $x^2 - n \cdot y^2 = -1$ (**2**), solutions are much rare.

For instance, if $n \leq 100$, this equation has solutions for only 20 values.

For how many $n \leq 5000000$, has the equation (**2**) solutions?

[**My timing: 70 sec**]

Answer **ced88747c71b6fca232e90081938299b**

304 Special Order I

2016-02-12 10:56:45 **by Philippe_57721** **6 xp** **Programming**

Consider the sequence of characters `ABCD`, each character being taken twice.

It is possible to order this sequence in such a way that:

- There is 1 character between 2 `A`s
- There are 2 characters between 2 `B`s
- There are 3 characters between 2 `C`s
- There are 4 characters between 2 `D`s

$$\mathtt{B}\underbrace{\mathtt{CD}}_{2}\mathtt{BA}\underbrace{\mathtt{C}}_{1}\mathtt{AD}$$

$$\mathtt{BC}\overbrace{\mathtt{DBA}}^{3}\mathtt{CAD}$$

$$\mathtt{BCD}\overbrace{\mathtt{BACA}}^{4}\mathtt{D}$$

Find the last such order in lexicographic order for the sequence `ABCDEFGHIJKLMNOPQRSTUVWXYZ_` (each character is taken twice).

[My timing: < 1 sec]

Answer 1dcb1078cfa7e70a6cd2225bc7c4eba5

305 Sum of divisors

2016-03-25 13:58:45	by Philippe_57721	5 xp	Math

$\sigma(n)$ is the function sum of divisors of n.

Find the digital sum of $\sigma(100000!)$.

You are given: digital sum of $\sigma(1000!) = 11349$.

[My timing: 15 sec]

Answer 2ed9a6c9f2bbd455d7a0ae2dee0e0a11

306 Golden Rod

2016-01-12 15:32:50	by elasolova	6 xp	Programming

PerfectStone Co. is an excavation company that is specialized in mining gold nodes. An unusual thing happened as they excavated a golden rod. On top of that, this stunning rod is really a long one! Now, they want to cut the rod in pieces and sell them to companies interested.

The length of the rod is 1000-units and they have to cut the rod into pieces of integer-sized units and sell all of them. They have given you a log that has the prices listed for each associated integer-size (prices for 1-unit to 1000-units).

All they ask from you is to maximize the revenue and supply here the best revenue attainable.

Answer f614214a11f8ffbd40bd0f4f79e1fb58

307 Special Order II

2016-02-12 10:57:03 **by Philippe_57721** **10 xp** **Programming**

The definitions are the same as in problem 304.

Find the **first** such order in lexicographic order for the sequence `ABCDEFGHIJKLMNOPQRST`.

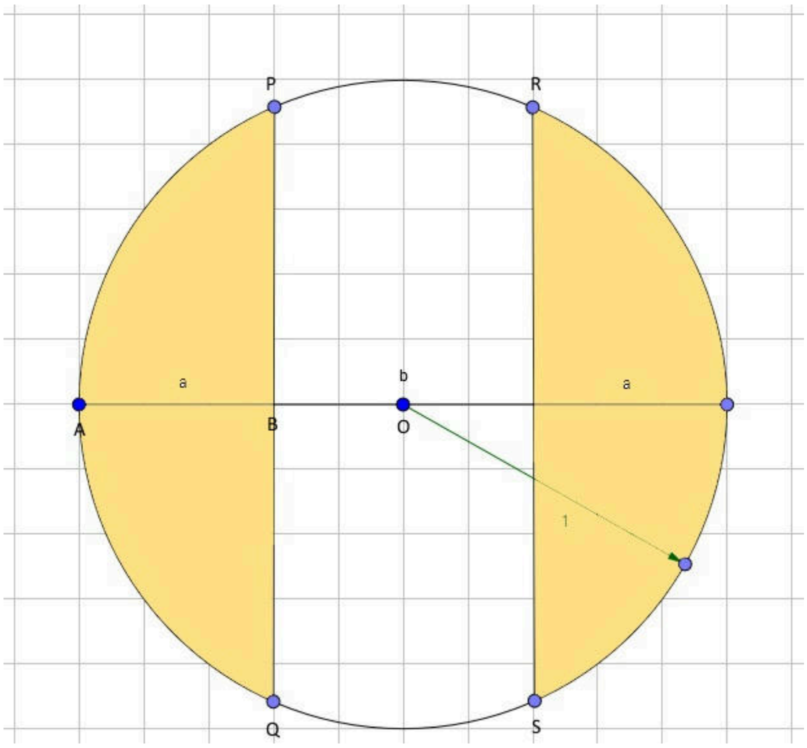
[My timing: 60 sec]

Answer **d1d3b9b0cd179edb7f7b739c63c9be7b**

308 Cutting the pie

2016-02-26 11:10:54 **by Philippe_57721** **6 xp** **Math**

A circular pie of radius 1 unit is divided in 3 equal (same area) parts by 2 lines perpendicular to its diameter.



What is the length of segment AB?

Give the result rounded to 10 places after the decimal point.

Answer 49ffe4f8f675fd4a096dbeccab392d70

309 Googolmania

2016-03-04 11:11:13 **by Philippe_57721** **8 xp** **Math**

Define $f(n)$ = the number of 0s before the last 1 in the binary representation of n. We have
$100 = 11\mathbf{00}100_2 \Rightarrow f(100) = 2$
$1000 = 11111\mathbf{0}1000_2 \Rightarrow f(1000) = 1$
$10000 = 100111\mathbf{000}10000_2 \Rightarrow f(10000) = 3$

Let $\gamma = 10^{100}$ (A googol).

Find $n_1 = f(10^\gamma)$ and $n_2 = f(10^{8\times\gamma})$.

Answer format: n_1, n_2

[My timing: instant]

Answer **7f9811515e64b910163871b31d398b83**

310 Nim operations

2016-01-26 11:59:04 **by sinan** **10 xp** **Math**

In mathematics, the nimbers, also called Grundy numbers, are introduced in combinatorial game theory, where they are defined as the values of nim heaps. They arise in a much larger class of games because of the Sprague-Grundy theorem. The nimbers are the ordinal numbers endowed with a new nimber addition and nimber multiplication, which are distinct from ordinal addition and ordinal multiplication. Visit the following page for more info on nimber addition and nimber multiplication:
Nimber - Wikipedia, the free encyclopedia

We also define Nimber square root such that it returns a number which, when the nimber multiplication applied to itself, returns the original number:
$s = \text{Nim\_sqrt}(n)$ such that $n = \text{Nim\_prod}(s, s)$.

Similarly Nimber inverse of a number is the one when the nimber multiplication is applied, returns 1:
$1 = \text{Nim\_prod}(n, \text{Nim\_inv}(n))$.

Evaluate the following using the above mentioned operations from $n =$ `0x100000` (hex) to `0x110000` (inclusive) and find the sum of the results.

$$\sqrt{\frac{n}{n + \sqrt{n}}}$$

Answer format: number
(in decimal)

[My timing: 6 s]

Answer ec8e8abb6f3a42615ebd135be47319f2

311 Nim equations

2016-01-26 11:59:24	by sinan	8 xp	Math

See problem 310 on Nim operations.

Find the 2 roots of the following equations where Nim operations are used only:

$1881x^2 + 1923x + 1938 = 0 \quad \text{where roots: } x_1 < 2^{13} < x_2$
$6273x^2 + 2935220086x + 2831770815 = 0 \quad \text{where roots: } x_3 < 2^{16} < x_4$
$6273x^2 + 12997147727796204837x + 11460394964710426743 = 0 \quad \text{where roots: } x_5 < 2^{24} < x_6$

Answer format: $x_1, x_2, x_3, x_4, x_5, x_6$

[My timing: < 15 s]

Answer c2241de598e8b366943fae029ac7e690

312 Laser beam

2016-03-11 11:05:49 **by Philippe_57721** **6 xp** **Math**

A rectangular box made of glass has the following dimensions: $1313 \times 1547 \times 1729$.

It is filled with transparent cubes of side 1.

A laser beam enters by the lower corner of the box $(0, 0, 0)$ and leaves by the upper corner $(1313, 1547, 1729)$.

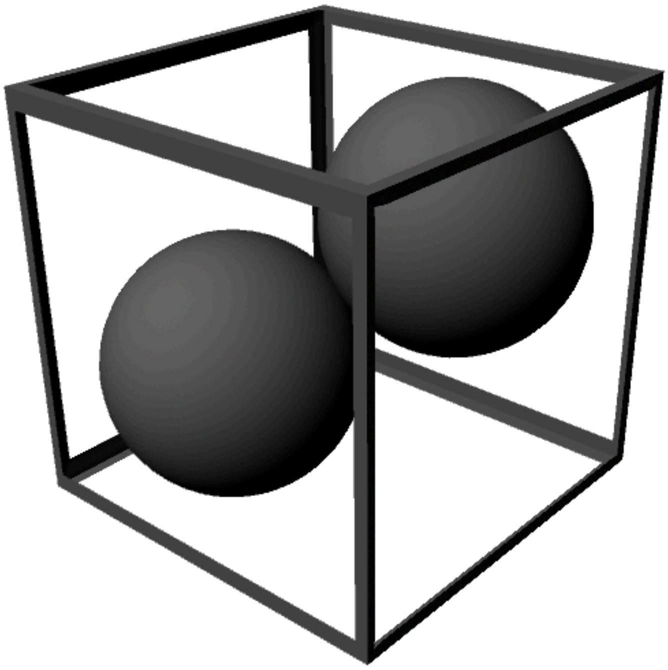
How many cubes does the beam cross?

Answer 640848630ce4c259085346b1a263c1bd

313 2-Sphere Packing

2016-03-28 09:15:41	**by elasolova**	**6 xp**	**Math**

Two equal sized spheres will be placed inside a unit-cube. What will be the maximum possible radius for spheres assuming that they are rigid bodies?



Round answer to 6 decimal places.

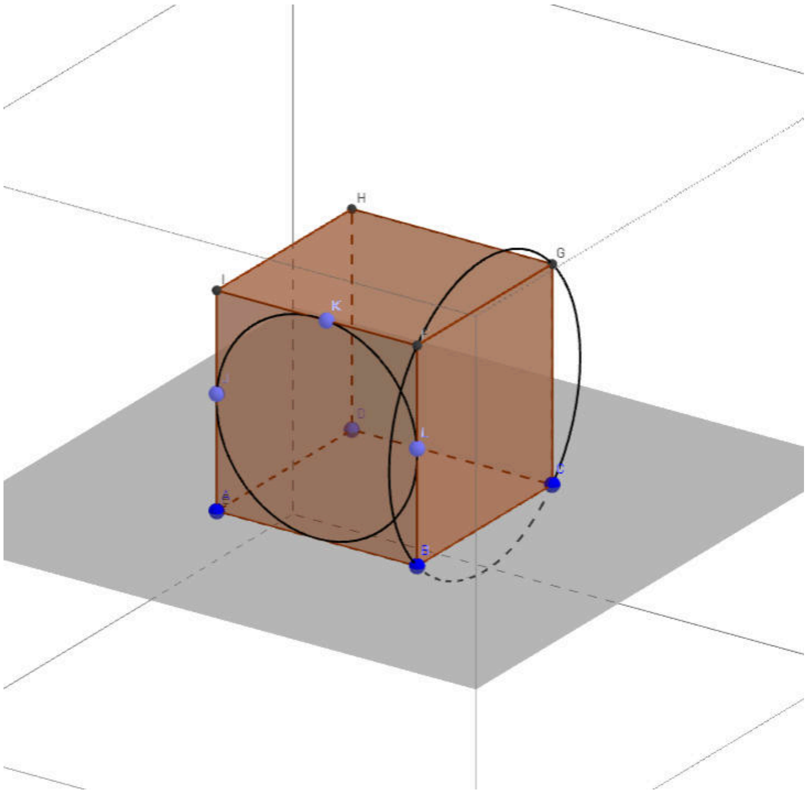
Note: Visualizing it will be helpful.

Answer 35352441e8c2ee92121e9a7feeeef697

314 Circles distance II

2016-06-03 10:04:54 **by Philippe_57721** **10 xp** **Math**

In a cube of side 1 unit, on one face we draw a circle inscribed in that face. On one adjacent face, we draw the circumscribed circle to that face.



Find the minimum distance between these 2 circles.

The answer is an algebric number.

Answer format: the minimal polynomial for this algebric number ..., a_3, a_2, a_1, a_0
where $P(x) = \cdots + a_3 \cdot x^3 + a_2 \cdot x_2 + a_1 \cdot x + a_0$ is the minimal polynomial

Example: `1,-1,-1` if the answer would be the golden ratio ϕ.

Answer 056cd160ef8ccf1fc276b73127e92719

315 Heronian numbers

2016-03-18 11:27:29 **by Philippe_57721** **7 xp** **Programming**

We say that an integer n is an Heronian number if it's the area of some Heronian triangle.

For instance, the following numbers are Heronian:
$10080 = \Delta(1224, 1153, 73)$
$71064 = \Delta(925, 831, 188)$

Find the first 2 Heronian numbers > 123456789.

Answer format: `n1,n2`

[My timing: 29 sec]

Answer **2323680c5e2eb4f998dc3508aa6409be**

316 Euler-constant like

2017-04-28 06:50:57 **by Philippe_57721** **13 xp** **Math**

What is $\displaystyle\sum_{0<1+4\times k\leq n} \frac{1}{1+4\times k} - \frac{\log(n)}{4}$ when $n \to \infty$.

Give your answer with 30 digits after the decimal point.

[My timing: instant]

Hint: there is a closed form.

Answer 72f8b60e89e21975d78e56a457420ec5

317 Heronian triangles in a range

2016-03-18 11:48:45	by sinan	8 xp	Math

How many (not necessarily primitive) Heronian triangles with sides a, b and c, and area in the range $[10000000, 10001000]$ are there?

Answer format: count, $\sum a_i, \sum b_i, \sum c_i$
where $a_i \leq b_i \leq c_i$

Example: `26,203,431,520` in the range $[0, 100]$.

[My timing: < 1 m]

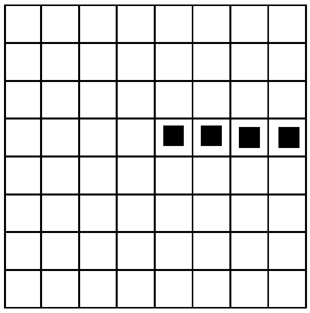
Answer 888ada1102c30bba68ea56a2260014bd

318 Cut it

2016-04-26 08:50:08	by sinan	10 xp	Programming

Cut the following square through the grid lines into 4 pieces in such a way that:

- each piece has one black box
- each piece is of similar shape by some rotation

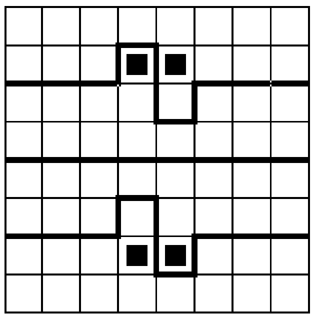


Each piece is to be identified as a bitmap. Bits are numbered from 63 to 0 starting from top line to bottom.

```
b63 b62 ... b56
b55 b54 ... b48
...
b7  b6  ... b0
```

After the cut, set the cells of a particular piece to 1 and others to 0 and get a 64-bit number as described above.

For example we would have the following numbers (in uppercase hex) for the following:



```
FFEF080000000000
0010F7FF00000000
00000000FFEF0800
000000000010F7FF
```

A cut like this is identified by **the smallest** number. So it would be `10F7FF` (leading zeroes are ignored) for the above example.

Answer format: `N1,N2,...`
(**the smallest** numbers for the appropriate cuts in uppercase hex and in ascending order)

Answer 9ebf2e554f39c3f4b516da14087a1485

319 The longest needle

2016-09-27 07:25:25	by sinan	10 xp	Math

We define a needle-like primitive triangle as follows:

- It has 3 vertices on lattice points
- It has no other lattice points inside or onside

Find such a triangle with the maximum possible perimeter in $x^2 + y^2 = R^2$ circle for $R = 123456789$.

Answer format: $a^2 + b^2 + c^2$
where a, b, c are the sides of the triangle

Example: 136 if $R = 5$ for instance for the triangle with vertices $(-4, -2), (1, 1), (4, 3)$.

[My timing: < 40 s]

Answer 03b6d2d60d8e980b9be06c4bbabf1a95

320 Bonus for p315

2016-09-27 07:25:50 by sinan **6 xp** Math

[1] We define a needle-like primitive **convex** quadrilateral as follows:

- It has 4 vertices on lattice points
- It has no other lattice points inside or onside

Find such a quadrilateral with the maximum possible perimeter in $x^2 + y^2 = R^2$ circle for $R = 10^8$?

Answer format: x_1, y_1, x_2, y_2
where $|x_i|$ >= $|y_i|$ and x_1 < x_2 (See the example below)

Example: `3,2,5,3` if $R = 5$.

There are more than one solutions. Enter the two vectors that identify them. For example $A(-4,-3), B(-1,-1), C(4,2), D(1,0)$ is a solution then the two vectors are $AB = DC = [3,2]$ and $BC = AD = [5,3]$.

Similarly, for $A(-4,-2), B(-1,0), C(4,3), D(1,1)$, $AB = DC = [3,2]$ and $BC = AD = [5,3]$.

[My timing: 30 s]

Answer 3a4d8c70768a2398928c973e3c97ab3e

[1] Actually Problem 319.

321 Minimum distance

2016-04-01 11:59:43 **by Philippe_57721** **12 xp** **Math**

If for the real a, b, c, d we have:
$a^2 + 4b^2 = 4$ and $cd = 4$.

what the minimum of $(a - c)^2 + (b - d)^2$?

Give your answer rounded to 20 digits after the decimal point.

Answer **27d631f41bc41931ebaa7306d4f909e2**

322 Imaginary tower

2016-04-15 11:37:03 **by Philippe_57721** **5 xp** **Math**

if $i^2 = -1$, what is the limit

$$L = i^{i^{i^{i^{\cdot^{\cdot}}}}}$$

Answer format: a, b
where $L = a + bi$, a and b rounded to 20 digits after the decimal point

Answer 30057adcc499ce74c39086419ca293dd

323 The biggest square

2016-04-02 06:57:39	by sinan	8 xp	Math

Consider the squares in $x^2 + y^2 = R^2$ circle with vertices on lattice points. The one with the biggest area would be $(R, 0), (0, R), (-R, 0), (0, -R)$.

Find the ones with the second biggest if $R = 87654321$.

Answer format: $\sum x_i, \sum y_i$, area
where $y_i > x_i > 0$ and are x, y coordinates of the one of the vertices.

Example: `2,4,40` if $R = 5$.

There are 2 squares with area = 40:
$(2, 4), (-4, 2), (-2, -4), (4, -2)$
$(4, 2), (-2, 4), (-4, -2), (2, -4)$

[My timing: < 1 s]

Answer dc4dfbd4b2c3383f30404467570035bc

324 Consecutive Zeros Mobius

2016-04-08 11:08:35 **by Philippe_57721** **8 xp** **Math**

It can easily be seen that there cannot be more than 3 consecutive integers such as $\mu(n_i) = \pm 1$, $\mu(n)$ being the Möbius function.

However, we can find arbitrary long sequences of consecutive integers with a Möbius value of 0.

Find 9 consecutive integers $n+1, n+2, \ldots, n+9$ such as $\mu(n+i) = 0$ (Each of them is divisible by a distinct prime square, the primes are the smallest possible).

Answer format: n

[My timing: 10 sec]

Answer **c31f1417d855e6e38dbc492f1fd83f87**

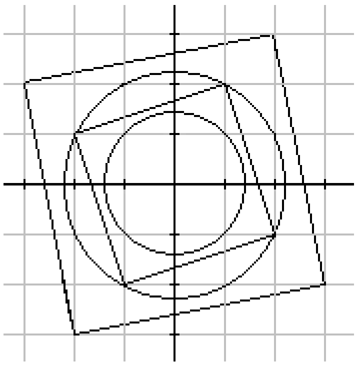
325 Homocentric circles

2016-04-16 08:01:54 **by sinan** **8 xp** **Math**

1. Set $R^2 = 2$.
2. Draw $x^2 + y^2 = R^2$ circle.
3. Next find the smallest square with vertices on lattice points such that it inscribes the circle but doesn't touch it.
4. Set $R^2 = a^2 + b^2$ where a, b are the coordinates of the one of any 4 vertices.
5. Go to step 2.

At the second iteration we would have $x^2 + y^2 = 5$ $(1^2 + 2^2 = 5)$.

At the third iteration we would have $x^2 + y^2 = 13$ $(2^2 + 3^2 = 13)$.



What would R^2 be equal to at the 50-th iteration?

Answer format: `Number`

Example: 13 for the third iteration.

[My timing: < 1 s]

Answer 16438933ae5e512b2fa7952099e964fe

326 Minimise it

2016-05-09 09:53:44	by sinan	8 xp	Math

Let A, B, C be the three lattice points inside $x^2 + y^2 = R^2$ circle such that the ratio $(c - a)/(a + b + c)$, where $a = |BC|, b = |AC|, c = |AB|$ and $a \leq b \leq c$ is minimised.

Solve it for $R = 10^9$.

Answer format: a^2, b^2, c^2
where $\gcd(a^2, b^2, c^2) = 1$

Example: `12544,12545,12545` for $R = 100$ with ratio $= 1.32859467950598133111134389 39\text{e}{-5}$, $A(57, 6), B(-40, -50), C(-40, 62)$.

[My timing: 200 ms]

Answer 04c3f33684bab1aa1a7283bf735bab08

327 Around the poles

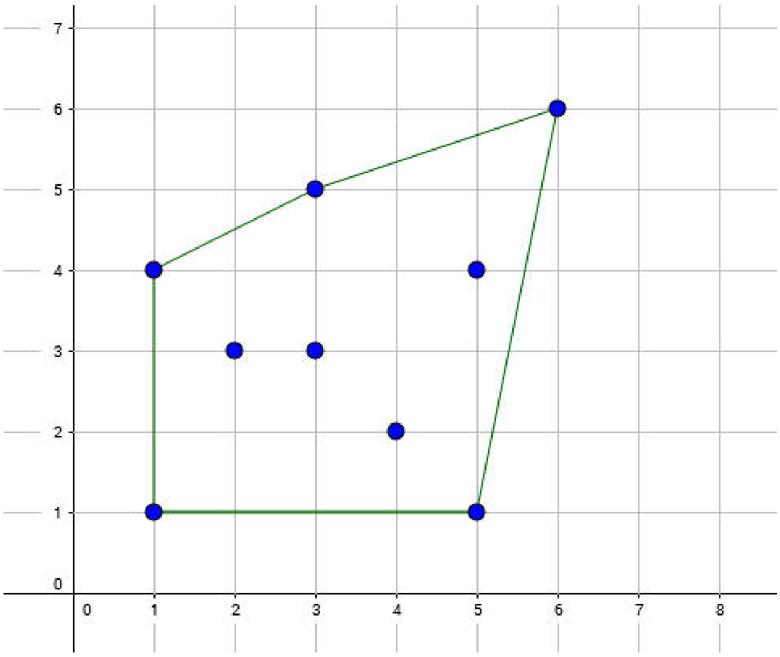
2016-05-13 06:08:03 **by Philippe_57721** **9 xp** **Programming**

Poles are disposed on the lattice plane. A rubber band surrounds them.

For instance, consider poles placed at the following points:

- $(1, 1)$
- $(1, 4)$
- $(2, 3)$
- $(3, 5)$
- $(3, 3)$
- $(4, 2)$
- $(5, 1)$
- $(5, 4)$
- $(6, 6)$

If we place a rubber band around these points, we obtain a polygon of area 18.



Now consider the following 1000 points:

- $P_1 = (141, 592)$
- $P_2 = (653, 589)$
- $P_3 = (793, 238)$
- $P_4 = (462, 643)$
- ...
- $P_{999} = (845, 68)$
- $P_{1000} = (772, 460)$

The coordinates are made of the digits of the decimal part of PI by group of 3.

What is the area of the corresponding polygon?

You are given: `830303` for first 100 points.

[My timing: < 1 sec]

Answer **f1f7b7ae193f0b2414b9572a823e04e2**

328 Lumberjacks

2016-04-22 11:13:57	by Philippe_57721	6 xp	Programming

A group of lumberjacks are gathered in a camp.

Each day, in turn, one of the lumberjacks stays in the camp to cook, while the others go to work in 2 teams.
- The sum of the ages of the 1st team equals the sum of the ages of the 2nd team.
- The ages of the lumberjacks are all different and are prime numbers.

What is the smallest possible number of lumberjacks and their ages (first solution in lexicographic order).

Answer format: `a1,a2,...,an`
in ascending order

[My timing: 2 sec]

Answer a70ac8c92c88007c2c8363161ac81166

329 Energy Potion

2016-06-18 12:40:50	**by elasolova**	**6 xp**	**Math**

A manufacturer has decided to produce an energy potion, to be named as Getafix. They are thinking about choosing a conical shaped can, for Getafix to be distinguishable from other products. Simply, they will cut the top of a regular cone.

So, they seek some advice on how to manufacture this product. Here is the list of requirements:
- They will create 1 liter beverages.
- The radius of bottom will be twice the top.
- Amount of material used for can will be as least as possible.

They want you to submit optimal height (`h`), bottom radius (`rb`), top radius (`rt`) all in centimeters and rounded to 3 decimal places.

Answer format: `h,rb,rt`

Answer **a670dbed665ad042e72c1761b43ff48c**

330 Graham Numbers

2016-04-29 11:44:07 **by Philippe_57721** **8 xp** **Math**

Let's define $a \uparrow [n]\, a = a \underbrace{\uparrow\uparrow \cdots \uparrow}_{n \text{ arrows}} a$ where $\uparrow$ represents the Knuth's arrow notation.

We define:

- $g_0 = 4$
- $g_1 = 3 \uparrow [g_0]\, 3$
- ...
- $g_{64} = 3 \uparrow [g_{63}]\, 3$

g_{64} is the notorious Graham's number.

The last digits of this humongous number are known:
$\cdots$03222348723967018485186439059104575627262464195387

We define $G(3) = g_{64}$ and the generalized Graham's number $G(p)$ where we replace all 3s with p.

What is the smallest prime number p such as G(p) contains the string '1415' in its last 100 digits.

Answer format: p,the last digits starting with '1415'

Example: `3,8485186439059104575627262464195387` if we ask for the smallest p such as $G(p)$ contains '8485'.

[My timing: 4 sec]

Answer 57019c7d4f097c5f4589d0abc44ef723

331 Minimal odds

2016-05-03 09:20:39 | **by sinan** | **8 xp** | **Math**

Let $x^2 - D \cdot y^2 = 4$ where D is not a square number and x, y are positive integers.

Find the minimal solution (the one with the smallest positive x, y pair) with **the biggest** odd y value for $D < 1000$.

Answer format: `x^2-D*y^2`

Example: `1523^2-61*195^2` for $D < 100$.

[**My timing: 4 s**]

Answer a919f85d3dc01eaf89e23638cd200e93

332 Heronian triples

2016-05-20 08:53:57	**by Philippe_57721**	**7 xp**	**Programming**

Consider the following triple of consecutive integers: $2701, 2702, 2703$.

It's the sides of an Heronian triangle.

$\triangle\,(2701, 2702, 2703) = 3161340$

Let's call it an Heronian triple.

Find the 100-th Heronian triple.

Answer format: the 1st element of the triple

You are given: 524173 for the 10-th element.

[My timing: tiny]

Answer 48a1f0ee05d35d1693397f67c44fd7a9

333 Buddy numbers

2016-05-06 15:21:49 **by Philippe_57721** **8 xp** **Programming**

Consider the couple of integers $(m, n) = (14, 224)$.

We have:
- $14 = 2^1 \times 7^1$
- $224 = 2^5 \times 7^1$
- $14 + 1 = 3^1 \times 5^1$
- $224 + 1 = 3^2 \times 5^2$

n, m have the same prime factors $\{2, 7\}$ and so have $n + 1, m + 1$ $\{3, 5\}$.

Let's call these pairs buddy numbers.

If $(14, 224)$ is the 3rd pair of buddy numbers, what is the 30-th?

[My timing: < 1 sec]

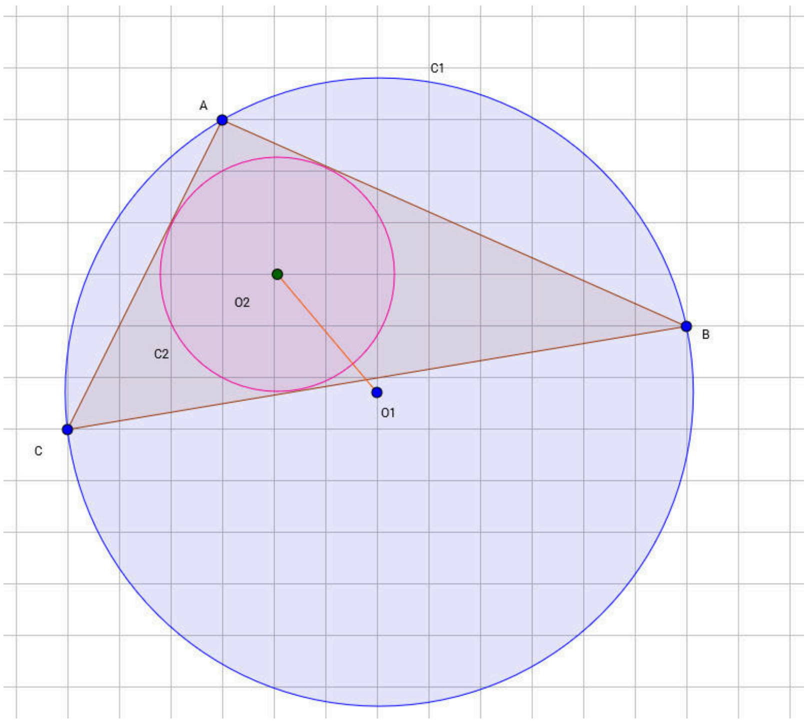
Note: Assume there exists only one pair of odd buddy numbers.

Answer **488907e4fdf16f31940b598d292faa80**

334 Circles distance I

2016-05-27 10:44:44 **by Philippe_57721** **9 xp** **Math**

In an **isosceles** triangle ABC, we draw the circumscribed circle C_1 with center O_1 and the inscribed circle C_2 with centre O_2.



If $AB = 520$, $AC = 520$ and $BC = 624$, the distance O_1O_2 is the integer 65.

How many **isosceles** integer triangles with perimeter $\leq 10^8$ are there such as the distance O_1O_2 is an integer.

Answer format: the count

Hint: you are given 469 for a perimeter of 10^4.

[My timing: < 90 sec]

Answer ea5dd3c19c9882175334ca7d9dd0df0d

335 Subtracting proper divisors

2016-06-10 11:12:52 **by Philippe_57721** **5 xp** **Programming**

Given an integer n, we apply the following process:
- search the largest proper divisor of n
- subtract this number from n
- repeat until we reach 1

Let $f(n)$ the number of steps before reaching 1.

Example with $n = 30$
- $n = 30 - 15$ ($15 =$ Largest proper divisor of 30)
- $n = 15 - 5$ ($5 =$ Largest proper divisor of 15)
- $n = 10 - 5$ ($5 =$ Largest proper divisor of 10)
- $n = 5 - 1$ ($1 =$ Largest proper divisor of 5)
- $n = 4 - 2$ ($2 =$ Largest proper divisor of 4)
- $n = 2 - 1$ ($1 =$ Largest proper divisor of 2)
- $n = 1$ Stop $\Rightarrow f(30) = 6$

Find $f(1000000!)$ ($1000000!$ i.e. $\text{Factorial}(1000000)$).

[My timing: 6 sec]

Answer **5cb25de9fefdf16838ab7f23ca010bf4**

336 Sum

2016-05-17 05:50:40	**by sinan**	**8 xp**	**Math**

Let $F(N) = \sum N/\text{GCD}(N, i)$ for $i = 1$ to N.

Find $F(12345678!) \bmod 10000004400000259$.

You are given $F(100!) \bmod 1000003 = 571693$.

[My timing: < 1 s]

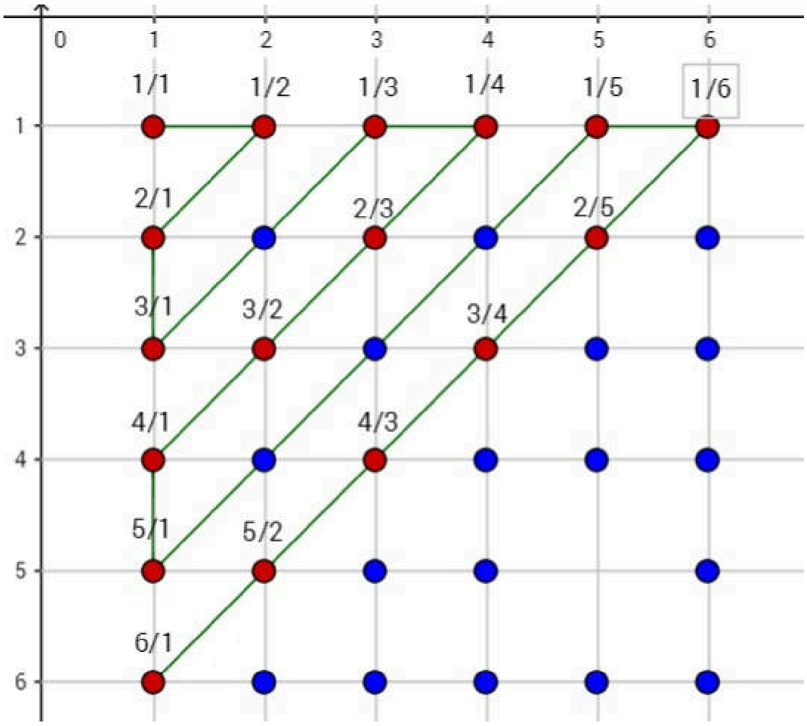
Answer 6968dde4c06b409cefc94f9609aab110

337 Diagonal Enumeration

2016-06-17 09:41:38 **by Philippe_57721** **10 xp** **Programming**

You certainly know of this nice proof of the equipotence between the set of integers and the set of rational numbers.

We consider an infinite grid numbered horizontally and vertically with integers:



We run through this grid according the above pattern and count each pair (i, j) where i and j are coprime.

1 - $\frac{1}{1} = 1$

2 - $\frac{1}{2}$

3 - $\frac{2}{1} = 2$

4 - $\frac{3}{1} = 3$

5 - $\frac{1}{3}$

6 - $\frac{1}{4}$

7 - $\frac{2}{3}$

8 - $\frac{3}{2}$

9 - $\frac{4}{3}$

10 - $\frac{5}{1} = 5$

11 - $\frac{1}{5}$

12 - $\frac{1}{6}$

13 - $\frac{2}{5}$

14 - $\frac{3}{4}$

15 - $\frac{4}{1} = 4$

16 - $\frac{5}{2}$

17 - $\frac{6}{1} = 6$

If we run through the upper diagonal matrix of a 6×6 grid, we get 17 rationals.

How many rationals can we count in the upper diagonal matrix of a $10^9 \times 10^9$ grid?

You are given: `303964542391` for a $10^6 \times 10^6$ grid.

[My timing: < 90 sec]

Answer **5269fa671014b44c63825572b41897b1**

338 Square divisors

2016-06-24 11:25:04 **by Philippe_57721** **6 xp** **Programming**

Among the divisors of 400, exactly 6 are perfect squares:
1, 4, 16, 25, 100, 400

How many divisors of $10^9!$ are perfect squares?

Answer format: the **20** last digits of this huge number before the trailing 0s

You are given: `54158086206534975488` for 1000!.

[My timing: 20 sec]

Beware of leading zeros

Answer d3dc6cab74b48e1b8b7d1f37831684e3

339 Multiplicative Order

2016-07-01 13:43:34	by Philippe_57721	7 xp	Programming

The multiplicative order $o(g, p)$ of a number g modulo p, (g and p coprime), is the smallest integer k such as $g^k = 1 \text{ modulo } p$.

For instance, $o(10, 73) = 8$.

For how many prime numbers $p < 4 \times 10^8$ the multiplicative order $o(10, p) < 100$?

Answer format: `count,sum`

Example: `55,11573` for a limit of 1000.

[My timing: 70 sec]

Answer 49090310921f45159058feca94bbf282

340 All steps

2016-05-23 14:41:52	by sinan	10 xp	Math

A number for which every pair of consecutive digits has a difference of one is called a step number. For example 234321 is a step number.

A pandigital number in a base B contains every digit from 0 to $B-1$ at least once.

Let $F(B, ND)$ denote the number of pandigital step numbers in base B **with *ND* or less digits**.

You are given:
$F(7, 100) \bmod 10^{20} = 17172910990334429 79$
$F(10, 1000) \bmod 10^{20} = 45956531342004698861$

What is $F(19, 1919^{19}) \bmod 10^{20}$?

[My timing: < 10 s]

Answer 83e729a4cc681269ca4118a25dd34d8a

341 Secret operation

2017-03-06 05:37:16 **by sinan** **6 xp** **Math**

Find the digits in the following multiplication:

```
      * * * * * * * * * * * *
                        * * *
      -----------------------
    * * * * * * * * * * * * *
  * * * * * * * * * * * * *
* * * * * * * * * * * * *
-----------------------------
* * * * * * * * * * * * * * *
```

Each star represents a prime number. There is only one solution.

Answer format: 999999999999*999=999999999999999

[My timing: < 1 m]

Answer 5294777382afdf0b31e4e85a352c2f8b

342 Alternate sums

2017-04-28 06:51:15	by Philippe_57721	4 xp	Math

How many different values can the expression $1 \pm 2 \pm 3 \pm \cdots \pm n$ take?

Give your answer for $n = 1000000$.

