

RoseCode

Unofficial Offline Edition

Problem Set From rosecode.neocities.org

Preface

Time may end, but hope will last forever.

About RoseCode

This document collects all the problems from <https://rosecodes.neocities.org> (RoseCode), which is a website that is along the lines of ProjectEuler. Unfortunately, only the remaining content of the problems can be seen on the website now, and a lot of useful information has been lost.

Due to various reasons, the content of some of the problems are incomplete, and some of them are even unworkable. In addition, the reference answer string is processed by the original answer (see next section), and quite a few answers are misplaced.

Despite all obstacles, the author tries his best to ensure that the content of the problems and the corresponding answers are complete and correct.

Note for the answer

The *answer* to each problem is processed from the original answer string. The following example Python code shows the process of getting the final answer.

```
from hashlib import md5

f = lambda x: md5(x.encode('ascii')).hexdigest().encode('ascii')).hexdigest()

ans = '123456'
print(f'{ans}\n{f(ans)}')
```

The running result is

```
123456
14e1b600b1fd579f47433b88e8d85291
```

This means that if the original answer string is **123456**, after processing the final answer string is **14e1b600b1fd579f47433b88e8d85291**.

1 Find a Prime

2009-09-06 00:46:15

by elasolova

5 xp

ProgrammingArchive

Prime numbers are numbers that are only divisible by 1 and themselves. The first prime number is 2. The 10000th is 104729.

Find the 78200th prime number.

Answer

e30928c4d3d1c530d66198f016081714

2 Fibonacci Run

2009-09-06 02:02:30

by elasolova

5 xp

ProgrammingArchive

Fibonacci terms are generated by adding previous two terms. So by starting with 1, 1 the sequence would be: 1, 1, 2, 3, 5, 8, 13, 21, ... and so on.

Find the first Fibonacci term that exceeds 100 in its digital sum (sum of digits).

Answer

5abae30126b49addb6b07816bd0f1e9a

3 Palindromic Primes

2009-09-06 15:08:15

by elasolova

6 xp

ProgrammingArchive

Palindromic prime is a prime number when reversed is the same prime. For example, 191 is one. Assume all one digit primes are palindromic.

Find sum of all palindromic primes not exceeding one million.

Answer **861b23fd3712b71971f9a803936904b4**

4 Distant Twin Primes

2009-09-08 01:46:01**by elasolova****6 xp****ProgrammingArchive**

The pair 3, 5 is special as it is the first prime pair to have difference of two (Twin primes). After this, comes pair 5, 7. The distance between these two pairs is 2 as it is calculated by subtracting the first member of each pair. Under 100, the pair 59, 61 has the longest length from such previous pair 41, 43 with the distance of 18.

Find longest distance of such prime pair, each member below one million, from such previous pair.

Answer **34e2cf40964474f394fc07e94bd32923**

5 Fractions to Decimals

2009-09-24 03:26:33

by elasolova

3 xp

ProgrammingArchive

Write a program that will accept a fraction of the form $\frac{N}{D}$, where N is the numerator and D is the denominator, that prints out the decimal representation. If the decimal representation has a repeating sequence of digits, it should be indicated by enclosing it in brackets. For example, $\frac{1}{3} = .3333333\dots$ is denoted as $.(3)$, and $\frac{41}{333} = .123123123\dots$ is denoted as $.(123)$.

Typical conversions are:

$$\frac{1}{3} = .(3)$$

$$\frac{22}{5} = 4.4$$

$$\frac{1}{7} = .(142857)$$

$$\frac{3}{8} = .375$$

$$\frac{45}{56} = .803(571428)$$

Sample Run

```
ENTER N,D: 1,7  
1/7 = .(142857)
```

What is the answer for the fraction $\frac{11}{59}$?

Source: 1993 USACO Qualifying Round

Answer

76a8ed8037d8c6ec28ba93cb5b6e9765

6 Concatable Prime Pairs

2009-09-24 03:57:20

by elasolova

5 xp

ProgrammingArchive

Pair 3, 7 is the first concatatable prime pair as 3, 7, 37 and 73 are all prime. When each member is below 100, there are 24 such pairs.

How many concatatable prime pairs exist when each member is below one thousand?

Answer **8545e38faf09ff054485fd8208ce3312**

7 Palindromic in base 4 and 6**2009-09-08 01:33:51****by elasolova****8 xp****ProgrammingArchive**

Number 21 in base 10 is special as it is the first two-digit number to have palindromic base 4 and base 6. We call a number palindromic if the number stays same when reversed. So, 21 in base 4 is 111 which is also palindromic. Similarly, in base 6 it is 33. Find sum of all such numbers in base 10 under ten million.

Note: All one digit numbers are assumed to be palindromic

Answer**7a88c52c85f9ec23ac975256d2ac42d1**

8 Equality in base 8 and base 9**2009-09-08 01:33:51****by elasolova****10 xp****ProgrammingArchive**

79 is 117 in base 8. When we reverse 79 we get 97. 97 is also 117 in base 9. Find the maximum number under 10 million that performs this peculiarity. Give your answer in base 8 or in base 9.

Answer **3138765a7685343f87427b42d3341c57**

9 Prime Cryptarithm

2009-09-24 03:22:57

by elasolova

6 xp

ProgrammingArchive

The following cryptarithm is a multiplication problem that can be solved by substituting digits from a specified set of N digits into the positions marked with *. If the set of prime digits 2, 3, 5, 7 is selected, the cryptarithm is called a PRIME CRYPTARITHM.

$$\begin{array}{r} * * * \\ \times * * \\ \hline * * * * \\ * * * * \\ \hline * * * * * \end{array}$$

Write a program that will find all solutions to the cryptarithm above for any subset of digits from the set 1, 2, 3, 4, 5, 6, 7, 8, 9.

Sample Run

ENTER A SET OF DIGITS: 23468

$$\begin{array}{r} 2 2 2 \\ \times 2 2 \\ \hline 4 4 4 \quad <3 more not shown> \\ 4 4 4 \\ \hline 4 8 8 4 \end{array}$$

The number of unique solutions = 4

Test your program with the prime digits 2, 3, 5, 7. What is the number of unique solutions?

Source: 1993 USACO Qualifying Round 1

Answer 28c8edde3d61a0411511d3b1866f0636

10 6x6 Checker Challenge

2009-09-24 03:57:33

by elasolova

5 xp

ProgrammingArchive

Examine the 6x6 checkerboard below and note that the six checkers are arranged on the board so that one and only one is placed in each row and each column, and there is never more than one in any diagonal. (Diagonals run from southeast to northwest and southwest to northeast and include all diagonals, not just the major two.)

Column	1	2	3	4	5	6
1		0				
2				0		
3						0
4	0					
5			0			
6					0	

The solution shown above is described by the sequence 2 4 6 1 3 5, which gives the column positions of the checkers for each row from 1 to 6.

ROW	1	2	3	4	5	6
COLUMN	2	4	6	1	3	5

This is one solution to the 6×6 Checker Challenge. Write a program that searches and finds all unique solution sequences to the 6×6 Checker Challenge. What is the total number of solutions found (including reflections and rotations)?

Source: 1993 USACO Qualifying Round 1

Answer [011ecee7d295c066ae68d4396215c3d0](#)

11 All Latin Squares

2009-09-24 03:12:19

by elasolova

7 xp

ProgrammingArchive

A square arrangement of numbers

```
1 2 3 4 5  
2 1 4 5 3  
3 4 5 1 2  
4 5 2 3 1  
5 3 1 2 4
```

is a 5×5 Latin Square because each whole number from 1 to 5 appears once and only once in each row and column.

Write a program that will compute the number of $N \times N$ Latin Squares whose first row is:

```
1 2 3 4 5 ... N
```

Sample Run

```
ENTER A WHOLE NUMBER (2-9): 4  
THE NUMBER OF 4 x 4 LATIN SQUARES IS 24.
```

What is the number of 5×5 Latin squares?

Source: 1993 USACO Qualifying Round 1

Answer b43e8a9faaf9e1e8334c13d6f20ff797

12 Farm Frenzy**2009-09-24 17:50:21****by elasolova****2 xp****MathArchive**

A farmer has 10 acres to plant in wheat and rye. He has to plant at least 7 acres. However, he has only 1200 to spend and each acre of wheat costs 200 to plant and each acre of rye costs 100 to plant. Moreover, the farmer has to get the planting done in 12 hours and it takes an hour to plant an acre of wheat and 2 hours to plant an acre of rye. If the profit is 500 per acre of wheat and 300 per acre of rye, how many acres of each should be planted to maximize profits? Give your answer as a pair in the form (x, y) where x : wheat, y : rye.

Answer **42f226cf66ec4c2b369d100f26a859d0**

13 Perfect Pitch

2009-09-24 18:16:43

by elasolova

3 xp

ProgrammingArchive

A perfect number is a positive integer that is equal to the sum of its proper divisors. For example, 6 is a perfect number because $6 = 1 + 2 + 3$.

8128 is also a perfect number. Find the next smallest perfect number that ends with 8128.

Answer

0f38cb3b329691ed5754ed9d5724eb2d

14 The 3n+1 Problem

2009-09-25 03:06:25

by elasolova

6 xp

ProgrammingArchive

Consider the following algorithm:

1. input n
2. print n
3. if $n = 1$ then STOP
4. if n is odd then $n \leftarrow 3n + 1$
5. else $n \leftarrow n/2$
6. GOTO 2

Given the input 22, the following sequence of numbers will be printed

22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

It is conjectured that the algorithm above will terminate (when a 1 is printed) for any integral input value. Despite the simplicity of the algorithm, it is unknown whether this conjecture is true. It has been verified, however, for all integers n such that $0 < n < 1000000$. (and, in fact, for many more numbers than this.)

Given an input n , it is possible to determine the number of numbers printed (including the 1). For a given n this is called the cycle-length of n . In the example above, the cycle length of 22 is 16.

For any two numbers i and j you are to determine the maximum cycle length over all numbers between i and j (inclusive).

What is the answer if $i = 1, j = 1000000$?

Answer 81b9dd0e0451918c42451dbd6ec71366

15 **Triplets****2009-09-25 04:23:01****by elasolova****5 xp****ProgrammingArchive**

Given:

- (1) X, Y and Z are different integers between 0 and 9 (both included)
- (2) $\frac{10 \times X + Y}{10 \times Y + Z} = \frac{X}{Z}$ (with infinite precision)

How many triplets are there?

Source: Prolog Contest 1 Ithaca

Answer**011ecee7d295c066ae68d4396215c3d0**

16 Swamp Kangaroo

2009-09-25 20:03:57

by elasolova

10 xp

ProgrammingArchive

Kangaroos are fascinating creatures. For one, the idea of carrying around their offspring in a pouch is very cute, and it reminds us of our own ways of transporting babies. For another, they can jump really far. That can be quite useful, in particular if you are a kangaroo stuck in a swamp with only small islands of land, and crocodiles swimming around. In that case, you would rather not land in the water. Besides mere jumping strength, it is also useful to have some computational power to compute how exactly to use that jumping strength to get where you need to go.

Here is how we model Kangaroo movement. Kangaroos can only move North-South or East-West; no other directions (such as diagonals). Kangaroos can jump any integer distance between 1 and 5 in one hop, which takes them one unit of time. However, after a longer jump, they have to rest before being able to jump again. Specifically, after jumping distance d , the Kangaroo has to rest $(d - 1)^2$ time units before being able to jump again. Also, if the next jump is in a different direction from the previous one, the Kangaroo takes an extra one time unit between the jumps to turn around.

The swamp will be described by a two-dimensional grid. Each entry is either water, denoted by a dot (.), or land, denoted by X. Two locations will be marked with special symbols. K denotes the initial position of the Kangaroo, and G is the goal the Kangaroo wants to reach (both of these are of course land). You are to find the shortest time in which the Kangaroo could get to the goal (if at all). It does not matter if the kangaroo is tired when it arrives at its destination; it doesn't have to rest.

What is the answer for the following input?

```
.....XXX.....XX.....X.X.XX.  
XK.....XXXXXXXX..XX.....  
X.....XXXXX....XX.X.X.X.....X.  
.....XXX....XX.....  
.....XX.....XX.....  
.....XX.....XX.....  
...XX.....XX.....XX.  
..XXXX.....X..  
.XXXX.....  
..XX.....XX.....X.....XXX..  
.....XX.....XX.X..X..X.XGX..  
.....XX.....XXX..
```

Source: USC Contest Fall 2008

Answer 30ec62934072bd5f3313cd90958318fa

17 Ski Lifts**2009-09-25 20:33:11****by elasolova****20 xp****ProgrammingArchive**

Climate changes made the rolling mountains of Alaska a perfect place for all-season skiing. To transform this region into a successful area for skiing, however, ski lifts are needed. People skiing do not like walking upwards. They want to ski downhill; when needed they are willing to ski on the same level for some time.

Optimal use of the area requires that, starting from an arbitrary point, a skier should be able to reach any other point just by skiing downhill or staying at the same level, and occasionally taking a ski lift.

A sufficient amount of ski lifts must be planned and constructed such as to fulfill this condition. On the other hand, building more ski lifts than necessary is a waste of money. What is the minimal number of ski lifts needed?

As ski lifts are built on high poles, we assume that a ski lift can be constructed from any place to any place, regardless of the terrain in between. A ski lift is unidirectional. It is important to know that in Alaska one is not allowed to ski in any other direction than North, South, East or West.

Input format

A line with two positive numbers w and l with $1 \leq w, l \leq 500$: the width and length of the area. w lines, each line containing l positive numbers $h(i, j)$, with $0 \leq h(i, j) \leq 10^9$, representing the height in each point of the area.

Input[ski.txt](#)

Source: Benelux Algorithm Programming Contest 2005

Answer**7b274337ad8441febbe6d566473e7d63**

18 Zebra Herd**2009-09-25 21:01:02****by elasolova****10 xp****ProgrammingArchive**

Zebras are very social animals. Like other members of the horse family, they form groups that tend to stick together and hang out fairly regularly, though not exclusively. (Humans also come to mind in this respect.) Lately, researchers have been trying to understand just how the communities of zebras evolve over time, what triggers changes, and so forth. Of course, all they have to go by is observations of where the zebras are over time. From that, we would like to figure out what are the most natural groups. The assumptions are that (a) if a zebra is part of a group, it tends to spend time close to others in that group, (b) if a zebra is not part of a group, it tends to spend time further away from others in that group, and (c) zebras do not change their group membership very often.

Let us make this more precise. You will be given a sequence of observations of zebras. For each observation time, you will have the exact location of each zebra. The distance between two zebras is exactly their Euclidean (straight-line) distance. We assume that there are exactly two groups of zebras in the herd, and will denote them by two colors. What we want to do is color each zebra either red or blue for each time step, expressing membership to one or the other group. To express assumption (c) above, we will assess a penalty of some given number c every time a zebra changes colors. To express assumptions (a) and (b), we look at the distance $d(i, j)$ between every pair i, j of zebras. If i and j are of the same color, then we assess a penalty of $a \cdot d(i, j)$ for this pair. If i and j are of opposite colors, then we assess a penalty of $-b \cdot d(i, j)$ (i.e., we give a bonus).

Thus, if you are given a proposed labeling of all zebras with either red or blue for each time step, you can compute how good an explanation of zebra activity it is. Your goal is to find the best possible labeling, in the sense that it has the smallest possible total penalty. But you will only need to output the total penalty of the labeling, not the labeling itself.

Input Format

The first line of data set contains two integers z, t , the number of zebras $2 \leq z \leq 10$, and the number of time steps $2 \leq t \leq 50$. Next comes a line with three floating point numbers $a, b, c \geq 0$, the penalty multipliers. This is followed by t lines, describing zebra positions. Each line contains $2z$ floating point numbers, giving the positions of the zebras in the form $x_1 \ y_1 \ x_2 \ y_2 \ \dots \ x_z \ y_z$. The first line contains the positions at time 1, the second line at time 2, and so forth.

Input[zebra.txt](#)

Give your answer rounded to two decimals like [wholepart].cd

Source: USC Contest Fall 2008

Answer**715176588d2e2019b5bab7024e9133c0**

19 Minimizing a DFA**2010-11-01 17:27:40****by elasolova****5 xp****ProgrammingArchive**

Here is a DFA given in the table form

State	a	b
1	2	4
2	1	4
3	1	4
4	3	3

In the minimal representation which states can be represented as just one state? What number is formed if you concatenate the state numbers you have found in increasing order?

Answer d9b1d7db4cd6e70935368a1efb10e377

20 Detimental

2010-11-01 17:35:43

by elasolova

4 xp

MathArchive

Let

$$A(x) = \begin{vmatrix} x & 2x & x^2 \\ x+1 & 2x+1 & x^2+1 \\ x+2 & 2x+2 & x^2+2 \end{vmatrix}$$

including the determinant operator.

Compute the sum of all $A(x)$ where x varies from 1 to 10^9 inclusive.**Answer** dcfcd07e645d245babe887e5e2daa016

21 Bit Sums**2009-09-26 15:35:24****by elasolova****5 xp****ProgrammingArchive**

Find the sum of the one hundred thousand smallest positive integers that use only the digits 1 and 0 and are perfectly divisible by 19. We're still dealing with base 10.

Answer **3b00360d0b24dd076621b681048454ea**

22 Happy Primes**2009-09-26 19:08:45****by elasolova****6 xp****ProgrammingArchive**

A happy number is built by adding the squares of its digits. Doing this permanently, the numbers will end in 1 or 4.

If a positive integer ends with 1, then it is a happy number.

The first few happy prime numbers are:

7, 13, 19, 23, 31, 79, 97, 103, 109, ...

What is the sum of the first 30000 happy prime numbers?

Answer**62a14055f8071f5c5f034c5a1f12ed7c**

23 **Divisible Hexagons****2009-09-26 19:03:21****by elasolova****2 xp****ProgrammingArchive**

A hexagonal number is computed as follows using the formula:

$$H(n) = n(2n - 1)$$

As shown, the first 5 hexagonal numbers are:

1, 6, 15, 28, 45

How many of the first 100000000 hexagonal numbers are divisible by all the numbers from 1 through 15?

Answer **8a615010f0469a53648706843d50122d**

24 Grande Facto

2009-10-12 22:50:57

by elasolova

4 xp

MathArchive

What is the sum of the digits of $50000!?$

Answer

574493e0d383d3df8bb6029e5fddce1d

25 Prime Triangle**2009-10-06 18:56:28****by elasolova****4 xp****ProgrammingArchive**

Given the Triangle of primes.

```
2
3 5
7 11 13
17 19 23 29
31 37 41 43 47
53 59 61 67 71 73
...
```

The sum of the 6th row is 384. Give the sum of the 1000th row.

Answer**8cab2a505eb34e919e18619c0e7d491c**

26 Passion of Pascal

2009-10-12 22:53:02

by elasolova

2 xp

ProgrammingArchive

What is the **digital sum of sum of numbers** in the **1000001th** row of Pascal Triangle?

Answer

62e3dde0ed5f930809b6303fbbe7493

27 Surjective Surge

2009-10-13 19:22:55

by elasolova

25 xp

MathArchive

Consider a function $f : A \rightarrow B$. What is the digital sum of number of surjective functions if $|A| = 1678$, $|B| = 1543$?

Answer dd4ef9c7c5b967575806f1d3e1000787

28 **Fruit Frenzy****2009-10-14 19:23:56****by elasolova****15 xp****MathArchive**

Assume you have 3 kinds of fruit in a basket: Apples, Bananas, Coconuts. And assume you have an unlimited source of these fruits. Then there are 15 ways to pick 4 fruits

4A, 4B, 4C
3A1B, 3A1C, 3B1A, 3B1C, 3C1A, 3C1B
2A2B, 2A2C, 2B2C
1A1B2C, 1A1C2B, 1B1C2A

Calculate how many ways to pick up 100000 fruits from 55555 kinds. What is the digital sum of this huge number?

Note: Problem reseted in Nov 26, 2009 as the answer in database was wrong.

Answer **0fd7aec2b5cb03dfd65400989ff1c564**

29 Dividing Cube**2009-10-21 01:20:58****by elasolova****2 xp****MathArchive**

How many integers between 1 and 2009 are divisible by a non-trivial cube (p^3 where $p > 1$)?

Note: Please solve by pencil and paper. It is very easy by programming.

Answer**588d0aca3271a7779caf6f0b20ad31b9**

30 Madequation**2009-10-30 02:11:33****by elasolova****25 xp****MathArchive**

Consider the equation

$$a + b + c + \dots + y + z = 100$$

(a to z : There are 26 variables in total)

How many solutions (number of all distinct $[a, b, c, \dots]$ lists) are there if all a, b, c, \dots are odd and positive?

Answer**80ec413696954ca897f01106f841d68f**

31 Game of Preference 1**2009-11-10 23:07:59****by elasolova****10 xp****Probability**

In the card game of preference, the deck of 32 cards(4-suit, 7-to-ace) is dealt among three players, so that every player gets 10 cards and the two remaining cards form the widow. How many ways are there to deal hands in such a way that the widow does not consist of two aces?

Answer**06935452afa35c1bf866eb1e886d1582**

32 Game of Preference 2**2009-11-10 23:08:06****by elasolova****10 xp****Probability**

Calculate the probability that in the game of preference (32 dealt among three players, 10 cards each, and 2 cards go to the widow) every player gets precisely one ace. Give the ratio in lowest terms.

Answer**ad691f5e186253d5bbf99003942a9d1d**

33 Relativement Primordiaux**2009-11-10 23:13:34****by elasolova****5 xp****MathArchive**

Consider the equation (x, y integers)

$$1773x + 1865y = 1$$

What is the smallest positive integer value for x ?

Answer**4e904ad3a6b3fa22cae1291c3818f31a**

34 Attente de Maximum**2009-11-10 23:14:55****by elasolova****5 xp****Probability**

You roll a pair of fair dice. Compute the expectation (expected value) of the maximum of the two results. Give the answer in lowest terms.

Info: [Expected Value: Wikipedia](#)

Answer**ed703dce39ebb5e07381353b77441f8a**

35 Steps to Hanoi

2009-11-11 16:47:34

by elasolova

3 xp

ProgrammingArchive

What is digital sum of the number of required moves to solve 300000-disk Tower of Hanoi puzzle?

Answer

5a1774856cca211d4ab8eb32c82b603f

36 Distinct Simple Graphs

2009-11-12 19:42:51

by elasolova

10 xp

MathArchive

What is the number of non-isomorphic (distinct) simple graphs on 28 nodes?

Answer

bf35b33faadfc76e8f16a2ee3f3dca2c

37 Comparisons in Mergesort**2009-11-13 16:43:48****by elasolova****8 xp****ProgrammingArchive**

Mergesort is a recursive sorting algorithm defined as:

- Assume length of input (n) is a power of two
- If $n = 1$, return the element
- If $n > 1$, break list into two
- call mergesort on each half
- merge the sorted lists

How many comparisons are required (in worst case) to sort a list with 2^{150} elements using mergesort?

Answer **814cd7505badaf54f6b874a201b7d4a7**

38 **Decipher****2010-11-05 20:18:05****by elasolova****14 xp****Crypto**

You have this rather mysterious message in decimal ascii form:

50 31 22 23 14 29 26 11 16 73 26 8 22 14 26 11 18 13 83 12 4 73 1 0 22 5 93 101

You only know that the key to decipher this message is 4 lettered. Find the secret message and type it.

Answer **dd8117d36227132688b265677ae47783**

39 F*ck World [BF]

2011-04-21 06:00:00

by elasolova

4 xp

Brainf**k

Write “Fuck World” in Brainfuck language.

Answer **4f0522feeda2c4ef50a61e7a476f9b26**

40 Lorem Ipsum

2009-11-19 05:02:35

by elasolova

3 xp

ProgrammingArchive

Count the number of unique words in this file: [ipsum.txt](#).

Answer [22ff51a85b792d59648febe87c7e8a89](#)

41 Output Input [BF]

2011-04-21 06:00:00

by elasolova

10 xp

Brainf**k

Write a Brainfuck code to output the input.

Answer

fabb925607c794f36b8cabafe74d75dc

42 Mean Square

2009-11-20 20:56:09

by elasolova

10 xp

MathArchive

What is the mean distance between two random points in a unit square? Give your answer to five decimal places.

Answer

8f59484a7283965be4c5e0e3ae6bdf92

43 Input Tuptuo [BF]

2011-04-26 06:00:00

by elasolova

10 xp

Brainf**k

Write a Brainfuck code that outputs the input in reverse.

Answer **1baefe74c697e441fa6013cde77ffa6e**

44 Dicemetrics**2009-11-22 03:22:46****by elasolova****10 xp****Probability**

A guy has 8 five-sided dice numbered from 1 to 5. A girl has 6 six-sided dice numbered from 1 to 6. They roll their dice and compare totals. The max total wins and it is a draw if sums are equal. What is the probability that the guy beats the girl? Round your answer to 9 decimal places.

Answer**04973eb7f37f4c9298885ef850e83de2**

45 Nonconsecutive Zeros

2009-11-22 23:40:01

by elasolova

6 xp

MathArchive

Derive a formula for the number of strings of length n over the alphabet $\{0, 1, 2\}$ which have no consecutive zeroes. What is the answer for $n = 150$?

Answer

d3c947523bcac173d492689cb227beab

46 Fibonacci [BF]

2011-04-26 06:00:00

by elasolova

13 xp

Brainf**k

Write a code in Brainfuck that given positive integer input n outputs n th Fibonacci number. **Assume 1st and 2nd are both 1.** Assume that the input will be between 1 and 13.

Answer

749d7d1b0ce950db32aa8b2cfbb9d513

47 State Lottery**2009-11-25 00:10:54****by elasolova****2 xp****MathArchive**

In this game of lottery 6 different numbers are chosen among 49. How many different combinations are there? Compute it, namely 49 choose 6.

Contributed by: **Gizmore**

Answer **0684891e5e50dc403248018b565c1996**

48 Lost Password**2009-11-25 08:03:11****by elasolova****12 xp****ProgrammingArchive**

I lost my password to a .rar file. It is heavily encrypted and I only get 73 passwords per second. Luckily I remember parts of my password, but I am not sure how long it would take.

The static string `Giz` appears somewhere or maybe does not.

There is definitely a fake 20th century birth year (only the year) somewhere.

There are 1-8 special characters (scattered and no multiple instances) from this charset: #+%&*!^; : -?/

Finally there is a word from my dictionary (65535 words) somewhere.

Note that any block can appear anywhere in the password. Could you please tell me how many days it will take to try every password if the speed is at 73 pass/s? Round your answer to 6 decimal places.

Contributed by: **Gizmore**

Answer**b5c492635f8821e5da83e6e70ef3f742**

49 **Binary Decode [BF]****2011-04-26 06:00:00****by elasolova****18 xp****Brainf**k**

Write a Brainfuck code that given a binary string outputs its decimal value. For simplicity only at max 8-bit binary values will be provided. Note that leading zeroes will be omitted.

Answer **244daf3838d7a55d445df40757cf9934**

50 Digital Sum [BF]

2011-04-26 06:00:00

by elasolova

9 xp

Brainf**k

Write a Brainfuck code that given positive integer n , outputs its digital sum meaning the sum of every digit.

Answer

80ecd7abc0579af356a6ca6fb24ec486

51 Totient [BF]

2011-04-26 06:00:00

by elasolova

28 xp

Brainf**k

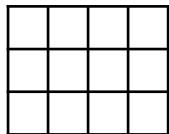
Write a Brainfuck code that given n , outputs n 's totient value. n will be random between 1 and 255.

Answer

f8ec53757476417c9350b1f6301bd85d

52 **RectoCount****2009-11-27 04:15:52****by elasolova****4 xp****ProgrammingArchive**

A rectangular grid of 4 by 3:



includes a total of 60 rectangles (including squares). How many rectangles are there in a 1453 by 1881 grid?

Answer **9c347a3af808909797b90fb1e9c97242**

53 String Decode [BF]

2011-04-26 06:00:00

by elasolova

15 xp

Brainf**k

Write a Brainfuck code that given a string, outputs hex code of given string.

Answer **9cc5e6129c7f78973f8c8bd0eb336032**

54 Be Careful!: Feel the Chemistry

2013-07-30 06:48:29

by elasolova

7 xp

Sequence

11 21 22 31 32 41 33 42 51 43 52 61 ?

Hint: *feel it*

Answer

1cdebc4405e3610c2b0ac08b81789cc9

55 Path to Palindromes

2009-12-02 07:30:11

by elasolova

6 xp

ProgrammingArchive

A number can be converted to a palindrome by summing the cube of each digit and repeating this procedure. For example 36 converts to 99 in 2 steps. Likewise 17 converts to 737 in 6 steps. We assume a number is not convertible in this way if it does not reach a palindrome in at most 100 steps. Below one million, which number has to undergo most steps (if there are more than one, assume smallest). Give your answer as `number.steps`

Answer e293d25b2e6f07a9fb373905cb4aae88

56 Convergents of e

2009-12-02 07:29:39

by elasolova

10 xp

ProgrammingArchive

Square root of 2 can be represented as $[1; (2)]$ where (2) means 2 repeats *ad infinitum*. In a similar way, $\sqrt{23} = [4; (1, 3, 1, 8)]$ (Note: [Continued Fraction](#))

Surprisingly,

$e = [2; 1, 2, 1, 1, 4, 1, 1, 6, 1, \dots, 1, 2k, 1, \dots]$.

The first ten terms in the sequence of convergents for e are:

2, 3, 8/3, 11/4, 19/7, 87/32, 106/39, 193/71, 1264/465, 1457/536, ...

Find the sum of digits in the numerator of the 1000th convergent of the continued fraction for e.

Note: An extension of Project Euler [Problem 65](#)

Answer 0e3a9753e6442ff8e2d954e523898a9c

57 Non-Repeating Digits

2009-12-02 08:05:02

by elasolova

6 xp

ProgrammingArchive

Write a counter function that counts from 1 to one million (inclusive) but only counts numbers whose digits don't repeat. As an example, 98, (99, 100, 101), 102 is a jump of 4. Submit total count of numbers and biggest jump. Format: total.jump

Answer dae0effa2bcebc**572983463f92ab797a**

58 Meaning of Life

2013-07-31 04:46:51

by elasolova

8 xp

Sequence

Have you ever seen the movie Gattaca?

All capitals.

Hint: *matching strand*?

Answer

0b373dddb5179af216202196b00cba32

59 Edges of Bipartite

2009-12-03 21:00:06

by elasolova

10 xp

MathArchive

Let $|V| = 1000$, and assume W a subset of V is selected at random from the collection of all subsets of V . Let us form the complete bipartite graph G with parts W and $V \setminus W$ (in other words, we connect two vertices if and only if one of them is in W while another is not). What is the expected number of edges in G ? Round to an integer if needed.

Answer f04ca46e8ee0bb079788ffaec7ebc5da

60 Pandigital Ends

2009-12-04 04:20:27

by elasolova

7 xp

Programming

A pandigital number makes use of the digits 1 through 9 at least once but not in any particular order. For example:

146523798 is a 9 digit pandigital.

What is the sum of values of i such that 3^i has its last 9 digits pandigital from 1 to 9, where $1 \leq i \leq 1000000$?

Answer 76bf00a22f55c046782907a6e4fcd63e

61 Factorial [BF]

2011-04-26 06:00:00

by elasolova

13 xp

Brainf**k

Write a Brainfuck code that given positive integer n , outputs its factorial.

Answer **941d23c1f6df779e74b09c7c965c010f**

62 n-Ovals Problem

2009-12-05 05:55:17

by elasolova

4 xp

MathArchive

Suppose that n ovals (an oval is a closed curve that does not cross over itself) are drawn on the plane such that no three ovals meet in a point and each pair of ovals intersects in exactly two points. How many distinct regions of the plane are created by n ovals? Provide a closed form answer including n .

Note: Use \wedge as the power operator. Do not use paranthesis (or spaces) anywhere. Use precedence rules instead. You should write terms decreasingly in terms of power of n .

Answer 19492447d71be6e366b47501a8141a6c

63 Parentheses Preference

2009-12-05 06:34:27

by elasolova

12 xp

ProgrammingArchive

Suppose we want to find the number of ways to parenthesize the expression

$a + b + c + \dots + y + z$ (26 terms)

so that a parenthesized form of the expression reflects the process of adding two terms. How many ways are there?

Answer **bafdf6f63860d3521249c98fbbaaf27e9**

64 Sort String [BF]

2011-04-26 06:00:00

by elasolova

10 xp

Brainf**k

Write a Brainfuck code that given string n , it sorts characters in string in increasingly alphabetic order then outputs this sorted string. Thus, given “haeusl” it outputs “achlsu”.

Answer 37ac82dfaf159fcfd623f7e3e894ee53

65 **Weird Dice****2009-12-10 01:38:52****by elasolova****3 xp****Probability**

Find two six sided dice, such that the probability of each sum from 2 to 12 is the same as two standard dice. Each side must have least one dot. Negative numbers are not allowed. There is another answer besides two standard 123456 dice. Format of the answer should be 111222,113344 (just an example) where first integer is the smaller and integers acquired by concatenation increasingly.

Answer **6f1e421bfc7159d4d48bed4c7fdeed5d**

66 Unique Chars [BF]

2011-04-26 06:00:00

by elasolova

15 xp

Brainf**k

Write a Brainfuck code that given input string x , outputs number of unique characters.

Answer

7bfcc85c0d74ff05806e0b5a0fa0c1df1

67 Prime Finder [BF]

2011-04-26 06:00:00

by elasolova

21 xp

Brainf**k

Write a Brainfuck code that given n , outputs n th prime. Assume n will be between 1 and 54.

Answer

a54c22fe5a0cf0edb9ab5299c8d60d54

68 Bob and Stephen**2010-01-10 04:00:13****by elasolova****2 xp****MathArchive**

Bob and Stephen are brothers. Stephen is twelve years older than Bob. What is the maximum possible age of Bob such that the ratio between ages of two brothers is an integer?

Answer **c8b2f17833a4c73bb20f88876219ddcd**

69 New Year's Equation

2010-01-10 04:02:42

by elasolova

4 xp

MathArchive

How many real solutions are there for the following equation:

$$x^{2010} + 2009 = 2008x$$

Answer

dcfcd07e645d245babe887e5e2daa016

70 New Testament

2010-01-10 04:08:01

by elasolova

5 xp

MathArchive

Given $x + y = < 2009$ and $y + z \leq 2010$. Suppose the number of non-negative integer solutions to these inequalities is in the form $2k(6k + 1)(6k + 2)$ with k positive, find k .

Answer d8827f2811ee6fbaa9a0fc087a1f8446

71 Be Careful!: Famous Ensemble

2013-07-31 04:09:52

by elasolova

6 xp

Sequence

5 85 987 4482 ?

Hint 1: *Think title*

Hint 2: *4 digits*

Answer

45e39d4eb3d1da13702f0b6e763d11f4

72 8-Puzzle Challenge

2010-01-20 06:24:07

by elasolova

12 xp

ProgrammingArchive

A 9-digit representation of 8-puzzle abcdefgh0 corresponds to:

```
a b c  
d e f  
g h
```

where 0 is the place of the empty cell.

Given 100 start states in [8puzzle.txt](#) compute the average of shortest path distance (that takes the least number of moves) to goal 253176408. You can use any of the suiting heuristic functions you like. Round your answer to 2 decimal places.

Answer a4d6a50b2f0276bda71f0e5f9bb1d190

73 Mouse Escape Time

2010-02-01 04:25:27

by elasolova

4 xp

MathArchive

Consider a mouse in a maze. There are four possible paths such that path 1 takes 5.32 minutes to escape, path 2 takes 3.12 minutes to escape, path 3 takes 8.24 minutes to return to same place and path 4 takes 2.35 minutes to return to same place. Given that the mouse chooses any of the four paths at random what is the expected time of escape in minutes? Round your answer to 4 decimal places.

Answer

2ce54e2f07f370941324911849396c28

74 String Shift [BF]

2011-04-26 06:00:00

by elasolova

5 xp

Brainf**k

Write a Brainfuck code that shifts input string chars by one and outputs it. Ex. javaist → kbwbjtu

Answer

b522a1532d429304912638c9f430b60c

75 Pragma Series: Question out of the Box**2012-06-15 22:46:32****by elasolova****4 xp****Hack**

Here is a simple question:

2+2=?

The problem here is that there is no usual way to submit the solution. You need to figure that out by yourself!!

Answer 011ecee7d295c066ae68d4396215c3d0

76 Pragma Series: Lucky Enough?

2012-06-15 16:42:30

by elasolova

11 xp

Hack

Here is a flash dice simulator. It is somewhat biased, however. You need to roll two 6s in order to get the secret code. If you do not hack somehow, you will never be able to get that lucky!! When you get the code submit like you did for problem 75.

Answer b6f20d75395e7531f07ec2e241a40d5d

77 Pragma Series: Unknown Decoder**2012-06-18 23:33:45****by elasolova****8 xp****Crypto**

In this problem we provide the answer. It is: javaist. However, the server will first decode the answer you submit. Unfortunately, the encode/decode algorithm used is unknown. However, we provide a basic flash tool to help you. Given an encoded text it provides the original data. Feel free to experiment and then encode the actual answer before submitting.

Note: The returned string of flash tool is only a crude report of actual encoded string. It is not totally dependable. So examine its source code, find the intermediate page and that way your experimentation with encoding will be more dependable.

Answer 0e21e3905108e19bcd1ebe6f27da07f7

78 Pragma Series: RGB Average**2012-06-21 22:41:47****by elasolova****12 xp****TimeRace**

p78.php creates a random png image. Your task is to detect the average RGB values for the image and POST it to the same page (p78.php) via `rgb=RAvg:GAvg:BAvg&submit=true`. Round each average to 2 decimal places. Note that you have 1 second to do so after you see the random image.

Answer **5a9e8b445c690639919892e311fd8649**

79 Pragma Series: Original Triangle**2012-06-23 18:51:42****by elasolova****13 xp****TimeRace**

In this problem you will be supplied with several thousands of random triangle vertice positions(in cartesian coordinates). Your task is to compute number of triangles that contain the origin.

[p79.php](#) supplies the list of triangles in the form Ax, Ay, Bx, By, Cx, Cy per line.(where ABC are the vertices). When you get the answer POST to same page (p79.php) as `amount=A&submit79=true`. You have 1.86 secs to do so.

Answer **5a9e8b445c690639919892e311fd8649**

80 Image Barcode

2012-06-23 21:34:51

by elasolova

5 xp

Crypto

Here is a barcode like image. Can you extract the code?

Answer **2752bf43f7f6efe2008f538796eee63a**

81 Pragma Series: Brute Interrogation Made Easier**2012-09-26 04:21:39****by elasolova****15 xp****Hack**

In this problem you need to interrogate an ex-agent prisoner. You can perform 4 actions. The prisoner's state is kept as a session variable on the server side and you will be able to see it once you get to the secret page. You also receive a text regarding the state.

These 4 actions translate to some basic arithmetic operations done on the variable. When the condition is met the prisoner will confess the secret code.

Note: You need to make the variable somehow 1000 to see the password. Brute force is not necessary from now on. It is simple reverse engineering.

Answer**85dc3b233cba0477ea6d82f4fbcc74e7**

82 Pragma Series: Sorted Radical Function**2012-06-27 22:42:46****by elasolova****12 xp****TimeRace**

The radical of n , $r(n)$, is the product of distinct prime factors of n . For example, $156 = 2 \times 2 \times 3 \times 13$, so $r(156) = 2 \times 3 \times 13 = 78$. If we calculate $r(n)$ for $1 \leq n \leq 10$, then sort them on $r(n)$, and sorting on n if the radical values are equal, we get:

1, 2, 4, 8, 3, 9, 5, 6, 7, 10

let $S(k)$ be the k th element on the sorted list. So $S(3) = 4$, $S(7) = 5$ etc. [p82.php](#) will print random M and N values where $r(i)$ is sorted for $1 \leq i \leq M$ and the answer is $S(N)$. POST to p82.php as $ans=S(N)&submit82=true$. You have 1.12 secs to calculate!!.

Note: M will be between 8000 and 10000, whereas N will be between 1000 and 3000.

Fix: Fixed index problem. Now the S array starts from 1 as described above instead of starting from 0.

Answer **5a9e8b445c690639919892e311fd8649**

83 Pragma Series: Binary or Not?

2012-06-28 23:26:45

by elasolova

15 xp

TimeRace

p83.php creates some random binary looking data. Your task is to decode that into english letters. (not necessarily forming a meaningful word!). Then POST as ans=letters&submit83=true. Time limit is 0.7 secs.

Answer 5a9e8b445c690639919892e311fd8649

84 BeCareful!: Grasshopper

2012-06-30 22:47:40

by elasolova

7 xp

MathArchive

On the real number axis, Goppy the Grasshopper can hop along (starting from 0) the axis either way, 364 or 715 units length. What can be the minimum distance of any hopped point to point 2010.

Answer

4e44f1ac85cd60e3caa56bfd4afb675e

85 Find The Next One

2019-01-30 07:01:05

by C_K_Yang

8 xp

Sequence

There is an integer sequence:

1, 8, 216, 13824, 4741632, 11119431168, 19421724672, 263128269312, 224188386112438272,
391914614322266112, ?, ...

Find the number which the question mark represents.

Answer

86 BeCareful!: MultCount**2012-07-01 21:30:48****by elasolova****4 xp****MathArchive**

For n bigger than 1, how many multiplications we need to perform (at least) to calculate n to the power 13?

Answer**4e44f1ac85cd60e3caa56bfd4afb675e**

87 BeCareful! 3-Slot Scales**2012-07-01 21:32:48****by elasolova****8 xp****MathArchive**

Looking the same, but having different weights, we have 50 balls. We also have a 3-slot scales. This scales can sort 3 balls from least to most in weight with a single measurement and it only accepts 3 balls (no more no less) at a time. The question then is, how many measurements (at minimum) we need to perform to find the 2nd lightest ball?

Answer **80ecd7abc0579af356a6ca6fb24ec486**

88 BeCareful!: Group Work

2012-07-01 13:38:00

by elasolova

5 xp

MathArchive

One of the professors in Math department gives a lot of homework. We have 3 particular students in this class called Alice, Ben, and Clark. When Alice and Ben works together they finish a day's homework in 5 hours. Clark and Alice together finish it in 3 hours as they kind of fancy each other :) and like to work together. Ben and Clark, together, finish in 4 hours. One day both Ben and Clark are sick and they cannot work. Alice has to do it alone. How long will it take (in hours)? Round to 2 decimal places.

Answer

f736b67909f706452fe35fe3f73dd38d

89 BeCareful! Couple HandShake

2012-07-01 17:36:15

by elasolova

8 xp

MathArchive

You are a reporter of Chicago Tribune Entertainment section. There was a celebrity couple-exclusive party in Chicago. It was a formal and fabulous one :) so everyone in the party had hanshaked others excluding their own partners. Your paparazzo were able to capture 144 handshakes. You know that they are the best in this job. So your director in the newspaper wants you to report how many couples there were in the party?

Answer

4c0d13d3ad6cc317017872e51d01b238

90 BeCareful!: UnHumDrum**2012-07-01 18:02:48****by elasolova****9 xp****MathArchive**

You are the head of a jewelry design firm. One of your clients asked you to design a specific brand of jewelry that will be called UnHumDrum. So here is the challenge. The materials will used are composed of 2 crimson red, 2 metallic blue, and 2 cyanic green beads. (there is no difference in these beads other than their color). You have to line these beads on a linear (not circular) silken strand and beads with same color must not be next to each other to avoid humdrum. (there comes the brand name :)) So in your catalog for this new brand, how many distinct products can you name?

Answer**4db87140662bd68076ef786f7163cedc**

91 Be careful!: Monthly Birthdays

2012-07-22 15:43:17

by elasolova

3 xp

MathArchive

What minimum number of friends do I need to have to be sure that at least 7 of my friends have their birthdays in same month?

Answer

8c9d4c8374b6277fbfddc0fa4e4fbe1f

92 Be careful!: Mega Box**2012-07-07 17:19:05****by elasolova****2 xp****MathArchive**

We have a mega box composed of small cubes attached together. The box is a 3d rectangular shape in that each dimension includes 9, 10, 11 cubes respectively ($9 \times 10 \times 11$ cubes included). We paint the whole surface of this box to red. The question is that how many cubes will not be painted at all?

Answer**40ece03de4fc621a653ad5d565e41701**

93 Be careful!: Rook Attack!

2012-07-08 16:43:59

by elasolova

6 xp

MathArchive

On a conventional chess board(8×8), we want to place 3 rooks. How many different placements are there so that they won't have a chance to attack each other?

Answer

f1a0ec25e9d0cfad11c7f6f21f072a63

94 BeCareful!: Bombastic Queue**2012-07-16 17:25:26****by elasolova****7 xp****MathArchive**

Mr. Bombastic wants to enqueue his students in a row. There are 7 students in the class that are queued according to their heights from shortest to longest currently. Mr. Bombastic has a taste of fashion and does not like orderly stuff. So he decides to disrupt the queue. However, the school directors only permit 2 but only 2 swaps starting from height order. A swap is between **any** two students. So how many different queue options does Mr. Bombastic have by exactly two successive swaps?

Note: There was a confusion about number of swaps available. It is EXACTLY two swaps. No more no less.

Answer **8cdf85a71d67ce91884d391eafb6704a**

95 BeCareful!: Prepare for War**2012-07-17 16:36:11****by elasolova****12 xp****MathArchive**

You have 10 distinct army units in a strategy game online. One of your allies warns you that you will witness an upcoming attack. This attack will simultaneously target 4 areas of your empire. As a commander, you want to evaluate your options. You decide that each area needs at least one army unit and also you have to deploy all of your 10 units no matter what. How many possible deployment options are there?

Note: There was a spacing error in the answer in the database. Problem fixed on 26 July 2012.

Answer **acb115ad95b90d6aca5460bff66c4cec**

96 Deep Compression

2012-07-24 14:50:53

by elasolova

6 xp

ProgrammingArchive

Some compression has been done on [this file](#). Solution lies deep inside.

Answer

6e86d53e3c80b57efc425c3d81837c52

97 Hitchhiker's Bad Luck

2013-07-31 04:36:17

by elasolova

12 xp

Sequence

10 29 36 bb 61 07 c3 82 90 b1 a6 ?

Hint 1: *think baseic*

Hint 2: *think as a whole*

Answer

d15509b48113ffaf089d780645d022ee

98 Be Careful!: Self Service**2012-09-02 00:58:40****by elasolova****8 xp****Sequence**

20,23,5,14,20,25,20,23,5,14,20,25,20,8,18,5,5,6,9,22,5,6,15,21,18,20,5,5,14,20,23,...

Get the rule for this sequence. Then what term waits longest to make its first appearance, tell me?

Answer**83a5a282f092aa7baf6982b54227bb54**

99 Be Careful!: Combination I

2012-09-02 01:17:36

by elasolova

5 xp

Sequence

88 op. 63 = 25

25 op. 9 = 16

38 op. 16 = 18

33 op. 18 = 15

32 op. 15 = x

Find the rule then find x, but think out of the box. op. is **not** a conventional operator.

Answer 7bfc85c0d74ff05806e0b5a0fa0c1df1

100 Be Careful!: Combination II**2012-09-02 02:09:44****by elasolova****5 xp****Sequence**

```
34 - 16 = 18
32 - 18 = 14
36 - 14 = 22
46 - 22 = x
64 - x = 40
75 - 40 = 35
50 - 35 = 15
35 - 15 = 20
```

Think out of the box again!! Submit x. “-” is not necessarily subtraction operator.

Answer **29bdbc822df2e6c13dcf4afe6913525f**

101 Aleration

2012-09-11 17:37:02

by elasolova

5 xp

Hack

Your task is to inject <script>alert()</script> into [this page](#). Note that there are some crude protection and illusions on the way. When you get the pass after injection just submit here.

Answer

ff7c3f88b0ec4cfffe5f9e3013659d889

102 App: Exeintro 1**2012-09-26 03:32:29****by elasolova****5 xp****Hack**

This is our starter exe challenge. The solution is obtained through a slight! modification. We are dealing with command line (so expect no UI for now) Feel free to discuss spoiler free thoughts on the forum.

[Download the App](#)**Answer** **7eec05929e2d0765a82247d358ff16a6**

103 App: Exeintro 2

2012-09-26 03:35:24

by elasolova

6 xp

Hack

Another intro exe challenge. Still we are dealing in command line. This one lacks some arguments obviously.

[Download the App](#)**Answer** 448d11c0814ddbdbb99a318c423ac141

104 App: ClickMania

2012-09-29 04:47:56

by elasolova

12 xp

Hack

Some clicking involved!

[Download the App](#)

Answer

7837471baab8142887d104e02df875f2

105 Number Cross

2012-10-03 02:15:19

by elasolova

7 xp

ProgrammingArchive

If you place 1 to 8 into this puzzle

```
* * *
* * *
* *
```

where no consecutive (+1 / -1) will be adjacent to each other (left, right, up, down, up-right, etc., all directions), you will have 6 solutions. Namely, one is this:

```
7 4 6
1 8 2
3 5
```

So the question is, how many solutions are there for 1 to 10 for this puzzle

```
* * *
* * *
* *
* *
```

Answer 0c010fdb9548da6b2e31539b3b9968b3

106 **Realistic I****2012-12-27 23:42:37****by elasolova****20 xp****Hack**

Here you have a [link](#) to an insurance website. This insurance company has suspended its services lately due to its underpopularity. Your friend John Milkway is a customer of this website and has already paid for his insurance plan. He wants his insurance back. Aware of your hacking skills he requested help. Help him to get what he wants.

Hint 1: *robots*

Hint 2: <Limit>

Select area to reveal hints if desired.

Answer **2223403e559fe371afb33a90cda28201**

107 **Realistic II****2012-12-31 23:30:34****by elasolova****20 xp****Hack**

This time javaist.com needs your help. There is a secret hacking team called Munati. They have chosen javaist.com as their newest target. Find exploit and login to their administration before it is too late.

[Link to Munati](#)

Hint 1: *schema*

NOTE: Problem is unavailable/unsolvable till a second notice due to database issues. Sorry for inconvenience...

Answer**0096c586ba3b4003636f607f62f4e357**

108 RSA Attack**2013-04-08 06:00:00****by sinan****10 xp****Crypto**

The following message has been encrypted using the RSA algorithm (<http://en.wikipedia.org/wiki/RSA>).

Message:

```
5270520056270255633349794102662146088700587067743698074774401192
3847352489345495988265259619286488631483317732994244643357354751
9287620596940036558335740983810271503064948492663425315576482108
0559744153567574381389598019296701861993253450006887163420818773
64884130165379971128518570467092626371748600039
```

For the conversion from text to number, treat a text of N bytes as $N \times 8$ -bit big endian number as in the following examples:

```
YOU = 89,79,85 -> hex: 594F55 decimal: 5853013
YOU CAN = 89,79,85,32,67,65,78 -> hex: 594F552043414E decimal: 25138499959341390
```

You are given the following RSA public key:

```
N =
6000448200243096997311180274736010139143866945550988201685002174
8739658862996422859619109213678968055145301683382976222210480135
9987695864415136344185954945286773854249588357461032878749312395
9570387283082631329238518405786607791478421823609345485233086548
71204913382882966214741283716047066022304286877
```

```
E =
5158023707740549673024298923137970117927796628442809247177892775
1105717909512676627114980499027410324845069540466241220902950388
3395360985550494492934002445385787731172407347263592136446777198
2606935670578340411244533070952883031679573061445213299472912441
18700665840856941580663774265885674204692471913
```

Can you decrypt the message?

Answer format: The secret phrase

Answer **5c75fb70c67ec3e8f0f73c7a393a6189**

109 Repeated Appereance**2013-08-01 06:00:00****by sinan****12 xp****Probability****by sinan**

How many times must a die with 6 faces be thrown, on average, until one face appears 4 times?

For example, one set of throws might be 1, 2, 1, 4, 3, 5, 2, 4, 6, 1, 3, 1.

Submit your answer as a/b , where a and b are two relatively prime, positive integers.

Answer**ae5ad70d0cb9f1587d8e028bf57ca717**

110 **Balls and Boxes****2013-11-27 20:08:42****by elasolova****15 xp****Probability**

A box contains r red balls and b blue balls. Balls are removed sequentially from the box (without replacement). What is the expected number of balls left in the box at the first instant at which all the remaining balls are of the same colour?

Give your answer in closed form and simplify as much as you can, use parenthesis where needed. After simplification, there will be a few possible answers depending on ordering of the terms and only one will be accepted so try all.

Answer **62dff1041b06f07a16160188891fc0d**

111 Histheorically?**2014-01-24 07:00:00****by elasolova****15 xp****Math**

Let's define a fast-growing sequence.

$$F_n = 2^{2^n} + 1, n \geq 1$$

Now let's calculate the sum s of all pairwise gcd (namely sum of greatest common divisors of all 2 element combinations). Now the sequence starts with $n = 1$ and goes up to $n = x$. Give the answer as a polynomial in x . Output format is (a, b, c) where $ax^2 + bx + c$ is the polynomial. Use reals with one digit after the decimal point.