[My timing: instant]

Answer e0cf6533cdae20632b6d6ca82135b2e7

343 Sum of digits equation

2016-07-08 14:05:26 **by Philippe_57721** **10 xp** **Programming**

We define $\mathrm{DS}(n) = n + \mathrm{SOD}(n)$ where $\mathrm{SOD}(n)$ is the sum of the digits of n (See problem 263).

It can be proved that $10^{13} + 1$ is the smallest integer such as the equation $\mathrm{DS}(x) = n$ has 3 solutions:

$$10^{13} + 1 = \begin{cases} \mathrm{DS}(9999999999892) \\ \mathrm{DS}(9999999999901) \\ \mathrm{DS}(10000000000000) \end{cases}$$

It can be proved too that $n = 10^{2222222222224} + 10000000000002$ is the smallest integer with 6 solutions.

Obviously, if x is a solution we must have $x < n$. Thus, we can write a solution as $n - x_i$.

Find these solutions.

Answer format: $x_1, x_2, x_3, x_4, x_5, x_6$
x_i in ascending order

Example: `1,100,109` for $10^{13} + 1$.

[My timing: < 1 sec]

Thanks to **sinan** who helped me to improve this problem.

Answer 73ac859ae64bd8b46973058fdd90c03a

344 Unreachable numbers

2016-07-15 09:44:18 **by Philippe_57721** **7 xp** **Programming**

Using four numbers 4 and the 4 operations $+$, $-$, $*$, $/$ we can build all numbers up to 9.

- $1 = ((4+4)-4)/4$
- $2 = (4/(4+4))*4$
- $3 = ((4+4)+4)/4$
- $4 = ((4-4)*4)+4$
- $5 = ((4*4)+4)/4$
- $6 = ((4+4)/4)+4$
- $7 = (4-(4/4))+4$
- $8 = ((4+4)+4)-4$
- $9 = ((4/4)+4)+4$
- $12 = (4-(4/4))*4$

$\Rightarrow$ 10 is the first not reachable.

With four numbers 5

- $1 = ((5+5)-5)/5$
- $2 = (5/5)+(5/5)$
- $3 = ((5+5)+5)/5$
- $4 = ((5*5)-5)/5$
- $5 = ((5-5)*5)+5$
- $6 = ((5*5)+5)/5$
- $7 = ((5+5)/5)+5$
- $9 = (5-(5/5))+5$

$\Rightarrow$ 8 is the first not reachable.

What are the first unreachable numbers when we use eight 4 and eight 5?

Answer format: u_4, u_5

Example: `10,8`

[My timing: 40 sec]

Answer ad532c2f878b3a62cdecd541e93d304f

345 Fraction decomposition

2016-08-05 14:50:59 **by Philippe_57721** **8 xp** **Programming**

There is a smallest n for which there exists a set of distinct integers $S = \{s_1, s_2, \ldots, s_n\}$ such as $\frac{17}{670} = \left(1 - \frac{1}{s_1}\right) \times \left(1 - \frac{1}{s_2}\right) \times \cdots \times \left(1 - \frac{1}{s_n}\right)$.

Actually, for this least n the set S is unique.

You are given that S is composed of the union of no more than 4 subsets of consecutive integers.

Find S.

Answer format: 'the smallest element'-'the largest element' comma separated for each subset

For instance, if $S = \{2, 3, 4, 5, 11, 12, 13\} = \{2, 3, 4, 5\} \cup \{11, 12, 13\}$, the answer would be: `2-5,11-13`

[My timing: 2 sec]

Answer **50039ee71010d762c7b498078bcb59c9**

346 Consecutive divisors

2016-07-22 11:31:27 **by Philippe_57721** **4 xp** **Programming**

Find the smallest integer with 720 divisors and the longest sequence of consecutive divisors.

For instance, for 24 divisors, the answer is $360 = \underbrace{1, 2, 3, 4, 5, 6}, 8, \ldots$

Answer format: `number,first divisor of the sequence,last divisor of the sequence`

Example: `360,1,6`

[My timing: 8 sec]

Answer **3d009e2d61a747c3ff532995bc1d6d62**

347 Nonaic Huffman Coding

2016-07-26 11:30:33 **by elasolova** **8 xp** **Programming**

Huffman Coding is a lossless data compression method that is based on the entropy of bit patterns in the data.

Usually for strings, conventional 8-bit (0-255) hashing is used for frequencies preceding actual coding. So, these 8-bit chunks directly correspond to well-know characters (such as a, b, c, etc., more specifically ASCII encoding). So, for this problem assume ASCII encoding for characters.

This problem asks to use Huffman coding scheme on this text file, but using 9-bit chunks for frequencies and then actual coding. Also, for this problem you do not need to account for additional information to ensure decompression.

File is currently 68337 bytes. Submit the compressed file size below in bytes (where file **does not** include any information for decompression).

Answer **98778d65a66805d892f4ab4543b439a7**

348 Heegner decompositions

2016-12-02 07:59:25 **by Philippe_57721** **10 xp** **Math**

Consider the field $\mathbb{F}_d$ of numbers $a + b \times \sqrt{-d}$ where a, b, d are integers and d is a squarefree positive number.

For some d, it can be proved that the factorization of the elements of $\mathbb{F}_d$ is unique.

The surprising fact about these special values is that there are only 9 of them:
$\{1, 2, 3, 7, 11, 19, 43, 67, 163\}$

They are called the Heegner numbers.

Can you find the smallest prime number $p > 10^{50}$ such as p can be written $p = x^2 + h \cdot y^2$ for each Heegner number h.

Answer format: $p \bmod 10^{50}, \sum x, \sum y$

Example: `2372241,606424,233349` for $p > 10^{10}$

$$
\begin{aligned}
10002372241 &= 23220^2 + 1 \times 97279^2 \\
10002372241 &= 90979^2 + 2 \times 29370^2 \\
10002372241 &= 79729^2 + 3 \times 34860^2 \\
10002372241 &= 95703^2 + 7 \times 10976^2 \\
10002372241 &= 82865^2 + 11 \times 16884^2 \\
10002372241 &= 52279^2 + 19 \times 19560^2 \\
10002372241 &= 90313^2 + 43 \times 6552^2 \\
10002372241 &= 44273^2 + 67 \times 10956^2 \\
10002372241 &= 47063^2 + 163 \times 6912^2
\end{aligned}
$$

For the equation $x^2 + y^2 = p$, we assume $x < y$.

[My timing: < 15 sec]

Answer **b532d4918f05b9a8ee61b629a5a1b24c**

349 Binary Quadratic Representation

2016-10-28 07:31:27	by Philippe_57721	8 xp	Programming

It can be proved that a prime number of form $5n+1$ has a unique representation by the binary quadratic form $x^2+3xy+y^2$.

For instance $10000000711 = 14365^2 + 3 \times 14365 \times 79734 + 79734^2$.

Find this representation for the first 100 prime numbers $\equiv 1 \mod 5$ and $> 10^{50}$.

Answer format: $\sum x, \sum y$
where $x < y$

[My timing: 7 sec]

Answer e43a7311d68c82e778e10bd42231414c

350 Squarefree factorisations

2016-08-12 14:56:52	**by Philippe_57721**	**6 xp**	**Programming**

Consider the number 420.

It can be decomposed in product of 2 squarefree factors in 4 different ways:
2×210
6×70
10×42
14×30

Let $d(n) =$ number of pairs (x, y) such as x, y squarefree, $x \cdot y = n$ and $x \leq y$.

Find $\sum_{k=1}^{20000000} d(k)$.

[My timing: 45 sec]

Answer 978f7f539e8c8fee3019581d1d47340d

351 Multiplicative persistence

2016-08-26 11:37:38 **by Philippe_57721** **9 xp** **Programming**

Given a positive integer, we multiply all its digits, we obtain a new number. We repeat the process until a fixed point.

Example:
Starting with $n = 277777788888899$, we have the following sequence:
$277777788888899 \Rightarrow 4996238671872 \Rightarrow 438939648 \Rightarrow 4478976 \Rightarrow 338688 \Rightarrow 27648 \Rightarrow 2688 \Rightarrow 768 \Rightarrow 336 \Rightarrow 54 \Rightarrow 20 \Rightarrow 0$
In this case, we have a chain of length 11.

This value is called the multiplicative persistence of n: $P(277777788888899) = 11$.

277777788888899 is actually the smallest integer with a multiplicative persistence of 11.

There are 5 15-digits numbers with a multiplicative persistenceof 11:
277777788888899
367777778888889
447777778888899
466777777888889
666677777788888

We only consider number in their canonical representation: digits in ascending order (as any permutation of a given number has the same multiplicative persistence as the original).

It's an open problem to prove or disprove that no integer has a multiplicative persistence greater than 11. This conjecture has been checked until 10^{500}.

How many 25-digits numbers with a multiplicative persistence of 11 are there?

Answer format: `count,sum`

Example: `5,2226788902344464` for 15-digits numbers.

[My timing: 60 sec]

Answer 2621aafb74aad71f8c8ac281cced4e9c

352 Periodic Kaprekar Numbers

2016-10-07 05:57:51 **by Philippe_57721** **8 xp** **Math**

A n-digits Kaprekar number, in base 10, is an integer such as its square can be split in 2 parts that add up to this number.

For instance:

$$324324324324^2 = \underbrace{105186267348}\underbrace{219138056976}$$
$$\text{and } 105186267348 + 219138056976 = 324324324324$$

We say that a string S is periodic if there is a substring T of S such as $S = T \cdots T$ where the number of repetitions of T is ≥ 2.

How many 68-digits periodic Kaprekar numbers are there?

Answer format: `count,sum`

Example: `8,41153193346` for 10-digits Kaprekar numbers $(1111111111, 2020202020, 3888938889, 4132841328, 5000050000, 5243952439, 9756097560, 9999999999)$

[My timing: < 1 sec]

Answer **f6b33827081dd1bc6ef7201d537ea97a**

353 Two ways decomposition

2017-01-13 08:39:56	by Philippe_57721	10 xp	Programming

Euler found that $635318657 = 59^4 + 158^4 = 133^4 + 134^4$.

And it was proved in 2004 that it's actually the smallest integer that can be expressed in two different ways as a sum of two 4th powers.

What is the smallest integer that can be written in 2 different ways as a sum of a 4th and a 5th power?
$n = a_1^4 + b_1^5 = a_2^4 + b_2^5$ where a_i and b_i are not perfect powers.

Answer format: n, a_1, b_1, a_2, b_2
with $a_1 < a_2$

[My timing: 45 sec]

Answer 73b74b5b38741289f774f3dab908a86a

354 In or out?

2016-09-09 06:53:30	by Philippe_57721	6 xp	Programming

Let the following pseudo-random generator:

```
static long seed = 641;
static long FastRandom()
{
    seed = (214013 * seed + 2531011) % 2147483647;
    return seed;
}
```

We consider the first 60000 values of this generator modulo 1000 as the coordinates of 10000 triangles:
$T_1 = (344, 302)(678, 941)(488, 674)$
$T_2 = (107, 545)(964, 568)(808, 239)$
...
$T_{10000} = (862, 679)(571, 263)(391, 426)$

How many times is the point $(499, 499)$ inside one of these triangles?

[My timing: < 1 sec]

Answer d388d6ce9a2b53aa22a0c7dc0cdc78af

355 Closed form

2016-09-30 09:58:50	**by Philippe_57721**	**5 xp**	**Math**

Let

$$\xi = \sqrt[16]{4870847 - \cfrac{1}{4870847 - \cfrac{1}{4870847 - \cfrac{1}{4870847 - \cdots}}}}$$

ξ can be written $\frac{a+\sqrt{b}}{c}$ for some coprime integers a, b, c.

Can you find them?

Answer format: `a,b,c`

Answer 2a3d4616e789219cfd2ccc082f212982

356 Gaussian factorization

2017-01-20 11:06:39	**by Philippe_57721**	**8 xp**	**Math**

Consider the Gaussian integer $\gamma = 14159 + 71828i$.

$\gamma = (0 + i) \times (1 + 2i) \times (2 + 3i) \times (5 - 8i) \times (283 - 920i)$
Each factor is a Gaussian prime number.

Can you find the Gaussian factorization of
$14159265358979323846626433 + 71828182845904523536028 74i$.

Answer format: $a_1, b_1/a_2, b_2/.../a_n, b_n$
where $a_i \geq 0$ and a_i in ascending order. To normalize the factors, we must have $|a_i| < |b_i|$.

Example: `0,1/1,2/2,3/5,-8/283,-920` for γ.

[My timing: instant]

Answer 66f7818856bc8695f0840a6febf6a7c1

357 FLT near miss

2016-11-04 12:41:24 by Philippe_57721 **7 xp** Programming

Since Euler, we know that Diophantine equation $x^3 + y^3 = z^3$ has no solution.

However, Ramanujan found an infinite set of solutions for equation $x^3 + y^3 = z^3 \pm 1$.

Here is his stunning solution. Let:
$X_1 = [9, 10, 12]$
$X_2 = [791, 812, 1010]$
$X_3 = [65601, 67402, 83802]$

And define $X_{n+3} = 82 \cdot X_{n+2} + 82 \cdot X_{n+1} - X_n$.

Let $x_n = X_n[1], y_n = X_n[2], z_n = X_n[3]$.

We have $x_n^3 + y_n^3 = z_n^3 + (-1)^{n+1}$.

For instance, $X_4 = [5444135, 5593538, 6954572]$ and indeed $5444135^3 + 5593538^3 = 6954572^3 - 1$.

Find solution for index 123456789101112.

Format answer: x, y, z modulo 10^{15}

[My timing: instant]

Answer d4dbb0f38f90fb36af1d6b5e4082b751

358 Contest

2016-08-30 13:47:14	by sinan	10 xp	Programming

10000 people attend a programming contest. The contest has 4 problems. The contestants who pass at least two out of four problems will be awarded.

Here are the statistics:
Problem 1: 5101 people pass
Problem 2: 4499 people pass
Problem 3: 1312 people pass
Problem 4: 148 people pass

Participation fee is 20. Those who pass two problems out of four are awarded with 30, those who pass three with 50 and those who pass all four with 100. The cost of organising this contest is 1000.

What is the minimum (`X`) and maximum (`Y`) profit that may be earned from this contest?

Answer format: `X,Y`

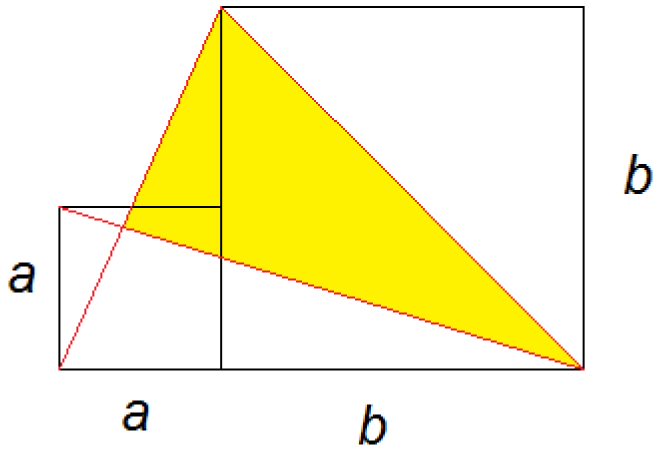
Thanks to **Philippe** for his helps in developing this problem.

Answer 33e8c0eb9fe8f3dfa9f6062a52b161a6

359 Smallest possible

2016-11-13 06:27:19 **by sinan** **6 xp** **Math**

There are two squares with sides equal to a and b as seen below. The sides a, b and the area of the painted triangle are equal to integers. What is the smallest possible area if $\sqrt{23} < b/a < \sqrt{24}$?



Answer format: `a,b,area`

[My timing: 5 s]

Answer 4acf2dec6a4bf614c57e6e8db7713fe0

360 Basic crypto

2016-08-18 08:41:09 **by sinan** **5 xp** **Crypto**

```
A <=> QQAA
Z <=> WgAA
```

Can you decode this?

```
VGgA ZWEA bnMA d2UA cmkA c2IA YXMA ZTYA NAAA
```

Answer 7466d1e13223c8904ca430f943b8e1c7

361 Polynomials

2016-08-22 12:30:15 **by sinan** **5 xp** **Math**

Let $f(x) = x^3 + a \cdot x^2 + b \cdot x + 18811938$ where a and b are integers.

If a, b can be selected in such a way that $f(x) = 0$ has only integer roots, what is the sum of $f(-1)$ for all the possibilities (i.e. all the distinct polynomials that meet the conditions).

Answer format: sum of $f_{a,b}(-1)$ for a and b that makes $f_{a,b}(x) = 0$

Answer 1d24022d9ddc5b359fd38a7273c4e468

362 Equilateral triangle

2016-09-30 09:59:10	by Philippe_57721	6 xp	Math

An equilateral triangle contains a point in its interior whose distances to the vertices are $3, 4, 5$.

What is the side of the triangle?

Give the answer rounded to 10 digits after the decimal part.

Answer 3052b9a24af87711251768af74ecad88

363 Best divisor

2016-10-14 07:53:53 **by Philippe_57721** **7 xp** **Programming**

20! possesses 41040 divisors.

Its largest divisor $< 10^{10}$ is 9999498600.

Find the largest divisor of 75! which is $< 10^{30}$.

[My timing: < 40 sec]

Answer c2262bf2b73ee33d0bc1033d1e5fb59e

364 Bonus for RC360

2016-10-14 07:54:08 **by Philippe_57721** **6 xp** **Programming**

Find integers n_1, n_2 such as:

- $n_1 \times n_2 = 75!$
- $n_1 < n_2$
- $n_2 - n_1$ is the smallest possible

Answer format: $n_2 - n_1$

[My timing: < 50 sec]

Answer 5df35e6682645651e89106b57a7908ec

365 Egyptian equation I

2016-09-23 07:57:22	by Philippe_57721	7 xp	Programming

The equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{n}$ with $0<x\leq y$ has 14 solutions for $n=12345$:

- $\frac{1}{12346}+\frac{1}{152411370}=\frac{1}{12345}$
- $\frac{1}{12348}+\frac{1}{50812020}=\frac{1}{12345}$
- $\frac{1}{12350}+\frac{1}{30492150}=\frac{1}{12345}$
- $\frac{1}{12354}+\frac{1}{16945570}=\frac{1}{12345}$
- $\frac{1}{12360}+\frac{1}{10172280}=\frac{1}{12345}$
- $\frac{1}{12370}+\frac{1}{6108306}=\frac{1}{12345}$
- $\frac{1}{12390}+\frac{1}{3398990}=\frac{1}{12345}$
- $\frac{1}{12420}+\frac{1}{2044332}=\frac{1}{12345}$
- $\frac{1}{12570}+\frac{1}{689674}=\frac{1}{12345}$
- $\frac{1}{13168}+\frac{1}{197520}=\frac{1}{12345}$
- $\frac{1}{14814}+\frac{1}{74070}=\frac{1}{12345}$
- $\frac{1}{16460}+\frac{1}{49380}=\frac{1}{12345}$
- $\frac{1}{19752}+\frac{1}{32920}=\frac{1}{12345}$
- $\frac{1}{24690}+\frac{1}{24690}=\frac{1}{12345}$

How many solutions are there for $n=2091129587$?

Answer format: count, $\sum x$, $\sum y$

Example: `14,200392,273453272` for $n=12345$.

[My timing: 12 sec]

Answer 3638d5bd96849d30e1fd2a55d3b9112e

366 Cyclic equation

2016-12-24 08:44:32 **by Philippe_57721** **8 xp** **Programming**

The equation $x \cdot y + y \cdot z + z \cdot x = n$ with $0 < x \leq y \leq z$ has 11 solutions for $n = 1234$:

- $1 \times 4 + 4 \times 246 + 246 \times 1 = 1234$
- $1 \times 12 + 12 \times 94 + 94 \times 1 = 1234$
- $1 \times 18 + 18 \times 64 + 64 \times 1 = 1234$
- $3 \times 8 + 8 \times 110 + 110 \times 3 = 1234$
- $4 \times 6 + 6 \times 121 + 121 \times 4 = 1234$
- $4 \times 21 + 21 \times 46 + 46 \times 4 = 1234$
- $8 \times 14 + 14 \times 51 + 51 \times 8 = 1234$
- $10 \times 13 + 13 \times 48 + 48 \times 10 = 1234$
- $10 \times 19 + 19 \times 36 + 36 \times 10 = 1234$
- $12 \times 14 + 14 \times 41 + 41 \times 12 = 1234$
- $18 \times 20 + 20 \times 23 + 23 \times 18 = 1234$

How many solutions are there for $n = 3000000$?

Answer format: count, $\sum x$, $\sum y$, $\sum z$

Example: `11,72,149,880` for $n = 1234$.

[My timing: 65 sec]

Answer ac51e8fc0956f1cc984b85b6271ce63e

367 Egyptian equation II

2016-09-23 07:57:40 | **by Philippe_57721** | **7 xp** | **Programming**

Let the diophantine equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$ with $x \leq y$.

How many solutions are there for $n = (1000000!)$?

As the answer is a BIG number, use the following condensed representation:
`(First 10 digits)[(number of remaining digits)](Last 10 digits)`

For instance, for 2^{127}, the representation is: `1701411834[19]5884105728`.

Example: `8583170787[2]5400804688` for $n = 100!$.

[My timing: < 1 sec]

Answer a5f4a44a25b41c7ab405898ce468791e

368 Fractions

2016-09-13 07:40:27	by sinan	8 xp	Math

Let p_i be the i-th prime.

Let $P(n)$ be the canonical product of the first n primes this way:
$\prod p_i^i$ for $i = 1$ to n.

Let $\#f(n)$ be the number of **unique** reduced fractions (of the form p/q) > 1 that can be constructed using the divisors of $P(n)$.

For example $\#f(2) = 2$ because
$P(2) = 2^1 \times 3^2 = 18$
The 2 fractions are $3/2$ and $9/2$.

What is $\#f(10^{10})$?

Answer format: rightmost 20 digits

[My timing: < 1 s]

Answer df709de2d2d3648e454946fbdb35a15c

369 Lonely runners

2016-09-15 15:35:30 **by Philippe_57721** **5 xp** **Programming**

n runners are on a circular track of length 1.

At $t = 0$, they are all in the same position and start to run in the same direction.

A runner R_i is said **lonely** at time t if he is at a distance of at least $1/n$ of every other runners.

There are 10 runners with the following speed: $13, 163, 353, 389, 487, 491, 563, 797, 857, 881$. A speed V_i means that the runner R_i completes the track in V_i unit of time.

At which time all runners have been lonely at least once? (The time t is discrete and should always be an integer)

Answer af65ec4985d3eb204c8c195444595c0e

370 DNA sequences alignment

2017-01-27 09:22:31 **by Philippe_57721** **8 xp** **Programming**

Given two sequences of nucleic acid, we try to align them the best way possible. We can insert any number of gaps in both sequences.

We use the following rules:
- if at a given position the acids are the same, we count $+2$
- if at a given position the acids are different, we count -1
- if there is a gap in one string, we count -2

For example, consider the two following sequences:
- `GAATTCAGTTA`
- `GGATCGA`

The best possible alignment (the one with the greater score) is:

```
GAATTCAGTTA
GGA-TC-G--A
```

which gives a score of 3
- `G G`: $+2$
- `A G`: -1
- `A A`: $+2$
- `T -`: -2
- `T T`: $+2$
- `C C`: $+2$
- `A -`: -2
- `G G`: $+2$
- `T -`: -2
- `T -`: -2
- `A A`: $+2$

You are given the 2 following sequences:

```
GTAATAGACTCGGAAACGCAACCGTCAGCAAAACGCGTTCGGTCGATCGT
AATATGTAAGATCCAATTAGGGCGACCTCTTGTGCGGTCAGTAGGAGTCT

ATAACTCTGAATCCCCCGACGTGTCGTGATGGGCGACGGACGGCACCCTT
AACGTGATCCTGAACTCCCGTGGGGACCGTTGTCGGTAATGCAGGGTGTG
```

What is the score for the best alignment and the number of acids identical in both aligned sequences?

Answer format: comma separated

Example: `3,6` for the sequences `GAATTCAGTTA` and `GGATCGA`.

[My timing: < 1 sec]

Answer 7a58aecc982ad52fb29496da2f71e580

371 Die Hard

2016-11-13 08:51:55 **by Philippe_57721** **8 xp** **Programming**

In the movie Die Hard 3, Officer John McLane has to stop a bomb to explode by using a conterweight of 4 gallons.

For this, he can only use 2 jugs of 5 and 3 gallons. He can do it in 6 moves:

```
[5] [0] // Pour 5 gallons in the 1st jug
[2] [3] // Fill the 2nd jug with the 1st
[2] [0] // Empty the 2nd jug
[0] [2] // Pour the 2 gallons from the 1st jug to the 2nd
[5] [2] // Pour 5 gallons the 1st jug
[4] [3] // Fill the 2nd jug with the 1st => We are done: 4 gallons remain in the 1st jug
```

- How many moves (M_1) would Officer McLane need to weight 641 gallons with 2 jugs of 19937 and 23209 gallons?
- Which capacity (C_2) between 1 and 19936 requieres the largest number of moves (M_2)?

Answer format: M_1, C_2, M_2

[My timing: 4 sec]

Answer **ba763325df68843e46db2e5804ec7dc2**

372 Die Hard - 3 Jugs

2016-11-17 10:35:43 | **by sinan** | **10 xp** | **Programming**

See the problem 371 by **Philippe**. This time we have 3 jugs with capacities 7, 31 and 1048575. What are the gallons that require the most number of moves?

Answer format: moves, $g_1, g_2, \ldots$
where $g_{i-1} < g_i$

Example: `20,54,57,58,62,66,67,70` if we have 3, 7 and 127 gallon jugs.

[My timing < 1 m and my memory requirement < 100 M]

Answer 6ed1ee25d294a8796ba3ff8d6dd624ab

373 Busy Beavers

2016-10-21 06:51:59 **by Philippe_57721** **7 xp** **Programming**

There is a special kind of Turing machines called the **Busy Beavers**.

A Busy Beaver is a machine defined for P symbols and Q states. Its purpose is to write as many symbols as possible on the tape.

Let's take an example with a 2 symbols and 2 states Busy Beaver.

It is described by a $P \times Q$ matrix.

```
 Symbol |      State-1     |      State-2
        | Write Move Next  | Write Move Next
-------------------------------------------
  0     |  1     R    2    |  1     L    1
  1     |  1     L    2    |  1     R    0
```

Each row corresponds to a symbol read on the tape. Each column corresponds to a state and contains a triple:

- Which symbol to write on the tape
- In which direction to move (Left or Right) on the tape
- Next state (State 0 means the Beaver halts)

Example: if we read a '0' on the tape, and we are is state 2, we write a '1' on the tape, the the Left and go to state 1.

We always starts with an empty tape (filled with '0'), and in state 1.

If we run this Busy Beaver, we get the following execution trace:

```
Step Curr Tape             Move Write Next
 0   1    00000{0}000000   R    1     2
 1   2    000001{0}00000   L    1     1
 2   1    00000{1}100000   L    1     2
 3   2    0000{0}1100000   L    1     1
 4   1    000{0}11100000   R    1     2
 5   2    0001{1}1100000   R    1     0
```

The beaver stops after 6 steps and in the end the tape contains 4 '1'.

This is actually the "best" 2×2 Busy Beaver (the one which writes the more '1' on a tape).

Let's take a 2×6 Busy Beaver, defined with the following matrix:

```
Symb |     State-1    |     State-2    |   State-3      |   State-4      |   State-5      |   State-6
     |Write Move Next|Write Move Next|Write Move Next|Write Move Next|Write Move Next|Write Move Next
----------------------------------------------------------------------------------------------------
 0   |   1   L      2 |  1    R      3 |  0    R      6 |  1    L      1 |  0    L      1 |  1    L      5
 1   |   1   L      1 |  1    R      2 |  1    R      4 |  0    R      5 |  1    R      3 |  1    L      0
```

You can verify that this beaver halts after 13 122 572 797 steps (took me 400 sec).

How many steps does it need to write 100000 '1'.

You are given:

- 3726 steps for 100 ‘1’
- 315587 steps for 1000 ‘1’

[My timing: 84 sec]

Answer **ca14d5fd86cc770f19828d3251f876ae**

374 De vermis mysteriis

2016-11-25 10:35:10 **by Philippe_57721** **15 xp** **Programming**

Professor Beklemishev defines a worm as follow:

We start with a list of non-negative integers $W_1 = [w_0, w_1, \dots, w_n]$ at step $m = 1$.

At step m
- If $w_n = 0$, then $W_m = [w_0, w_1, \dots, w_{n-1}]$ (We chop its head)
- Otherwise, let $k = \text{argmax}_{i<n}(w_i < w_n)$

If k exists let $G = [w_0, \dots, w_k]$, $B = [w_{k+1}, \dots, w_{n-1}]$.
If k does not exist let $G = []$ (an empty list), $B = [w_0, \dots, w_{n-1}]$.
Then $W_m = G + B + \dots + B$, $(m+1)$ copies of B.

Here is the evolution of the worm [1,1]
1 - $[1, 1]$
2 - $[1, 0, 1, 0, 1, 0]$
3 - $[1, 0, 1, 0, 1]$
4 - $[1, 0, 1, 0, 0, 0, 0, 0, 0]$
5 - $[1, 0, 1, 0, 0, 0, 0, 0]$
6 - $[1, 0, 1, 0, 0, 0, 0]$
7 - $[1, 0, 1, 0, 0, 0]$
8 - $[1, 0, 1, 0, 0]$
9 - $[1, 0, 1, 0]$
10 - $[1, 0, 1]$
11 - $[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$
...
24 - $[1]$
25 - $[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$
...
50 - $[0]$
51 - $[]$

He proved that every worm evolves eventually to an empty list. This cannot be proved in Peano. The number of steps to reach an empty list is not calculable.

At which step does the worm $W_1 = [1, 1, 1]$ turns into $[1, 1]$? (It's a VERY VERY LARGE number)

[My timing: < 1 sec]

Answer d4dc4fe316ff838127e8df965f2b372d

375 Slaying the hydra

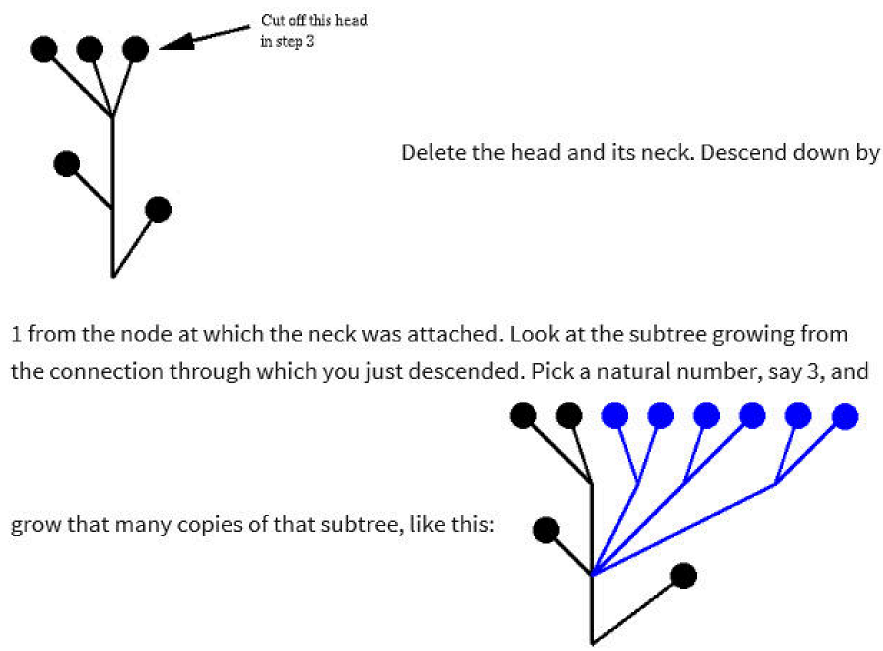
2018-11-09 01:12:36 **by Philippe_57721** **12 xp** **Programming**

An hydra is a rooted tree.

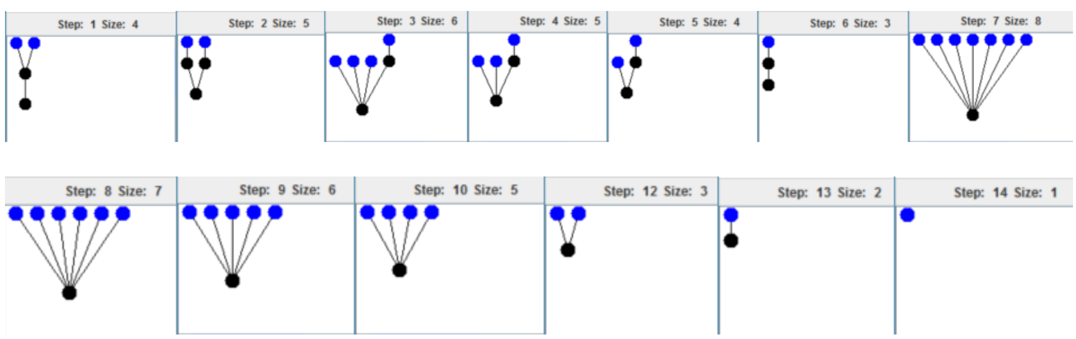
Hercules' task is to slay the hydra by choping all its heads. But the hydra's heads grow again with the following rules. At the step n:

- if Hercules cuts a head growing from the root, the hydra doesn't grow any head.
- if Hercules cuts a head attached to a given node, this head is deleted.

The node at which the head was attached is copied n times on the node just below.



Here is the evolution of a small hydra:



We define the size of an hydra as the number of its heads and nodes. The hydra in the above picture as a size of 4 at step 1.

We can represent an hydra as a string as follow:

- `n` A leaf
- `n(..)` A node, inside the parenthesis its children

For instance, in the above picture the first 3 hydras are:
`n(n(nn))`
`n(n(n)n(n))`
`n(nnnn(n))`

We always cut the left most heads.

If we consider all the hydras of size 5, after 100 steps, the maximum sizes reached are the following:

- `n(n(n(n(n))))` - 5686
- `n(n(n(nn)))` - 341
- `n(n(n(n)n))` - 288
- `n(n(n(n))n)` - 21
- `n(n(nn(n)))` - 70
- `n(n(nnn))` - 156
- `n(n(nn)n)` - 9
- `n(n(n)n(n))` - 6
- `n(n(n)nn)` - 5
- `n(nn(n(n)))` - 67
- `n(nn(nn))` - 20
- `n(nn(n)n)` - 5
- `n(nnn(n))` - 5
- `n(nnnn)` - 4

(The initial size before steps is not counted)

The 5 largest sizes are (in descending order): 5686, 341, 288, 156, 70.

What are the 5 largest sizes reached among all the 6-size hydras after 100 steps?

Answer format: comma delimited list in descending order

[My timing: 55 sec]

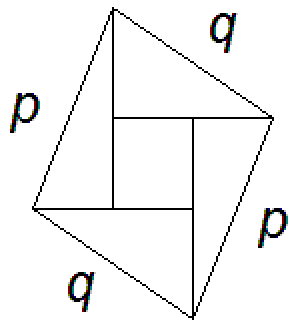
Credit to Andrej Bauer for the images

Answer 37110a0845899a3c2e4dc8ef32d34586

376 Twins

2016-12-05 08:22:06	by sinan	8 xp	Programming

Consider **integer sided** Pythagorean triangles with their hypotenus and shorter leg equal to **twin primes** (*). Using four of such triangles and a rectangle with sides also equal to **twin primes** (*), we draw a parallelogram as seen below.



Find such parallelograms with their area less than 10^{19}?

Answer format: count,sum_of_areas mod 2^{64}

Example: `1,169` (for area < 10^3).

[My timing: < 1 m]

* Note that **twin primes** here does NOT mean a pair of primes with a difference of 2, rather any twin primes pair where a twin prime means **a prime number that is either 2 less or 2 more than another prime number**. For example for the triple (5, 12, 13), we have 5 and 13 which are two twin primes.