Answer**029c808e80134d14ccbe18d8e192e94a**

112 Mirror primes 2**2014-05-20 06:19:32****by sinan****5 xp****Math**

Mirror (Palindromic) prime is a prime number when reversed is the same prime. Assume all one digit primes are mirror primes.

Find the sum of all mirror primes with all **prime digits** not exceeding 10^{20} .

Answer format: count , sum

Example: 8,2227 for all mirror primes with all **prime digits** not exceeding 10^3 (2, 3, 5, 7, 353, 373, 727, 757).

Answer**b9e9537ea7e4e8d39d069a7a03f7b68c**

113 Odd coefficients**2014-05-20 18:27:02****by Philippe_57721****10 xp****Math**by **Philippe_57721**Let $P[X, n] = (1 + X + X^2)^n$

$$P[X, 7] = X^{14} + 7X^{13} + 28X^{12} + 77X^{11} + 161X^{10} + 266X^9 + 357X^8 + 393X^7 + 357X^6 + 266X^5 + 161X^4 + 77X^3 + 28X^2 + 7X + 1$$

It contains 11 odd coefficients: 1, 7, 77, 161, 357, 393, 357, 161, 77, 7, 1

How many odd coefficients are there in $P[X, 6700417]$?

[My timing: < 100 ms]

Answer**26c2e58c3a6556738930d45dff1e687d**

114 From End to Beginning**2014-05-21 07:10:08****by sinan****8 xp****Math**

Let $a(n)$ be the n -th term of a sequence.

We have the following relation:

$$a(n) = 3 \cdot a(n - 1) - 2 \cdot a(n - 2) + 1 \cdot a(n - 3)$$

What is $a(1)$ if we have the following values:

$$a(10^{100} + 0) = 0$$

$$a(10^{100} + 1) = 1$$

$$a(10^{100} + 2) = 2$$

Submit your answer as $a(1) \bmod 10^9$.

Problem maintainer: **sinan**

Answer **81ff5274a990a2c94a76cad063e2cca0**

115 A special order

2014-05-22 17:48:13

by sinan

6 xp

Programming

Order the numbers 1 to 66 in such a way that sum of any 2 consecutive numbers is a square number. In case there are multiple solutions, find the lexicographically first solution.

Answer format: concatenate every 5th number starting with the 1st. (i.e: 1st, 6th, 11th, ..., 66th)

Example: 831116 for 1 to 16 (8, 1, 15, 10, 6, 3, 13, 12, 4, 5, 11, 14, 2, 7, 9, 16)

Answer

908bab3ef7771f48cba7e30af2250628

116 The biggest square (harder)

2016-04-12 08:30:48

by sinan

10 xp

Math

See p319.¹ The same definition and the same answer format. Find those with **100th biggest area** for $R = 10^{10} + 19$.

[My timing: < 10 s]

Answer

c95cc286fc710445c123f141cb186ff9

¹ Actually [Problem 323](#).

117 Pure partitions

2014-05-22 17:55:28

by Philippe_57721

10 xp

Math

Author **Philippe_57721**

A partition of n is pure if it can express every number between 1 and n in exactly one way.

There are 8 pure partitions of 11:

- 6 3 1 1 = 1 (1 + 1) 3 (3 + 1) (3 + 1 + 1) 6 (6 + 1) (6 + 1 + 1) (6 + 3) (6 + 3 + 1) (6 + 3 + 1 + 1)
- 6 2 2 1 = 1 2 (2 + 1) (2 + 2) (2 + 2 + 1) 6 (6 + 1) (6 + 2) (6 + 2 + 1) (6 + 2 + 2) (6 + 2 + 2 + 1)
- 6 1 1 1 1 1 = 1 (1 + 1) (1 + 1 + 1) (1 + 1 + 1 + 1) (1 + 1 + 1 + 1 + 1) 6 (6 + 1) (6 + 1 + 1) (6 + 1 + 1 + 1) (6 + 1 + 1 + 1 + 1)
- 4 4 2 1 = 1 2 (2 + 1) 4 (4 + 1) (4 + 2) (4 + 2 + 1) (4 + 4) (4 + 4 + 1) (4 + 4 + 2) (4 + 4 + 2 + 1)
- 4 4 1 1 1 = 1 (1 + 1) (1 + 1 + 1) 4 (4 + 1) (4 + 1 + 1) (4 + 1 + 1 + 1) (4 + 4) (4 + 4 + 1) (4 + 4 + 1 + 1)
- 3 3 3 1 1 = 1 (1 + 1) 3 (3 + 1) (3 + 1 + 1) (3 + 3) (3 + 3 + 1) (3 + 3 + 1 + 1) (3 + 3 + 3) (3 + 3 + 3 + 1) (3 + 3 + 3 + 1 + 1)
- 2 2 2 2 2 1 = 1 2 (2 + 1) (2 + 2) (2 + 2 + 1) (2 + 2 + 2) (2 + 2 + 2 + 1) (2 + 2 + 2 + 2) (2 + 2 + 2 + 1) (2 + 2 + 2 + 2 + 2) (2 + 2 + 2 + 2 + 2 + 1)
- 1 1 1 1 1 1 1 1 1 1 = 1 (1 + 1) (1 + 1 + 1) (1 + 1 + 1 + 1) (1 + 1 + 1 + 1 + 1) (1 + 1 + 1 + 1 + 1 + 1) (1 + 1 + 1 + 1 + 1 + 1 + 1) (1 + 1 + 1 + 1 + 1 + 1 + 1 + 1) (1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1) (1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1)

How many pure partitions of 46079 are there?

[My timing: < 100 ms]

Answer a6d157db60dca7917186fe31a891a97a

118 A special number

2014-05-25 06:30:15

by sinan

4 xp

Programming

Find the biggest number whose any 4 consecutive digits sum to a square number and whose any decimal digit is not used more than 5 times.

For example 522007 is such a number: $5 + 2 + 2 + 0 = 9$, $2 + 2 + 0 + 0 = 4$, $2 + 0 + 0 + 7 = 9$.

Answer e8dfc63a77ae08b2699743eae1dcbd3b

119 Simplify**2014-05-25 16:12:15****by Philippe_57721****8 xp****Math**

Let's define:

$$R2(x) = x^{\frac{1}{2}} \text{ (Square root)}$$

$$R3(x) = x^{\frac{1}{3}} \text{ (Cubic root)}$$

We can simplify the following expression:

$R3(R3(2) - 1)$ as

$$R3(1/9) - R3(2/9) + R3(4/9)$$

LaTeX code

$$\sqrt[3]{\sqrt[3]{2} - 1} = \sqrt[3]{1/9} + \sqrt[3]{2/9} \sqrt[3]{4/9}$$

Can you simplify $R2(R3(3645) - R3(2916))$?

LaTeX code

$$\sqrt{\sqrt[3]{3645} - \sqrt[3]{2916}}$$

Format answer: a linear combination of radicands in increasing order using functions R2, R3 and without spaces.

Hint: The answer involves only small integers ...

Answer e743f75be0ebbeb7a3da7b5697d3154b

120 A secret conversation**2014-05-28 09:29:42****by sinan****25 xp****Crypto**

Alice and Bob have the following RSA keys:

```
N_bob =
2843320233792542251053357394068088686668524975752636080093537940
4832527514764874620628848105181825527933880102457169730719923244
6420151499813375523635289637990354962034693574665424583399133573
0583501632679081431737342001818666796295631297455573102373116440
3029788484704936984714437598885024284453243844454411794042986345
2670573135811710297531056679013108703134713805990185300653031156
4180092399756052224998536204529486287954472923574337261846322017
890592838801890702039

E_bob =
2291100640658304898982037736299578548614658498374369307423059101
7753909793205446710905103866626852874140382358105613997851675753
6616368494790527495119970061488145079661142528335377919451554688
6154257484268335107367291548215957325536731933203947342597628607
0538102530714442994442646163672237217358454654896394943346204459
0665520882478282704144094054610779463057719130141630088734441774
4643471937489057261616554090115047189679240761528659032842549170
461539414559766262563

N_alice =
5686640467585084502106714788136177373337049951505272160187075880
9665055029529749241257696210363651055867760204914339461439846489
2840302999626752018801395151071678016047051400133923004670186013
9988684396623348180130983267413984406693145186191663434333107341
4271159645546495517290635898239652290125489708412903139175746507
8882509062160255488830168051051374738819714402801269575387131769
8468196572196081277369024478340281325985826876817304126462736931
162058746229329458443

E_alice =
5532204639026134156498081092471471626870989697962125080810121970
0937589706597407035452447205974152734582820015569746612727007206
2814426400049844220852036436793883376956881347807180964186086536
6805991700540784345980142718081120280543581590237686483113697701
650936698498444544372555441703117867313078723749470914317101582
9520122197714627967971392769730377826127068782925537780549266077
97687180839901208440133776665269835168092927057505751662270100
621679600058468454699
```

Alice sends Bob the following message:

```
M_AtoB =
1396364112413623642374178953014019904568061507434558505408267555
8231810369233960158591102037668709444252754006802542256490072017
0291609574950964015823622596731476468470628411816615719782348534
7273167553704712490006234283555813449834048635758312335972910209
5849958053264948388306938543259511664955402250394909679552507339
```

2828805017197137725781299915845926173013085987451784686823149814
4313379547441762202636503884816587950694638762568426358881638049
169947376779771961136

What should Bob send back to Alice?

Input format: The first 100 digits of the message encrypted using Alice's public key

[My timing: < 1 m]

Answer **a5acdbbbb14723ff5a0a81b6763e02c1**

121 In a distance**2014-05-29 12:01:53****by Philippe_57721****6 xp****Programming****Author Philippe_57721**

We are reading the decimal part of PI by slices of 10 digits:

1415926535, 8979323846, 2643383279, 5028841971, ...

In the first 20000000 digits, how many slices are the closest of 1415926535 (not included) in the sense of the Damerau-Levenshtein distance?

Answer format: Count,(Slice with the closest euclidian distance)

Example: 3 , 1145726635 (for the first 2000000 digits)

[**My timing: 35 sec**]

PS: As there are two slightly different algorithms for this distance, the problem is using the restricted edit distance version.

Answer**247298a7f5fa1474d5e5961150f87106**

122 Smallest difference**2014-06-01 16:48:34****by Philippe_57721****5 xp****Math**

For $1 \leq x, y \leq 1000$, the smallest possible value of $|x \cdot \text{PI} - y \cdot \text{E}|$ is reached for $(x, y) = (777, 898)$ ($\text{PI} = 3.141592\dots$, $\text{E} = 2.71828\dots$)

Which pair $(x, y) \leq 10^{20}$ does yield the smallest value?

Answer format: x, y

Answer**eb7779a6bc6f3bebff839f0bac9ff4de**

123 **Creatures****2014-06-04 18:22:30****by sinan****12 xp****Programming**

Let there be creatures with different N colors ($1, \dots, N$) for $N > 7$. Let $\text{num}(i)$ be the number of creatures with i -th color on a specific day.

The following events occur within an N -day period.

On the i -th day of the period:

number of creatures with i -th color increase by 1000 more than the sum of existing creatures with $(i + 1)$ -th, $(i + 2)$ -th and twice the $(i + 3)$ -th colors.

$$\text{num}(i) = \text{num}(i) + \text{num}(i + 1) + \text{num}(i + 2) + 2 \cdot \text{num}(i + 3) + 1000$$

number of creatures with $(i + 4)$ -th color becomes 17 more than twice the existing creatures of that color.

$$\text{num}(i + 4) = 2 \cdot \text{num}(i + 4) + 17$$

All the $(i + 5)$ -th colored creatures but one change their color to i -th color (i.e. one of them doesn't change its color).

number of creatures with $(i + 6)$ -th color quadruples.

$$\text{num}(i + 6) = 4 \cdot \text{num}(i + 6)$$

Find the total number of creatures after 10^{19} days if $N = 11$ and initially there is 1 creature of each color.

Assume that the first day is the first day of the period.

Note: $(i + n)$ -th color should mean $((i + n) - N)$ -th color if $i + n > N$. For example if $i = 9$ then $(i + 4)$ -th should mean 2nd ($9 + 4 - 11 = 2$)

Input format: total mod 10^9

Example: 619784765 for 100th day when $N = 10$.

[My timing: < 1 s]

Answer **02ddab418694e6c02bd6f5ec23fb71d**

124 Slices of PI E - Part 1**2014-06-05 18:03:06****by Philippe_57721****7 xp****Programming****Author Philippe_57721**

We are reading the decimal parts of PI and E by slices of 10 digits:

PI = 1415926535, 8979323846, 2643383279, 5028841971, ...

E = 7182818284, 5904523536, 0287471352, 6624977572, ...

What is the “first” slice appearing in both constants?

“first” means it requires the minimum numbers of digits in the decimal parts.

For slices of 5 digits, the answer would be: 32823

[My timing: < 3 sec]

Answer 08299d7d2db9272ad0d7271947bd3b32

125 A Game of Risk

2014-06-07 07:59:33

by elasolova

12 xp

Probability

Two friends, Arya and Bran are being held captive in separate places. They request a trial by game from their captivators.

Here is the deal: Arya has to choose a random number between 1 and 101010101 (inclusive). Bran has to choose also but between 1 and 303030303 (inclusive). There is no possible communication between them and they will only be freed if the sum of their numbers is odd. Otherwise they are doomed forever.

What is the probability of them being freed? Give the answer as an irreducible fraction.

Answer ce05b49bc7beea9ccfe3e3ac43d75c7c

126 Bombastic Queue - revisited**2014-06-08 14:03:25****by sinan****15 xp****Math**

Mr. Bombastic wants to enqueue his students in a row. There are 50 students in the class that are queued according to their heights from shortest to longest currently.

Mr. Bombastic has a taste of fashion and does not like orderly stuff. So he decides to disrupt the queue. However, the school directors only permit 20 but only 20 swaps.

A swap is between any two students. So how many different queue options does Mr. Bombastic have such that at least 30 or more students are not at their original places after exactly twenty successive swaps?

[My timing: < 1 s]

Answer **97ca9584f8ecc7df6e786176701c3ac6**

127 Factorial Fine Factorisations**2014-06-09 15:40:34****by Philippe_57721****8 xp****Math**

A factorisation of N in factors is fine if each factor contains only 1 prime number.

$6! = 720$ possesses 10 fine factorisations:

- 5 9 16
- 3 3 5 16
- 2 5 8 9
- 2 3 3 5 8
- 4 4 5 9
- 3 3 4 4 5
- 2 2 4 5 9
- 2 2 3 3 4 5
- 2 2 2 2 5 9
- 2 2 2 2 3 3 5

How many fine factorisations are there for $10000!$.

Give the digital sum of this large number.

[My timing: 1 sec]

Answer**3e2b8b6ab4cf4d52ceb2158dd118ad68**

128 Flip-flop Bit Sets

2014-06-14 06:32:42

by Philippe_57721

10 xp

Math

Let B be the binary digits of an integer N . $B = \{b_1, b_2, b_3, \dots, b_n\}$, b_1 is the least significant bit and $b_n = 1$.

Consider the subsets of B whose elements are alternately 0 and 1.

Example: $N = 40$, $B = \{0, 0, 0, 1, 0, 1\}$.

There are 11 such flip-flop subsets (including the empty set):

Index	Values
[]	{}
[1, 4]	{0, 1}
[1, 6]	{0, 1}
[2, 4]	{0, 1}
[2, 6]	{0, 1}
[3, 4]	{0, 1}
[3, 6]	{0, 1}
[5, 6]	{0, 1}
[1, 4, 5, 6]	{0, 1, 0, 1}
[2, 4, 5, 6]	{0, 1, 0, 1}
[3, 4, 5, 6]	{0, 1, 0, 1}

How many flip-flop subsets are there for $N = 1234567891011121314$?

[My timing: < 100 ms]

Answer

2c1904be55334a7dc8159abfa3dd06b5

129 **Crazy World Cup****2014-06-15 23:26:23****by elasolova****15 xp****Math**

World cup 2014 got crazier then ever.

Each new day a new team registers to world cup with an insane match schedule. Team registered at day i dictates that they will only be able to play i days and then they will take rest for i days after and this pattern will continue on.

In an example team registered at day 1 will have an availability schedule like:

10101010... (where 1 denotes available)

Availability for Team registered in day 2:

011001100...

Availability for Team registered in day 3:

00111000111...

So on and so forth ...

The administration wants to match up all teams at i th day. So, number of available teams at day i must be even to do that.

The question is just for mathematical enthusiasm and it may make no sense. So, how many days will NOT be valid for such a matching if the world cup lasted for 10^{18} days?

Answer**ca283dd1b30f520848c50620d8be43b8**

130 Mills-like Constant**2014-06-19 17:55:56****by Philippe_57721****10 xp****Math**

In 1947, William Mills proved the following theorem:

There exists a real number M such as $\lceil M^{3^k} \rceil$ is a prime number for all $k \geq 1$ ($\lceil x \rceil$ is the integer part of x).

Find the smallest real P such as $\lceil P^{2^k} \rceil$ is prime for k in $\{1, 2, 3, 4, 5, 6, 7\}$.

Answer format: Round your answer to 30 digits after the decimal point

[My timing: < 1 sec]

Answer **8fd9723df80b95e1edcab5716f1399b0**

131 Exploring the Champernowne Constant**2014-06-22 16:40:59****by Philippe_57721****8 xp****Programming**

The Champernowne constant is the real number whose decimal part is the concatenation of all positive integers:

$C = 0.1234567891011121314151617181920\dots$

At which index does appear the first occurrence of the substring 803396454804396?

Hint: The substring 91 appears at index 9.

[My timing: < 100 ms]

Answer **475d2de6c4c9996aac8e2336192106da**

132 Perfectly protected pillars

2014-06-27 18:04:46

by Philippe_57721

10 xp

Programming

In the main hall of the King's palace there are pillars disposed in a rectangular grid $N \times M$.

The king wants to dispose 4 guards at the 4 directions of each pillar. A guard at the east of a pillar is at the west of the next pillar, etc ...

Being mathematically inclined, the king wants that for each pillar, there are exactly 2 guards looking at it and 2 guards turning their back.

Here is an example with 2×3 pillars (and 17 guards):

How many possible configurations are there for 4×6 pillars?

You are given: 374 configurations for 2×3 pillars.

[My timing: 25 sec]

Answer d3ae2cb07ed2d42845c786b144fd88c4

133 Find a recurrence**2014-07-01 14:10:31****by sinan****20 xp****Programming**

See the problem 130.¹ Find a recurrence relation for $M \times 8$ grid for $M > 0$.

Then how many configurations are there for $8^{88} \times 8$ grid?

Input format: enter the rightmost 20 digits

Hint: Solve the p130 first.² Once solved, you can access to a reasonably fast code in the problem forum. If you opt to use it, you need to improve it to solve this one. Then you need to calculate the solutions for $M = 1, 2, \dots, 72$ to find a recurrence equation.

[My timing: 45 s]

Answer**2ba729c4a035e52e521f1b1780d01140**

¹ Actually [Problem 132](#).

² Actually [Problem 132](#).

134 Fibonacci Partitions**2014-07-03 18:28:26****by Philippe_57721****8 xp****Programming**

105 is the smallest integer which can be decomposed in 10 ways as sum of distinct Fibonacci numbers:

- 1 2 5 8 13 21 55
- 1 2 5 8 34 55
- 1 2 5 8 89
- 1 2 13 34 55
- 1 2 13 89
- 3 5 8 13 21 55
- 3 5 8 34 55
- 3 5 8 89
- 3 13 34 55
- 3 13 89

What is the smallest integer with 5000 decompositions?

[My timing: 80 sec]

Answer**130a700b5b21f11c778dbd0ed6b7e127**

135 PPT PPH**2014-07-04 10:29:53****by sinan****8 xp****Programming**

Consider the primitive Phytagorean triangles with an hypotenuse equal to a Pythagorean prime (i.e. a prime of the form $4n + 1$).

How many such triangles are there if their hypotenuses are between 10^{19} and $10^{19} + 10^6$?

Input format: count, sum of the smallest sides

Example: 11,248 for such triangles with hypotenuses in the range [1, 100].

[My timing: < 8 s]

Answer**25ec37758f913b9dbe38e4af367d5e76**

136 Prime factorization

2014-07-07 13:34:05

by gerrob

6 xp

Math

Let $n = \text{product}(p = 2, 2 \times 10^{13}, \text{GCD}((p - 1)! + 1)/p, p))$, where p runs on primes in the $[2, 2 \times 10^{13}]$ interval. Find the prime factorization of n .

Answer format: p_1, p_2, \dots (the prime factors (in increasing order) separated by commas)

Answer ff52fc43f71e5c9512fb27dc89b77520

137 Prime factorization II**2014-07-07 17:17:58****by gerrob****10 xp****Math**

Let $n = a^3 + b^3 + c^3 - 3 \cdot a \cdot b \cdot c$ for $a = 6^{58}$, $b = 9^{77}$ and $c = 10^{107}$. Find the largest prime factor of n .

Answer format: the first 15 digits of p (where p is the largest prime factor of n)

Answer **30a875a3e60b92c8d3d60f084b379e82**

138 Prime factorization III**2014-07-08 17:41:52****by gerrob****20 xp****Math**

Factorize

```
n=607276355939551395694982966044853363251342292177823373417587
84713877172821406508105108904580762643099722451038071790577629
66146955927832350716773086591287243257004809800687612881198923
12964954781339200674374192464853793312653411362168644077995148
82737480091560848572467009817477287506184262337350899238591586
60165210989162315162237390274079347400510539774234802757173409
900779266325776828376103361629
```

if you know that $n = p \cdot q \cdot r \cdot s \cdot t$ (where $p < q < r < s < t$ are primes) and $a(k) = p^k + q^k + r^k + s^k + t^k$ is also given for $k = 1, 2, 3, 4$.

```
a(1)=453306060916445726446955842710001266645415882171369174540
545924454461622792521197
```

```
a(2)=412388085910145742639378306590809549247806487325798372900
59245194079972906154093103939592496434982514076077514851921775
290041532158103511462156470745708110025037
```

```
a(3)=376457151856844823991928212346099344993790369186748265217
38654613892459171474496511969709382919811744994610512839233935
48261024280834398924731925789449892141131187714843927528816158
972950469861033059950606994804630313833406221358401579103605
```

```
a(4)=344837785052229879929633968218624753055360189677589857384
06632682262315858765640669324039738708246441562105120956216321
57297123761957784882209548382514367318717411453259136909844948
96073358196417987776533296882038572085793093406337685989015708
73462550791352914700063280706923742527650079450664756326761568
5644489544816085
```

Answer format: $L(p), L(q), L(r), L(s), L(t)$ (where $L(x)$ is the first five digits of x)

Answer **880efa4b35b2beed4d9ea492c5efd358**

139 **Towers of Hanoi****2014-07-10 11:03:51****by sinan****10 xp****Programming**

There is a 30 disk layout as shown below (from bottom to top) - **not necessarily reached by way of always making the optimal moves:**

1	21	13	8	5	3	2	1												
2	25	16	9	4															
3	30	29	28	27	26	24	23	22	20	19	18	17	15	14	12	11	10	7	6

Target is to move all the disks to the second peg. To reach that target, what would be the layout after 10^9 **optimal** moves and what would be total of the moves if we assign 12 for a move from first to second, 13 for a move from first to third, etc. (possible moves would be 12, 13, 21, 23, 31, 32)?

Input format: 1st peg, 2nd peg, 3rd peg, total

Example: 10965, 321, 874, 2191 for the following

First layout for a 10-disk configuration:

1	10	9	8	5	4
2	7	6	3		
3	2	1			

The layout after 100 moves targeting to move all the disks to the 3rd peg:

1	10	9	6	5
2	3	2	1	
3	8	7	4	

total = 2191

Answer **158c546adea42d68fc2a75f2a4566c2b**

140 Exploring the Champernowne Constant - Ultimate**2014-07-10 18:01:47****by Philippe_57721****12 xp****Programming**

C is the Champernowne constant (See problem 129).¹

Define $K(n) = [C \cdot (10^{10^n})]$ where $[x]$ is the integer part of x (the first 10^n digits of C).

Example: $K(1) = 1234567891$.

What is the digital sum of the digital sum of $K(100)$?

You are given:

$K(10) = 44567901235$

$K(15) = 4441049382716054$

$K(20) = 441486267257142575580$

[My timing: < 100 ms]

Answer e6d29d6863a7820ae06871d24dd09913

¹ Actually Problem 131.

141 Prime factorization IV**2014-07-11 17:57:36****by gerrob****20 xp****Math**

$p = 79888923798664049$ divides $n = a^b + c^d$ for some $0 < a, b, c, d < 18000$ (integers) where $a < c$. Find n , so the a, b, c, d quadruplet. The solution is unique.

Answer format: a,b,c,d (where $a < c$)

[My timing: 61 seconds]

Answer **c522cf2facbf8732e867b3d7405c76f5**

142 Arbitrage**2014-07-11 18:14:13****by gerrob****8 xp****Math**

On an event there are three possible outcomes. The provided odds on these are: 2, 3 and 7. Suppose that you have $m = 614889782588491410$ (money), and you can place bets on all of these 3 outcomes. What is the maximal profit that you can reach (independently from the occurring outcome)? The answer is an integer.

Answer format: profit

Answer **0aff4e37c4438dee4c18c592ccd28bca**

143 Lattice points

2014-07-14 13:37:43

by sinan

8 xp

Math

For a, b, c, d all being positive integers, we have the following:

$$L_1 : y = (b/a) \cdot x$$

$$L_2 : y = -(a/b) \cdot x + a$$

L_1 and L_2 cross at a $P(c, d)$ point.

The distances from P to origin $(0, 0)$ and to $(0, a)$ and to $(b, 0)$ points are all integers.

Let $f(R)$ denote the number of (a, b, c, d) solutions if P is inside $x^2 + y^2 = R^2$ circle.

You are given: $f(10^5) = 3782$.

What is $f(10^{19})$?

[My timing: < 100 ms]

Answer

4201ed1ab8e63702376c1c1d41894748

144 Find a recurrence 2**2014-07-15 11:34:04****by sinan****5 xp****Math**

Find a recurrence relation for the following sequence:

1, 2, 3, 5, 8, 3, 11, 14, 25, 39, 64, 25, 89, 114, 203, 317, 520, 203, 723, 926, ...

Then what is the 10^{100} -th number modulo 10^9 ?

Answer **782c636f4af8f269b023f4e616a0ceed**

145 Equilateral triangles

2014-07-15 15:02:39

by sinan

10 xp

Math

Assume a D point inside an ABC equilateral triangle with $AD = x$, $BD = y$, $CD = z$, $0 < x \leq y \leq z < 100$ and x, y, z are all integers.

Let the length of the sides (s) of the triangle be a function of x, y, z : $s = f(x, y, z)$.

Find all such triangles.

Input format: `cnt sum of s's` (3 digits after decimal point)

Example: 13,59.831 for $0 < x \leq y \leq z < 5$.

Answer b256f46666c9d0e16b2e7e9b6caa3649

146 Equilateral triangles 2**2014-07-18 11:59:34****by sinan****15 xp****Math**

Assume a D point on the AB side of an ABC equilateral triangle with $AD = x$, $BD = y$, $CD = z$, $0 < x < y < z$ and x, y, z are all integers.

Let the sides of the ABC triangle be equal to $s = x + y$.

Find all such triangles for $s < 10^8$.

Input format: `cnt, sum of z's, sum of s's`

Example: 35, 1853, 2081 for $s < 100$.

[My timing: < 10 s]

Answer **9ae3990a735c5d057ce0a48914f94314**

147 What else

2014-07-20 15:14:55

by Philippe_57721

8 xp

Programming

The function T is defined as:

```
T(x,y,z) =  
    if x <= y  
        return y  
    else  
        return T(T(x-1,y,z),T(y-1,z,x),T(z-1,x,y))
```

The function U is defined as:

$U(n) = T(n, 0, n+1)$

We consider the number of times the **else** clause in the T function is invoked when computing $U(n)$ (no memoization). Let $E(n)$ this number.

You are given $E(5) = 223$.

What is $E(100)$?

Answer format: Give the digital sum of this huge number

Answer 9a93bc40e53f31e10ff47d97bd93c4a2

148 Highly composite numbers decomposition**2014-07-26 06:27:05****by Philippe_57721****8 xp****Programming**

36 can be written in 9 ways as product of factors > 1:

- {36}
- {3, 12}
- {4, 9}
- {6, 6}
- {2, 18}
- {3, 3, 4}
- {2, 3, 6}
- {2, 2, 9}
- {2, 2, 3, 3}

A number is highly composite if it has more divisors than all smaller numbers.

What is the first highly composite number which can be written in more than 10^8 ways as product of factors > 1?

Answer format: n, (number of products)

Example: 20160, 1261 (for a threshold of 1000)

[My timing: 45 sec]

Answer**1fc82c619c401a0544708b1945b802b0**

149 Number decomposition again

2014-07-28 09:20:44

by sinan

10 xp

Programming

Let P be the product of first 20 primes.

$$P = \text{prod}(i = 1, 20, \text{prime}(i)) = 557940830126698960967415390$$

In how many ways can $200 \cdot P$ be written as product of factors > 1 ?

[My timing: < 10 s]

Answer

fa5770f838653162e4bd3cb64ca35a67

150 **Towers of Hanoi revisited****2014-08-02 13:04:50****by Philippe_57721****10 xp****Programming**

In a game of Hanoi with n disks, we can consider the layout after m moves as a permutation of $\{1, \dots, n\}$.

For instance, with 15 disks, after 12345 moves the layout is:

- Peg-1 : 7 8 9 10 11 12 15
- Peg-2 : 1 4 5 6 13 14
- Peg-3 : 2 3

corresponding to permutation: 7 8 9 10 11 12 15 1 4 5 6 13 14 2 3

The index of this permutation in lexicographic order is 563569656784.

With 70 disks, what is the index of the permutation corresponding to the layout after 123456789101112131415 moves?

[My timing: < 100 ms]

Answer **c60eafe2663ec705c125f4eaf967f0e5**

151 Similar triangles**2014-08-03 11:02:55****by sinan****8 xp****Math**

For a, b, c, d all being positive integers, consider 2 similar triangles with the following lengths:

$T_1 : a, b, c$ ($a < b < c$)

$T_2 : b, c, d$ ($b < c < d$)

What is the smallest pair of such triangles for which the ratio $(a + b + c)/(a + b - c)$ first exceeds 10^{10} ?

Input format: perimeter of T1 (a+b+c)

Answer **a1742ea582aebc415eda487e97c7a5bc**

152 A lattice points puzzle**2014-08-06 10:54:52****by sinan****10 xp****Math**

Consider $A(x_a, y_a), B(x_b, y_b), C(x_c, y_c), D(x_d, y_d)$ lattice points in the first quadrant (i.e. coordinates with integer $x, y: x \geq 0$ and $y \geq 0$).

AB segment is parallel to DC and AD parallel to BC .

Consider N points M_1, M_2, \dots, M_N on BC segment that split BC into N equal segments:

For $x_c > x_b$ and $y_c > y_b$,

$$M_1(x_b + 1 \cdot (x_c - x_b)/N, y_b + 1 \cdot (y_c - y_b)/N),$$

$$M_2(x_b + 2 \cdot (x_c - x_b)/N, y_b + 2 \cdot (y_c - y_b)/N),$$

...

$$M_N(x_b + N \cdot (x_c - x_b)/N, y_b + N \cdot (y_c - y_b)/N) = C(x_c, y_c).$$

M_1, M_2, \dots, M_N are **not necessarily lattice points** but all the line segments drawn from A to M_1, M_2, \dots, M_N cross BD segment at lattice points.

What is the smallest area of $ABCD$ you can get for $N = 1000$?

Input format: Area($ABCD$) mod 10^9

Answer**d8079566f3a5c22981ec25a6f432e31e**

153 A best approximation**2014-08-06 11:01:16****by sinan****10 xp****Math**

For a, b, c being positive **coprime** integers and $a < b < c$, consider the following 2 integer sided squares:

$S_1 : A(0, 0); B(c, 0); C(c, c); D(0, c)$

$S_2 : A'(a, 0); B'(a + b, a); C'(b, a + b); D'(0, b)$

What is the best approximation for $\text{Area}(S_1)/\text{Area}(S_2) = 2$ if $\text{Area}(S_2) < 10^{24}$? In other words find the (a, b, c) triples that minimize $|\text{Area}(S_1)/\text{Area}(S_2) - 2|$.

Input format: enter the list of “a” values separated by commas (possibly more than one)

Example: a1,a2,...,an ($a_1 < a_2 < \dots < a_n$)

Answer**d86a7941ba160e7a5c7d9db75e3b665c**

154 A boring sequence

2014-08-07 17:31:46

by Philippe_57721

12 xp

Math

Let $P(x) = x^{19} + 6$

Let $G(n) = \text{GCD}(P(n), P(n + 1))$

The sequence $\{G(1), G(2), G(3), \dots\}$ seems constant and equal to 1 for an awful long time ...

What is the first n for which $G(n) > 1$?

Answer format: n

[My timing: < 5 sec]

Answer**aa0405a3757e36455aadd556c35da575**

155 Hint for 152**2014-08-10 15:33:10****by Philippe_57721****5 xp****Programming**

If we apply the permutation P with index I (in lexicographic order) to the string S for the following values:

```
I = 366899275
S = "-EHILN00STTU"
```

we get $P = \{10, 3, 2, 1, 9, 7, 5, 12, 11, 4, 8, 6\}$

=> "THE-SOLUTION" (Take the 10th char, then the 3rd, ... in 1-based origin)

Apply the permutation P with index I to the string S for the following values:

```
I =
6044912527595411749579227181311120239399976240987501298038504481
5846361032595450296127862666721532739781384807770642238
S =
-----AAABBBDEEEEEEEFHHILLLLLMMNNNN0000000000PPPRRRSSSSSTTTTTUUVWYY"
```

and you'll get a good lead to solve problem 152.¹

Answer format: the string

Answer **858c57b45de337f451b135866ff7c36c**

¹ Actually [Problem 154](#).

156 Brute interrogation - revisited

2014-08-17 09:09:44

by sinan

10 xp

Programming

See the following code:

```
function(N)
begin
    UP=101
    DOWN=211
    FACTOR=-2
    temp=0
    loop for each decimal digit of N from left to right
    begin // loop
        if digit is equal to 1; add UP to temp
        else if digit is equal to 2; subtract DOWN from temp
        else if digit is equal to 3; multiply temp by FACTOR
    end // loop
    return temp
end
```

What is the smallest N for which $\text{function}(N)$ returns 100000001?

[My timing: < 1 m]

Answer 6f0b32018ad04bb380409912ce1e89c8

157 Morley's triangles - Warming up

2014-08-17 12:46:19

by Philippe_57721

7 xp

Math

A beautiful theorem of elementary geometry only discovered in 1898 by Frank Morley states: *In any triangle, the three points of intersection of the adjacent angle trisectors form an equilateral triangle.*

What is the side of the Morley's triangle for the integer sided triangle: (20424, 24167, 29791). It's a rational number.

Answer format: P/Q
(in irreducible form)

Answer 4ac8fad9b1873593c3956aa24e4cc609

158 Morley's triangles - Practise

2014-08-17 12:50:19

by Philippe_57721

12 xp

Programming

Find an integer sided isosceles triangle

- whose perimeter is between 94000 and 95000
- whose Morley's triangle has a rational side

Answer format: $a, b, p/q$

- $a < b$ are the 2 sides of the isosceles triangle (a and b are coprime)
- p/q is the side of the Morley's triangle in irreducible form

[My timing: 15 min ...]

Answer

3ea0a97e5080c5e7d76875812cc18d48

159 End of Fibonacci

2014-08-19 02:58:11

by elasolova

6 xp

Math

Find the smallest $n > 0$ such that $\text{Fibonacci}(n)$ ends in nine nines.

$F(1) = 1, F(2) = 1, F(3) = 2$

by Buri

Answer `be8e77c6e27147cd5bf9b9c071ebcd08`

160 Programming exercise - Counting

2014-08-24 06:09:36

by Philippe_57721

8 xp

Programming

Let the following function

```
function(n)
begin
    count = 0
    for i in 1 to n
        for j in (i+1) to n
            if 1 = GCD(i,j)
                count = count+1
    return count
end
```

What is function(123456789).

You are given: function(10^5) = 3039650753.

[My timing: 80 sec]

PS: I slightly changed the parameter (and the answer) as the original solution might be easily found on the web.

Answer d8abcc59bbe09d9031523f0d5d68b682

161 Hunting the polygons

2014-08-28 05:14:04

by Philippe_57721

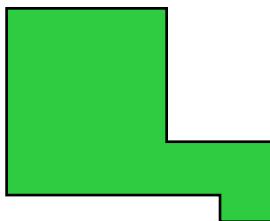
12 xp

Programming

A golygon is a polygon:

- whose all angles are right
- whose lengths are the consecutive integers 1, 2, 3, ... in that order

There is 1 non-crossing golygon with 8 sides:



It's area is 52.

Find the minimum and maximum area amongst all non-crossing 24 sides golygons.

Answer format: min,max

Example: 384,664 (for 16 sides golygons)

[My timing: 30 sec]

Answer 2c055674fd21f210928e25c312a4ef97

162 Quasi-perfect numbers**2014-09-05 08:28:05****by Philippe_57721****7 xp****Programming**

An integer is perfect if it is equal to the sum of its proper divisors.

Let say an integer is quasi-perfect if it can be written as a sum of some distinct of its proper divisors.

For instance, $N = 150$ as the following proper divisors:

1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75

and can be written as:

- $1 + 3 + 5 + 6 + 10 + 50 + 75$
- $1 + 3 + 6 + 10 + 25 + 30 + 75$
- $1 + 3 + 6 + 15 + 50 + 75$
- $2 + 3 + 5 + 10 + 25 + 30 + 75$
- $2 + 3 + 5 + 15 + 50 + 75$
- $2 + 3 + 15 + 25 + 30 + 75$
- $5 + 15 + 25 + 30 + 75$
- $10 + 15 + 50 + 75$
- $25 + 50 + 75$

Find the first such decomposition for 1000768 (in lexicographic order).

Answer format: comma separated list of divisors in ascending order

Example: 1,3,5,6,10,50,75

[My timing: 50 sec]

Answer 1c7c4710a5cddfcc911c41803fc0a90d

163 Quasi-perfect numbers (revisited)**2014-09-06 07:22:22****by gerrob****15 xp****Programming**

See [Quasi-perfect numbers problem](#) from Philippe. This time find the first decomposition in lexicographic order for $N = 30380400$.

Answer format: $P \bmod 98961651023$

(where P is the product of all terms in the decomposition)

Example: 3375000 (for $N = 150$)

[My timing: 24 seconds]

Answer**fa3a0795556d4c2922dbb4d1998e6e30**

164 A sum with constraints**2014-09-14 16:28:01****by Philippe_57721****7 xp****Programming**

Find the sum of all integers with at most 12 digits and containing exactly one ‘1’, two ‘2’, three ‘3’ and four ‘4’ in their digital representation (e.g.: 23313447424).

You are given: 121970799987802920 (for all integers with at most 10 digits and containing exactly one ‘1’, two ‘2’ and three ‘3’)

[My timing: 15 sec]

Answer**ea24d9d11dc6d7772c3d6dbf6a5d34ff**

165 Consecutive numbers**2014-09-15 12:24:01****by sinan****15 xp****Programming**

For $N > 0$, consider the ways to write N as the sum of consecutive integers from a to b where $b \geq a > 0$.

$$N = a + (a + 1) + (a + 2) + \dots + (b - 1) + b$$

For example:

$$N = 9 = 4 + 5 = 2 + 3 + 4$$

$$N = 10 = 1 + 2 + 3 + 4$$

Let N be the product of first 42 primes:

$$N = \text{prod}(i = 1, 42, \text{prime}(i))$$

$$N = 5397346292805549782720214077673687806275517530364350655459511599582614290$$

How can we write N in such a way that $(b - a)$ is **maximized**?

Input format: a, b

Example:

1, 20 for the product of first 4 primes:

$$N = 2 \times 3 \times 5 \times 7 = 210 = 1 + 2 + 3 + \dots + 20$$

Answer**4e4502a6fd2ce6db3233f573803bea66**

166 GCD Arithmetic Mean**2014-09-20 14:04:32****by Philippe_57721****7 xp****Math**

Let's define $A(n) = \text{Arithmetic mean of } \text{GCD}(n, k) \text{ for } k \text{ in } \{1, \dots, n\}$.

Find $A(30!)$.

You are given $A(1000) = 17/2$.

Answer format: P/Q

[My timing: 36 sec]

Answer**5578e1d2a1278c5b517fb12a98fbd123**

167 GCD Harmonic Mean**2014-09-20 14:05:28****by Philippe_57721****8 xp****Math**

Let's define $H(n) = \text{Harmonic mean of } \text{GCD}(n, k) \text{ for } k \text{ in } \{1, \dots, n\}$.

For the record, if $V = \{V_1, V_2, \dots, V_n\}$ the harmonic mean of $V = n/(1/V_1 + 1/V_2 + \dots + 1/V_n)$.

Find $H(25!)$.

You are given $H(1050) = 52500/22403$.

Answer format: P/Q

[My timing: 5 sec]

Answer**d6674a5c27ed26d2c266d4a843076fea**

168 Hendeca divisibility**2014-09-27 17:32:56****by Philippe_57721****10 xp****Programming**

It can be proved that there is a smallest integer h such as for any sequence of k (greater or equal to h) consecutive integers there is always one whose digital sum is divisible by 11.

It is not true for less than h consecutive integers.

What is h ?

What is the first exception, i.e. the smallest e such as the $(h - 1)$ consecutive integers starting with e have their digital sum not divisible by 11?

Answer format: h, e

Answer**b0eaa924044418df29602b7de3d84852**

169 Circle Festival

2014-09-28 10:58:49

by elasolova

8 xp

Math

A huge crowd is celebrating the new season.

There are N people (numbered from 1 to N) in the crowd and they gather in a **circle** in sequence.

They decide to play a game like this:

It starts with 1st person. He eliminates person 2 with a wooden sword and gives the sword to 3. Then 3 eliminates 4 and sword is passed to 5. So each person eliminates the one next to himself and passes the sword to the other next. The winner of the game is the last person standing.

Which number is the winner if $N = 9 * 10^{29}$?

Hint: Deduce a formula from smaller cases.

Answer 23e2455505f38cd37e060afe37c87ebd

170 A modular equation**2014-10-05 16:19:06****by Philippe_57721****12 xp****Programming**

Solve the modular equation

$$314159265358979323846^x = 271828182845904523536 \pmod{3506457161893013907641318390893283649218125103}$$

Answer format: x

[My timing: 45 sec]

Answer f6aa44213e9d8471c3a216fa8be34c96

171 Too long...

2014-10-09 08:58:47

by sinan

10 xp

Hack

Just get the correct value and enter as the answer! Try it

The JavaScript code for this problem is as follows:

```

function f(N)
{
    var a,b,c,d,e,f,g,h,s=0,tb,tc,td,te,tf,tg,th;
    for(a=0;a<=N;a+=1){
        for(b=0;b<=N;b+=2){
            tb=a+b;if(tb>N)break;
            for(c=0;c<=N;c+=3){
                tc=tb+c;if(tc>N)break;
                for(d=0;d<=N;d+=4){
                    td=tc+d;if(td>N)break;
                    for(e=0;e<=N;e+=5){
                        te=td+e;if(te>N)break;
                        for(f=0;f<=N;f+=6){
                            tf=te+f;if(tf>N)break;
                            for(g=0;g<=N;g+=7){
                                tg=tf+g;if(tg>N)break;
                                for(h=0;h<=N;h+=8){
                                    th=tg+h;if(th>N)break;
                                    if(th==N)
                                    {
                                        s++;
                                        if((s&0x3FFF)==0)
                                        {
                                            var ans=prompt('Taking too long...
Continue?', 'Yes');
                                            if(ans != 'Yes') return (0-
document.myform.myinput.value);
                                        }
                                    }
                                }
                            }
                        }
                    }
                }
            }
        }
    }
    return s;
}

function try_it()
{
    var N=document.myform.myinput.value;
    if (N==f(1234)){

```

```
    alert("Right! Just submit it below.");
}else{
    alert("Wrong");
}
}
```

Answer b73ff51339812bbabaa1de71e25cd4a2

172 Programming exercise - Fixed points

2014-10-11 08:41:23

by Philippe_57721

10 xp

Programming

The function F is defined by

$$F(1) = 1$$

$$F(3) = 3$$

$$F(2 \cdot n) = F(n)$$

$$F(4 \cdot n + 1) = 2F(2 \cdot n + 1) - F(n)$$

$$F(4 \cdot n + 3) = 3F(2 \cdot n + 1) - 2F(n)$$

A fixed point is n such as $F(n) = n$.

How many fixed points less or equal to 10^{100} have F?

Answer format: the count

[My timing: < 100 ms]

Answer

ea7e1472196dd23a82602b2b9b2c51fa

173 **Cube painting****2014-10-14 10:06:03****by sinan****10 xp****Math**

Consider the number of ways to paint the faces of a cube using n distinct colors.

Let $d(i)$ be i -th decimal digit of $41152/333333 = 0.123456123456\dots$ i.e. $d(1) = 1, d(5) = 5, d(9) = 3$, etc.

Let $F(n)$ be the number of different cubes that can be obtained if it is allowed that upto $d(i)$ faces can be painted with the i -th color. $i = 1, 2, \dots, n$.

What is $F(1001)$?

Example: 415 for $F(5)$.

Answer**30736c4a411ce324908fad408b72e1c8**

174 **Next 2****2014-10-14 11:22:05****by sinan****5 xp****Sequence**

23800-728-5852-36176-23800-3360-3312-78624-25536-5408-988-192192-?-?

Input format: number1-number2

Hint: This sequence is a tweaked form of a sequence that otherwise can be found easily (even in OEIS).

Another hint: It has been tweaked in such a way that

OEIS: $f(0, 1) - f(1, 1) - f(2, 1) - \dots$ This: $f(0, 0) - f(1, 0) - f(2, 0) - \dots$ Yet another hint: solve p188¹**Answer** **a2d643035def3800e946f56a7b139a3d**

¹ Actually Problem 190.

175 Pandigital primes**2014-10-16 18:29:04****by Philippe_57721****4 xp****Math**

We have $19 = 201$ in base 3.

Can you find all the prime numbers p such as there is a base b in which p can be written with all digits $\{0, \dots, b - 1\}$ used exactly once?

Digit 0 on the left is allowed.

Answer format: p1, p2, ...

(in ascending order)

Answer**6cf8c44959f790ea9b0f088b94a72f2b**

176 GCDs**2014-10-16 18:30:02****by Philippe_57721****4 xp****Math**

What is the GCD of all numbers $(n^{37} - n)$ when $n \geq 1$?

Answer format: the GCD

Answer**c0cf95f0dc dc1baf ed14499e05bd4e07**

177 Find it

2014-10-17 13:48:28

by sinan

8 xp

Hack

Just get the correct value and enter as the answer! Try it

The JavaScript code for this problem is as follows:

```
var M=13;

function func(X,N)
{
    if(((X-1)%M)==0){
        return (func(((X-1)/M)*(M-2),N-1));
    } else if(N>0){
        return (-1);
    } else {
        return (X);
    }
}

function test()
{
    var t=document.myform.myinput.value;
    t>>=30;
    t>>=17; // answer < 2^47
    if (t == 0)
    {
        t=func(document.myform.myinput.value,M);
    }
    if(t>0)
        alert("Right! Just submit it below.");
    else
        alert("Wrong");
}
```

Answer **6d7c5709003d40c36a11b0b16cc7e561**

178 Find the maximum

2014-10-24 17:24:48

by Philippe_57721

8 xp

Math

The Diophantine equation $n^2 - n \cdot m - m^2 = 1$, ($n, m > 0$) has infinitely many solutions.

If $n, m < 10^{15}$, what is the maximum of $n^2 + m^2$?

Answer**b3602fe156a5dacb82b7c14c70f517d9**

179 How many solutions

2014-11-02 17:17:47

by Philippe_57721

8 xp

Math

Consider the diophantine equation $x^2 + 3 * y^2 = z^2$ ($0 < x, y, z$ and coprime).

How many solutions are there with $z \leq 10^9$?

You are given:

- 276 solution for $z < 1000$
- 2767 solution for $z < 10000$
- 27574 solution for $z < 100000$

Answer format: the count

[My timing: 120 sec]

Answer

7999e24efa33e6ea8d9a25e15f894d16

180 How many points**2014-11-03 12:03:58****by sinan****12 xp****Math**

For real numbers $a, k > 0.0$, let $F(a, k)$ denotes the number of lattice points inside the OAB triangle with the following vertices:

$$O(0.0, 0.0)$$

$$A(a, 0.0)$$

$$B(0.0, k \cdot a)$$

For example:

$$F(5.0, 5.0) = 81 \text{ for } O(0.0, 0.0), A(5.0, 0.0), B(0.0, 25.0).$$

$$\text{What is } F\left(10^{16} + 1, 2/\left(\sqrt{5} + 1\right)\right)?$$

[My timing: 1 s]

Answer**9cbe139c05aeb03b1f3597fe3715d4d9**

181 How many points 2**2014-11-04 10:35:03****by sinan****8 xp****Math**

For real numbers $a, m > 0.0$, let $F(a, m)$ denote the number of lattice points inside the OAB tringle with the following vertices:

$$O(0.0, 0.0)$$

$$A(a, 0.0)$$

$B(x_b, y_b)$ which is the intersection point of the following lines:

$$L_1 : y = x/m$$

$$L_2 : y = m \cdot (x - a)$$

For example:

$F(8.0, 3.0) = 19$ for $O(0.0, 0.0), A(8.0, 0.0), B(9.0, 3.0)$.

What is $F(123456789101112, 2/(\sqrt{5} - 1))$?

[My timing: 1 s]

Note: You are advised to solve p178 first.¹

Answer**903f453f0844edb8b9bf416854d407b4**

¹ Actually Problem 180.

182 Tubes**2014-11-06 07:26:05****by sinan****20 xp****Programming**

Consider 6 tubes named A, B, C, D, E, F with capacities 10, 10, 7, 7, 7, 7 respectively.

In the first state they contain the following unit amounts of a certain liquid (A to F):

9, 7, 6, 5, 0, 0

And the goal state is as follows (A to F):

9, 2, 2, 2, 5, 7

Treat each state as a hex number (FEDCBA) and assign a 6-digit number for them as in the following example:

The first state = 005679

The last state = 752229

We need to reach to the goal state with the minimum number of steps.

Let A be the eventual array of size k with the states (from first to last) in it:

$A = [005679, \dots, 752229]$

where $A[i]$ is the state at the i -th phase, $i = 1, \dots, k$.

Let N be the hex number obtained by concatenating the hex digits of the above array:

$N = 005679\dots752229$ or

$N = A[1] \cdot B^{k-1} + A[2] \cdot B^{k-2} + \dots + A[k] \cdot B^0$ where $B = 16^6$.

Find the smallest N . (i.e. Find the lexicographically first, shortest path.)

Answer format: $N \bmod 1000000007$

Example:

Capacities: 7, 6, 5, 4, 4, 4

First: 7, 5, 0, 0, 0, 0

Last: 3, 2, 3, 4, 0, 0

1 : 7, 5, 0, 0, 0, 0 $s = 000057$

2 : 3, 5, 0, 4, 0, 0 $s = 004053$

3 : 3, 5, 4, 0, 0, 0 $s = 000453$

4 : 3, 6, 3, 0, 0, 0 $s = 000363$

5 : 3, 2, 3, 4, 0, 0 $s = 004323$

$N = 000057004053000453000363004323$ (hex)

$N \bmod 1000000007 = 10356901$ (decimal)

[My timing: 25 s]

Answer **45a1acb875e00bb1d45c6b96ccb0be9a**

183 Run it (javascript)

2014-11-08 10:39:29

by sinan

10 xp

Programming

Here's a javascript code in action. Experiment with it and submit the value for $M = 1000000000$.

[My timing: 42 s]

M =

M=20

```
a=2 b=2 k=1 nt=1
a=4 b=4 k=2 nt=2
a=6 b=3 k=2 nt=3
a=6 b=6 k=3 nt=4
a=8 b=8 k=4 nt=5
a=12 b=4 k=3 nt=6
a=12 b=6 k=4 nt=7
```

found 7 solutions

The JavaScript code for this problem is as follows:

```
function func()
{
    var M=myform.myinput.value;
    var nt=0;
    var outputDiv = document.getElementById("outputDiv");
    outputDiv.innerHTML = "<pre>";
    if (M<=0 || M>99) {
        alert("Should be >0 and <100");
    } else {
        outputDiv.innerHTML += "<font size=+2 ><b>M=" + M + "</b></font><br>";
        for (var a=1; a<M; a++){
            for (var b=1; b<=a; b++){
                var ab=a+b;
                if (ab>=M) break;
                var k=Math.floor(0.5+(a*b)/ab);
                var diff=Math.abs(a*b-k*ab);
                if (diff==0){
                    nt++;
                    outputDiv.innerHTML += "a=" + a + " b=" + b + " k=" + k + " nt=" + nt + "<br>";
                }
            }
        }
        outputDiv.innerHTML += "<br><b>found " + nt + " solutions</b>";
    }
    outputDiv.innerHTML += "</pre>";
}
```

Answer

ba6b80a01d4472dbbc29516d571fb2d5

184 ones and twos**2014-11-11 17:03:50****by Philippe_57721****7 xp****Programming**

There is one integer N such as $N \times 2^{100}$ is a 100-digits number containing only 1's and 2's.

Can you find it.

[My timing: < 100 ms]

Answer**7c7cbf2dc04676bc626a8d8649fc8852**

185 Dividing triples

2014-11-16 09:27:27

by Philippe_57721

7 xp

Math

Find all the triples (a, b, c) with $0 < a < b < c$ such as:

$(a - 1) \cdot (b - 1) \cdot (c - 1)$ divides $a \cdot b \cdot c - 1$.

Answer format: (a1,b1,c1)(a2,b2,c2)...

($a1 < a2 \dots$)

Answer

cfc3a56fdcf56276be2b74019ee73b7

186 **Tubes 2****2014-11-17 14:33:18****by gerrob****24 xp****Programming**

See p180 from Sinan.¹ Now there are 10 tubes named with $A, B, C, D, E, F, G, H, I, J$ with capacities 14, 13, 12, 12, 10, 10, 7, 7, 7, 7 respectively. The first state is (A to J) 10, 9, 11, 10, 9, 8, 5, 5, 4, 2. The goal state is (A to J) 13, 12, 11, 11, 4, 6, 6, 0, 6, 4. Encode each state as a hex number JIHGFEDCBA, so the starting state is 245589AB9A, the ending state is 460664BBCD.

We need to reach to the goal state with the minimum number of steps.

Let A be the eventual array of size k with the states (from first to last) in it:

$A = [245589AB9A, \dots, 460664BBCD]$

where $A[i]$ is the state at the i -th phase, $i = 1, \dots, k$.

Let N be the hex number obtained by concatenating the hex digits of the above array:

$N = 245589AB9A\dots460664BBCD$ or

$N = A[1] \cdot B^{k-1} + A[2] \cdot B^{k-2} + \dots + A[k] \cdot B^0$ where $B = 16^{10} = 2^{40}$.

Find the smallest N . (i.e. Find the lexicographically first, shortest path.)

Answer format: $N \bmod 1000000007$

[My timing: 60 sec] (using less than 1GB of Ram)

Answer **e7a0b25841cb0e7799f438f5ca332ea9**

¹ Actually [Problem 182](#).

187 Dividing sextuplets**2014-11-20 10:20:18****by gerrob****10 xp****Programming**

There is an obvious generalization of p183 from Philippe.¹ Find all $1 < a < b < c < d < e < f$ (integer) sextuplets for that $(a - 1) \cdot (b - 1) \cdot (c - 1) \cdot (d - 1) \cdot (e - 1) \cdot (f - 1)$ divides $a \cdot b \cdot c \cdot d \cdot e \cdot f - 1$. You can easily check that this is a valid sextuplet:

[3, 5, 17, 257, 65555, 226112997].

Answer format: `cnt,s`

where `cnt` is the number of solutions and `s` is the sum of $a + b + c + d + e + f$ values over all solutions.

[My timing: < 1 sec]

Answer**5715d4489daa7c557f18c3ee33b3f268**

¹ Actually [Problem 185](#).

188 Dividing septuplets**2014-11-20 10:28:55****by gerrob****12 xp****Programming**

Find all $1 < a < b < c < d < e < f < g$ (integer) septuplets for that $(a - 1) \cdot (b - 1) \cdot (c - 1) \cdot (d - 1) \cdot (e - 1) \cdot (f - 1) \cdot (g - 1)$ divides $a \cdot b \cdot c \cdot d \cdot e \cdot f \cdot g - 1$.

Answer format: `cnt,s`

where `cnt` is the number of solutions and `s` is the sum of $a + b + c + d + e + f + g$ values over all solutions.

[My timing: 2 min]

Answer **0e7d27f0743a50572e7a2867ebfc7033**

189 Smallest partition**2014-11-23 10:23:32****by Philippe_57721****10 xp****Math****Proposition:**

For any partition of $\{3, \dots, n\} = A \cup B$ the equation $x \cdot y = z$ where x, y, z (not necessarily distinct) are all in A or all in B has always a solution.

Find the least integer n such as this proposition is true.

[My timing: pencil and paper]

Answer **a345ba5a87227571c02d216368d858e6**

190 Bruteforce it**2014-11-24 08:51:05****by sinan****20 xp****Programming**

Here is an hex dump of a password protected data:

```
50 4b 03 04 0a 00 01 00 00 00 ae 60 6b 45 68 81
7f 1e 18 00 00 00 0c 00 00 00 01 00 00 00 01 3e
24 52 bf a8 02 52 f9 7a 98 64 65 da e2 4e 27 a7
f4 22 2c 77 6b 7b fd
```

I believe you can easily figure out what it is and brute force the password using the following hints:

- Static string “sinan” is a part of the password (any uppercase lowercase combination is possible)
- It has a 3-digit decimal number in it [000 – 999]
- 4 distinct characters are also present in it from the set {‘(‘,’)’; ‘:; ‘; ‘_’; ‘.’; ‘; ‘,’}
- The above mentioned parts may appear in any order (i.e. 6! possibilities)

Answer format: enter the decrypted data

Note: The decrypted data is an hint for the problem 172.¹ So this problem is more rewarding than its points.

[My timing: 460 s]

Answer **03b4aea6bfbf3faa105f27ae6fb0e8fa**

¹ Actually [Problem 174](#).

191 Non attacking queens

2014-12-07 19:20:08

by Philippe_57721

8 xp

Programming

On a 5×5 chessboard, there are 8 ways to place 3 white queens and 5 black queens so that no queen of a given color can attack a queen of another color.

One position is

```
+---+---+---+---+
| W | W |   |   |
+---+---+---+---+
|   |   |   | B | B |
+---+---+---+---+
|   |   |   |   | B |
+---+---+---+---+
| W |   |   |   |   |
+---+---+---+---+
|   |   | B | B |   |
+---+---+---+---+
```

How many ways are there to place 5 white queens and 6 black queens on a 6×6 chessboard the same way?

Answer format: Count/White queens positions/Black queens positions
for the 1st solution in lexicographic order.

A position is given as: (i, j) where i, j are the row and column index starting from the upper left cell, index starting at 1.

Example: 8/(1,1)(1,2)(4,1)/(2,4)(2,5)(3,5)(5,3)(5,4)
for 3 white and 5 black queens on a 5×5 chessboard.

[My timing: 40 sec]

Answer 2be117d6f72ae808edab7d7b9fa5e81e

192 The Best Sub-sequence

2014-12-21 17:43:15

by Philippe_57721

8 xp

Programming

Consider the decimal part of PI by groups of 2 digits.

We get the following sequence for the 20 first elements:

14, 15, 92, 65, 35, 89, 79, 32, 38, 46, 26, 43, 38, 32, 79, 50, 28, 84, 19, 71

Subtract from each element of this sequence 50:

-36, -35, 42, 15, -15, 39, 29, -18, -12, -4, -24, -7, -12, -18, 29, 0, -22, 34, -31, 21

You can verify that the sub-sequence starting at index 3 and ending at index 7 (42, 15, -15, 39, 29) has the largest sum: 110.

Now group the decimal part of PI by groups of 10 digits and subtract $10^{10}/2$ from each element.

We get the sequence:

$S = -3584073465, 3979323846, -2356616721, 28841971, 1939937510,$

$820974944, 923078164, -4371379101, 3628034825, -1578829321, \dots$

What is the sub-sequence with the largest sum in the first 5000000 elements of S ?

Answer format: Sum, Starting index, Ending index

Example: 110, 3, 7

[My timing: 2 sec] (Computation of PI not included!)

Answer**90e000007b4e78fac882ac2d3051c819**

193 No remainder

2014-12-25 16:45:07

by sinan

10 xp

Math

Let $F(n) = 11 + 105^n$.

Let S be the serie of $F(n)$ values for $n = 1, 2, \dots$:

116, 11036, 1157636, 121550636, 12762815636, 1340095640636, ...

What is the 10^4 -th term **that can be divided by 27126986671934931497 with no remainder**.

Answer format: the term mod 10^9

[My timing: 3 s]

Answer 6907587da01f0ffaee64f29c55e1c8c2

194 Trains

2014-12-27 09:58:34

by Philippe_57721

7 xp

Programming

On a circular track, three trains travel at the same speed: one unit per second.

The track has a length of 641 units.

- Train no. 1 has a length of 11 units and occupies the position 0 to 10. It travels clockwise
- Train no. 2 has a length of 13 units and occupies the position 100 to 112. It travels counter-clockwise
- Train no. 3 has a length of 17 units and occupies the position 200 to 216. It travels clockwise

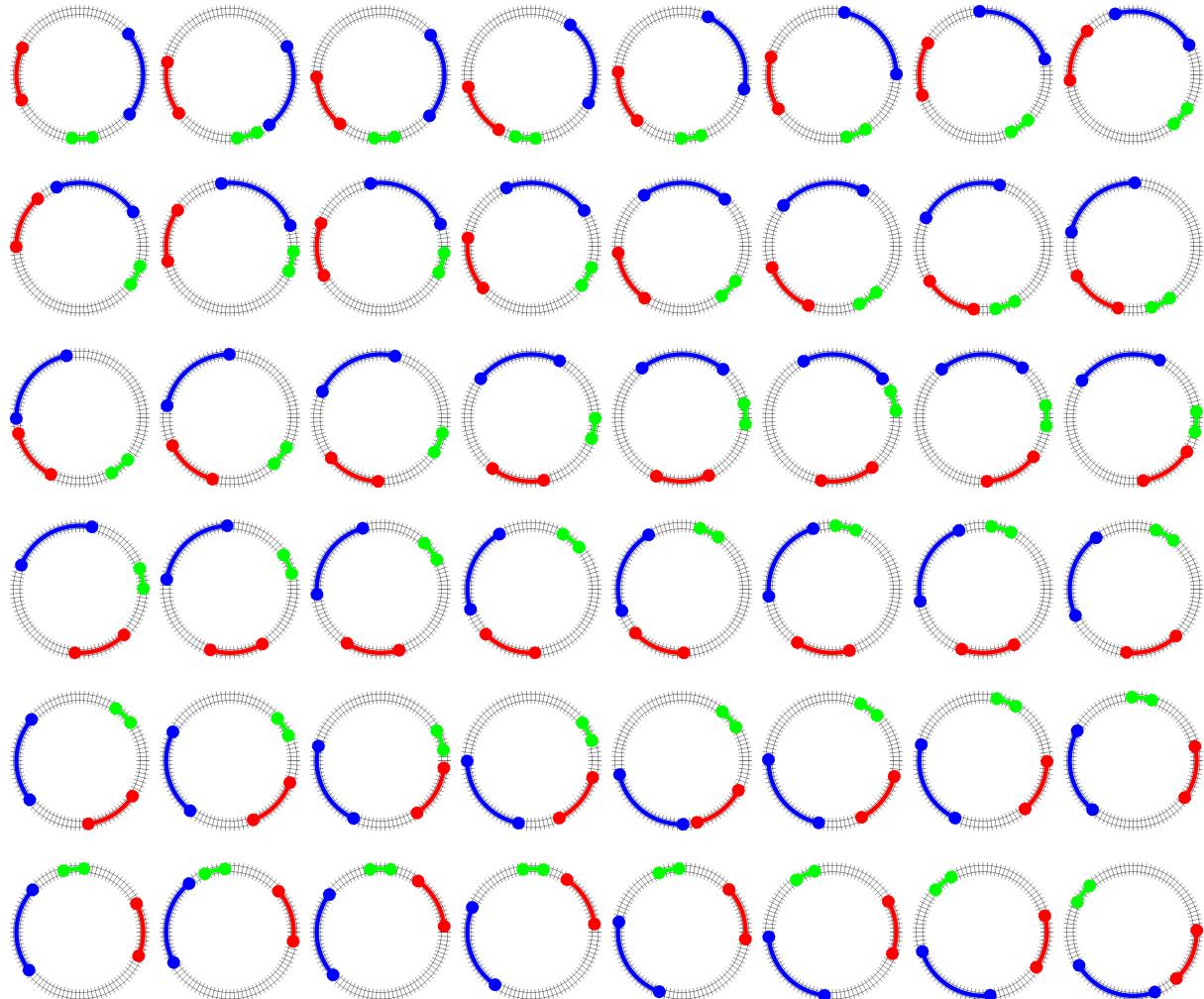
When two trains collide, they collide elastically, and go in the opposite direction at the same speed.

The trains start to move at $T = 0$.

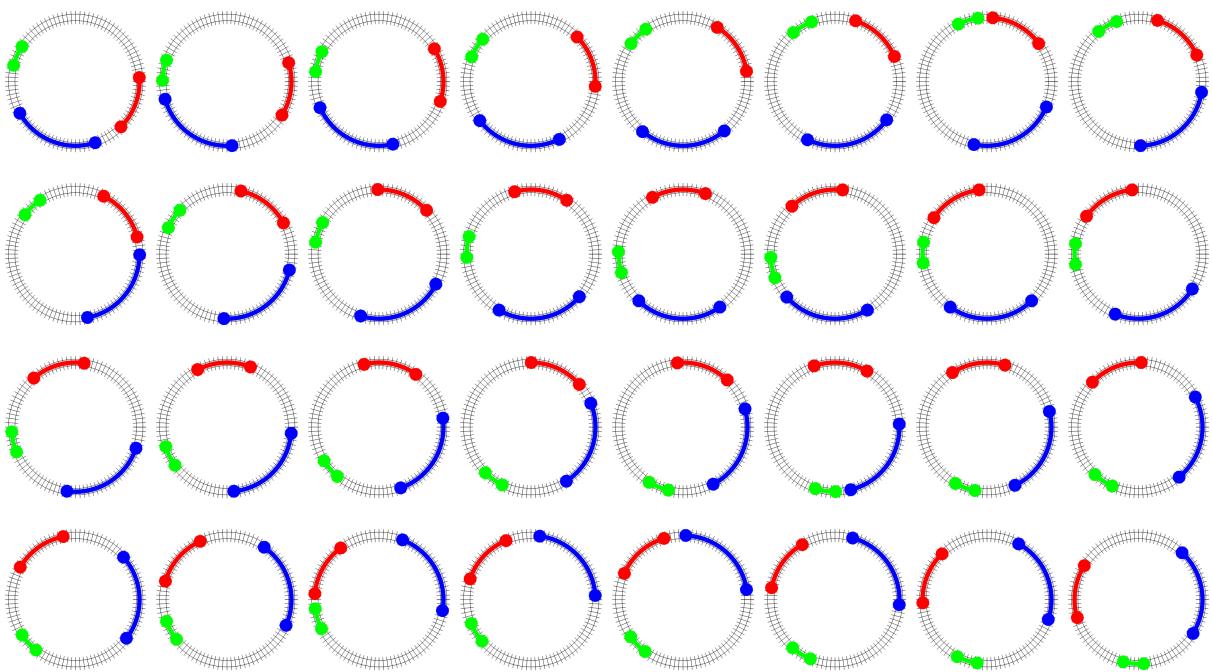
At which T do the trains occupy the same position again?

[My timing: 5 sec]

Here is an example:¹



¹Selected key frames from the original GIF.



(Image credit: apgoucher. I will remove it if there is a copyright issue)

Answer e3138a4b634d2999696df71280c942c2

195 Antipodean numbers

2015-01-04 18:53:35

by Philippe_57721

7 xp

Programming

A number is antipodean if it is equal to its rotation by 180°: you type such a number on a calculator and turn it upside down, you read the same value.

- 1961 is such a number

How many antipodean numbers with a number of digits less or equal to 17 are there and what is their sum?

Answer format: Count, Sum

You are given: 14404, 5947658589706 for 9 digits or less.

[My timing: 50 sec]

Answer

3795546a18e4350d1760f72a011f5fbe

196 Meeting time**2015-01-06 11:39:57****by sinan****8 xp****Math**

There are 2 points (A and B) 100 kms apart. A vehicle (VA) starts to travel from point A to point B and another (VB) from B to A at the same time (00:00:00).

VA travels at a speed (km/h) equal to the distance-to- B +13 at any time while VB travels at the speed of 61 km/h at any time.

So for example when VA is 91.1 kms away from B , its speed would be 104.1 km/h at that time.

At what time do the two vehicles meet?

Answer format: HH:MM:SS:ms

(rounded to milliseconds)

Example: 01:11:17:691

Answer **13ec87ce779e984a43eccf42266b3c7e**

197 A Fractal Sequence**2015-01-10 07:15:05****by Philippe_57721****7 xp****Programming**

The application T operates on a sequence S as:

$$T(S) = \{1, S[1], 2, S[2], \dots, S[n], n + 1\}$$

For instance:

$$T(\{10, 20, 30\}) = \{1, 10, 2, 20, 3, 30, 4\}$$

Consider the infinite sequence $S = T(T(T(T(\dots(\{1\})\dots))))$. T is applied infinitely.

What is $S[12345678910111213]$?

[My timing: < 100 ms]

Answer**eaf6173aad3ee5d6eddec5b384c2386f**

198 A special grid**2015-01-14 09:58:28****by sinan****6 xp****Programming**

Consider an $N \times N$ grid ($N < 16$) where hex digits 1 to N are used only once in each row or column and treat the whole grid as a number starting from first row to the last.

What is the smallest number for $N = 13$?

Answer format: last 2 rows as uppercase hex

Example for $N = 5$:

```
1, 2, 3, 4, 5
2, 1, 4, 5, 3
3, 4, 5, 1, 2
4, 5, 2, 3, 1
5, 3, 1, 2, 4
```

Number = 1234521453345124523153124

Answer = 4523153124

Answer **7712803abea4d16c2598c4db2aa11526**

199 A special sequence**2015-01-14 10:46:41****by sinan****4 xp****Math**

1, 17, 283, 48405761, 21491955953121731720119, ?

Hint: solve p196 first.¹**Answer****1822e0f42054d5aa316026798312a8cd**

¹ Actually Problem 198.

200 **2-derangements****2015-01-16 16:45:50****by Philippe_57721****10 xp****Programming**

A derangement is a well known kind of permutation with no fixed point.

Define a 2-derangement as a pair P_1, P_2 of derangements where: $P_1[i] \neq P_2[i]$ for all i .

Here are 2 examples of 2-derangements for 5 elements:

2 4 5 3 1
4 5 1 2 3

4 5 1 2 3
2 4 5 3 1

How many 2-derangements are there for 208 elements?

As the answer is a BIG number, use the following condensed representation:

First 10 digits [number of remaining digits] Last 10 digits

For instance, for 2^{127} , the representation is:

1701411834[19]5884105728

[My timing: 40 sec]

Answer**27c1456beebd0bd571af0311091e4737**

201 **Divisibility****2015-01-17 17:10:10****by sinan****12 xp****Programming**

How many numbers below 10^{15} can be divided by sum of their digits?

For example, 54 is such a number, the sum of its digits is $5 + 4 = 9$, 9 can divide 54.

Answer format: number

Example: 32 if below 10^2 or 11871 if below 10^5 .

[My timing: 6 s]

Answer **2ee2dfffa650b06cdd86f7d726e54acf**

202 Run it (javascript)

2015-01-22 09:45:50

by sinan

15 xp

Math

Here's a javascript code in action. Experiment with it and submit the value for $M = 100$ and $N = 200$.

Use the following condensed representation:

First 10 digits[number of remaining digits]Last 10 digits

For instance, for 2^{127} , the representation is:

1701411834[19]5884105728

M = N = **Run it**

M=5, N=5

COUNT=771

The JavaScript code for this problem is as follows:

```
/*
See the following pseudo code:

define global variables ROWS, COLS and COUNT
define GRID as a ROWS by COLS matrix and initialize it to zero

recursive_function(row=1, col=1, row_max=1)
begin
    set GRID[row, col] to 1
    if col+1 is less than or equal to COLS
    begin // if
        if GRID[row, col+1] is equal to zero; call recursive_function(row, col+1,
row_max)
    end // if
    else if row_max is equal to ROWS; add 1 to COUNT

    if row+1 is less than or equal to ROWS and GRID[row+1, col] is equal to zero;
        then call recursive_function(row+1, col, row_max+1)

    if row-1 is greater than zero and GRID[row-1, col] is equal to zero;
        then call recursive_function(row-1, col, row_max)
    set GRID[row, col] to 0
end

main(M,N)
begin
    ROWS=M
    COLS=N
    COUNT=0
    call recursive_function(row=1, col=1, row_max=1)
    return COUNT
end
*/
```

```

var ROWS;
var COLS;
var GRID = new Array();
var COUNT;

function recursive_function(row,col,row_max)
{
    GRID[row-1][col-1]=1;
    if(col+1<=COLS){
        if(GRID[row-1][col+1-1]==0) recursive_function(row,col+1,row_max);
    } else if (row_max==ROWS) COUNT++;
        if (row+1<=ROWS && GRID[row+1-1][col-1]==0) recursive_function(row+1,col,row_max+1);
        if (row>1 && GRID[row-1-1][col-1]==0) recursive_function(row-1,col,row_max);
    GRID[row-1][col-1]=0
}

function main()
{
    var M=myform.myM.value;
    var N=myform.myN.value;
    if (M<=0 || M>=10 || N<=0 || N>=10) {
        alert("Should be >0 and <10");
    } else {
        ROWS=M;
        COLS=N;
        COUNT=0;
        for (i=0;i<ROWS;i++){
            GRID[i]=new Array();
            for (j=0;j<COLS;j++){
                GRID[i][j]=0;
            }
        }
        recursive_function(1,1,1);
        outputDiv.innerHTML = "<pre><font size=+2 \\\><b\\\>M=" + M + ", N=" + N + "</b\\\></font\\\>" ;
        outputDiv.innerHTML += "<b\\\>COUNT=" + COUNT + "</b\\\></pre>";
    }
}

```

Answer d8451dd59c2fda41729a367f13fcd1bd

203 Balanced triangles

2015-01-23 14:19:32

by Philippe_57721

7 xp

Programming

Starting with a sequence of signs + and -, we form a “pascal-like” triangle applying the following rules:

- the 1st row is the initial sequence
- each next row is built by combining each 2 consecutive signs from the previous row, with the classical rules ($+ \otimes + \Rightarrow +$, $+ \otimes - \Rightarrow -$, etc.)

For instance, starting with ‘-+--’, we obtain the following triangle:

```
- + - -
- - +
+ -
-
```

Such a triangle is said balanced if it contains the same number of + and -. The following triangle is balanced:

```
- + - +
- - -
+ +
+
```

Find the number of sequences with 23 signs containing the largest number of + which produce a balanced triangle and give the last one (lexicographic order).

Answer format : count, sequence of + and - without spaces

PS: count is the number of sequences with that largest number of +.

For 7 signs, the answer would be: 5, +--+--

[My timing: 100 sec]

Answer f3aab82c97287883eeab217f9751f175

204 Happy numbers

2015-01-24 21:07:19

by gerrob

5 xp

Math

Let $f(n)$ be the sum of the squares of the digits of n . Iterate this process, if we can reach 1 in this sequence then we say n is a happy number. $H(n)$ denotes the number of (positive) happy numbers up to n . For example $n = 736$ is a happy number as the iterated sequence is 736, 94, 97, 130, 10, 1.

Find $H(12345678)$.

Example: $H(1000) = 143$.

Answer

cda22fae4535a2af0b8cc838359eabcb

205 Happy numbers II

2015-01-24 21:09:40

by gerrob

8 xp

Math

Find $H(2^{300})$ so the number of happy numbers up to 2^{300} .

Answer c43fb08a56292827c2c6933b11e323a5

206 **Balanced triangles (revisited)**

2015-01-24 22:37:18

by gerrob

10 xp

Programming

See p201 from Philippe.¹ Find the answer for 47 signs!

Answer format: count, sequence of + and - without spaces
where count is the number of sequences with that largest number of +.

[My timing: 2 min]

Answer **8654c39acfb3e3cc9533888c6c57b118**

¹ Actually [Problem 203](#).

207 Laver Tables

2015-01-30 07:28:02

by Philippe_57721

8 xp

Programming

Laver tables, discovered by Richard Laver, are very intriguing mathematical objects.

Although their definition is elementary, some of their properties cannot be proved in the classical set theory (ZFC), but require (so far) some hypotheses about large cardinals.

Let's define the operation \star for integers in range $[1\dots n]$ (n is a power of 2) by the following axioms:

$$x \star 1 = (1 + x) \bmod n$$

$$x \star (y \star z) = (x \star y) \star (x \star z)$$

Here is the Laver table for $n = 8$:

1	2	3	4	5	6	7	8
+ - + - + - + - + - + - + - + - +							
1 2 4 6 8 2 4 6 8							
+ - + - + - + - + - + - + - + - +							
2 3 4 7 8 3 4 7 8							
+ - + - + - + - + - + - + - + - +							
3 4 8 4 8 4 8 4 8							
+ - + - + - + - + - + - + - + - +							
4 5 6 7 8 5 6 7 8							
+ - + - + - + - + - + - + - + - +							
5 6 8 6 8 6 8 6 8							
+ - + - + - + - + - + - + - + - +							
6 7 8 7 8 7 8 7 8							
+ - + - + - + - + - + - + - + - +							
7 8 8 8 8 8 8 8 8							
+ - + - + - + - + - + - + - + - +							
8 1 2 3 4 5 6 7 8							
+ - + - + - + - + - + - + - + - +							

You'll notice that, except for the last one, all rows are periodic.

Find the periodic part of the 1st row for $n = 2^{16}$.

Answer format: comma delimited list of values

Example: for $n = 8$, the answer would be: 2, 4, 6, 8.

N.B.: It can be proved that the periodicity of the 1st row is unbounded. But the first n for which the period is greater than the one you'll find for 2^{16} is greater than $A(9, A(8, A(8, 255)))$. A is the Ackerman function ...

[My timing: < 1 s]

Answer 8c7d5b4304bc480b373d3febe5b19a98

208 Laver Tables II**2015-02-02 18:56:06****by Philippe_57721****15 xp****Programming**

See problem 205.¹

Find the periodic part of the 1st row for $n = 2^{23}$.

Answer format: to keep the answer short, give the first difference of the vector of values divided by 2

For instance, for 2^{10} , the first row is:

2, 12, 14, 240, 242, 252, 254, 768, 770, 780, 782, 1008, 1010, 1020, 1022, 1024

So the answer would be:

5, 1, 113, 1, 5, 1, 257, 1, 5, 1, 113, 1, 5, 1, 1

[My timing: 32 sec]

Answer**885382d1ebc7fcff661f2f3af8ba4250**

¹ Actually [Problem 207](#).

209 Huge Fibonacci**2015-01-31 16:15:31****by sinan****5 xp****Math**

Let $\text{Fib}(N)$ be N -th Fibonacci number with $\text{Fib}(1) = 1$ and $\text{Fib}(2) = 1$.

What is $\text{Fib}(\text{Fib}(1000000)) \bmod 1000000007$?

[My timing: 2 s]

Answer **3938bb50dc7ba66da0db5120836de83b**

210 Trigonometric equations

2015-02-01 13:37:04

by gerrob

5 xp

Math

Solve in real variables:

$$(\sin x)^2 + (\cos y)^2 = y^2$$

$$(\sin y)^2 + (\cos x)^2 = x^2$$

Submit all solution in lexicographic increasing way.

Answer format: $(x_1, y_1), (x_2, y_2), \dots$

Here lex. sorting means that $(a, b) < (c, d)$ if $a < c$ or $\{a = c \text{ and } b < d\}$. You can assume that all solution is in fact rational, so submit them as $x = p/q$, where $q > 0$, $\gcd(p, q) = 1$ (and not use q if x is integer).

Example: $(-1/2, 0), (3/4, 7)$

Answer

06469901718bc2eae5427ad21bd9a6b1

211 **Enumerate the rationals****2015-02-06 12:47:10****by Philippe_57721****7 xp****Programming**

Consider the function $F(X) = 1/(2 \cdot [X] - X + 1)$ where $[X]$ is the integer part of X .

It is quite remarkable that the sequence $\{0, F(0), F(F(0)), F(F(F(0))), \dots\}$ contains every non-negative rational exactly once. $\{0, F(0), F(F(0)), F(F(F(0))), \dots\} = \mathbb{Q}_+$.

We note $F(0)^n = F(F(\dots(F(0))\dots))$, function F is applied n times.

What is $F(0)^{1413121110987654321}$

Find n such as $F(0)^n = 1415926535/5772156649$

Answer format: N/D, Index

[My timing: < 100 ms]

Answer**e2929fff9a1d65b51a35b2ce94a4a43b**

212 Maximising a Ratio

2015-02-08 09:27:23

by sinan

10 xp

Programming

Consider a Pythagorean right-angled triangle with lengths a , b , and c ($a < b < c$).

Its Perimeter: $P = a + b + c$. Its Area: $A = a \cdot b / 2$.

Find the (a, b, c) triple with maximum A/P ratio if $P \leq 10^{17}$.

Answer format: a, b, c

Example: 280, 294, 406 if $P \leq 10^3$

$A = 41160, P = 980, A/P = 42$

[My timing: 40 s]

Answer 0e949fcfb999ff1ff848e21149a5def

213 Subsets**2015-02-08 11:37:04****by sinan****6 xp****Programming**

Let S be the set of numbers up to 40. $S = \{1, 2, \dots, 40\}$.

Note the following subsets:

S_1 is the prime numbers in S .

S_2 is the semiprime numbers in S .

S_3 is the all other numbers of S .

Now consider 4 subsets all with 4 distinct numbers.

A is a subset of S_1 .

B is a subset of S_2 .

C is a subset of S_3 .

D is also a subset of S_3 but C and D are disjoint.

Find the number of A, B, C and D subsets such that

- they all sum to the same number
- the smallest element of $A <$ that of $B <$ that of $C <$ that of D

Example:

$$A = \{5, 19, 23, 37\}$$

$$B = \{6, 14, 26, 38\}$$

$$C = \{8, 12, 28, 36\}$$

$$D = \{16, 18, 20, 30\}$$

$$5 + 19 + 23 + 37 = 6 + 14 + 26 + 38 = 8 + 12 + 28 + 36 = 16 + 18 + 20 + 30 = 84$$

$$5 < 6 < 8 < 16$$

This counts as one solution.

[My timing: 21 s]

Answer 671b7c6d831d522c204b0a453579f832

214 Subsets II**2015-02-08 20:45:53****by gerrob****10 xp****Programming**

See p211 from Sinan,¹ same problem, but for the first 256 positive integers, so for $S = \{1, 2, \dots, 256\}$.

[My timing: 3 seconds]

(without any trick using roughly 210 MB of Ram, the answer fits in 64 bits.)

Answer a5d11669f58f0ed29b12a598682489b5

¹ Actually [Problem 213](#).

215 Compositorial plus one

2015-02-08 21:39:47

by gerrob

6 xp

Math

Let $c(n) = n!/n\#$ be the n -th compositorial number, where $n\#$ is the product of primes up to n . For example $c(7) = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 / (2 \times 3 \times 5 \times 7) = 24$.

$S(p) = \{n : c(n) + 1 \text{ is divisible by } p\}$.

$H(p) = |S(p)|$, so the size of $S(p)$.

Example: $H(5) = 4$ as $S(5) = \{4, 5, 6, 7\}$.

Find several values of $H(p)$ (where p is prime).

Answer format: H(6031001),H(6873371),H(9103859),H(6133003),H(7834447)

Answer e816ca2c258bf2a7af780d67713ba73a

216 Shift Me Down

2015-02-13 15:59:39

by elasolova

4 xp

Sequence

3 4 6 8 12 ?

Answer

f93b8bbbac89ea22bac0bf188ba49a61

218 Prime 11-tuples

2015-02-13 16:11:01

by sinan

4 xp

Programming

S is a set of 11 prime numbers such that

- the average is also a prime number
- the biggest element < 75

How many such S are there and what is the last S ?

Answer format: cnt;p1,p2,...,p11

($p_1 < p_2 < \dots < p_{11}$)

Answer **59b52cca72cc5aca7eaee289af3247c5**

219 Girls and boys

2015-02-13 16:47:22

by Philippe_57721

5 xp

Math

At school, little boys are mischievous: they pull girls' hair, they kick. Girls don't like boys and prefer to be with girls.

In a classroom a teacher decides to place children in row in such a way that each girl gives the hand to at least one other girl.

B G G B - good

B G B G - not good

How many possibilities are there in a classroom with 40 children.

Answer**f21fd32134cef2996463d1cde5ba3aa7**

220 Minimising a Ratio

2015-02-21 10:18:53

by sinan

5 xp

Math

Let $N = 345 + 456 = F1 \cdot F2$ ($F1 < F2$).

Find $F1$ and $F2$ proper divisors of N that minimise $(F2 - F1)/N$.

Answer format: F1,F2

where F1 and F2 is entered using the following condensed representation:

Leading 10 digits[number of digits not shown]trailing 10 digits

For example, 2^{127} in this representation would be shown as

1701411834[19]5884105728

Answer

31779bda880b78154467cddd7f2dfffc5

221 Prime change

2015-02-21 10:19:21

by sinan

6 xp

Programming

There are some coins worth 1, 2, 5, 11, 23, 47, 101, 211 and bills worth 503, 1009, 2003, 5003, 10007.

In how many ways can you pay 12345, if you are allowed to use **upto 47 coins?**

[My timing: 15 s]

Answer

1ad527b8e44ab56ef2e8b3b9c59d219e

222 Conspicuous buildings

2015-02-20 11:35:12

by Philippe_57721

9 xp

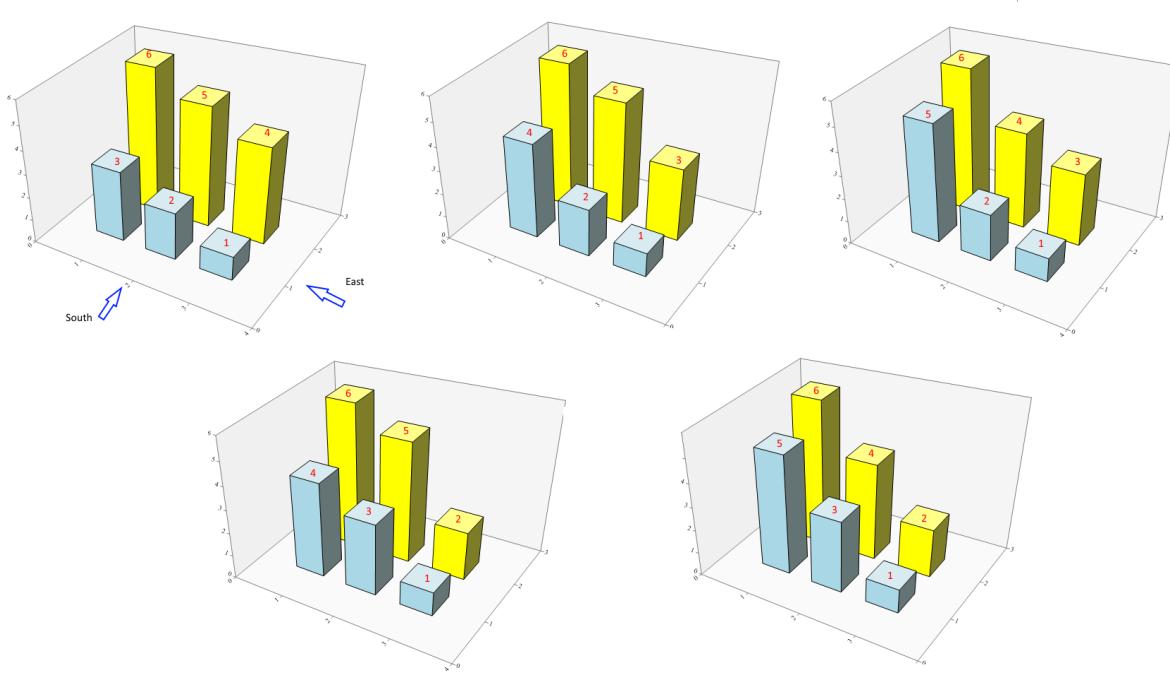
Programming

In a city, all the buildings are located in the same district.

This district is a rectangle of size W by H , its sides are labelled South, East, North and West.

- Each building occupies a 1-unit square, there are $W \times H$ buildings.
- Their heights are all integers from 1 to $W \times H$.
- When we look at the buildings facing South face or East face, we can see each building, partly or totally. No building is hidden behind a higher one.

For a 2×3 rectangle, there are five possible arrangements for the buildings.



How many possible arrangements are there for a 15×20 rectangle?

Answer format: the count in compressed format as in problem 198¹

[My timing: < 100 ms]

Answer a57abb9af4a3c8dd29c2284b2148dad3

¹ Actually Problem 200.

223 Fusible Numbers I**2015-02-27 14:06:15****by Philippe_57721****6 xp****Programming**

Fusible numbers are an extraordinary set of rational numbers. You can learn more [here](#).

They are defined by the following rules:

- We define the operation \sim between 2 reals as $a \sim b = (a + b + 1)/2$
- 0 is a fusible number
- c is a fusible number if there exist fusible numbers a and b such as $c = a \sim b$ AND $|a - b| < 1$

Here are some examples:

$$1/2 = 0 \sim 0$$

$$1 = (0 \sim 0) \sim (0 \sim 0)$$

$$5/4 = (0 \sim 0) \sim ((0 \sim 0) \sim (0 \sim 0))$$

It is easy to see that all fusible numbers other than 0 can be written $p/2^q$ for some integers p and q where p is odd.

Find all fusible numbers in the interval $]1/2, 5/4[$ (bounds excluded) with a denominator ≤ 128 .

Answer format: comma separated list in ascending order

(We represent a fusible number $p/2^q$ as $p.q$, e.g.: $3/4 = 3.2$)

Example: 15.4, 63.6, 1.0, 5.1

[My timing: < 1 sec]

Answer 39cfedb769400f229f0d367595ae7a60

224 Fusible Numbers II

2015-02-27 14:06:29

by Philippe_57721

10 xp

Programming

We define a canonical representation of a fusible number as a representation with only 0.

For instance, the shortest canonical representations of 21/16 are:

- (((0~0)~0)~(0~0))~(0~0)
- (((0~0)~0)~0)~((0~0)~0)
- (((0~0)~0)~0)~(0~(0~0))
- ((0~(0~0))~(0~0))~(0~0)
- ((0~(0~0))~0)~((0~0)~0)
- ((0~(0~0))~0)~(0~(0~0))
- ((0~0)~((0~0)~0))~(0~0)
- ((0~0)~0)~(((0~0)~0)~0)
- ((0~0)~0)~((0~(0~0))~0)
- ((0~0)~0)~(0~((0~0)~0))
- ((0~0)~0)~(0~(0~(0~0)))
- (0~((0~0)~0))~((0~0)~0)
- (0~((0~0)~0))~(0~(0~0))
- (0~(0~(0~0)))~((0~0)~0)
- (0~(0~0))~(((0~0)~0)~0)
- (0~(0~0))~((0~(0~0))~0)
- (0~(0~0))~(0~((0~0)~0))
- (0~(0~0))~(0~(0~(0~0)))
- (0~0)~(((0~0)~0)~(0~0))
- (0~0)~((0~(0~0))~(0~0))
- (0~0)~((0~0)~(0~(0~0)))

and the last in alphabetic order is (0~(0~(0~0)))~(0~(0~0)).

What is the last (in alphabetic order) shortest canonical representation of 111/64?

Hint: The number of 0s in a representation is 13.

[My timing: 4 sec]

Answer 3da9a492d7de6699c611846d69a45798

225 Fusible Numbers III

2015-02-27 14:06:42

by Philippe_57721

8 xp

Programming

A fusible number can be written in several ways as $a \sim b$ for some fusible numbers a and b ($a \leq b$)

For instance $101/64 = (3/4 \sim 45/32)(15/16 \sim 39/32)(31/32 \sim 19/16)$.

Find all such representations for $111/64$

Answer format: comma separated list of $a \sim b$ where $a \leq b$ and first terms in ascending order
if $(ai \sim bi)$ and $(aj \sim bj)$ are 2 representations with $i < j$, then $ai < aj$.

A fusible $a = p/2^q$ is represented as $p.q$ (see problem 220).¹

Example: $3.2 \sim 45.5, 15.4 \sim 39.5, 31.5 \sim 19.4$
for $101/64$.

[My timing: 1 sec]

Answer**184ba57d97d750bb365565ba1f27b7d4**

¹ Actually Problem 223.

226 P-automorphic numbers

2015-03-06 15:04:39

by Philippe_57721

6 xp

Programming

A number is *p-automorphic* if the last digits of its *p*-th power are the number itself.

For instance 68751 is a 3-automorphic number:

$$68751^3 = 3249653517\mathbf{68751}$$

There are 10 10-digits 6-automorphic numbers:

- $1787109376^6 = 32576678364650828374994886894920831594093799601\mathbf{787109376}$
- $2000000001^6 = 64000000192000000240000000160000000060000000012\mathbf{000000001}$
- $3787109376^6 = 2950170318650991258032575341918572870335539112113\mathbf{787109376}$
- $4000000001^6 = 409600000614400000384000000128000000024000000024\mathbf{000000001}$
- $5787109376^6 = 37563853963345670599003034369316678034196984424625787109376$
- $6000000001^6 = 46656000046656000019440000004320000000540000000036\mathbf{000000001}$
- $7787109376^6 = 222975757734371332908041996970049236323178429737137787109376$
- $8000000001^6 = 2621440001966080000614400000102400000096000000048\mathbf{000000001}$
- $8212890625^6 = 306885405287085842075678332463439801358617842197418212890625$
- $9787109376^6 = 878874068949224372247665088145076247737279875049649787109376$

How many 50-digits 73-automorphic numbers are there?

Answer format: count,sum

Example: 10,57148437509

for 10-digits 6-automorphic numbers.

[My timing: 3 sec]

Answer 111167427cd8295481e964148140d50b

227 A long period**2015-03-02 13:37:40****by gerrob****8 xp****Math**

Define the following sequence: $a(0) = (\sin(2\pi/n))^2$ and for $k > 0$, $a(k) = 4 \cdot a(k-1) \cdot (1 - a(k-1))$. For odd n value this sequence is periodical!

Find the period for $n = 314159265358979323846264338327950288419716939937510582097$.

Answer 113c04a6709a703859b2d6276488345c

228 Hyperbolic prime triples

2015-03-13 15:35:23

by Philippe_57721

6 xp

Programming

There are (probably infinitely) many triples of prime numbers $P_1 < P_2 < P_3$ such as $P_i + P_j \cdot P_k$ is a square number for all combinations (i, j, k) with $i \neq j \neq k$.

Here is an example: 4049, 126001, 130051.

Find the triple with the largest $P_3 < 180000$.

Answer format: comma separated list

Example: 4049, 126001, 130051

[My timing: 95 sec]

Answer f05e9dd8eddf49feb70afae21c746515

229 Meanderings

2015-03-20 09:27:42

by Philippe_57721

4 xp

Programming

A ant walks exploring each lattice point in the first quadrant of the plane with the following pattern:

37	→	38	→	39	→	40	→	41	→	42	→	43
↑												↓
36	←	35	←	34	←	33	←	32	←	31		44
											↑	↓
17	→	18	→	19	→	20	→	21		30		45
↑							↓		↑		↓	
16	←	15	←	14	←	13		22	←	29	←	46
							↑	↓	↑		↓	
5	→	6	→	7		12		23		28		47
↑			↓		↑		↓		↑		↓	
4	←	3		8		11		24		27		48
			↑		↓		↑		↓		↑	↓
1	→	2		9	→	10		25	→	26		49

The 1st point visited (index = 1) is (0, 0)

The 10th point is (3, 0)

The 100000th point is (143, 316)

What is the 10^{17} -th point?

What is the index of the point (718281828, 141592653)?

Answer format: x,y/index

Example: 143,316/51594074

for 10^5 and (7182, 1415).

[My timing: < 100 ms]

Answer 691be549e68a17df7aac35d46c884caa

230 Anti constitutionnal amendment**2015-04-03 09:57:33****by Philippe_57721****4 xp****Programming**

The word **anticonstitutionnellement**, meaning unconstitutionally is the longest word in French.

How many subsequences can you find for this word?

For example, the word ‘nine’ has 14 subsequences: (empty string included)
”, ‘e’, ‘i’, ‘n’, ‘ie’, ‘in’, ‘ne’, ‘ni’, ‘nn’, ‘ine’, ‘nie’, ‘nin’, ‘nne’, ‘nine’

[My timing: 90 sec]

Answer f6ceccccb8ee326bf73fc64d3f73a216

231 F-sequences**2015-03-27 10:38:59****by Philippe_57721****10 xp****Programming**

Given two sequences $X[1]\dots X[n]$ and $Y[1]\dots Y[m]$ ($n \leq m$), we say that the former is a subsequence of the latter if there is a sequence $1 \leq i[1] < i[2] < \dots < i[n] \leq m$ such as $X[j] = Y[i[j]]$ for all $1 \leq j \leq n$.

E.g: 'PAIN' is a subsequence of 'DIOPHANTINE'.

We say a sequence $S[1]\dots S[n]$ is a F-sequence if no sequence $S[i]\dots S[2 \cdot i]$ ($2 \cdot i \leq n$) is a subsequence of any sequence $S[j]\dots S[2 \cdot j]$ ($i < j$ and $2 \cdot j \leq n$).

What is the length of longest F-sequence over the alphabet {A, B}?

Find the 1st F-sequence in lexicographic order over the alphabet {A, B, C} with length 80.

Hint: Here an example of a F-sequence of length 40:

AACBABB BBBB CCCCCCCCCC BBBB BBBB BBBB BBBB BA

Answer format: count, sequence

[My timing: < 100 ms]

Answer**c0f662afe30b93fb1c89d5044e226261**

232 Congruent numbers**2015-04-10 07:35:36****by Philippe_57721****12 xp****Programming**

How many squarefree numbers are congruent in the range $[10^7, 10^7 + 1000]$?

Hint: Assume the BSD conjecture.

Answer format: count,sum modulo 10^7

[My timing: 45 sec]

Answer**1ee2af07c54c9acf0b0d8fb5844edf3b**

233 Integer Averages

2015-04-17 09:54:33

by Philippe_57721

6 xp

Programming

Let define a sequence S as follow:

- $S[1] = 1$
- $S[n + 1] =$ the first integer not in $S[1\dots n]$ such as the average of $S[1\dots n + 1]$ is an integer.

Find $S[2357111317192329]$.

[My timing: < 100 ms]

Answer 5072ee14c435036270d679ab189bb5a0

234 Meeting point**2015-04-02 09:02:39****by sinan****10 xp****Math**

There are 2 particles moving on the unit circle ($x^2 + y^2 = R^2$ and $R = 1$).

Particle A moves clockwise at a speed equal to $2 + x$ while particle B moves counter-clockwise at a speed equal to $2 + x$, where x and y are the coordinate values of the moving particle.

At $T = 0$ time, A is located at $(-1, 0)$ and B at $(0, -1)$, so the speeds would be 1 and 2 for A and B respectively.

Where do they first meet and when?

Answer format: X,Y,elapsed time

X, Y rounded to 3 digits after decimal point. elapsed time is a decimal number (rounded to 12 digits after decimal point).

Answer**433a9ce8cbae494af44f05925e2bb9b9**

235 **Totient Proportionality****2015-04-05 15:02:36****by Min_25****15 xp****Math**

Let $C(N)$ be the number of positive integer $n \leq N$ such that

$$\frac{n}{\phi(n)} = \frac{\phi(n)}{\phi(\phi(n))},$$

where $\phi(n)$ is the totient function.

You are given that $C(10^5) = 342$.

Find $C(10^{15})$.

[My timing: 10.9 sec]

Answer **c0794ebc58f691d538260f93e4f37c11**

236 Inverse Sigma Function

2015-04-05 00:59:29

by Min_25

8 xp

Math

Find the sum of n such that $\sigma(n) = 13!$, where $\sigma(n)$ is the sum of divisors function.

Example: 104530 for $7!$.

[My timing: 4.6 s]

Answer

e0bcad5f84744d8a11a9f60b183b506a

237 Programming exercise - Fractions madness

2015-04-24 10:07:03

by Philippe_57721

10 xp

Programming

Consider the sequence of fractions:

$$F = \left\{ \frac{23}{95}, \frac{57}{23}, \frac{17}{39}, \frac{130}{17}, \frac{11}{14}, \frac{35}{11}, \frac{19}{13}, \frac{1}{19}, \frac{35}{2}, \frac{13}{7}, \frac{7}{1} \right\}$$

We define the following algorithm (n is an integer):

```
func(n) {
    i = 0;
    Start:
    j = 0;
    while (F[j]*n not integer) {
        j++;
    }
    i++;
    n = n*F[j];
    if (n has only factors 2 and 3) {
        print(i);
    }
    goto Start;
}
```

The first values func(6) yield are:

- 10, 24, 46, ...

Find the 100th value.

You are given the 20-th element: 185500.

[My timing: < 100 ms]

Answer 8f670dd1464d9f034a81426095c9f838

238 Phinary representation

2015-04-30 15:43:17

by Philippe_57721

10 xp

Math

$\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.

Any positive integer can be represented as a sum of powers of ϕ with integer exponents:

- $1 = \phi^0$
- $2 = \phi^1 + \phi^{-2}$
- $3 = \phi^2 + \phi^{-2}$
- $4 = \phi^2 + \phi^0 + \phi^{-2}$
- $5 = \phi^3 + \phi^{-1} + \phi^{-4}$
- $6 = \phi^3 + \phi^1 + \phi^{-4}$
- $7 = \phi^4 + \phi^{-4}$
- $8 = \phi^4 + \phi^0 + \phi^{-4}$
- $9 = \phi^4 + \phi^1 + \phi^{-2} + \phi^{-4}$
- $10 = \phi^4 + \phi^2 + \phi^{-2} + \phi^{-4}$

If we add the constraint that no exponent are consecutive, this representation is unique. This representation is called the phinary representation of an integer.

For an integer n , we define $M(n)$ as the sum of all exponents in its phinary representation:

$$M(641) = \sum 641_{\text{base-}\varphi} = \sum \{13, 9, 7, 5, 2, 0, -3, -11, -14\} = 8$$

In the range $[1, 10000000]$, find the minimum for M , the first integer which reaches the minimum, the maximum, and the first integer which reaches the maximum.

Answer format: $M_{\min}, n_{\min}, M_{\max}, n_{\max}$
 $-17, 77, 8, 46$ for the range $[1, 100]$.

[My timing: 10 sec]

P.S: Thanks to **sinan** for his suggestions which made this problem more interesting.

Answer

0c7b49c9371501d8a089298098f52cd1

239 Find the period

2015-05-14 21:37:28

by Philippe_57721

8 xp

Programming

The period of a prime p is the period of the fraction $1/p$.

For example,

$$\text{Period}(79) = 13 \quad \left(\frac{1}{79} = 0.\underbrace{0126582278481\dots}_{13} \right)$$

Find the smallest prime with a period of 798.

Answer

c20dd77c6da3e3c3794b7f539e008edb

240 Large factorization

2015-05-07 13:25:50

by Philippe_57721

10 xp

Math

We have:

$$\begin{aligned} 2^{10000000000^2} + 1 &= f_1 \cdot f_2 \\ &= \underbrace{41776\cdots01}_{1505149979 \text{ digits}} \cdot \underbrace{41776\cdots05}_{1505149979 \text{ digits}} \end{aligned}$$

Find the last 10 digits of f_1 and f_2 .

Answer format: $f_1 \bmod 10^{10}$, $f_2 \bmod 10^{10}$

Leading zeros are ignored

Answer 4373f0631fbffbb19dd89938596d20cb2

241 Smooth factorisation

2015-05-22 06:51:56

by Philippe_57721

8 xp

Programming

Let's define a smooth factorization of $n!$ as $n! = f_1 \times f_2 \times \dots \times f_n$ with:

$$f_i \leq f_{i+1}$$

f_n = the largest prime factor of $n!$

Examples:

$$\begin{aligned}13! &= 2 \times 3 \times 3 \times 5 \times 5 \times 6 \times 6 \times 6 \times 7 \times 8 \times 8 \times 11 \times 13 \\&= 3 \times 4 \times 4 \times 4 \times 5 \times 5 \times 6 \times 6 \times 6 \times 6 \times 7 \times 11 \times 13 \\14! &= 4 \times 4 \times 4 \times 4 \times 4 \times 5 \times 5 \times 6 \times 7 \times 7 \times 9 \times 9 \times 11 \times 13\end{aligned}$$

Find the last (in numeric lexicographic order) smooth factorization of 28!.