Answer 412c2ce1c56d567bb0ceadeecf79cedc

377 Pandigital never prime

2016-12-16 14:04:40 **by Philippe_57721** **5 xp** **Programming**

Consider the number 100020. If you change one of its digits, you never get a prime number:

$\mathbf{1}00020 = 2^2 \times 3 \times 5 \times 1667$
$\mathbf{2}00020 = 2^2 \times 5 \times 73 \times 137$
$\mathbf{3}00020 = 2^2 \times 5 \times 7 \times 2143$
$\mathbf{4}00020 = 2^2 \times 3 \times 5 \times 59 \times 113$
$\mathbf{5}00020 = 2^2 \times 5 \times 23 \times 1087$
$\mathbf{6}00020 = 2^2 \times 5 \times 19 \times 1579$
$\mathbf{7}00020 = 2^2 \times 3^2 \times 5 \times 3889$
$\mathbf{8}00020 = 2^2 \times 5 \times 13 \times 17 \times 181$
$\mathbf{9}00020 = 2^2 \times 5 \times 11 \times 4091$

$1\mathbf{0}0020 = 2^2 \times 3 \times 5 \times 1667$
$1\mathbf{1}0020 = 2^2 \times 5 \times 5501$
$1\mathbf{2}0020 = 2^2 \times 5 \times 17 \times 353$
$1\mathbf{3}0020 = 2^2 \times 3 \times 5 \times 11 \times 197$
$1\mathbf{4}0020 = 2^2 \times 5 \times 7001$
$1\mathbf{5}0020 = 2^2 \times 5 \times 13 \times 577$
$1\mathbf{6}0020 = 2^2 \times 3^2 \times 5 \times 7 \times 127$
$1\mathbf{7}0020 = 2^2 \times 5 \times 8501$
$1\mathbf{8}0020 = 2^2 \times 5 \times 9001$
$1\mathbf{9}0020 = 2^2 \times 3 \times 5 \times 3167$

$10\mathbf{0}020 = 2^2 \times 3 \times 5 \times 1667$
$10\mathbf{1}020 = 2^2 \times 5 \times 5051$
$10\mathbf{2}020 = 2^2 \times 5 \times 5101$
$10\mathbf{3}020 = 2^2 \times 3 \times 5 \times 17 \times 101$
$10\mathbf{4}020 = 2^2 \times 5 \times 7 \times 743$
$10\mathbf{5}020 = 2^2 \times 5 \times 59 \times 89$
$10\mathbf{6}020 = 2^2 \times 3^2 \times 5 \times 19 \times 31$
$10\mathbf{7}020 = 2^2 \times 5 \times 5351$
$10\mathbf{8}020 = 2^2 \times 5 \times 11 \times 491$
$10\mathbf{9}020 = 2^2 \times 3 \times 5 \times 23 \times 79$

$100\mathbf{0}20 = 2^2 \times 3 \times 5 \times 1667$
$100\mathbf{1}20 = 2^3 \times 5 \times 2503$
$100\mathbf{2}20 = 2^2 \times 5 \times 5011$
$100\mathbf{3}20 = 2^5 \times 3 \times 5 \times 11 \times 19$
$100\mathbf{4}20 = 2^2 \times 5 \times 5021$
$100\mathbf{5}20 = 2^3 \times 5 \times 7 \times 359$
$100\mathbf{6}20 = 2^2 \times 3^2 \times 5 \times 13 \times 43$
$100\mathbf{7}20 = 2^4 \times 5 \times 1259$
$100\mathbf{8}20 = 2^2 \times 5 \times 71^2$
$100\mathbf{9}20 = 2^3 \times 3 \times 5 \times 29^2$

$100\mathbf{00} = 2^5 \times 5^5$
$100\mathbf{01}0 = 2 \times 5 \times 73 \times 137$
$100\mathbf{02}0 = 2^2 \times 3 \times 5 \times 1667$
$100\mathbf{03}0 = 2 \times 5 \times 7 \times 1429$
$100\mathbf{04}0 = 2^3 \times 5 \times 41 \times 61$
$100\mathbf{05}0 = 2 \times 3 \times 5^2 \times 23 \times 29$
$100\mathbf{06}0 = 2^2 \times 5 \times 5003$
$100\mathbf{07}0 = 2 \times 5 \times 10007$
$100\mathbf{08}0 = 2^4 \times 3^2 \times 5 \times 139$
$100\mathbf{09}0 = 2 \times 5 \times 10009$

$10002\mathbf{0} = 2^2 \times 3 \times 5 \times 1667$
$10002\mathbf{1} = 29 \times 3449$
$10002\mathbf{2} = 2 \times 13 \times 3847$
$10002\mathbf{3} = 3 \times 7 \times 11 \times 433$
$10002\mathbf{4} = 2^3 \times 12503$
$10002\mathbf{5} = 5^2 \times 4001$
$10002\mathbf{6} = 2 \times 3^2 \times 5557$
$10002\mathbf{7} = 23 \times 4349$
$10002\mathbf{8} = 2^2 \times 17 \times 1471$
$10002\mathbf{9} = 3 \times 33343$

What is the smallest pandigital number (in base 10) with that property? *

* All the numbers you get when changing one digit must be pandigital too.

[My timing: < 1 sec]

Answer **18004655cfcda8896f9e2228032dc76a**

378 Mutated worm

2017-04-17 16:15:22 by sinan **10 xp** Programming

See the problem 374 by **Philippe**. We slightly modify the evolution code in that it creates m copies instead of $(m+1)$ at m-th step. So the evolution of $[1,1]$ now becomes like the following:
$1 : [1,1]$
$2 : [1,0,1,0]$
$3 : [1,0,1]$
$4 : [1,0,0,0,0,0]$
$5 : [1,0,0,0,0]$
$6 : [1,0,0,0]$
$7 : [1,0,0]$
$8 : [1,0]$
$9 : [1]$
$10 : [0,0,0,0,0,0,0,0,0,0]$
$11 : [0,0,0,0,0,0,0,0,0]$
$12 : [0,0,0,0,0,0,0,0]$
$13 : [0,0,0,0,0,0,0]$
$14 : [0,0,0,0,0,0]$
$15 : [0,0,0,0,0]$
$16 : [0,0,0,0]$
$17 : [0,0,0]$
$18 : [0,0]$
$19 : [0]$
$20 : []$

At which step does the worm $W_1 = [1,2]$ turn into $[]$?

Answer format: rightmost 20 digits

[My timing < 1 s]

Answer 484d22a01d606a03dd1c5ea79ff0b49c

379 Red flag

2017-02-27 06:44:19 **by sinan** **6 xp** **Math**

Unfortunately the picture `https://rosecode.neocities.org/uploads/Turkish_flag.png` was lost.

Letter	Measure	Length
G	Width	
A	Distance between the centre of the outer crescent and the seam of the white band	1/2 G
B	Diameter of the outer circle of the crescent	1/2 G
C	Distance between the centres of the inner and outer circles of the crescent	1/16 G
D	Diameter of the inner circle of the crescent	2/5 G
E	Distance between the inner circle of the crescent and the circle around the star	1/3 G
F	diameter of the circle around the star	1/4 G
L	Length	3/2 G
M	Width of the white hem at the hoist	1/30 G

If the total area of the flag is equal to 1 (that means $G \times (L + M) = 1$) what is the area of red part rounded to 10 digits after the decimal point?

Note that the dotted circle around the star is drawn for illustration purpose and actually is a part of the red area.

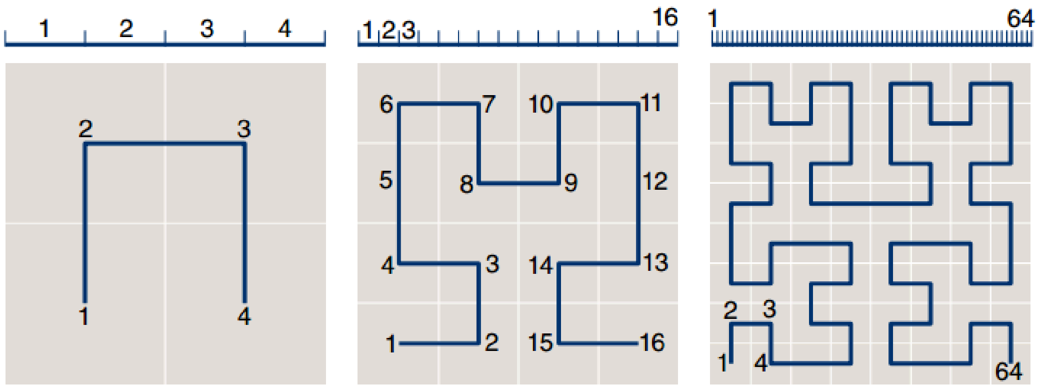
Answer 7df30b4d380ffbd3c3a12b9ce419c880

380 Exploring the Hilbert Curve

2017-02-03 09:25:54 **by Philippe_57721** **8 xp** **Math**

The Hilbert curve is a space-filling curve described by the great German mathematician David Hilbert in 1891.

It is defined as the limit of the following process:
We start with a square of side 1 that we divide in 4 parts.
We join the center of each sub-square as shown in the picture.
We repeat the process by dividing the square in 16 parts.
The Hilbert curve is the curve obtained when continuing this process infinitely.



A space-filling curve evolves through successive stages of refinement as it grows to cover the area of a square. This illustration is a redrawing of the first published diagram of such a curve; the original appeared in an 1891 paper by David Hilbert. The idea behind the construction is to divide a line segment into four intervals and divide a square into four quadrants, then establish a correspondence between the points of corresponding intervals and quadrants. The process continues with further recursive subdivisions.

As this curve encounters every point in the square, we can establish a bijection between the square and the segment $[0, 1]$, as shown in the picture.

What is the point (x, y) which corresponds to the value $\pi - 3$ in the interval $[0, 1]$?
What point z in the interval $[0, 1]$ corresponds to the point $(e - 2, \gamma)$ in the quare? (γ is the Euler Mascheroni constant)

Answer format: `x,y,z`
truncate the values at 10 digits after the decimal point

Hint: You are given the point in the square corresponding to $\sqrt{2} - 1$ in the interval $[0, 1]$:
$(0.4472924291, 0.9498609981)$

[My timing: < 1 sec]

Credit to Brian Hayes for the pictures

Answer 23cdd928ce804e3eed9fb4e442954844

381 Cube puzzle

2017-01-06 10:19:58	by Philippe_57721	4 xp	Probability

A solid cube has its 6 faces painted in black. It is cut into 1000 smaller cubes (each face evenly divided in 10). These little cubes are placed in a bag and one is selected randomly. This selected cube is then rolled like a die.

What is the probability that it lands with a black face up?

Answer format: `P/Q`
an irreductible fraction

Answer 57a7d6de1978e8480375c8cd3632b8ab

382 A special subset

2017-02-07 07:48:01	by sinan	8 xp	Programming

Let S_N be the set of positive numbers less or equal to N, $S_N = \{1, 2, ..., N\}$.

Consider the subsets with the following properties:
- none of the elements is a prime number
- none of the elements contains any prime digit(s)
- none of any 2 elements' difference is a prime number
- none of any 2 elements' difference contains any prime digit(s)

Find the subset of S_{100000} with the maximal sum.

Answer format: `sum`

Example: `20` for S_{10} subsets:
$\{1\}, \{1, 9\}, \{\mathbf{1}, \mathbf{9}, \mathbf{10}\}, \{1, 10\}, \{4\}, \{4, 8\}, \{4, 10\}, \{6\}, \{6, 10\}, \{8\}, \{8, 9\}, \{9\}, \{9, 10\}, \{10\}$

[My timing: < 1 m]

Answer b4ae75793af3df9055bb64a09e3b745f

383 Alleviated factorisation

2017-02-17 08:31:59	**by Philippe_57721**	**9 xp**	**Programming**

$P < Q$ are two prime numbers.

You are given:

$$\begin{aligned} P \times Q &= 25021280035441905627372198214765572382628368011 64 \backslash \\ &\quad 29486119232882711861280541724576724725825255100 63 \\ P \otimes Q &= 10896394514466713067376536898727387282419639480038 \end{aligned}$$

($\otimes$ is the XOR operation)

Can you find P and Q ?

Answer format: `P,Q`

[**My timing:** 75 **sec**]

Answer 3d7ce3a60f9449ab9581f34e7744e319

384 Denesting

2017-02-10 08:15:12 **by Philippe_57721** **7 xp** **Math**

Denesting is the process of simplifying an expression with embedded n-th roots to obtain an equivalent expression with a lesser level of roots.

Ramanujan was a master in this matter and provided some amazing examples:

$$\sqrt[3]{\sqrt[3]{2}-1} = \left(\sqrt[3]{\frac{1}{9}} - \sqrt[3]{\frac{2}{9}} + \sqrt[3]{\frac{4}{9}}\right) \times \frac{1}{3}$$

$$\sqrt{\sqrt[3]{5}-\sqrt[3]{4}} = \left(\sqrt[3]{2} + \sqrt[3]{20} - \sqrt[3]{25}\right) \times \frac{1}{3}$$

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}} = \left(3 + \sqrt[4]{5} + \sqrt{5} + \sqrt[4]{125}\right) \times \frac{1}{2}$$

Can you denest the following expression: $\sqrt{\sqrt[3]{1404}-\sqrt[3]{875}}$?

The solution has the form $\left(\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}\right) \times \frac{p}{q}$ with $a < b < c$ and p, q coprime.

Answer format: `a,b,c,p,q`

[My timing: 35 sec]

Answer dafb34086c6aec6bfd6ef81e478cf1a6

385 Rational as Prime Egyptian Fractions

2017-02-24 09:09:16 **by Philippe_57721** **7 xp** **Programming**

Conjecture
For any positive rational r there is a finite set of prime numbers P_r^+ such that $\sum_{p\in P_r^+} \frac{1}{p+1} = r$.

For instance $\frac{3}{14} = \frac{1}{5+1} + \frac{1}{23+1} + \frac{1}{167+1}$.

Can you verify this conjecture for $\frac{25}{61}$?

Answer format: $p_1, p_2, \ldots, p_n$
if $\frac{25}{61} = \frac{1}{p_1+1} + \frac{1}{p_2+1} + \cdots + \frac{1}{p_n+1}$

As there are generally more than one solution, give the 1st one in lexicographic order.

Limit your search to prime numbers < 100000.

[My timing: 35 sec]

Answer 7bfa0a7be39f3b3b02584a1c8062a789

386 Squarefree Factorisations

2017-03-03 07:51:13 **by Philippe_57721** **8 xp** **Programming**

Let's n an integer with the following factorisation: $n = a_1^{e_1} \times a_2^{e_2} \times \cdots \times a_p^{e_p}$, where a_i are squarefree and $\forall i \in \{1, \ldots, p-1\}$, a_i divides a_{i+1}.

For instance:
$56 = 2^2 \times 14^1$
$5040 = 2^2 \times 6^1 \times 210^1$
$526773121875 = 3^2 \times 15^3 \times 1155^1 \times 15015^1$

It can be proved that this factorisation is unique.

For such a factorisation, let's consider all the divisors of n: $a_1^{f_1} \times a_2^{f_2} \times \cdots \times a_p^{f_p}$ where $0 \leq f_i \leq e_i$.

Define $\sigma'(n) = \sum_d (d)$ where d runs over the divisors of n as defined above.
$\sigma'(5040) = 1 + 2 + 4 + 6 + 12 + 24 + 210 + 420 + 840 + 1260 + 2520 + 5040 = 10339.$

We say that n is a champion if the ratio $\frac{\sigma'(n)}{n}$ is greater than any ratio $\frac{\sigma'(m)}{m}$ with $m < n$.

Here are the first 10 champions:
1. $1 \Rightarrow 2$
2. $24 \Rightarrow 2.04166666666667$
3. $48 \Rightarrow 2.1875$
4. $96 \Rightarrow 2.26041666666667$
5. $192 \Rightarrow 2.296875$
6. $384 \Rightarrow 2.31510416666667$
7. $768 \Rightarrow 2.32421875$
8. $1152 \Rightarrow 2.3515625$
9. $2304 \Rightarrow 2.37022569444444$
10. $4608 \Rightarrow 2.37955729166667$

What is the 66-th champion?

[My timing: 5 sec]

Answer de5bd6dcc93ff229be36c8d8af40a4b1

387 Double mirror

2017-03-10 07:40:16	by Philippe_57721	9 xp	Programming

Consider the number 35490. It can be written as:
2, 21, 2, 1 in base 23
1, 2, 21, 2 in base 32

Find the smallest number that can be written as:
$d_1, d_2, \ldots, d_9$ in base b_1b_2
$d_9, d_8, \ldots, d_1$ in base b_2b_1
$(b_1, b_2 \in \{1, \ldots, 9\})$

Answer format: `number,base1`
where `base1` $<$ `base2`

Example: `35490,23`

[My timing: 4 sec]

Answer 92f44f935e4a4f63184ecedf82eb9bb3

388 A mysterious machine

2017-03-17 07:46:03	by Philippe_57721	5 xp	Math

Given a string representing a positive integer, the machine outputs another string using the following rules:

Rule 1: If the input is $2X$, the output will be X.
e.g.: $M(234) = 34$

Rule 2: If the input is $3X$ and $M(X) = Y$ the output will be $Y2Y$.
e.g.: $M(327) = 727$

Find X such as $M(X) = X$
Find Y such as $M(Y) = Y2Y$
Find Z such as $M(Z) = 7Z$

Answer format: `X,Y,Z`

[My timing: pencil and paper]

Answer 6d184baaeb6dbb0288a25437d05cf699

389 Fair partition

2017-03-24 07:45:47	by Philippe_57721	6 xp	Programming

The number $10! = 3628800$ has a set D of 270 divisors.

We can find 2 sets D_1 and D_2 such as
- the product of each element of D_1 with each element of D_2 generates D
- the cardinals of the sets D_1 and D_2 are the closest possible

$D_1 = \{1, 3, 5, 9, 15, 25, 27, 45, 75, 81, 135, 225, 405, 675, 2025\}$ and $|D_1| = 15$
$D_2 = \{1, 2, 4, 7, 8, 14, 16, 28, 32, 56, 64, 112, 128, 224, 256, 448, 896, 1792\}$ and $|D_2| = 18$

Find a fair partition for 100!.

Answer format: $|D_1|, |D_2|$
(in ascending order)

Example: `15,18`

[My timing: 54 sec]

Answer 72bf4dbb1ae382cf8936cf17da0cc7e9

390 Radical triangles

2017-02-20 08:08:48	by sinan	6 xp	Math

For positive integers a, b, c ($a \leq b \leq c$), let $T(N)$ denote the number of triangles with sides $\sqrt{a}$, $\sqrt{b}$, and $\sqrt{c}$ for $c \leq N$.

What is $T(100000)$?

Example: `145048` for $N = 100$.

[My timing: < 1 m]

Answer 8a131aacc98cf8d55ba690c3671e011b

391 Not a sum of distinct squares

2017-04-07 06:44:02	by Philippe_57721	5 xp	Programming

It is known since Lagrange that every positive integer can be represented as the sum of integer squares, 4 integers being enough.

But, if we add the constraint that all squares must be distinct, some integers are not representable as such a sum. This set is finite.

How many positive integers can not be represented by a sum of distinct squares, and what is their sum? (You are given that the largest one if less than 5000)

Answer format: `Count,Sum`

[My timing: 15 s]

Answer e3288ee7769c09f280ec23aaafa31e41

392 Beatty sequences

2017-03-31 06:50:34	by Philippe_57721	5 xp	Programming

There is a beautiful theorem by Samuel Beatty which states:

If α and β are positive irrational numbers such as $\frac{1}{\alpha} + \frac{1}{\beta} = 1$ then $A = \{a \in \mathbb{N}, \lfloor n \cdot \alpha \rfloor\}$ and $B = \{n \in \mathbb{N}, \lfloor n \cdot \beta \rfloor\}$ are a partition of N: $N = A \cup B$ and $A \cap B = \varnothing$.

Let's A the vector contained in this file data389.txt.

If A is a Beatty sequence, can you find the corresponding value α.

Answer format: α
truncated at 6 positions after the decimal point

[My timing: instant]

Answer d58c9e1f9a93d19ea15fa0f903b581c1

393 A distant prime number

2017-04-07 06:44:25 **by Philippe_57721** **5 xp** **Math**

We know the the 10^9-th prime number is 22801763489.

What is the 10^{200}-th prime number?

Answer format: The first 30 digits of that number

[My timing: 6 sec]

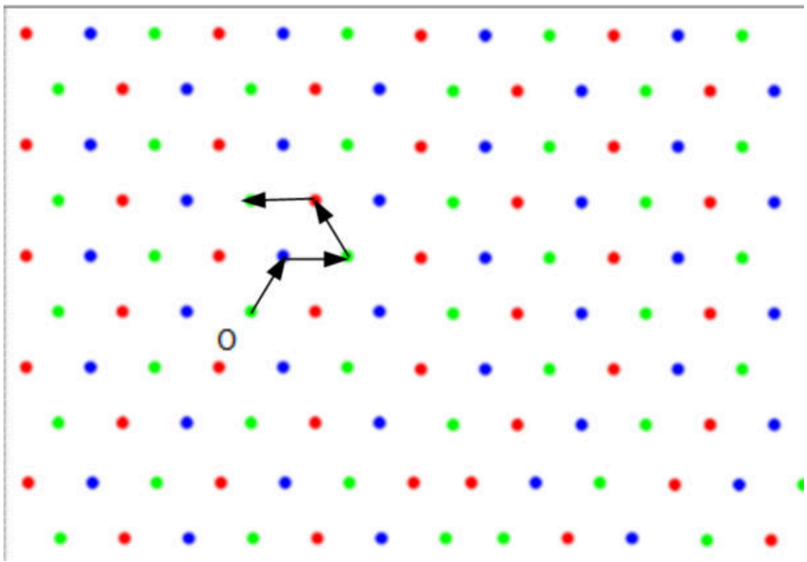
Answer 7b48313401051752b760814f3a8f7b2f

394 Self avoiding paths

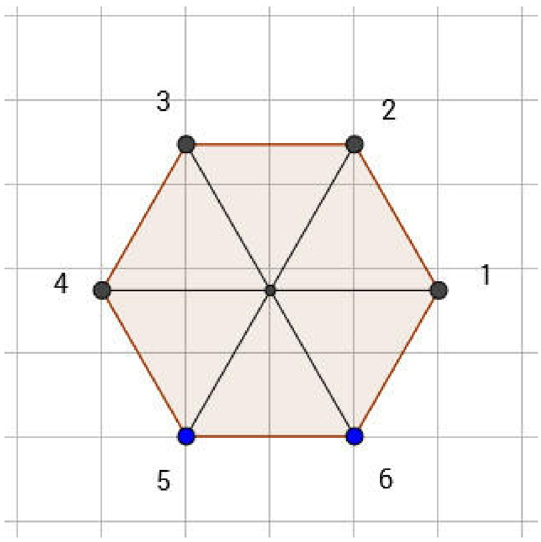
2017-04-13 14:19:32 **by Philippe_57721** **7 xp** **Programming**

In the equilateral triangle lattice plan, let's point O be the origin.

A self avoiding path is built as follow:
- We start from O.
- At each step we choose one of the 6 neighbour points which has not been visited yet.



We set a value on the 6 vectors starting from a given point as follow:



We can thus associate a number to each path: the concatenation of the values of the vectors in that path.

For instance, the path given in the 1st figure has the value: 2134.

How many self avoiding paths with 12 points are there?
How many of these paths correspond to a palindromic value?

Answer format: `Count-paths,Count-palindromic`

You are given `618,30` for 5 points.

[My timing: 30 sec]

Answer 3eee75da5b2f0014f7791905ba582a45

395 Maximize XOR

2017-04-21 06:59:22	by Philippe_57721	6 xp	Programming

Let $2^k \leq p < q \leq 2^{k+1} - 1$.

We are looking for pairs (p, q) such as
- $p \otimes q$ is maximal ($\otimes$ is the XOR operator)
- p, q are both either a prime or a square number

How many such pairs are there for $k = 26$?

Answer format: `count,first p,first q`

Example: `3,1307,1764` for $k = 10$.

[My timing: 42 sec]

Answer e3a4219d7f48191ee8369cf42abb7d04

396 Hidden title

2017-05-05 04:09:16 **by Philippe_57721** **6 xp** **Programming**

Consider the amazing Tupper's formula:

$$\frac{1}{2} < \left\lfloor \mathrm{mod}\left(\left\lfloor \frac{y}{17} \right\rfloor 2^{-17\lfloor x \rfloor - \mathrm{mod}(\lfloor y \rfloor, 17)}, 2 \right) \right\rfloor$$

Plot the set of points (x, y) where $0 \leq x \leq 105$ and $k \leq y \leq k + 16$.

```
k =
18103678631912000062470282381747441976757893373198
```

Answer format: the text hidden in the plot

Answer 33c080d31af03976ce0a4a69545d85fb

397 Prime Pandigital Step Numbers

2017-05-12 06:49:22 **by Philippe_57721** **6 xp** **Programming**

A step number is a number whose digits are in ascending order. For example: 1223455567 is a step number.

A pandigital step number is a step number which contains all digits (except 0) at least once. For example: 1122345556777899 is a pandigital step number.

Find the smallest (S) and largest (L) 30-digits prime pandigital step numbers.

Answer format: `S,L`

Example: `111111111111223456789,123456777788888888899` for 21 digits.

[My timing: 65 sec]

Answer **26347a11bfa10a887d1827427bb12c38**

398 The Goodstein Sequence

2017-05-19 08:03:19	by Philippe_57721	7 xp	Programming

We define the hereditary base-n representation of a number as follow:
For instance, if $n = 266$, and base $= 2$, its hereditary base-2 representation is:

$$\begin{aligned} 266 &= 2^1 + 2^3 + 2^8 \\ &= 2^1 + 2^{1+2} + 2^{2^3} \\ &= 2^1 + 2^{1+2} + 2^{2^{1+2}} \end{aligned}$$

We express n in base 2 and recursively every exponent in base 2.

Let's define the G_k sequence as follow:
$G_1(n) = n$
$G_2(n) =$ Take the hereditary base-2 representation of n, replace each 2s with 3s and substract 1
$G_{k(n)} =$ Take the hereditary base-k representation of $G_{k-1}(n)$, replace each ks with $(k+1)$s and substract 1

Here the first values for $G_k(6)$
$G_1(6) = 6$
$G_2(6) = 29$
$G_3(6) = 257$
$G_4(6) = 3125$
$G_5(6) = 46655$
$G_6(6) = 98039$
$G_7(6) = 187243$
$G_8(6) = 332147$

Find $G_{50}(13)$.

[My timing: < 1 sec]

Answer 2d79a1fff4547dcf6b67c3816da7020c

399 Factorial divisibility

2017-05-27 07:09:28	by Philippe_57721	8 xp	Programming

Let $S(n)$ be the smallest integer such as n divides $S(n)!$.

For instance $S(9) = 6$ for $6! \equiv 0 \bmod 9$.

Consider the set A of numbers of form $2^{e_1} \times 3^{e_2} \times 5^{e_3} \times 7^{e_4} \times 11^{e_5} \times 13^{e_6} \times 17^{e_7} \times 19^{e_8}$.

What is $\sum_{n \in A'} S(n)$ where A' contains the first 10^8 elements of A.

You are given: `45499522` when A' contains the first 10^6 elements of A.

[My timing: 120 sec]

PS 1: We consider $S(1) = 0$.
PS 2: There is a solution in less than 15 sec.

Answer be65cad4adb708babf6efd4cd21811eb

400 How close

2017-06-09 07:41:33 **by Philippe_57721** **6 xp** **Programming**

Using only prime numbers less than 30, the closest (less than) we can get from $10^{10} - 1$ is:
$9999928224 = 2^5 \times 3^5 \times 11 \times 13 \times 17 \times 23^2$

Using only prime numbers less than 50, how close can you get from $10^{15} - 1$?

[**My timing: 60 sec**]

Answer bd6d4409c9c064d7213c7947d309681e

401 A Big Sum

2017-06-01 14:52:19 **by Philippe_57721** **8 xp** **Math**

Let $S(n) = \sum_{i \leq n} \left\lfloor i \times \sqrt{2} \right\rfloor$ where $\lfloor x \rfloor =$ integer part of x.

For instance $S(100) = 7092$.

What is $S(10^{20})$?

[My timing: instant]

Thanks to **czp001** who gave me the idea!

Answer 6db6bbc1fd53b43dc7912445d0a30bc9

402 Ordinal numbers I

2017-06-16 12:21:03 **by Philippe_57721** **12 xp** **Programming**

Georg Cantor made many brillant discoveries, but the ones I find the most fascinating are in the realm of infinite sets, especially the Ordinal Numbers.

They are a new kind of numbers with some strange arithmetic rules:
$(\omega+1)+(\omega^2+\omega\cdot 2+1)=\omega^2+\omega\cdot 2+1$
$(\omega^2+\omega\cdot 2+1)+(\omega+1)=\omega^2+\omega\cdot 3+1$

Let the following ordinal numbers:
$\sigma_1=\omega^2$
$\sigma_2=\omega+1$
$\sigma_3=\omega\cdot 2+2$
$\sigma_4=\omega\cdot 4+3$
$\sigma_5=\omega\cdot 8+4$

If we add the five σ_i ordinal numbers using all possible 120 permutations and sort them (using the ordinal number order)

- how many different sums can we obtain
- what is the 1-st (lowest) value
- what is the 17-th value

Answer format: comma delimited

The ordinal numbers are represented using the Cantor Normal Form

- the symbol ω is represented with a `w`
- exponentiation is represented with a `^`
- the symbol $\cdot$ is represented with a `.`
- all spaces are removed

$\omega^2+\omega\cdot 3+1 \rightarrow$ `w^2+w.3+1`

[My timing: < 1 s]

Answer **84a73b5bfa3c2771f717cbc84cad51ec**

403 Ordinal Numbers II

2017-06-16 12:21:23 **by Philippe_57721** **5 xp** **Programming**

For definitions, See problem 402.

What is $(\omega^2+\omega+1)^{\omega^3+\omega^2+\omega+2}$.

[My timing: < 1 sec]

Answer **b048ae49976898300debd845b0e6a28e**

404 Powerful Numbers I

2017-06-23 07:22:18 **by Philippe_57721** **9 xp** **Math**

A number is said powerful if all its prime factors appear with an exponent greater than 1.

A **pure powerful number** is a powerful number that is not a perfect square.

Two pure powerful numbers can be consecutive:
$12167\ (23^3), 12168\ (2^3 \times 3^2 \times 13^2)$

Find the 11-th pair $(m, m+1)$ of consecutive pure powerful number — $(12167, 12168)$ being the 1st.

Answer format: `m`

[My timing: < 30 sec]

Answer **3b6d39661401eb0a5b070155537a7762**

405 Squarefree Numbers

2017-07-07 07:11:37 **by Philippe_57721** **7 xp** **Math**

A squarefree number is a number not divisible by a perfect square (other than 1).

How many squarefree numbers ≤ 123456789123456789 are there?

Answer format: `count`

[**My timing:** 55 sec]

Answer 1f3770fee89dd48b8e9f439b57cd18bd

406 Powerful Numbers II

2017-06-23 07:59:41 by Min_25 **10 xp** Math

Find all powerful numbers $\leq 10^{25}$.

Answer Format: `count,sum`

Example: `214122,716436326576527` (for $\leq 10^{10}$)

[My timing: 0.5 sec]

Answer c932d8df5f991b85f24a06f0e1e219f9

407 Carmichael chains

2017-06-29 15:03:40 **by Philippe_57721** **6 xp** **Programming**

A Carmichael number is a chain if when we remove repeatedly its largest factor, we still obtain a Carmicheal number.

174470590282430768272287350512321 is a Carmichael chain of length 10 (it has 10 factors).

$174470590282430768272287350512321 = 7 \times 13 \times 31 \times 61 \times 181 \times 541 \times 2161 \times 187921 \times 3570481 \times 7140961$
$24432368456070656074481761 = 7 \times 13 \times 31 \times 61 \times 181 \times 541 \times 2161 \times 187921 \times 3570481$
$6842878720281848881 = 7 \times 13 \times 31 \times 61 \times 181 \times 541 \times 2161 \times 187921$
$36413592521761 = 7 \times 13 \times 31 \times 61 \times 181 \times 541 \times 2161$
$16850343601 = 7 \times 13 \times 31 \times 61 \times 181 \times 541$
$31146661 = 7 \times 13 \times 31 \times 61 \times 181$
$172081 = 7 \times 13 \times 31 \times 61$
$2821 = 7 \times 13 \times 31$
All these numbers are Carmichael numbers.

Of course, we stop at 3 factors, as a Carmichael number can't have less than 3 factors.

Find the next Carmichael chain of length 10. We assume that no factor is greater than 10^7.

[My timing: 60 sec]

Answer 954e1e0ac73c5720df4e3bf88cf75ddb

408 Mertens Equations

2017-09-15 01:10:56 **by Philippe_57721** **7 xp** **Programming**

The Mertens function is the summatory function of the Möbius function.

$$M(n) = \sum_{k \leq n} \mu(k)$$

Find the first i such as $M(i) = -9876$ and the first j such as $M(j) = +9876$.

Answer format: `i,j`

You are given: `12855874,10193871` for ± 1234.

[My timing: 60 sec]

Answer 47aa39cb641a77fd2f7acbd360db3931

409 Infinite String

2017-05-26 14:48:56 **by Min_25** **12 xp** **Math**

Let s_0 be the string "01" and $s_i = s_{i-1} + s_{i-1}[0 : i]$ for $i \geq 1$, where $s[0 : i]$ means the first i characters of s.

For example, $s_1 =$ "010", $s_2 =$ "01001" and $s_3 =$ "01001010".

Let $C(i)$ be the number of "1" in the $s_i[0 : i]$.

You are given that $C(10) = 4$, $C(10^4) = 3576$ and $C(10^8) = 35807401$.

Find $C(10^{18})$.

[My Timing: instant]

Answer 9b3781857afac58efe20d1b50890eb15

410 Sum of three cubes

2017-07-14 06:54:46 **by Philippe_57721** **7 xp** **Programming**

It is conjectured that all positive integers, not of the form $9 \times k \pm 4$ can be written as a sum of 3 cubes, probably in infinitely many ways.

Some decompositions are hard to discover.

For instance, the smallest decomposition of 74 was only discovered in 2016:
$74 = -284650292555885^3 + 66229832190556^3 + 283450105697727^3$

Decompositions for all integers up to 100 are known, except for 33 and 42.

Find all the decompositions of 1026 where the absolute value of each cube is $< 10^5$.

Answer format: `Count,X,Y,Z`
where $X < Y < Z$ is the triple with the largest absolute values for X

You are given: `6,-14900,10849,12664` for 993 and a threshold of 20000.
$993 = -14900^3 + 10849^3 + 12664^3$
$993 = -11450^3 - 4127^3 + 11626^3$
$993 = -8339^3 + 6121^3 + 7051^3$
$993 = -2528^3 + 673^3 + 2512^3$
$993 = -1007^3 - 842^3 + 1174^3$
$993 = -2^3 + 1^3 + 10^3$

[My timing: 60 sec]

Answer **a749d17124fec0a22d2c730b17d0c0bf**

411 A Big Factorial

2017-08-05 05:21:23 **by Philippe_57721** **6 xp** **Programming**

What is the digital sum of 1500000!.

You are given: 1938780 for 100000!.

[My timing: 120 sec]

Answer **075f1e96f56fbf9a85f2fbb214ee6127**

412 Magic square of squares

2017-07-21 06:51:45	**by Philippe_57721**	**6 xp**	**Programming**

It is a open problem to know whether a 3×3 magic square containing only squares exists or not.

We say a square is **almost magic** if all the sums are equal except for the anti-diagonal.

The following square is almost magic:

$$\begin{bmatrix} 174^2 & 381^2 & 138^2 \\ 291^2 & 222^2 & 246^2 \\ 282^2 & 6^2 & 339^2 \end{bmatrix}$$

What is the first (lexicographic order) almost magic square with all elements squares of numbers ≤ 200.

Answer format: comma separated list of the square root of each element
(all values must be distinct)

Exemple: `174,381,138,291,222,246,282,6,339` for the above example.

[My timing: 25 sec]

Answer **51a3d4f5a4965d6dbd71770de4609c18**

413 Standard Nim

2017-06-24 15:40:27	by Min_25	8 xp	Math

Let $C(n, k)$ be the number of integer solutions $(x_1, \ldots, x_k)$ such that $\bigoplus_{i=1}^{k} x_i = 0$ and $0 \le x_i < n$ for each i. Here, $x \oplus y$ means x xor y.

Let $F(n)$ be the n-th Fibonacci number: $F(0) = 0$, $F(1) = 1$, and $F(i) = F(i-1) + F(i-2)$ for $i \ge 2$.

Let

$$S(n) := \sum_{i=1}^{n} \sum_{j=1}^{n} C(F(i), F(j)).$$

You are given $S(6) = 2153296$ and $S(7) = 18998620089329$.

Find $S(92)$ modulo $10^9 + 7$.

[My timing: 0.3 seconds]

Answer b96adeae380a26e518c15019fe3f0bbe

414 Symmetric Equation I

2017-07-28 08:22:25 **by Philippe_57721** **7 xp** **Programming**

Here is the smallest known solution in positive integers for the Diophantine equation

$$\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y}=4$$

$x = 43736[\cdots]772036$
$y = 368751[\cdots]055579$
$z = 1544768[\cdots]277999$
(The $\cdots$ represent 70 digits!)

If we allow one variable to be negative, we can find smaller solutions.

For example, $(x, y, z) = (-1, 4, 11)$ and $(-5, 9, 11)$ are solutions of the above equation.

Find the first solution to $\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y}=38$ with $x < 0 < y < z$ (coprime integers).

Answer format: `x,y,z`

[My timing: 15 sec]

Answer 7273c1fd6fc33ca8e3f9157ad2cd391e

415 Symmetric Equation II

2017-07-28 08:22:40 **by Philippe_57721** **10 xp** **Math**

See problem 414.

Find the first solution to $\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y} = 466$ with $x < 0 < y < z$ (coprime integers).

Answer format: `x,y,z`

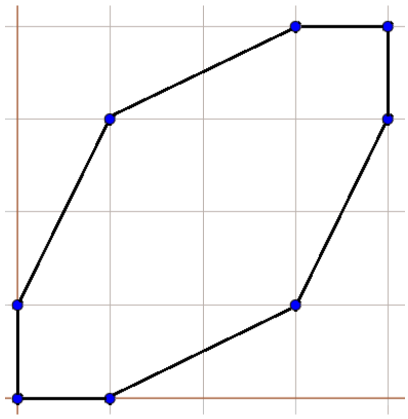
[My timing: less than 1 sec]

Answer 55734a6df7f41bdf96a7c6c5032c617a

416 Convex numbers

2017-09-05 02:21:52 **by sinan** **8 xp** **Programming**

Take the number 134431001 as an example. We can make pairs using the 2 digits following one another: $[(1,3),(3,4),(4,4),(4,3),(3,1),(1,0),(0,0),(0,1)]$



In general for a k-digit number like $d_1 d_2 \cdots d_k$ we can have a $(k-1)$-gon: $[(d_1,d_2),(d_2,d_3),\ldots,(d_{k-1},d_k)]$

If the drawn polygon is a convex one then we will call this kind of number as **convex number**.

How many **positive** convex numbers are there?