Answer format: comma separated list of factors

Example: 5,6,6,6,6,6,6,7,7,8,8,10,10,11,13,17,19 for 19!.

[My timing: 70 sec]

Answer**84b6c2c4c5e758c8957e9df95fedaa96**

242 Generalized Pisano Period 1**2015-05-07 04:56:35****by Min_25****8 xp****Math**

Let $F_k(n)$ be a linear recurrence sequence such that

$$\begin{aligned}F_k(n) &= 1 \quad (1 \leq n \leq k), \\F_k(n) &= \sum_{i=1}^k F_k(n-i) \quad (n > k).\end{aligned}$$

For example, $F_2(1) = 1$, $F_2(2) = 1$ and $F_2(3) = 2$.

Let $P(k, m)$ be the (minimum) period of $F_k(n)$ modulo m .

You are given that $P(2, 10) = 60$ and $P(2, 10^9) = 1\,500\,000\,000$.

Find $P(10, 10^9)$.

[My timing: instant]

Answer **4ef3d650778b312c9f165d97216eb1a5**

243 Generalized Pisano Period 2**2015-05-07 04:56:58****by Min_25****15 xp****Math**

Let $F_k(n)$ be a linear recurrence sequence such that

$$\begin{aligned}F_k(n) &= 1 \quad (1 \leq n \leq k), \\F_k(n) &= \sum_{i=1}^k F_k(n-i) \quad (n > k).\end{aligned}$$

For example, $F_2(1) = 1$, $F_2(2) = 1$ and $F_2(3) = 2$.

Let $P(k, m)$ be the (minimum) period of $F_k(n)$ modulo m .

You are given that $P(2, 10) = 60$ and $P(2, 10^9) = 1\,500\,000\,000$.

Find $P(50, 10^9)$.

Note: The answer is a 29 digit number.

[My timing: 42 sec (PyPy)]

Answer**1ecd8de7460356ed39e0e4133252b61d**

244 Brazilian numbers

2015-05-29 09:59:20

by Philippe_57721

8 xp

Programming

A number is said brazilian if it can be written with the same ‘digit’ for some base ($0 < \text{digit} < \text{base}$).

For instance, the following numbers are brazilian:

$$2222 = (2, 2, 2, 2)_{10}$$

$$1281 = (7, 7, 7)_{13}$$

$$33500852988 = (127, 127, 127, 127)_{641}$$

How many brazilian numbers $\leq 100\,000\,000\,000$ are there? (We consider only numbers with at least 3 ‘digits.’)

You are given:

114 numbers $\leq 1\,000$

617 numbers $\leq 10\,000$

3016 numbers $\leq 100\,000$

14387 numbers $\leq 1\,000\,000$

[My timing: 4.7 sec]

Answer 3aa7c660b1f5591170a837c69c60bfb6

245 Weighted triangles

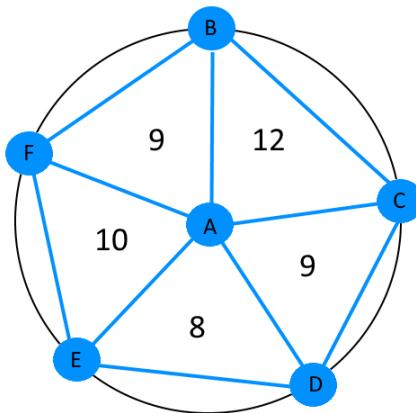
2015-06-05 03:48:11

by Philippe_57721

5 xp

Math

5 points are placed on a circle. Consecutive integers 1 to 6 are placed on the center of the circle and on each point. Joining each consecutive pair of points with the center, we define 5 triangles. The sums of integers for each triangle are: 12, 9, 8, 10, 9. You can guess that the integers are respectively (center first, points clock-wise): 2, 4, 6, 1, 5, 3.



Now, we are using 11 points.

The sums inside the triangles are respectively: 26, 22, 26, 22, 14, 16, 22, 26, 22, 24.

Find the values for each point (same order as in the example). There are more than one solution.

Answer format: comma separated lists between slashes (in lexicographic order)

Example: 1,2,3/3,2,1

Answer

f898a16ad01044fead55081c20408009

246 A special set

2015-05-25 04:42:34

by sinan

10 xp

Programming

Let S be a set like $\{1, \dots, M\}$ with cardinality N .

Any element but the first can be written as a sum of some two lesser and not necessarily distinct elements.

How many such sets can you find if $M = 1881$ and $N = 15$?

Input format: cnt, S_1001

(S_1001 means 1001st such set)

Example: cnt, {1, 2, 3, 6, 7, 13, 26, 52, 104, 208, 209, 418, 836, 1045, 1881}

if asked 101st such set.

Sets are ordered comparing each element from 1 to M :

100: {1, 2, 3, 6, 7, 13, 26, 52, 104, 208, 209, 418, 627, 1254, 1881}

101: {1, 2, 3, 6, 7, 13, 26, 52, 104, 208, 209, 418, 836, 1045, 1881}

Here, there is a difference at 13th elements.

[My timing: 2 s]

Answer**fb0b5b31ac3afb49b24217fcf2d3aaff**

247 Crossings and nestings

2015-06-12 09:03:07

by Philippe_57721

12 xp

Math

Consider the permutation $\{2, 10, 1, 8, 9, 4, 6, 3, 5, 7\}$.

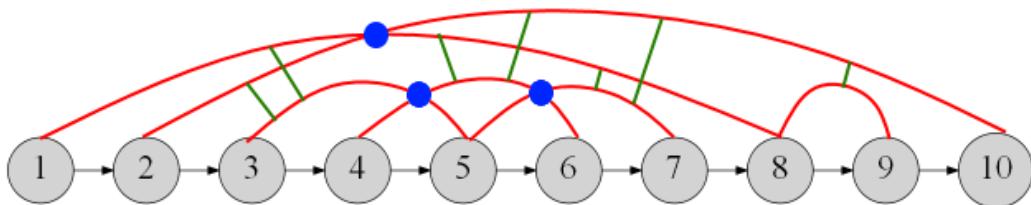
It can be decomposed in 4 “runs” of ascending values:

- 2, 10
- 1, 8, 9
- 4, 6
- 3, 5, 7

Let's draw the following graph:

- 10 nodes (1 to 10)
- we join by an edge each consecutive node in the same run

We get the following graph:



We count how many times edges are crossing: 3 (blue dots)

- [1, 8] crosses [2, 10]
- [3, 5] crosses [4, 6]
- [4, 6] crosses [5, 7]

We count how many edges are nested below another one: 7 (green lines)

- [3, 5] nested below [2, 10]
- [3, 5] nested below [1, 8]
- [4, 6] nested below [2, 10]
- [4, 6] nested below [1, 8]
- [5, 7] nested below [2, 10]
- [5, 7] nested below [1, 8]
- [8, 9] nested below [2, 10]

Among all the permutations of $\{1, 2, \dots, 10\}$ how many have exactly 4 crossings and 8 nestings?

What is the latest in lexicographic order?

Answer format: count/permutation

(as comma separated list)

Example: 99/1,2,3

[My timing: 20 sec]

Answer **2336c357e289f2f5256e1c32c48909a6**

248 Smooth factorisation (hard)**2015-05-22 23:15:58****by gerrob****20 xp****Math**

See the problem r237 from Philippe!¹ Find the answer for $n = 450$, as the answer would be quite long, give only $\sum_{i=1}^n f[i]^6$.

Example: 80930279 for $n = 19$.

[My timing: 58 sec.]

Answer **51e4c7ccba556721e4970b6746e683f6**

¹ Actually [Problem 241](#).

249 Friend Exponent

2015-06-02 16:18:49

by Min_25

15 xp

Math

For given two real numbers α and β , a positive integer n is called k -friend exponent of α for β if α^n and β shares the first k -significant digits.

For example, $n = 9$ is a 1-friend exponent of π for e because $\pi^9 = 29809.099333446\cdots$ and $e = 2.71828\cdots$ shares “2”.

Let $E(\alpha, \beta, k)$ denote the minimum k -friend exponent of α for β . It can be verified that $E(\pi, e, 1) = 9$, $E(\pi, e, 2) = 19$, $E(\pi, e, 5) = 430881$ and $E(\pi, e, 10) = 2448322414$.

Find $E(\pi, e, 50)$.

[My timing: 0.5 sec. (PyPy)]

Answer

c5f69d009f1ce0587d773f8db076c9bb

250 Polyominoes 1

2015-07-03 12:24:49

by Philippe_57721

11 xp

Programming

A polyomino is a geometric figure obtained by joining equal squares edge by edge. We consider that one side of each square is painted: when comparing two polyominoes for symmetry, the painted sides must always stay up. Two polyominoes are equal if they differ only by a rotation of a multiple of $\frac{\pi}{2}$.

To sort the polyominoes, we define the “canonical value” of a polyomino as follow:

- A polyomino is represented by a matrix of 0 and 1.
- We consider all 4 rotations and keep those where the matrix is such as the number of rows is \leq the number of columns.
- We add a column of 0 at the end, flatten the values and consider them as a binary representation of some integer. The canonical value is **the smallest of these integers**.

For example, consider the polyomino



It can be represented either by $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ or by $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

The first representation yields the values $\{0, 0, 1, 0, 1, 1, 1, 0\} = 46$. The second representation yields the values $\{1, 1, 1, 0, 1, 0, 0, 0\} = 232$. Thus, the canonical value for this polyomino is 46.

With 4 squares, there are 7 possible polyominoes (sorted by their canonical values):

- 30



- 46



- 54



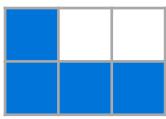
- 78



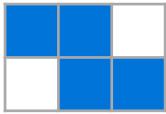
- 108



- 142



- 198



How many polyominoes with 8 squares are there?

What is the canonical value of the 666-th polyomino?

Format answer: Count,Value

[My timing: 40 sec]

Answer **74029fadf52b6bd2e1db689c9d8c7506**

251 Polyominoes 2**2015-07-17 13:44:51****by Philippe_57721****13 xp****Programming**

For the definitions, see problem 247.¹ Among all the 8-squares polyominoes, which ones can cover perfectly a 4×8 rectangle? (A polyomino covers perfectly a rectangle if we can tile without hole or overlap, the rectangle with multiple copies of that polyomino.)

Answer format : comma separated list of canonical values (ascending order)

Example: 126, 190, 222, 238, 798, 2110

for covering a 2×6 rectangle with 6-squares polyominoes.

[My timing: 6 sec]

Answer **ec0fb152d0168b4f876e3342ab6e2dd2**

¹ Actually Problem 250.

252 Sort

2015-06-19 11:02:12

by Philippe_57721

6 xp

Math

Consider the following HUGE numbers:

$$a = \text{googol}!$$

$$b = \text{Skewes' number} = e^{e^{e^{79}}}$$

$$c = 2^{3^{4^5}}$$

$$d = 6^{5^{4^3}}$$

$$e = 4!!!!$$

$$f = p_{\text{googol}\#} \quad (p_n\# \text{ is the primorial function})$$

Sort these numbers in ascending order.

Answer format: comma separated list

Example: f, e, d, c, b, a

For the record, the primorial function $p_n\#$ is the product of the first n prime numbers.

$$p_5\# = 2 \times 3 \times 5 \times 7 \times 11 = 2310$$

Answer**db361498f85de7014be5d4aeaf909afe**

253 Colored coins

2015-06-26 12:51:33

by Philippe_57721

7 xp

Programming

$T(n)$ is the n -th triangular number.

$T(n)$ coins are arranged in a triangle. We color each coin with 4 different colors such as no two touching coins have the same color.

For $n = 2$, there are 24 possible configurations:

```
+---+ +---+ +---+ +---+ +---+ +---+
| 1 | | 1 | | 1 | | 1 | | 1 | | 1 |
|2 3| |2 4| |3 2| |3 4| |4 2| |4 3|
+---+ +---+ +---+ +---+ +---+ +---+  

+---+ +---+ +---+ +---+ +---+ +---+
| 2 | | 2 | | 2 | | 2 | | 2 | | 2 |
|1 3| |1 4| |3 1| |3 4| |4 1| |4 3|
+---+ +---+ +---+ +---+ +---+ +---+  

+---+ +---+ +---+ +---+ +---+ +---+
| 3 | | 3 | | 3 | | 3 | | 3 | | 3 |
|1 2| |1 4| |2 1| |2 4| |4 1| |4 2|
+---+ +---+ +---+ +---+ +---+ +---+  

+---+ +---+ +---+ +---+ +---+ +---+
| 4 | | 4 | | 4 | | 4 | | 4 | | 4 |
|1 2| |1 3| |2 1| |2 3| |3 1| |3 2|
+---+ +---+ +---+ +---+ +---+ +---+
```

We represent a configuration:

```
+---+
| A |
| B C |
|D E F|
+---+
```

as A,BC,DEF.

How many configurations are there for $n = 10$

What is the 10000000-th (in lexicographic order)?

Example: 192/1,32,214

if we ask the 20-th configuration for $n = 3$.

[My timing: 7 sec]

Answer e5b0cb04de243669196cf8426dcae991

254 Semiprimes**2015-07-10 10:57:10****by Philippe_57721****12 xp****Programming**

An integer is semiprime if it is the product of 2 (not necessarily distinct) prime numbers.

There are 34 semiprimes below 100:

4, 6, 9, 10, 14, 15, 21, 22, 25, 26, 33, 34, 35, 38, 39, 46, 49, 51, 55, 57, 58, 62, 65, 69, 74, 77, 82, 85, 86, 87, 91, 93, 94, 95

How many semiprimes are there below 30 000 000 000?

[My timing: 60 sec]

Answer **d6f3ef9fc98c27fc08ed9c0fbc2ab78d**

255 Complex Base 1

2015-08-01 08:13:27

by Philippe_57721

15 xp

Math

A complex integer is a number $p + q \times i$ ($i^2 = -1$) where p and q are integers.

It can be proved that every complex integer has a unique representation in base $i - 1$ with 'digits' 0 or 1.

For instance, we have:

$$2 = 0 \times (-1 + i)^0 + 0 \times (-1 + i)^1 + 1 \times (-1 + i)^2 + 1 \times (-1 + i)^3 \quad (1100)$$

$$\begin{aligned} 6 = & 0 \times (-1 + i)^0 + 0 \times (-1 + i)^1 + 1 \times (-1 + i)^2 + 1 \times (-1 + i)^3 + 1 \times (-1 + i)^4 + \dots \\ & + 0 \times (-1 + i)^5 + 1 \times (-1 + i)^6 + 1 \times (-1 + i)^7 + 1 \times (-1 + i)^8 \quad (111011100) \end{aligned}$$

Actually, 6 is the smallest natural integer whose representation contains 6 digits to '1'.

Let's define the function $R(n)$ for an natural integer n as follow: consider the digits of n in base $i - 1$ as the binary representation of an integer.

Thus:

$$R(2) = 12$$

$$R(6) = 476$$

Find the smallest integer a , whose representation contains 60 digits.

Find the smallest integer b , whose representation contains 93 digits to '1'.

Answer format: a,R(a),b,R(b)

[My timing: instant]

Answer 565879355f5ec90aa60c99a7ba22a45f

256 Complex Base 2

2015-08-01 08:14:10

by Philippe_57721

15 xp

Programming

The definitions are the same as in problem 252.¹

Consider the following pseudo-random generator:

```
static long seed = 641;
static long FastRandom()
{
    seed = (214013 * seed + 2531011) % 43097251273073;
    return seed;
}
```

Let V the sequence of the first 100000 values of FastRandom.

You are given the following values:

1	139713344
10	34286987424384
100	2133420965406
1000	31159015095342
10000	29256942849607
100000	42800860506246

Find the sum of $R(n)$ for all n in V .

[My timing: 12 sec]

Answer b96cc8fdab20499d7ffcb06d03260653

¹ Actually Problem 255.

257 Pattern Locks

2015-07-17 13:45:07

by Philippe_57721

7 xp

Programming

Pattern locks are a popular mechanism to secure our cell phones.

Given a grid 3×3 , the user define a path by joining some points on that grid:

1	2	3
4	5	6
7	8	9

We add the following constraints:

- the path starts with point 1
- we visit every point exactly once

One possible path is: 1, 4, 3, 5, 6, 2, 7, 8, 9.

Actually, there are exactly 5040 possible paths with the above constraints. There are 2 paths which reach the maximal length:

- 1, 6, 7, 2, 9, 4, 3, 8, 5
- 1, 8, 3, 4, 9, 2, 7, 6, 5

The maximal length is 16.652...

A path starting with 1, 3, 2 would not be valid, for when joining point 1 and 3 we have visited point 2.

How many possible paths are there on a 3×4 grid?

Which paths reach the maximal length?

Answer format: count/(comma separated list of points)

Example: 5040/1,6,7,2,9,4,3,8,5/1,8,3,4,9,2,7,6,5
for a 3×3 grid.

[My timing: 40 sec]

Answer ac4c1be08f1d7ba33327f51183d93ff7

258 P-sets**2015-08-07 08:48:24****by Philippe_57721****8 xp****Programming**

Let P a finite set of prime numbers.

A P -set is the set of all numbers whose prime factors are in P .

For instance, if $P = \{2, 3, 5\}$, the corresponding P -set is
 $\{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 27, 30, 32, 36, \dots\}$.

The 100000th number of this P -set is 29014219670751100192948224000000000000000.

What is the 100000000th number in the $\{2, 3, 5, 7, 11\}$ -set?

[My timing: 40 sec]

Answer **6fab782829208d2d114c46e306d2ea4f**

259 **Alphabet permutations****2015-07-24 11:50:16****by Philippe_57721****6 xp****Probability**

The classical latin alphabet {**a**, b, c, d, **e**, f, g, h, **i**, j, k, l, m, n, **o**, p, q, r, s, t, **u**, v, w, x, **y**, z} contains 20 consonants and 6 **vowels**.

What is the probability that in a permutation of this alphabet, there are no consecutive vowels?

Answer format: P/Q

with P and Q coprime.

[My timing: < 1 sec]

Answer**ee7c1d30528d58345a78d06331a3a9c3**

260 Matchstick math

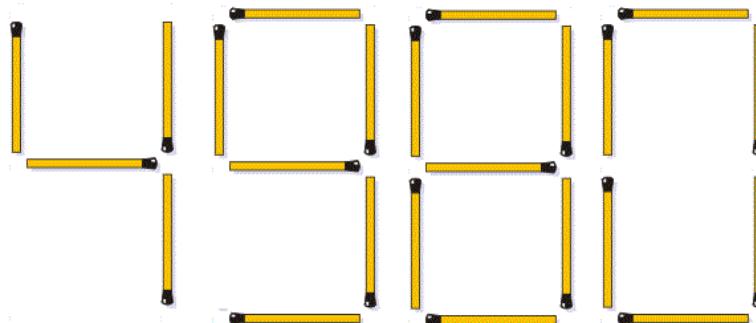
2015-07-31 08:56:32

by sinan

5 xp

Math

See the following number:



What is the biggest number you can get if you are allowed to move 1, 2, 3 or 4 matchsticks?

Answer format: n1,n2,n3,n4

Notes:

- You should look for the ways to increase the number of digits. For example out of “8”, you can make “51” by moving 2 sticks.
- Ignore any spacing issues after the move, think all the digits as a part of the one new number.

Answer ed3c36764ecf0e17dd4ed04e3de66db8

261 A slow converging serie**2015-09-27 15:49:28****by Philippe_57721****12 xp****Math**

P is the set of prime numbers.

Let $S = \sum_{p \in P} \frac{1}{1+p^2} = 0.3890595553\dots$

Find the value of S with a precision of 50 digits after the decimal point (truncated not rounded).

[My timing: 15 sec]

Answer**55ff5d2af96d514bde27a073bca5f6b3**

262 Matchsticks prime

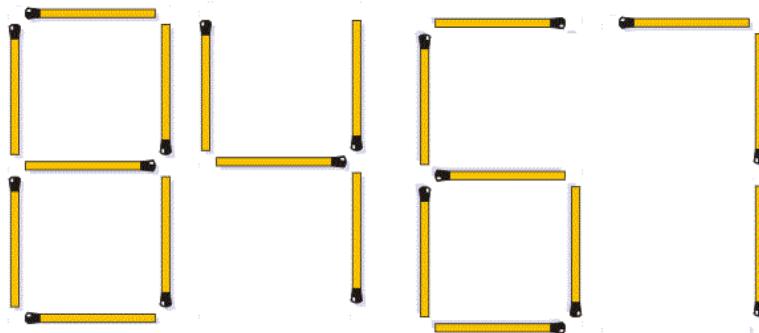
2015-08-22 07:07:32

by Philippe_57721

6 xp

Programming

Using 20 matchsticks, we can write the following prime number:



How many different prime numbers can you write using exactly 20 matchsticks?

[My timing: < 200 ms]

N.B.: Non significative 0s are not allowed, thus: 0019 is not valid.

Answer**bedc8b5da097e4f6a7a4e0a0681b9771**

263 **Digit Sum Sequence****2015-08-02 02:41:12****by Min_25****12 xp****Math**

Let $x_1 = 1$ and $x_i = x_{i-1} + \text{sum_of_digits}(x_{i-1})$ for $i > 1$.

You are given that $x_{10} = 62$, $x_{100} = 1205$ and $x_{100000} = 2609882$.

Find $x_{10^{17}}$.

[My timing: 0.2 sec.]

Answer**bbd61f255581afe1721496ab570c43f5**

264 Anti-Pythagorean equation

2015-08-14 04:46:41

by Philippe_57721

8 xp

Math

We have

$$1040^{-2} + 4095^{-2} = 1008^{-2}$$

How many solutions are there to the “anti-pythagorean” equation $x^{-2} + y^{-2} = z^{-2}$ with x, y, z coprime and $x \leq y$ and $x \leq 10^{12}$?

Answer format: count, $\sum z$

[My timing: 9 sec]

Answer**36b2088bd7a9db01d82fe1cc39eeaa10**

265 Anti-Pythagorean equation 2

2015-08-14 04:54:21

by Min_25

12 xp

Math

See the problem 261 from Philippe for the description.¹

In this problem, find the answer for $x \leq 10^{18}$.

[My timing: 0.8 sec.]

Answer

fe300e68829e970bdc5a4cb56a83f575

¹ Actually [Problem 264](#).

266 Parallelepipedic Box

2015-08-28 09:48:22

by Philippe_57721

7 xp

Math

The parallelepipedic box of dimensions $2 \times 3 \times 6$ has a diagonal of 7.

A parallelepipedic box with integer sides has a diagonal of 100000.

Let x, y, z its dimensions with $x \leq y \leq z$.

Find all the possible triples x, y, z .

Answer format: count, $\sum z$

[My timing: 10 sec]

Answer**45d20c53c61d79e58387c9f17e297689**

267 The Schinzel-Sierpinski conjecture

2015-09-04 11:37:30

by Philippe_57721

4 xp

Math

This conjecture states:

Every rational $\frac{n}{d}$ can be written $\frac{p+1}{q+1}$ where p, q are prime numbers.

For instance: $\frac{22}{7} = \frac{43+1}{13+1}$, $\frac{355}{113} = \frac{2129+1}{677+1}$.

Find the representation for $\frac{2^{19}-1}{2^{19}}$.

Answer format: p, q

give the smallest pair p, q .

[My timing: < 1 sec]

Answer**c76e5252fc2160fd4afccf02f291d3c6**

268 Meeting probability

2015-08-25 05:41:10

by sinan

10 xp

Math

There is a grid of roads ($M \times N$). Two people, one at $A(0, 0)$ and the other at $B(N, M)$, start to walk from A to B and B to A respectively at the same time. They both walk the shortest path to their destination. For instance, if you come from B to A , you must always move from East to West (left) or North to South (down).

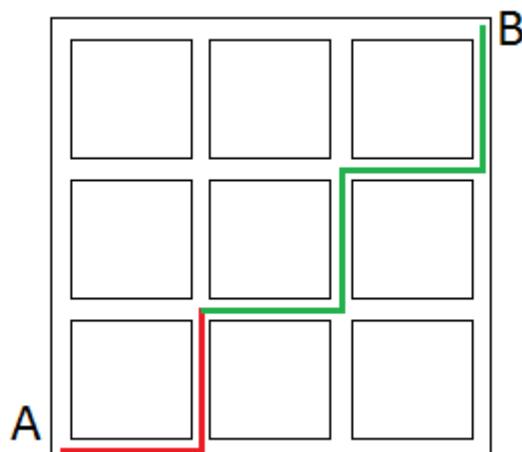
The man at B walks faster than the one at A by a factor of k ($v_B = k \cdot v_A$ where $k > 1$).

If they walk in any **possible** direction equally likely on a 19×23 grid, what is the probability that they meet each other on their way if k is equal to $4/3$ (1) or $6/5$ (2)?

Answer Format: $P_1/Q_1, P_2/Q_2$

Example: Answer=5/14, 5/28 for $k = 2$ and $k = 3/2$ on 3×3 grid.

The following shows a possible meeting for $k = 2$:



Answer 48377a799d0df282b4cb35a4652fc420

269 Counting the zeros**2015-09-11 11:05:04****by Philippe_57721****9 xp****Programming**

Consider the integers in range [111, 122] in binary form:

```
1 1 0 1 1 1 1  
1 1 1 0 0 0 0  
1 1 1 0 0 0 1  
1 1 1 0 0 1 0  
1 1 1 0 0 1 1  
1 1 1 0 1 0 0  
1 1 1 0 1 0 1  
1 1 1 0 1 1 0  
1 1 1 0 1 1 1  
1 1 1 1 0 0 0  
1 1 1 1 0 0 1
```

and count how many times the digit 0 appears. There are 28 zeros.

How many times does the digit 0 appear in the range [12345678910111213, 31211101987654321]?

[My timing: < 100 ms]

Answer **34fe1bd23852aa4af8268fb79f98fedd**

270 Vectors Sums

2015-11-01 06:06:58

by Philippe_57721

8 xp

Math

V is a vector $\in \{-1, 1\}^n$.

We define the following n sums:

$$S_j = \sum_{i=1}^j V_i - \sum_{i=j+1}^n V_i \quad \text{for } 1 \leq j \leq n$$

For example if $V = (-1, -1, 1, 1, 1)$ then $S = (-3, -5, -3, -1, 1)$.

We define then $T_j = 1$ if $S_j > 0$ and 0 otherwise. Thus $T = (0, 0, 0, 0, 1)$.

There are 2^n possible vectors V .

How many different vectors T are there?

Give your answer for $n = 50$.

Hint: there are 2728 possibilities for $n = 15$.

Answer 39c05f8b98eb919ef0f22c215b101f5b

271 Triangle Fair Split

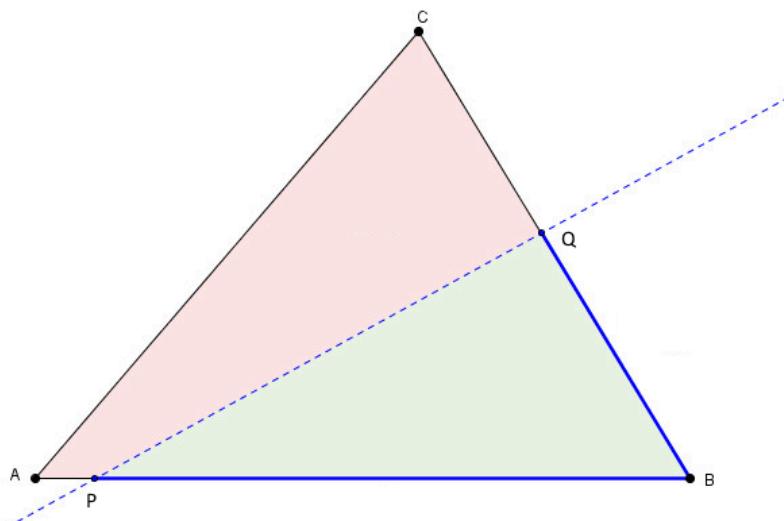
2015-09-19 03:41:24

by Philippe_57721

10 xp

Math

For a triangle, there is always at least one line that splits simultaneously the perimeter and the area in two equal parts.



You are given a triangle of sides 7182, 8182, 8459.

Find how many lines split fairly this triangle and give their length (line PQ on the picture) in ascending order (comma delimited, rounded to 5 digits after the decimal point).

Caveat: If 2 distinct lines have the same length, they will be counted as 2 solutions.

Answer 913cc08eceafb0bffc1c870d87df811d

272 Functional Equation

2015-10-10 12:21:37

by Philippe_57721

7 xp

Math

f is a function such as

- $f : [0, \infty[\rightarrow \mathbb{R}$
- f is strictly increasing
- $f(x) > -\frac{1}{x}$
- $f(x) \cdot f(f(x) + \frac{1}{x}) = 1$ for all $x > 0$

What is $f(1)$?

Give your answer rounded to 10 digits after the decimal point.

Answer 44f73401c571b0e00ca741f9b9e4d3cc

273 Diophantine Equation

2015-10-03 13:04:46

by Philippe_57721

8 xp

Math

Solve the diophantine equation (Euler)

$$2^{200} = 7 \cdot x^2 + y^2 \text{ with } x, y \text{ odd } > 0$$

Answer format: x, y

[My timing: < 100 ms]

Answer 00bfa1db27c71433b903fbbf8534684f

274 Beautiful permutations

2015-10-25 05:32:05

by Philippe_57721

15 xp

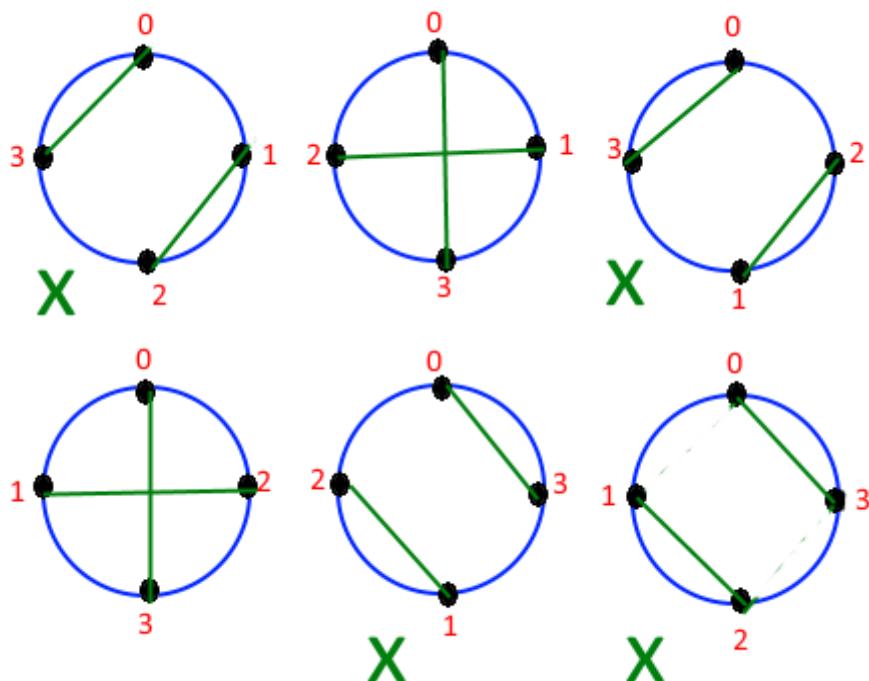
Math

n is an integer > 2 .

The $n + 1$ points of a permutation of $0, 1, \dots, n$ are placed on a circle.

The permutation is said beautiful if for any 4 numbers $0 \leq a, b, c, d \leq n$ with $a + c = b + d$, the chord joining a and c does not intersect the chord joining b and d .

For $n = 3$, there are 4 beautiful permutations:



How many beautiful permutations are there for $n = 1000$?

You are given:

$C = 6$ for $n = 4$

$C = 10$ for $n = 5$

[My timing: < 1 s]

Answer 037a6f5a4cfcbdf7d5a6d1c4e465a686

275 Counting the solutions

2015-10-17 19:15:36

by Philippe_57721

7 xp

Math

How many pairs of positive integers $(a, b) \leq 10^9$ are there such as

$$\frac{a^2}{2ab^2 - b^3 + 1}$$

is an integer.

[My timing: instant]

Answer

458e90eac17694d1b240d3d172512405

276 Merge**2015-10-22 17:05:23****by sinan****10 xp****Hack**

I'm sure you can make something out of this. **Pay attention to the title!**

The background image code for this problem is as follows:

```
background-image:url('
data:image/bmp;base64,
Qk1+AQAAAAAAAD4AAAAoAAAAIAAAAoAAAABAAEAAAAAEABA
AAAAAAAAAAAAA0wEAADsB
AABQSwMEFAABAAA0ZBWR9XAp3AwAAAAJAAAAEAAAypRj
qizuI0Jynv8UeR0BStDrYee20F7as2tqC
uitUVXHmECq1ycLTsyC6fu8IUP/gUEsDBBQAAQAAADmQV
kccxXIuQgAADYAAAABAAAAM1AP8NKjrG/W
ycRbRewGuIGx5gdK/hgWxaYtlMKKwy2kXEkS6Koqo/+hUb
EDWktWzzD/Y2nQKkQlYXb/7lZhPY/VBL
AwQUAAEAAA5kFZHktMJ32YAAAbAAAAQAAADUwmjixRkWj
MCRwKdPomcAjXe6rPet0Hdz+le67Mh+J
l1BPk/xFh005j+W2U49yPHeGhmY7aIofM4zfhcBn+1gRw
Ym3WX6KTnluvWuFDce9qChAxXKHgidEux+rh
mh08H/oul3ZFww9QSwECHgAFBAMCAQ==
')
```

Answer c9d8de3dadb95a84f050e2ab42a8817e

277 Lenstra's Numbers

2015-11-07 17:30:43

by Philippe_57721

8 xp

Math

We have $1233 = 12^2 + 33^2$.

In homage to Hendrik Lenstra who first noticed this property let's call such integers Lenstra's numbers.

Find the first 36-digits Lenstra's number i.e. n such as:

$$n = d_1d_2\cdots d_{36} = (d_1d_2\cdots d_{18})^2 + (d_{19}d_{20}\cdots d_{36})^2.$$

[My timing: < 1 sec]

Answer **643d4465aecff239163bb90a5cef181c**

278 "...THISISTHEEND"**2016-01-01 08:25:43****by sinan****15 xp****Math**

A random generator emits letters based on the English letter frequencies as given below: Out of 10000 letters, it would emit 1203 E's, 910 T's, 812 A's, 768 O's, 731 I's 695 N's, 628 S's 602 R's, 592 H's, 432 D's, 398 L's, 288 U's, 271 C's, 261 M's, 230 F's, 211 Y's, 209 W's, 203 G's, 182 P's, 149 B's, 111 V's, 69 K's, 17 X's, 11 Q's, 10 J's, and 7 Z's.

The generated letters are concatenated to a string starting from an empty string.

What is the probability that we would see a "...THISISTHEEND" before seeing any "...THIS...", "...IS...", "...THE..." or "...END..."?

Answer format: N/D
(irreducible fraction)

Check point:

Answer=1/245 to see a "...THEEND" before seeing any "...THE..." or "...END..." if it were to emit E,T,N,H,D with equal probabilities.

Answer**9778e6bd33c1fee59b921d0b97865210**

279 Equations system 1

2015-11-14 22:51:59

by Philippe_57721

5 xp

Math

Solve the system:

$$\frac{4x^2}{1+4x^2} = y$$

$$\frac{4y^2}{1+4y^2} = z$$

$$\frac{4z^2}{1+4z^2} = x$$

with $x, y, z \neq 0$.

Answer format: x, y, z

(in form p/q as they are rational numbers)

Answer 45a9579813ee6b5bffa8136876389297

280 Equations system 2**2015-11-14 22:52:34****by Philippe_57721****7 xp****Math**

Given n nonnegative real numbers $a_1 \leq a_2 \cdots \leq a_n$ we have:

$$\sum_{i=1}^n a_i = 96$$

$$\sum_{i=1}^n a_i^2 = 144$$

$$\sum_{i=1}^n a_i^3 = 216$$

Find n , a_1 and a_n .

Answer format: n, a_1, a_n

(in form p/q as a_i are rational numbers)

Answer**a75c59831a952477eb099b4103e4c3b0**

281 Big Numbers

2015-11-09 08:47:07

by sinan

8 xp

Crypto

```
2669036874315126775104877548238488463891833998332712460583098453881000,  
0484301744523614366479893134367535543909264822062412679066093167567155,  
1354048607368638427346804918325229520841941616290566812653826024545117,  
1350648907706272463683405891107264553969125237442323337687390942483673,  
2509362553029436312532246910309862404167219972001633567328214012282722
```

You need to make something out of these numbers.

Answer d223c9a68135220dfec7027f3ca53efa

282 Wilson quotient**2015-11-22 14:32:39****by Philippe_57721****9 xp****Programming**

Given a prime number p , we define the Wilson quotient as:

$$W_p = \frac{(p-1)!+1}{p}$$

and its residue modulo p as:

$$w_p = W_p \text{ modulo } p \text{ in the interval } -\frac{p}{2} \leq w_p < \frac{p}{2}.$$

Find $w_{200000039}$.

[My timing: 45 sec]

Answer**72e323a4d237ed4785a0ccd9f04ae874**

283 **Champions**

2016-01-29 08:55:31

by Philippe_57721

13 xp

Programming

Consider the sequence $S(n)$ defined by:

$$S(1) = 2$$

$$S(n) = S(n-1) + \text{GCD}(S(n-1), n - 1 + (-1)^n) \quad (n > 1)$$

The first values are:

2, 4, 5, 6, 9, 12, 13, 14, 21, 22, 23, 24, 25, 26, 39, 40, 45, 54, 55, 60, ...

We calculate the vector of first differences $D(n) = S(n+1) - S(n)$:

Then, we select in this vector each value greater than all the previous ones:

2, 3, 7, 13, 43, ...

Find the 32nd element of this sequence.

You are given:

- $M(1) = 2$
 - $M(2) = 3$
 - $M(10) = 2659$

[My timing: 7 min]

Some solvers found it under 1 min.

There is a fascinating conjecture about this sequence of champions: each champion is the larger prime of a twin pair of primes

Answer 59f2a193f7f8a5245452f76b6361e738

284 **Awesome algorithm****2015-11-28 17:12:59****by Philippe_57721****7 xp****Math**

Given a vector A of rational numbers, we define the following transformation:

$$B = \{1 \times (A_1 - A_2), 2 \times (A_2 - A_3), 3 \times (A_3 - A_4), \dots\}$$

We repeat the process:

$$C = \{1 \times (B_1 - B_2), 2 \times (B_2 - B_3), 3 \times (B_3 - B_4), \dots\}$$

And so on ...

Consider now the vector

$$Z = \{B_1, C_1, D_1, \dots\}$$

If $A = \{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$ find the first index such as the denominator of Z_i is $> 10^{50}$.

Answer format: index,denominator

[My timing: < 1 sec]

Answer**ec7cbf506c9cbcdee1aa599be064ebce**

285 Caesarian shift

2015-12-01 07:11:39

by sinan

8 xp

Crypto

You need to decode the following string:

yparjcfwnixigfwzt

Here's the pseudo code to get this encoded string:

```
function encode(string, n=29101923, offset=10^9)
begin
    len=length of string
    let array be an array with offset+len elements
    for i=1 to n
        begin
            array[i]=1
        end
    for i=n+1 to offset+len
        begin
            array[i]=sum of previous n elements
        end
    for i=1 to len
        begin
            encoded_string[i]=string[i] shifted by (array[offset+i] mod 26)
        end
    return encoded_string
end
```

Example:

encode("solution", 101, 10^5) returns "bvktehbs".

[My timing: < 1 m]

Answer 800f2ef92f2bb49ba2d5b2d19414a3e5

286 **Twelve faces of different colors****2016-02-08 07:33:02****by sinan****15 xp****Math**

Let there be N distinct colors.

Let $a_n = (a_{n-1} + a_{n-2}) \bmod 9$ with $a_1 = a_2 = 1$.

Consider the ways to paint all the faces of dodecahedron using these N distinct colors. **If selected, you can use up to $a_i + 2$ but no less than 2 of i -th color where $i = 1$ to N .**

What is the total number of ways to paint a dodecahedron if $N = 1000$?

Answer format: number

Example: 652706 if $N = 5$ or 130992447 if $N = 10$.

[My timing: 2 s]

Answer **31b1023f835b61bfe65763095b21bb0f**

287 Shifted digit

2015-12-15 11:31:32

by sinan

6 xp

Math

Consider the numbers in base-12 that can be written with the decimal digits only (0 to 9).

Of those, find **the smallest** number which, after a ROR 1 (RO*state to R*ight) operation, gives a number equal to 8 times the original number.

Example: 2497 becomes 7249 which is 3 times the original
 $(7249)_{12} / (2497)_{12} = 3$

Answer format: the number in base-12

See also: [Circular shift - Wikipedia, the free encyclopedia](#)

Answer

dee3651b30302aa78a762487b9064d02

288 Which one is the best, e or Pi ?

2016-01-11 09:29:54

by sinan

8 xp

Math

e and Pi are transcendental numbers meaning they cannot be the roots of an algebraic equation with rational coefficients.

So we look for some approximations such that the root of an quadratic equation is as close as possible.

Let the equation be given with the A, B, C triples: $A \cdot x^2 + B \cdot x + C = 0$ where A, |B| and |C| are integers $< 10^{10}$ and > 0 .

Find the (A_e, B_e, C_e) and (A_{Pi}, B_{Pi}, C_{Pi}) triples and corresponding roots (root_e and root_{Pi}), and calculate the following ratios:

$$R_e = |\text{e} - \text{root}_e|/\text{e}$$

$$R_{Pi} = |\text{Pi} - \text{root}_{Pi}|/\text{Pi}$$

What are the best triples for e and Pi?

Answer format: $A_e, B_e, C_e, A_{Pi}, B_{Pi}, C_{Pi}$
(the triples with the smallest ratio)

See also:

[Transcendental number](#) - Wikipedia, the free encyclopedia

[Integer relation algorithm](#) - Wikipedia, the free encyclopedia

Answer

cc866b17490238d00d82d60c2d493edd

289 A Diophantine Equation

2015-12-08 07:17:56

by sinan

12 xp

Math

$$x^2 + y^2 - k \cdot x \cdot y = 1 \quad (\text{1})$$

where k is an integer > 1 and $0 < x < y$.

Let $\text{pol}(n)$ be a monic polynomial of degree n :

$$k^n + a_{n-1} \cdot k^{n-1} + \dots + a_1 \cdot k + a_0$$

If the first solution to the above equation (1) is $(1, k)$ pair then the solution to n -th would be $(\text{pol}(n-1), \text{pol}(n))$.

What is the 100-th solution?

Answer format: $sum_1 + sum_2$

where $sum_1 = \sum a_i^3$ for $\text{pol}(99)$ and $sum_2 = \sum a_i^3$ for $\text{pol}(100)$.

Example: -17402

(875 - 18277) for 10-th solution.

[My timing: < 1 s]

See also:

[Monic polynomial](#) - Wikipedia, the free encyclopedia

[Diophantine equation](#) - Wikipedia, the free encyclopedia

Answer**2e71305a0d7bb9d59b0af232ab2e6864**

290 Concurrent lines

2015-12-11 15:01:33

by Philippe_57721

8 xp

Math

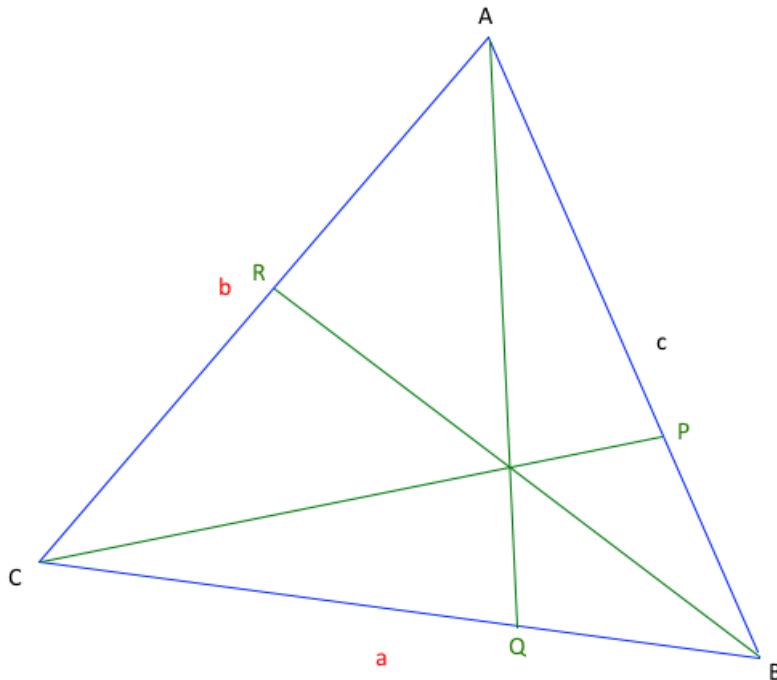
A triangle ABC has the following dimensions:

- $BC = a = 1033$
- $AC = b = 1553$
- $AB = c = 1973$

We choose 3 points:

- P on AB at a distance p of A ($0 < p < c$)
- Q on BC at a distance q of B ($0 < q < a$)
- R on AC at a distance r of C ($0 < r < b$)

(p, q, r are integers)



How many triples (p, q, r) are there such as the lines AQ , BR and CP are concurrent?

Answer format: Count, $\sum p$, $\sum q$, $\sum r$

You are given: 4,214,202,206 for the triangle 101,103,107.

[My timing: < 20 sec]

Answer 6d12fe7551e438131d7c41b7bbaf84a0

291 First sudoku grid

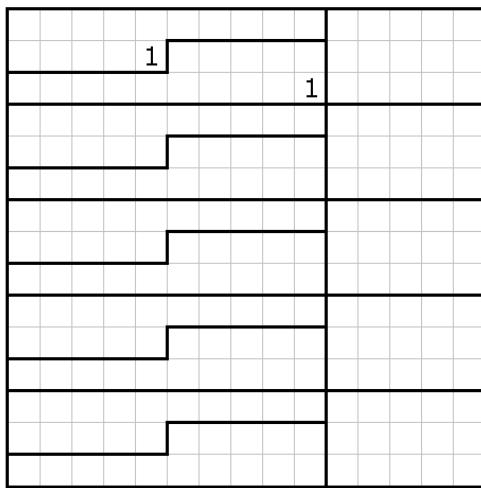
2015-12-22 10:19:11

by sinan

8 xp

Programming

Using lowercase hex digits 1 to f, fill the grid given in the picture in such a way to get the lowest possible hex number when you concatenate the lines starting from top to bottom.



Answer format: last 2 lines
(last 30 digits of the number)

[My timing: < 10 s]

Answer 19a410bc0a9b3c37ecc2cad39ae8d1a8

292 Bonus for p286

2015-12-15 13:49:09

by sinan

6 xp

Math

¹ Consider the following Diophantine equation: $x^2 + y^2 - k \cdot x \cdot y = 1$ where $k > 1$ and $0 \leq x < y$.If the first solution is $(0, 1)$, what is the k th solution for $k = 3^{4^5}$?Answer format: $x\%M, y\%M$ where $M = 1000000007$

[My timing: < 1 ms]

Answer

5a2c36c95d3fa650c0eac1568b532f17

¹ Actually [Problem 289](#).

293 Decode it (javascript)

2015-12-21 09:15:47

by sinan

8 xp

Crypto

X5dzkDFkg5BnNUEchPVkYb87f7X1QN0MDR6U1pzQLjCPHULNPT0KJRsRHZu
 iJd0eYTohf5kbLA/bcLmgeWmMGmomEpC

Enter the key to decode it.

Key = Decode it

Key=1

Not a key!

The JavaScript code for this problem is as follows:

```
function func()
{
    var keyStr = "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz0123456789+/";
    var A=parseInt(myform.myinput.value);
    var K=404;
    var KK=K*K;
    var AA=A*A;
    var encodedText = document.getElementById('encoded_text').innerHTML;
    var outputDiv = document.getElementById("outputDiv");
    outputDiv.innerHTML = "<pre>";
    if (A<=0) {
        alert("Should be >0");
    } else {
        outputDiv.innerHTML += "<font size=+2 ><b>Key="+A+"</b></font><br><br>";
        // K*A^2+K*x-A-x^2=0 : x^2-Kx+A-KA^2=0
        var delta=Math.sqrt(KK+4*(AA-1)*K);
        var x=Math.floor((K+delta)/2);
        for(var ii=-1; ii<2; ++ii){
            var t=x+ii;
            var tt=t*t;
            var z1=K*(AA+t), z2=(tt+A);
            if (z1==z2){
                var arr=new Array();
                var i=0, keylen=0, s=t, cd=0;
                while (s>0) {
                    arr[keylen++]=s%10;
                    s=Math.floor(s/10);
                }
                outputDiv.innerHTML += "Decoded: ";
                var j=keylen, k=0;
                for(i=0; i<encodedText.length; ++i) {
                    var inx=keyStr.indexOf(encodedText.charAt(i));

```

```
        if(inx<0) continue;
        if(--j<0)
            j+=keylen;
        inx-=arr[j];
        if (inx<0)
            inx+=64;
        outputDiv.innerHTML += keyStr.charAt(inx);
        if(i>0 && (i&63)==0)
            outputDiv.innerHTML += "<br>";
        cd+=k++*keyStr.charCodeAt(inx);
    }
    if (cd==360117)
        outputDiv.innerHTML += "<br><br><font color=\"green\"/>Decoded
successfully.</font/><br>";
    else
        outputDiv.innerHTML += "<br><br><font color=\"red\"/>Decoded
something but it doesn't seem to be OK.</font/><br>";
    break;
}
outputDiv.innerHTML += "<br>";
if (ii==2)
    outputDiv.innerHTML += "<br><font color=\"red\"/>Not a key!</font/>
<br>";
outputDiv.innerHTML += "</pre>";
}
```

Answer aee3adb3a618b282b6a43547e1e2441b

294 Ratio of triangles areas

2015-12-24 07:57:48

by Philippe_57721

9 xp

Math

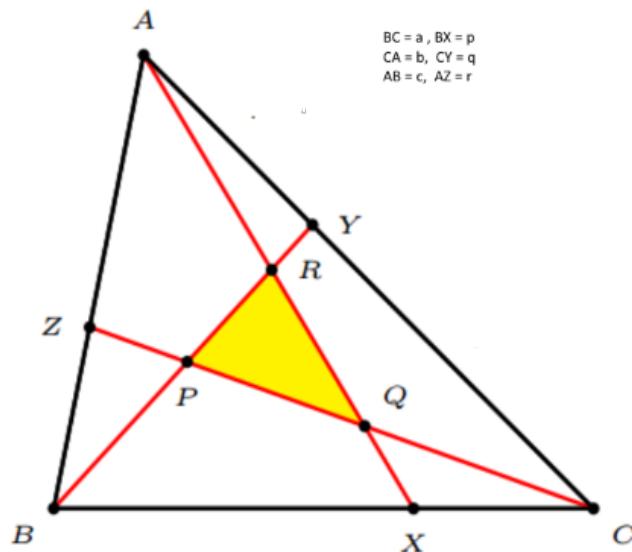
A Heronian triangle ABC has the following dimensions:

- $BC = a = 203$
- $CA = b = 265$
- $AB = c = 372$

We choose 3 points:

- point X on BC at a distance p of B
- point Y on AC at a distance q of A
- point Z on AB at a distance r of C

(p, q, r are integers)



For how many triples (p, q, r) the ratio $\text{Area}(ABC)/\text{Area}(PQR)$ is an integer?

Answer format: Count/p,q,r

for the smallest ratio

You are given: 6/2,3,2 for the triangle (3,4,5).

[My timing: 10 sec]

Answer f8c39b511d329baba9514342cbc3de17

295 The Hurwitz equation

2016-01-01 17:36:37

by Philippe_57721

8 xp

Math

The Hurwitz equation is the following diophantine equation. For fixed $n, k > 0$:

$$x_1^2 + x_2^2 + \cdots + x_n^2 = kx_1x_2\cdots x_n \quad \text{for } k, n \in \mathbb{N}^*$$

It can be proved that when this equation has solutions, then $k \leq n$.

Can you find all the k such as this equation has solution for $n = 61$?

Answer format: comma delimited list of k_s

You are given: 1, 4, 5, 13, 23 for $n = 23$.

[My timing: 90 sec]

PS: There are much faster solutions than mine ...