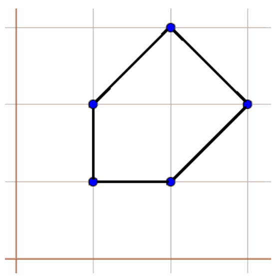
Answer format: `count,sum`

[My timing: < 1 m]

Notes:

- Any two neighbouring line segments cannot be collinear.
- All the generated pairs must be distinct.
- Some polygons can be generated by more than one number.

For example: $112321, 123211, 232112, 321123, 211232$. Since the numbers are different, they are all counted.



Answer 4920c89b2107f92cd5ac26c5a597c90e

417 Generating Seeds

2017-07-16 05:35:47	by Min_25	8 xp	Math

Let D be a positive integer.

Suppose that we would like to find all nonnegative integer solutions (x, y, z) of

$$x^2 + D = y \cdot z.$$

Let's assume that (x, y, z) is a solution of the above equation. Then, it can be verified that $(x + y, y, 2x + y + z)$ and $(x + z, 2x + y + z, z)$ are also solutions of the equation. Let's define this generating process as the *evolution* of (x, y, z).

Surprisingly, we can find all solutions uniquely by choosing some seeds $S_D = \{(x_1, y_1, z_1), \ldots, (x_k, y_k, z_k)\}$ and *evolving* them repeatedly (a *seed* is a solution of the equation).

For example, when $D = 2$, we can choose S_2 as $S_2 = \{(0, 1, 2), (0, 2, 1)\}$.

Let $C(D)$ be the minimum number of seeds needed to enumerate all nonnegative integer solutions of the equation.

It can be verified that $C(2) = 2$, $C(3) = 3$ and $C(100) = 18$.

Let $S(n) := \sum_{D=1}^{n} C(D)$. You are given $S(10) = 40$ and $S(100) = 1714$.

Find $S(3 \times 10^7)$.

[My Timing: 14.8 seconds (PyPy)]

Answer	0eb1affb9e606571bed2b0473822a94e

418 Fifty-Fifty

2017-07-26 07:17:47 by Min_25 **8 xp** Math

Let $p_{4,3}(n)$ be the smallest prime p such that $p > n$ and $p \equiv 3 \pmod 4$ and let

$$p_{4,3}^k(n) := \begin{cases} p_{4,3}^{k-1}(p_{4,3}(n)) & \text{if } k > 1 \\ p_{4,3}(n) & \text{if } k = 1 \end{cases}.$$

Let $F(n) := \frac{n-1}{2}! \bmod n$.

For example, $F(3) = 1$ and $F(47) = 46$.

Let $S(n, m) := \sum_{k=1}^{m} F(p_{4,3}^k(n))$.

You are given $S(1, 32) = 1856$, $S(10^7, 32) = 150008239$ and $S(10^{12}, 32) = 18000000012930$.

Find $S(10^{18}, 32)$.

[My timing: 1.8 seconds (PyPy)]

Note: It would be hard without **PARI/GP**.

Answer 1306ef2697ec63221f6057ccce0d0987

419 Bonus for p416

2017-08-04 12:14:24 **by Min_25** **8 xp** **Math**

Let $r_3(n) = \#\{(x, y, z) \in \mathbb{Z}^3 \mid x^2 + y^2 + z^2 = n\}$.

For example, $r_3(0) = 1$, $r_3(1) = 6$ and $r_3(100) = 30$.

Let $S(n, m) = \sum_{k=0}^{m-1} r_3(n + k)$.

It can be verified that $S(1, 100) = 4168$ and $S(10^8, 100) = 6410310$.

Find $S(10^{17}, 100)$.

[My timing: 2.6 seconds (PyPy)]

Note: It would be hard without **PARI/GP** and some **papers**.

Answer cb648b1b3a04152fa503b4813e1e6448

420 Into the 4th dimension

2017-09-01 01:01:46	by Philippe_57721	5 xp	Math

Consider a sphere whose square of its radius is n and centered at $(0,0,0,0)$ in the 4-th dimension space. For a given n, the sphere touches a certain number of integer lattice points.

For instance, if $n = 7$, it touches the point $(1,2,1,1)$.

Let $t(n)$ the number of integer lattice points touched by the sphere with square of radius n.

Find $\sum_{k=0}^{100} t(10^{17}+k)$.

You are given $\sum_{k=0}^{100} t(1000+k) = 1042664$.

[My timing: 9 sec]

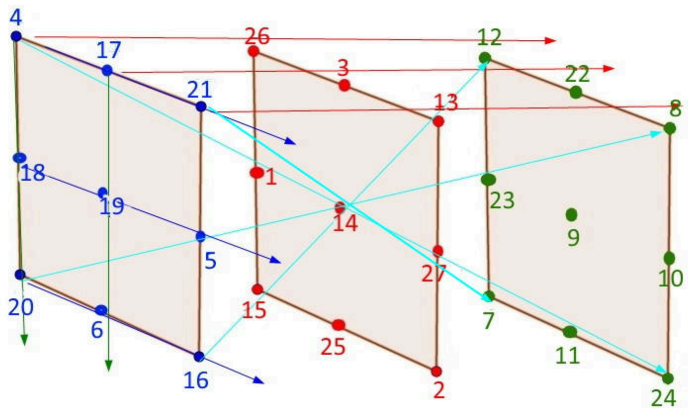
Answer 19bc582ed6e64925484663c1e6a81b5e

421 Enumerating the magic cubes

2017-08-11 08:06:12 **by Philippe_57721** **8 xp** **Programming**

A magic cube is the 3-D equivalent of a magic square.

Here is an exemple of a $3 \times 3 \times 3$ magic cube with all integers from 1 to 27:



- The sum of all rows and columns in the 3 directions are equals

$4 + 26 + 12 = 42$
$4 + 18 + 20 = 42$
$4 + 17 + 21 = 42$

- The sum of the 4 main diagonals are equals

$4 + 14 + 24 = 42$
$21 + 14 + 7 = 42$
$16 + 14 + 12 = 42$
$20 + 14 + 8 = 42$

How many $3 \times 3 \times 3$ magic cubes with integers in $\{1, ..., 27\}$ are there?

Answer format: `Count:comma separated list of values for the cube of rank 123`

Example: `999:4,17,21,18,19,5,20,6,16,26,3,13,1,14,27,15,25,2,12,22,8,23,9,10,7,11,24` (corresponds to the cube in the above picture)

[My timing: 2 min]

PS: We ignore symmetries: facing the 6 faces of this example, we get 6 distinct solutions. To rank the cubes, represent them as a 27 elements vectors (as in the above example), then sort the vectors in lexicographic order.

Answer 296868f9e04f72b73c3462bb9389ae74

422 Expected Universe I

2017-07-31 16:19:07	**by elasolova**	**6 xp**	**Probability**

Initially the Universe does not exist. The God (or whatever you name it) sits idle within nothingness. Luckily the God has a little friend that keeps asking questions every day but just one for each day.

The little asker has 3 questions of the form:
(1) Will you say YES to my next question?
(2) Will you say NO to my next question?
(3) Does the Universe exist now?

(a) God can only answer as YES or NO. God always answers immediately within the same day.
(b) The answer to (3) is always deterministic, namely there is only one choice always, in default case being always NO, but can only be forced to YES through a sequence of (1)s and (2)s.
(c) If (1) or (2) asked right after (3) or at start, the answer is nondeterministic, namely God can answer at will as YES or NO. This is the only case for God to be nondeterministic.
(d) A sequence of (1)s and (2)s which goes on after the preceeding (1) or (2) is deterministic, and God has to answer possible successive (1)s and (2)s accordingly and never cheat.
(e) If a (3) appears at the end of a sequence of (1)s and (2)s, the answer is obviously deterministic. But as noted in (c), a (1) or (2) appearing after a (3) or (3)s, will again cause a nondeterministic answer from God.
(f) These rules go on until the God is forced to answer as YES to Universe existence question, and the Universe finally emerges.

Here is a sample run to get familiarized with the concept:
A: Does the Universe exist now?
G: NO (initially it does not exist!)
A: Will you say YES to my next question?
G: NO (nondeterministic, can say either …)
A: Will you say YES to my next question?
G: NO (deterministic, has to say NO)
A: Will you say NO to my next question?
G: NO (deterministic)
A: Will you say NO to my next question?
G: YES (deterministic)
A: Does the Universe exist now?
G: NO (deterministic!)
A: Will you say YES to my next question?
G: YES (nondeterministic as form (3) breaks determinism)
A: Does the Universe exist now?
G: YES (finally!)

So the asker's choices are uniformly random, and also the God's will (when possible) is uniformly random. With these final assumptions, what is the expected number of days needed for the Universe to emerge? Round your answer to 3 decimal places.

Answer 3acfd93e8745bf57eb673b05352a5499

423 Expected Universe II

2017-07-31 16:19:29 **by elasolova** **6 xp** **Probability**

Initially the Universe does not exist. However, each day either a matter or anti-matter emerges uniformly randomly. The Universe is born when matter count exceeds that of anti-matter for the first time. In this case, what is the probability that the Universe emerges within **365** days?

Give the result as an exact fraction in lowest terms.

Answer c2fedfdd7459c664b861cf2f9ccfe9df

424 Friedman Numbers

2017-08-18 08:07:09 **by Philippe_57721** **10 xp** **Programming**

A Friedman number is a number which can be written using **all its digits exactly once** and a combination of the following operations:
`+` (addition)
`-` (subtraction)
`*` (multiplication)
`/` (division)
`^` (exponentiation)
`,` (concatenation) (*)

Examples:
```
5776 = 76^(7-5) = 76 7 5 - ^
8092 = (90^2)-8 = 90 2 ^ 8 -
11943 = (11^3-4)*9 = 11 3 ^ 4 - 9 *
16225 = ((52^2)*6)+1 = 52 2 ^ 6 * 1 +
```
Expressions are represented in postfix notation.

Let's call the set of terms used in a decomposition, the **primitive factors**.

The Friedman number 1395 has two sets of primitive factors:
$\{15, 93\}$
$\{5, 9, 31\}$
For we have the following decompositions:
$1395 = 15 \times 93$
$1395 = 5 \times 9 \times 31$

In the range $[13800, 13900]$, find the Friedman number with the most primitive factors For each primitive factors of this number, find the corresponding decompositions and give the first one in alphabetic order (in postfix notation). (The primitive factors are sorted in lexicographic order: first by length, then by values.)

Answer format: `comma separated list for primitive factors 1:decomposition 1|...`

Example: `2,9,16:16 9 2 ^ *|1,2,6,9:6 9 1 - 2 / ^` in the range $[1200, 1400]$.

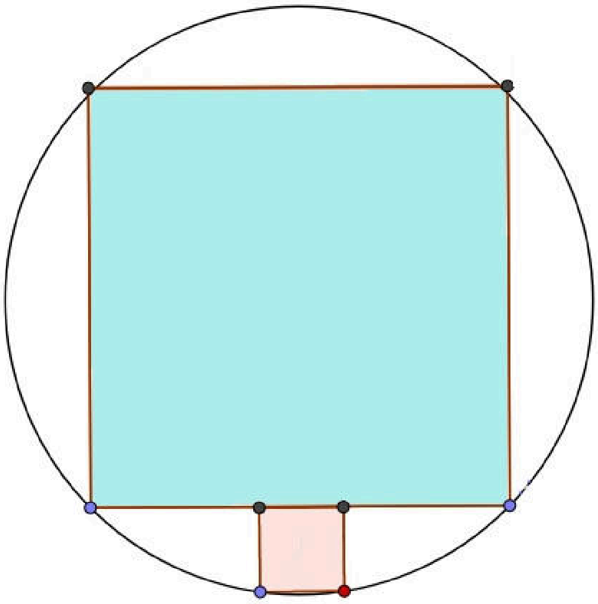
[My timing: 100 sec]

* Concatenation can only be used to build the integers used in expressions, but not in expressions.

Answer 693547c406afb18b2b146fcaa4848054

425 Squares ratio

2017-08-26 00:33:22	**by Philippe_57721**	**5 xp**	**Math**



A = area of the large square
a = area of the small square

What is $\frac{A}{a}$?

Answer c81fb13777b701cb8ce6cdb7f0661f1b

426 Vampire Numbers

2017-09-08 01:17:55 **by Philippe_57721** **5 xp** **Programming**

A vampire number is a $2k$-digits number n such as there exist two k-digits numbers a and b such as:

- $n = a \times b$
- the digits of a and b are exactly the digits of n

Example: $1001295697 = 19001 \times 52697$.

The two factors a and b are called, quite logically, the fangs.

Find the first and last 10-digits vampire numbers **not containing the digit 0** with prime fangs.

Anwer format: `first,last`

Example: `11722657,79168819` (for 8 digits)
$11722657 = 2267 \times 5171, \quad 79168819 = 8171 \times 9689$

[My timing: 82 sec]

Hint: The first digits of *last* are $945\cdots$

Answer 886607381f97896e844fda111130fec0

427 BrainForce

2017-08-29 10:54:53	**by sinan**	**8 xp**	**Hack**

You need to force your brain to solve this one :)

Answer **4612ccc394501d69700ba51ace9eeb5a**

428 Bandpass audio signal

2018-05-18 02:33:29	by elasolova	8 xp	Crypto

An audio signal is intercepted. It is known that the signal includes double-side band suppressed carrier modulation. It is known that the target channel uses carrier frequency as 4000 Hz with a bandwidth of 1500 Hz.

More mathematically, a bandpass signal is formulated as:
$s(t) = m(t)\cos(2\pi f_c t)$

In such a case there is a single channel broadcasting the message $m(t)$ at carrier frequency f_c.

In case of multiple channels broadcasting it becomes:
$s(t) = m_1(t)\cos(2\pi f_1 t) + m_2(t)\cos(2\pi f_2 t) + \cdots$

So, given $s(t)$ as this audio signal, can you recover $m(t)$ within the bandwidth $[2500 \text{ Hz}, 5500 \text{ Hz}]$?

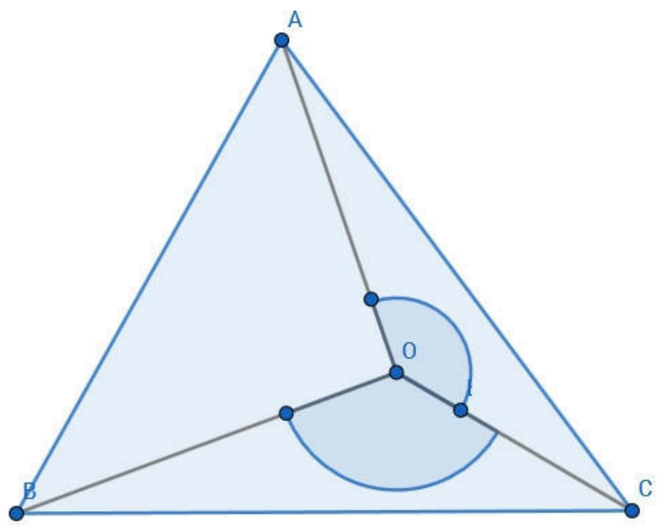
Please provide the keyword you hear in lower case.

Answer e238bee7ff15e80ec43df8a0d04c6180

429 Triangulation

2017-09-29 03:39:15 **by Philippe_57721** **6 xp** **Math**

A point O is choosen inside an equilateral triangle ABC.



The angle $\widehat{AOC}$ is 105°.
The angle $\widehat{BOC}$ is 115°.

We use the segments OA, OB, OC to build a new triangle.

What are the angles of this triangle?

Answer format: the sorted (ascending order) angles in degree, comma separated

Answer a46f1ade6a44cc5ec79e7b4bcac2aadd

430 Segments

2017-09-23 23:17:30 **by sinan** **6 xp** **Programming**

Let $\mathrm{fib}(n)$ be n-th Fibonacci number. $\mathrm{fib}(1) = 1$, $\mathrm{fib}(2) = 1$ and $\mathrm{fib}(n) = \mathrm{fib}(n-1) + \mathrm{fib}(n-2)$ for $n > 2$.

Let $f(n) = 1 + (\mathrm{fib}(n) \bmod 5)$.

Let v_N be the vector of $f(i)$ values for $i = 1$ to N. For example $v_3 = [2, 2, 3]$.

Let T_N be the sum of the elements of V_N. For example $T_3 = 7$.

Consider an ABC triangle and a D point that bisects BC side: $|BD| = |CD|$.

Draw the AD segment and assume $N - 1$ points on it that further creates N segments such that the lengths of these subsegments are proportional to the elements of v_N starting from A to D.
$s_i = v_N[i]/T_N \cdot |AD|$ for $i = 1$ to N

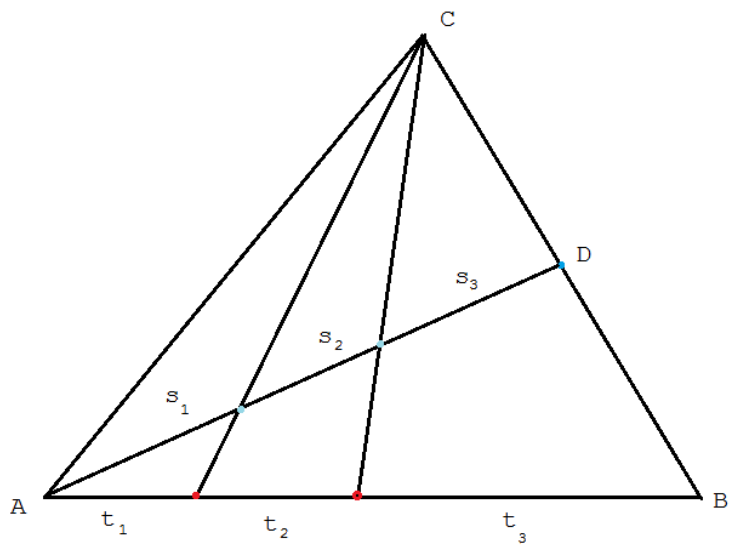
Now draw the line segments from C to AB that cross these $N - 1$ points on AD segment. So now we have N subsegments on AB side as well.

Let the lengths of these subsegments on the AB side be given as t_i for $i = 1$ to N starting from A to B.

Find the sum of ratios $|AB|/t_i$ for $i = 1$ to N where $N = 1919$.

Answer format: `p/q`
(reduced fraction)

You are given that the answer = `251/21` for $N = 3$.



Answer b82155fb1e7e7e91eecd43a7313eaa02

431 N-th Prime

2018-06-28 21:35:23 **by Philippe_57721** **6 xp** **Programming**

Let $P_n(k)$ the k-th prime number.

For instance, $P_n(1000000) = 15485863$.

Find $P_n(3141592653589793)$.

[My timing: 40 sec]

Answer 5133b521dae4df5d3693f0089566534d

432 Subareas

2017-11-01 03:20:36 **by sinan** **8 xp** **Programming**

Let $\mathrm{fib}(n)$ be n-th Fibonacci number. $\mathrm{fib}(1) = 1$, $\mathrm{fib}(2) = 1$ and $\mathrm{fib}(n) = \mathrm{fib}(n-1) + \mathrm{fib}(n-2)$ for $n > 2$.

Let $f(n) = 1 + (\mathrm{fib}(n) \bmod 5)$.

Let v_{M+N} be the vector of $f(i)$ values for $i = 1$ to $M + N$. For example $v_{2+2} = [2, 2, 3, 4]$.

Let T_M be the sum of the first M elements of V_{M+N} and T_N be the sum of the last N. For example $T_M = 4$ and $T_N = 7$ for $M = 2$ and $N = 2$.

Consider an ABC triangle with $M - 1$ points on AB side and $N - 1$ points on BC side.

Draw $M - 1$ line segments from C to the $M - 1$ points on the A_B side thereby creating M line segments on the AB side proportional to the first M values of V_{M+N} starting from A to B.
$|AB|_j = v_{M+N}[j] \cdot |AB|/T_M$ for $j = 1$ to M

Similarly draw $N - 1$ line segments from A to the $N - 1$ points on the BC side thereby creating N line segments on the BC side proportional to the last N values of V_{M+N} starting from B to C.
$|BC|_i = v_{M+N}[M + i] \cdot |BC|/T_N$ for $i = 1$ to N

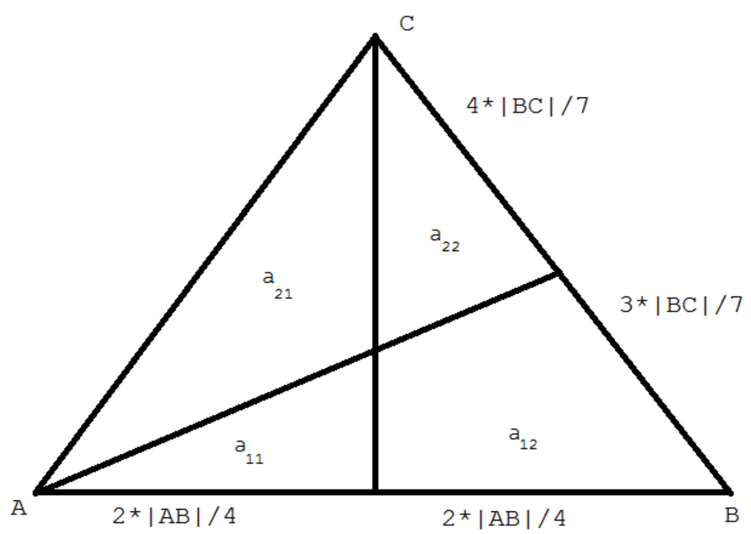
So now we have $M \times N$ subareas named a_{ij} where $1 \le i \le N$ and $1 \le j \le M$.

Let's assume that the area of ABC triangle is equal to 1.

Find the values of these $M \times N$ subareas for $M = 19$ and $N = 23$, sort them in ascending order and submit the answer as per the format given below.

Answer format: $\mathrm{area}_{\mathrm{first}}, \mathrm{aream}_{\mathrm{edian}}, \mathrm{area}_{\mathrm{last}}$
where areas are in the form of `p/q` (reduced fraction)

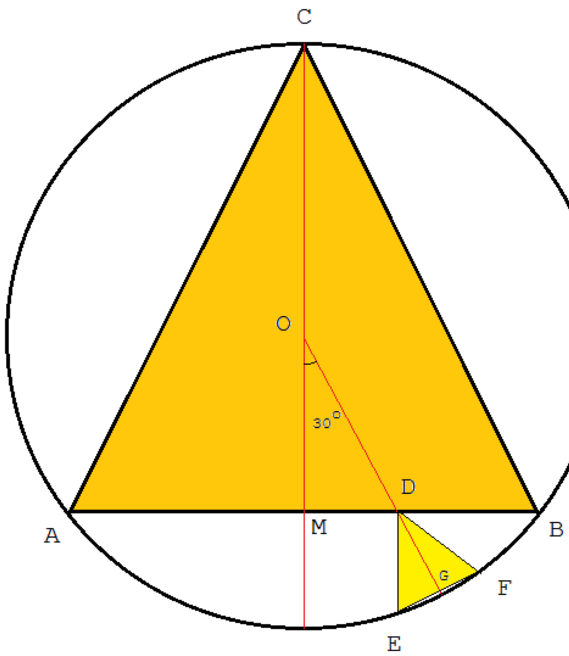
You are given that the answer = `3/22,1/4,4/11` for $M = 2$ and $N = 2$.
Areas: $a_{11} = 3/22$, $a_{12} = 45/154$; $a_{21} = 4/11$, $a_{22} = 16/77$.



Answer 829525c84d29c06171ec3f8f28d6f352

433 Triangles ratio

2017-10-01 00:57:17 **by sinan** **5 xp** **Math**



ABC is an equilateral triangle. DEF is another equilateral triangle with its two vertices (E, F) on the circumcircle of ABC and its third vertex (D) on the AB segment. O is the circumcenter of ABC.

What is the ratio $\text{area}(ABC)/\text{area}(DEF)$?

Answer format: number rounded to 5 digits after the decimal point

Answer **3604304b10145855f6d4d1137121e52d**

434 Plowing the field

2017-09-22 01:06:20 **by Philippe_57721** **5 xp** **Programming**

10 farmers $(A, B, \ldots, J)$ are plowing a field.

A and B working together can plow the field in 2 days.
B and C working together can plow the field in 3 days.
C and D working together can plow the field in 5 days.
D and E working together can plow the field in 7 days.
E and F working together can plow the field in 11 days.
F and G working together can plow the field in 13 days.
G and H working together can plow the field in 17 days.
H and I working together can plow the field in 19 days.
I and J working together can plow the field in 23 days.
J working alone can plow the field in 29 days.

How many days does it take if all the farmers are working together?

Answer format: `P/Q`
(irreductible fraction)

Answer 60f925966a1529370633fc11e7b64d6c

435 Equations

2017-10-09 09:19:47 **by sinan** **6 xp** **Math**

The following equations are given:

- $x^2 + y^2 + x \cdot y = a$
- $y^2 + z^2 + y \cdot z = b$
- $z^2 + x^2 + z \cdot x = c$

Find **positive** (x, y, z) triples when $a = \sqrt{i+1}, b = \sqrt{i+2}, c = \sqrt{i+3}$ for $i = 1$ to N where $N = 1923$.

Answer format: $\sum x, \sum y, \sum z$
where each sum is a number rounded to 10 digits after the decimal point

Example: `2.4103394172,2.1120641648,2.7525737652` for $N = 3$.

Answer **4a23ad8f82464593805e7e523a0ec9c3**

436 First semiprime

2017-10-06 01:23:01	by Philippe_57721	6 xp	Math

What is the least positive integer a such as $a^4 + 2^{1002}$ is semiprime?

[My timing: < 1 sec]

Answer e77a86da0bbccff2cce55db4b444a43c

437 Harmonic variations

2017-11-17 12:08:22 **by Philippe_57721** **12 xp** **Programming**

It is well known that the Harmonic serie $\sum_{n\geq 1}\frac{1}{n}$ is divergent.

A more surprising result is that if we remove all numbers containing at least one digit 9 the corresponding serie is convergent.

What is $\sum_{n\in S_{1,2,1,4}}\frac{1}{n}$.
Where $S_{1,2,1,4}$ is the set of all numbers with exactly one digit 2 and one digit 4 in their base-10 representation.
$S_{1,2,1,4} = \{24, 42, 124, 142, 204, 214, 234, 240, 241, 243, 245, ...\}$

Answer format: give the result with the first 20 digits after the decimal point

[My timing: < 1 sec]

Answer 9b67edbc52342e9cb478f98ad90b7bb0

438 Best Matrices Multiplication

2017-10-13 01:49:58 **by Philippe_57721** **6 xp** **Programming**

Let said the cost for the product of 2 matrices of size (p, q) and (q, r) is $p \times q \times r$.

Let's take 4 matrices with respective sizes:
$A : (50, 20)$
$B : (20, 1)$
$C : (1, 10)$
$D : (10, 100)$

We have 5 different ways to multiply these matrices with very different costs:
$A \times (B \times (C \times D)) = 103000$
$A \times ((B \times C) \times D) = 120200$
$(A \times B) \times (C \times D) = 7000$
$(A \times (B \times C)) \times D = 60200$
$((A \times B) \times C) \times D = 51500$
The smallest cost is 7000.

We take 50 matrices with respective sizes:

```
(8737, 1458) (1458, 1629) (1629, 5104) (5104, 8634) (8634, 9493)
(9493, 1380) (1380,  898) ( 898, 3036) (3036, 5012) (5012, 2405)
(2405, 5697) (5697, 6520) (6520, 4777) (4777, 4031) (4031, 3752)
(3752, 7848) (7848, 7540) (7540, 2624) (2624, 2436) (2436, 4598)
(4598, 1020) (1020, 9050) (9050, 7031) (7031, 8692) (8692,   48)
(  48,  396) ( 396, 2747) (2747, 4224) (4224, 9760) (9760, 8525)
(8525,  391) ( 391,  714) ( 714, 4590) (4590, 1455) (1455, 5437)
(5437, 4039) (4039, 6210) (6210, 7036) (7036, 5152) (5152, 4300)
(4300, 3511) (3511, 2484) (2484,  449) ( 449, 5683) (5683, 9809)
(9809, 4691) (4691, 3968) (3968, 3950) (3950, 5605) (5605, 1983)
```

What is the smallest cost for multiplying these matrices?

Answer format: `cost`

[My timing: < 1 sec]

Answer 3690123cd007ffae6b1c5bdd515663a3

439 Equilateral triangles 3

2017-10-30 04:08:20 **by liuguangxi** **9 xp** **Math**

Assume a point D **inside** an equilateral triangle ABC of side length t and with $AD = x$, $BD = y$, $CD = z$, $0 < x \leq y \leq z$, which is represented as quadruple (x, y, z, t).

If x, y, z, t are all integers and they are coprime, the quadruple (x, y, z, t) is called primitive integer solution. For example, $(57, 65, 73, 112)$ is such a solution. And it's known that there are infinitely many solutions of primitive integer quadruple (x, y, z, t).

Find all such solutions for $t \leq 5000$.

Answer format: [number of solutions],[sum of $x + y + z + t$ of all solutions]

Example: `3,1237` for $t \leq 200$
(explanation: there are total 3 solutions $(57, 65, 73, 112)$, $(73, 88, 95, 147)$, $(43, 147, 152, 185)$)

Answer b2a3528f1e5482cfbf3a555ecb44b43a

440 Special squarefree sum

2017-11-13 03:19:39 **by liuguangxi** **12 xp** **Math**

A squarefree number is called special if it can be expressed as the sum of two positive squares. For example, 65 $(= 1^2 + 8^2)$ is a special squarefree number while 15 is not.

Let $S(N)$ be the sum of all special squarefree numbers from 1 to N. You are given that $S(10) = 17$, $S(1000) = 81665$, $S(10^6) = 61081126084$.

Find $S(10^{12})$.

Answer 22a016522b8ac46cdc2fe5627b8d370c

441 Enhanced perfect power

2017-11-06 02:59:43 **by liuguangxi** **9 xp** **Math**

A perfect power n is called enhanced if there exists two integers $m > 0$ and $k > 2$ such that $n = m^k$. The enhanced perfect powers no more than 100 are $1, 8, 16, 27, 32, 64, 81$.

Let $S(N)$ be the sum of all enhanced perfect powers no more than N. You are given $S(100) = 229$, $S(10000) = 99352$ and $S(10^8) \bmod 1000000007 = 553225565$.

Find $S(10^{100}) \bmod 1000000007$.

Answer **f478879b29e0495fde8227fca7f8b9d9**

442 Triple pandigital

2017-10-23 03:59:06	**by liuguangxi**	**6 xp**	**Programming**

Consider a 10-digit pandigital number n in base 10. If we take first and last half of n, two 5-digit numbers x and y are got. Specially, square of x and square of y are also both 10-digit pandigital.

Find all pandigital numbers n satisfying above properties. Note: leading zero is not allowed in any numbers.

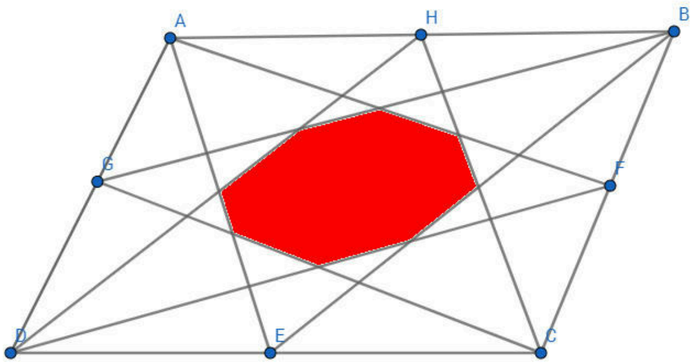
Answer format: [count],[sum]
(sample input: `1,1023456789`)

Answer **65a1f095a7bf0f1f16925e1ae1d89c37**

443 Polygons ratio

2017-10-20 01:31:10 | **by Philippe_57721** | **6 xp** | **Math**

$ABCD$ is a parallelogram.



From each vertex, we join the middle of the 2 opposites sides
F is the middle of BC
E is the middle of CD
We draw the lines AF and AE

We draw thus 8 lines which define an octogon.

What is $\frac{\text{Area(Octogon)}}{\text{Area(Parallelogram)}}$.

Give your answer rounded to 10 digits after the decimal point.

Answer 590f32e54e7b7b4163ec2c202f87fcdc

444 Kolakoski Sequence

2017-11-29 18:06:37 **by Min_25** **12 xp** **Math**

Let $K(a, b)$ be the infinite sequence such that:

- The first element is a,
- It consists of a and b, and
- Its run length sequence is equal to $K(a, b)$.

Here, for example, we assume that the run length sequence of $[1, 1, 1, 2, 3, 4, 4, 5]$ is $[3, 1, 1, 2, 1]$.

We can verify that

- $K(2, 3) = [2, 2, 3, 3, 2, 2, 2, 3, 3, 3, 2, \ldots]$, and
- $K(2, 4) = [2, 2, 4, 4, 2, 2, 2, 2, 4, 4, 4, 4, 2, \ldots]$.

Let $C_{a,b}(N)$ be the number of a in the first N elements of $K(a, b)$.

For example, $C_{2,4}(4) = 2$ and $C_{2,4}(5) = 3$.

Let $S(M, N) := \sum_{a=2}^{M-1} \sum_{b=a+1}^{M} C_{a,b}(N)$.

You are given $S(5, 100) = 300$ and $S(10, 10^6) = 17856847$.

Find $S(100, 10^{15})$.

Answer 902e53f7063b9d7ace6e26819cc697e8

445 Central binomial coefficients

2017-11-20 03:33:37 **by liuguangxi** **8 xp** **Math**

The binomial coefficients can be arranged in triangular form like this:

```
              1
            1   1
          1   2   1
        1   3   3   1
      1   4   6   4   1
    1   5   10  10  5   1
  1   6   15  20  15  6   1
1   7   21  35  35  21  7   1
            ......
```

It is known as Pascal's triangle.

The middle coefficients of the even-numbered rows (row numbering starts from zero) form a sequence $a_n(n \geq 0) : 1, 2, 6, 20, 70, 252, 924, 3432, 12870, 48620, 184756, \ldots$. It has been proven that no number a_n with $n > 4$ is squarefree.

Let $f(n)$ be n divided by largest squarefree divisor of n. For example, $24 = 2^3 \times 3$ and its largest squarefree divisor is 6, so $f(24) = 24/6 = 4$.

Let $S(N)$ be the sum of all $f(a_n)$ for n from 1 to N. You are given $S(10) = 1+1+2+1+6+2+4+3+2+2 = 24$, $S(100) = 429083$ and $S(10^3) \bmod 1000000007 = 258384246$.

Find $S(10^6) \bmod 1000000007$.

Answer 913e4e8f7ea6c186005d59c03c2831e4

446 A big sum 2

2017-11-27 02:59:05 **by liuguangxi** **12 xp** **Math**

Let

$$S(m,n)=\sum_{k=1}^{m}\sum_{x=1}^{n}\left(x^k\cdot\left\lfloor\frac{\pi x}{k}\right\rfloor^k\right)$$

where $\lfloor x\rfloor$ is the largest integer less than or equal to x.

You are given $S(1,10)=1182$, $S(3,100)=16716537706541$ and $S(10,10^4) \bmod 1000000007 = 211872856$.

Find $S(20,10^{20}) \bmod 1000000007$.

Hint: solve Problem 401 — A Big Sum first and there are some clues in the secret forum.

Answer **a10332a0e65dfad81d51b0f18251e66a**

447 Sum of powers

2017-10-27 07:47:50 **by Philippe_57721** **6 xp** **Programming**

Find $\left(\sum_{i=1}^{10^{1000}} i^{100}\right)$ modulo 1000000007.

[My timing: < 3 sec]

Answer **e705b34db09f5847f1815eb900450939**

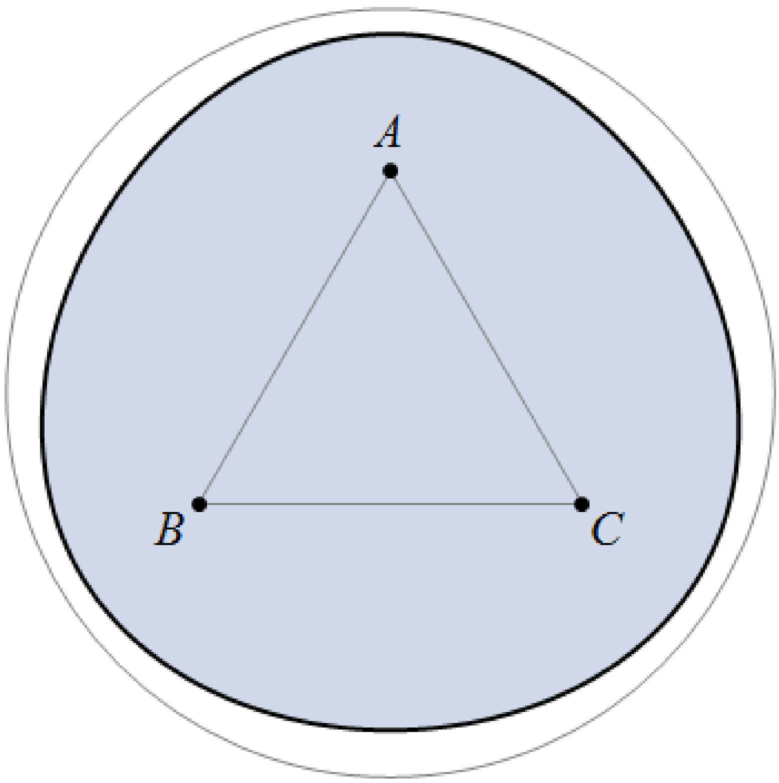
448 Generalized ellipse 1

2017-12-11 03:00:08 **by liuguangxi** **8 xp** **Math**

It is known that a circle is a curve in a plane that the distance from a given point is constant for every point on the curve. And an ellipse is a curve in a plane surrounding two focal points such that the sum of the distances to the two focal points is constant for every point on the curve.