Answer 5415465a1e6996d5d66aea6041a390a0

296 **Farey polygons I**

2016-01-08 16:15:50

by Philippe_57721

7 xp

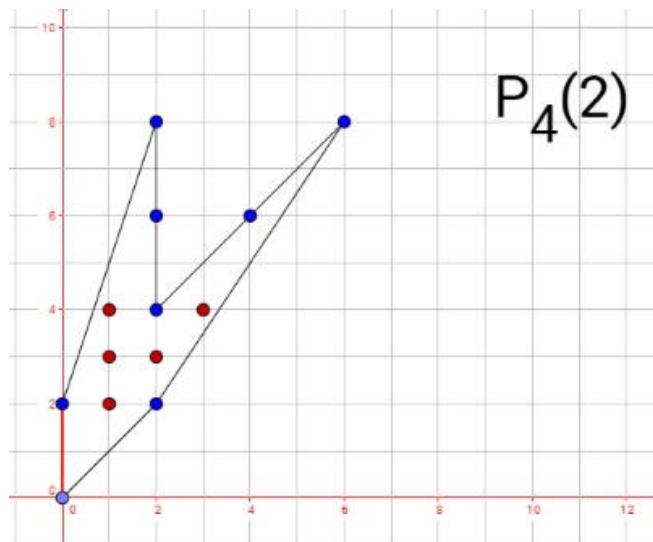
Programming

Consider the [Farey sequence](#) of order n .

For instance $F_4 = \left\{ \frac{0}{0}, \frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1} \right\}$.

We define $F_n(z)$ by applying a zoom z i.e. multiplying each term by z .

We consider each term of the sequence $F_n(z)$ as a point and join these points respecting the order. We obtain the Farey polygon $P_n(z)$ of order n and zoom z



$P_4(2)$ contains 5 points in its interior.

Find the first Farey polygon $P_n(2)$ whose inside contains (strict) more than 2000 lattice points.

Answer format: n, count

[My timing: 40 sec]

Answer **387a2f6e02eef5869aa07d3ac19dae8f**

297 **Farey polygons II**

2016-01-15 12:14:34

by Philippe_57721

6 xp

Math

What is the area of the Farey polygon (See [problem 296](#)) of order 10000000 and zoom 1?

You are given: 1522 for the Farey polygon of order 100.

[My timing: 40 sec]

Answer

57282e01a0afb8ce024e0777bb34b8d4

298 Decode it 2 (javascript)

2015-12-28 07:42:40

by sinan

8 xp

Crypto

```
4pxLgKsp+pVENasG+apWefzl9Ko/RbD7vGgUTbz7wm8UTb3lt5xNi0A3r5hS
N0c0ApxRd+w48FdHefLu`
```

Enter the key to decode it.

Key =

Key=12345678

Not a key!

The JavaScript code for this problem is as follows:

```
function func()
{
    var keyStr = "ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz0123456789+/";
    var Key=parseInt(myform.myinput.value);
    var encodedText = document.getElementById('encoded_text').innerHTML;
    var outputDiv = document.getElementById("outputDiv");
    outputDiv.innerHTML = "<pre>";
    if (Key<=0) {
        alert("Should be >0");
    } else {
        var arr=new Array();
        var i=0, keylen=0, s=Key, cd=0;
        while (s>0) {
            arr[keylen++]=s%10;
            s=Math.floor(s/10);
        }
        if(keylen!=8) {
            alert("key should be 8 digits long");
        } else {
            outputDiv.innerHTML += "<font size=+2 ><b>Key="+Key+"</b></font><br><br>";
            var inx=keylen;
            while (1) {
                for (i=s=0; i<keylen; ++i) s+=arr[i];
                if (--inx<0) inx+=keylen;
                arr[inx]=s;
                if (s==Key) {
                    outputDiv.innerHTML += "Decoded: ";
                    var j=keylen, k=0;
                    for(i=0; i<encodedText.length; ++i) {
                        var inx=keyStr.indexOf(encodedText.charAt(i));
                        if (inx<0) continue;
```

```
        if(--j<0)
            j+=keylen;
        inx+=arr[j]&63;
        if (inx>=64)
            inx-=64;
        outputDiv.innerHTML += keyStr.charAt(inx);
        if(i>0 && (i&63)==0)
            outputDiv.innerHTML += "<br>";
        cd+=++k*keyStr.charCodeAt(inx);
    }
    if (cd==281077)
        outputDiv.innerHTML += "<br><br><font color=\"green\">Decoded
successfully.</font><br>";
    else
        outputDiv.innerHTML += "<br><br><font color=\"red\">Decoded
something but it doesn't seem to be OK.</font><br>";
        break;
    } else if (s>Key) {
        outputDiv.innerHTML += "<br><br><font color=\"red\">Not a key!
</font><br>";
        break;
    }
}
outputDiv.innerHTML += "<br>";
}
outputDiv.innerHTML += "</pre>";
}
```

Answer c5a4591951353e248873e43f6ee020a6

299 a*b=c*d where a>b and c<d

2015-12-24 08:48:05

by sinan

10 xp

Sequence

1,3,6,1,6,21,82,9,81,...,?

Note: This is partly a stegano challenge. And think of the title as a hint.

Answer

e068e1c49217cecc3644bebc48900512

300 Squares dissection

2016-01-22 10:58:37

by Philippe_57721

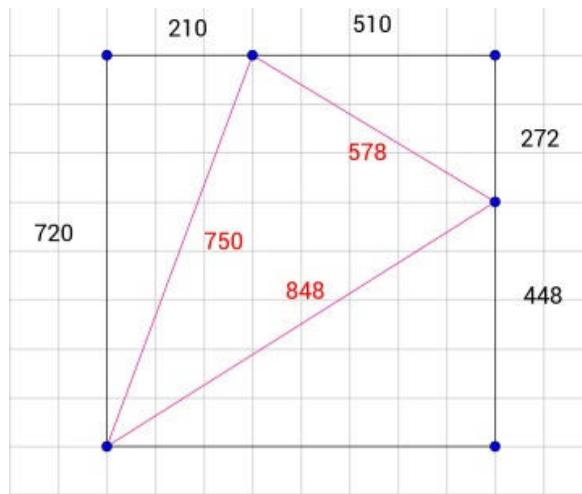
8 xp

Math

Consider the square with side = 720.

We can dissect it in 3 Pythagorean triangles:

- (720, 210, 750)
- (510, 272, 578)
- (448, 720, 848)



We represent this dissection as: $c_1, c_2, c_3 = 720, 210, 272$ ($c_2 \leq \frac{c_1}{2}$)

How many squares with integer sides ≤ 20000 can be dissected this way?

Answer format: Count, $\sum c_1, \sum c_2, \sum c_3$

You are given: 3, 2040, 1035, 569 for a threshold of 1000.

[My timing: 24 sec]

Answer

64917581018c373c22db086f1e1df652

301 Integer medians

2016-02-05 11:59:29

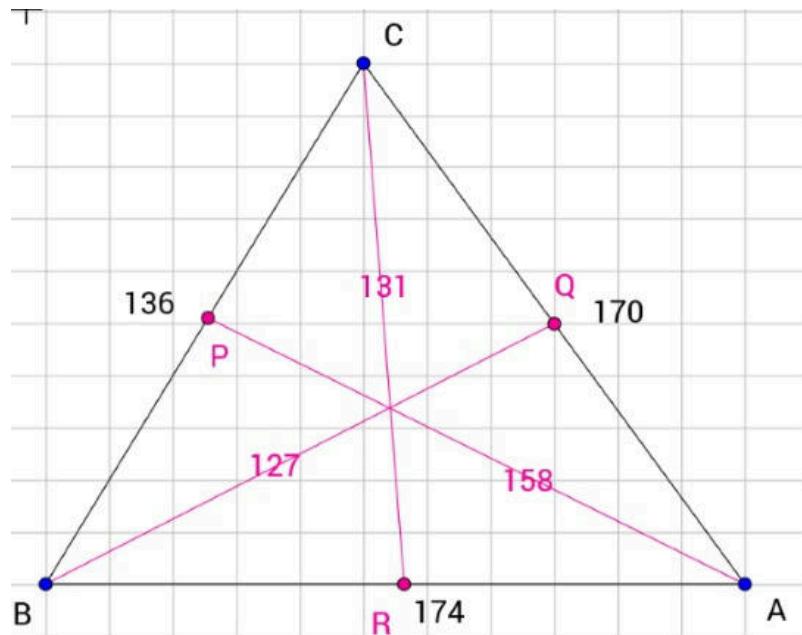
by Philippe_57721

7 xp

Programming

The following integer triangle with sides 136, 170, 174 has medians with integer length:

- $AP = 158$
- $BQ = 131$
- $CR = 127$



Find all the integer triangles with perimeter ≤ 10000 having integer medians. (We consider only primitive triangles: if (a, b, c) is a solution so is $(k \cdot a, k \cdot b, k \cdot c)$. We count only (a, b, c)).

Answer format: Count, $\sum a$, $\sum b$, $\sum c$
 (a, b, c) being the sides of the triangles)

You are given: 2, 390, 432, 490 for a threshold of 1000.

[My timing: 80 sec]

Answer 9c270d2df827a3f6bf5acb9d32682b1d

302 A 5-way recurrence

2015-12-30 07:20:29

by sinan

6 xp

Math

$$\begin{aligned}a_{n+1} &= 1 \cdot a_n - 1 \cdot b_n + 2 \cdot c_n - 3 \cdot d_n + 5 \cdot e_n - 8 \\b_{n+1} &= 1 \cdot b_n - 2 \cdot c_n + 3 \cdot d_n - 5 \cdot e_n + 8 \cdot a_n - 13 \\c_{n+1} &= 2 \cdot c_n - 3 \cdot d_n + 5 \cdot e_n - 8 \cdot a_n + 13 \cdot b_n - 21 \\d_{n+1} &= 3 \cdot d_n - 5 \cdot e_n + 8 \cdot a_n - 13 \cdot b_n + 21 \cdot c_n - 34 \\e_{n+1} &= 5 \cdot e_n - 8 \cdot a_n + 13 \cdot b_n - 21 \cdot c_n + 34 \cdot d_n - 55 \\a_0 &= b_0 = c_0 = d_0 = e_0 = 1\end{aligned}$$

What is n -th solution for $n = 3^{4^5}$?

Answer format: $a_n \% M, b_n \% M, c_n \% M, d_n \% M, e_n \% M$
where $M = 10000019$

Note: The answer should consist of positive numbers.

Answer 76fe6137c46092422a7a2d45b3116ede

303 The Pell-Fermat equation

2016-02-19 11:01:53

by Philippe_57721

5 xp

Math

It is well known that the Pell-Fermat equation $x^2 - n \cdot y^2 = 1$ (1) has always solutions when n is not square. But for the symmetrical equation $x^2 - n \cdot y^2 = -1$ (2), solutions are much rare.

For instance, if $n \leq 100$, this equation has solutions for only 20 values.

For how many $n \leq 5000000$, has the equation (2) solutions?

[My timing: 70 sec]

Answer ced88747c71b6fca232e90081938299b

304 Special Order I

2016-02-12 10:56:45

by Philippe_57721

6 xp

Programming

Consider the sequence of characters ABCD, each character being taken twice.

It is possible to order this sequence in such a way that:

- There is 1 character between 2 As
- There are 2 characters between 2 Bs
- There are 3 characters between 2 Cs
- There are 4 characters between 2 Ds

B CD BA C AD
 $\underbrace{2}_{\text{ }} \underbrace{3}_{\text{ }} \underbrace{1}_{\text{ }}$
BC $\underbrace{\text{DBA CAD}}_{\text{ 4 }}$
BCD $\underbrace{\text{BACA D}}_{\text{ }}$

Find the last such order in lexicographic order for the sequence ABCDEFGHIJKLMNOPQRSTUVWXYZ_ (each character is taken twice).

[My timing: < 1 sec]

Answer 1dcb1078cfa7e70a6cd2225bc7c4eba5

305 Sum of divisors

2016-03-25 13:58:45

by Philippe_57721

5 xp

Math

$\sigma(n)$ is the function sum of divisors of n .

Find the digital sum of $\sigma(100000!)$.

You are given: digital sum of $\sigma(1000!) = 11349$.

[My timing: 15 sec]

Answer

2ed9a6c9f2bbd455d7a0ae2dee0e0a11

306 **Golden Rod****2016-01-12 15:32:50****by elasolova****6 xp****Programming**

PerfectStone Co. is an excavation company that is specialized in mining gold nodes. An unusual thing happened as they excavated a golden rod. On top of that, this stunning rod is really a long one! Now, they want to cut the rod in pieces and sell them to companies interested.

The length of the rod is 1000-units and they have to cut the rod into pieces of integer-sized units and sell all of them. They have given you a [log](#) that has the prices listed for each associated integer-size (prices for 1-unit to 1000-units).

All they ask from you is to maximize the revenue and supply here the best revenue attainable.

Answer**f614214a11f8ffbd40bd0f4f79e1fb58**

307 Special Order II

2016-02-12 10:57:03

by Philippe_57721

10 xp

Programming

The definitions are the same as in [problem 304](#).

Find the **first** such order in lexicographic order for the sequence ABCDEFGHIJKLMNOPQRST.

[My timing: 60 sec]

Answer**d1d3b9b0cd179edb7f7b739c63c9be7b**

308 Cutting the pie

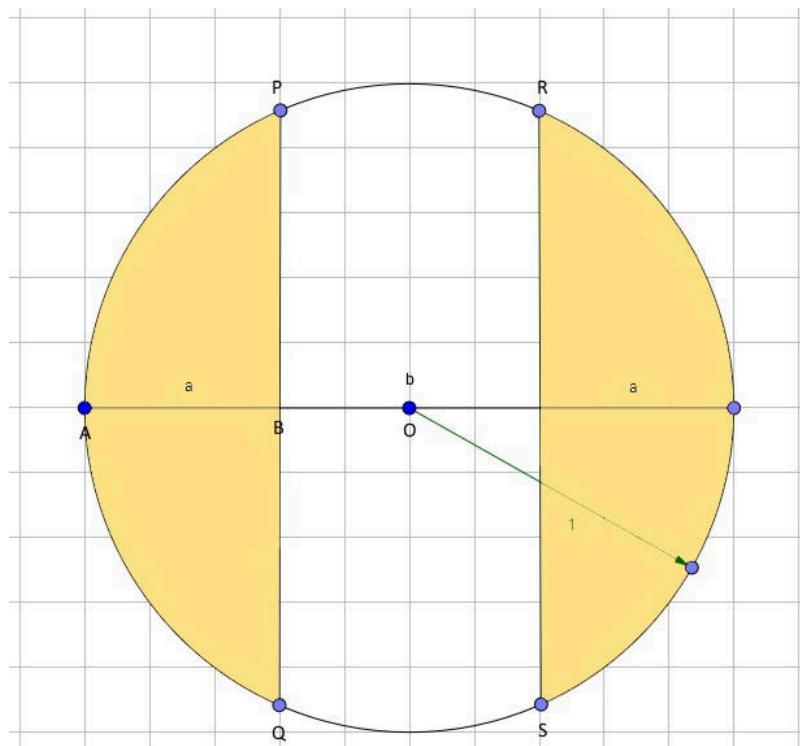
2016-02-26 11:10:54

by Philippe_57721

6 xp

Math

A circular pie of radius 1 unit is divided in 3 equal (same area) parts by 2 lines perpendicular to its diameter.



What is the length of segment AB ?

Give the result rounded to 10 places after the decimal point.

Answer 49ffe4f8f675fd4a096dbeccab392d70

309 Googolmania

2016-03-04 11:11:13

by Philippe_57721

8 xp

Math

Define $f(n)$ = the number of 0s before the last 1 in the binary representation of n . We have

$$100 = 1100100_2 \Rightarrow f(100) = 2$$

$$1000 = 111101000_2 \Rightarrow f(1000) = 1$$

$$10000 = 10011100010000_2 \Rightarrow f(10000) = 3$$

Let $\gamma = 10^{100}$ (A googol).

Find $n_1 = f(10^\gamma)$ and $n_2 = f(10^{8 \times \gamma})$.

Answer format: n_1, n_2

[My timing: instant]

Answer 7f9811515e64b910163871b31d398b83

310 Nim operations

2016-01-26 11:59:04

by sinan

10 xp

Math

In mathematics, the nimbers, also called Grundy numbers, are introduced in combinatorial game theory, where they are defined as the values of nim heaps. They arise in a much larger class of games because of the Sprague-Grundy theorem. The nimbers are the ordinal numbers endowed with a new nimmer addition and nimmer multiplication, which are distinct from ordinal addition and ordinal multiplication. Visit the following page for more info on nimmer addition and nimmer multiplication:

[Nimber - Wikipedia, the free encyclopedia](#)

We also define Nimber square root such that it returns a number which, when the nimmer multiplication applied to itself, returns the original number:

$s = \text{Nim_sqrt}(n)$ such that $n = \text{Nim_prod}(s, s)$.

Similarly Nimber inverse of a number is the one when the nimmer multiplication is applied, returns 1:
 $1 = \text{Nim_prod}(n, \text{Nim_inv}(n))$.

Evaluate the following using the above mentioned operations from $n = 0x100000$ (hex) to $0x110000$ (inclusive) and find the sum of the results.

$$\sqrt{\frac{n}{n + \sqrt{n}}}$$

Answer format: number

(in decimal)

[My timing: 6 s]

Answer ec8e8abb6f3a42615ebd135be47319f2

311 Nim equations

2016-01-26 11:59:24

by sinan

8 xp

Math

See [problem 310](#) on Nim operations.

Find the 2 roots of the following equations where Nim operations are used only:

$$1881x^2 + 1923x + 1938 = 0 \quad \text{where roots: } x_1 < 2^{13} < x_2$$

$$6273x^2 + 2935220086x + 2831770815 = 0 \quad \text{where roots: } x_3 < 2^{16} < x_4$$

$$6273x^2 + 12997147727796204837x + 11460394964710426743 = 0 \quad \text{where roots: } x_5 < 2^{24} < x_6$$

Answer format: $x_1, x_2, x_3, x_4, x_5, x_6$

[My timing: < 15 s]

Answer

c2241de598e8b366943fae029ac7e690

312 **Laser beam****2016-03-11 11:05:49****by Philippe_57721****6 xp****Math**

A rectangular box made of glass has the following dimensions: $1313 \times 1547 \times 1729$.

It is filled with transparent cubes of side 1.

A laser beam enters by the lower corner of the box $(0, 0, 0)$ and leaves by the upper corner $(1313, 1547, 1729)$.

How many cubes does the beam cross?

Answer**640848630ce4c259085346b1a263c1bd**

313 **2-Sphere Packing**

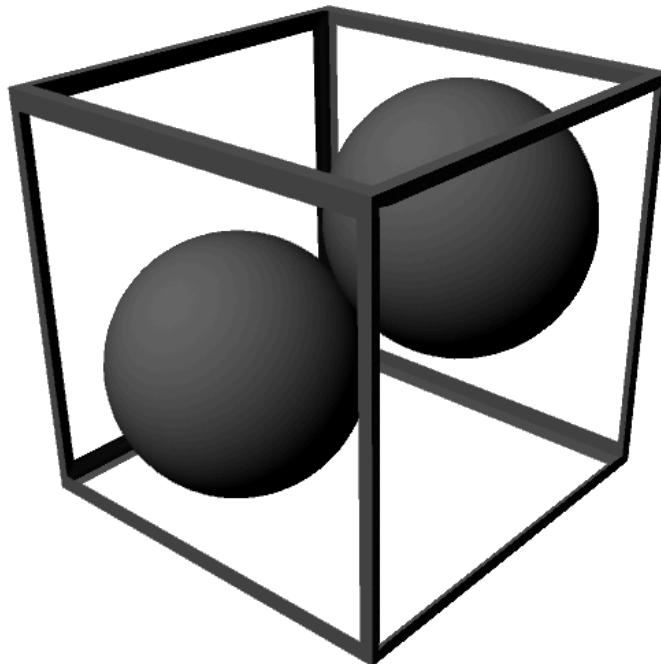
2016-03-28 09:15:41

by elasolova

6 xp

Math

Two equal sized spheres will be placed inside a unit-cube. What will be the maximum possible radius for spheres assuming that they are rigid bodies?



Round answer to 6 decimal places.

Note: Visualizing it will be helpful.

Answer **35352441e8c2ee92121e9a7feeeef697**

314 Circles distance II

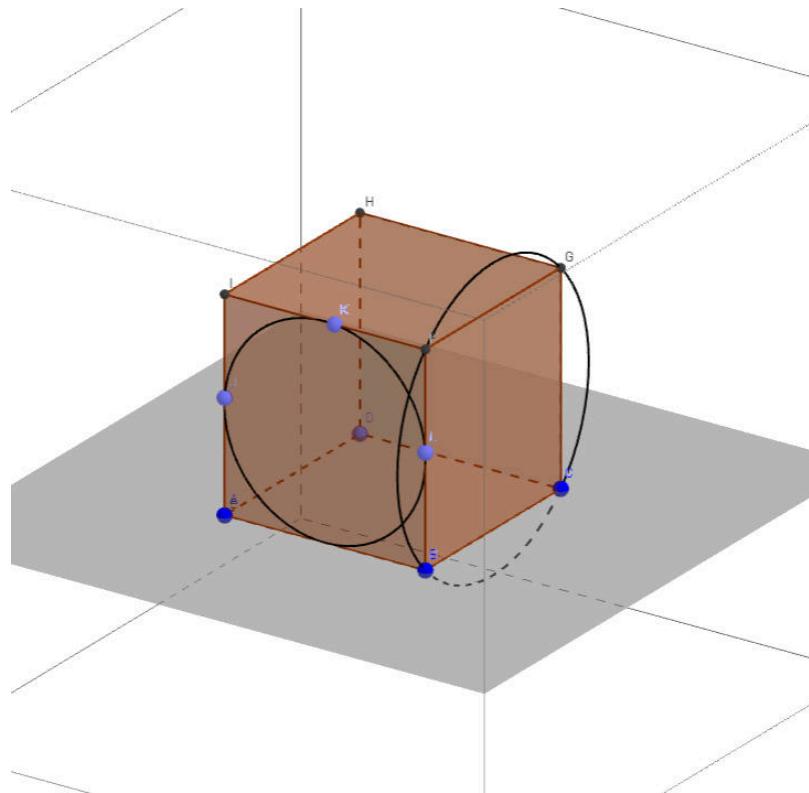
2016-06-03 10:04:54

by Philippe_57721

10 xp

Math

In a cube of side 1 unit, on one face we draw a circle inscribed in that face. On one adjacent face, we draw the circumscribed circle to that face.



Find the minimum distance between these 2 circles.

The answer is an algebraic number.

Answer format: the minimal polynomial for this algebraic number ..., a_3, a_2, a_1, a_0
where $P(x) = \dots + a_3 \cdot x^3 + a_2 \cdot x^2 + a_1 \cdot x + a_0$ is the minimal polynomial

Example: 1, -1, -1

if the answer would be the golden ratio ϕ .

Answer 056cd160ef8ccf1fc276b73127e92719

315 Heronian numbers

2016-03-18 11:27:29

by Philippe_57721

7 xp

Programming

We say that an integer n is an Heronian number if it's the area of some [Heronian triangle](#).

For instance, the following numbers are Heronian:

$$10080 = \Delta(1224, 1153, 73)$$

$$71064 = \Delta(925, 831, 188)$$

Find the first 2 Heronian numbers > 123456789 .

Answer format: n1,n2

[My timing: 29 sec]

Answer

2323680c5e2eb4f998dc3508aa6409be

316 Euler-constant like

2017-04-28 06:50:57

by Philippe_57721

13 xp

Math

What is $\sum_{0 < 1 + 4 \times k \leq n} \frac{1}{1 + 4 \times k} - \frac{\log(n)}{4}$ when $n \rightarrow \infty$

Give your answer with 30 digits after the decimal point.

[My timing: instant]

Hint: there is a closed form.

Answer 72f8b60e89e21975d78e56a457420ec5

317 Heronian triangles in a range

2016-03-18 11:48:45

by sinan

8 xp

Math

How many (not necessarily primitive) Heronian triangles with sides a , b and c , and area in the range [10000000, 10001000] are there?

Answer format: count, $\sum a_i$, $\sum b_i$, $\sum c_i$
where $a_i \leq b_i \leq c_i$

Example: 26, 203, 431, 520 in the range [0, 100].

[My timing: < 1 m]

Answer 888ada1102c30bba68ea56a2260014bd

318 Cut it

2016-04-26 08:50:08

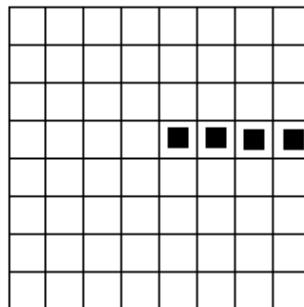
by sinan

10 xp

Programming

Cut the following square through the grid lines into 4 pieces in such a way that:

- each piece has one black box
- each piece is of similar shape by some rotation

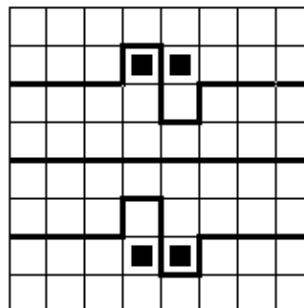


Each piece is to be identified as a bitmap. Bits are numbered from 63 to 0 starting from top line to bottom.

```
b63 b62 ... b56
b55 b54 ... b48
...
b7 b6 ... b0
```

After the cut, set the cells of a particular piece to 1 and others to 0 and get a 64-bit number as described above.

For example we would have the following numbers (in uppercase hex) for the following:



```
FFEF080000000000
0010F7FF00000000
00000000FFEF0800
000000000010F7FF
```

A cut like this is identified by **the smallest** number. So it would be **10F7FF** (leading zeroes are ignored) for the above example.

Answer format: N1,N2,...

(**the smallest** numbers for the appropriate cuts in uppercase hex and in ascending order)

Answer **9ebf2e554f39c3f4b516da14087a1485**

319 The longest needle

2016-09-27 07:25:25

by sinan

10 xp

Math

We define a needle-like primitive triangle as follows:

- It has 3 vertices on lattice points
- It has no other lattice points inside or on side

Find such a triangle with the maximum possible perimeter in $x^2 + y^2 = R^2$ circle for $R = 123456789$.

Answer format: $a^2 + b^2 + c^2$

where a, b, c are the sides of the triangle

Example: 136 if $R = 5$ for instance for the triangle with vertices $(-4, -2), (1, 1), (4, 3)$.

[My timing: < 40 s]

Answer 03b6d2d60d8e980b9be06c4bbabf1a95

320 Bonus for p315**2016-09-27 07:25:50****by sinan****6 xp****Math**

¹ We define a needle-like primitive **convex** quadrilateral as follows:

- It has 4 vertices on lattice points
- It has no other lattice points inside or onside

Find such a quadrilateral with the maximum possible perimeter in $x^2 + y^2 = R^2$ circle for $R = 10^8$?

Answer format: x_1, y_1, x_2, y_2

where $|x_i| \geq |y_i|$ and $x_1 < x_2$ (See the example below)

Example: 3, 2, 5, 3 if $R = 5$.

There are more than one solutions. Enter the two vectors that identify them. For example $A(-4, -3), B(-1, -1), C(4, 2), D(1, 0)$ is a solution then the two vectors are $AB = DC = [3, 2]$ and $BC = AD = [5, 3]$.

Similarly, for $A(-4, -2), B(-1, 0), C(4, 3), D(1, 1)$, $AB = DC = [3, 2]$ and $BC = AD = [5, 3]$.

[My timing: 30 s]

Answer**3a4d8c70768a2398928c973e3c97ab3e**

¹ Actually [Problem 319](#).

321 Minimum distance

2016-04-01 11:59:43

by Philippe_57721

12 xp

Math

If for the real a, b, c, d we have:

$$a^2 + 4b^2 = 4 \text{ and } c \cdot d = 4$$

what the minimum of $(a - c)^2 + (b - d)^2$?

Give your answer rounded to 20 digits after the decimal point.

Answer 27d631f41bc41931ebaa7306d4f909e2

322 Imaginary tower

2016-04-15 11:37:03

by Philippe_57721

5 xp

Math

if $i^2 = -1$, what is the limit

$$L = i^{i^{i^{i^{\dots}}}}$$

Answer format: a, b

where $L = a + bi$, a and b rounded to 20 digits after the decimal point.

Answer

30057adcc499ce74c39086419ca293dd

323 The biggest square

2016-04-02 06:57:39

by sinan

8 xp

Math

Consider the squares in $x^2 + y^2 = R^2$ circle with vertices on lattice points. The one with the biggest area would be $(R, 0), (0, R), (-R, 0), (0, -R)$.

Find the ones with the second biggest if $R = 87654321$.

Answer format: $\sum x_i, \sum y_i, \text{area}$

where $y_i > x_i > 0$ and are x, y coordinates of the one of the vertices.

Example: **2, 4, 40** if $R = 5$.

There are 2 squares with area = 40:

(**2, 4**), $(-4, 2), (-2, -4), (4, -2)$

(**4, 2**), $(-2, 4), (-4, -2), (2, -4)$

Answer**dc4dfbd4b2c3383f30404467570035bc**

324 Consecutive Zeros Möbius**2016-04-08 11:08:35****by Philippe_57721****8 xp****Math**

It can easily be seen that there cannot be more than 3 consecutive integers such as $\mu(n_i) = \pm 1$, $\mu(n)$ being the [Möbius function](#).

However, we can find arbitrary long sequences of consecutive integers with a Möbius value of 0.

Find 9 consecutive integers $n + 1, n + 2, \dots, n + 9$ such as $\mu(n + i) = 0$ (Each of them is divisible by a distinct prime square, the primes are the smallest possible).

Answer format: n

[My timing: 10 sec]

Answer**c31f1417d855e6e38dbc492f1fd83f87**

325 Homocentric circles

2016-04-16 08:01:54

by sinan

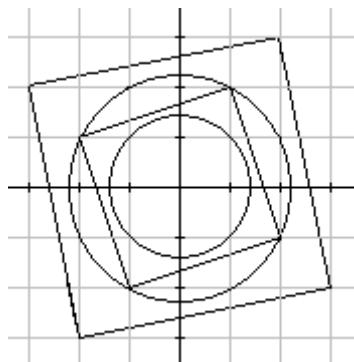
8 xp

Math

1. Set $R^2 = 2$.
2. Draw $x^2 + y^2 = R^2$ circle.
3. Next find the smallest square with vertices on lattice points such that it inscribes the circle but doesn't touch it.
4. Set $R^2 = a^2 + b^2$ where a, b are the coordinates of the one of any 4 vertices.
5. Go to step 2.

At the second iteration we would have $x^2 + y^2 = 5$ ($1^2 + 2^2 = 5$).

At the third iteration we would have $x^2 + y^2 = 13$ ($2^2 + 3^2 = 13$).



What would R^2 be equal to at the 50-th iteration?

Answer format: Number

Example: 13 for the third iteration.

[My timing: < 1 s]

Answer 16438933ae5e512b2fa7952099e964fe

326 Minimise it**2016-05-09 09:53:44****by sinan****8 xp****Math**

Let A, B, C be the three lattice points inside $x^2 + y^2 = R^2$ circle such that the ratio $(c - a)/(a + b + c)$, where $a = |BC|$, $b = |AC|$, $c = |AB|$ and $a \leq b \leq c$ is minimised.

Solve it for $R = 10^9$.

Answer format: a^2, b^2, c^2

where $\text{gcd}(a^2, b^2, c^2) = 1$

Example: 12544, 12545, 12545 for $R = 100$ with ratio = 1.328594679505981331113438939e - 5, $A(57, 6)$, $B(-40, -50)$, $C(-40, 62)$.

[My timing: 200 ms]

Answer **04c3f33684bab1aa1a7283bf735bab08**

327 Around the poles

2016-05-13 06:08:03

by Philippe_57721

9 xp

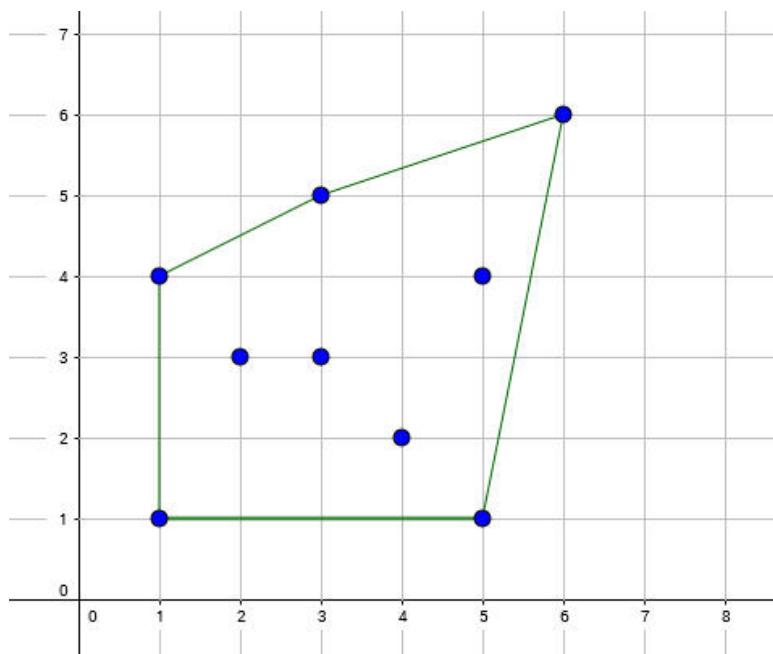
Programming

Poles are disposed on the lattice plane. A rubber band surrounds them.

For instance, consider poles placed at the following points:

- (1, 1)
- (1, 4)
- (2, 3)
- (3, 5)
- (3, 3)
- (4, 2)
- (5, 1)
- (5, 4)
- (6, 6)

If we place a rubber band around these points, we obtain a polygon of area 18.



Now consider the following 1000 points:

- $P_1 = (141, 592)$
- $P_2 = (653, 589)$
- $P_3 = (793, 238)$
- $P_4 = (462, 643)$
- ...
- $P_{999} = (845, 68)$
- $P_{1000} = (772, 460)$

The coordinates are made of the digits of the decimal part of PI by group of 3.

What is the area of the corresponding polygon?

You are given: 830303 for first 100 points.

[My timing: < 1 sec]

Answer

f1f7b7ae193f0b2414b9572a823e04e2

328 Lumberjacks**2016-04-22 11:13:57****by Philippe_57721****6 xp****Programming**

A group of lumberjacks are gathered in a camp.

Each day, in turn, one of the lumberjacks stays in the camp to cook, while the others go to work in 2 teams.

- The sum of the ages of the 1st team equals the sum of the ages of the 2nd team.
- The ages of the lumberjacks are all different and are prime numbers.

What is the smallest possible number of lumberjacks and their ages (First solution in lexicographic order).

Answer format: a₁, a₂, ..., a_n
in ascending order

[My timing: 2 sec]

Answer **a70ac8c92c88007c2c8363161ac81166**

329 Energy Potion

2016-06-18 12:40:50

by elasolova

6 xp

Math

A manufacturer has decided to produce an energy potion, to be named as Getafix. They are thinking about choosing a conical shaped can, for Getafix to be distinguishable from other products. Simply, they will cut the top of a regular cone.



So, they seek some advice on how to manufacture this product. Here is the list of requirements:

- They will create 1 liter beverages.
- The radius of bottom will be twice the top.
- Amount of material used for can will be as least as possible.

They want you to submit optimal height (h), bottom radius (rb), top radius (rt) all in centimeters and rounded to 3 decimal places.

Answer format: h, rb, rt

Answer a670dbed665ad042e72c1761b43ff48c

330 Graham Numbers

2016-04-29 11:44:07

by Philippe_57721

8 xp

Math

Let's define $a \uparrow [n] a = \underbrace{a \uparrow \uparrow \dots \uparrow a}_{n \text{ arrows}}$ where \uparrow represents the [Knuth's arrow notation](#).

We define:

- $g_0 = 4$
- $g_1 = 3 \uparrow [g_0] 3$
- ...
- $g_{64} = 3 \uparrow [g_{63}] 3$

g_{64} is the notorious Graham's number.

The last digits of this humongous number are known:

...03222348723967018485186439059104575627262464195387

We define $G(3) = g_{64}$ and the generalized Graham's number $G(p)$ where we replace all 3s with p .

What is the smallest prime number p such as $G(p)$ contains the string '1415' in its last 100 digits.

Answer format: p ,the last digits starting with '1415'

Example: 3,8485186439059104575627262464195387

if we ask for the smallest p such as $G(p)$ contains '8485'.

[My timing: 4 sec]

Answer 57019c7d4f097c5f4589d0abc44ef723

331 Minimal odds**2016-05-03 09:20:39****by sinan****8 xp****Math**

Let $x^2 - D \cdot y^2 = 4$ where D is not a square number and x, y are positive integers.

Find the minimal solution (the one with the smallest positive x, y pair) with **the biggest odd y value** for $D < 1000$.

Answer format: $x^2 - D \cdot y^2$

Example: $1523^2 - 61 \cdot 195^2$ for $D < 100$.

[My timing: 4 s]

Answer **a919f85d3dc01eaf89e23638cd200e93**

332 Heronian triples

2016-05-20 08:53:57

by Philippe_57721

7 xp

Programming

Consider the following triple of consecutive integers: 2701, 2702, 2703.

It's the sides of an [Heronian triangle](#).

$$\triangle(2701, 2702, 2703) = 3161340$$

Let's call it an Heronian triple.

Find the 100th Heronian triple.

Answer format: the 1st element of the triple

You are given: 524173 for the 10th element.

[My timing: tiny]

Answer**48a1f0ee05d35d1693397f67c44fd7a9**

333 Buddy numbers

2016-05-06 15:21:49

by Philippe_57721

8 xp

Programming

Consider the couple of integers $(m, n) = (14, 224)$.

We have:

- $14 = 2^1 \times 7^1$
- $224 = 2^5 \times 7^1$
- $14 + 1 = 3^1 \times 5^1$
- $224 + 1 = 3^2 \times 5^2$

n, m have the same prime factors $\{2, 7\}$ and so have $n + 1, m + 1$ $\{3, 5\}$.

Let's call these pairs buddy numbers.

If $(14, 224)$ is the 3rd pair of buddy numbers, what is the 30th?

[My timing: < 1 sec]

Note: Assume there exists only one pair of odd buddy numbers.

Answer 488907e4fdf16f31940b598d292faa80

334 Circles distance I

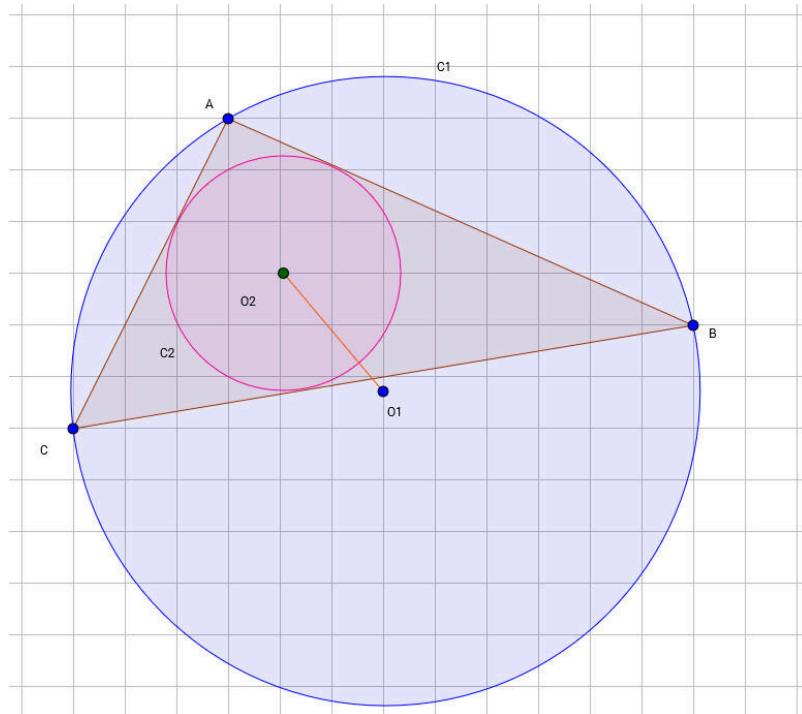
2016-05-27 10:44:44

by Philippe_57721

9 xp

Math

In an **isosceles** triangle ABC , we draw the circumscribed circle C_1 with center O_1 and the inscribed circle C_2 with centre O_2 .



If $AB = 520$, $AC = 520$ and $BC = 624$, the distance O_1O_2 is the integer 65.

How many **isosceles** integer triangles with perimeter $\leq 10^8$ are there such as the distance O_1O_2 is an integer.

Answer format: the count

Hint: you are given 469 for a perimeter of 10^4 .

[My timing: < 90 sec]

Answer ea5dd3c19c9882175334ca7d9dd0df0d

335 Subtracting proper divisors

2016-06-10 11:12:52

by Philippe_57721

5 xp

Programming

Given an integer n , we apply the following process:

- search the largest proper divisor of n
- subtract this number from n
- repeat until we reach 1

Let $f(n)$ the number of steps before reaching 1.

Example with $n = 30$

- $n = 30 - 15$ ($15 =$ Largest proper divisor of 30)
- $n = 15 - 5$ ($5 =$ Largest proper divisor of 15)
- $n = 10 - 5$ ($5 =$ Largest proper divisor of 10)
- $n = 5 - 1$ ($1 =$ Largest proper divisor of 5)
- $n = 4 - 2$ ($2 =$ Largest proper divisor of 4)
- $n = 2 - 1$ ($1 =$ Largest proper divisor of 2)
- $n = 1$ Stop $\Rightarrow f(30) = 6$

Find $f(1000000!)$ ($1000000!$ i.e. $\text{Factorial}(1000000)$).

[My timing: 6 sec]

Answer**5cb25de9fefdf16838ab7f23ca010bf4**

336 Sum

2016-05-17 05:50:40

by sinan

8 xp

Math

Let $F(N) = \sum N/\text{GCD}(N, i)$ for $i = 1$ to N .

Find $F(12345678!)$ mod 10000004400000259.

You are given $F(100!) \text{ mod } 1000003 = 571693$.

[My timing: < 1 s]

Answer

6968dde4c06b409cefc94f9609aab110

337 Diagonal Enumeration

2016-06-17 09:41:38

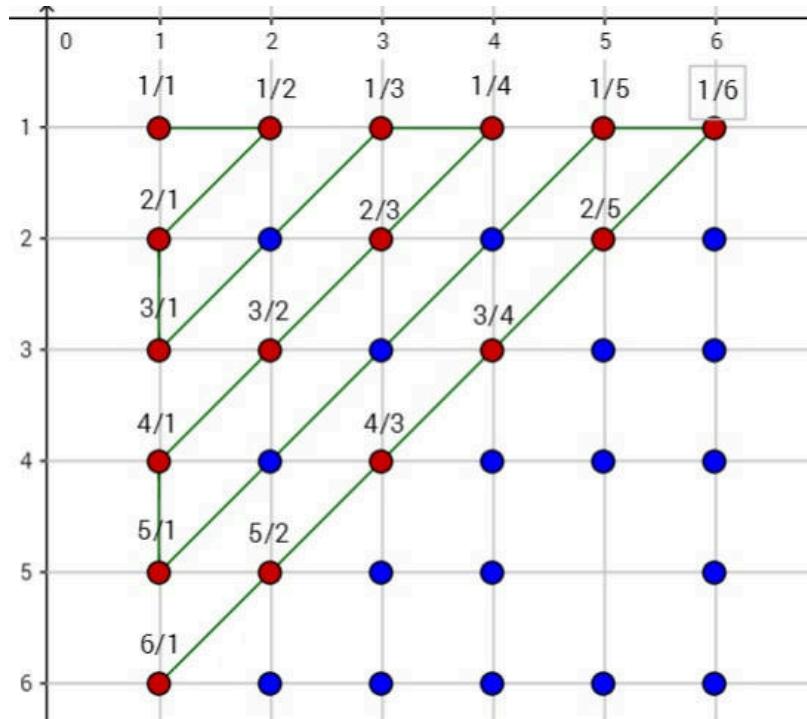
by Philippe_57721

10 xp

Programming

You certainly know of this nice proof of the equipotence between the set of integers and the set of rational numbers.

We consider an infinite grid numbered horizontally and vertically with integers:



We run through this grid according the above pattern and count each pair (i, j) where i and j are coprime.

$$1 - \frac{1}{1} = 1$$

$$2 - \frac{1}{2}$$

$$3 - \frac{2}{1} = 2$$

$$4 - \frac{3}{1} = 3$$

$$5 - \frac{1}{3}$$

$$6 - \frac{1}{4}$$

$$7 - \frac{2}{3}$$

$$8 - \frac{3}{2}$$

$$9 - \frac{4}{3}$$

$$10 - \frac{5}{1} = 5$$

$$11 - \frac{1}{5}$$

$$12 - \frac{1}{6}$$

$$13 - \frac{2}{5}$$

$$14 - \frac{3}{4}$$

$$15 - \frac{4}{1} = 4$$

$$16 - \frac{5}{2}$$

$$17 - \frac{6}{1} = 6$$

If we run through the upper diagonal matrix of a 6×6 grid, we get 17 rationals.

How many rationals can we count in the upper diagonal matrix of a $10^9 \times 10^9$ grid?

You are given: 303964542391 for a $10^6 \times 10^6$ grid.

[My timing: < 90 sec]

Answer

5269fa671014b44c63825572b41897b1

338 **Square divisors****2016-06-24 11:25:04****by Philippe_57721****6 xp****Programming**

Among the divisors of 400, exactly 6 are perfect squares:

- 1, 4, 16, 25, 100, 400

How many divisors of $10^9!$ are perfect squares?

Answer format: the **20** last digits of this huge number before the trailing 0s

You are given: 54158086206534975488 for 1000!.

[My timing: 20 sec]

Beware of leading zeros

Answer**d3dc6cab74b48e1b8b7d1f37831684e3**

339 Multiplicative Order

2016-07-01 13:43:34

by Philippe_57721

7 xp

Programming

The multiplicative order $o(g, p)$ of a number g modulo p , (g and p coprime), is the smallest integer k such as $g^k \equiv 1 \pmod{p}$.

For instance, $o(10, 73) = 8$.

For how many prime numbers $p < 4 \times 10^8$ the multiplicative order $o(10, p) < 100$?

Answer format: count, sum

Example: 55, 11573 for a limit of 1000.

[My timing: 70 sec]

Answer

49090310921f45159058feca94bbf282

340 All steps**2016-05-23 14:41:52****by sinan****10 xp****Math**

A number for which every pair of consecutive digits has a difference of one is called a step number. For example 234321 is a step number.

A pandigital number in a base B contains every digit from 0 to $B - 1$ at least once.

Let $F(B, ND)$ denote the number of pandigital step numbers in base B with **ND or less digits**.

You are given:

$$F(7, 100) \bmod 10^{20} = 1717291099033442979$$

$$F(10, 1000) \bmod 10^{20} = 45956531342004698861$$

What is $F(19, 1919^{19}) \bmod 10^{20}$?

[My timing: < 10 s]

Answer**83e729a4cc681269ca4118a25dd34d8a**

341 Secret operation

2017-03-06 05:37:16

by sinan

6 xp

Math

Find the digits in the following multiplication:

$$\begin{array}{r} * * * * * * * * * * \\ * * * \\ \hline - - - - - \\ * * * * * * * * * * \\ * * * * * * * * * * \\ * * * * * * * * * * \\ \hline - - - - - \\ * * * * * * * * * * * \end{array}$$

Each star represents a prime number. There is only one solution.

Answer format: 999999999999*999=9999999999999999

[My timing: < 1 m]

Answer 5294777382afdf0b31e4e85a352c2f8b

342 Alternate sums

2017-04-28 06:51:15

by Philippe_57721

4 xp

Math

How many different values can the expression $1 \pm 2 \pm 3 \pm \cdots \pm n$ take?

Give your answer for $n = 1000000$.

[My timing: instant]

Answer

e0cf6533cdae20632b6d6ca82135b2e7

343 Sum of digits equation**2016-07-08 14:05:26****by Philippe_57721****10 xp****Programming**

We define $DS(n) = n + \text{SOD}(n)$ where $\text{SOD}(n)$ is the sum of the digits of n (See [problem 263](#)).

It can be proved that $10^{13} + 1$ is the smallest integer such as the equation $DS(x) = n$ has 3 solutions:

$$10^{13} + 1 = \begin{cases} DS(99999999999892) \\ DS(9999999999901) \\ DS(100000000000000) \end{cases}$$

It can be proved too that $n = 10^{222222222224} + 10000000000002$ is the smallest integer with 6 solutions.

Obviously, if x is a solution we must have $x < n$. Thus, we can write a solution as $n - x_i$.

Find these solutions.

Answer format: $x_1, x_2, x_3, x_4, x_5, x_6$

x_i in ascending order

Example: 1, 100, 109 for $10^{13} + 1$.

[**My timing: < 1 sec**]

Thanks to **sinan** who helped me to improve this problem.

Answer **73ac859ae64bd8b46973058fdd90c03a**

344 **Unreachable numbers****2016-07-15 09:44:18****by Philippe_57721****7 xp****Programming**

Using four numbers 4 and the 4 operations $+, -, *, /$ we can build all numbers up to 9.

- $1 = ((4 + 4) - 4)/4$
- $2 = (4/(4 + 4)) * 4$
- $3 = ((4 + 4) + 4)/4$
- $4 = ((4 - 4) * 4) + 4$
- $5 = ((4 * 4) + 4)/4$
- $6 = ((4 + 4)/4) + 4$
- $7 = (4 - (4/4)) + 4$
- $8 = ((4 + 4) + 4) - 4$
- $9 = ((4/4) + 4) + 4$
- $12 = (4 - (4/4)) * 4$

$\Rightarrow 10$ is the first not reachable.

With four numbers 5

- $1 = ((5 + 5) - 5)/5$
- $2 = (5/5) + (5/5)$
- $3 = ((5 + 5) + 5)/5$
- $4 = ((5 * 5) - 5)/5$
- $5 = ((5 - 5) * 5) + 5$
- $6 = ((5 * 5) + 5)/5$
- $7 = ((5 + 5)/5) + 5$
- $9 = (5 - (5/5)) + 5$

$\Rightarrow 8$ is the first not reachable.

What are the first unreachable numbers when we use eight 4 and eight 5?

Answer format: u4, u5

Example: 10, 8

[My timing: 40 sec]

Answer **ad532c2f878b3a62cdecd541e93d304f**

345 Fraction decomposition**2016-08-05 14:50:59****by Philippe_57721****8 xp****Programming**

There is a smallest n for which there exists a set of distinct integers $S = \{s_1, s_2, \dots, s_n\}$ such as
 $\frac{17}{670} = \left(1 - \frac{1}{s_1}\right) \times \left(1 - \frac{1}{s_2}\right) \times \dots \times \left(1 - \frac{1}{s_n}\right)$

Actually, for this least n the set S is unique.

You are given that S is composed of the union of no more than 4 subsets of consecutive integers.

Find S .

Answer format: 'the smallest element'-'the largest element' comma separated for each subset

For instance, if $S = \{2, 3, 4, 5, 11, 12, 13\} = \{2, 3, 4, 5\} \cup \{11, 12, 13\}$, the answer would be:
2-5,11-13.

[My timing: 2 sec]

Answer 50039ee71010d762c7b498078bcb59c9

346 Consecutive divisors**2016-07-22 11:31:27****by Philippe_57721****4 xp****Programming**

Find the smallest integer with 720 divisors and the longest sequence of consecutive divisors.

For instance, for 24 divisors, the answer is $360 = \underbrace{1, 2, 3, 4, 5, 6, 8, \dots}$

Answer format: number,first divisor of the sequence,last divisor of the sequence

Example: 360,1,6

[My timing: 8 sec]

Answer **3d009e2d61a747c3ff532995bc1d6d62**

347 Nonaic Huffman Coding

2016-07-26 11:30:33

by elasolova

8 xp

Programming

Huffman Coding is a lossless data compression method that is based on the entropy of bit patterns in the data.

Usually for strings, conventional 8-bit (0-255) hashing is used for frequencies preceding actual coding. So, these 8-bit chunks directly correspond to well-known characters (such as a, b, c, etc., more specifically ASCII encoding). So, for this problem assume ASCII encoding for characters.

This problem asks to use Huffman coding scheme on this [text file](#), but using 9-bit chunks for frequencies and then actual coding. Also, for this problem you do not need to account for additional information to ensure decompression.

File is currently 68337 bytes. Submit the compressed file size below in bytes (where file **does not** include any information for decompression).

Answer 98778d65a66805d892f4ab4543b439a7

348 Heegner decompositions

2016-12-02 07:59:25

by Philippe_57721

10 xp

Math

Consider the field \mathbb{F}_d of numbers $a + b \times \sqrt{-d}$ where a, b, d are integers and d is a squarefree positive number.

For some d , it can be proved that the factorization of the elements of \mathbb{F}_d is unique.

The surprising fact about these special values is that there are only 9 of them:

$\{1, 2, 3, 7, 11, 19, 43, 67, 163\}$

They are called the [Heegner numbers](#).

Can you find the smallest prime number $p > 10^{50}$ such as p can be written $p = x^2 + h \cdot y^2$ for each Heegner number h .

Answer format: $p \bmod 10^{50}$, $\sum x$, $\sum y$

Example: 2372241, 606424, 233349 for $p > 10^{10}$

$$\begin{aligned} 10002372241 &= 23220^2 + 1 \times 97279^2 \\ 10002372241 &= 90979^2 + 2 \times 29370^2 \\ 10002372241 &= 79729^2 + 3 \times 34860^2 \\ 10002372241 &= 95703^2 + 7 \times 10976^2 \\ 10002372241 &= 82865^2 + 11 \times 16884^2 \\ 10002372241 &= 52279^2 + 19 \times 19560^2 \\ 10002372241 &= 90313^2 + 43 \times 6552^2 \\ 10002372241 &= 44273^2 + 67 \times 10956^2 \\ 10002372241 &= 47063^2 + 163 \times 6912^2 \end{aligned}$$

For the equation $x^2 + y^2 = p$, we assume $x < y$.

[My timing: < 15 sec]

Answer **b532d4918f05b9a8ee61b629a5a1b24c**

349 Binary Quadratic Representation

2016-10-28 07:31:27

by Philippe_57721

8 xp

Programming

It can be proved that a prime number of form $5n + 1$ has a unique representation by the binary quadratic form $x^2 + 3xy + y^2$.

For instance $10000000711 = 14365^2 + 3 \times 14365 \times 79734 + 79734^2$.

Find this representation for the first 100 prime numbers $\equiv 1$ modulo 5 and $> 10^{50}$.

Answer format: $\sum x, \sum y$

where $x < y$

[My timing: 7 sec]

Answer**e43a7311d68c82e778e10bd42231414c**

350 Squarefree factorisations

2016-08-12 14:56:52

by Philippe_57721

6 xp

Programming

Consider the number 420.

It can be decomposed in product of 2 squarefree factors in 4 different ways:

2×210

6×70

10×42

14×30

Let $d(n)$ = number of pairs (x, y) such as x, y squarefree, $x \cdot y = n$ and $x \leq y$.

Find $\sum_{k=1}^{20000000} d(k)$.

[My timing: 45 sec]

Answer**978f7f539e8c8fee3019581d1d47340d**

351 Multiplicative persistence

2016-08-26 11:37:38

by Philippe_57721

9 xp

Programming

Given a positive integer, we multiply all its digits; we obtain a new number. We repeat the process until a fixed point.

Example:

Starting with $n = 277777788888899$, we have the following sequence:

$277777788888899 \Rightarrow 4996238671872 \Rightarrow 438939648 \Rightarrow 4478976 \Rightarrow 338688 \Rightarrow 27648 \Rightarrow 2688 \Rightarrow 768 \Rightarrow 336 \Rightarrow 54 \Rightarrow 20 \Rightarrow 0$

In this case, we have a chain of length 11.

This value is called the multiplicative persistence of n

$$P(277777788888899) = 11$$

277777788888899 is actually the smallest integer with a multiplicative persistence of 11.

There are 5 15-digits numbers with a multiplicative persistence of 11:

277777788888899

367777778888889

447777778888899

466777777888889

666677777788888

We only consider numbers in their canonical representation: digits in ascending order (as any permutation of a given number has the same multiplicative persistence as the original).

It's an open problem to prove or disprove that no integer has a multiplicative persistence greater than 11. This conjecture has been checked until 10^{500} .

How many 25-digits numbers with a multiplicative persistence of 11 are there?

Answer format: count,sum

Example: 5,2226788902344464 for 15-digits numbers.

[My timing: 60 sec]

Answer 2621aafb74aad71f8c8ac281cced4e9c

352 Periodic Kaprekar Numbers**2016-10-07 05:57:51****by Philippe_57721****8 xp****Math**

A n -digits [Kaprekar number](#), in base 10, is an integer such as its square can be split in 2 parts that add up to this number.

For instance:

$$324324324324^2 = \underbrace{105186267348}_{\text{and } 105186267348} \underbrace{219138056976}_{= 324324324324}$$

We say that a string S is periodic if there is a substring T of S such as $S = T \cdots T$ where the number of repetitions of T is ≥ 2 .

How many 68-digits periodic Kaprekar numbers are there?

Answer format: count,sum

Example: 8,41153193346

for 10-digits Kaprekar numbers (1111111111, 2020202020, 3888938889, 4132841328, 5000050000, 5243952439, 9756097560, 9999999999)

[My timing: < 1 sec]

Answer**f6b33827081dd1bc6ef7201d537ea97a**

353 Two ways decomposition**2017-01-13 08:39:56****by Philippe_57721****10 xp****Programming**

Euler found that $635318657 = 59^4 + 158^4 = 133^4 + 134^4$.

And it was proved in 2004 that it's actually the smallest integer that can be expressed in two different ways as a sum of two 4th powers.

What is the smallest integer that can be written in 2 different ways as a sum of a 4th and a 5th power?
 $n = a_1^4 + b_1^5 = a_2^4 + b_2^5$ where a_i and b_i are not perfect powers.

Answer format: n, a_1, b_1, a_2, b_2
with $a_1 < a_2$.

[My timing: 45 sec]

Answer**73b74b5b38741289f774f3dab908a86a**

354 In or out?

2016-09-09 06:53:30

by Philippe_57721

6 xp

Programming

Let the following pseudo-random generator:

```
static long seed = 641;
static long FastRandom()
{
    seed = (214013 * seed + 2531011) % 2147483647;
    return seed;
}
```

We consider the first 60000 values of this generator modulo 1000 as the coordinates of 10000 triangles:

$$T_1 = (344, 302)(678, 941)(488, 674)$$

$$T_2 = (107, 545)(964, 568)(808, 239)$$

...

$$T_{10000} = (862, 679)(571, 263)(391, 426)$$

How many times is the point (499, 499) inside one of these triangles?

[My timing: < 1 sec]

Answer d388d6ce9a2b53aa22a0c7dc0cdc78af

355 **Closed form****2016-09-30 09:58:50****by Philippe_57721****5 xp****Math**

Let

$$\xi = \sqrt[16]{4870847 - \frac{1}{4870847 - \frac{1}{4870847 - \frac{1}{4870847 - \dots}}}}$$

 ξ can be written $\frac{a+\sqrt{b}}{c}$ for some coprime integers a, b, c .

Can you find them?

Answer format: a, b, c

Answer**2a3d4616e789219cf2ccc082f212982**

356 Gaussian factorization

2017-01-20 11:06:39

by Philippe_57721

8 xp

Math

Consider the Gaussian integer $\gamma = 14159 + 71828i$.

$$\gamma = (0 + i) \times (1 + 2i) \times (2 + 3i) \times (5 - 8i) \times (283 - 920i)$$

Each factor is a Gaussian prime number.

Can you find the Gaussian factorization of

$$1415926535897932384626433 + 7182818284590452353602874i$$

Answer format: $a_1, b_1/a_2, b_2/\dots/a_n, b_n$

where $a_i \geq 0$ and a_i in ascending order. To normalize the factors, we must have $|a_i| < |b_i|$.

Example: $0, 1/1, 2/2, 3/5, -8/283, -920$ for γ .

[My timing: instant]

Answer

66f7818856bc8695f0840a6febfb6a7c1

357 **FLT near miss****2016-11-04 12:41:24****by Philippe_57721****7 xp****Programming**

Since Euler, we know that Diophantine equation $x^3 + y^3 = z^3$ has no solution.

However, Ramanujan found an infinite set of solutions for equation $x^3 + y^3 = z^3 \pm 1$.

Here is his stunning solution. Let:

$$X_1 = [9, 10, 12]$$

$$X_2 = [791, 812, 1010]$$

$$X_3 = [65601, 67402, 83802]$$

And define $X_{n+3} = 82 \cdot X_{n+2} + 82 \cdot X_{n+1} - X_n$.

Let $x_n = X_n[1], y_n = X_n[2], z_n = X_n[3]$.

We have $x_n^3 + y_n^3 = z_n^3 + (-1)^{n+1}$.

For instance, $X_4 = [5444135, 5593538, 6954572]$ and indeed $5444135^3 + 5593538^3 = 6954572^3 - 1$.

Find solution for index 123456789101112.

Format answer: x, y, z modulo 10^{15}

[My timing: instant]

Answer**d4dbb0f38f90fb36af1d6b5e4082b751**

358 Contest**2016-08-30 13:47:14****by sinan****10 xp****Programming**

10000 people attend a programming contest. The contest has 4 problems. The contestants who pass at least two out of four problems will be awarded.

Here are the statistics:

Problem 1: 5101 people pass

Problem 2: 4499 people pass

Problem 3: 1312 people pass

Problem 4: 148 people pass

Participation fee is 20. Those who pass two problems out of four are awarded with 30, those who pass three with 50 and those who pass all four with 100. The cost of organising this contest is 1000.

What is the minimum (X) and maximum (Y) profit that may be earned from this contest?

Answer format: X,Y

Thanks to **Philippe** for his helps in developing this problem.

Answer **33e8c0eb9fe8f3dfa9f6062a52b161a6**

359 Smallest possible

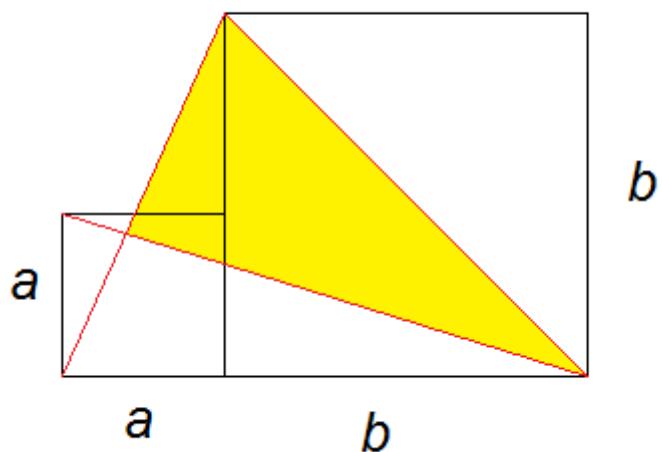
2016-11-13 06:27:19

by sinan

6 xp

Math

There are two squares with sides equal to a and b as seen below. The sides a , b and the area of the painted triangle are equal to integers. What is the smallest possible area if $\sqrt{23} < b/a < \sqrt{24}$?



Answer format: a, b, area

[My timing: 5 s]

Answer 4acf2dec6a4bf614c57e6e8db7713fe0

360 Basic crypto

2016-08-18 08:41:09

by sinan

5 xp

Crypto

A <=> QQAA
Z <=> WgAA

Can you decode this?

VGgA ZWEA bnMA d2UA cmkA c2IA YXMA ZTYA NAAA

Answer **7466d1e13223c8904ca430f943b8e1c7**

361 Polynomials**2016-08-22 12:30:15****by sinan****5 xp****Math**

Let $f(x) = x^3 + a \cdot x^2 + b \cdot x + 18811938$ where a and b are integers.

If a, b can be selected in such a way that $f(x) = 0$ has only integer roots, what is the sum of $f(-1)$ for all the possibilities (i.e. all the distinct polynomials that meet the conditions).

Answer format: sum of $f_{a,b}(-1)$ for a and b that makes $f_{a,b}(x) = 0$

Answer **1d24022d9ddc5b359fd38a7273c4e468**

362 Equilateral triangle

2016-09-30 09:59:10

by Philippe_57721

6 xp

Math

An equilateral triangle contains a point in its interior whose distances to the vertices are 3, 4, 5.

What is the side of the triangle?

Give the answer rounded to 10 digits after the decimal part.

Answer

3052b9a24af87711251768af74ecad88

363 Best divisor**2016-10-14 07:53:53****by Philippe_57721****7 xp****Programming**

$20!$ possesses 41040 divisors.

Its largest divisor $< 10^{10}$ is 9999498600.

Find the largest divisor of $75!$ which is $< 10^{30}$.

[My timing: < 40 sec]

Answer**c2262bf2b73ee33d0bc1033d1e5fb59e**

364 Bonus for RC360**2016-10-14 07:54:08****by Philippe_57721****6 xp****Programming**

Find integers n_1, n_2 such as:

- $n_1 \cdot n_2 = 75!$
- $n_1 < n_2$
- $n_2 - n_1$ is the smallest possible

Answer format: $n_2 - n_1$

[My timing: < 50 sec]

Answer**5df35e6682645651e89106b57a7908ec**

365 Egyptian equation I

2016-09-23 07:57:22

by Philippe_57721

7 xp

Programming

The equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$ with $0 < x \leq y$ has 14 solutions for $n = 12345$:

- $\frac{1}{12346} + \frac{1}{152411370} = \frac{1}{12345}$
- $\frac{1}{12348} + \frac{1}{50812020} = \frac{1}{12345}$
- $\frac{1}{12350} + \frac{1}{30492150} = \frac{1}{12345}$
- $\frac{1}{12354} + \frac{1}{16945570} = \frac{1}{12345}$
- $\frac{1}{12360} + \frac{1}{10172280} = \frac{1}{12345}$
- $\frac{1}{12370} + \frac{1}{6108306} = \frac{1}{12345}$
- $\frac{1}{12390} + \frac{1}{3398990} = \frac{1}{12345}$
- $\frac{1}{12420} + \frac{1}{2044332} = \frac{1}{12345}$
- $\frac{1}{12570} + \frac{1}{689674} = \frac{1}{12345}$
- $\frac{1}{13168} + \frac{1}{197520} = \frac{1}{12345}$
- $\frac{1}{14814} + \frac{1}{74070} = \frac{1}{12345}$
- $\frac{1}{16460} + \frac{1}{49380} = \frac{1}{12345}$
- $\frac{1}{19752} + \frac{1}{32920} = \frac{1}{12345}$
- $\frac{1}{24690} + \frac{1}{24690} = \frac{1}{12345}$

How many solutions are there for $n = 2091129587$?

Answer format: count, $\sum x, \sum y$

Example: 14, 200392, 273453272 for $n = 12345$.

[My timing: 12 sec]

Answer

3638d5bd96849d30e1fd2a55d3b9112e

366 Cyclic equation

2016-12-24 08:44:32

by Philippe_57721

8 xp

Programming

The equation $x \cdot y + y \cdot z + z \cdot x = n$ with $0 < x \leq y \leq z$ has 11 solutions for $n = 1234$:

- $1 \times 4 + 4 \times 246 + 246 \times 1 = 1234$
- $1 \times 12 + 12 \times 94 + 94 \times 1 = 1234$
- $1 \times 18 + 18 \times 64 + 64 \times 1 = 1234$
- $3 \times 8 + 8 \times 110 + 110 \times 3 = 1234$
- $4 \times 6 + 6 \times 121 + 121 \times 4 = 1234$
- $4 \times 21 + 21 \times 46 + 46 \times 4 = 1234$
- $8 \times 14 + 14 \times 51 + 51 \times 8 = 1234$
- $10 \times 13 + 13 \times 48 + 48 \times 10 = 1234$
- $10 \times 19 + 19 \times 36 + 36 \times 10 = 1234$
- $12 \times 14 + 14 \times 41 + 41 \times 12 = 1234$
- $18 \times 20 + 20 \times 23 + 23 \times 18 = 1234$

How many solutions are there for $n = 3000000$?

Answer format: count, $\sum x, \sum y, \sum z$

Example: 11, 72, 149, 880 for $n = 1234$.

[My timing: 65 sec]

Answer ac51e8fc0956f1cc984b85b6271ce63e

367 Egyptian equation II

2016-09-23 07:57:40

by Philippe_57721

7 xp

Programming

Let the diophantine equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$ with $x \leq y$.

How many solutions are there for $n = 1000000!$?

As the answer is a BIG number, use the following condensed representation:
(First 10 digits)[(number of remaining digits)](Last 10 digits)

Example: 8583170787[2]5400804688 for $n = 100!$.

[My timing: < 1 sec]

Answer**a5f4a44a25b41c7ab405898ce468791e**

368 **Fractions****2016-09-13 07:40:27****by sinan****8 xp****Math**

Let p_i be the i -th prime.

Let $P(n)$ be the canonical product of the first n primes this way:

$$\prod p_i^i \text{ for } i = 1 \text{ to } n.$$

Let $\#f(n)$ be the number of **unique** reduced fractions (of the form $p/q > 1$) that can be constructed using the divisors of $P(n)$.

For example $\#f(2) = 2$ because

$$P(2) = 2^1 \times 3^2 = 18$$

The 2 fractions are $3/2$ and $9/2$.

What is $\#f(10^{10})$?

Answer format: rightmost 20 digits

[My timing: < 1 s]

Answer**df709de2d2d3648e454946fdbb35a15c**

369 **Lonely runners****2016-09-15 15:35:30****by Philippe_57721****5 xp****Programming**

n runners are on a circular track of length 1.

At $t = 0$, they are all in the same position and start to run in the same direction.

A runner R_i is said **lonely** at time t if he is at a distance of at least $1/n$ of every other runners.

There are 10 runners with the following speed: 13, 163, 353, 389, 487, 491, 563, 797, 857, 881. A speed V_i means that the runner R_i completes the track in V_i unit of time.

At which time all runners have been lonely at least once? (The time t is discrete and should always be an integer)

Answer**af65ec4985d3eb204c8c195444595c0e**

370 DNA sequences alignment

2017-01-27 09:22:31

by Philippe_57721

8 xp

Programming

Given two sequences of nucleic acid, we try to align them the best way possible. We can insert any number of gaps in both sequences.

We use the following rules:

- if at a given position the acids are the same, we count +2
- if at a given position the acids are different, we count -1
- if there is a gap in one string, we count -2

For example, consider the two following sequences:

- GAATTCAGTTA
- GGATCGA

The best possible alignment (the one with the greater score) is:

```
GAATTCAGTTA
GGA-TC-G--A
```

which gives a score of 3

- G G : +2
- A G : -1
- A A : +2
- T - : -2
- T T : +2
- C C : +2
- A - : -2
- G G : +2
- T - : -2
- T - : -2
- A A : +2

You are given the 2 following sequences:

```
'GTAATAGACTCGAACCGCAACCGTCAGCAAACCGCTTGGTCGATCGT
AATATGTAAGATCCAATTAGGGCGACCTCTTGTGCGGTAGTAGGAGTCT'

'ATAACTCTGAATCCCCGACGTGTCGTGATGGGCACGGACGGCACCCCTT
AACGTGATCCTGAACTCCCCTGGGGACCGTTGTCGGAATGCAGGGTGTG'
```

What is the score for the best alignment and the number of acids identical in both aligned sequences?

Answer format: comma separated

Example: 3,6 for the sequences GAATTCAGTTA and GGATCGA.

[My timing: < 1 sec]

Answer 7a58aecc982ad52fb29496da2f71e580

371 Die Hard**2016-11-13 08:51:55****by Philippe_57721****8 xp****Programming**

In the movie [Die Hard 3](#), Officer John McLane has to stop a bomb to explode by using a conterweight of 4 gallons.

For this, he can only use 2 jugs of 5 and 3 gallons. He can do it in 6 moves:

```
[5] [0] // Pour 5 gallons in the 1st jug
[2] [3] // Fill the 2nd jug with the 1st
[2] [0] // Empty the 2nd jug
[0] [2] // Pour the 2 gallons from the 1st jug to the 2nd
[5] [2] // Pour 5 gallons the 1st jug
[4] [3] // Fill the 2nd jug with the 1st => We are done: 4 gallons remain in the
1st jug
```

- How many moves (M_1) would Officer McLane need to weight 641 gallons with 2 jugs of 19937 and 23209 gallons?
- Which capacity (C_2) between 1 and 19936 requieres the largest number of moves (M_2)?

Answer format: M_1, C_2, M_2

[My timing: 4 sec]

Answer**ba763325df68843e46db2e5804ec7dc2**

372 Die Hard - 3 Jugs**2016-11-17 10:35:43****by sinan****10 xp****Programming**

See the [problem 371](#) by **Philippe**. This time we have 3 jugs with capacities 7, 31 and 1048575. What are the gallons that require the most number of moves?

Answer format: moves, g_1, g_2, \dots

where $g_{i-1} < g_i$

Example: 20, 54, 57, 58, 62, 66, 67, 70 if we have 3, 7 and 127 gallon jugs.

[My timing < 1 m and my memory requirement < 100 M]

Answer **6ed1ee25d294a8796ba3ff8d6dd624ab**

373 Busy Beavers

2016-10-21 06:51:59

by Philippe_57721

7 xp

Programming

There is a special kind of [Turing machines](#) called the **Busy Beavers**.

A Busy Beaver is a machine defined for P symbols and Q states. Its purpose is to write as many symbols as possible on the tape.

Let's take an example with a 2 symbols and 2 states Busy Beaver.

It is described by a $P \times Q$ matrix.

Symbol	State-1			State-2		
	Write	Move	Next	Write	Move	Next
0	1	R	2	1	L	1
1	1	L	2	1	R	0

Each row corresponds to a symbol read on the tape. Each column corresponds to a state and contains a triple:

- Which symbol to write on the tape
- In which direction to move (Left or Right) on the tape
- Next state (State 0 means the Beaver halts)

Example: if we read a '0' on the tape, and we are in state 2, we write a '1' on the tape, go Left and go to state 1.

We always start with an empty tape (filled with '0'), and in state 1.

If we run this Busy Beaver, we get the following execution trace:

Step	Curr	Tape	Move	Write	Next
0	1	00000{0}000000	R	1	2
1	2	000001{0}00000	L	1	1
2	1	00000{1}100000	L	1	2
3	2	0000{0}1100000	L	1	1
4	1	000{0}11100000	R	1	2
5	2	0001{1}1100000	R	1	0

The beaver stops after 6 steps and in the end the tape contains 4 '1'.

This is actually the "best" 2×2 Busy Beaver (the one which writes the most '1' on a tape).

Let's take a 2×6 Busy Beaver, defined with the following matrix:

Symb	State-1		State-2		State-3		State-4		State-5		State-6	
	Write	Move	Next	Write	Move	Next	Write	Move	Next	Write	Move	Next
0	1	L	2	1	R	3	0	R	6	1	L	1
1	1	L	1	1	R	2	1	R	4	0	R	5

You can verify that this beaver halts after 13 122 572 797 steps (took me 400 sec).

How many steps does it need to write 100000 '1'?

You are given:

- 3726 steps for 100 '1'
- 315587 steps for 1000 '1'

[My timing: 84 sec]

Answer

ca14d5fd86cc770f19828d3251f876ae

375 Slaying the hydra

2018-11-09 01:12:36

by Philippe_57721

12 xp

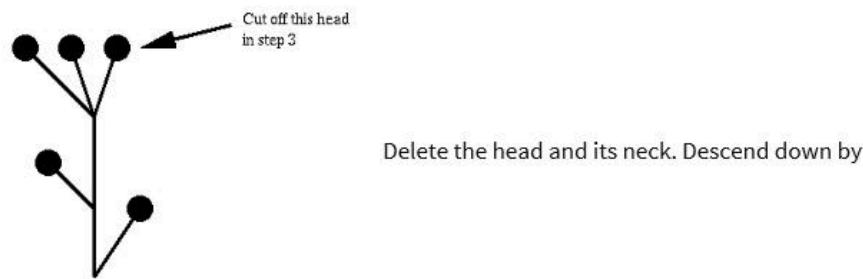
Programming

An hydra is a rooted tree.

Hercules' task is to slay the hydra by chopping all its heads. But the hydra's heads grow again with the following rules. At the step n :

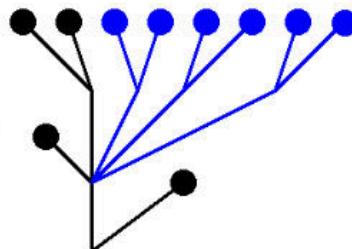
- if Hercules cuts a head growing from the root, the hydra doesn't grow any head.
- if Hercules cuts a head attached to a given node, this head is deleted.

The node at which the head was attached is copied n times on the node just below.

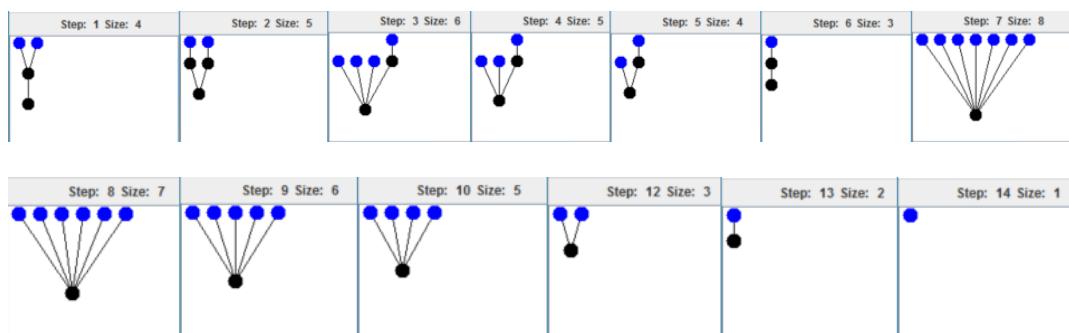


1 from the node at which the neck was attached. Look at the subtree growing from the connection through which you just descended. Pick a natural number, say 3, and

grow that many copies of that subtree, like this:



Here is the evolution of a small hydra:



We define the size of an hydra as the number of its heads and nodes. The hydra in the above picture has a size of 4 at step 1.

We can represent an hydra as a string as follow:

- n A leaf
- n(. .) A node, inside the parenthesis its children

For instance, in the above picture the first 3 hydras are:

$n(n(nn))$
 $n(n(n)n(n))$
 $n(nnnn(n))$

We always cut the left most heads.

If we consider all the hydras of size 5, after 100 steps, the maximum sizes reached are the following:

- $n(n(n(n(n))))$ - 5686
- $n(n(n(nn)))$ - 341
- $n(n(n(n)n))$ - 288
- $n(n(n(n))n)$ - 21
- $n(n(nn(n)))$ - 70
- $n(n(nnn))$ - 156
- $n(n(nn)n)$ - 9
- $n(n(n)n(n))$ - 6
- $n(n(n)nn)$ - 5
- $n(nn(n(n)))$ - 67
- $n(nn(nn))$ - 20
- $n(nn(n)n)$ - 5
- $n(nnn(n))$ - 5
- $n(nnnn)$ - 4

(The initial size before steps is not counted)

The 5 largest sizes are (in descending order): 5686, 341, 288, 156, 70.

What are the 5 largest sizes reached among all the 6-size hydras after 100 steps?

Answer format: comma delimited list in descending order

[My timing: 55 sec]

Credit to Andrej Bauer for the images

Answer **37110a0845899a3c2e4dc8ef32d34586**

376 Twins

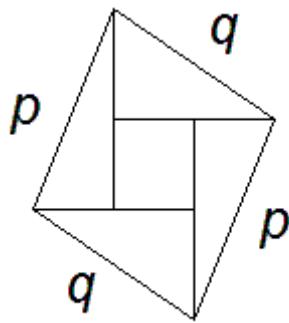
2016-12-05 08:22:06

by sinan

8 xp

Programming

Consider **integer sided** Pythagorean triangles with their hypotenuse and shorter leg equal to **twin primes** (*). Using four of such triangles and a rectangle with sides also equal to **twin primes** (*), we draw a parallelogram as seen below.



Find such parallelograms with their area less than 10^{19} ?

Answer format: count,sum_of_areas mod 264

Example: 1, 169 (for area $< 10^3$).

[My timing: < 1 m]

* Note that **twin primes** here does NOT mean a pair of primes with a difference of 2, rather any twin primes pair where a twin prime means a **prime number that is either 2 less or 2 more than another prime number**. For example for the triple (5, 12, 13), we have 5 and 13 which are two twin primes.

Answer 412c2ce1c56d567bb0ceadeecf79cedc

377 Pandigital never prime**2016-12-16 14:04:40****by Philippe_57721****5 xp****Programming**

Consider the number 100020. If you change one of its digits, you never get a prime number:

$$100020 = 2^2 \times 3 \times 5 \times 1667$$

$$200020 = 2^2 \times 5 \times 73 \times 137$$

$$300020 = 2^2 \times 5 \times 7 \times 2143$$

$$400020 = 2^2 \times 3 \times 5 \times 59 \times 113$$

$$500020 = 2^2 \times 5 \times 23 \times 1087$$

$$600020 = 2^2 \times 5 \times 19 \times 1579$$

$$700020 = 2^2 \times 3^2 \times 5 \times 3889$$

$$800020 = 2^2 \times 5 \times 13 \times 17 \times 181$$

$$900020 = 2^2 \times 5 \times 11 \times 4091$$

$$100020 = 2^2 \times 3 \times 5 \times 1667$$

$$110020 = 2^2 \times 5 \times 5501$$

$$120020 = 2^2 \times 5 \times 17 \times 353$$

$$130020 = 2^2 \times 3 \times 5 \times 11 \times 197$$

$$140020 = 2^2 \times 5 \times 7001$$

$$150020 = 2^2 \times 5 \times 13 \times 577$$

$$160020 = 2^2 \times 3^2 \times 5 \times 7 \times 127$$

$$170020 = 2^2 \times 5 \times 8501$$

$$180020 = 2^2 \times 5 \times 9001$$

$$190020 = 2^2 \times 3 \times 5 \times 3167$$

$$100020 = 2^2 \times 3 \times 5 \times 1667$$

$$101020 = 2^2 \times 5 \times 5051$$

$$102020 = 2^2 \times 5 \times 5101$$

$$103020 = 2^2 \times 3 \times 5 \times 17 \times 101$$

$$104020 = 2^2 \times 5 \times 7 \times 743$$

$$105020 = 2^2 \times 5 \times 59 \times 89$$

$$106020 = 2^2 \times 3^2 \times 5 \times 19 \times 31$$

$$107020 = 2^2 \times 5 \times 5351$$

$$108020 = 2^2 \times 5 \times 11 \times 491$$

$$109020 = 2^2 \times 3 \times 5 \times 23 \times 79$$

$$100020 = 2^2 \times 3 \times 5 \times 1667$$

$$100120 = 2^3 \times 5 \times 2503$$

$$100220 = 2^2 \times 5 \times 5011$$

$$100320 = 2^5 \times 3 \times 5 \times 11 \times 19$$

$$100420 = 2^2 \times 5 \times 5021$$

$$100520 = 2^3 \times 5 \times 7 \times 359$$

$$100620 = 2^2 \times 3^2 \times 5 \times 13 \times 43$$

$$100720 = 2^4 \times 5 \times 1259$$

$$100820 = 2^2 \times 5 \times 71^2$$

$$100920 = 2^3 \times 3 \times 5 \times 29^2$$

100000 = $2^5 \times 5^5$
100010 = $2 \times 5 \times 73 \times 137$
100020 = $2^2 \times 3 \times 5 \times 1667$
100030 = $2 \times 5 \times 7 \times 1429$
100040 = $2^3 \times 5 \times 41 \times 61$
100050 = $2 \times 3 \times 5^2 \times 23 \times 29$
100060 = $2^2 \times 5 \times 5003$
100070 = $2 \times 5 \times 10007$
100080 = $2^4 \times 3^2 \times 5 \times 139$
100090 = $2 \times 5 \times 10009$

100020 = $2^2 \times 3 \times 5 \times 1667$
100021 = 29×3449
100022 = $2 \times 13 \times 3847$
100023 = $3 \times 7 \times 11 \times 433$
100024 = $2^3 \times 12503$
100025 = $5^2 \times 4001$
100026 = $2 \times 3^2 \times 5557$
100027 = 23×4349
100028 = $2^2 \times 17 \times 1471$
100029 = 3×33343

What is the smallest [pandigital number](#) (in base 10) with that property? *

* All the numbers you get when changing one digit must be pandigital too.

[My timing: < 1 sec]

Answer **18004655cfcd8896f9e2228032dc76a**

378 Mutated worm

2017-04-17 16:15:22

by sinan

10 xp

Programming

See the [problem 374](#) by **Philippe**. We slightly modify the evolution code in that it creates m copies instead of $(m + 1)$ at m -th step. So the evolution of $[1, 1]$ now becomes like the following:

```
1 : [1, 1]
2 : [1, 0, 1, 0]
3 : [1, 0, 1]
4 : [1, 0, 0, 0, 0, 0]
5 : [1, 0, 0, 0, 0]
6 : [1, 0, 0, 0]
7 : [1, 0, 0]
8 : [1, 0]
9 : [1]
10 : [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
11 : [0, 0, 0, 0, 0, 0, 0, 0, 0]
12 : [0, 0, 0, 0, 0, 0, 0, 0]
13 : [0, 0, 0, 0, 0, 0, 0]
14 : [0, 0, 0, 0, 0, 0]
15 : [0, 0, 0, 0, 0]
16 : [0, 0, 0, 0]
17 : [0, 0, 0]
18 : [0, 0]
19 : [0]
20 : []
```

At which step does the worm $W_1 = [1, 2]$ turn into $[]$?

Answer format: rightmost 20 digits

[My timing < 1 s]

Answer 484d22a01d606a03dd1c5ea79ff0b49c

379 Red flag

2017-02-27 06:44:19

by sinan

6 xp

Math

Unfortunately the picture “https://rosecodes.neocities.org/uploads/Turkish_flag.png” was lost.

Letter	Measure	Length
G	Width	
A	Distance between the centre of the outer crescent and the seam of the white band	1/2 G
B	Diameter of the outer circle of the crescent	1/2 G
C	Distance between the centres of the inner and outer circles of the crescent	1/16 G
D	Diameter of the inner circle of the crescent	2/5 G
E	Distance between the inner circle of the crescent and the circle around the star	1/3 G
F	diameter of the circle around the star	1/4 G
L	Length	3/2 G
M	Width of the white hem at the hoist	1/30 G

If the total area of the flag is equal to 1 (that means $G \cdot (L + M) = 1$) what is the area of red part rounded to 10 digits after the decimal point?

Note that the dotted circle around the star is drawn for illustration purpose and actually is a part of the red area.

Answer **7df30b4d380ffbd3c3a12b9ce419c880**

380 Exploring the Hilbert Curve

2017-02-03 09:25:54

by Philippe_57721

8 xp

Math

The [Hilbert curve](#) is a space-filling curve described by the great German mathematician David Hilbert in 1891.

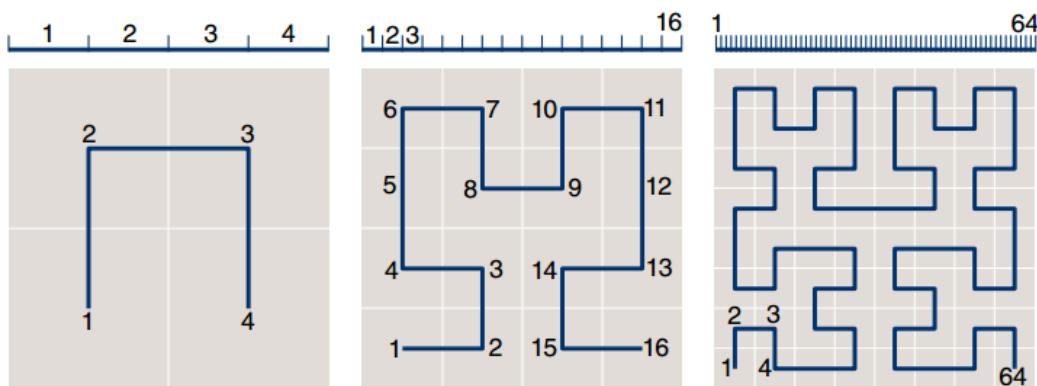
It is defined as the limit of the following process:

We start with a square of side 1 that we divide in 4 parts.

We join the center of each sub-square as shown in the picture.

We repeat the process by dividing the square in 16 parts.

The Hilbert curve is the curve obtained when continuing this process infinitely.



A space-filling curve evolves through successive stages of refinement as it grows to cover the area of a square. This illustration is a redrawing of the first published diagram of such a curve; the original appeared in an 1891 paper by David Hilbert. The idea behind the construction is to divide a line segment into four intervals and divide a square into four quadrants, then establish a correspondence between the points of corresponding intervals and quadrants. The process continues with further recursive subdivisions.

As this curve encounters every point in the square, we can establish a bijection between the square and the segment $[0, 1]$, as shown in the picture.

What is the point (x, y) which corresponds to the value $\pi - 3$ in the interval $[0, 1]$?

What point z in the interval $[0, 1]$ corresponds to the point $(e - 2, \gamma)$ in the square? (γ is the Euler Mascheroni constant)

Answer format: x, y, z

truncate the values at 10 digits after the decimal point

Hint:

You are given the point in the square corresponding to $\sqrt{2} - 1$ in the interval $[0, 1]$:
 $(0.4472924291, 0.9498609981)$

[My timing: < 1 sec]

Credit to Brian Hayes for the pictures

Answer 23cdd928ce804e3eed9fb4e442954844

381 **Cube puzzle****2017-01-06 10:19:58****by Philippe_57721****4 xp****Probability**

A solid cube has its 6 faces painted in black. It is cut into 1000 smaller cubes (each face evenly divided in 10). These little cubes are placed in a bag and one is selected randomly. This selected cube is then rolled like a die.

What is the probability that it lands with a black face up?

Answer format: P/Q

an irreducible fraction

Answer**57a7d6de1978e8480375c8cd3632b8ab**

382 A special subset**2017-02-07 07:48:01****by sinan****8 xp****Programming**

Let S_N be the set of positive numbers less or equal to N , $S_N = \{1, 2, \dots, N\}$.

Consider the subsets with the following properties:

- none of the elements is a prime number
- none of the elements contains any prime digit(s)
- none of any 2 elements' difference is a prime number
- none of any 2 elements' difference contains any prime digit(s)

Find the subset of S_{100000} with the maximal sum.

Answer format: sum

Example: 20 for S_{10} subsets:

{1}, {1, 9}, {1, 9, 10}, {1, 10}, {4}, {4, 8}, {4, 10}, {6}, {6, 10}, {8}, {8, 9}, {9}, {9, 10}, {10}

[My timing: < 1 m]

Answer b4ae75793af3df9055bb64a09e3b745f

383 Alleviated factorisation**2017-02-17 08:31:59****by Philippe_57721****9 xp****Programming**

$P < Q$ are two prime numbers.

You are given:

$$\begin{aligned}P \times Q = & 2502128003544190562737219821476557238262836801164 \\& 2948611923288271186128054172457672472582525510063 \\P \otimes Q = & 10896394514466713067376536898727387282419639480038\end{aligned}$$

(\otimes is the XOR operation)

Can you find P and Q ?

Answer format: P, Q

[My timing: 75 sec]

Answer **3d7ce3a60f9449ab9581f34e7744e319**

384 Denesting

2017-02-10 08:15:12

by Philippe_57721

7 xp

Math

Denesting is the process of simplifying an expression with embedded n-th roots to obtain an equivalent expression with a lesser level of roots.

Ramanujan was a master in this matter and provided some amazing examples:

$$\begin{aligned}\sqrt[3]{\sqrt[3]{2} - 1} &= \left(\sqrt[3]{\frac{1}{9}} - \sqrt[3]{\frac{2}{9}} + \sqrt[3]{\frac{4}{9}} \right) \times \frac{1}{3} \\ \sqrt{\sqrt[3]{5} - \sqrt[3]{4}} &= (\sqrt[3]{2} + \sqrt[3]{20} - \sqrt[3]{25}) \times \frac{1}{3} \\ \sqrt[4]{\frac{3 + 2\sqrt[4]{5}}{3 - 2\sqrt[4]{5}}} &= (3 + \sqrt[4]{5} + \sqrt{5} + \sqrt[4]{125}) \times \frac{1}{2}\end{aligned}$$

Can you denest the following expression: $\sqrt{\sqrt[3]{1404} - \sqrt[3]{875}}$?

The solution has the form $(\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}) \times \frac{p}{q}$ with $a < b < c$ and p, q coprime.

Answer format: a,b,c,p,q

[My timing: 35 sec]

Answer

dafb34086c6aec6bfd6ef81e478cf1a6

385 Rational as Prime Egyptian Fractions

2017-02-24 09:09:16

by Philippe_57721

7 xp

Programming

Conjecture

For any positive rational r there is a finite set of prime numbers P_r^+ such that $\sum_{p \in P_r^+} \frac{1}{p+1} = r$.

For instance $\frac{3}{14} = \frac{1}{5+1} + \frac{1}{23+1} + \frac{1}{167+1}$.

Can you verify this conjecture for $\frac{25}{61}$?

Answer format: p_1, p_2, \dots, p_n

if $\frac{25}{61} = \frac{1}{p_1+1} + \frac{1}{p_2+1} + \dots + \frac{1}{p_n+1}$

As there are generally more than one solution, give the 1st one in lexicographic order.

Limit your search to prime numbers < 100000.

[My timing: 35 sec]

Answer 7bfa0a7be39f3b3b02584a1c8062a789

386 Squarefree Factorisations

2017-03-03 07:51:13

by Philippe_57721

8 xp

Programming

Let's n an integer with the following factorisation: $n = a_1^{e_1} \times a_2^{e_2} \times \dots \times a_p^{e_p}$, where a_i are squarefree and $\forall i \in \{1, \dots, p-1\}$, a_i divides a_{i+1} .

For instance:

$$56 = 2^2 \times 14^1$$

$$5040 = 2^2 \times 6^1 \times 210^1$$

$$526773121875 = 3^2 \times 15^3 \times 1155^1 \times 15015^1$$

It can be proved that this factorisation is unique.

For such a factorisation, let's consider all the divisors of n : $a_1^{f_1} \times a_2^{f_2} \times \dots \times a_p^{f_p}$ where $0 \leq f_i \leq e_i$.

Define $\sigma'(n) = \sum_d (d)$ where d runs over the divisors of n as defined above.

$$\sigma'(5040) = 1 + 2 + 4 + 6 + 12 + 24 + 210 + 420 + 840 + 1260 + 2520 + 5040 = 10339.$$

We say that n is a champion if the ratio $\frac{\sigma'(n)}{n}$ is greater than any ratio $\frac{\sigma'(m)}{m}$ with $m < n$.

Here are the first 10 champions:

1. $1 \Rightarrow 2$
2. $24 \Rightarrow 2.04166666666667$
3. $48 \Rightarrow 2.1875$
4. $96 \Rightarrow 2.26041666666667$
5. $192 \Rightarrow 2.296875$
6. $384 \Rightarrow 2.31510416666667$
7. $768 \Rightarrow 2.32421875$
8. $1152 \Rightarrow 2.3515625$
9. $2304 \Rightarrow 2.37022569444444$
10. $4608 \Rightarrow 2.37955729166667$

What is the 66-th champion?

[My timing: 5 sec]

Answer **de5bd6dcc93ff229be36c8d8af40a4b1**

387 Double mirror**2017-03-10 07:40:16****by Philippe_57721****9 xp****Programming**

Consider the number 35490. It can be written as:

2, 21, 2, 1 in base 23

1, 2, 21, 2 in base 32

Find the smallest number that can be written as:

d_1, d_2, \dots, d_9 in base $b_1 b_2$

d_9, d_8, \dots, d_1 in base $b_2 b_1$

($b_1, b_2 \in \{1, \dots, 9\}$)

Answer format: `number,base1`

where `base1 < base2`

Example: 35490, 23

[My timing: 4 sec]

Answer **92f44f935e4a4f63184ecedf82eb9bb3**

388 A mysterious machine**2017-03-17 07:46:03****by Philippe_57721****5 xp****Math**

Given a string representing a positive integer, the machine outputs another string using the following rules:

Rule 1: If the input is $2X$, the output will be X .

e.g.: $M(234) = 34$

Rule 2: If the input is $3X$ and $M(X) = Y$ the output will be $Y2Y$.

e.g.: $M(327) = 727$

Find X such as $M(X) = X$

Find Y such as $M(Y) = Y2Y$

Find Z such as $M(Z) = 7Z$

Answer format: X, Y, Z

[My timing: pencil and paper]

Answer**6d184baaeb6dbb0288a25437d05cf699**

389 Fair partition**2017-03-24 07:45:47****by Philippe_57721****6 xp****Programming**

The number $10! = 3628800$ has a set D of 270 divisors.

We can find 2 sets D_1 and D_2 such as

- the product of each element of D_1 with each element of D_2 generates D
- the cardinals of the sets D_1 and D_2 are the closest possible

$$D_1 = \{1, 3, 5, 9, 15, 25, 27, 45, 75, 81, 135, 225, 405, 675, 2025\} \text{ and } |D_1| = 15$$

$$D_2 = \{1, 2, 4, 7, 8, 14, 16, 28, 32, 56, 64, 112, 128, 224, 256, 448, 896, 1792\} \text{ and } |D_2| = 18$$

Find a fair partition for $100!$.

Answer format: $|D_1|, |D_2|$

(in ascending order)

Example: 15, 18

[My timing: 54 sec]

Answer **72bf4dbb1ae382cf8936cf17da0cc7e9**

390 Radical triangles**2017-02-20 08:08:48****by sinan****6 xp****Math**

For positive integers a, b, c ($a \leq b \leq c$), let $T(N)$ denote the number of triangles with sides \sqrt{a} , \sqrt{b} , and \sqrt{c} for $c \leq N$.

What is $T(100000)$?

Example: 145048 for $N = 100$.

[My timing: < 1 m]

Answer **8a131aacc98cf8d55ba690c3671e011b**

391 Not a sum of distinct squares

2017-04-07 06:44:02

by Philippe_57721

5 xp

Programming

It is known since [Lagrange](#) that every positive integer can be represented as the sum of integer squares, 4 integers being enough.

But, if we add the constraint that all squares must be distinct, some integers are not representable as such a sum. This set is finite.

How many positive integers can not be represented by a sum of distinct squares, and what is their sum?
(You are given that the largest one if less than 5000)

Answer format: Count , Sum

[My timing: 15 s]

Answer e3288ee7769c09f280ec23aaafa31e41

392 Beatty sequences

2017-03-31 06:50:34

by Philippe_57721

5 xp

Programming

There is a beautiful theorem by [Samuel Beatty](#) which states:

If α and β are positive irrational numbers such as $\frac{1}{\alpha} + \frac{1}{\beta} = 1$ then $A = \{a \in \mathbb{N}, \lfloor n \cdot \alpha \rfloor\}$ and $B = \{n \in \mathbb{N}, \lfloor n \cdot \beta \rfloor\}$ are a partition of N : $N = A \cup B$ and $A \cap B = \emptyset$.

Let's A the vector contained in this file [data389.txt](#).

If A is a Beatty sequence, can you find the corresponding value α .

Answer format: α

truncated at 6 positions after the decimal point.

[My timing: instant]

Answer**d58c9e1f9a93d19ea15fa0f903b581c1**

393 A distant prime number**2017-04-07 06:44:25****by Philippe_57721****5 xp****Math**

We know the the 10^9 -th prime number is 22801763489.

What is the 10^{200} -th prime number?

Answer format: The first 30 digits of that number

[My timing: 6 sec]

Answer**7b48313401051752b760814f3a8f7b2f**

394 Self avoiding paths

2017-04-13 14:19:32

by Philippe_57721

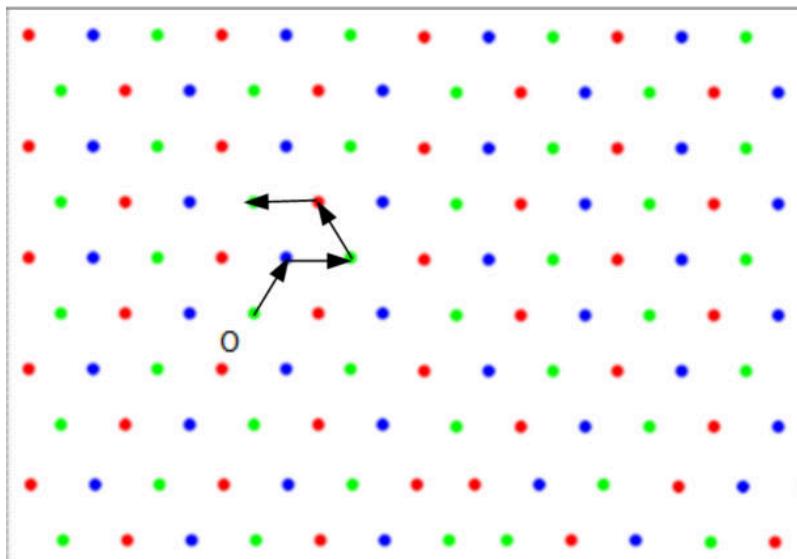
7 xp

Programming

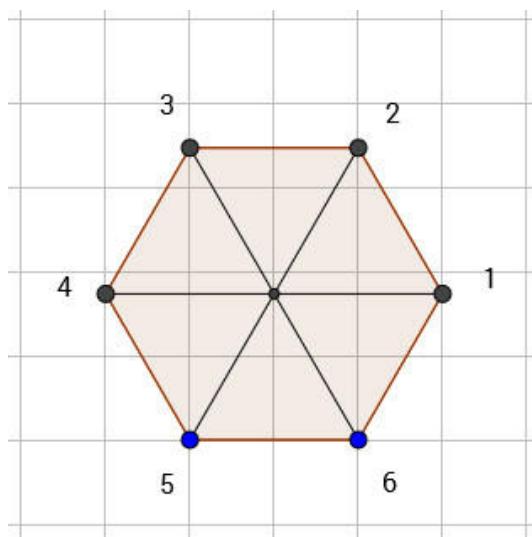
In the equilateral triangle lattice plan, let's point O be the origin.

A self avoiding path is built as follow:

- We start from O .
- At each step we choose one of the 6 neighbour points which has not been visited yet.



We set a value on the 6 vectors starting from a given point as follow:



We can thus associate a number to each path: the concatenation of the values of the vectors in that path.

For instance, the path given in the 1st figure has the value: 2134.

How many self avoiding paths with 12 points are there?

How many of these paths correspond to a palindromic value?

Answer format: Count-paths, Count-palindromic

You are given 618,30 for 5 points.

[My timing: 30 sec]

Answer 3eee75da5b2f0014f7791905ba582a45

395 Maximize XOR

2017-04-21 06:59:22

by Philippe_57721

6 xp

Programming

Let $2^k \leq p < q \leq 2^{k+1} - 1$.

We are looking for pairs (p, q) such as

- $p \otimes q$ is maximal (\otimes is the XOR operator)
- p, q are both either a prime or a square number

How many such pairs are there for $k = 26$?

Answer format: count,first p,first q

Example: 3,1307,1764 for $k = 10$.

[My timing: 42 sec]

Answer**e3a4219d7f48191ee8369cf42abb7d04**

396 Hidden title

2017-05-05 04:09:16

by Philippe_57721

6 xp

Programming

Consider the amazing Tupper's formula:

$$\frac{1}{2} < \left\lfloor \text{mod}\left(\left\lfloor \frac{y}{17} \right\rfloor 2^{-17\lfloor x \rfloor - \text{mod}(\lfloor y \rfloor, 17)}, 2\right) \right\rfloor$$

Plot the set of points (x, y) where $0 \leq x \leq 105$ and $k \leq y \leq k + 16$.

```
k =
18103678631912000062470282381747441976757893373198
86435384963567843211312931211913675581074713889534
005789379675441499470450575532393376026086675639
79479602976376683335698251880808911605667287509150
04068115678628489149512637203885271143388014539045
70710336898701369407801663926977881040872681066426
17283868280916627888347962832772249277391186206061
40236546487592898274058357130776778346128459449558
45786918834227702457154420543255799489565393910464
89136316751596075567372059735592342126648150418628
790330350241623669114319450430013312
```

Answer format: the text hidden in the plot

Answer 33c080d31af03976ce0a4a69545d85fb

397 Prime Pandigital Step Numbers**2017-05-12 06:49:22****by Philippe_57721****6 xp****Programming**

A step number is a number whose digits are in ascending order

For example: 1223455567 is a step number

A pandigital step number is a step number which contains all digits (except 0) at least once

For example: 1122345556777899 is a pandigital step number

Find the smallest (S) and largest (L) 30-digits prime pandigital step numbers.

Answer format: S, L

Example: 11111111111223456789, 1234567778888888899 for 21 digits.

[My timing: 65 sec]

Answer**26347a11bfa10a887d1827427bb12c38**

398 The Goodstein Sequence

2017-05-19 08:03:19

by Philippe_57721

7 xp

Programming

We define the hereditary base- n representation of a number as follow:

For instance, if $n = 266$, and base = 2, its hereditary base-2 representation is:

$$\begin{aligned} 266 &= 2^1 + 2^3 + 2^8 \\ &= 2^1 + 2^{1+2} + 2^{2^3} \\ &= 2^1 + 2^{1+2} + 2^{2^{1+2}} \end{aligned}$$

We express n in base 2 and recursively every exponent in base 2.