We can generalize the definition of ellipse to n-ellipse. Given n points (called foci) in a plane, an n-ellipse is the locus of all points of the plane whose sum of distances to the n foci is a constant.

There is an equilateral triangle ABC of side length 1 in a plane. A 3-ellipse curve is the set of all points of the same plane whose sum of distances to the three points A, B and C is 3 (see the thick curve line in the picture). And the curve can be put inside the circle with a radius of 1.



Find the enclosed area and circumference of the closed curve above. Give your answer rounded to 6 digits after the decimal point.

Answer format: [area],[circumference]

Example: `3.141593,6.283185` for a circle with a radius of 1.

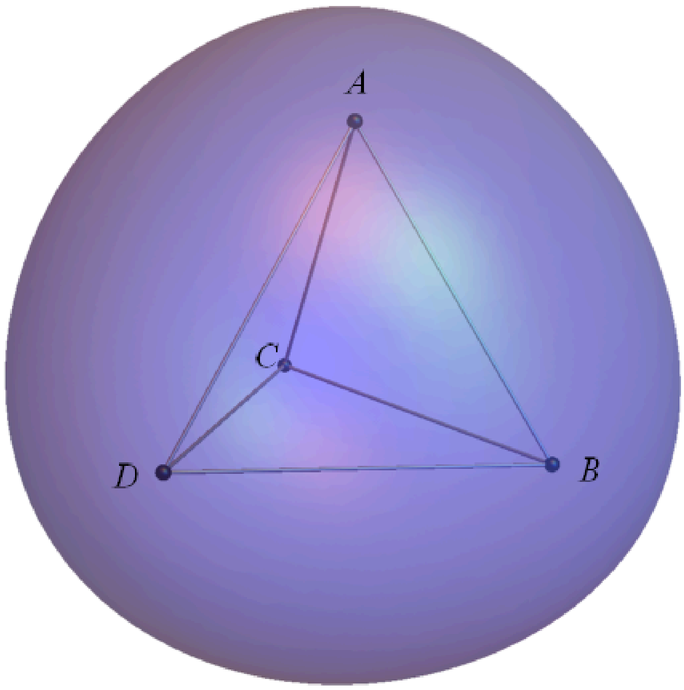
Answer 6162aca21a7ea112a5919c5000b6e369

449 Generalized ellipse 2

2017-12-18 03:00:58	**by liuguangxi**	**10 xp**	**Math**

The definition of n-ellipse is as below: given n points (called foci) in a plane, an n-ellipse is the locus of all points of the plane whose sum of distances to the n foci is a constant. We can generalize the definition from plain to space, and the set of all points satisfying the constraint become a surface.

There is a tetrahedron $ABCD$ of edge length 1 in a space. A 4-ellipse surface is the set of all points of the same space whose sum of distances to the 4 points A, B, C and D is 4 (see the blue solid in the picture).



Find the enclosed volume and surface area of the closed surface above. Give your answer rounded to 6 digits after the decimal point.

Answer format: [volume],[surface area]

Example: `4.188790,12.566371` for a sphere of radius 1.

Answer 7fd2c8c8fb08ac4c5e867c0f5151fb9b

450 Best Matrices Multiplication 2

2017-11-21 02:53:39 **by Min_25** **9 xp** **Math**

Let A be an infinite sequence of matrices $A := (A_1, A_2, \dots)$.

A_i has x_i rows and x_{i+1} columns, where

$$x_n = 5000 + (r_n \bmod 1000)$$

and

$$r_1 = 2, \quad r_n = r_{n-1}^2 \bmod 98765431 \ (n \geq 2).$$

Let $C(n)$ be the minimum cost of matrix multiplication of $A_1, \dots, A_n$. (see Problem 438 for the cost.)

For example, $x_1 = 5002$, $x_{11} = 5053$, $C(3) = 257423728320$ and $C(10) = 1290936750576$.

Find $C(6543)$.

Answer 014553c7b8c0828b8bc348d4a4cf108a

451 Machin Like Formulae

2017-12-01 01:27:44 **by Philippe_57721** **10 xp** **Programming**

One of the most famous formulae for computing π is $\pi/4 = 4 \cdot \arctan(1/5) - \arctan(1/239)$ due to John Machin (1706) who used it to compute 100 decimal places.

We can rewrite it as $\arctan(1/239) = -\arctan(1/1) + 4 \cdot \arctan(1/5)$.

It turns out that many integers like 239 have an inverse which can be decomposed as a linear combination of Arctan of inverses.

A few examples:

$\arctan(\frac{1}{3}) = \arctan(1) - \arctan(\frac{1}{2})$
$\arctan(\frac{1}{7}) = -\arctan(1) + 2 \cdot \arctan(\frac{1}{2})$
$\arctan(\frac{1}{8}) = \arctan(1) - \arctan(\frac{1}{2}) - \arctan(\frac{1}{5})$
$\arctan(\frac{1}{13}) = \arctan(1) - \arctan(\frac{1}{2}) - \arctan(\frac{1}{4})$
$\arctan(\frac{1}{17}) = -\arctan(1) + 2 \cdot \arctan(\frac{1}{2}) - \arctan(\frac{1}{12})$
$\arctan(\frac{1}{18}) = \arctan(1) - 2 \cdot \arctan(\frac{1}{2}) + \arctan(\frac{1}{5})$
$\arctan(\frac{1}{21}) = \arctan(\frac{1}{4}) - \arctan(\frac{1}{5})$
$\arctan(\frac{1}{30}) = \arctan(1) - \arctan(\frac{1}{2}) - \arctan(\frac{1}{4}) - \arctan(\frac{1}{23})$
$\arctan(\frac{1}{31}) = \arctan(\frac{1}{5}) - \arctan(\frac{1}{6})$
$\arctan(\frac{1}{32}) = -\arctan(1) + 2 \cdot \arctan(\frac{1}{2}) - \arctan(\frac{1}{9})$

The irreductible numbers, those which cannot be decomposed this way are called Størmer numbers. Thus, the inverse of every number which is not a Størmer number can be expressed as a linear combinaison of Arctan of inverse of smaller irreductible integers.

What makes John Machin's formula particularily efficient is the fact that it contains few terms and the numbers involved are not too small: the convergence of the Arctan serie is then quite fast.

Let's define the cost of a formula as the sum of the inverse of the distinct integers without 1.
$C(32) = \frac{1}{2} + \frac{1}{9} + \frac{1}{32} = 0.642361$

Numbers like 21 or 31 are not suitable for a computation of π as they do not include the number 1 in their decomposition, their cost will be infinity.

What are the 10 most efficient numbers less than 20000?

Answer format: comma separated list in ascending order of cost

Example: `41,46,75,17,32,7,68,93,43,57` (for numbers less than 100).

$C(41) = 0.6077235772 \to \arctan(\frac{1}{41}) = \arctan(1) - 2 \cdot \arctan(\frac{1}{2}) + 2 \cdot \arctan(\frac{1}{12})$
$C(46) = 0.6421095008 \to \arctan(\frac{1}{46}) = -\arctan(1) + 2 \cdot \arctan(\frac{1}{2}) - \arctan(\frac{1}{12}) - \arctan(\frac{1}{27})$
$C(75) = 0.6421212121 \to \arctan(\frac{1}{75}) = -\arctan(1) + 2 \cdot \arctan(\frac{1}{2}) - \arctan(\frac{1}{12}) - \arctan(\frac{1}{22})$
$C(17) = 0.6421568627 \to \arctan(\frac{1}{17}) = -\arctan(1) + 2 \cdot \arctan(\frac{1}{2}) - \arctan(\frac{1}{12})$
$C(32) = 0.6423611111 \to \arctan(\frac{1}{32}) = -\arctan(1) + 2 \cdot \arctan(\frac{1}{2}) - \arctan(\frac{1}{9})$
$C(7) = 0.6428571429 \to \arctan(\frac{1}{7}) = -\arctan(1) + 2 \cdot \arctan(\frac{1}{2})$

$C(68) = 0.6813725490 \to \arctan\left(\frac{1}{68}\right) = 2 \cdot \arctan(1) - 3 \cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{6}\right)$

$C(93) = 0.6899193548 \to \arctan\left(\frac{1}{93}\right) = \arctan(1) - 2 \cdot \arctan\left(\frac{1}{2}\right) + \arctan(16) - \arctan\left(\frac{1}{80}\right)$

$C(43) = 0.6899224806 \to \arctan\left(\frac{1}{43}\right) = \arctan(1) - 2 \cdot \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{6}\right)$

$C(57) = 0.7175438596 \to \arctan\left(\frac{1}{57}\right) = -2 \cdot \arctan(1) + 3 \cdot \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right)$

[My timing: < 10 sec]

Answer **085776fb8a3e94c5daa676bbfe727bfd**

452 A Staggering Sequence

2017-11-10 03:05:49	by Philippe_57721	8 xp	Programming

Let the following sequence:

- $c_1 = 2$
- $c_n = \frac{c_{n-1} \times (c_{n-1} + n + 140)}{n} \quad (n > 1)$

What is the first index n such as c_n is not an integer?
How many digits does the integer part of c_n have?

Answer format: `n,number of digits`

[My timing: < 1 sec]

Answer aafe1a17bf4bbb02ecc84873ab315baf

453 Value of dot product

2017-12-25 02:40:56 **by liuguangxi** **7 xp** **Probability**

There are two random vectors $\boldsymbol{a} = [a_1, a_2]$ and $\boldsymbol{b} = [b_1, b_2]$. Let s be the dot product of these two vectors, that is $s = \boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2$.

Assume a_1, a_2, b_1, b_2 are mutually independent random variables of the uniform distribution on the interval $[0, 2]$. Find the probability of value s less than 5 and the expected value of s. Give your answer rounded to 8 digits after the decimal point.

Answer format: [probability],[expected value]
(sample input: `0.12345678,1.23456789`)

Thanks to **baihacker** for the idea.

Answer e296012720ce31335b4dcdab915d8d72

454 Equilateral triangles 4

2018-01-01 03:05:35 **by liuguangxi** **9 xp** **Probability**

Assume a point D **inside** an equilateral triangle ABC of side length $\sqrt{3}$. Then we can use the three segments AD, BD and CD to build a new triangle PQR.

When point D is in the center of triangle ABC, the area of triangle PQR is $\sqrt{3}/4$, which is the maximum value for all possible positions of D; and the circumference of PQR is 3, which is the minimum value. Moreover, when point D is close to the vertex of triangle ABC, the area of triangle PQR is close to zero; and the circumference of PQR is close to its maximum value $2\sqrt{3}$.

Assuming that point D is chosen randomly (with **uniform distribution**) within triangle ABC, find the expected area and circumference of triangle PQR. Give your answers rounded to 8 digits after the decimal point.

Answer format: [expectation of area],[expectation of circumference]
(sample input: `0.43301270,3.46410162`)

Answer **286d9511dccba0077fb462470fc19cae**

455 A cubic form

2018-01-10 08:19:35 **by sinan** **8 xp** **Math**

Let $D(x, y, z) = x^3 + y^3 + z^3 - 3 \cdot x \cdot y \cdot z$.

It can be proven that for any integers $x_1, y_1, z_1, x_2, y_2, z_2$ there exist integers x_3, y_3, z_3 such that: $D(x_1, y_1, z_1) \cdot D(x_2, y_2, z_2) = D(x_3, y_3, z_3)$.

Let N be the number of (x_3, y_3, z_3) triples of the form $x_3 \leq y_3 \leq z_3$ for $D(x_1, y_1, z_1) \cdot D(x_2, y_2, z_2) = D(x_3, y_3, z_3)$.

Find the N and the sum of all z_3 values for $D(34040, 34238, 35404) \cdot D(34551, 34564, 34567)$.

Answer format: `N,sum`

Example: `12,548` for $D(1, 2, 3) \cdot D(11, 12, 13)$.

[My timing: < 1 m]

Answer 6aefb260a93a59a569429a78e1f0b396

456 Automatic typewriter

2018-01-08 03:01:12 **by liuguangxi** **10 xp** **Probability**

An automatic typewriter can generate a random digit string by generating a sequence of random digits and concatenating them together. Each digit is chosen independently from 0 to 9 with probability $1/55$, $2/55$, $3/55$, $4/55$, $5/55$, $6/55$, $7/55$, $8/55$, $9/55$ and $10/55$, respectively. The generation is stopped as soon as a specific pattern occurs in the random string.

$D(n)$ is a string consisting of first n digits of π. For example, $D(3) = 314$, $D(10) = 3141592653$.

Let $E(n)$ be the expected length of the generated random string for the string pattern $D(n)$. For example, $E(3) = 4159.375$, $E(10) = 104702034619.87625\cdots$. Usually $E(n)$ is a decimal number.

Let $S(n)$ be $E(n)$ rounded to the nearest integer. You are given $S(3) = 4159$, $S(10) = 104702034620$ and $S(50) = 18123304041728207905585295690813920900131725610455 83$.

Find $S(10000)$.

As the answer is a very big number, use the following condensed representation:
(First 10 digits)[(number of remaining digits)](Last 10 digits)
For instance, the representation of 2^{127} is: `1701411834[19]5884105728`.

Answer format: [condensed representation of $S(10000)$]

Example: `1812330404[32]2561045583` for $S(50)$.

Answer **9adbf1cf04df928c09cd7d4d4253dd2f**

457 Broken DNA

2017-11-24 00:39:25 **by Philippe_57721** **7 xp** **Programming**

A DNA molecule is made of 2 strands of nitrogenous bases: `A`, `C`, `G` or `T`.

Each base in the 1st strand is associated with one in the 2nd according the following pairing rules:

- `A` with `T`
- `C` with `G`

Example:

```
T G C T A A C C G G
| | | | | | | | | |
A C G A T T G G C C
```

The 2 strands are separated and broken into fragments of random length:

```
TGCTAACCGG => TGCT,AACC,GG
ACGATTGGCC => ACGAT,TGG,CC
```

Then all the fragments are shuffled. For instance: `CC,TGG,GG,TGCT,ACGAT,AACC`.

You are given the following 65 fragments:

```
CCGTCCGTTGCCC, TGTCTTGAGCTTCGTGCTGAGCACATGTAATCATAC, TTATT
TGTACAAGCGCGTTTAATGTTATACCAGCATTAATCTACTC
GTGAGTGAATGGGTTATAAACCGGCTGTACGGCTGTTGGGTGTACTGGAGCTACTGCCA
AGATGGTCAACCCTGCCACATCTCCTCAACATC, ACCTCTTCTCGCCTTCACCAGCAAGCATGATT
CGAAGCATTGAACAATTT, ACCGAACGACTGACATACGCTTTTAAATTCTACCAGTTGGGACGGTGTAGAG
ACGTAGGGTATACATTAAACAACCGTTATAGATAT
TTTACACTTCTTGTCTTTCCACCGCAATGTCTGTATTCTAAGACGCA
GGCCGGCTCTAACCTCGAGACCTATAAATCCAAATTGTAGCTTCAAG, CACACT
ACATCTGACAGTGTGGAAAAAGACGTTATGTTAGCGGGGT, GTGAGTAC
ATTTGGATCGCCTGTCGCGTGATTATTAAATCGGGCATAGCTG
CACATTCTTCCAACGCCACAAAATGCTGACTTCTAA, GCCATTTCGCGCCCGGCCGAGATTGGAGCTCTGGATATTT
CGACTCGTGTA, CAATCGCGGATATCTTGAGTATACG, ATTAAGCACCCTCGCTAAGGGCGGAAC
TAACTCGGTGCTTCGTAACTTGTTAAATGGCTTGCTGACTGTATGCGAAAATTTA, CATTAGTATGTGGAGAAGAGC
ATACAGATTTAATAATACGTAGCTAGGGAGCCACTCATGGTGTAAGAAGGTTGCG
AGTTCGCTGCAACTCCGGGCAGAA, GTGTTTTACGACTGAAGATTTGTAGACTGTCACACCTTTTTCTGCAA
AAAGGTGGCGTTACAGACATAAGATTCTGCGTGCAGCAAACAGTAAGTGGAGACTGAG, ATTACGTTAATC, CGCC
CCGACTGGCAGGCAACGGGAAATGTGAAGAACAG, AATAATTTAGCCCGTATCGACCACAGGT, ATCCCTCG
CGCGTGCCTATGCCGACCATATTGCATCCCATATGTAATTTGTTGGCAATATCTATAT
GGGCCTAAACGAGTTCTGTCAAAGGCTCAT, AGGTTTAACATCGA
CACTTACCCAATATTTGGCCGACATGCCGACAACCCACATGACCTCGATG, CCCCGGCGTACAGCTAAAAATAGGCA
CAGGTGTTAGCGCCTATAGAACTCATATGCATTCAAGGAG, GGCT
CGCTTAATCGCCCGGCAGTTATCTTCCGCAGC
CAAGGTCCCGCGAGTAGGCCCGGATTTGCTCAAGACAGTTTCCGAGTAGCAGC, GATAACTCGGTGGCACCA
ACGGTATC, TGGTAACAATGCTGCCAAGAG
GCATCCACTATTCATTGCGGATTAGTGGGTTCCAGGGCGCTCATCC, AGGGCATGTG
CGTCGAATAAACATGTTCGCGCAAATTACAATATGGTCGTAATTAGATGAGATTGAGCCA
GGAAGTGGTCGTTCGTACTAACTATTGAGCCACCGTGGTGCGCACGGATACGGCTGGTATA
CAAAAGAAGTCCTGCACACCGTGTTAAACCTAGCGGACAGCGCACT, AATACCGGGGGAGCCTACGTCT
TAAGTTCCT, GAGTTGTAGTATGTCTAAATTATTATGCATCG
```

```
TACAATCGCCCCAGCGAATTAGCGGGCCGTCAATAGAAGGCGTCGGGCTGA
GACTCGTTTTCTTCAGGACGTGTGGCACA
CGACGTTGAGGCCCGTCTTTTATGGCCCCCTCGGATGCAGATAATGCAATTAGGT, GTGTCCACCGA, TAGTC
CGTCGTTTGTCATTCACCTCT, AAATTTTCCCCATCGCGCGTCTGCGTAGGTGATAAGTAACGCCTAATCACC
TTAAAAGGGGTAGCGCGCAGAC, CCGGTAAAGCGCG, ATGTCCA, TTGACCATTGTTACGACGGTTCTCTA
TAATTCGTGGGAGCGATTCC, CCGTACACGGGGCCGCATGTCGATTTTTATCCGTACAGAACTCGAAGCA
```

Your task is to reconstruct the original DNA molecule from the fragments.

Answer format: the MD5 hash (lowercase hexa) of the comma separated fragments for the 1st strand in alphabetic order (The solution is unique)

Example: `cfe6feb5eb9e4d176ac4bdc81b722ede` (MD5 for `ACGAT,TGG,CC`).

[My timing: instant]

Answer 608e076ec7d3d9f400369190d880adc7

458 Sum of phi

2018-01-15 03:01:26 **by liuguangxi** **11 xp** **Math**

Let

$$S(n) = \sum_{i=1}^{n}\sum_{j=1}^{n}\varphi(i \cdot j)$$

where $\varphi(x)$ is Euler's totient function.

You are given $S(2) = \varphi(1 \times 1) + \varphi(1 \times 2) + \varphi(2 \times 1) + \varphi(2 \times 2) = 5$, $S(50) = 696335$ and $S(5000) \bmod 1000000007 = 397838656$.

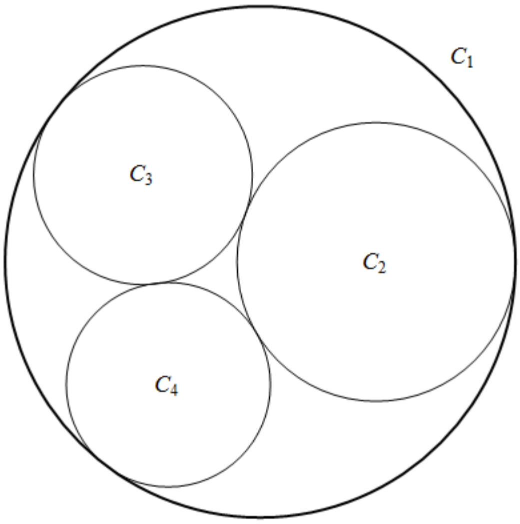
Find $S(50000000) \bmod 1000000007$.

Answer **be569d112c425ee7c155293356912323**

459 Integral circle packings 1

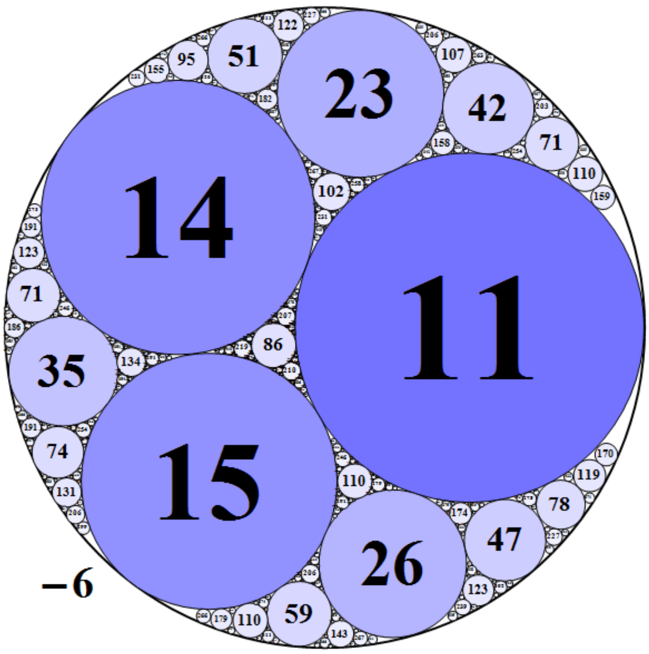
2018-01-29 03:09:00 **by liuguangxi** **9 xp** **Math**

Three circles C_2, C_3 and C_4 are placed inside a larger circle C_1 such that each pair of circles is tangent to one another and the inner circles do not overlap.



We can repeatedly fill the interstices between mutually tangent circles with further tangent circles and generate a circle packing. Denote the curvature of a circle be the reciprocal of its radius. Interestingly, every circle in such a packing would have integer curvature if the curvatures of four mutually tangent circles C_1, C_2, C_3 and C_4 are integers, and we call such a packing an integral circle packing.

The figure below shows an integral circle packing with curvatures of four circles -6, 11, 14 and 15 (each circle is labeled with its curvature and only part of circles are shown).



Define an integer quadruple (a, b, c, d) with $a < 0 < b \le c \le d$ representing the curvatures of initial four mutually tangent circles C_1, C_2, C_3 and C_4, which determines the integral circle packing. The negative value here represents the negative curvature of the bounding circle. For example, the above integral circle packing can be represented as $(-6, 11, 14, 15)$.

If greatest common divisor of a, b, c, d is 1, the quadruple (a, b, c, d) is called **primitive** quadruple. A quadruple (a, b, c, d) is a **root** quadruple if the curvature of any circle other than initial four circles in the circle packing is not less than d. For example, $(-6, 11, 14, 15)$ is a root quadruple but $(-6, 11, 14, 23)$ not, because there exists curvature value 15 in the circle packing which is less than 23.

All curvatures (negative signs are omitted) of an integral circle packing form an infinite integer set. Especially there are infinite primes in the set if the quadruple for packing is primitive. For example, for primitive quadruple $(-3, 4, 12, 13)$, the primes in the set are $3, 13, 37, 61, 109, 157, 181, 229$ and so on. Let $\mathrm{SP}(a, b, c, d, n)$ be sum of distinct primes no more than n in the set with quadruple (a, b, c, d). For instance, $\mathrm{SP}(-3, 4, 12, 13, 100) = 114$. Let $S(t, n)$ be sum of all $\mathrm{SP}(a, b, c, d, n)$ with **primitive root** quadruple (a, b, c, d) satisfying $d \le t$.

You are given $S(10, 100) = \mathrm{SP}(-1, 2, 2, 3, 100) + \mathrm{SP}(-2, 3, 6, 7, 100) + \mathrm{SP}(-3, 5, 8, 8, 100) + \mathrm{SP}(-4, 8, 9, 9, 100) = 299 + 251 + 90 + 147 = 787$.

Find $S(50, 10^7)$.

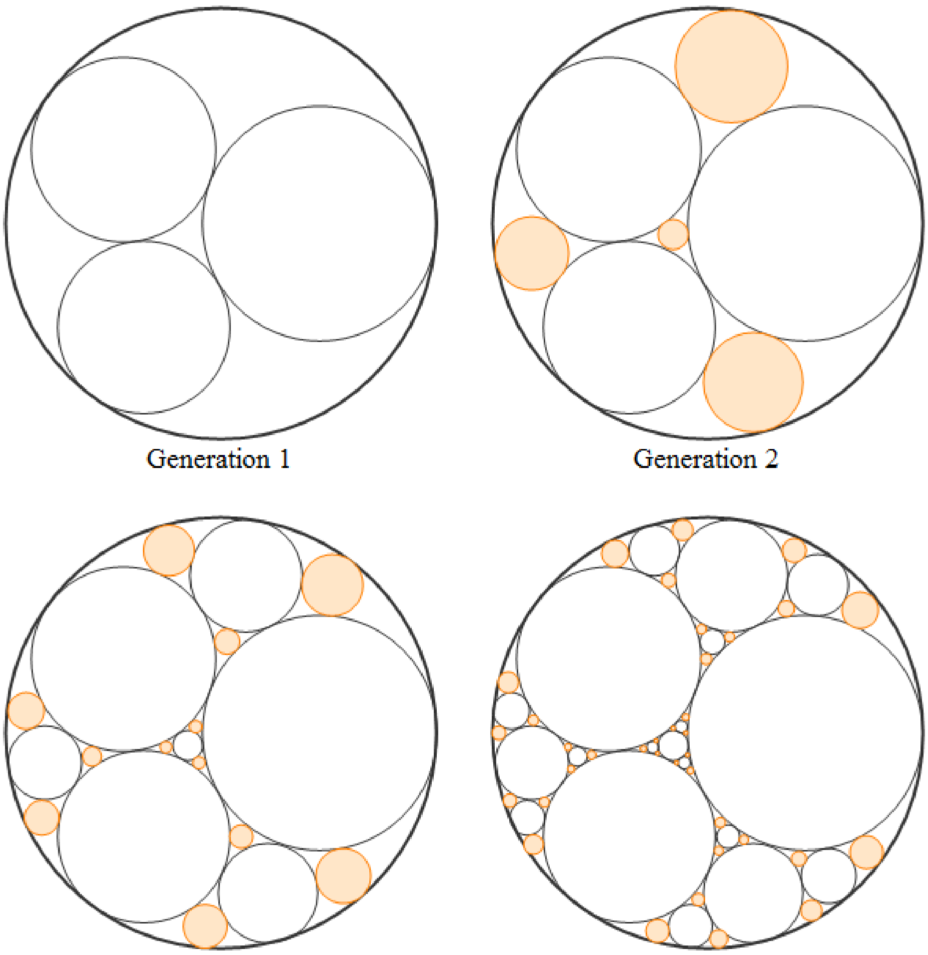
Answer 0be24f127ddbbb3bfc31506ca5052e82

460 Integral circle packings 2

2018-02-05 03:49:55 **by liuguangxi** **10 xp** **Math**

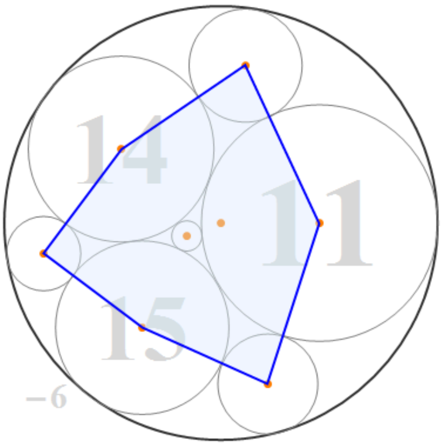
See Problem 459 — Integral circle packings 1 for the concept of integral circle packing and quadruple.

According to the circles generated sequence in a circle packing, define the generation as below: the 4 mutually tangent circles are the first generation; fill the 4 interstices between them with 4 tangent circles forming the second generation; and repeatedly fill the interstices between mutually tangent circles of previous generation with further tangent circles forming the new generation. The circles of the first 4 generations are shown below. Actually the number of new generated circles in the n-th ($n \geq 2$) generation is $4 \times 3^{n-2}$.

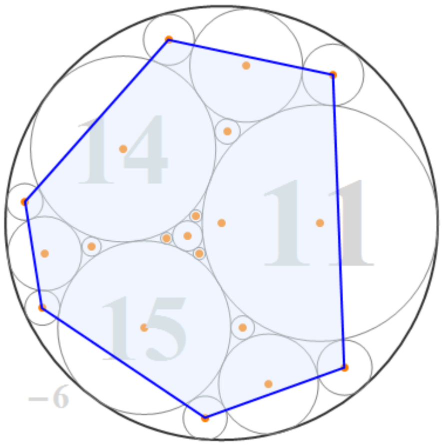


Generation 1 Generation 2 Generation 3 Generation 4

For a certain generation in a circle packing, a convex hull can be formed from the set of center points of all circles. The figures below show the convex hull (blue polygon) of the 2nd and 3rd generation for integral circle packing with quadruple $(-6, 11, 14, 15)$, respectively.



Generation 2



Generation 3

It can be calculated that the area of convex hull of the 2nd generation is 670/23023, and that of the 3 rd generation is 45650065/866875849. In fact, the area of convex hull of any generation for any integral circle packing is a rational number.

Define $R(a, b, c, d, n)$ be the ratio of the area of convex hull of the n-th generation to the area of the bounding circle with quadruple (a, b, c, d). For example, $R(-6, 11, 14, 15, 2) = (670/23023)/(\pi/6^2) = 0.3334767170$, and $R(-6, 11, 14, 15, 3) = (45650065/866875849)/(\pi/6^2) = 0.6034442099$. Obviously the area ratio tends to 1 when n tends to infinity. Let $S(t, n)$ be sum of all $R(a, b, c, d, n)$ with **primitive root** quadruple (a, b, c, d) satisfying $d \leq t$.

You are given $S(10, 3) = R(-1, 2, 2, 3, 3) + R(-2, 3, 6, 7, 3) + R(-3, 5, 8, 8, 3) + R(-4, 8, 9, 9, 3) = 2.2572820988$ (rounded to 10 digits after the decimal point).

Find $S(50, 10)$. Give your answer rounded to 8 digits after the decimal point.

Answer 09ce2ac8475da7e93e4700a552410480

461 Convergents of infinite sum

2018-02-12 03:20:16 **by liuguangxi** **9 xp** **Math**

Define a sequence b_n as below: $b_0 = c, b_n = b_{n-1}^2 - 2\ (n \geq 1)$, where c is a positive integer greater than or equal to 3.

Let infinite sum s be

$$s = \sum_{n=0}^{\infty} \frac{1}{\prod_{k=0}^{n} b_k} = \frac{1}{b_0} + \frac{1}{b_0 b_1} + \frac{1}{b_0 b_1 b_2} + \frac{1}{b_0 b_1 b_2 b_3} + \cdots$$

It can be proved that the infinite sum is convergent and is an irrational number for all possible values of c.

s can be represented as an infinite continued fraction and corresponding convergents are denoted by p_n/q_n ($n \geq 0, p_n$ and q_n are coprime). For example, for $c = 6$, $s = 0.1715728752\cdots$, and the first several convergents are $p_0/q_0 = 0/1, p_1/q_1 = 1/5, p_2/q_2 = 1/6, p_3/q_3 = 5/29$ and so on.

Let $P(c, n)$ and $Q(c, n)$ be numerator and denominator of the n-th convergents p_n/q_n of s with value c, respectively. For instance, $P(6, 3) = 5$, $Q(6, 3) = 29$. Given Fibonacci sequence f_n defined as $f_1 = 1$, $f_2 = 1$, $f_n = f_{n-1} + f_{n-2} (n \geq 3)$. The value of this sequence is no less than 3 from the 4-th item.

Let the sum $\mathrm{SP}(m, n) = \sum_{i=4}^{m} P(f_i, n)$ and $\mathrm{SQ}(m, n) = \sum_{i=4}^{m} Q(f_i, n)$. You are given $\mathrm{SP}(5, 10) = 606$, $\mathrm{SQ}(5, 10) = 2784$, $\mathrm{SP}(10, 100) \bmod 1000000007 = 774200907$, $\mathrm{SQ}(10, 100) \bmod 1000000007 = 830200702$.

Find $\mathrm{SP}(10^5, 10^{18})$ and $\mathrm{SQ}(10^5, 10^{18})$, both modulo 1000000007.

Answer format: [$\mathrm{SP}(10^5, 10^{18})$],[$\mathrm{SQ}(10^5, 10^{18})$]

Example: `774200907,830200702` for $\mathrm{SP}(10, 100)$ and $\mathrm{SQ}(10, 100)$.

Thanks to **czp** for the idea.

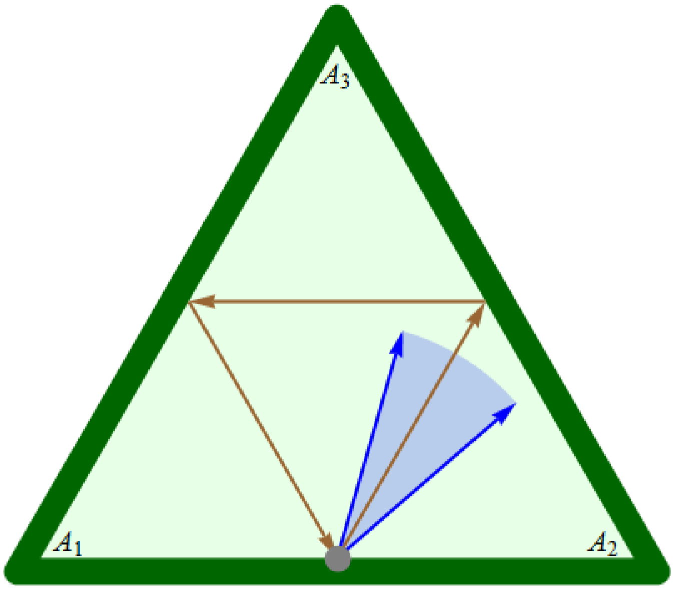
Answer 6007c8c4a4b13b257984183d5ac3944d

462 Polygonal billiards

2018-02-26 02:57:23 **by liuguangxi** **8 xp** **Math**

Given a **regular** n-sided polygon ($n \geq 3$) billiard table and its vertices are labeled $A_1, A_2, \ldots, A_n$ in anticlockwise direction. A billiard ball is placed in the middle of the edge A_1A_2. When it is launched at a certain angle, it keeps moving and reflects off the sides of the table. Suppose the table and the ball are ideal, and reflection and incidence angles are the same when bouncing off the edge.

Let's take an equilateral triangle table ($n = 3$) as an example.



When the ball is launched at angle 60 degrees, it hits the edge A_2A_3 and A_3A_1, and just returns to start point. For the n-sided polygon, there exists a continuous launch angle range such that the ball hits edge $A_2A_3, A_3A_4, \ldots, A_{n-1}A_n$ in turn and return to edge A_1A_2 at last. For the equilateral triangle table, the minimum launch angle is 40.893394649131 degrees, and the maximum one is 73.897886248014 degrees. See blue arrows in above figure.

Let $D(n)$ be the length of launch angle range satisfying above rule for a **regular** n-sided polygon, in degrees. You are given $D(3) = 33.004491598883$, $D(10) = 1.866152295275$ and $D(50) = 0.072102098755$.

Find $\sum_{n=3}^{2018} D(n)$. Give your answer rounded to 10 digits after the decimal point.

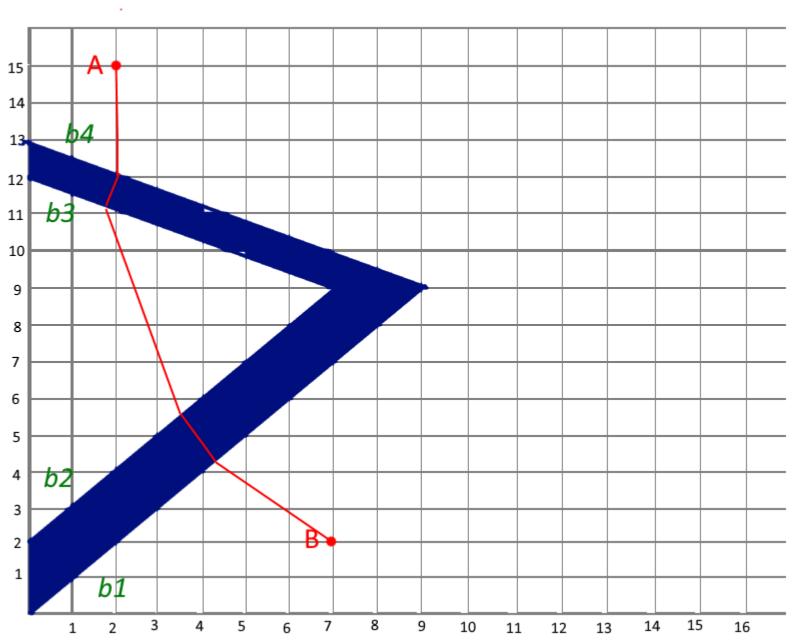
Answer aa97198a9bdb2abe99617ce5e722a290

463 Shortest Road

2017-12-29 01:10:54 **by Philippe_57721** **7 xp** **Math**

A river forming a bend has the following shape:

- bank b_1 passes at points $(0, 0)$ and $(9, 9)$
- bank b_2 passes at point $(0, 2)$ and is parallel to b_1
- bank b_4 passes at points $(0, 13)$ and $(9, 9)$
- bank b_3 passes at point $(0, 12)$ and is parallel to b_4



We want to build a road between points $A(2, 15)$ and $B(7, 2)$. The constraint is that bridges over the river must be perpendicular to the banks. What is the shortest possible distance (A, B)?

Give your answer rounded to 10 digits after the decimal point.

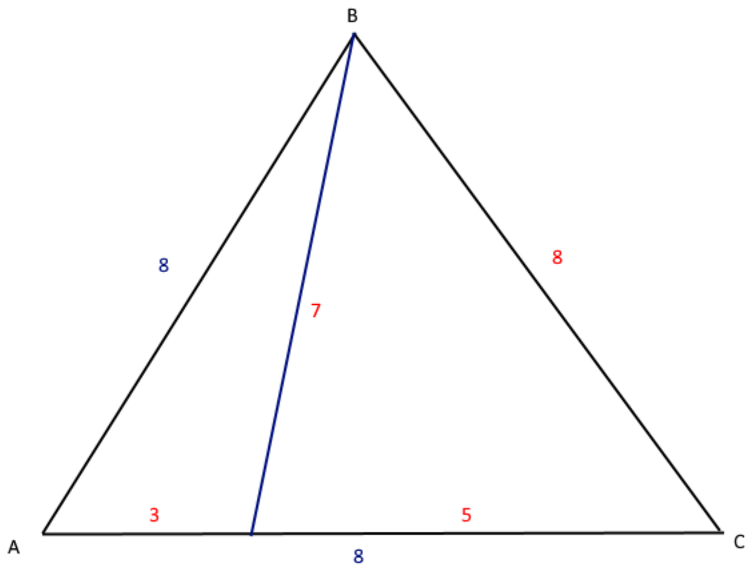
Answer 9a0b515377a1ecd56f43bcc0f64ce85a

464 Decomposable Triangles

2018-01-12 03:23:13 **by Philippe_57721** **7 xp** **Programming**

An integer equilateral triangle is decomposable if it can be split in 2 integer triangles.

For instance, the triangle $(8, 8, 8)$ can be decomposed as $(8, 3, 7) + (8, 5, 7)$.



How many triangles (`T`) with side $\leq 10^7$ are decomposable?
Which triangle (`L`) has the largest number of decompositions? (if multiple possibilities, give the largest one)
How many decompositions (`D`) does it possess?

For instance, the triangle $(2592, 2592, 2592)$ has 3 decompositions:

- $(2592, 2457, 297) + (2592, 2457, 2295)$
- $(2592, 2322, 702) + (2592, 2322, 1890)$
- $(2592, 2268, 972) + (2592, 2268, 1620)$

Answer format: `T,L,D`

You are given: `364,960,8` for triangles with side ≤ 1000.

[My timing: 20 sec]

Answer 655da325d3f92fe58efc3e679e875638

465 Random in 3-4-5

2018-03-07 00:41:43	by sinan	8 xp	Probability

ABC is a right triangle with sides 3, 4 and 5. There are 3 random points (D, E and F) on the three sides assumed to be uniformly distributed: D on AB, E on BC and F on AC.

Draw the three lines segments from vertices to these random points: AE, BF and CD. The intersection of AE and BF is the point P. The intersection of AE and CD is the point Q. The intersection of CD and BF is the point R.

What is expected area of the PQR triangle?

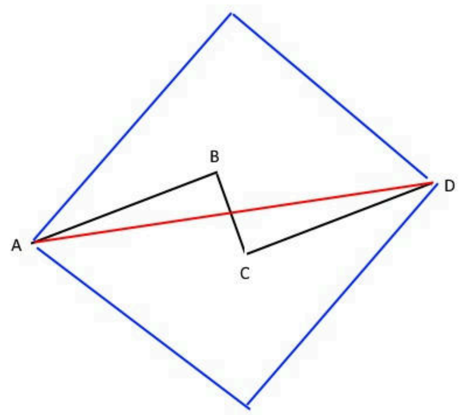
Answer format: decimal number rounded to 10 digits after the decimal point.

Answer 5187e5172794f0b5944c7837190ef6af

466 Special Squares

2018-01-19 01:03:29 **by Philippe_57721** **7 xp** **Programming**

Two segments AB and CD of length a are joined by a perpendicular BC of length b. The segment AD is the diagonal of a square of side c.



For instance, if $a = 23$ and $b = 94$, then c is an integer $c = 74$.

In the picture, points B and C fall inside the square, but it is not necessarily the case in general.

How many coprime pairs (a, b) produce an integer side square where the side $c \leq 10^7$?

Answer format: count, $\sum a, \sum b$

You are given: `161,36737,73474` for a threshold of 1000.

[My timing: 1 sec]

Answer 50460f0a65840681106e359bb63ac032

467 Magical Beans

2018-01-26 08:02:43 **by Philippe_57721** **6 xp** **Probability**

Jack has 10 magical beans.

On day 0, he plants his beans in his garden.

Each day, each bean has a probability $1/2$ to grow into a beanstalk.

What is the expected number of days required for all the beans turn into beanstalks?

Answer format: `P/Q` where P and Q are coprime

Answer 65fb2fffe031797b89481edd94bdf636

468 Displacement of permutation

2018-03-05 01:47:52 **by liuguangxi** **7 xp** **Math**

Let p be a permutation of the set $\{1, 2, 3, ..., n\}$. We call the sum $D_k(p) = \sum_{i=1}^{n} |p_i - i|^k$ the total displacement with k-th power of p. For example, if $p = (4, 2, 1, 3)$, $D_1(p) = |4-1| + |2-2| + |1-3| + |3-4| = 6$, $D_2(p) = |4-1|^2 + |2-2|^2 + |1-3|^2 + |3-4|^2 = 14$, $D_3(p) = |4-1|^3 + |2-2|^3 + |1-3|^3 + |3-4|^3 = 36$.

For any positive integer n, assume every possible permutation p of the set $\{1, 2, 3, ..., n\}$ is uniformly distributed. Let $E(n)$ be the expected value of $D_1(p) + D_2(p) + D_3(p)$. You are given $E(2) = 3$, $E(3) = 40/3$, $E(5) = 432/5$, and all $E(n)$ are rational number. Let $F(n)$ be the numerator of $E(n)$ with $E(n)$ be fraction in lowest terms. So, $F(2) = 3$, $F(3) = 40$, $F(5) = 432$ and so on.

Let $S(n) = \sum_{i=1}^{n} F(i)$. Find $S(12345678987654321) \bmod 1000000007$.

Thanks to **baihacker** for the idea.

Answer **c72510d48521a0d462bdf106d0ad3e10**

469 Permutation Order I

2018-02-02 01:50:24 **by Philippe_57721** **7 xp** **Programming**

Let $\mathfrak{S}(n)$ be the set of all permutations of $[1..n]$.

The order of a permutation p is the smallest integer e such as $p^e = \mathrm{Id}$. We apply the permutation e times to itself and get the identity permutation $\{1, 2, ..., n\}$.

For $\mathfrak{S}(4)$:
$\mathrm{Order}(1\ 2\ 3\ 4) = 1$
$\mathrm{Order}(1\ 2\ 4\ 3) = 2$
$\mathrm{Order}(1\ 3\ 2\ 4) = 2$
$\mathrm{Order}(1\ 3\ 4\ 2) = 3$
$\mathrm{Order}(1\ 4\ 2\ 3) = 3$
$\mathrm{Order}(1\ 4\ 3\ 2) = 2$
$\mathrm{Order}(2\ 1\ 3\ 4) = 2$
$\mathrm{Order}(2\ 1\ 4\ 3) = 2$
$\mathrm{Order}(2\ 3\ 1\ 4) = 3$
$\mathrm{Order}(2\ 3\ 4\ 1) = 4$
$\mathrm{Order}(2\ 4\ 1\ 3) = 4$
$\mathrm{Order}(2\ 4\ 3\ 1) = 3$
$\mathrm{Order}(3\ 1\ 2\ 4) = 3$
$\mathrm{Order}(3\ 1\ 4\ 2) = 4$
$\mathrm{Order}(3\ 2\ 1\ 4) = 2$
$\mathrm{Order}(3\ 2\ 4\ 1) = 3$
$\mathrm{Order}(3\ 4\ 1\ 2) = 2$
$\mathrm{Order}(3\ 4\ 2\ 1) = 4$
$\mathrm{Order}(4\ 1\ 2\ 3) = 4$
$\mathrm{Order}(4\ 1\ 3\ 2) = 3$
$\mathrm{Order}(4\ 2\ 1\ 3) = 3$
$\mathrm{Order}(4\ 2\ 3\ 1) = 2$
$\mathrm{Order}(4\ 3\ 1\ 2) = 4$
$\mathrm{Order}(4\ 3\ 2\ 1) = 2$

What is the largest order for the elements of $\mathfrak{S}(85)$?
For which permutation do we get this order? Give the index in lexicographic order (0-origin) of the 1-st one.

Answer format: Maximal order,Index modulo 10^9

Example: `420,257453673` for $\mathfrak{S}(20)$.

[My timing: 40 sec]

Answer 843d31bf2feb25024458949644f20a5b

470 Permutation Order II

2018-02-08 21:25:16	by Philippe_57721	7 xp	Programming

See Problem 469 — Permutation Order I for definitions.

Let $\mathfrak{A}(n,p)$ be the number of permutations of $[1..n]$ with order p.

We have:
$\mathfrak{A}(7,1) = 1$
$\mathfrak{A}(7,2) = 231$
$\mathfrak{A}(7,3) = 350$
$\mathfrak{A}(7,4) = 840$
$\mathfrak{A}(7,5) = 504$
$\mathfrak{A}(7,6) = 1470$
$\mathfrak{A}(7,7) = 720$
$\mathfrak{A}(7,10) = 504$
$\mathfrak{A}(7,12) = 420$

Find the last 10 significant (before trailing 0s) digits of $\mathfrak{A}(1000,641)$.

[My timing: < 100 ms]

Answer 51e5b0e2ca5d7900153312a44552534a

471 Kimberling Sequence

2018-02-23 01:12:30 **by Philippe_57721** **6 xp** **Programming**

The Kimberling sequence is defined as follow.

We start with $S_0 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...\}$. We build S_{k+1} from S_k: for i in $[1..k]$, we take $S_k[k+i]$ and $S_k[k-i]$, ignoring $S_k[k]$, then we take the rest of the sequence.

The first iterations are:
0 $(\mathbf{1})$ 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
1 2 $(\mathbf{3})$ 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
2 4 2 $(\mathbf{5})$ 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
3 6 2 7 (4) 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
4 8 7 9 2 $(\mathbf{10})$ 6 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
5 6 2 11 9 12 $(\mathbf{7})$ 13 8 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
6 13 12 8 914 11 $(\mathbf{15})$ 2 16 6 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
7 2 11 16 14 6 917$(\mathbf{8})$ 18 12 19 13 20 21 22 23 24 25 26 27 28 29 30...
8 18 17 12 9 19 6 13 14 $(\mathbf{20})$ 16 21 11 22 2 23 24 25 26 27 28 29 30...
9 16 14 21 13 11 6 22 19 2 $(\mathbf{9})$ 23 12 24 17 25 18 26 27 28 29 30...
10 23 2 12 19 24 22 17 6 25 11 $(\mathbf{18})$ 13 26 21 27 14 28 16 29 30...

The Kimberling sequence is formed by the diagonal elements.
$K = 1, 3, 5, 4, 10, 7, 15, 8, 20, 9, 18, 24, 31, 14, 28, 22, 42, 35, 33, 46, 53, 6, 36, 23, 2, 55, 62, 59, 76, 65, ...$

It is conjectured that K contains exactly all the integers.

Let $O(n)$ the index of the 1st occurrence of n in K: $K_{O(n)} = n$.

One can verify that:
$O(2) = 25$
$O(3) = 2$
$O(19) = 49595$

Find $O(16268)$.

[My timing: 20 sec]

Answer dff7734f7b9c3f737334217b8f250ced

472 Palindromic Infinite Sequence

2018-02-15 21:52:59	by Philippe_57721	9 xp	Programming

An infinite sequence $\{S_1, S_2, S_3, S_4, ...\}$ is said palindromic if for every n_0 there is $n \geq n_0$ such as $\{S_1, S_2, S_3, S_4, ..., S_n\}$ is palindromic.

It can be proved that for any irrational α, the sequence $\lfloor \alpha \cdot n \rfloor - \lfloor \alpha \cdot (n-1) \rfloor$ for $n \geq 1$ is palindromic. $\lfloor x \rfloor$ is the integer part of x.

For the irrational $\mathrm{e} = 2.71828\cdots$, this sequence is palindromic for following indexes:

- 1 – 1
- 2 – 4
- 3 – 11
- 4 – 18
- 5 – 25
- 6 – 32
- 7 – 71

Find the 10^9-th index. Give your answer modulo 10^{10}.

You are given `3352820698` the 10^2-th index.

[My timing: 15 sec]

Answer af3a30e52d39ebf8c439e0cf7d406185

473 An Unusual Development

2018-03-08 21:32:37 **by Philippe_57721** **6 xp** **Programming**

It can be proved that any real number $x \in (0,1]$ can be written uniquely in the form:

$$x = \frac{1}{a_1} - \frac{1}{a_1 \times a_2} + \frac{1}{a_1 \times a_2 \times a_3} - \cdots$$

a_i being a sequence of strictly increasing integers.

For example, $\mathrm{e} - 2 = \frac{1}{1} - \frac{1}{1\times3} + \frac{1}{1\times3\times6} - \frac{1}{1\times3\times6\times14} + \frac{1}{1\times3\times6\times14\times142} - \cdots$.

What is a_{100} when developping the Champernowne constant $C = 0.123456789101112\cdots$.

Give your answer in compressed form:
`10 first digits[omitted digits count]10 last digits`

Example: `1701411834[19]5884105727` for $M_{127} = 2^{127} - 1$.

[My timing: 7 sec]

Answer 99ed4b98170957f55ea00877eb7d03b2

474 Palindromic Decomposition

2018-03-02 00:46:23 **by Philippe_57721** **8 xp** **Programming**

It has recently been proved (2016) that:
For $g \geq 5$ every integer can be decomposed as a sum of 3 palindromes in base g.

For instance: $123456789 = 40904 + 4708074 + 118707811$ (base 10).

How many such decompositions $n = x + y + z$ and $0 < x \leq y \leq z$ are there for $n = 314159263$ (in base 10).

Answer format: `count,x,y,z`
decomposition with the largest x

[My timing: 15 sec]

Answer 9e3cc28a763f44eb1705dd6e23957767

475 Magic Prime Square

2018-04-06 00:50:46 **by Philippe_57721** **6 xp** **Programming**

$\begin{bmatrix} 11 & 29 & 43 & 73 \\ 31 & 67 & 17 & 41 \\ 61 & 13 & 59 & 23 \\ 53 & 47 & 37 & 19 \end{bmatrix}$ is the first (in lexicographic order) 4×4 magic square using the smallest distinct prime numbers > 10.

Find the first (in lexicographic order) 4×4 magic square using the smallest distinct prime numbers > 100.

Answer format: comma separated list of the matrix elements

Example: `11,29,43,73,31,67,17,41,61,13,59,23,53,47,37,19`

[My timing: 70 sec]

Answer c1bb823df7f282eadbaaf67a6fec0a45

476 Records in Good Sequences

2018-03-23 01:55:15 **by Philippe_57721** **6 xp** **Programming**

A sequence S of positive integers, not necessarily distinct is a good sequence if:
- when $k \in S$, then $(k-1) \in S$ for $k \geq 2$
- the first occurence of $k-1$ in S appears before the last occurence of k

Examples: $\{3,1,3,1,2\}$ is not a good sequence: the 1st occurence of 2 is after the last 3.

The following sequences are good:
$\{2,2,1,3,2\}$
$\{1,3,2,1,3\}$
$\{3,1,1,2,3\}$
$\{3,2,3,1,2\}$
$\{1,2,1,3,1\}$

In a sequence $S=\{a_1,a_2,a_3,...,a_n\}$, a_j is a record if it is greater than all elements before it: $\forall i<j, a_i<a_j$.

The sequence $\{4,7,5,1,6,8,2,3\}$ contains 3 records: $4,7,8$.

Amongst all good sequences with 100 elements in $\{1,2,3,...,100\}$, how many have 3 records?

Answer format: give the last 10 significant digits
(before trailing 0s)

Answer ee71fd30f11214e88c6ac0eabcb4762c

477 Maxima in Good Sequences

2018-03-29 22:58:30	**by Philippe_57721**	**6 xp**	**Programming**

See Problem 476 — Records in Good Sequences for definitions.

Amongst all the good sequences with 100 elements in $\{1, 2, 3, \ldots, 100\}$, how many have a maximum element $= 50$?

Answer format: give the answer modulo 10^{10}

Answer 0555dd3414c53c10e729504fb2e94404

478 A > B > C > A

2018-03-16 01:44:30 **by Philippe_57721** **7 xp** **Programming**

Given 2 dice A, B, we define $\mathfrak{P}(A, B)$ as the probability of rolling a strictly higher value with A than B. If $A = \{2, 2, 4, 4, 9, 9\}$ and $B = \{1, 1, 6, 6, 8, 8\}$ we have $\mathfrak{P}(A, B) = \frac{5}{9} > \frac{1}{2}$.

Thus the probability to win with A is greater than with B. We write: $A > B$.

We add a third die $C = \{3, 3, 5, 5, 7, 7\}$. We have: $\mathfrak{P}(B, C) = \frac{5}{9}$, $\mathfrak{P}(C, A) = \frac{5}{9}$.

Therefore: $A > B > C > A$.

A triple of dice is called non-transitive if: $A > B > C > A$, $\mathfrak{P}(A, B) = \mathfrak{P}(B, C) = \mathfrak{P}(C, A)$.

Of course if (A, B, C) is such a triple, so is (B, C, A) and (C, A, B). These 3 triples are considered equivalent.

We consider dice with values in range $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

How many dictinct non-transitive triples are there?
There are triples where all faces for A have a distinct value. What is the last one in lexicographic order?
Between the 3 possible representation for a triple (A, B, C), (B, C, A) and (C, A, B), we choose the one where A is the first in lexicographic order.

Answer format: `Count/(die1)(die2)(die3)`

Example: `9999/(9,9,9,9,9,9)(9,9,9,9,9,9)(9,9,9,9,9,9)`

You are given there are 104 distinct non-transitive triples for the set $\{1, 2, 3, 4, 5, 6\}$.

[My timing: 80 sec]

Answer a83d0d923d03665982e1f8ed9be964ac

479 Interesting numbers

2018-02-27 09:25:56	by sinan	7 xp	Math

Let n be a number with $2d$ digits. Let a be the first d digits of this number and b the last d digits.

For example:
$n = 140400 \quad d = 3 \quad a = 140 \quad b = 400$

This number has an interesting property such that $n = b^2 - a^2$.

How many numbers less than 10^{50} have this interesting property?

Answer format: $\text{cnt}, \sum n$

[My timing: ~ 2 s]

Answer 4a7a6cfc44e94cf0181dd3e9bb783796

480 Counting modulo pairs

2018-03-12 02:35:13	by liuguangxi	10 xp	Math

Let $f(m)$ denote the number of integer pairs (x, m) such that there exists a positive integer y satisfies $x^y \equiv 0 \pmod{m}$, where x and m are both positive integers and $1 \leq x \leq m$. For example, $f(12) = 2$, as two pairs $(6, 12)$ and $(12, 12)$ meet the condition.

Define $S(n) = \sum_{m=1}^{n} f(m)$. You are given $S(10) = 16$ and $S(1000) = 5764$.

Find $S(10^{14})$.

Thanks to **baihacker** for the idea.

Answer e29f327859e90507b3a26a7a57f6d440

481 Subset of a Farey Sequence

2018-04-13 00:52:42 **by Philippe_57721** **7 xp** **Programming**

Let $\gamma = 0.5772156649\cdots$ the Euler-Mascheroni constant.

How many terms in the Farey sequence of order n are there such as: $\frac{a}{b} < \gamma$ for $n = 10^9$?

You are given 175452524024 for $n = 10^6$.

[My timing: 30 sec]

Answer 417e0c2cde5c9281a937880014d17a4b

482 Prüfer Code I

2018-04-20 01:03:36 **by Philippe_57721** **9 xp** **Programming**

The Prüfer code is a very compact way to represent a labeled tree.

We can represent a tree as a string with the following rules: (Pseudo BNF)
```
tree = node | node '(' tree [ ',' tree ] ')'
node = an integer
```

For examples:
```
1(2(3(4,5)))
1(2(3),4,5)
```

Consider a labeled tree with n nodes labeled $1, 2, ..., n$ in sequential order.

We can see its Prüfer code as the base 10 representation of some integer.

For instance, the tree `1(2(3,4(5)),6(7),8)` has the following Prüfer code $\{2, 4, 2, 1, 6, 1\} = 242161$.

Find all labeled trees with 10 nodes whose Prüfer code correspond to a prime number. Among these trees find those whose Prüfer code contain distinct values.

Answer format: sum of the values,(`/` delimited list of the trees in string representation — lexicographic order)

Example: `11006162,1(2(3(4(5(6)))),7(8))/1(2(3(4(5))),6(7(8)))` for 8 nodes.

[My timing: instant]

Answer 2f744e2ab79523f6a11f655e0d1f6376

483 Prüfer Code II

2018-05-04 00:55:24	by Philippe_57721	7 xp	Programming

For definitions See problem 482.

The file rc481.txt contains 1000 Prüfer codes.

Which code produces the labeled tree whose string representation is the largest in lexicographic order?

Answer format: the string representation corresponding to that code

Example:
`1(3,9(8(25(28(26(7(12(10(19(4,6),24(11,17))),20(14(21(5,23(13(29)))))),22)))),18(15)),30(16(2),27))`
This is the string representation for the 1st record of the file.

[My timing: instant]

Answer 8e1bdb9dce388b5a26ee47512ed66a3d

484 Period of permutation

2018-03-25 22:18:33	by liuguangxi	10 xp	Math

Let $i_1, i_2, i_3, \ldots, i_n$ be a permutation of the set $\{1, 2, 3, \ldots, n\}$ where n is a positive integer. A permutation can also be viewed as a function f. We denote this permutation by the 2-by-n array

$$f = \begin{pmatrix} 1 & 2 & \cdots & n \\ i_1 & i_2 & \cdots & i_n \end{pmatrix}$$

It maps number 1 to i_1, number 2 to i_2 and so on. When this function operates on a permutation of set $1, 2, 3, \ldots, n$, that is $x_1, x_2, x_3, \ldots, x_n$, a new permutation $y_1, y_2, y_3, \ldots, y_n$ can be obtained. Here $y_1 = f(x_1), y_2 = f(x_2), \ldots, y_n = f(x_n)$.

For example, when $n = 4$ and there is a permutation $2, 4, 1, 3$. We make this permutation operate on a permutation $1, 2, 3, 4$ continuously, and it will be

$$1, 2, 3, 4 \to 2, 4, 1, 3 \to 4, 3, 2, 1 \to 3, 1, 4, 2 \to 1, 2, 3, 4 \to \cdots$$

Obviously after 4 operations the result permutation returns to the initial one.

Let $F(l, p)$ be the number of permutations of the set $\{1, 2, 3, \ldots, l\}$ which makes the permutation $1, 2, 3, \ldots, l$ returns to itself after p operations. For instance, $F(4, 3) = 9$, as there are 9 permutations meet the condition: $(1, 2, 3, 4)$, $(1, 3, 4, 2)$, $(1, 4, 2, 3)$, $(2, 3, 1, 4)$, $(2, 4, 3, 1)$, $(3, 1, 2, 4)$, $(3, 2, 4, 1)$, $(4, 1, 3, 2)$, $(4, 2, 1, 3)$. You are also given $F(5, 5) = 25$, $F(10, 6) = 625176$, $F(20, 11) = 609493248001$.

Let $S(L, P) = \sum_{l=1}^{L} \sum_{p=1}^{P} F(l, p)$. Find $S(10000, 1000) \bmod 1000000007$.

Thanks to **baihacker** for the idea.

Answer d601163db5ef263ef2c847eeddfdf98e

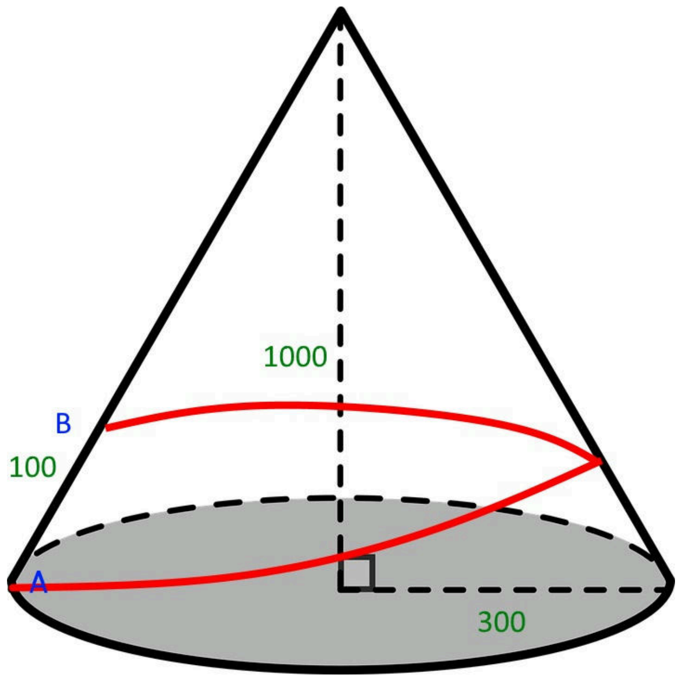
485 Climbing the mountain

2018-04-27 02:29:55 **by Philippe_57721** **7 xp** **Math**

A mountain has the shape of a cone. It has a height of 1000 and the radius of the base is 300.

A trail with the shortest length possible goes from point A to point B (A and B are on the same slant at a distance of 100).

We can show that the trail starts to go uphill then downhill.



What are the length of the uphill part (`u`) and of the downhill part (`d`)?

Answer format: `u,d`
rounded to 10 digits after the decimal point

Answer c78068a9140737f81e443204377d5bb2

486 Product of tuple

2018-04-01 23:38:33	by liuguangxi	10 xp	Math

Let $S(p, n)$ denote the number of n-tuples of positive integers whose product is no larger than p. For example, $S(4, 2) = 8$, as there are eight 2-tuples $(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (3, 1), (4, 1)$ meet the condition. You are also given $S(6, 4) = 39$, $S(10, 10) = 571$, $S(100, 100) \bmod 1000000007 = 959337187$.

Find $S(10^{12}, 10^{12}) \bmod 1000000007$.

Thanks to **baihacker** for the idea.

This problem was inspired by Problem 452 at Project Euler (https://projecteuler.net/problem=452).

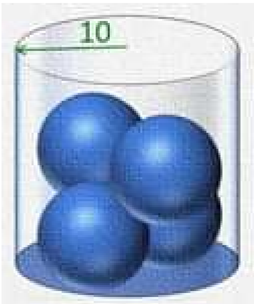
Answer 986a23786b940f1dfdd3ca8e6b373aa5

487 Crammed spheres

2018-05-12 00:43:18	by Philippe_57721	6 xp	Math

A cylinder has a radius of 10.

We place 4 spheres of radius 5 inside the cylinder.



What is the volume of water needed to cover exactly the spheres?

Give your answer rounded to 10 digits after the decimal point.

Answer 18d73f932a250ff94b1c87351b8ef939

488 Leading digits of a power

2018-05-18 07:23:59	by Philippe_57721	7 xp	Programming

Let e_n the sequence of integers such as the decimal development of 2^{e_n} starts with the following digits: $66666\cdots$

For instance we have:
$e_1 = 224296 \rightarrow 2^{224296} = 66666468242741993302\cdots$
$e_5 = 620220 \rightarrow 2^{620220} = 66666969606590729636\cdots$

What is $e_{1000000}$?

[My timing: 40 sec]

Answer 4a2302d414aee38953bdb632eb67d0da

489 Good sequences revisited

2018-05-03 03:54:42 **by liuguangxi** **10 xp** **Programming**

Let $F(N, M, R, D)$ be the number of good sequences which meet the following condition:
(1) The sequence has N elements
(2) The maximum element in the sequence is equal to M
(3) The sequence contains R records
(4) Each number occurs no more than D times in the sequence

The good sequence and records are the same definition in Problem 476.

For example, $F(4, 2, 2, 2)$ is 3. Only three sequences $\{1, 1, 2, 2\}$, $\{1, 2, 1, 2\}$, $\{1, 2, 2, 1\}$ meet the conditions:
(1) 4 elements
(2) Maximum element = 2
(3) 2 records
(4) Each number occurs no more than 2 times

Notice $\{1, 2, 1, 1\}$ is not in $F(4, 2, 2, 2)$ because number 1 occurs 3 times; $\{2, 1, 2, 2\}$ is not in $F(4, 2, 2, 2)$ because it contains only 1 record and number 2 occurs 3 times; $\{2, 3, 1, 2\}$ is not in $F(4, 2, 2, 2)$ because the maximum element is 3.

You are given $F(11, 8, 3, 2) = 394581$ and $\mathrm{F}(20, 13, 7, 4) \bmod 1000000007 = 342894563$.

Find $F(80, 47, 13, 10) \bmod 1000000007$.

Thanks to **C_K_Yang** for the idea.

Answer a324b65544963902c1917142fca1a9df

490 Index of a Farey sequence

2018-04-25 17:30:36	by liuguangxi	8 xp	Math

Let $F_{\text{idx}}(n,k)$ denote the k-th term in the Farey sequence of order n. For example, the Farey sequence of order 5 is $\{0/1, 1/5, 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5, 1/1\}$, and the 7-th term is $3/5$, so $F_{\text{idx}}(5,7) = 3/5$. You are also given $F_{\text{idx}}(12,31) = 7/11$, $F_{\text{idx}}(123,3141) = 82/121$.

Find $F_{\text{idx}}(1234567890, 314159265358979323)$. The answer should be represented as an irreducible fraction.

Thanks to **czp** for the idea.

Answer 673f681e97557a3fddaf4efcdc1f8dae

491 Generalized Pythagorean Equation

2018-05-26 03:22:08 **by Philippe_57721** **7 xp** **Programming**

Consider the Diophantine equation $x^2 + 13xy + y^2 = z^2$.

How many solutions are there with $0 < x < y < 10^7$ (x, y, z coprime).

Answer format: count, $\sum x, \sum y, \sum z$

You are given: `28989,580750261,1448965526,3481052101` for a threshold of 10^5.

[My timing: 20 sec]

Answer 157c3c24ffc82eebc393f394e27b8cd8

492 A self referential sequence

2018-07-27 00:50:30 **by Philippe_57721** **8 xp** **Programming**

The sequence $G = \{1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, ...\}$ is defined by: G_n = number of occurences of n in the sequence.

$G_4 = 3 \Rightarrow$ the number 4 occurs 3 times in the sequence.

What is $m = G_{123456789101112}$?
Find the smallest n such as $G_n = 123456654321$.

You are given `1684,128276792` for `123456,123321`.

Answer format: `m,n`

[My timing: 1.5 sec]

Answer 21810181572b26acc2d08a5d9d11d22d

493 Prime permutations

2018-06-01 00:36:41 **by Philippe_57721** **5 xp** **Programming**

A permutation P of $\{1, 2, ..., n\}$ is said prime if:

- $P_1 = 1$
- $P_n = n$
- $\forall i < n$, $P_i + P_{i+1}$ is a prime number

$\{1, 6, 5, 8, 9, 2, 3, 4, 7, 10\}$ is a prime permutation.

How many prime permutations are there for $n = 21$?

[My timing: 30 sec]

Answer bcd5fb98fc1a168dc3960a3fa3e5b254

494 High degree Diophantine equation

2018-06-15 02:03:24 **by Philippe_57721** **5 xp** **Math**

How many triples (x, y, z) with $0 < x < y < z \leq 123456789$ and (x, y, z) coprime are there such as: $(x^4 + y^4 + z^4)^2 = 2 \times (x^8 + y^8 + z^8)$.

Answer format: `count`

[My timing: 3 sec]

Answer 1e526b820e3747f9338d1f97ae208f59

495 Sum of factorials

2018-06-21 23:29:59 **by liuguangxi** **9 xp** **Math**

Let

$$S(n) = \sum_{k=1}^{n} k!$$

You are given $S(10) = 4037913$, $S(100) \bmod 1000000000039 = 947219250081$.

Find $S(314159265358) \bmod 1000000000039$.

Answer 7d2c11b3facf37df69813c5695521c3b

496 Fuzzy squaring

2018-06-08 01:22:11 **by Philippe_57721** **7 xp** **Programming**

Let x_0 a rational number > 1. We define $x_{n+1} = x_n \cdot \lceil x_n \rceil$ where $\lceil x \rceil$ is the smallest integer $\geq x$.

It is conjectured that $\forall x_0, x_n$ is eventually an integer for n large enough.

For instance, if $x_0 = \frac{8}{7}$, $x_1 = \frac{16}{7}$, $x_2 = \frac{48}{7}$, $x_3 = 48$. The sequence starting with $\frac{8}{7}$ produces an integer after 3 steps.

How many steps are necessary if we start with $\frac{3012}{3011}$?

You are given 18 steps for $\frac{6}{5}$.

[My timing: 5 sec]

Answer dac79350b1eb7b44c7b6bc35c14ac824

497 m-excellent pairs

2018-07-06 02:55:39 **by Philippe_57721** **6 xp** **Math**

Let ν a positive irrational number and m a positive integer.

A pair of integers (a, b) is said m-*good* if $a \cdot \lceil \nu \cdot b \rceil - b \cdot \lfloor \nu \cdot a \rfloor = m$ ($\lceil x \rceil$ and $\lfloor x \rfloor$ are respectively the smallest integer $\geq x$ and the largest integer $\leq x$).

A pair of positive integers (a, b) is said m-*excellent* if (a, b) is m-*good* and neither $(a - b, b)$ nor $(a, b - a)$ are m-*good*.

How many m-*excellent* pairs ($m = 123456789101112$) are there if $\nu = 0.5772156649015314\cdots$?

Answer **3a6481020729c7c00ef44da6fe7533ad**

498 Streaks

2018-06-11 02:57:46	by sinan	5 xp	Probability

This problem is about Freecell streaks (in this site in particular).

A run of wins is called a **streak**.

The standard Freecell game has 8 columns of cards and 4 free cells. This version is commonly referred to as 8×4. There is a wide range of other variants too. The number of columns of cards may range from 4 –13, and the number of free cells may range from zero to 10. Thus, a game with 13 columns of cards and no free cells is referred to as a 13×0.

The game is currently scaled so that when your streak is small, you are automatically given somewhat easier games. As your streak increases, the difficulty level will go up until at a streak of 50 you will be playing complete random deals (difficulty level 10). Here's the progression:

Streak	Difficulty
0–9	5
10–19	6
20–29	7
30–39	8
40–49	9
≥ 50	10 (random)

There is a unique degree of "winnability" for each variant. 12-sum games such as the standard 8×4 are usually pretty winnable. But when you trend down into the < 10 sum games and such, winnability gets pretty dicey. 5×4 games, a 9-sum variant, are only rarely winnable.

At any level there are 2^{15} different games for any variant (numbered 0 to 32767).

What is average streak length for 10×2 variant assuming you can win any "winnable" game dealed randomly? The number of unwinnable games by level for 10×2 given as below:

Level	Count
5	1
6	2
7	5
8	6
9	12
10	18

Input format: streak length rounded to 15 digits after the decimal point

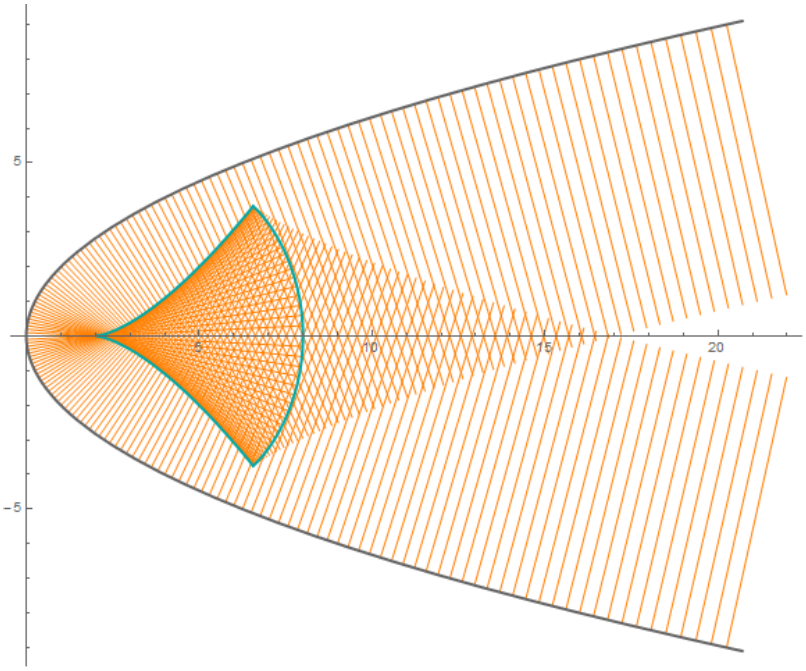
See also: streaks info

Answer e8780c0dc0620e401b1f6cd9b82cfc96

499 Ginkgo leaf

2018-08-01 22:40:43	**by liuguangxi**	**8 xp**	**Math**

There is a parabola with equation $y^2 = 4x$. A line segment of length 8 moves along the parabola, with one end on the curve, and always perpendicular to the curve. The figure below shows the trajectory of the line segment when one end of it moving through the whole curve.



In the whole process of moving the line segment, some points were swept exactly 3 times. Interestingly, these points form a connected region which looks like a ginkgo leaf (blue line around in the figure).

Find the area of the "ginkgo leaf". Give your answer rounded to 10 digits after the decimal point.

Thanks to **czp** for the idea.