Let's define the G_k sequence as follow:

$$G_1(n) = n$$

$G_2(n)$ = Take the hereditary base-2 representation of n , replace each 2s with 3s and subtract 1

$G_{k(n)}$ = Take the hereditary base-k representation of $G_{k-1}(n)$, replace each ks with $(k+1)s$ and subtract 1

Here the first values for $G_k(6)$

$$G_1(6) = 6$$

$$G_2(6) = 29$$

$$G_3(6) = 257$$

$$G_4(6) = 3125$$

$$G_5(6) = 46655$$

$$G_6(6) = 98039$$

$$G_7(6) = 187243$$

$$G_8(6) = 332147$$

Find $G_{50(13)}$.

[My timing: < 1 sec]

Answer

2d79a1fff4547dcf6b67c3816da7020c

399 Factorial divisibility

2017-05-27 07:09:28

by Philippe_57721

8 xp

Programming

Let $S(n)$ be the smallest integer such as n divides $S(n)!$.

For instance $S(9) = 6$ for $6! \equiv 0 \pmod{9}$.

Consider the set A of numbers of form $2_1^e \times 3_2^e \times 5_3^e \times 7_4^e \times 11_5^e \times 13_6^e \times 17_7^e \times 19_8^e$.

What is $\sum_{n \in A'} S(n)$ where A' contains the first 10^8 elements of A .

You are given: 45499522 when A' contains the first 10^6 elements of A .

[My timing: 120 sec]

PS 1: We consider $S(1) = 0$.

PS 2: There is a solution in less than 15 sec.

Answer**be65cad4adb708babf6efd4cd21811eb**

400 How close

2017-06-09 07:41:33

by Philippe_57721

6 xp

Programming

Using only prime numbers less than 30, the closest (less than) we can get from $10^{10} - 1$ is:

$$9999928224 = 2^5 \times 3^5 \times 11 \times 13 \times 17 \times 23^2$$

Using only prime numbers less than 50, how close can you get from $10^{15} - 1$?

[My timing: 60 sec]

Answer**bd6d4409c9c064d7213c7947d309681e**

401 A Big Sum

2017-06-01 14:52:19

by Philippe_57721

8 xp

Math

Let $S(n) = \sum_{i \leq n} \lfloor i \cdot \sqrt{2} \rfloor$ where $\lfloor x \rfloor$ = integer part of x .

For instance $S(100) = 7092$.

What is $S(10^{20})$?

[My timing: instant]

Thanks to **czp001** who gave me the idea!

Answer**6db6bbc1fd53b43dc7912445d0a30bc9**

402 **Ordinal numbers I**

2017-06-16 12:21:03

by Philippe_57721

12 xp

Programming

Georg Cantor made many brilliant discoveries, but the ones I find the most fascinating are in the realm of infinite sets, especially the [Ordinal Numbers](#).

They are a new kind of numbers with some strange arithmetic rules:

$$(\omega + 1) + (\omega^2 + \omega \cdot 2 + 1) = \omega^2 + \omega \cdot 2 + 1$$

$$(\omega^2 + \omega \cdot 2 + 1) + (\omega + 1) = \omega^2 + \omega \cdot 3 + 1$$

Let the following ordinal numbers:

$$\sigma_1 = \omega^2$$

$$\sigma_2 = \omega + 1$$

$$\sigma_3 = \omega \cdot 2 + 2$$

$$\sigma_4 = \omega \cdot 4 + 3$$

$$\sigma_5 = \omega \cdot 8 + 4$$

If we add the five σ_i ordinal numbers using all possible 120 permutations and sort them (using the ordinal number order)

- how many different sums can we obtain
- what is the 1st (lowest) value
- what is the 17-th value

Answer format: comma delimited

The ordinal numbers are represented using the [Cantor Normal Form](#)

- the symbol ω is represented with a `w`
- exponentiation is represented with a `^`
- the symbol \cdot is represented with a `.`
- all spaces are removed

$$\omega^2 + \omega \cdot 3 + 1 \rightarrow w^2+w.3+1$$

[My timing: < 1 s]

Answer**84a73b5bfa3c2771f717cbc84cad51ec**

403 **Ordinal Numbers II**

2017-06-16 12:21:23

by Philippe_57721

5 xp

Programming

For definitions, See [problem 402](#).

What is $(\omega^2 + \omega + 1)^{\omega^3 + \omega^2 + \omega + 2}$.

[My timing: < 1 sec]

Answer **b048ae49976898300debd845b0e6a28e**

404 Powerful Numbers I**2017-06-23 07:22:18****by Philippe_57721****9 xp****Math**

A number is said **powerful** if all its prime factors appear with an exponent greater than 1.

A **pure powerful number** is a powerful number that is not a perfect square.

Two pure powerful numbers can be consecutive:

12167 (23^3), 12168 ($2^3 \times 3^2 \times 13^2$)

Find the 11-th pair ($m, m + 1$) of consecutive pure powerful number — (12167, 12168) being the 1st.

Answer format: m

[My timing: < 30 sec]

Answer **3b6d39661401eb0a5b070155537a7762**

405 Squarefree Numbers

2017-07-07 07:11:37

by Philippe_57721

7 xp

Math

A [squarefree number](#) is a number not divisible by a perfect square (other than 1).

How many squarefree numbers ≤ 123456789123456789 are there?

Answer format: count

[My timing: 55 sec]

Answer**1f3770fee89dd48b8e9f439b57cd18bd**

406 Powerful Numbers II

2017-06-23 07:59:41

by Min_25

10 xp

Math

Find all powerful numbers $\leq 10^{25}$.

Answer Format: count,sum

Example: 214122,716436326576527 (for $\leq 10^{10}$)

[My timing: 0.5 sec]

Answer**c932d8df5f991b85f24a06f0e1e219f9**

407 Carmichael chains**2017-06-29 15:03:40****by Philippe_57721****6 xp****Programming**

A [Carmichael number](#) is a chain if when we remove repeatedly its largest factor, we still obtain a Carmichael number.

174470590282430768272287350512321 is a Carmichael chain of length 10 (it has 10 factors).

$174470590282430768272287350512321 = 7 \times 13 \times 31 \times 61 \times 181 \times 541 \times 2161 \times 187921 \times 3570481 \times 7140961$

$24432368456070656074481761 = 7 \times 13 \times 31 \times 61 \times 181 \times 541 \times 2161 \times 187921 \times 3570481$

$6842878720281848881 = 7 \times 13 \times 31 \times 61 \times 181 \times 541 \times 2161 \times 187921$

$36413592521761 = 7 \times 13 \times 31 \times 61 \times 181 \times 541 \times 2161$

$16850343601 = 7 \times 13 \times 31 \times 61 \times 181 \times 541$

$31146661 = 7 \times 13 \times 31 \times 61 \times 181$

$172081 = 7 \times 13 \times 31 \times 61$

$2821 = 7 \times 13 \times 31$

All these numbers are Carmichael numbers.

Of course, we stop at 3 factors, as a Carmichael number can't have less than 3 factors.

Find the next Carmichael chain of length 10. We assume that no factor is greater than 10^7 .

[My timing: 60 sec]

Answer**954e1e0ac73c5720df4e3bf88cf75ddb**

408 Mertens Equations**2017-09-15 01:10:56****by Philippe_57721****7 xp****Programming**

The [Mertens function](#) is the summatory function of the [Möbius function](#).

$$M(n) = \sum_{k \leq n} \mu(k)$$

Find the first i such as $M(i) = -9876$ and the first j such as $M(j) = +9876$.

Answer format: i, j

You are given: 12855874, 10193871 for ± 1234 .

[My timing: 60 sec]

Answer**47aa39cb641a77fd2f7acbd360db3931**

409 Infinite String

2017-05-26 14:48:56

by Min_25

12 xp

Math

Let s_0 be the string “01” and $s_i = s_{i-1} + s_{i-1}[0 : i]$ for $i \geq 1$, where $s[0 : i]$ means the first i characters of s .

For example, $s_1 = “010”$, $s_2 = “01001”$ and $s_3 = “01001010”$.

Let $C(i)$ be the number of “1” in the $s_i[0 : i]$.

You are given that $C(10) = 4$, $C(10^4) = 3576$ and $C(10^8) = 35807401$.

Find $C(10^{18})$.

[My Timing: instant]

Answer **9b3781857afac58efe20d1b50890eb15**

410 Sum of three cubes**2017-07-14 06:54:46****by Philippe_57721****7 xp****Programming**

It is conjectured that all positive integers, not of the form $9 \times k \pm 4$ can be written as a sum of 3 cubes, probably in infinitely many ways.

Some decompositions are hard to discover.

For instance, the smallest decomposition of 74 was only discovered in 2016:

$$74 = -284650292555885^3 + 66229832190556^3 + 283450105697727^3$$

Decompositions for all integers up to 100 are known, except for 33 and 42.

Find all the decompositions of 1026 where the absolute value of each cube is $< 10^5$.

Answer format: Count,X,Y,Z

where $X < Y < Z$ is the triple with the largest absolute values for X

You are given: 6, -14900, 10849, 12664 for 993 and a threshold of 20000.

$$993 = -14900^3 + 10849^3 + 12664^3$$

$$993 = -11450^3 - 4127^3 + 11626^3$$

$$993 = -8339^3 + 6121^3 + 7051^3$$

$$993 = -2528^3 + 673^3 + 2512^3$$

$$993 = -1007^3 - 842^3 + 1174^3$$

$$993 = -2^3 + 1^3 + 10^3$$

[My timing: 60 sec]

Answer a749d17124fec0a22d2c730b17d0c0bf

411 A Big Factorial**2017-08-05 05:21:23****by Philippe_57721****6 xp****Programming**

What is the digital sum of 1500000!.

You are given: 1938780 for 100000!.

[My timing: 120 sec]

Answer **075f1e96f56fbf9a85f2fbb214ee6127**

412 Magic square of squares

2017-07-21 06:51:45

by Philippe_57721

6 xp

Programming

It is an open problem to know whether a 3×3 magic square containing only squares exists or not.

We say a square is **almost magic** if all the sums are equal except for the anti-diagonal.

The following square is almost magic:

$$\begin{bmatrix} 174^2 & 381^2 & 138^2 \\ 291^2 & 222^2 & 246^2 \\ 282^2 & 6^2 & 339^2 \end{bmatrix}$$

What is the first (lexicographic order) almost magic square with all elements squares of numbers ≤ 200 .

Answer format: comma separated list of the square root of each element
(All values must be distinct)

Exemple: 174, 381, 138, 291, 222, 246, 282, 6, 339 for the above example.

[My timing: 25 sec]

Answer**51a3d4f5a4965d6dbd71770de4609c18**

413 Standard Nim**2017-06-24 15:40:27****by Min_25****8 xp****Math**

Let $C(n, k)$ be the number of integer solutions (x_1, \dots, x_k) such that $\bigoplus_{i=1}^k x_i = 0$ and $0 \leq x_i < n$ for each i . Here, $x \oplus y$ means x xor y .

Let $F(n)$ be the n -th Fibonacci number: $F(0) = 0$, $F(1) = 1$, and $F(i) = F(i - 1) + F(i - 2)$ for $i \geq 2$.

Let

$$S(n) := \sum_{i=1}^n \sum_{j=1}^n C(F(i), F(j)).$$

You are given $S(6) = 2153296$ and $S(7) = 18998620089329$.

Find $S(92)$ modulo $10^9 + 7$.

[My timing: 0.3 seconds]

Answer**b96adeae380a26e518c15019fe3f0bbe**

414 Symmetric Equation I**2017-07-28 08:22:25****by Philippe_57721****7 xp****Programming**

Here is the smallest known solution in positive integers for the Diophantine equation

$$\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y} = 4$$

$x = 43736[\dots]772036$

$y = 368751[\dots]055579$

$z = 1544768[\dots]277999$

(The ... represent 70 digits!)

If we allow one variable to be negative, we can find smaller solutions.

For example, $(x, y, z) = \{-1, 4, 11\}$ and $\{-5, 9, 11\}$ are solutions of the above equation.

Find the first solution to $\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y} = 38$ with $x < 0 < y < z$ (coprime integers).

Answer format: x, y, z

[My timing: 15 sec]

Answer **7273c1fd6fc33ca8e3f9157ad2cd391e**

415 Symmetric Equation II**2017-07-28 08:22:40****by Philippe_57721****10 xp****Math**

See [problem 414](#).

Find the first solution to $\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y} = 466$ with $x < 0 < y < z$ (coprime integers).

Answer format: x, y, z

[My timing: less than 1 sec]

Answer**55734a6df7f41bdf96a7c6c5032c617a**

416 Convex numbers

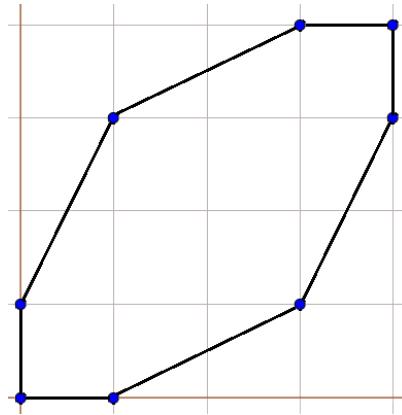
2017-09-05 02:21:52

by sinan

8 xp

Programming

Take the number 134431001 as an example. We can make pairs using the 2 digits following one another:
 $[(1, 3), (3, 4), (4, 4), (4, 3), (3, 1), (1, 0), (0, 0), (0, 1)]$



In general for a k -digit number like $d_1d_2\cdots d_k$ we can have a $(k - 1)$ -gon:
 $[(d_1, d_2), (d_2, d_3), \dots, (d_{k-1}, d_k)]$

If the drawn polygon is a convex one then we will call this kind of number as **convex number**.

How many **positive** convex numbers are there?

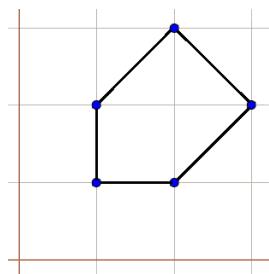
Answer format: count, sum

[My timing: < 1 m]

Notes:

- Any two neighbouring line segments cannot be collinear.
- All the generated pairs must be distinct.
- Some polygons can be generated by more than one number.

For example: 112321, 123211, 232112, 321123, 211232. Since the numbers are different, they are all counted.



Answer 4920c89b2107f92cd5ac26c5a597c90e

417 Generating Seeds**2017-07-16 05:35:47****by Min_25****8 xp****Math**

Let D be a positive integer.

Suppose that we would like to find all nonnegative integer solutions (x, y, z) of

$$x^2 + D = y \cdot z.$$

Let's assume that (x, y, z) is a solution of the above equation. Then, it can be verified that $(x + y, y, 2x + y + z)$ and $(x + z, 2x + y + z, z)$ are also solutions of the equation. Let's define this generating process as the *evolution* of (x, y, z) .

Surprisingly, we can find all solutions uniquely by choosing some seeds $S_D = \{(x_1, y_1, z_1), \dots, (x_k, y_k, z_k)\}$ and *evolving* them repeatedly (a *seed* is a solution of the equation).

For example, when $D = 2$, we can choose S_2 as $S_2 = \{(0, 1, 2), (0, 2, 1)\}$.

Let $C(D)$ be the minimum number of seeds needed to enumerate all nonnegative integer solutions of the equation.

It can be verified that $C(2) = 2$, $C(3) = 3$ and $C(100) = 18$.

Let $S(n) := \sum_{D=1}^n C(D)$. You are given $S(10) = 40$ and $S(100) = 1714$.

Find $S(3 \times 10^7)$.

[My Timing: 14.8 seconds (PyPy)]

Answer**0eb1affb9e606571bed2b0473822a94e**

418 **Fifty-Fifty****2017-07-26 07:17:47****by Min_25****8 xp****Math**

Let $p_{4,3}(n)$ be the smallest prime p such that $p > n$ and $p \equiv 3 \pmod{4}$ and let

$$p_{4,3}^k(n) := \begin{cases} p_{4,3}^{k-1}(p_{4,3}(n)) & \text{if } k > 1 \\ p_{4,3}(n) & \text{if } k = 1 \end{cases}.$$

Let $F(n) := \frac{n-1}{2}! \bmod n$.

For example, $F(3) = 1$ and $F(47) = 46$.

Let $S(n, m) := \sum_{k=1}^m F(p_{4,3}^k(n))$.

You are given $S(1, 32) = 1856$, $S(10^7, 32) = 150008239$ and $S(10^{12}, 32) = 18000000012930$.

Find $S(10^{18}, 32)$.

[My timing: 1.8 seconds (PyPy)]

Note: It would be hard without PARI/GP.

Answer **1306ef2697ec63221f6057ccce0d0987**

419 Bonus for p416

2017-08-04 12:14:24

by Min_25

8 xp

Math

Let $r_3(n) = \#\{(x, y, z) \in \mathbb{Z}^3 \mid x^2 + y^2 + z^2 = n\}$.

For example, $r_3(0) = 1$, $r_3(1) = 6$ and $r_3(100) = 30$.

Let $S(n, m) = \sum_{k=0}^{m-1} r_3(n+k)$.

It can be verified that $S(1, 100) = 4168$ and $S(10^8, 100) = 6410310$.

Find $S(10^{17}, 100)$.

[My timing: 2.6 seconds (PyPy)]

Note: It would be hard without PARI/GP and some papers.

Answer

cb648b1b3a04152fa503b4813e1e6448

420 Into the 4th dimension

2017-09-01 01:01:46

by Philippe_57721

5 xp

Math

Consider a sphere whose square of its radius is n and centered at $(0, 0, 0, 0)$ in the 4th dimension space. For a given n , the sphere touches a certain number of integer lattice points.

For instance, if $n = 7$, it touches the point $(1, 2, 1, 1)$.

Let $t(n)$ the number of integer lattice points touched by the sphere with square of radius n .

Find $\sum_{k=0}^{100} t(10^{17} + k)$.

You are given $\sum_{k=0}^{100} t(1000 + k) = 1042664$.

[My timing: 9 sec]

Answer 19bc582ed6e64925484663c1e6a81b5e

421 Enumerating the magic cubes

2017-08-11 08:06:12

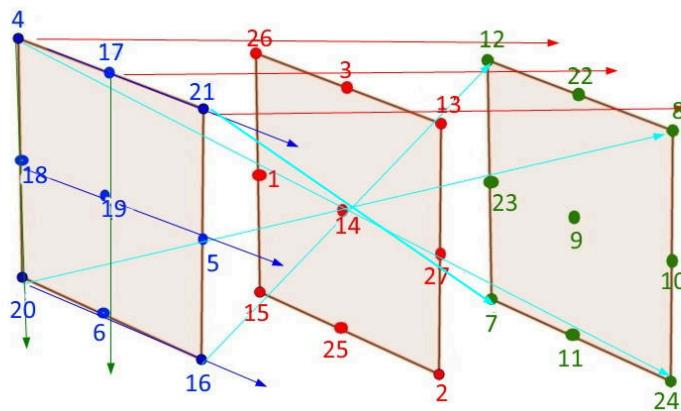
by Philippe_57721

8 xp

Programming

A magic cube is the 3-D equivalent of a magic square.

Here is an example of a $3 \times 3 \times 3$ magic cube with all integers from 1 to 27:



- The sum of all rows and columns in the 3 directions are equals

$$4 + 26 + 12 = 42$$

$$4 + 18 + 20 = 42$$

$$4 + 17 + 21 = 42$$

- The sum of the 4 main diagonals are equals

$$4 + 14 + 24 = 42$$

$$21 + 14 + 7 = 42$$

$$16 + 14 + 12 = 42$$

$$20 + 14 + 8 = 42$$

How many $3 \times 3 \times 3$ magic cubes with integers in $\{1, \dots, 27\}$ are there?

Answer format: Count:comma separated list of values for the cube of rank 123

Example: 999:4,17,21,18,19,5,20,6,16,26,3,13,1,14,27,15,25,2,12,22,8,23,9,10,7,11,24

(Corresponds to the cube in the above picture)

[My timing: 2 min]

PS: We ignore symmetries: facing the 6 faces of this example, we get 6 distinct solutions. To rank the cubes, represent them as a 27 elements vectors (as in the above example), then sort the vectors in lexicographic order.

Answer 296868f9e04f72b73c3462bb9389ae74

422 Expected Universe I

2017-07-31 16:19:07

by elasolova

6 xp

Probability

Initially the Universe does not exist. The God (or whatever you name it) sits idle within nothingness. Luckily the God has a little friend that keeps asking questions every day but just one for each day.

The little asker has 3 questions of the form:

- (1) Will you say YES to my next question?
- (2) Will you say NO to my next question?
- (3) Does the Universe exist now?

- (a) God can only answer as YES or NO. God always answers immediately within the same day.
- (b) The answer to (3) is always deterministic, namely there is only one choice always, in default case being always NO, but can only be forced to YES through a sequence of (1)s and (2)s.
- (c) If (1) or (2) asked right after (3) or at start, the answer is nondeterministic, namely God can answer at will as YES or NO. This is the only case for God to be nondeterministic.
- (d) A sequence of (1)s and (2)s which goes on after the preceding (1) or (2) is deterministic, and God has to answer possible successive (1)s and (2)s accordingly and never cheat.
- (e) If a (3) appears at the end of a sequence of (1)s and (2)s, the answer is obviously deterministic. But as noted in (c), a (1) or (2) appearing after a (3) or (3)s, will again cause a nondeterministic answer from God.
- (f) These rules go on until the God is forced to answer as YES to Universe existence question, and the Universe finally emerges.

Here is a sample run to get familiarized with the concept:

- A: Does the Universe exist now?
- G: NO (initially it does not exist!)
- A: Will you say YES to my next question?
- G: NO (nondeterministic, can say either ...)
- A: Will you say YES to my next question?
- G: NO (deterministic, has to say NO)
- A: Will you say NO to my next question?
- G: NO (deterministic)
- A: Will you say NO to my next question?
- G: YES (deterministic)
- A: Does the Universe exist now?
- G: NO (deterministic!)
- A: Will you say YES to my next question?
- G: YES (nondeterministic as form (3) breaks determinism)
- A: Does the Universe exist now?
- G: YES (finally!)

So the asker's choices are uniformly random, and also the God's will (when possible) is uniformly random. With these final assumptions, what is the expected number of days needed for the Universe to emerge? Round your answer to 3 decimal places.

Answer 3acfd93e8745bf57eb673b05352a5499

423 Expected Universe II

2017-07-31 16:19:29

by elasolova

6 xp

Probability

Initially the Universe does not exist. However, each day either a matter or anti-matter emerges uniformly randomly. The Universe is born when matter count exceeds that of anti-matter for the first time. In this case, what is the probability that the Universe emerges within 365 days?

Give the result as an exact fraction in lowest terms.

Answer

c2fedfdd7459c664b861cf2f9ccfe9df

424 Friedman Numbers

2017-08-18 08:07:09

by Philippe_57721

10 xp

Programming

A Friedman number is a number which can be written using **all its digits exactly once** and a combination of the following operations:

- + (addition)
- (subtraction)
- * (multiplication)
- / (division)
- \wedge (exponentiation)
- , (concatenation) (*)

Examples:

$$5776 = 76^{\wedge}(7-5) = 76 \ 7 \ 5 \ - \ ^$$

$$8092 = (90^{\wedge}2)-8 = 90 \ 2 \ ^ \ 8 \ -$$

$$11943 = (11^{\wedge}3-4)*9 = 11 \ 3 \ ^ \ 4 \ - \ 9 \ *$$

$$16225 = ((52^{\wedge}2)*6)+1 = 52 \ 2 \ ^ \ 6 \ * \ 1 \ +$$

Expressions are represented in [postfix notation](#).

Let's call the set of terms used in a decomposition, the **primitive factors**.

The Friedman number 1395 has two sets of primitive factors:

{15, 93}

{5, 9, 31}

For we have the following decompositions:

$$1395 = 15 \times 93$$

$$1395 = 5 \times 9 \times 31$$

In the range [13800, 13900], find the Friedman number with the most primitive factors. For each primitive factors of this number, find the corresponding decompositions and give the first one in alphabetic order (in postfix notation). (The primitive factors are sorted in lexicographic order: first by length, then by values)

Answer format: comma separated list for primitive factors 1:decomposition 1|...

Example: 2,9,16:16 9 2 \wedge *|1,2,6,9:6 9 1 - 2 / \wedge

in the range [1200, 1400]

[My timing: 100 sec]

* Concatenation can only be used to build the integers used in expressions, but not in expressions.

Answer 693547c406afb18b2b146fcaa4848054

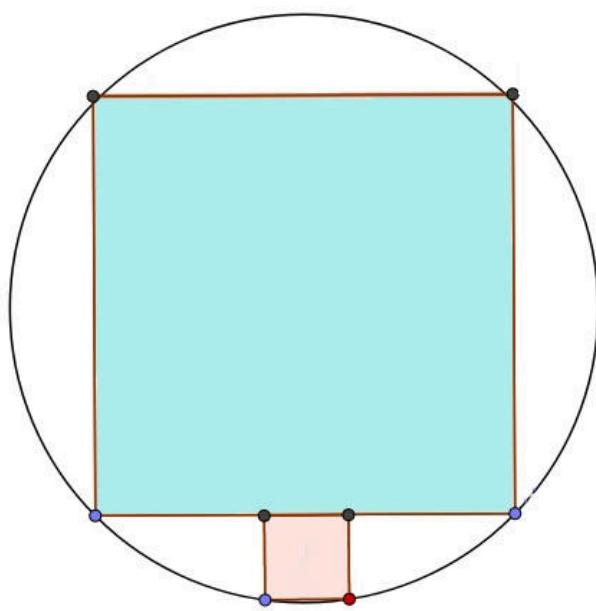
425 Squares ratio

2017-08-26 00:33:22

by Philippe_57721

5 xp

Math

 A = area of the large square a = area of the small squareWhat is $\frac{A}{a}$?**Answer** c81fb13777b701cb8ce6cdb7f0661f1b

426 Vampire Numbers

2017-09-08 01:17:55

by Philippe_57721

5 xp

Programming

A vampire number is a $2k$ -digits number n such as there exist two k -digits numbers a and b such as:

- $n = a \times b$
- the digits of a and b are exactly the digits of n

Example: $1001295697 = 19001 \times 52697$.

The two factors a and b are called, quite logically, the fangs.

Find the first and last 10-digits vampire numbers **not containing the digit 0** with prime fangs.

Answer format: `first, last`

Example: 11722657, 79168819 (for 8 digits)

$11722657 = 2267 \times 5171$, $79168819 = 8171 \times 9689$

[My timing: 82 sec]

Hint: The first digits of last are 945...

Answer**886607381f97896e844fda111130fec0**

427 BrainForce

2017-08-29 10:54:53

by sinan

8 xp

Hack

You need to force your brain to solve this one :)

Answer **4612ccc394501d69700ba51ace9eeb5a**

428 Bandpass audio signal

2018-05-18 02:33:29

by elasolova

8 xp

Crypto

An audio signal is intercepted. It is known that the signal includes double-side band suppressed carrier modulation. It is known that the target channel uses carrier frequency as 4000 Hz with a bandwidth of 1500 Hz.

More mathematically, a bandpass signal is formulated as:

$$s(t) = m(t) \cos(2\pi f_c t)$$

In such a case there is a single channel broadcasting the message $m(t)$ at carrier frequency f_c .

In case of multiple channels broadcasting it becomes:

$$s(t) = m_1(t) \cos(2\pi f_1 t) + m_2(t) \cos(2\pi f_2 t) + \dots$$

So, given $s(t)$ as this [audio signal](#), can you recover $m(t)$ within the bandwidth [2500 Hz, 5500 Hz]?

Please provide the keyword you hear in lower case.

Answer e238bee7ff15e80ec43df8a0d04c6180

429 Triangulation

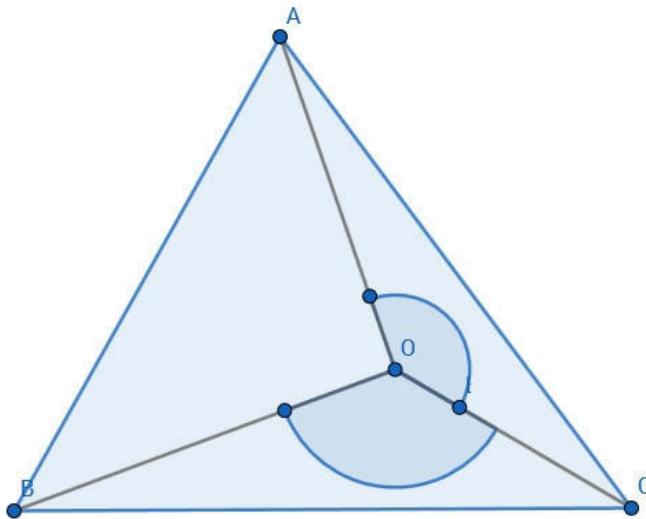
2017-09-29 03:39:15

by Philippe_57721

6 xp

Math

A point O is chosen inside an equilateral triangle ABC



The angle \widehat{AOC} is 105° .

The angle \widehat{BOC} is 115° .

We use the segments OA, OB, OC to build a new triangle.

What are the angles of this triangle?

Answer format: the sorted (ascending order) angles in degree, comma separated

Answer a46f1ade6a44cc5ec79e7b4bcac2aadd

430 Segments

2017-09-23 23:17:30

by sinan

6 xp

Programming

Let $\text{fib}(n)$ be n -th Fibonacci number. $\text{fib}(1) = 1$, $\text{fib}(2) = 1$ and $\text{fib}(n) = \text{fib}(n - 1) + \text{fib}(n - 2)$ for $n > 2$.

Let $f(n) = 1 + (\text{fib}(n) \bmod 5)$.

Let v_N be the vector of $f(i)$ values for $i = 1$ to N . For example $v_3 = [2, 2, 3]$.

Let T_N be the sum of the elements of V_N . For example $T_3 = 7$.

Consider an ABC triangle and a D point that bisects BC side: $|BD| = |CD|$.

Draw the AD segment and assume $N - 1$ points on it that further creates N segments such that the lengths of these subsegments are proportional to the elements of v_N starting from A to D .

$$s_i = v_N[i]/T_N \cdot |AD| \text{ for } i = 1 \text{ to } N$$

Now draw the line segments from C to AB that cross these $N - 1$ points on AD segment. So now we have N subsegments on AB side as well.

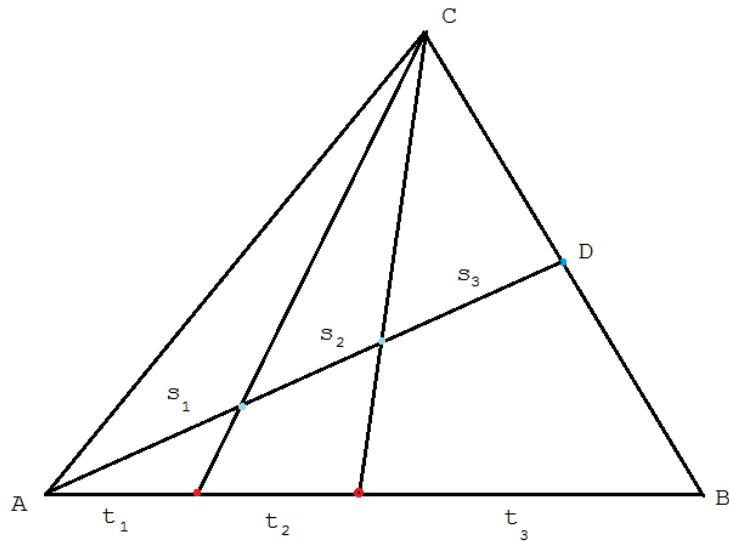
Let the lengths of these subsegments on the AB side be given as t_i for $i = 1$ to N starting from A to B .

Find the sum of ratios $|AB|/t_i$ for $i = 1$ to N where $N = 1919$.

Answer format: p/q

(reduced fraction)

You are given that the answer = $251/21$ for $N = 3$.



Answer b82155fb1e7e7e91eec43a7313eaa02

431 N-th Prime

2018-06-28 21:35:23

by Philippe_57721

6 xp

Programming

Let $P_n(k)$ the k -th prime number.

For instance, $P_n(1000000) = 15485863$.

Find $P_n(3141592653589793)$.

[My timing: 40 sec]

Answer**5133b521dae4df5d3693f0089566534d**

432 Subareas

2017-11-01 03:20:36

by sinan

8 xp

Programming

Let $\text{fib}(n)$ be n -th Fibonacci number. $\text{fib}(1) = 1$, $\text{fib}(2) = 1$ and $\text{fib}(n) = \text{fib}(n - 1) + \text{fib}(n - 2)$ for $n > 2$.

Let $f(n) = 1 + (\text{fib}(n) \bmod 5)$.

Let v_{M+N} be the vector of $f(i)$ values for $i = 1$ to $M + N$. For example $v_{2+2} = [2, 2, 3, 4]$.

Let T_M be the sum of the first M elements of V_{M+N} and T_N be the sum of the last N . For example $T_M = 4$ and $T_N = 7$ for $M = 2$ and $N = 2$.

Consider an ABC triangle with $M - 1$ points on AB side and $N - 1$ points on BC side.

Draw $M - 1$ line segments from C to the $M - 1$ points on the AB side thereby creating M line segments on the AB side proportional to the first M values of V_{M+N} starting from A to B .

$$|AB|_j = v_{M+N}[j] \cdot |AB| / T_M \text{ for } j = 1 \text{ to } M$$

Similarly draw $N - 1$ line segments from A to the $N - 1$ points on the BC side thereby creating N line segments on the BC side proportional to the last N values of V_{M+N} starting from B to C .

$$|BC|_i = v_{M+N}[M + i] \cdot |BC| / T_N \text{ for } i = 1 \text{ to } N$$

So now we have $M \cdot N$ subareas named a_{ij} where $1 \leq i \leq N$ and $1 \leq j \leq M$.

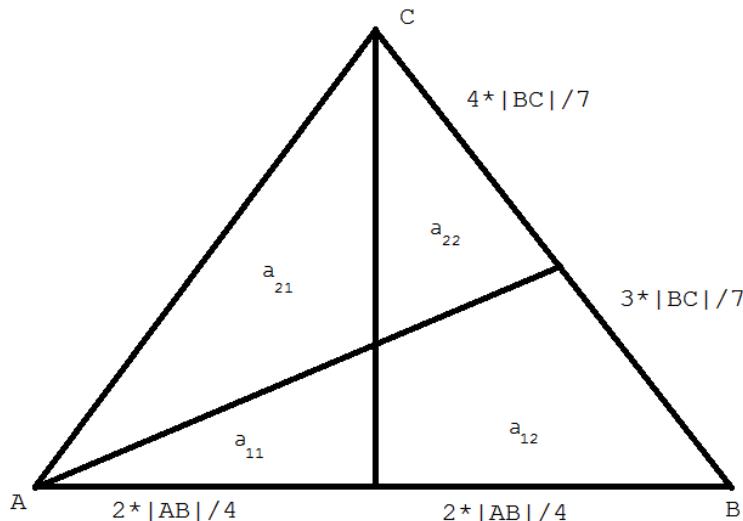
Let's assume that the area of ABC triangle is equal to 1.

Find the values of these $M \cdot N$ subareas for $M = 19$ and $N = 23$, sort them in ascending order and submit the answer as per the format given below.

Answer format: $\text{area}_{\text{first}}, \text{area}_{\text{median}}, \text{area}_{\text{last}}$
where areas are in the form of p/q (reduced fraction)

You are given that the answer = $3/22, 1/4, 4/11$ for $M = 2$ and $N = 2$.

Areas: $a_{11} = 3/22, a_{12} = 45/154, a_{21} = 4/11, a_{22} = 16/77$.



Answer

829525c84d29c06171ec3f8f28d6f352

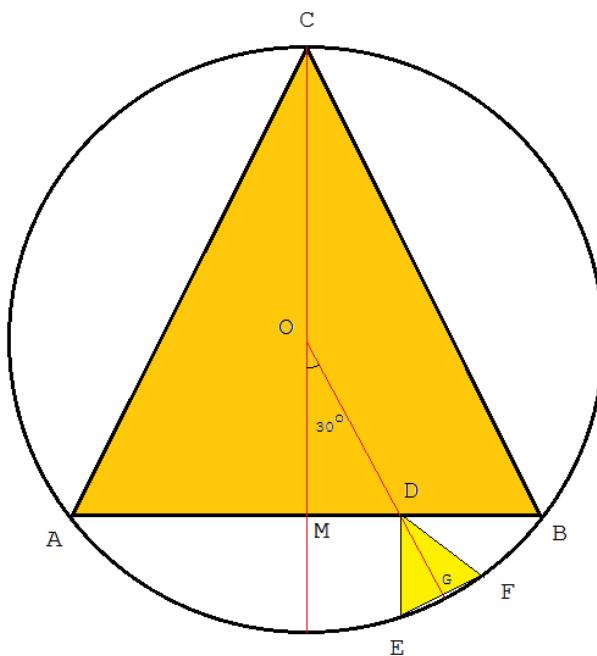
433 Triangles ratio

2017-10-01 00:57:17

by sinan

5 xp

Math



ABC is an equilateral triangle. DEF is another equilateral triangle with its two vertices (E, F) on the circumcircle of ABC and its third vertex (D) on the AB segment. O is the circumcenter of ABC .

What is the ratio $\text{area}(ABC)/\text{area}(DEF)$?

Answer format: number rounded to 5 digits after the decimal point

Answer 3604304b10145855f6d4d1137121e52d

434 Plowing the field

2017-09-22 01:06:20

by Philippe_57721

5 xp

Programming

10 farmers (A, B, \dots, J) are plowing a field.

A and B working together can plow the field in 2 days.

B and C working together can plow the field in 3 days.

C and D working together can plow the field in 5 days.

D and E working together can plow the field in 7 days.

E and F working together can plow the field in 11 days.

F and G working together can plow the field in 13 days.

G and H working together can plow the field in 17 days.

H and I working together can plow the field in 19 days.

I and J working together can plow the field in 23 days.

J working alone can plow the field in 29 days.

How many days does it take if all the farmers are working together?

Answer format: P/Q

(irreducible fraction)

Answer 60f925966a1529370633fc11e7b64d6c

435 Equations

2017-10-09 09:19:47

by sinan

6 xp

Math

The following equations are given:

- $x^2 + y^2 + x \cdot y = a$
- $y^2 + z^2 + y \cdot z = b$
- $z^2 + x^2 + z \cdot x = c$

Find **positive** (x, y, z) triples when $a = \sqrt{i+1}$, $b = \sqrt{i+2}$, $c = \sqrt{i+3}$ for $i = 1$ to N where $N = 1923$.

Answer format: $\sum x, \sum y, \sum z$

where each sum is a number rounded to 10 digits after the decimal point

Example: 2.4103394172, 2.1120641648, 2.7525737652 for $N = 3$.

Answer

4a23ad8f82464593805e7e523a0ec9c3

436 First semiprime

2017-10-06 01:23:01

by Philippe_57721

6 xp

Math

What is the least positive integer a such as $a^4 + 2^{1002}$ is [semiprime](#)?

[My timing: < 1 sec]

Answer**e77a86da0bbccff2cce55db4b444a43c**

437 Harmonic variations

2017-11-17 12:08:22

by Philippe_57721

12 xp

Programming

It is well known that the Harmonic serie $\sum_{n \geq 1} \frac{1}{n}$ is divergent.

A more surprising result is that if we remove all numbers containing at least one digit 9 the corresponding serie is convergent.

What is $\sum_{n \in S_{1,2,1,4}} \frac{1}{n}$.

Where $S_{1,2,1,4}$ is the set of all numbers with exactly one digit 2 and one digit 4 in their base-10 representation.

$$S_{1,2,1,4} = \{24, 42, 124, 142, 204, 214, 234, 240, 241, 243, 245, \dots\}$$

Answer format: give the result with the first 20 digits after the decimal point

[My timing: < 1 sec]

Answer

9b67edbc52342e9cb478f98ad90b7bb0

438 Best Matrices Multiplication

2017-10-13 01:49:58

by Philippe_57721

6 xp

Programming

Let said the cost for the product of 2 matrices of size (p, q) and (q, r) is $p \times q \times r$.

Let's take 4 matrices with respective sizes:

$A : (50, 20)$

$B : (20, 1)$

$C : (1, 10)$

$D : (10, 100)$

We have 5 different ways to multiply these matrices with very different costs:

$$A \times (B \times (C \times D)) = 103000$$

$$A \times ((B \times C) \times D) = 120200$$

$$(A \times B) \times (C \times D) = 7000$$

$$(A \times (B \times C)) \times D = 60200$$

$$((A \times B) \times C) \times D = 51500$$

The smallest cost is 7000.

We take 50 matrices with respective sizes:

```
(8737, 1458) (1458, 1629) (1629, 5104) (5104, 8634) (8634, 9493)
(9493, 1380) (1380, 898) (898, 3036) (3036, 5012) (5012, 2405)
(2405, 5697) (5697, 6520) (6520, 4777) (4777, 4031) (4031, 3752)
(3752, 7848) (7848, 7540) (7540, 2624) (2624, 2436) (2436, 4598)
(4598, 1020) (1020, 9050) (9050, 7031) (7031, 8692) (8692, 48)
(48, 396) (396, 2747) (2747, 4224) (4224, 9760) (9760, 8525)
(8525, 391) (391, 714) (714, 4590) (4590, 1455) (1455, 5437)
(5437, 4039) (4039, 6210) (6210, 7036) (7036, 5152) (5152, 4300)
(4300, 3511) (3511, 2484) (2484, 449) (449, 5683) (5683, 9809)
(9809, 4691) (4691, 3968) (3968, 3950) (3950, 5605) (5605, 1983)
```

What is the smallest cost for multiplying these matrices?

Answer format: cost

[My timing: < 1 sec]

Answer 3690123cd007ffae6b1c5bdd515663a3

439 Equilateral triangles 3

2017-10-30 04:08:20

by liuguangxi

9 xp

Math

Assume a point D **inside** an equilateral triangle ABC of side length t and with $AD = x$, $BD = y$, $CD = z$, $0 < x \leq y \leq z$, which is represented as quadruple (x, y, z, t) .

If x, y, z, t are all integers and they are coprime, the quadruple (x, y, z, t) is called primitive integer solution. For example, $(57, 65, 73, 112)$ is such a solution. And it's known that there are infinitely many solutions of primitive integer quadruple (x, y, z, t) .

Find all such solutions for $t \leq 5000$.

Answer format: [number of solutions],[sum of $x + y + z + t$ of all solutions]

Example: 3, 1237 for $t \leq 200$

(explanation: there are total 3 solutions $(57, 65, 73, 112)$, $(73, 88, 95, 147)$, $(43, 147, 152, 185)$)

Answer**b2a3528f1e5482cfbf3a555ecb44b43a**

440 Special squarefree sum

2017-11-13 03:19:39

by liuguangxi

12 xp

Math

A [squarefree number](#) is called special if it can be expressed as the sum of two positive squares. For example, 65 ($= 1^2 + 8^2$) is a special squarefree number while 15 is not.

Let $S(N)$ be the sum of all special squarefree numbers from 1 to N . You are given that $S(10) = 17$, $S(1000) = 81665$, $S(10^6) = 61081126084$.

Find $S(10^{12})$.

Answer

22a016522b8ac46cdc2fe5627b8d370c

441 Enhanced perfect power

2017-11-06 02:59:43

by liuguangxi

9 xp

Math

A [perfect power](#) n is called enhanced if there exists two integers $m > 0$ and $k > 2$ such that $n = m^k$. The enhanced perfect powers no more than 100 are 1, 8, 16, 27, 32, 64, 81.

Let $S(N)$ be the sum of all enhanced perfect powers no more than N . You are given $S(100) = 229$, $S(10000) = 99352$ and $S(10^8) \bmod 1000000007 = 553225565$.

Find $S(10^{100}) \bmod 1000000007$.

Answer

f478879b29e0495fde8227fca7f8b9d9

442 **Triple pandigital****2017-10-23 03:59:06****by liuguangxi****6 xp****Programming**

Consider a 10-digit [pandigital number](#) n in base 10. If we take first and last half of n , two 5-digit numbers x and y are got. Specially, square of x and square of y are also both 10-digit pandigital.

Find all pandigital numbers n satisfying above properties. Note: leading zero is not allowed in any numbers.

Answer format: [count],[sum]

(sample input: 1,1023456789)

Answer **65a1f095a7bf0f1f16925e1ae1d89c37**

443 Polygons ratio

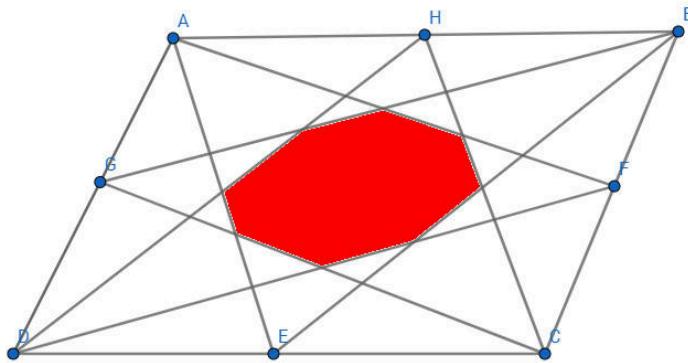
2017-10-20 01:31:10

by Philippe_57721

6 xp

Math

$ABCD$ is a parallelogram.



From each vertex, we join the middle of the 2 opposite sides

F is the middle of BC

E is the middle of CD

We draw the lines AF and AE

We draw thus 8 lines which define an octagon.

What is $\frac{\text{Area}(\text{Octagon})}{\text{Area}(\text{Parallelogram})}$.

Give your answer rounded to 10 digits after the decimal point.

Answer 590f32e54e7b7b4163ec2c202f87fcdc

444 Kolakoski Sequence**2017-11-29 18:06:37****by Min_25****12 xp****Math**

Let $K(a, b)$ be the infinite sequence such that:

- The first element is a ,
- It consists of a and b , and
- Its run length sequence is equal to $K(a, b)$.

Here, for example, we assume that the run length sequence of $[1, 1, 1, 2, 3, 4, 4, 5]$ is $[3, 1, 1, 2, 1]$.

We can verify that

- $K(2, 3) = [2, 2, 3, 3, 2, 2, 2, 3, 3, 3, 2, \dots]$, and
- $K(2, 4) = [2, 2, 4, 4, 2, 2, 2, 2, 4, 4, 4, 4, 2, \dots]$.

Let $C_{a,b}(N)$ be the number of a in the first N elements of $K(a, b)$.

For example, $C_{2,4}(4) = 2$ and $C_{2,4}(5) = 3$.

Let $S(M, N) := \sum_{a=2}^{M-1} \sum_{b=a+1}^M C_{a,b}(N)$.

You are given $S(5, 100) = 300$ and $S(10, 10^6) = 17856847$.

Find $S(100, 10^{15})$.

Answer **902e53f7063b9d7ace6e26819cc697e8**

445 Central binomial coefficients

2017-11-20 03:33:37

by liuguangxi

8 xp

Math

The binomial coefficients can be arranged in triangular form like this:

		1					
	1		1				
1		2		1			
1	1		3	3	1		
1	1	4	6	4	1		
1	1	5	10	10	5	1	
1	1	6	15	20	15	6	1
1	1	7	21	35	35	21	7
							1
						

It is known as Pascal's triangle.

The middle coefficients of the even-numbered rows (row numbering starts from zero) form a sequence $a_n (n \geq 0)$: 1, 2, 6, 20, 70, 252, 924, 3432, 12870, 48620, 184756, It has been proven that no number a_n with $n > 4$ is squarefree.

Let $f(n)$ be n divided by largest squarefree divisor of n . For example, $24 = 2^3 \times 3$ and its largest square-free divisor is 6, so $f(24) = 24/6 = 4$.

Let $S(N)$ be the sum of all $f(a_n)$ for n from 1 to N . You are given $S(10) = 1 + 1 + 2 + 1 + 6 + 2 + 4 + 3 + 2 + 2 = 24$, $S(100) = 429083$ and $S(10^3) \bmod 1000000007 = 258384246$.

Find $S(10^6) \bmod 1000000007$.

Answer 913e4e8f7ea6c186005d59c03c2831e4

446 A big sum 2

2017-11-27 02:59:05

by liuguangxi

12 xp

Math

Let

$$S(m, n) = \sum_{k=1}^m \sum_{x=1}^n \left(x^k \cdot \left\lfloor \frac{\pi x}{k} \right\rfloor^k \right)$$

where $\lfloor x \rfloor$ is the largest integer less than or equal to x .You are given $S(1, 10) = 1182$, $S(3, 100) = 16716537706541$ and $S(10, 10^4) \bmod 1000000007 = 211872856$.Find $S(20, 10^{20}) \bmod 1000000007$.Hint: solve [Problem 401 - A Big Sum](#) first and there are some clues in the secret forum.**Answer****a10332a0e65dfad81d51b0f18251e66a**

447 Sum of powers**2017-10-27 07:47:50****by Philippe_57721****6 xp****Programming**

Find $\left(\sum_{i=1}^{10^{100}} i^{100} \right)$ modulo 1000000007.

[My timing: < 3 sec]

Answer**e705b34db09f5847f1815eb900450939**

448 Generalized ellipse 1

2017-12-11 03:00:08

by liuguangxi

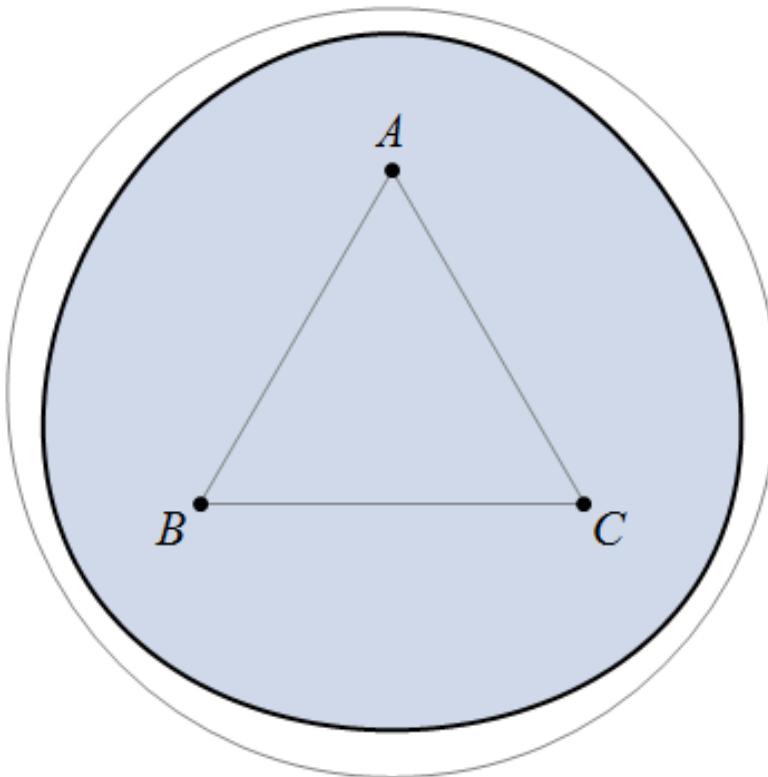
8 xp

Math

It is known that a circle is a curve in a plane that the distance from a given point is constant for every point on the curve. And an ellipse is a curve in a plane surrounding two focal points such that the sum of the distances to the two focal points is constant for every point on the curve.

We can generalize the definition of ellipse to *n*-ellipse. Given *n* points (called foci) in a plane, an *n*-ellipse is the locus of all points of the plane whose sum of distances to the *n* foci is a constant.

There is an equilateral triangle *ABC* of side length 1 in a plane. A 3-ellipse curve is the set of all points of the same plane whose sum of distances to the three points *A*, *B* and *C* is 3 (see the thick curve line in the picture). And the curve can be put inside the circle with a radius of 1.



Find the enclosed area and circumference of the closed curve above. Give your answer rounded to 6 digits after the decimal point.

Answer format: [area],[circumference]

Example: 3.141593, 6.283185 for a circle with a radius of 1.