Answer 3e1171b9016f01cb75e9879acf4ef5a2

500 Binary function

2018-07-09 03:33:48 **by liuguangxi** **8 xp** **Math**

Let binary function $f(x, y) = (xy - 1)/(x + y - 1)$. There are infinite (x, y) pairs of positive integers such that $f(x, y)$ is integer. For example, $f(1, 1) = 0$, $f(3, 5) = 2$, $f(5, 17) = 4$ and so on.

Find all such (x, y) pairs for $1 \le x \le y \le 50000000$.

Answer format: [number of pairs],[sum of x of all pairs],[sum of y of all pairs]

Example: `258,56772,129736` for $1 \le x \le y \le 1000$.

Answer 238cc36edad2557dc0b5a2c14ad1742a

501 Hyper factorial

2018-07-13 02:46:55 by Philippe_57721 **7 xp** Programming

We define the hyper factorial function $!^!$ as $n!^! = 1^1 \times 2^2 \times \cdots \times n^n$.

What is $200000000!^! \bmod 2147483647$.

You are given: $60629238 \equiv 1000000!^! \bmod 2147483647$.

[My timing: 60 sec]

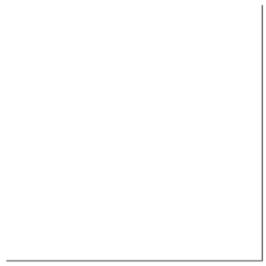
Answer 6285e28a9b337838bac4d894a3e399d7

502 Dragon curve

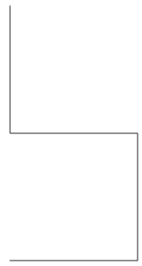
2018-08-03 02:08:54 **by Philippe_57721** **8 xp** **Programming**

The Dragon curve is a fractal curve obtained by repeatedly appending a rotated version of itself.

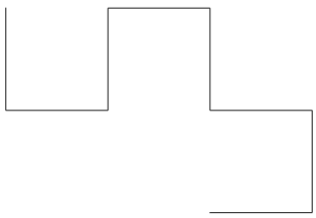
Here are the first 4 iterations of this curve:



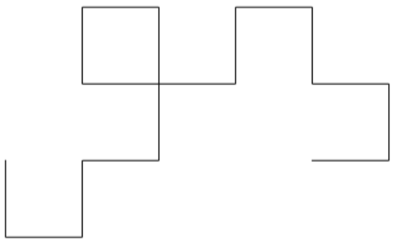
Iteration **1**



Iteration **2**



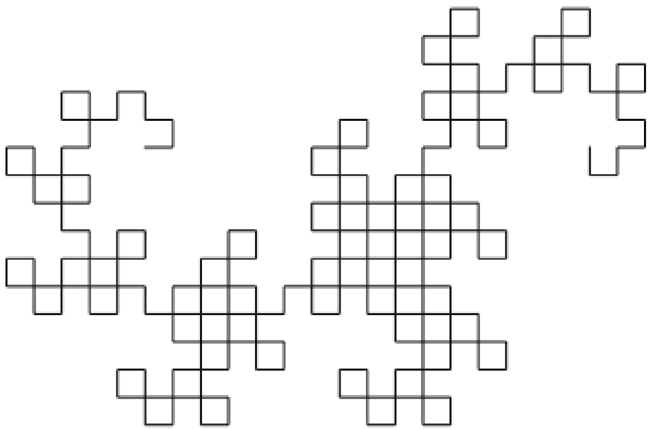
Iteration **3**



Iteration **4**

At each iteration (starting at 4), this curve contains closed squares grouped in contiguous blocks.

For instance, at iteration 8 we have:



The number of squares in each group starting from the initial point of the curve are: 1, 3, 6, 14, 33, 6, 3, 1.

What is the number of squares in each group at iteration 15?

Answer format: comma delimited list

Example: `1,3,6,14,33,6,3,1` at iteration 8.

[My timing: 30 sec]

Answer **e552d4957bb5f32c9aa86f1dd207fb14**

503 Product of phi

2018-07-24 19:26:15	**by liuguangxi**	**8 xp**	**Math**

Given two positive integers n and k, define function

$$T(n,k)=\prod_{a_1=1}^{k}\prod_{a_2=1}^{k}\cdots\prod_{a_n=1}^{k}\varphi\left(\prod_{i=1}^{n}a_i\right)$$

where $\varphi(x)$ is Euler's totient function.

For example, $T(2,3)=\varphi(1\times 1)\cdot\varphi(1\times 2)\cdot\varphi(1\times 3)\cdot\varphi(2\times 1)\cdot\varphi(2\times 2)\cdot\varphi(2\times 3)\cdot\varphi(3\times 1)\cdot\varphi(3\times 2)\cdot\varphi(3\times 3)=192$. And you are given $T(10,10) \bmod 1000000007=7103300$.

Find $T(10^7,10^7) \bmod 1000000007$.

Thanks to **C_K_Yang** for the idea.

Answer **e44dba9d127ad3398c003afa3b2dd304**

504 Kostka numbers

2018-08-31 00:43:59	by Philippe_57721	6 xp	Programming

Let λ a sequence of non increasing integers. It can be seen as the size of a jagged array.

Let μ a sequence of $\sum \lambda_i$ consecutive integers starting with 1.

We fill in the array with the elements of μ using the following rules:
- in each row the elements must be in ascending order (equality permitted)
- in each column, the element are strictly in acending order

We defined $K(\lambda, \mu)$ as the number of such arrays. It is called the Kostka number.

If $\lambda = \{3, 2\}$ and $\mu = \{1, 2, 3, 3, 4\}$, there are 3 possible arrays:

```
1  3  3          1  2  3          1  2  4
2  4             3  4             3  3
```

Therefore $K(\{3, 2\}, \{1, 2, 3, 3, 4\}) = 3$.

What is $K(\{6, 4, 4, 3\}, \{1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9\}$?

[My timing: 4 sec]

Answer de6180a7b68846f3374ef4e13213c3ab

505 Find the partition

2018-09-14 01:03:23 **by Philippe_57721** **6 xp** **Programming**

A partition of an integer n will be represented by a non increasing sequence of integers.

The partitions of 7 in lexicographic order are:
- 1 - 1 1 1 1 1 1 1
- 2 - 2 1 1 1 1 1
- 3 - 2 2 1 1 1
- 4 - 2 2 2 1
- 5 - 3 1 1 1 1
- 6 - 3 2 1 1
- 7 - 3 2 2
- 8 - 3 3 1
- 9 - 4 1 1 1
- 10 - 4 2 1
- 11 - 4 3
- 12 - 5 1 1
- 13 - 5 2
- 14 - 6 1
- 15 - 7

and in anti-lexicographic order:
- 1 - 7
- 2 - 6 1
- 3 - 5 2
- 4 - 5 1 1
- 5 - 4 3
- 6 - 4 2 1
- 7 - 4 1 1 1
- 8 - 3 3 1
- 9 - 3 2 2
- 10 - 3 2 1 1
- 11 - 3 1 1 1 1
- 12 - 2 2 2 1
- 13 - 2 2 1 1 1
- 14 - 2 1 1 1 1 1
- 15 - 1 1 1 1 1 1 1

The 10th partition is 4, 2, 1 in the 1st case and 3, 2, 1, 1 in the 2nd.

What are the 123456789th partition of 123 in lexicographic and anti-lexicographic order?

Answer format: (comma delimited values)/(comma delimited values)

Example: `4,2,1/3,2,1,1`

[My timing: 11 sec]

Answer **31d31a067d76e25a7afe513b579110df**

506 Special Pythagorean Triples

2018-09-21 03:23:39	by Philippe_57721	7 xp	Programming

$(430, 1824, 1874)$ is a special Pythagorean triple.

Indeed, we have:
$430^2 + 2133331200 = 46190^2$
$1824^2 + 2133331200 = 46224^2$
$1874^2 + 2133331200 = 46226^2$

We call **special** a Pythagorean triple if there exists a positive integer t such as for each element e of the triple $e^2 + t$ is a square.

There can exist multiple integers for a given triple. For instance, the triple $(24531, 30600, 39219)$ is special with the values 133453264 and 119200745664.

Find all special Pythagorean triples (a, b, c) with $a + b + c < 2000000$ (a triple is counted for each distinct t).

Answer format: Count, $\sum t$

You are given: `18,17172343737` for a threshold of 10000.

[My timing: < 1 min.]

Answer 10c63916a11c61133b4ac0e9ad7a6853

507 Side of a square

2018-08-16 08:27:25 **by Philippe_57721** **4 xp** **Math**

A point is choosen inside a square.

From that point 4 lines are drawn to the middle of each side of the square, defining 4 regions.

The areas of these regions are 72108967, 65123128, 69011033.

What is the side of the square?

Answer ef6b2ac6e6235f2e7ab71da8fd0bed73

508 Yet Another Diophantine Equation

2018-09-28 02:12:26 **by Philippe_57721** **6 xp** **Programming**

The Diophantine equation $x^2 - 6xy + y^2 = 1$ has inifinitly many solutions in positive integers $x \leq y$.

Find the 10^{100}-th.

Answer format: `x,y`
give the **last 10 digits** for each variable

Answer 89d9252b5e08dd6f5f1efab9ce48aac6

509 Sum of two squares

2019-03-29 03:36:26 **by Philippe_57721** **6 xp** **Math**

Find all solutions of $x^2 + y^2 = 1234567891011121613 \cdot (x - y)$ in positive integers.

Answer format: $x_1, y_1 / \ldots\ x_n, y_n$
where x_i are in ascending order

Example: `387,225/850,225` for the equation $x^2 + y^2 = 1237 \cdot (x - y)$.

[My timing: instant]

Answer 42cc027d449a12e78132078812834e63

510 Deficiency 128

2018-08-28 03:28:57 **by Min_25** **11 xp** **Math**

Let $S(N)$ be the sum of n $(1 \le n \le N)$ such that $\sigma(n) - 2n = -128$, where $\sigma(n)$ is the sum of divisors function.

For example, $S(10^7) = 11033992$.

Find $S(10^{15})$.

Answer cf43ee049a3d772b8a38d73b4e5a8920

511 Higher order recursive sequence

2018-09-12 20:49:16 **by liuguangxi** **7 xp** **Math**

Define a sequence an as below: $a_1 = 17$, $a_{n+1} = \frac{a_n^4+6a_n^2+1}{4a_n(a_n^2+1)}$ $(n \geq 1)$. Then $a_2 = 10657/2465, a_3 = 2134495165562497/1571545212141185, \ldots$. Obviously all numbers in this sequence are rational.

For a prime p and a fully reduced fraction a/b, define $Q(a/b, p)$ to be the smallest positive q for which $ab \equiv q \pmod{p}$. For example $Q(3/5, 109) = 66$, because $5 \times 66 = 330 \equiv 3 \pmod{109}$ and 66 is the smallest positive such number.

You are given $Q(a_2, 1000000007) = 877890477, Q(a_{10}, 1000000007) = 16332768$.

Find the sum of $Q(a_{1234567891011121314}, p)$ over all primes p between 2000000000 and 2000100000.

Thanks to **czp** for the idea.

Answer **d85471b108d971c3f9a9c4d7db9d63ea**

512 Abundance 128

2018-08-28 03:29:10 **by Min_25** **11 xp** **Math**

Let $S(N)$ be the sum of n $(1 \le n \le N)$ such that $\sigma(n) - 2n = 128$, where $\sigma(n)$ is the sum of divisors function.

For example, $S(10^7) = 25029284$.

Find $S(10^{15})$.

Answer e2f68a37bf705413b53ce73c786f38f5

513 Bell Numbers Modulo Factorial

2018-09-06 06:05:27 **by Min_25** **8 xp** **Math**

Let $B(n)$ be the n-th Bell number, which can be computed as $B(0) = 1$ and $B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k$ for $n \geq 0$.

For example, $B(5) = 52$ and $B(10) = 115975$.

Let $f(N, M) := B(N) \bmod M$.

You can verify that $f(5, 24) = 4$, $f(10, 8!) = 35335$ and $f(10^4, 12!) = 82759235$.

Find $f(10^7, 30!) \bmod (10^9 + 7)$.

Answer e7f6268322d32df76ffe47432a0f893d

514 Trees gracious numberings

2018-10-06 03:27:21 **by Philippe_57721** **6 xp** **Programming**

A gracious numbering of a tree with N vertices is defined as follow:
- each vertex is numbered by a number in $[0..N-1]$
- each edge being numbered by the difference, in absolute value, between the values of its two vertices, the values of the edges contain all integers in $[1..N-1]$ exactly once.

For the tree `A(B,C(D,E,F))`, here is an example of gracious numbering:

```
[A]  3
 +-B  4
 +-[C]  5
    +-D  2
    +-E  1
    +-F  0
```

You can verify:

```
[AB] = 1
[AC] = 2
[CD] = 3
[CE] = 4
[CF] = 5
```

It is conjectured that there exists a gracious numbering for all trees.

How many gracious numberings are there for the following tree: `A(B(C,D(E,F)),G(H,I(J,K)))`?

```
[A]
 +-[B]
 |  +-C
 |  +-[D]
 |     +-E
 |     +-F
 +-[G]
    +-H
    +-[I]
       +-J
       +-K
```

[My timing: 20 sec]

Answer 18b5a74712d19dbd46089a1a6490251d

515 Runs Resistance

2018-09-29 07:22:04	by Min_25	11 xp	Math

Let $A_0(n)$ be the array of the binary representation of positive integer n.

For example, $A_0(1) = [1]$, $A_0(2) = [1, 0]$ and $A_0(11) = [1, 0, 1, 1]$.

Let $R(A)$ be the function that returns the array of run-lengths of the given array A.

For example, $R([1, 0]) = [1, 1]$, $R([1, 0, 1, 1]) = [1, 1, 2]$ and $R([1, 0, 0, 0, 1]) = [1, 3, 1]$.

Let $A_k(n) := R(A_{k-1}(n))$ for $k \geq 1$, and $M(n)$ be the minimum integer k (≥ 0) such that $|A_k(n)| = 1$, where $|A|$ denotes the length of A.

Finally, let $S(n) := \sum_{i=1}^{n} M(i)$.

You can verify that $M(1) = 0$, $M(2) = 2$, $M(3) = 1$, $M(11) = 4$, $S(100) = 346$ and $S(1234567) = 6717538$.

Find $S(1234567654321)$.

Answer 1ae472b6d78f10365f60c339f8b0f652

516 Proportion of pandigital numbers

2018-10-12 02:15:07	by liuguangxi	10 xp	Math

A pandigital number in base 10 is an integer that contains each of the digits from 0 to 9 (leading zero is not considered). Clearly there cannot be any pandigital numbers below 1023456789. In other words, 1023456789 is the smallest pandigital number. Pandigital numbers become more and more common as number gets bigger. In fact, the least number for which the proportion of pandigital numbers first reaches 1% is 1982653472320.

Given two positive integers m, n and $m > n$, Let $G(m, n)$ be the least number for which the proportion of pandigital numbers is at least n/m. For instance, $G(100, 1) = 1982653472320$, $G(10, 1) = 108552671412935708$.

Find $\sum_{i=2}^{100} G(i, i-1)$. Since the answer can be very large, give your answer modulo 1000000033.

Thanks to **C_K_Yang** for the idea.

Answer 7580f389e3bb583ff47dc5a9a3dfcd02

517 Binary sequences

2018-10-19 00:45:04 **by Philippe_57721** **7 xp** **Math**

Let $F(m, n, r, s)$ be the number of binary sequences containing:
- m times the digit '0'
- n times the digit '1'
- r times the sequence '00'
- s times the sequence '11'

For instance, $F(4, 5, 2, 3) = 24$:
- 0 0 0 1 0 1 1 1 1
- 0 0 0 1 1 0 1 1 1
- 0 0 0 1 1 1 0 1 1
- 0 0 0 1 1 1 1 0 1
- 0 0 1 0 0 1 1 1 1
- 0 0 1 1 0 0 1 1 1
- 0 0 1 1 1 0 0 1 1
- 0 0 1 1 1 1 0 0 1
- 0 1 0 0 0 1 1 1 1
- 0 1 1 0 0 0 1 1 1
- 0 1 1 1 0 0 0 1 1
- 0 1 1 1 1 0 0 0 1
- 1 0 0 0 1 1 1 1 0
- 1 0 0 1 1 1 1 0 0
- 1 0 1 1 1 1 0 0 0
- 1 1 0 0 0 1 1 1 0
- 1 1 0 0 1 1 1 0 0
- 1 1 0 1 1 1 0 0 0
- 1 1 1 0 0 0 1 1 0
- 1 1 1 0 0 1 1 0 0
- 1 1 1 0 1 1 0 0 0
- 1 1 1 1 0 0 0 1 0
- 1 1 1 1 0 0 1 0 0
- 1 1 1 1 0 1 0 0 0

What is $F(777, 555, 333, 111)$?

Answer format: the last 10 digits before the trailing 0 of this large number

You are given 7741694224 for $F(77, 55, 33, 11)$.

Answer 3852fccf67d025c68cf038bd197026e3

518 Hypergraph Intro

2019-02-06 03:11:52 **by elasolova** **3 xp** **Math**

In mathematics, hypergraph is a generalization of a graph in which an edge can join any number of vertices (denoted as m), therefore introducing hyperedges.

In that case, the number of hyperedges of size m that a complete hypergraph with n many vertices has a more general formula, which boils down to well-know formula $\frac{n(n-1)}{2}$ for $m = 2$.

Given that a complete hypergraph with $n = 11$ number of vertices, it can be shown that the number of hyperedges of size $m = 3$ is 165. Then, what is the answer for $n = 1007$ and $m = 35$?

Note: This is rather an easy problem with an immediate solution, but serves as a conceptual introduction to prospective problems related to hypergraphs.

Answer **36d725a03c77bda265a490cd93dc9648**

519 Largest root of a polynomial

2018-10-26 02:28:44 **by Philippe_57721** **5 xp** **Math**

Let $P_n(X) = X^5 + F_{2n}X^4 + 2(F_{2n} - 2F_{n+1}^2)X^3 + 2F_{2n}(F_{2n} - 2F_{n+1}^2)X^2 + F_{2n}^2 X + F_{2n}^3$ (F_n is the n-th Fibonacci number).

Example: $P_4(X) = X^5 + 21X^4 - 58X^3 - 1218X^2 + 441X + 9261$.

It can be proved that equation $P_n(X) = 0$ has only integer roots.

Find the largest positive root of $P_{1000}(X) = 0$ (Give your answer mod 10^{10}).

Answer 0b6f76c165c5585eac51b291882e509b

520 q-Factorial

2018-11-06 20:49:04 **by liuguangxi** **8 xp** **Math**

For n a positive integer, the q-factorial is defined by

$$\begin{aligned}[n]_q! &= \prod_{k=1}^{n} [k]_q = \prod_{k=1}^{n} \sum_{i=0}^{k-1} q^i \\ &= 1(1+q)(1+q+q^2)\cdots(1+q+\cdots+q^{n-1})\end{aligned}$$

Let $S(n,q) = [n]_q!$. You are given $S(5,2) = 9765$, $S(100,7) \bmod 1000000000039 = 850054509459$.

Find $S(123456789876, 7) \bmod 1000000000039$.

Answer 8aa1b38d42e7f6e041cc9ba33bf03d6f

521 Factorial-like Product

2018-11-14 02:17:52	by Min_25	9 xp	Math

Let $R(p, k) := \left(\prod_{i=1}^{p-1}(i^k + 1)\right) \bmod p$.

Find $\sum_{3 \le p \le 10^{10}} R(p, 20181026)$, where p is prime.

Answer 3d637b69fab3d2a487116091991246fd

522 Fibonacci Partitions Revisited

2018-11-08 06:25:20 **by Philippe_57721** **0 xp** **Programming**

Let $F = \{1, 2, 3, 5, 8, 13, 21, 34, 55, ...\}$ the sequence of distinct Fibonacci numbers.

The number 1000 can be decomposed in 15 ways as a sum of elements of F

- $5, 14$
- $3, 4, 14$
- $5, 12, 13$
- $1, 2, 4, 14$
- $3, 4, 12, 13$
- $5, 10, 11, 13$
- $1, 2, 4, 12, 13$
- $3, 4, 10, 11, 13$
- $5, 8, 9, 11, 13$
- $1, 2, 4, 10, 11, 13$
- $3, 4, 8, 9, 11, 13$
- $5, 6, 7, 9, 11, 13$
- $1, 2, 4, 8, 9, 11, 13$
- $3, 4, 6, 7, 9, 11, 13$
- $1, 2, 4, 6, 7, 9, 11, 13$

For each decomposition, we give the indexes in F (0-origin):
$\{5, 12, 13\} \Rightarrow 1000 = F_5 + F_{12} + F_{13} = 13 + 377 + 610$

Find the number of decompositions of 1234568 and give the middle one (if there is n decompositions, give the decomposition $\lfloor n/2 \rfloor$). The 1st one has index 0. The decompositions are sorted by length, then lexicographically.

Answer format: count/(comma delimited list of indexes)

You are given: `15/3,4,10,11,13` for $n = 1000$.

[My timing: 50 sec]

Answer 3b236df9fead529dfe3ab682b478ce38

523 Sum of bigomega

2018-11-30 19:42:32	by liuguangxi	8 xp	Math

Arithmetic function $\Omega(n)$ is the total number of prime factors of n, counting prime factors with multiplicity. For example, $\Omega(3) = 1$, $\Omega(6) = 2$, $\Omega(72) = 5$. Let $S(n) = \sum_{x=1}^{n} \Omega(x)$. You are given $S(10) = 15$, $S(1000) = 2877$.

Find $S(10^{13})$.

Thanks to **baihacker** for the idea.

Answer 51dac4bff56799427878789ccdbc8a66

524 Number of Distinct Unordered Factorization

2018-11-21 03:45:56	by Min_25	7 xp	Math

Let $C(n)$ denote the number of factorizations of n into distinct factors greater than 1.

For example, $C(24) = 5$ since $24 = 2 \times 12 = 2 \times 3 \times 4 = 3 \times 8 = 4 \times 6$.

Let $S(n) := \sum_{i=2}^{n} C(n)$. You are given $S(10) = 12$ and $S(10^4) = 99733$.

Find $S(10^{12})$.

Answer 2d44a12d4dfadedb8b04ebd1ba65c761

525 Sum of largest prime factors revisited

2018-12-28 17:42:42 **by liuguangxi** **9 xp** **Math**

Let $f(n)$ be the largest prime factor of n. For example $f(1) = 0$, $f(6) = 3$ and $f(30) = 5$.

Define $S(k,n) = \sum_{x_1=1}^{n} \sum_{x_2=1}^{n} \cdots \sum_{x_k=1}^{n} f\left(\prod_{i=1}^{k} x_i\right)$.

You are given $S(2,3) = f(1 \times 1) + f(1 \times 2) + f(1 \times 3) + f(2 \times 1) + f(2 \times 2) + f(2 \times 3) + f(3 \times 1) + f(3 \times 2) + f(3 \times 3) = 21$, $S(3,10) = 4790$, $S(6,10) = 5697722$.

Find $S(9, 10^9) \bmod 10^9$.

Thanks to **baihacker** for the idea.

This problem was inspired by Problem 642 at Project Euler (https://projecteuler.net/problem=642).

Answer **c5ee640c7f942386d5d29b532f41ad31**

526 Mysterious Asymptotic Behavior

2018-11-18 04:15:34 **by Min_25** **0 xp** **Math**

Let $C(N)$ be the number of positive integers $n \geq 2$ such that $n \cdot (\text{largest prime factor of } n) \leq N$.

You are given that $C(10) = 3$, $C(10^6) = 9107$, $C(10^{11}) = 34113192$ and $C(10^{12}) = 188014194$.

Find $C(10^{16})$.

Answer 5cd6eb297de9398d79515fe95d0af868

527 Mysterious Asymptotic Behavior 2

2018-11-19 05:21:03 **by Min_25** **0 xp** **Math**

Let $C(N)$ be the number of positive integers $n \geq 2$ such that $n \cdot (\text{largest prime factor of } n)^2 \leq N$.

You are given that $C(10) = 1$, $C(10^6) = 1228$, $C(10^{11}) = 888908$ and $C(10^{12}) = 3613513$.

Find $C(10^{18})$.

Answer bf733aabf4f4294f7c73e96483896e21

528 One chord and two segments

2018-12-06 08:52:43 **by Philippe_57721** **7 xp** **Math**

Three segments, $[AB]$ of length a, $[BC]$ of length b and $[CD]$ of length c are perpendicular to each other.

The circle passing by the points A, B, D has a radius r.

How many triples (a, b, c) such as $1 \leq a, b, c \leq 1000$ and $\text{GCD}(a, b, c) = 1$ are there such as r is an integer?

Answer format: count, $\sum a, \sum b, \sum c$

You are given: `4,30,18,10` for a threshold of 10.

Answer 0c70e5c1a13f2f0de1a2be8986a3c639

529 1-2-3 type Pythagorean triangles

2018-12-03 22:45:08	by C_K_Yang	8 xp	Math

An integer-sided triangle is a 1-2-3 type Pythagorean triangle if its three sides a, b, c satisfy the condition: $1 \times a^2 + 2 \times b^2 = 3 \times c^2$.

For example, A triangle with three sides $\{5, 13, 11\}$ is a 1-2-3 type Pythagorean triangle because $1 \times 5^2 + 2 \times 13^2 = 3 \times 11^2$.

Find all distinct 1-2-3 type Pythagorean triangles with perimeter ≤ 25000000.

Answer format: count, $\sum$perimeter

You are given: `595,315428` for a threshold of 1000.

Answer acbbb9361d2f4d1a78f736cc738418ee

530 Range for random sequence

2019-01-15 05:20:26	by liuguangxi	8 xp	Probability

There are n $(n > 1)$ independent and identically distributed random variables whose cumulative distribution function is

$$F(x) = \begin{cases} 0 & : x < 0 \\ x^2 & : 0 \le x \le 1 \\ 1 & : x > 1 \end{cases}$$

Let R be the range for this random variables (difference between maximum and minimum). Let $E(n)$ be the expected value of R. Obviously $E(n)$ is between 0 and 1, furthermore it is rational number. You are given $E(2) = 4/15$ and $E(3) = 2/5$.

Find $E(20)$. Give your answer rounded to 10 digits after the decimal point.

Thanks to **baihacker** for the idea.

Answer 82a1a7fc40d7c005226824d06d9b40a1

531 Sum for random sequence

2019-01-23 22:12:49	**by liuguangxi**	**8 xp**	**Probability**

A sequence generator start generating independent and identically distributed random variables whose cumulative distribution function is

$$F(x) = \begin{cases} 0 & : x < 0 \\ x^2 & : 0 \leq x \leq 1 \\ 1 & : x > 1 \end{cases}$$

The cumulative sum is updated for every new generated number. When the sum is larger than 1, the generator stops. Let C denotes the count of generated numbers when it stops. For example, the probability for $C = 3$ is $7/45$.

Find the expected value of C. Give your answer rounded to 10 digits after the decimal point.

Thanks to **baihacker** for the idea.

Answer **0d55f2da2739bf119492802d335b0675**

532 Throwing dice

2018-12-29 02:05:11 **by Philippe_57721** **6 xp** **Programming**

With 10 6-faces dice (numbered 1 to 6), we can obtain all sums between 10 and 60.

The sum 35 is the *champion*. It can be obtain in 4395456 ways. it's actually the sum that can be obtained in the largest number of ways.

What is the champion if we use $n = 100$ dice ?

Answer format: Champion, Number of ways

Example: `35,4395456`

Answer **b924370091b8da38346340216b30031d**

533 Every day is a holiday revisited

2019-02-22 01:23:50 **by liuguangxi** **9 xp** **Math**

On planet J, a year lasts for D days. Holidays are defined by the two following rules.
1. At the beginning of the reign of the current Emperor, his birthday is declared a holiday from that year onwards.
2. If both the day before and after a day d are holidays, then d also becomes a holiday.

Initially there are no holidays. Let $f(n, d)$ be the number of methods such that all the days of the year are holidays after exactly d days, with the year lasts for n days.

You are given $f(4, 2) = 4$, $f(10, 5) = 240$, $f(50, 100) \bmod 1000000007 = 263154905$.

Find $f(1000, 10^{12}) \bmod 1000000007$.

Thanks to **baihacker** for the idea.

This problem was inspired by Problem 645 at Project Euler (https://projecteuler.net/problem=645).

Answer **5b4d7b5648d1d38485d0850de6f75d69**

534 The number of pandigital number

2018-12-18 01:42:47 **by C_K_Yang** **7 xp** **Math**

In mathematics, a pandigital number is an integer that in a given base B has each digit $\{0, 1, 2, 3, \ldots, B-1\}$ at least once.

Given two positive integers B and L, let $C(B, L)$ be the number of all the L-digit pandigital numbers in base B. For example, $C(3, 3) = 4$, there are only four 3-digit pandigital numbers in base 3, namely $(102)_3$, $(120)_3$, $(201)_3$ and $(210)_3$.

Now we define a new function $D(B, N) = \sum_{L=1}^{N} C(B, L)$.

You are given $D(5, 10) = 4865472$ and $D(123, 321) \bmod 1000000007 = 844902472$.

Find $D(12345678, 87654321) \bmod 1000000007$.

Note: leading zeroes are not allowed.

Answer **fe3110eb5af8f132db9b643d63a75c27**

535 The sum of pandigital number

2018-12-18 01:43:17 **by C_K_Yang** **9 xp** **Math**

In mathematics, a pandigital number is an integer that in a given base B has each digit $\{0, 1, 2, 3, ..., B - 1\}$ at least once.

Given two positive integers B and L, let $S(B, L)$ denote the sum of all the L-digit pandigital numbers in base B. For example, $S(3, 3) = 66$, the sum of four 3-digit pandigital numbers in base 3 is 66 since $(102)_3 + (120)_3 + (201)_3 + (210)_3 = (2110)_3 = (66)_{10}$.

Now we define a new function $H(B, N) = \sum_{L=1}^{N} S(B, L)$.

You are given $H(5, 10) = 24678960400080$ and $H(123, 321) \bmod 1000000007 = 372652728$.

Find $H(12345678, 87654321) \bmod 1000000007$.

Note: leading zeroes are not allowed.

Answer **ed49378173eec010291909883428227f**

536 Zumkeller Numbers

2018-12-27 02:04:55 **by Philippe_57721** **0 xp** **Programming**

A *Zumkeller* number is a number such as the set of its divisors can be partitioned in two subsets of equal sum.

For instance, $\text{divisors}(108) = E = \{1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108\}$ and $E = A \cup B$ with $A = \{2, 12, 18, 108\}$ and $B = \{1, 3, 4, 6, 9, 27, 36, 54\}$, $\sum_{a \in A} a = \sum_{b \in B} b = 140$.

Find all Zumkeller numbers in the range $[439050, 439100]$.

Answer format: `count,sum`

[My timing: 6 sec]

Answer 968786c262221a2fdf72e2b85271c1b0

537 Skipping squares revisited

2019-04-08 00:19:48 **by liuguangxi** **7 xp** **Math**

For positive integer k ($k > 1$), we begin a sum s at 0 and repeatedly apply a process: we add one integer to s from 1 to k with equal probability. The process ends when s is a perfect square. For example, if s goes through $0, 2, 3, 5, 7, 9$, the process ends at $s = 9$, and two squares 1 and 4 were skipped over.

Let $E(k)$ be the expected number of perfect squares skipped over when the process finishes for parameter k. For example, $E(2) = 0.6832612735$ and $E(10) = 4.9658004390$, both rounded to 10 digits after the decimal point.

Find $E(100)$. Give your answer rounded to 10 digits after the decimal point.

This problem was inspired by Problem 648 at Project Euler (https://projecteuler.net/problem=648).

Answer **8ffb7a6f011b7da4ebcf8eef241379bf**

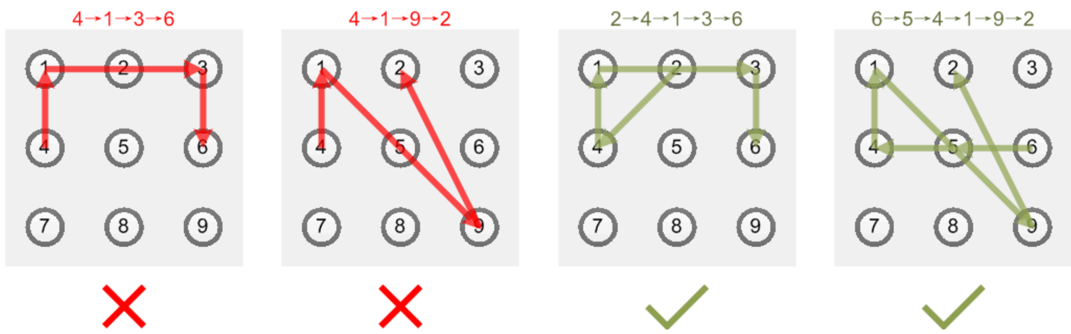
538 Counting unlock patterns

2019-10-08 20:25:33 **by liuguangxi** **9 xp** **Math**

Given a square tiled $n \times n$ key lock screen and two integers k_1 and k_2 , where $1 \leq k_1 \leq k_2 \leq n^2$. Here are the rules for a valid pattern:

- Each pattern must connect at least k_1 keys and at most k_2 keys.
- All the keys must be distinct.
- If the line connecting two consecutive keys in the pattern passes through any other keys, the other keys must have previously selected in the pattern. No jumps through non selected key are allowed.
- The order of keys used matters.

For $n = 3$, here are some invalid and valid patterns:



- Invalid move: $4 \to 1 \to 3 \to 6$. Line $1 \to 3$ passes through key 2 which had not been selected in the pattern.
- Invalid move: $4 \to 1 \to 9 \to 2$. Line $1 \to 9$ passes through key 5 which had not been selected in the pattern.
- Valid move: $2 \to 4 \to 1 \to 3 \to 6$. Line $1 \to 3$ is valid because it passes through key 2, which had been selected in the pattern.
- Valid move: $6 \to 5 \to 4 \to 1 \to 9 \to 2$. Line $1 \to 9$ is valid because it passes through key 5, which had been selected in the pattern.

Define $S(n, k_1, k_2)$ be the total number of unlock patterns of the $n \times n$ key lock screen, which consist of minimum of k_1 keys and maximum k_2 keys. You are given $S(3, 4, 9) = 389112$, which is the well-known total number of possible Android unlock patterns.

Find $S(5, 4, 25)$.

Thanks to **baihacker** for the idea.

Answer 747917eb3bb56efca7e81ad5aefd191d

539 Egyptian fractions

2019-01-05 21:55:38 **by C_K_Yang** **10 xp** **Math**

Let $A(n)$ be the number of Egyptian fractions for which $\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_k} = 3$ (for any $k > 0$) with $0 < x_1 < x_2 < \cdots < x_k \leq n$.

For example, $A(24) = 1$, there is only one Egyptian fraction which is equal to 3.

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{15} + \frac{1}{18} + \frac{1}{20} + \frac{1}{24} = 3$$

You are given $A(30) = 25$.

Find $A(200)$.

Answer c9bd875dd8c102d452e8a34fa28820ff

540 Tesseract Pentatope Picking

2019-02-04 18:00:38 **by elasolova** **0 xp** **Probability**

First, some definitions are given:

- Simplex is the generalization of line-segments (1d) or triangles (2d) to n-dimensions. Tetrahedron is the well-known 3-simplex. Pentatope then is the 4-simplex.
- Similarly, hypercube is the generalization of squares (2d) or cubes (3d) to n-dimensions. Tesseract is the 4-dimensional hypercube.

Then the question builds up as follows:
In line line picking, the expected length of picked line-segment is shown to be 13.
Up one dimension, namely in square triangle picking, the expected area of picked triangle is analytically given as $\frac{11}{144}$.
Up one more, namely in cube tetrahedron picking it gets a bit complicated, but there is again an analytic answer for the expected volume of picked tetrahedron, $\frac{3977}{216000} - \frac{\pi^2}{2160}$.

What about going one more up, namely tesseract pentatope picking? Since an analytic answer is hard to find, devise a numerical solution that gives the expected content of picked pentatope rounded to 6 decimal places.

There might be some inaccuracy in my calculation for the time being …

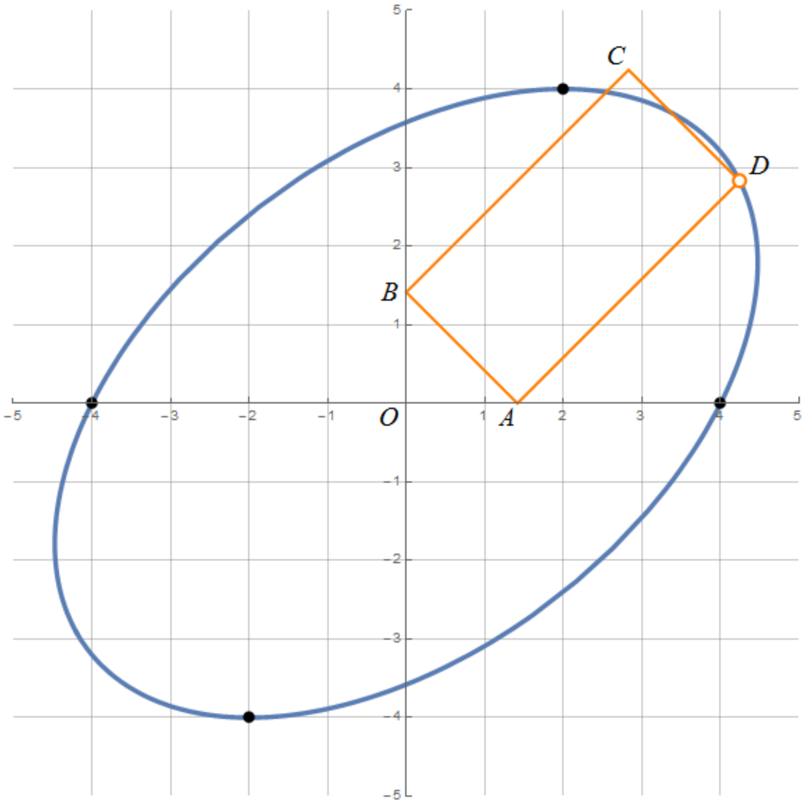
Answer **066f5c7650cb359a42a98a6df6356cd4**

541 Lattice points on the trace 1

2019-11-19 20:46:18 **by liuguangxi** **8 xp** **Math**

Given a rectangle $ABCD$ with side $AB = CD = a$ and side $BC = AD = b$. A curve is traced by point D of the rectangle as point A slides along the x-axis and point B slides along the y-axis.

The curve with $a = 2$ and $b = 4$ is shown below.



There are 4 lattice points $(-4,0)$, $(-2,-4)$, $(4,0)$, $(2,4)$ on the curve above. Let $C(a,b)$ be the number of distinct lattice points on the curve with the rectangle side a and b. For example, $C(2,4) = 4$, $C(5,5) = 12$ and $C(6,3) = 4$.

Define $S(n) = \sum_{a=1}^{n} \sum_{b=1}^{n} C(a,b)$. You are given $S(10) = 432$, $S(50) = 10960$ and $S(100) = 44128$.

Find $S(10^8)$.

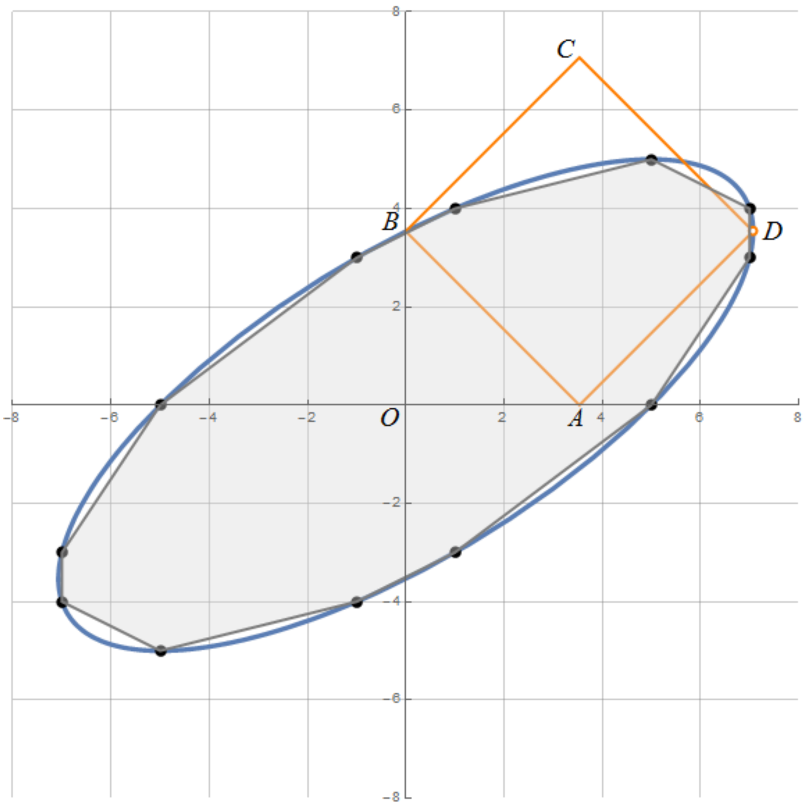
Answer d9b87ea258ffcba5bd940ffff25ea101

542 Lattice points on the trace 2

2019-11-19 20:52:20 **by liuguangxi** **8 xp** **Math**

Given a square $ABCD$ with side length a. A curve is traced by point D of the square as point A slides along the x-axis and point B slides along the y-axis.

The curve with $a = 5$ is shown below.



For positive integer a, it can be shown that at least 4 lattice points are on the curve. An inscribed polygon can be formed by the lattice points on the curve. Let $R(a)$ be the ratio of the area of the inscribed polygon to the area of the closed curve with parameter a. Then $R(5) = 0.942197263104$.

For $a \leq 100$, the maximum value of $R(a)$ is 0.992825486421, with $a = 65$.

Find the maximum value of $R(a)$ and corresponding value a for $a \leq 10^{15}$. If there are two or more values of a taking the maximum value R, give the smallest a.

Answer format: [value of a],[value of $R(a)$ (rounded to 12 digits after the decimal point)]

Example: `65,0.992825486421` for $a \leq 100$.

Answer 8526c1ed3352028c2f729ac24953c9f6

543 Number divisible by a range of numbers

2019-03-04 16:15:27	by C_K_Yang	8 xp	Programming

Given two positive integers a and b with $a \leq b$, let $P(N, a, b)$ denote the number of numbers not greater than N and divisible by at least one number between a and b (inclusive).

For example, $P(15, 3, 5) = 9$
- 3 can divide $3, 6, 9, 12, 15$
- 4 can divide $4, 8, 12$
- 5 can divide $5, 10, 15$

There are 9 numbers not greater than 15 and divisible by at least one number between 3 and 5, namely $3, 4, 5, 6, 8, 9, 10, 12, 15$.

You are given $P(10^8, 21, 40) = 41982923$.

Find $P(10^{17}, 61, 120)$.

Answer 2e2eea4cf1963d05b3809165febf6f2c

544 Spatial Awareness

2019-08-12 02:55:29	**by elasolova**	**6 xp**	**Math**

Usually, in data processing multidimensional data (i.e. a multidimensional array) is vectorized and then processed. However, because of vectorization the spatial information in the data is lost.

For this problem, assume that you receive 1000 distinct values after a vectorization of a 4D array and you have no other information whatsoever. Then, how many possible spatial states are there for the original data? Give your answer in mod 100000009.

Note: Assume that an array of size $1 \times 1 \times 1 \times 1000$ or any other combination involving 1, will also count as a 4D array.

Answer 9f287d589c478f59ff659fbcd85b1c74

545 Distinct powers revisited

2019-03-16 08:15:18 **by C_K_Yang** **10 xp** **Programming**

Consider all integer combinations of a^b for $2 \le a \le 5$ and $1 \le b \le 5$:

$$2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32$$
$$3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243$$
$$4^1 = 4, 4^2 = 16, 4^3 = 64, 4^4 = 256, 4^5 = 1024$$
$$5^1 = 5, 5^2 = 25, 5^3 = 125, 5^4 = 625, 5^5 = 3125$$

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 18 distinct terms:

$$2, 3, 4, 5, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125$$

Given two integers $N > 1$ and $M > 0$, let $R(N, M)$ be the number of distinct terms in the sequence generated by a^b for $2 \le a \le N$ and $1 \le b \le M$. For example, $R(5, 5) = 18$, $R(10^2, 10^3) = 92715$ and $R(10^5, 10^6) = 99813359409$.

Find the last 20 digits of $R(10^{21}, 11^{21})$.

This problem is a harder version of Project Euler Problem29 (https://projecteuler.net/problem=29).

Answer **b68b986b18c38afdb425510782b820d7**

546 Pandigital representation of a number

2019-04-12 01:10:10 **by Philippe_57721** **7 xp** **Programming**

Using the 9 digits 1, 2, ..., 9 in that order and the 5 operations `+-*/^` (digits can be concatened too) it is possible to express some integers.

For instance, we have:
`38706 = -12+3^4*(567-89)`

Can you find this representation for 88205? (The solution is unique)

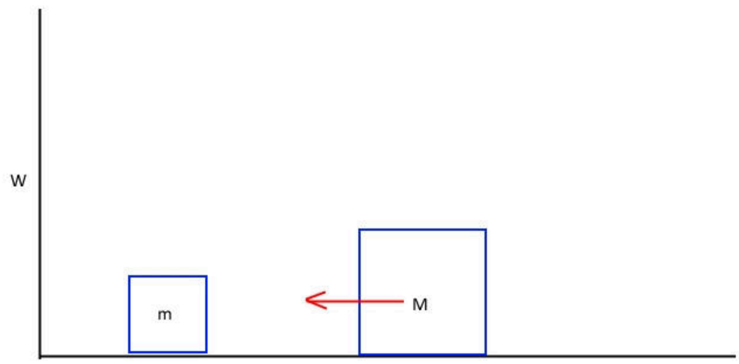
Answer format: as in the above example, without spaces and the minimum parentheses

Answer 6ab4562e5bdbd849250997df08e1201c

547 Collisions

2019-04-26 01:03:10	by Philippe_57721	8 xp	Math

Two blocks of mass m and M ($m < M$) are initially still.



We push the block M in the direction of the block m. The block m will hit the wall and bounces back toward M. After a certain number of collisions of m between M and the wall, M will move to the right indefinitely.

Given that $m = 1, M = 123456789$, how many collisions are there?

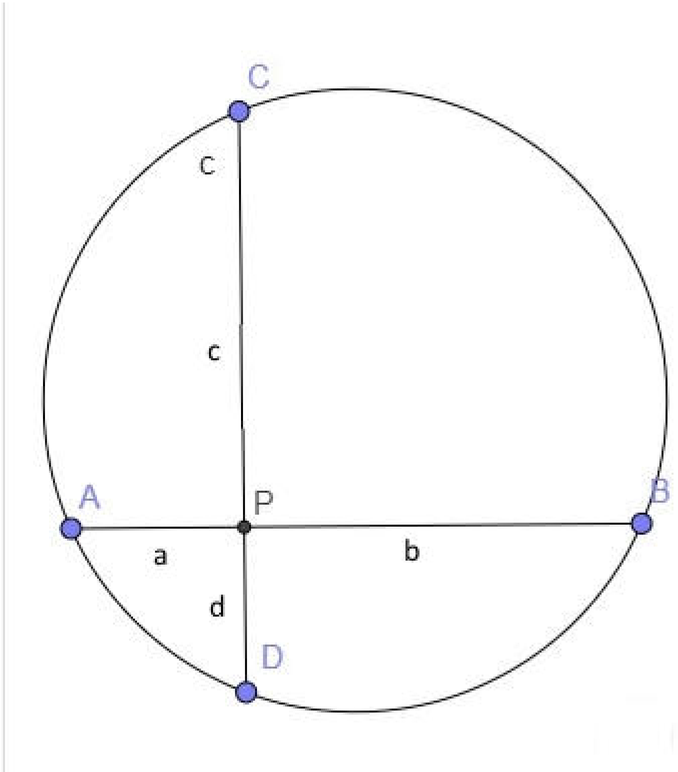
It is assumed that the collisions are perfect: no friction, no energy loss.

Answer 3885e7d745f7a780484eb1182532bbf7

548 Intersecting chords

2019-05-17 01:12:39 **by Philippe_57721** **6 xp** **Math**

In a circle, two perpendicular chords AB and CD intersect in P.



We have the following lengths:
- $\|AP\| = a$
- $\|PB\| = b$
- $\|CP\| = c$
- $\|PD\| = d$

and $a < b, c < d, a < c$.

Given that a, b, c, d are positive integers $\leq$ 5000, how many distinct circles (distinct radius) can we found?

You are given: 1913 for a threshold of 100.

[My timing: 60 sec]

Answer b0236506212d871c03a1580f5bafb61d

549 Stone game

2019-05-10 20:47:06	by C_K_Yang	9 xp	Math

Alice and Bob play a game with some piles of stones denoted by an array $[a_1, a_2, a_3, ...]$ where a_i is the number of stones in the i-th pile. They take stones by turns and **Alice** always goes first. In each turn they must take **at least one stone** and at most a certain number of stones from a single pile. The game begins with taking stones from the first pile. If the first pile becomes empty, they are permitted to take stones from the second pile. If the second pile becomes empty, they are permitted to take stones from the third pile, and so on. The player who can not make a valid move loses.

We define an array $[a_1, a_2, a_3, ...]$ as the winning configuration if the first player (Alice) can force a win no matter what the second player (Bob) does, or the losing configuration if Bob can force a win no matter what Alice does. We also assume that Alice and Bob always play perfectly.

For example, $[2, 1]$ is a winning configuration if they can take at most 2 stones in his/her turn. The strategy is explained below.
(1) Alice takes one stone in the first pile.
(2) Bob takes one stone in the first pile.
(3) Alice takes one stone in the second pile.
(4) Bob can not make a valid move, Alice wins!!!

However, $[1, 2]$ is a losing configuration if they are allowed to take at most 2 stones in a single move. The strategy is described below.
(1) Alice takes one stone in the first pile.
(2) Bob takes two stones in the second pile.
(3) Alice can not make a valid move, Alice loses!!!

We define the function $S(K, M, N)$ as the number of winning configurations $[a_1, a_2, a_3, ...]$ that satisfy the following conditions:
(1) There are K elements in the array(K piles of stones).
(2) $1 \leq a_i \leq M$ for $1 \leq i \leq K$.
(3) Alice and Bob can take no more than N stones in a single move.

You are given $S(3, 4, 2) = 39$, $S(5, 5, 3) = 2084$ and $S(25, 25, 8) \bmod 1000000007 = 313496033$.

Find $S(10^{18}, 11^{18}, 1234567890) \bmod 1000000007$.

Thanks to **liuguangxi** for promoting this problem to larger K, M, N.

Answer b182fa6de7fc957c81dd2c705a1538b8

550 Sum of the numbers with a fixed digit sum

2019-05-29 19:44:44	by C_K_Yang	9 xp	Programming

Let $F(B, L, D)$ be the sum of all the L-digit numbers in base B, the digit sum of which is D.

For example, $F(3, 3, 3) = 79$. In base 3, there are five 3-digit numbers the digit sum of which is 3. The sum of these five numbers is 79: $(102)_3 + (111)_3 + (120)_3 + (201)_3 + (210)_3 = (2221)_3 = (79)_{10}$.

Now we define a new function $S(B, L, N) = \sum_{p \le N, p \text{ is prime}} F(B, L, p)$.

For example, $S(5, 5, 11) = 964140$ and $S(56, 20, 101) \equiv 913761394 \pmod{1000004321}$.

Find $S(556678, 999, 12345701) \bmod 1000004321$.

Note: Leading zeroes are not allowed for a L-digit number in base B.

Answer 6a9047dd2c1f5d7c5f61539b408b2ec0

551 A specific sum over all subsets of a set

2019-06-09 18:07:22 **by C_K_Yang** **9 xp** **Math**

Given two positive integers N and M with $0 < N \leq M$, let $I(N, M)$ be the set which contains all natural numbers between N and M (inclusive). For example, $I(3, 6) = \{3, 4, 5, 6\}$.

We define the product of a set as the product of all elements in that set, written as $P(\{...\})$ where $\{...\}$ is a specific set. For example, $P(I(3, 6)) = 3 \times 4 \times 5 \times 6 = 360$.

Finally, We define a function $C(N, M, K)$ as the sum of product of sets over all $I(N, M)$'s subsets which contain exactly K elements.

In other words, $C(N, M, K) = \displaystyle\sum_{S \in I(N,M),\ \mathrm{len}(S)=K} P(S)$.

You are given $C(10, 20, 3) = 549450$ and $C(100, 200, 10) \bmod (10^{18} + 3) = 491824643147614094$.

Find $\sum_{i=1}^{500} C(10^{20}, 11^{20}, i)$. Give your answer modulo $(10^{18} + 3)$.

[My timing: 4 s (Python)]

Answer **b76f1f2b5792d11379f3d67e9af92700**

552 Bonus for p550

2019-06-18 20:15:56	**by C_K_Yang**	**8 xp**	**Math**

Given two natural numbers a and b with $1 < a \le b$, let $J(a, b, N)$ be the set which contains all natural numbers not greater than N and not divisible by any number between a and b (inclusive). For example, $J(4, 6, 10) = \{1, 2, 3, 7, 9\}$

- 4 can divide 4 and 8
- 5 can divide 5 and 10
- 6 can divide 6

Hence $4, 5, 6, 8$ and 10 will not appear in the set which $J(4, 6, 10)$ represents. Although they are all not greater than 10.

We define the product of a set as the product of all elements in that set, written as $P(\{...\})$ where $\{...\}$ is a specific set. For example, $P(J(4, 6, 10)) = 1 \times 2 \times 3 \times 7 \times 9 = 378$.

Finally, We define a function $D(a, b, N, K)$ as the sum of product of sets over all $J(a, b, N)$'s subsets which contain exactly K elements.

In other words, $D(a, b, N, K) = \sum_{S \in J(a,b,N),\ \mathrm{len}(S)=K} P(S)$.

You are given $D(5, 8, 20, 3) = 46035$ and $D(7, 12, 3000, 10) \bmod (10^{18} + 3) = 736056499082019203$.

Find $\sum_{i=1}^{150} D(11, 20, 11^{20}, i)$. Give your answer modulo $(10^{18} + 3)$.

[My timing: 18 s (Python)]

Answer 4b78d38d24c4b94ece4c59a68c7d51ca

553 Kostka Numbers 2

2019-10-23 23:39:53 **by sinan** **8 xp** **Math**

By a partition of m, we mean an n-tuple: $p = [a_1, a_2, ..., a_n]$ of positive integers with $a_1 \geq a_2 \geq \cdots \geq a_n$ and $a_1 + a_2 + \cdots + a_n = m$.

So to any partion p of m, a Ferrer's diagram consisting of m boxes is associated, arranged in n rows in such a way that i-th row contains a_i boxes.

Let c be another partition of m with k-tuple: $c = [b_1, b_2, ..., b_k]$ of positive integers with $b_1 \geq b_2 \geq \cdots \geq b_k$ and $b_1 + b_2 + \cdots + b_k = m$.

If $p = [a_1, a_2, ..., a_n]$ and $c = [b_1, b_2, ..., b_k]$ are two arbitrary partitions of m, then by a semi-standard Young tableau of shape p and content c, we mean any distribution of the numbers $1, 2, ..., m$ in the boxes of the associated Ferrer's diagram of p in such a way that

1. every row is non-decreasing;
2. every column is (strictly) increasing; and
3. for any $1 \leq i \leq m$, the multiplicity of i in the distribution is b_i.

For example if $m = 6$: $p = [3, 2, 1]$ and $c = [2, 2, 2]$ (which means we have 2 of 1's, 2 of 2's and 2 of 3's, totally 6 of them).

```
1 1 2
2 3
3

1 1 3
2 2
3
```

The number of all semi-standard Young tableaux of shape p and content c is denoted by $K(p, c)$ here and it is called the Kostka coefficient or the Kostka number.

Let $p = [21, 13, 8, 5, 3]$. Find the following:
$K(p, c_1)$ where $c_1 = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$
$K(p, c_2)$ where $c_2 = [2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2]$
$K(p, c_3)$ where $c_3 = [3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 2]$
$K(p, c_4)$ where $c_4 = [4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 2]$
$K(p, c_5)$ where $c_5 = [5, 5, 5, 5, 5, 5, 5, 5, 5, 5]$

Answer format: $K(p, c_1), K(p, c_2), K(p, c_3), K(p, c_4), K(p, c_5)$

Answer 12e8e84459709a220834363aca0152df

554 Primorial

2019-09-26 20:40:58	by liuguangxi	12 xp	Math

For a positive integer n, the primorial $n\#$ is defined as the product of all primes $\leq n$: $2\# = 2, 3\# = 6, 6\# = 30, \ldots$.

Define $F(n,p) = \begin{cases} n\# \bmod p & , \quad \text{if } n < p, \\ \frac{1}{p} \times (n\#) \bmod p \ , & \quad \text{if } n \geq p. \end{cases}$

For example, $F(3,7) = 2 \times 3 \bmod 7 = 6$ and $F(7,3) = 2 \times 5 \times 7 \bmod 3 = 1$.

For $P = 998244353$, You are given $F(143, P) = 675572163$ and $F(10^9, P) = 574463500$.

Find $F(10^{13}, P)$.

[My timing: ~ 20 s for 10^{13}, ~ 65 s for 10^{14}]

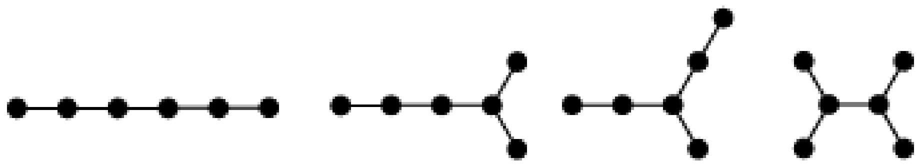
Answer fe807eface99f4ac172bcc7b898b5600

555 Distinct trees I

2019-08-17 00:03:46 **by C_K_Yang** **9 xp** **Math**

Let $Q(n, m)$ be the number of distinct trees with n nodes where each node can have no more than m edges connected to it. Two trees are not considered distinct if they are isomorphic.

For example, $Q(6, 3) = 4$. Four distinct trees are shown below.



You are given $Q(15, 4) = 4347$ and $Q(50, 7) \equiv 646618404 \pmod{1000000007}$.

Find $Q(1000, 15) \bmod 1000000007$.

Answer **dd0edc085ebcca1cafa9ca39a2930df5**

556 A Simpler Goodstein Sequence

2019-08-13 10:25:58	by Philippe_57721	0 xp	Math

Given a positive integer n, let the following process:
1) We start with base 2
2) Express n in the current base
3) Increment the base by 1
4) Take the number n in this new base and decrement it by 1
5) Go to step 2 until $n = 0$

Here an example for $n = 4$

Step	Number	Base	Digits	Comments
1	4	2	$1, 0, 0$	
2	8	3	$2, 2$	$8 = 1 \times 3^2 + 0 \times 3^1 + 0 - 1$
3	9	4	$2, 1$	$9 = 2 \times 4^1 + 2 - 1$
4	10	5	$2, 0$	
5	11	6	$1, 5$	
6	11	7	$1, 4$	
7	11	8	$1, 3$	
8	11	9	$1, 2$	
9	11	10	$1, 1$	
10	11	11	$1, 0$	
11	11	12	11	
12	10	13	10	
13	9	14	9	
14	8	15	8	
15	7	16	7	
16	6	17	6	
17	5	18	5	
18	4	19	4	
19	3	20	3	
20	2	21	2	
21	1	22	1	
22	0	23	0	Done!

We reach the value 0 in 22 steps. Let write $G(4) = 22$.

What is $G(1024)$? Give your answer modulo 1000000007.

PS: This is a variant of Problem 396.[1]

[1] Actually Problem 398.

Answer 549864c4aa8cc916404162feb6984073

557 The Two Cities

2019-09-01 09:28:55 **by Philippe_57721** **7 xp** **Math**

A traveler goes from city A to city B.

When traveling uphill, he moves at speed u km/h
When traveling on flat road, he moves at speed f km/h
When traveling downhill, he moves at speed d km/h (of course $0 < u < f < d$)

To go from A to B, it takes T_1 hours.
To go from B to A, it takes T_2 hours.

For some triples (u, f, d) these information are sufficient to determinate the distance between A and B (for instance $(30, 35, 42)$).

How many such triples of positive integers (u, f, d) with $\text{GCD}(u, f, d) = 1$ are there if $d < 10^8$?

[My timing: 40 sec]

All values in this problem (speed or distance) are assumed to be integers.

Answer de0686b9d7ced3405a599e8b90202016

558 Number of solutions to a Diophantine Equation I

2019-08-30 18:47:55 **by C_K_Yang** **8 xp** **Math**

Consider the following Diophantine Equation: $x^2 - y^2 - z^2 - xy - yz - zx = 0$.

We call an integer triple (x, y, z) a primitive solution if it is a solution to the Diophantine Equation with $\gcd(x, y, z) = 1$.

For example, $(3, 1, 1)$ is a primitive solution to the above Diophantine Equation:
$3^2 - 1^2 - 1^2 - 3 \times 1 - 1 \times 1 - 1 \times 3 = 0$

In fact, there are only five primitive solutions for $0 < z \leq y \leq x \leq 100$, namely $(3, 1, 1)$, $(19, 11, 1)$, $(37, 19, 5)$, $(61, 29, 11)$ and $(91, 41, 19)$. The sum of these five primitive solutions is 349, i.e. $\sum x + y + z$.

How many primitive solutions are there to the above Diophantine Equation for $0 < z \leq y \leq x \leq 10^{13}$? What is the sum of all these primitive solutions?

Answer format: count, $(\sum x + y + z) \bmod 10^9$

You are given: `479,3971343` for a threshold of 10000.

Answer **47469eb26d8ca25d155c9f5de61d8505**

559 Number of solutions to a Diophantine Equation II

2019-09-09 20:51:07 **by C_K_Yang** **8 xp** **Math**

Consider the following Diophantine Equation: $x^2 + y^2 + z^2 - xy - yz - zx - x - y - z = 0$.

If x, y, z are all integers, then it is a solution to the Diophantine Equation.

For example, $x = 4, y = 6, z = 9$ is one solution to the above Diophantine Equation:
$4^2 + 6^2 + 9^2 - 4 \times 6 - 6 \times 9 - 9 \times 4 - 4 - 6 - 9 = 0$.

How many solutions are there to the above Diophantine Equation for $0 < x \le y \le z \le 10^{14}$? What is the sum of all these solutions?

Answer format: count, $(\sum x + y + z) \bmod 10^9$

You are given: `12082,178899731` for a threshold of 10000.

Answer a769cf65af91e968827a77f46f1c6c26

560 Balanced numbers revisited

2019-09-21 07:39:49	by C_K_Yang	9 xp	Programming

A positive integer with k (decimal) digits is called balanced if its first $\left\lceil \frac{k}{2} \right\rceil$ digits sum to the same value as its last $\left\lceil \frac{k}{2} \right\rceil$ digits, where $\lceil x \rceil$, pronounced ceiling of x, is the smallest integer $\geq x$, thus $\lceil \pi \rceil = 4$ and $\lceil 5 \rceil = 5$.

For example, 77, 101, 13722 and 585774 are balanced numbers.

Find the number and the sum of all balanced numbers n for $0 < n \leq 11^{1111}$ and give your answer modulo 1000000007.

Answer format: `count,sum`
(both modulo 1000000007)

You are given: `634521,744405873` for a threshold of 12345678.

[My timing: 1 s]

Answer 52924012a87c12822a1faf7c734303b3

561 Grid painting

2019-10-14 00:17:18	by sinan	8 xp	Programming

Let there be a grid with M rows and N columns. The cells are numbered starting from 0 (topmost left) to $M \times N - 1$ (bottommost right) and some cells are painted to yellow randomly using the following code:

```
uint8 grid[NROWS*NCOLS] = {0};

void FillGrid(const int NROWS, const int NCOLS, const int NCELLS)
{
    int NCellsPainted = (NCELLS+1)>>1;
    uint painted = 0;
    uint32 seed = 10001;
    while (painted < NCellsPainted)
    {
        seed = seed * 10001 + 1001;
        uint rowcol = seed % NCELLS;
        if (grid[rowcol] == 0)
        {
            grid[rowcol] = 1; // [row,col] painted
            ++painted;
        }
    }
}
```

Consider as an example a 6×7 grid filled with 20 painted cells by generating random numbers in the range $[0, 40)$:

0	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	32	33	34
35	36	37	38	39	40	41

Notice that some cells are painted contiguously (on the same row the cells to the right or to the left of a painted cell or on the same column upper/lower cells are also painted). Let's find such randomly painted blocks with more than 1 cell and treat them as some sets. As for the example we would have the following two sets:
$S_1 = \{0, 1, 2, 3, 4, 5, 7, 8, 11, 15\}$
$S_2 = \{17, 23, 24, 25, 28, 29, 30\}$
S_1 has 10 elements that sum to 56 and S_2 has 7 elements that sum to 176.

How many such sets would we have if we have a 9991×1001 grid filled with 5000000 painted cells by generating random integers in the range $[0, 10000000)$?
What would be the set number of the set with the maximal number of elements?
What would be the set number of the set with the maximal sum of elements?

Note that the sets are sorted by their smallest elements and numbered accordingly.

Answer format: `a,b,c,d,e,f,g`
where
a is the number of sets
b is the set number of the set with the maximal number of elements
c is the number of elements of the set S_b
d is the sum of elements of the set S_b
e is the set number of the set with the maximal sum of elements
f is the number of elements of the set S_e
g is the sum of elements of the set S_e

Example: `2,1,10,56,2,7,176` for the 6×7 grid above.

[My timing: < 10 s]

Answer df57cac5372dbc5a6ab1bb24894f760e

562 Least number divisible by each number of a sequence

2019-10-19 18:49:10	by C_K_Yang	8 xp	Math

Given two integers $m > 0$ and $n > 0$, let $L(m, n)$ be the number of sequences which satisfy the following properties:

- Each number in the sequence is a natural number.
- The least number divisible by each number of the sequence is m.
- The length of the sequence is n.
- Any two adjacent terms in the sequence are co-prime.

For example, $L(4, 3) = 6$, there are only 6 co-prime sequences: $\{1, 1, 4\}$, $\{1, 4, 1\}$, $\{2, 1, 4\}$, $\{4, 1, 1\}$, $\{4, 1, 2\}$ and $\{4, 1, 4\}$.

You are given $L(10!, 10) \equiv 800548088 \pmod{1000000007}$ and $L(100!, 100) \equiv 73129372 \pmod{1000000007}$.

Find $L(10^{12}!, 10^{12}) \bmod 1000000007$.

Answer 17395b280e332232578d56da7a2c3ca4

563 The number with digit sum 10

2019-10-31 22:30:05 **by C_K_Yang** **10 xp** **Math**

Writing down the numbers which have a digit sum of 10 in ascending order, we get:
$19, 28, 37, 46, 55, 64, 73, 82, 91, 109, 118, \ldots$

Let $f(n)$ be the n-th occurrence of the digit sum 10. For example, $f(1) = 19$, $f(10) = 109$ and $f(100) = 1423$.

Let $S(k) = \sum_{n=1}^{k} f(n)$. For example $S(100) = 67582$ and $S(10^5) = 178986850167799$.

Find $\sum_{i=1}^{100} S(11^i)$.

Give your answer modulo $(10^{18} + 3)$.

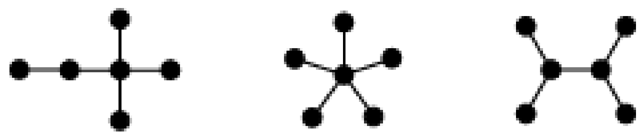
Answer 54dc20c76c6e9fb602e07cedb72d7323

565 Distinct trees II

2019-11-29 17:01:50 **by C_K_Yang** **8 xp** **Math**

Let $T(n, m)$ be the number of distinct trees with n nodes where the distance between any two nodes is no more than m. The distance between two vertices p and q is the number of edges we will come across as we travel from vertex p to vertex q in the shortest path. Two trees are not considered distinct if they are isomorphic.

For example, $T(6, 3) = 3$. Three distinct trees are shown below.



You are given $T(15, 4) = 128$ and $T(50, 7) \equiv 270742753 \pmod{1000000007}$.

Find $T(1000, 15) \bmod 1000000007$.

Answer **6e620c59fd033b2bdfa3415ae0292a12**

566 Tetration

2019-12-09 17:08:54 **by C_K_Yang** **7 xp** **Math**

The tetration of a number a by a positive integer b, denoted by $a \uparrow\uparrow b$ or ${}^b a$, is recursively defined by:
$a \uparrow\uparrow 1 = a$
$a \uparrow\uparrow (k+1) = a^{a \uparrow\uparrow k}$

Thus we have e.g. $2 \uparrow\uparrow 2 = 4$ and $3 \uparrow\uparrow 3 = 7625597484987$.

Let $P_{n,i}$ be the i-th prime number among all prime numbers $> n$ such that $P_{10,1} = 11$, $P_{30,2} = 37$ and $P_{100,3} = 107$.

Let $\mathrm{MOD}(n, m) = n$ modulo m.

Further define $f(k) = \sum_{i=1}^{k} \mathrm{MOD}\big(i \uparrow\uparrow i, P_{k,i}\big)$.

For example $f(3) = \mathrm{MOD}(1 \uparrow\uparrow 1, 5) + \mathrm{MOD}(2 \uparrow\uparrow 2, 7) + \mathrm{MOD}(3 \uparrow\uparrow 3, 11) = 14$.

You are given $f(100) = 20245$.

Find $f(10^6)$.

Answer e818d3161bd84ffe7c21552c6b937a09

567 Minimum logarithmic pair

2019-12-19 17:14:59 **by C_K_Yang** **10 xp** **Math**

For any two integers $m > 1$ and $n > 1$, we call an integer pair (m, n) a minimum logarithmic pair if there does **not** exist two integers a and b with $1 < a < m$ and $1 < b < n$ such that $\log_a(b) = \log_m(n)$.

For example, $(4, 9)$ is not a minimum logarithmic pair because $\log_4(9) = \log_2(3)$. Also $(3, 9)$ is not a minimum logarithmic pair since $\log_3(9) = \log_2(4)$. However $(2, 3)$, $(3, 4)$ and $(2, 4)$ are all minimum logarithmic pairs.

Let $D(N)$ be the sum of $m + n$ over all minimum logarithmic pairs (m, n) for $1 < m, n \leq N$.

You are given $D(10) = 818$, $D(100) = 982052$ and $D(10000) = 999830459302$.

Find $D(11^{111})$ and give your answer modulo $10^{18} + 3$.

Answer 9248079081914d2c0faf8f21fdc53ce4

568 Shuffling Cards revisited

2019-12-29 22:40:34 **by C_K_Yang** **9 xp** **Probability**

A standard deck of 52 playing cards, which consists of thirteen ranks (Ace, Two, …, Ten, King, Queen and Jack) each in four suits (Clubs, Diamonds, Hearts and Spades), is randomly shuffled. A *connection* is defined as two cards with the same rank appearing next to each other. Let us call a rank *perfect-k* if k connections occur in that rank after the shuffle.

For examples, the following card arrangement has one perfect-0, one perfect-1, and one perfect-2 (where `S` = spade, `H` = Heart, `D` = diamond, and `C` = club).

```
[7D][5C][7C][2S][7H][2D][2H][7S][2C][5D][5S][5H]
```

Rank 2 is perfect-1 because there is only one connection between `2D` and `2H`, Rank 5 is perfect-2 because one connection occurs between `5D` and `5S`, another connection occurs between `5S` and `5H`. Rank 7 is perfect-0 since no connections occur.

Let $E(m, n)$ be the probability that the number of perfect-i ranks is always prime for all non-zero number of perfect-i ranks after a random shuffle if we use a deck of playing cards which consists of m ranks, each in n suits.

You are given $E(5, 3) \approx 0.55272$ and $E(13, 4) \approx 0.26393$ (A standard deck of 52 playing cards).

Find $E(10, 8)$ and give your answer rounded to 12 places after the decimal point.

Answer **a5c7e7cd7411c0d328f65af353d70104**

569 Maximal Unity

2020-03-02 00:51:05	by sinan	6 xp	Math

Let S_N be the set of positive numbers less or equal to N. For example $S_5 = \{1, 2, 3, 4, 5\}$.

Find the subset of S_{100} with maximal number of elements ($\boldsymbol{MN}$) such that $\mathrm{sum}(1/e_i) = 1$ for $i = 1$ to MN and the sum $T = \mathrm{sum}(i \cdot e_i)$ for $i = 1$ to MN is maximal.

Answer format: `MN,T`

Example: `3,26` for the subset $S = \{e_1 = 2, e_2 = 3, e_3 = 6\}$ of S_6.

Answer 4bca322860c89e701c10b91726a83ff0

570 The number of divisors of cube-free numbers

2020-04-09 18:47:25	by C_K_Yang	8 xp	Math

A cube-free number is a positive integer which is divisible by no cubic number other than 1. Given a number $n > 0$, $f(n)$ is defined as follows:

$$f(n) = \begin{cases} \sigma_0(n) & \text{if } n \text{ is cube-free} \\ b(n) & \text{otherwise} \end{cases}$$

where $\sigma_0(n)$ is the number of divisors of n, $b(n) \equiv \sigma_0\left(c(n)^2\right)$ and $c(n)$ is the maximum number p such that p^3 divides n.

For example, $f(28) = 6$ and $f(432) = 9$.

Explanation: 28 is a cube-free number, $f(28) = \sigma_0(28) = 6$. However, 432 is not a cube-free number and 216 is the maximum cubic number which divides 432. $c(432) = 6$ and $b(432) = \sigma_0(6^2) = 9$.

Let $S(N) = \sum_{n=1}^{N} f(n)$. You are given $S(10^8) = 1223779154$.

Find $S(12345678987654321)$.

Answer 1f85c2826ca6bd51bd0f0b19152a8c9d

571 Balls in the boxes

2020-05-19 22:30:01 **by C_K_Yang** **9 xp** **Math**

We have several blue balls and red balls and we want to put them into boxes.

The rules for placing the balls into boxes are described below.
- Each box contains at least 2 balls and at most 6 balls.
- Each box contains at least 1 red ball and at most 4 blue balls.
- Balls are different from each other, even they have the same color.
- The order of the box doesn't matter. The order of each ball in the box doesn't matter.

For example, if we have 3 blue balls and 2 red balls. There are 7 different ways to put them into boxes.
(1) $\{B1, R1\}, \{B2, B3, R2\}$
(2) $\{B1, R2\}, \{B2, B3, R1\}$
(3) $\{B2, R1\}, \{B1, B3, R2\}$
(4) $\{B2, R2\}, \{B1, B3, R1\}$
(5) $\{B3, R1\}, \{B1, B2, R2\}$
(6) $\{B3, R2\}, \{B1, B2, R1\}$
(7) $\{B1, B2, B3, R1, R2\}$

Let $W(B, R)$ be the number of different ways to place B blue balls and R red balls into the boxes.

You are given $W(3, 2) = 7$, $W(5, 4) = 1510$ and $W(31, 27) \equiv 336072473 \pmod{1000000007}$.

Find $W(3141, 2718) \bmod 1000000007$.

Answer 6beb14c4c8c769c366b9e278f056bec2