Answer 6162aca21a7ea112a5919c5000b6e369

449 Generalized ellipse 2

2017-12-18 03:00:58

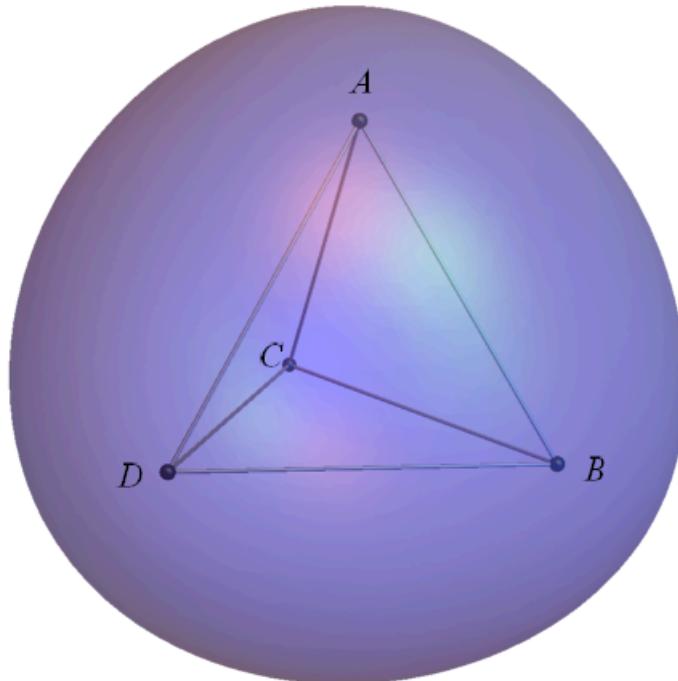
by liuguangxi

10 xp

Math

The definition of *n*-ellipse is as below: given n points (called foci) in a plane, an *n*-ellipse is the locus of all points of the plane whose sum of distances to the *n* foci is a constant. We can generalize the definition from plain to space, and the set of all points satisfying the constraint become a surface.

There is a tetrahedron $ABCD$ of edge length 1 in a space. A 4-ellipse surface is the set of all points of the same space whose sum of distances to the 4 points A, B, C and D is 4 (see the blue solid in the picture).



Find the enclosed volume and surface area of the closed surface above. Give your answer rounded to 6 digits after the decimal point.

Answer format: [volume],[surface area]

Example: 4.188790, 12.566371 for a sphere of radius 1.

Answer 7fd2c8c8fb08ac4c5e867c0f5151fb9b

450 Best Matrices Multiplication 2

2017-11-21 02:53:39

by Min_25

9 xp

Math

Let A be an infinite sequence of matrices $A := (A_1, A_2, \dots)$.

A_i has x_i rows and x_{i+1} columns, where

$$x_n = 5000 + (r_n \bmod 1000)$$

and

$$r_1 = 2, \quad r_n = r_{n-1}^2 \bmod 98765431 \quad (n \geq 2).$$

Let $C(n)$ be the minimum cost of matrix multiplication of A_1, \dots, A_n . (see [Problem 438](#) for the cost.)

For example, $x_1 = 5002$, $x_{11} = 5053$, $C(3) = 257423728320$ and $C(10) = 1290936750576$.

Find $C(6543)$.

Answer

014553c7b8c0828b8bc348d4a4cf108a

451 Machin Like Formulae

2017-12-01 01:27:44

by Philippe_57721

10 xp

Programming

One of the most famous formulae for computing π is $\pi/4 = 4 \cdot \arctan(1/5) - \arctan(1/239)$ due to John Machin (1706) who used it to compute 100 decimal places.

We can rewrite it as $\arctan(1/239) = -\arctan(1/1) + 4 \cdot \arctan(1/5)$.

It turns out that many integers like 239 have an inverse which can be decomposed as a linear combination of Arctan of inverses.

A few examples:

$$\arctan\left(\frac{1}{3}\right) = \arctan(1) - \arctan\left(\frac{1}{2}\right)$$

$$\arctan\left(\frac{1}{7}\right) = -\arctan(1) + 2 \cdot \arctan\left(\frac{1}{2}\right)$$

$$\arctan\left(\frac{1}{8}\right) = \arctan(1) - \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{5}\right)$$

$$\arctan\left(\frac{1}{13}\right) = \arctan(1) - \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{4}\right)$$

$$\arctan\left(\frac{1}{17}\right) = -\arctan(1) + 2 \cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{12}\right)$$

$$\arctan\left(\frac{1}{18}\right) = \arctan(1) - 2 \cdot \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right)$$

$$\arctan\left(\frac{1}{21}\right) = \arctan\left(\frac{1}{4}\right) - \arctan\left(\frac{1}{5}\right)$$

$$\arctan\left(\frac{1}{30}\right) = \arctan(1) - \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{4}\right) - \arctan\left(\frac{1}{23}\right)$$

$$\arctan\left(\frac{1}{31}\right) = \arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{6}\right)$$

$$\arctan\left(\frac{1}{32}\right) = -\arctan(1) + 2 \cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{9}\right)$$

The irreducible numbers, those which cannot be decomposed this way are called Størmer numbers. Thus, the inverse of every number which is not a Størmer number can be expressed as a linear combination of Arctan of inverse of smaller irreducible integers.

What makes John Machin's formula particularly efficient is the fact that it contains few terms and the numbers involved are not too small: the convergence of the Arctan serie is then quite fast.

Let's define the cost of a formula as the sum of the inverse of the distinct integers without 1.

$$C(32) = \frac{1}{2} + \frac{1}{9} + \frac{1}{32} = 0.642361$$

Numbers like 21 or 31 are not suitable for a computation of π as they do not include the number 1 in their decomposition, their cost will be infinity.

What are the 10 most efficient numbers less than 20000?

Answer format: comma separated list in ascending order of cost

Example: 41, 46, 75, 17, 32, 7, 68, 93, 43, 57 (for numbers less than 100).

$$C(41) = 0.6077235772 \rightarrow \arctan\left(\frac{1}{41}\right) = \arctan(1) - 2 \cdot \arctan\left(\frac{1}{2}\right) + 2 \cdot \arctan\left(\frac{1}{12}\right)$$

$$C(46) = 0.6421095008 \rightarrow \arctan\left(\frac{1}{46}\right) = -\arctan(1) + 2 \cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{12}\right) - \arctan\left(\frac{1}{27}\right)$$

$$C(75) = 0.6421212121 \rightarrow \arctan\left(\frac{1}{75}\right) = -\arctan(1) + 2 \cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{12}\right) - \arctan\left(\frac{1}{22}\right)$$

$$C(17) = 0.6421568627 \rightarrow \arctan\left(\frac{1}{17}\right) = -\arctan(1) + 2 \cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{12}\right)$$

$$C(32) = 0.6423611111 \rightarrow \arctan\left(\frac{1}{32}\right) = -\arctan(1) + 2 \cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{9}\right)$$

$$C(7) = 0.6428571429 \rightarrow \arctan\left(\frac{1}{7}\right) = -\arctan(1) + 2 \cdot \arctan\left(\frac{1}{2}\right)$$

$$C(68) = 0.6813725490 \rightarrow \arctan\left(\frac{1}{68}\right) = 2 \cdot \arctan(1) - 3 \cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{6}\right)$$

$$C(93) = 0.6899193548 \rightarrow \arctan\left(\frac{1}{93}\right) = \arctan(1) - 2 \cdot \arctan\left(\frac{1}{2}\right) + \arctan(16) - \arctan\left(\frac{1}{80}\right)$$

$$C(43) = 0.6899224806 \rightarrow \arctan\left(\frac{1}{43}\right) = \arctan(1) - 2 \cdot \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{6}\right)$$

$$C(57) = 0.7175438596 \rightarrow \arctan\left(\frac{1}{57}\right) = -2 \cdot \arctan(1) + 3 \cdot \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right)$$

[My timing: < 10 sec]

Answer **085776fb8a3e94c5daa676bbfe727bfd**

452 A Staggering Sequence**2017-11-10 03:05:49****by Philippe_57721****8 xp****Programming**

Let the following sequence:

- $c_1 = 2$
- $c_n = c_{n-1} \cdot (c_{n-1} + n + 140)/n$

What is the first index n such as c_n is not an integer?

How many digits does the integer part of c_n have?

Answer format: n, number of digits

[My timing: < 1 sec]

Answer aafe1a17bf4bbb02ecc84873ab315baf

453 Value of dot product

2017-12-25 02:40:56

by liuguangxi

7 xp

Probability

There are two random vectors $\mathbf{a} = [a_1, a_2]$ and $\mathbf{b} = [b_1, b_2]$. Let s be the dot product of these two vectors, that is $s = \mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2$.

Assume a_1, a_2, b_1, b_2 are mutually independent random variables of the uniform distribution on the interval $[0, 2]$. Find the probability of value s less than 5 and the expected value of s . Give your answer rounded to 8 digits after the decimal point.

Answer format: [probability],[expected value]
(sample input: 0.12345678,1.23456789)

Thanks to **baihacker** for the idea.

Answer

e296012720ce31335b4dcda915d8d72

454 Equilateral triangles 4

2018-01-01 03:05:35

by liuguangxi

9 xp

Probability

Assume a point D **inside** an equilateral triangle ABC of side length $\sqrt{3}$. Then we can use the three segments AD , BD and CD to build a new triangle PQR .

When point D is in the center of triangle ABC , the area of triangle PQR is $\sqrt{3}/4$, which is the maximum value for all possible positions of D ; and the circumference of PQR is 3, which is the minimum value. Moreover, when point D is close to the vertex of triangle ABC , the area of triangle PQR is close to zero; and the circumference of PQR is close to its maximum value $2\sqrt{3}$.

Assuming that point D is chosen randomly (with **uniform distribution**) within triangle ABC , find the expected area and circumference of triangle PQR . Give your answers rounded to 8 digits after the decimal point.

Answer format: [expectation of area],[expectation of circumference]
(sample input: 0.43301270,3.46410162)

Answer**286d9511dccba0077fb462470fc19cae**

455 A cubic form

2018-01-10 08:19:35

by sinan

8 xp

Math

Let $D(x, y, z) = x^3 + y^3 + z^3 - 3 \cdot x \cdot y \cdot z$.

It can be proven that for any integers $x_1, y_1, z_1, x_2, y_2, z_2$ there exist integers x_3, y_3, z_3 such that:
 $D(x_1, y_1, z_1) \cdot D(x_2, y_2, z_2) = D(x_3, y_3, z_3)$

Let N be the number of (x_3, y_3, z_3) triples of the form $x_3 \leq y_3 \leq z_3$ for $D(x_1, y_1, z_1) \cdot D(x_2, y_2, z_2) = D(x_3, y_3, z_3)$.

Find the N and the sum of all z_3 values for $D(34040, 34238, 35404) \cdot D(34551, 34564, 34567)$.

Answer format: N, sum

Example: 12,548 for $D(1, 2, 3) \cdot D(11, 12, 13)$

[My timing: < 1 m]

Answer**6aefb260a93a59a569429a78e1f0b396**

456 Automatic typewriter

2018-01-08 03:01:12

by liuguangxi

10 xp

Probability

An automatic typewriter can generate a random digit string by generating a sequence of random digits and concatenating them together. Each digit is chosen independently from 0 to 9 with probability $1/55$, $2/55$, $3/55$, $4/55$, $5/55$, $6/55$, $7/55$, $8/55$, $9/55$ and $10/55$, respectively. The generation is stopped as soon as a specific pattern occurs in the random string.

$D(n)$ is a string consisting of first n digits of π . For example, $D(3) = 314$, $D(10) = 3141592653$.

Let $E(n)$ be the expected length of the generated random string for the string pattern $D(n)$. For example, $E(3) = 4159.375$, $E(10) = 104702034619.87625\dots$. Usually $E(n)$ is a decimal number.

Let $S(n)$ be $E(n)$ rounded to the nearest integer. You are given $S(3) = 4159$, $S(10) = 104702034620$ and $S(50) = 1812330404172820790558529569081392090013172561045583$.

Find $S(10000)$.

As the answer is a very big number, use the following condensed representation:

(First 10 digits)[(number of remaining digits)](Last 10 digits)

For instance, the representation of 2^{127} is: 1701411834[19]5884105728.

Answer format: [condensed representation of $S(10000)$]

Example: 1812330404[32]2561045583 for $S(50)$.

Answer**9adbf1cf04df928c09cd7d4d4253dd2f**

457 Broken DNA

2017-11-24 00:39:25

by Philippe_57721

7 xp

Programming

A DNA molecule is made of 2 strands of nitrogenous bases: A, C, G or T.

Each base in the 1st strand is associated with one in the 2nd according the following pairing rules:

- A with T
- C with G

Example:

```
T G C T A A C C G G
| | | | | | | |
A C G A T T G G C C
```

The 2 strands are separated and broken into fragments of random length:

TGCTAACCGG => TGCT, AACC, GG

ACGATTGGCC => ACGAT, TGG, CC

Then all the fragments are shuffled. For instance: CC, TGG, GG, TGCT, ACGAT, AACC.

You are given the following 65 fragments:

```
CCGTCCGTTGCC, TGTCTTGAGCTTCGTGCTGAGCACATGTAATCATAC, TTATT
TGTACAAGCGCCTTAATGTTATACAGCATTAACTACTC
GTGAGTGAATGGGTTATAAACCGGCTGTACGGCTGTTGGGTGACTGGAGCTACTGCCA
AGATGGTCAACCCGCCACATCTCTAACATC, ACCTCTCTGCCTCACAGCAAGCATGATT
CGAAGCATTGAAACAATT, ACCGAACGACTGACATACGCTTTAAATTCTACCAGTTGGACGGTAGAG
ACGTAGGGTATACATTAACAAACCGTTATAGATAT
TTTACACTTCTGTCTTCCACCGCAATGTCTGATTCTAACGCA
GCCGGCTCTAACCTCGAGACCTATAAATCAAATTGTTAGCTTCAAG, CACACT
ACATCTGACAGTGTGGAAAAAGACGTTATGTTAGCGGGGT, GTGAGTAC
ATTTGGATCGCTGTGCGTGATTATTAATCGGGCATAGCTG
CACATTCTCCAACGCCACAAATGCTGACTTCAA, GCCATTTCGCCCGGAGATTGGAGCTGGATATTT
CGACTCGTGTA, CAATCGCGGATATCTTGAGTATACG, ATTAAGCACCCCTCGCTAACGGCGAAC
TAACTCGGTCTCGTAACTTGTAAATGGCTTGACTGTATGCGAAAATTAA, CATTAGTATGTGGAGAACAGC
ATACAGATTAAATAACAGTAGCTAGGGAGCCACTCATGGTGAAGAAGGTTGCG
AGTTCGCTGCAACTCCGGGAGAA, GTGTTTACGACTGAAGATTGTTAGACTGTCACACCTTTCTGCAA
AAAGGTGGCGTTACAGACATAAGATTCTGCGTGAGCAAACAGTAAGTGGAGACTGAG, ATTACGTTAAC, CGCC
CCGACTGGCAGGCAACGGGAAATGTAAGAACAG, AATAATTAGCCCGTATCGACCACAGGT, ATCCCTCG
CGCGTGCCTATGCCGACCATATTGCATCCCATATGTAATTGTTGGCAATATCTATAT
GGGCCTAACGAGTTCTGCAAGGCTCAT, AGGTTAACATCGA
CACTTACCAATATTGCCGACATGCCGACAACCCACATGACCTCGATG, CCCCGCGTACAGCTAAAATAGGCA
CAGGTGTTAGCGCCTATAGAACTCATATGCATTCAAGGAG, GGCT
CGCTTAATGCCCGGAGTTATCTCCGAGC
CAAGGTCCCGGAGTAGGCCGGATTGCTCAAGACAGTTCCGAGTAGCAGC, GATAACTCGGTGGCACCA
ACGGTATC, TGGTAACAATGCTGCAAGAG
GCATCCACTATTGCGGATTAGTGGGTTCCAGGGCGCTCATCC, AGGGCATGTG
CGTCGAATAAACATGTTCGCGAAATTACAATATGGCGTAATTAGATGAGATTGAGGCCA
GGAAGTGGTCGTTGTAACACTATTGAGCCACCGTGGTGGCACGGATACGGCTGGTATA
CAAAGAAGTCCTGCACACCGTGTAAACCTAGCGGACAGCGCACT, AATACGGGGGAGCCTACGTCT
TAAGTTCCCT, GAGTTGAGTATGCTAAATTATTATGCATCG
TACAATGCCCGAGCGAATTAGCGGGCGTCAATAGAAGGCGTGGGCTGA
```

```
GACTCGTTTCTTCAGGACGTGTGGCACA  
CGACGTTGAGGCCGTCTTATGGCCCCCTCGGATGCAGATAATGCAATTAGGT, GTGTCCACCGA, TAGTC  
CGTCGTTGTCATTACCTCT, AAATTTCCCCATCGCGCTCTCGTAGGTGATAAGTAACGCCAATCACC  
TTAAAAGGGTAGCGCGCAGAC, CCGGTAAAGCGCG, ATGTCCA, TTGACCATTGTTACGACGGTTCTCTA  
TAATTCGTGGGAGCGATTCC, CCGTACACGGGCCGATGTCGATTTTATCCGTACAGAACTCGAAGCA
```

Your task is to reconstruct the original DNA molecule from the fragments.

Answer format: the MD5 hash (lowercase hexa) of the comma separated fragments for the 1st strand in alphabetic order (The solution is unique)

Example: cfe6feb5eb9e4d176ac4bdc81b722ede (MD5 for ACGAT, TGG, CC).

[My timing: instant]

Answer **608e076ec7d3d9f400369190d880adc7**

458 Sum of phi**2018-01-15 03:01:26****by liuguangxi****11 xp****Math**

Let

$$S(n) = \sum_{i=1}^n \sum_{j=1}^n \varphi(i \cdot j)$$

where $\varphi(x)$ is Euler's totient function.You are given $S(2) = \varphi(1 \times 1) + \varphi(1 \times 2) + \varphi(2 \times 1) + \varphi(2 \times 2) = 5$, $S(50) = 696335$ and $S(5000) \bmod 1000000007 = 397838656$.Find $S(50000000) \bmod 1000000007$.**Answer** **be569d112c425ee7c155293356912323**

459 Integral circle packings 1

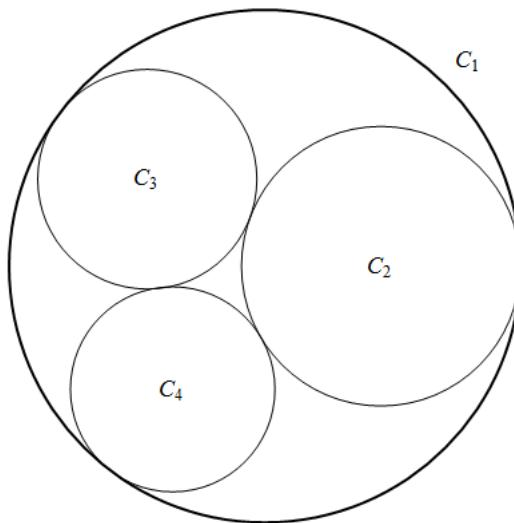
2018-01-29 03:09:00

by liuguangxi

9 xp

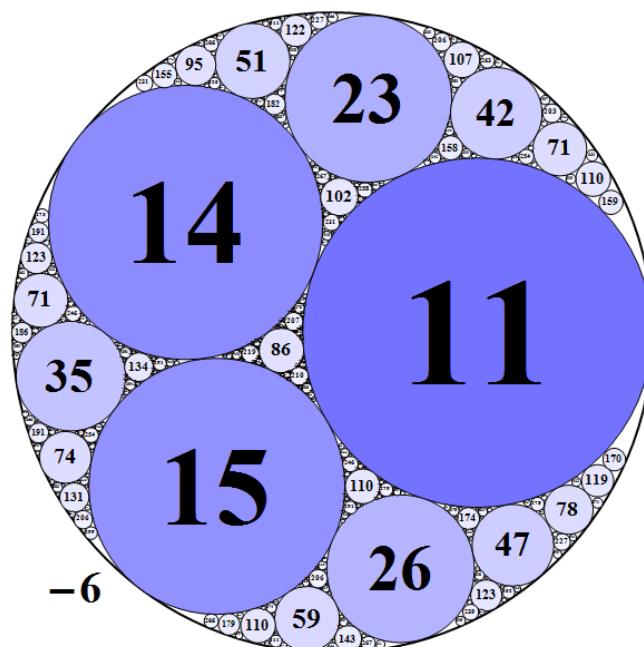
Math

Three circles C_2 , C_3 and C_4 are placed inside a larger circle C_1 such that each pair of circles is tangent to one another and the inner circles do not overlap.



We can repeatedly fill the interstices between mutually tangent circles with further tangent circles and generate a circle packing. Denote the curvature of a circle be the reciprocal of its radius. Interestingly, every circle in such a packing would have integer curvature if the curvatures of four mutually tangent circles C_1, C_2, C_3 and C_4 are integers, and we call such a packing an integral circle packing.

The figure below shows an integral circle packing with curvatures of four circles $-6, 11, 14$ and 15 (each circle is labeled with its curvature and only part of circles are shown).



Define an integer quadruple (a, b, c, d) with $a < 0 < b \leq c \leq d$ representing the curvatures of initial four mutually tangent circles C_1, C_2, C_3 and C_4 , which determines the integral circle packing. The negative value here represents the negative curvature of the bounding circle. For example, the above integral circle packing can be represented as $(-6, 11, 14, 15)$.

If greatest common divisor of a, b, c, d is 1, the quadruple (a, b, c, d) is called **primitive** quadruple. A quadruple (a, b, c, d) is a **root** quadruple if the curvature of any circle other than initial four circles in the circle packing is not less than d . For example, $(-6, 11, 14, 15)$ is a root quadruple but $(-6, 11, 14, 23)$ not, because there exists curvature value 15 in the circle packing which is less than 23.

All curvatures (negative signs are omitted) of an integral circle packing form an infinite integer set. Especially there are infinite primes in the set if the quadruple for packing is primitive. For example, for primitive quadruple $(-3, 4, 12, 13)$, the primes in the set are 3, 13, 37, 61, 109, 157, 181, 229 and so on. Let $SP(a, b, c, d, n)$ be sum of distinct primes no more than n in the set with quadruple (a, b, c, d) . For instance, $SP(-3, 4, 12, 13, 100) = 114$. Let $S(t, n)$ be sum of all $SP(a, b, c, d, n)$ with **primitive root** quadruple (a, b, c, d) satisfying $d \leq t$.

You are given $S(10, 100) = SP(-1, 2, 2, 3, 100) + SP(-2, 3, 6, 7, 100) + SP(-3, 5, 8, 8, 100) + SP(-4, 8, 9, 9, 100) = 299 + 251 + 90 + 147 = 787$.

Find $S(50, 10^7)$.

Answer **0be24f127ddb3bfc31506ca5052e82**

460 Integral circle packings 2

2018-02-05 03:49:55

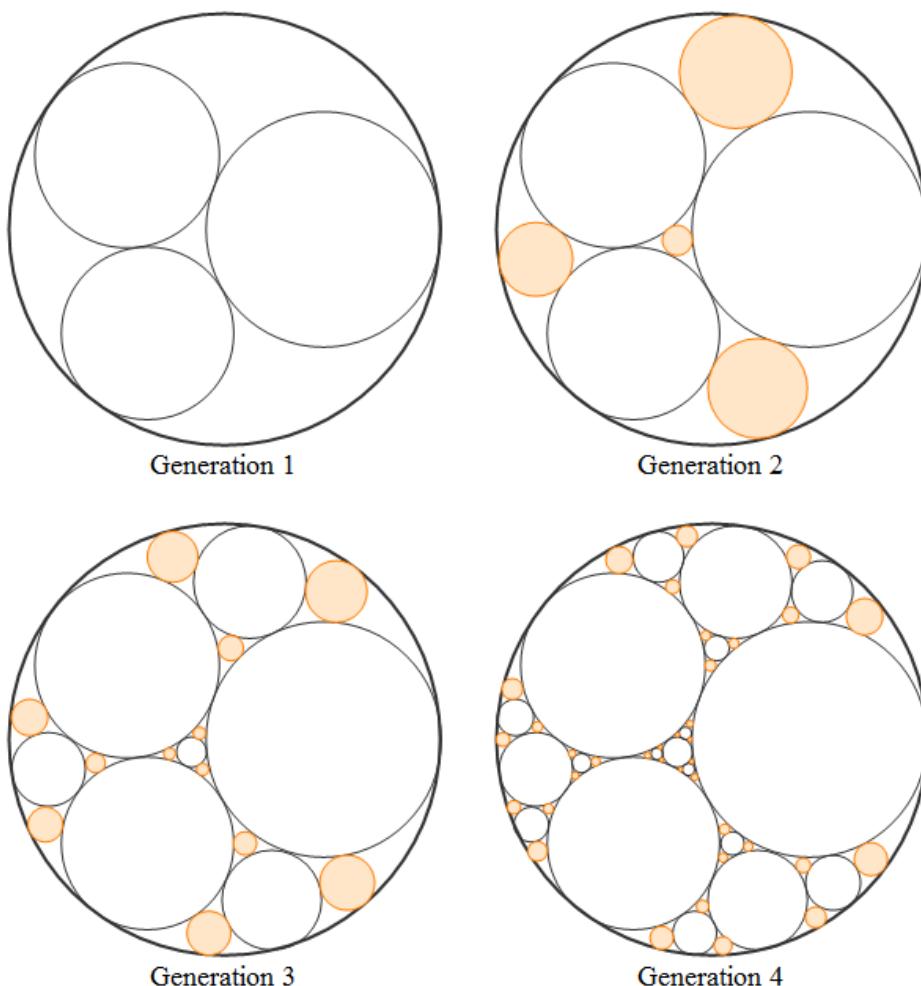
by liuguangxi

10 xp

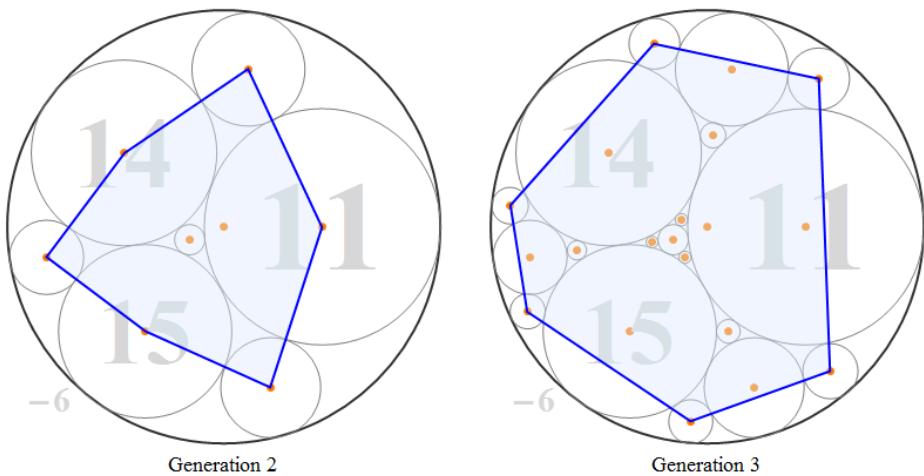
Math

See [Problem 459 — Integral circle packings 1](#) for the concept of integral circle packing and quadruple.

According to the circles generated sequence in a circle packing, define the generation as below: the 4 mutually tangent circles are the first generation; fill the 4 interstices between them with 4 tangent circles forming the second generation; and repeatedly fill the interstices between mutually tangent circles of previous generation with further tangent circles forming the new generation. The circles of the first 4 generations are shown below. Actually the number of new generated circles in the n -th ($n \geq 2$) generation is $4 \times 3^{n-2}$.



For a certain generation in a circle packing, a convex hull can be formed from the set of center points of all circles. The figures below show the convex hull (blue polygon) of the 2nd and 3rd generation for integral circle packing with quadruple $(-6, 11, 14, 15)$, respectively.



It can be calculated that the area of convex hull of the 2nd generation is $670/23023$, and that of the 3rd generation is $45650065/866875849$. In fact, the area of convex hull of any generation for any integral circle packing is a rational number.

Define $R(a, b, c, d, n)$ be the ratio of the area of convex hull of the n -th generation to the area of the bounding circle with quadruple (a, b, c, d) . For example, $R(-6, 11, 14, 15, 2) = (670/23023)/(\pi/62) = 0.3334767170$, and $R(-6, 11, 14, 15, 3) = (45650065/866875849)/(\pi/62) = 0.6034442099$. Obviously the area ratio tends to 1 when n tends to infinity. Let $S(t, n)$ be sum of all $R(a, b, c, d, n)$ with **primitive root** quadruple (a, b, c, d) satisfying $d \leq t$.

You are given $S(10, 3) = R(-1, 2, 2, 3, 3) + R(-2, 3, 6, 7, 3) + R(-3, 5, 8, 8, 3) + R(-4, 8, 9, 9, 3) = 2.2572820988$ (rounded to 10 digits after the decimal point).

Find $S(50, 10)$. Give your answer rounded to 8 digits after the decimal point.

Answer **09ce2ac8475da7e93e4700a552410480**

461 Convergents of infinite sum

2018-02-12 03:20:16

by liuguangxi

9 xp

Math

Define a sequence b_n as below: $b_0 = c, b_n = b_{n-1}^2 - 2$ ($n \geq 1$), where c is a positive integer greater than or equal to 3.

Let infinite sum s be

$$s = \sum_{n=0}^{\infty} \frac{1}{\prod_{k=0}^n b_k} = \frac{1}{b_0} + \frac{1}{b_0 b_1} + \frac{1}{b_0 b_1 b_2} + \frac{1}{b_0 b_1 b_2 b_3} + \dots$$

It can be proved that the infinite sum is convergent and is an irrational number for all possible values of c .

s can be represented as an infinite continued fraction and corresponding convergents are denoted by p_n/q_n ($n \geq 0, p_n$ and q_n are coprime). For example, for $c = 6, s = 0.1715728752\dots$, and the first several convergents are $p_0/q_0 = 0/1, p_1/q_1 = 1/5, p_2/q_2 = 1/6, p_3/q_3 = 5/29$ and so on.

Let $P(c, n)$ and $Q(c, n)$ be numerator and denominator of the n -th convergents p_n/q_n of s with value c , respectively. For instance, $P(6, 3) = 5, Q(6, 3) = 29$. Given Fibonacci sequence f_n defined as $f_1 = 1, f_2 = 1, f_n = f_{n-1} + f_{n-2}$ ($n \geq 3$). The value of this sequence is no less than 3 from the 4-th item.

Let the sum $SP(m, n) = \sum_{i=4}^m P(f_i, n)$ and $SQ(m, n) = \sum_{i=4}^m Q(f_i, n)$. You are given $SP(5, 10) = 606, SQ(5, 10) = 2784, SP(10, 100) \bmod 1000000007 = 774200907, SQ(10, 100) \bmod 1000000007 = 830200702$.

Find $SP(10^5, 10^{18})$ and $SQ(10^5, 10^{18})$, both modulo 1000000007.

Answer format: [SP($10^5, 10^{18}$)], [SQ($10^5, 10^{18}$)]

Example: 774200907, 830200702 for $SP(10, 100)$ and $SQ(10, 100)$.

Thanks to **czp** for the idea.

Answer

6007c8c4a4b13b257984183d5ac3944d

462 Polygonal billiards

2018-02-26 02:57:23

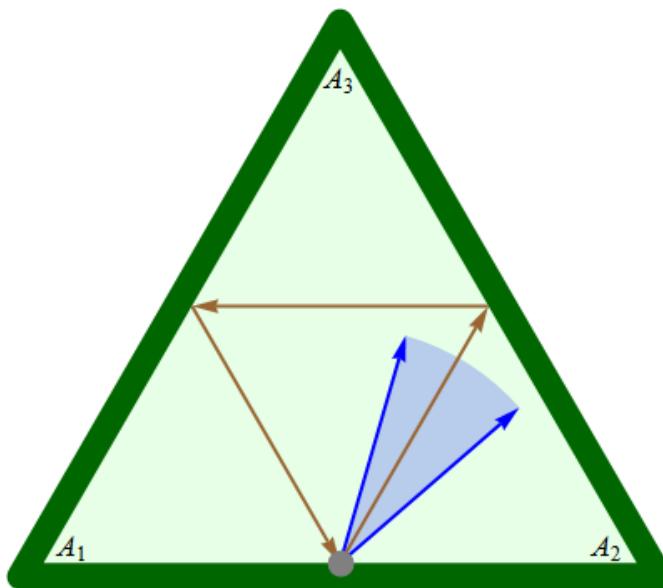
by liuguangxi

8 xp

Math

Given a **regular** n -sided polygon ($n \geq 3$) billiard table and its vertices are labeled A_1, A_2, \dots, A_n in anticlockwise direction. A billiard ball is placed in the middle of the edge A_1A_2 . When it is launched at a certain angle, it keeps moving and reflects off the sides of the table. Suppose the table and the ball are ideal, and reflection and incidence angles are the same when bouncing off the edge.

Let's take an equilateral triangle table ($n = 3$) as an example.



When the ball is launched at angle 60 degrees, it hits the edge A_2A_3 and A_3A_1 , and just returns to start point. For the n -sided polygon, there exists a continuous launch angle range such that the ball hits edge $A_2A_3, A_3A_4, \dots, A_{n-1}A_n$ in turn and return to edge A_1A_2 at last. For the equilateral triangle table, the minimum launch angle is 40.893394649131 degrees, and the maximum one is 73.897886248014 degrees. See blue arrows in above figure.

Let $D(n)$ be the length of launch angle range satisfying above rule for a regular n -sided polygon, in degrees. You are given $D(3) = 33.004491598883$, $D(10) = 1.866152295275$ and $D(50) = 0.072102098755$.

Find $\sum_{n=3}^{2018} D(n)$. Give your answer rounded to 10 digits after the decimal point.

Answer aa97198a9bdb2abe99617ce5e722a290

463 Shortest Road

2017-12-29 01:10:54

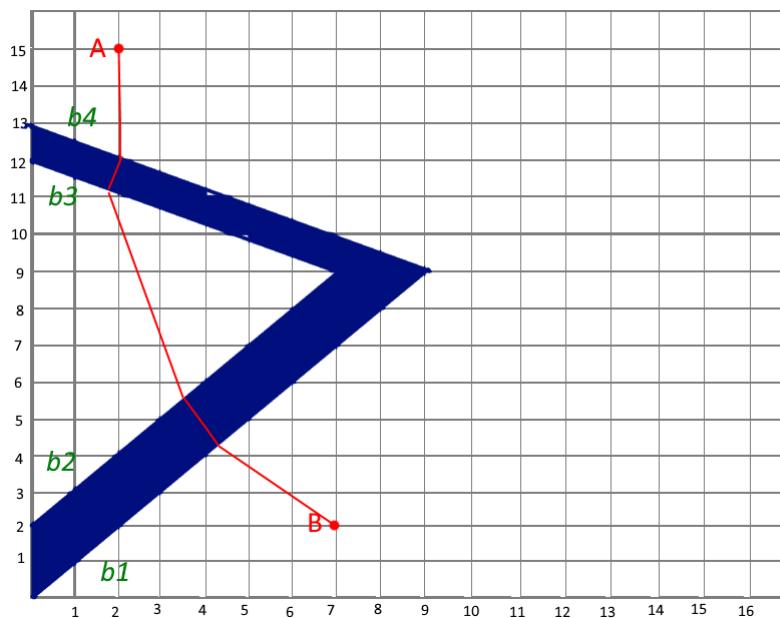
by Philippe_57721

7 xp

Math

A river forming a bend has the following shape:

- bank b_1 passes at points $(0, 0)$ and $(9, 9)$
- bank b_2 passes at point $(0, 2)$ and is parallel to b_1
- bank b_4 passes at points $(0, 13)$ and $(9, 9)$
- bank b_3 passes at point $(0, 12)$ and is parallel to b_4



We want to build a road between points $A(2, 15)$ and $B(7, 2)$. The constraint is that bridges over the river must be perpendicular to the banks. What is the shortest possible distance (A, B) ?

Give your answer rounded to 10 digits after the decimal point.

Answer **9a0b515377a1ecd56f43bcc0f64ce85a**

464 Decomposable Triangles

2018-01-12 03:23:13

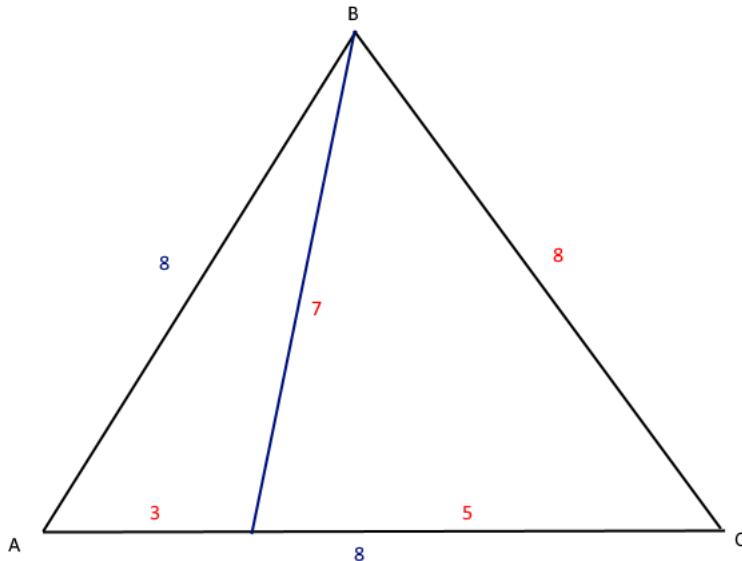
by Philippe_57721

7 xp

Programming

An integer equilateral triangle is decomposable if it can be split in 2 integer triangles.

For instance, the triangle (8, 8, 8) can be decomposed as (8, 3, 7) + (8, 5, 7).



How many triangles (T) with side $\leq 10^7$ are decomposable?

Which triangle (L) has the largest number of decompositions? (if multiple possibilities, give the largest one)

How many decompositions (D) does it possess?

For instance, the triangle (2592, 2592, 2592) has 3 decompositions:

- (2592, 2457, 297) + (2592, 2457, 2295)
- (2592, 2322, 702) + (2592, 2322, 1890)
- (2592, 2268, 972) + (2592, 2268, 1620)

Answer format: T, L, D

You are given: 364,960,8 for triangles with side ≤ 1000 .

[My timing: 20 sec]

Answer 655da325d3f92fe58efc3e679e875638

465 Random in 3-4-5

2018-03-07 00:41:43

by sinan

8 xp

Probability

ABC is a right triangle with sides 3, 4 and 5. There are 3 random points (D, E and F) on the three sides assumed to be uniformly distributed: D on AB , E on BC and F on AC .

Draw the three lines segments from vertices to these random points: AE , BF and CD . The intersection of AE and BF is the point P . The intersection of AE and CD is the point Q . The intersection of CD and BF is the point R .

What is expected area of the PQR triangle?

Answer format: decimal number rounded to 10 digits after the decimal point.

Answer 5187e5172794f0b5944c7837190ef6af

466 Special Squares

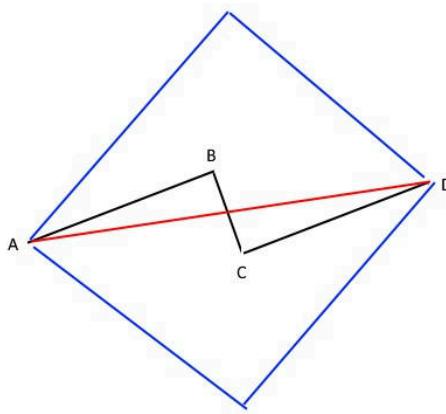
2018-01-19 01:03:29

by Philippe_57721

7 xp

Programming

Two segments AB and CD of length a are joined by a perpendicular BC of length b . The segment AD is the diagonal.



For instance, if $a = 23$ and $b = 94$, then c is an integer $c = 74$.

In the picture, points B and C fall inside the square, but it is not necessarily the case in general.

How many coprime pairs (a, b) produce an integer side square where the side $c \leq 10^7$?

Answer format: count, $\sum a, \sum b$

You are given: 161, 36737, 73474 for a threshold of 1000.

[My timing: 1 sec]

Answer 50460f0a65840681106e359bb63ac032

467 **Magical Beans****2018-01-26 08:02:43****by Philippe_57721****6 xp****Probability**

Jack has 10 magical beans.

On day 0, he plants his beans in his garden.

Each day, each bean has a probability $1/2$ to grow into a beanstalk.

What is the expected number of days required for all the beans turn into beanstalks?

Answer format: P/Q where P and Q are coprime

Answer **65fb2fffe031797b89481edd94bdf636**

468 Displacement of permutation

2018-03-05 01:47:52

by liuguangxi

7 xp

Math

Let p be a permutation of the set $\{1, 2, 3, \dots, n\}$. We call the sum $D_{k(p)} = \sum_{i=1}^n |p_i - i|^k$ the total displacement with k -th power of p . For example, if $p = (4, 2, 1, 3)$, $D_1(p) = |4 - 1| + |2 - 2| + |1 - 3| + |3 - 4| = 6$, $D_2(p) = |4 - 1|^2 + |2 - 2|^2 + |1 - 3|^2 + |3 - 4|^2 = 14$, $D_3(p) = |4 - 1|^3 + |2 - 2|^3 + |1 - 3|^3 + |3 - 4|^3 = 36$.

For any positive integer n , assume every possible permutation p of the set $\{1, 2, 3, \dots, n\}$ is uniformly distributed. Let $E(n)$ be the expected value of $D_1(p) + D_2(p) + D_3(p)$. You are given $E(2) = 3$, $E(3) = 40/3$, $E(5) = 432/5$, and all $E(n)$ are rational number. Let $F(n)$ be the numerator of $E(n)$ with $E(n)$ be fraction in lowest terms. So, $F(2) = 3$, $F(3) = 40$, $F(5) = 432$ and so on.

Let $S(n) = \sum_{i=1}^n F(i)$. Find $S(12345678987654321) \bmod 1000000007$.

Thanks to **baihacker** for the idea.

Answer

c72510d48521a0d462bdf106d0ad3e10

469 Permutation Order I

2018-02-02 01:50:24

by Philippe_57721

7 xp

Programming

Let $\mathfrak{S}(n)$ be the set of all permutations of $[1..n]$.

The order of a permutation p is the smallest integer e such as $p^e = \text{Id}$. We apply the permutation e times to itself and get the identity permutation $\{1, 2, \dots, n\}$.

For $\mathfrak{S}(4)$:

Order(1 2 3 4) = 1

Order(1 2 4 3) = 2

Order(1 3 2 4) = 2

Order(1 3 4 2) = 3

Order(1 4 2 3) = 3

Order(1 4 3 2) = 2

Order(2 1 3 4) = 2

Order(2 1 4 3) = 2

Order(2 3 1 4) = 3

Order(2 3 4 1) = 4

Order(2 4 1 3) = 4

Order(2 4 3 1) = 3

Order(3 1 2 4) = 3

Order(3 1 4 2) = 4

Order(3 2 1 4) = 2

Order(3 2 4 1) = 3

Order(3 4 1 2) = 2

Order(3 4 2 1) = 4

Order(4 1 2 3) = 4

Order(4 1 3 2) = 3

Order(4 2 1 3) = 3

Order(4 2 3 1) = 2

Order(4 3 1 2) = 4

Order(4 3 2 1) = 2

What is the largest order for the elements of $\mathfrak{S}(85)$?

For which permutation do we get this order? Give the index in lexicographic order (0-origin) of the 1st one.

Answer format: Maximal order,Index modulo 10^9

Example: 420,257453673 for $\mathfrak{S}(20)$.

[My timing: 40 sec]

Answer 843d31bf2feb25024458949644f20a5b

470 Permutation Order II**2018-02-08 21:25:16****by Philippe_57721****7 xp****Programming**

See [Problem 469 - Permutation Order I](#) for definitions.

Let $\mathfrak{A}(n, p)$ be the number of permutations of $[1..n]$ with order p .

We have:

$\mathfrak{A}(7, 1) = 1$
 $\mathfrak{A}(7, 2) = 231$
 $\mathfrak{A}(7, 3) = 350$
 $\mathfrak{A}(7, 4) = 840$
 $\mathfrak{A}(7, 5) = 504$
 $\mathfrak{A}(7, 6) = 1470$
 $\mathfrak{A}(7, 7) = 720$
 $\mathfrak{A}(7, 10) = 504$
 $\mathfrak{A}(7, 12) = 420$

Find the last 10 significant (before trailing 0s) digits of $\mathfrak{A}(1000, 641)$.

[My timing: < 100 ms]

Answer**51e5b0e2ca5d7900153312a44552534a**

471 Kimberling Sequence

2018-02-23 01:12:30

by Philippe_57721

6 xp

Programming

The Kimberling sequence is defined as follow.

We start with $S_0 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$. We build S_{k+1} from S_k : for i in $[1..k]$, we take $S_k[k+i]$ and $S_k[k-i]$, ignoring $S_k[k]$, then we take the rest of the sequence.

The first iterations are:

- 0 (1) 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
- 1 2 (3) 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
- 2 4 2 (5) 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
- 3 6 2 7 (4) 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
- 4 8 7 9 2 (10) 6 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
- 5 6 2 11 9 12 (7) 13 8 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
- 6 13 12 8 9 14 11 (15) 2 16 6 17 18 19 20 21 22 23 24 25 26 27 28 29 30...
- 7 2 11 16 14 6 9 17 (8) 18 12 19 13 20 21 22 23 24 25 26 27 28 29 30...
- 8 18 17 12 9 19 6 13 14 (20) 16 21 11 22 2 23 24 25 26 27 28 29 30...
- 9 16 14 21 13 11 6 22 19 2 (9) 23 12 24 17 25 18 26 27 28 29 30...
- 10 23 2 12 19 24 22 17 6 25 11 (18) 13 26 21 27 14 28 16 29 30...

The Kimberling sequence is formed by the diagonal elements.

$K = 1, 3, 5, 4, 10, 7, 15, 8, 20, 9, 18, 24, 31, 14, 28, 22, 42, 35, 33, 46, 53, 6, 36, 23, 2, 55, 62, 59, 76, 65, \dots$

It is conjectured that K contains exactly all the integers.

Let $O(n)$ the index of the 1st occurrence of n in K : $K_{O(n)} = n$.

One can verify that:

$$O(2) = 25$$

$$O(3) = 2$$

$$O(19) = 49595$$

Find $O(16268)$.

[My timing: 20 sec]

Answer **dff7734f7b9c3f737334217b8f250ced**

472 Palindromic Infinite Sequence**2018-02-15 21:52:59****by Philippe_57721****9 xp****Programming**

An infinite sequence $\{S_1, S_2, S_3, S_4, \dots\}$ is said palindromic if: for every n_0 there is $n \geq n_0$ such as $\{S_1, S_2, S_3, S_4, \dots, S_n\}$ is palindromic.

It can be proved that for any irrational α , the sequence $\lfloor \alpha \cdot n \rfloor - \lfloor \alpha \cdot (n-1) \rfloor$ for $n \geq 1$ is palindromic. $\lfloor x \rfloor$ is the integer part of x .

For the irrational $e = 2.71828\dots$, this sequence is palindromic for following indexes:

- 1 – 1
- 2 – 4
- 3 – 11
- 4 – 18
- 5 – 25
- 6 – 32
- 7 – 71

Find the 10^9 -th index. Give your answer modulo 10^{10} .

You are given 3352820698 the 10^2 -th index.

[My timing: 15 sec]

Answer**af3a30e52d39ebf8c439e0cf7d406185**

473 An Unusual Development

2018-03-08 21:32:37

by Philippe_57721

6 xp

Programming

It can be proved that any real number $x \in (0, 1]$ can be written uniquely in the form:

$$x = \frac{1}{a_1} - \frac{1}{a_1 \times a_2} + \frac{1}{a_1 \times a_2 \times a_3} - \dots$$

a_i being a sequence of strictly increasing integers.

For example, $e - 2 = \frac{1}{1} - \frac{1}{1 \times 3} + \frac{1}{1 \times 3 \times 6} - \frac{1}{1 \times 3 \times 6 \times 14} + \frac{1}{1 \times 3 \times 6 \times 14 \times 142} - \dots$.

What is a_{100} when developing the Champernowne constant $C = 0.123456789101112\dots$.

Give your answer in compressed form: 10 first digits[omitted digits count]10 last digits

Example: 1701411834[19]5884105727 for $M_{127} = 2^{127} - 1$.

[My timing: 7 sec]

Answer**99ed4b98170957f55ea00877eb7d03b2**

474 Palindromic Decomposition

2018-03-02 00:46:23

by Philippe_57721

8 xp

Programming

It has recently been proved (2016) that:

For $g \geq 5$ every integer can be decomposed as a sum of 3 palindromes in base g .

For instance: $123456789 = 40904 + 4708074 + 118707811$ (base 10).

How many such decompositions $n = x + y + z$ and $0 < x \leq y \leq z$ are there for $n = 314159263$ (in base 10).

Answer format: count,x,y,z

decomposition with the largest x

[My timing: 15 sec]

Answer 9e3cc28a763f44eb1705dd6e23957767

475 Magic Prime Square

2018-04-06 00:50:46

by Philippe_57721

6 xp

Programming

$$\begin{bmatrix} 11 & 29 & 43 & 73 \\ 31 & 67 & 17 & 41 \\ 61 & 13 & 59 & 23 \\ 53 & 47 & 37 & 19 \end{bmatrix}$$
 is the first (in lexicographic order) 4×4 magic square using the smallest distinct prime numbers > 10 .

Find the first (in lexicographic order) 4×4 magic square using the smallest distinct prime numbers > 100 .

Answer format: comma separated list of the matrix elements

Example: 11,29,43,73,31,67,17,41,61,13,59,23,53,47,37,19

[My timing: 70 sec]

Answer**c1bb823df7f282eadbaaf67a6fec0a45**

476 **Records in Good Sequences**

2018-03-23 01:55:15

by Philippe_57721

6 xp

Programming

A sequence S of positive integers, not necessarily distinct is a good sequence if:

- when $k \in S$, then $(k - 1) \in S$ for $k \geq 2$
- the first occurrence of $k - 1$ in S appears before the last occurrence of k

Examples: $\{3, 1, 3, 1, 2\}$ is not a good sequence: the 1st occurrence of 2 is after the last 3.

The following sequences are good:

$\{2, 2, 1, 3, 2\}$

$\{1, 3, 2, 1, 3\}$

$\{3, 1, 1, 2, 3\}$

$\{3, 2, 3, 1, 2\}$

$\{1, 2, 1, 3, 1\}$

In a sequence $S = \{a_1, a_2, a_3, \dots, a_n\}$, a_j is a record if it is greater than all elements before it: $\forall i < j, a_i < a_j$.

The sequence $\{4, 7, 5, 1, 6, 8, 2, 3\}$ contains 3 records: 4, 7, 8.

Amongst all good sequences with 100 elements in $\{1, 2, 3, \dots, 100\}$, how many have 3 records?

Answer format: give the last 10 significant digits

(before trailing 0s)

Answer ee71fd30f11214e88c6ac0eabcb4762c

477 Maxima in Good Sequences**2018-03-29 22:58:30****by Philippe_57721****6 xp****Programming**

See [Problem 476 - Records in Good Sequences](#) for definitions.

Amongst all the good sequences with 100 elements in $\{1, 2, 3, \dots, 100\}$, how many have a maximum element = 50?

Answer format: give the answer modulo 10^{10}

Answer**0555dd3414c53c10e729504fb2e94404**

478 A > B > C > A

2018-03-16 01:44:30

by Philippe_57721

7 xp

Programming

Given 2 dice A, B , we define $\mathfrak{P}(A, B)$ as the probability of rolling a strictly higher value with A than B . If $A = \{2, 2, 4, 4, 9, 9\}$ and $B = \{1, 1, 6, 6, 8, 8\}$ we have $\mathfrak{P}(A, B) = \frac{5}{9} > \frac{1}{2}$.

Thus the probability to win with A is greater than with B . We write: $A > B$.

We add a third die $C = \{3, 3, 5, 5, 7, 7\}$. We have: $\mathfrak{P}(B, C) = \frac{5}{9}$, $\mathfrak{P}(C, A) = \frac{5}{9}$.

Therefore: $A > B > C > A$.

A triple of dice is called non-transitive if: $A > B > C > A$, $\mathfrak{P}(A, B) = \mathfrak{P}(B, C) = \mathfrak{P}(C, A)$.

Of course if (A, B, C) is such a triple, so is (B, C, A) and (C, A, B) . These 3 triples are considered equivalent.

We consider dice with values in range $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

How many distinct non-transitive triples are there?

There are triples where all faces for A have a distinct value. What is the last one in lexicographic order?

Between the 3 possible representation for a triple (A, B, C) , (B, C, A) and (C, A, B) , we choose the one where A is the first in lexicographic order.

Answer format: Count/(die1)(die2)(die3)

Example: 9999/(9,9,9,9,9,9)(9,9,9,9,9,9)(9,9,9,9,9,9)

You are given there are 104 distinct non-transitive triples for the set $\{1, 2, 3, 4, 5, 6\}$.

[My timing: 80 sec]

Answer a83d0d923d03665982e1f8ed9be964ac

479 Interesting numbers

2018-02-27 09:25:56

by sinan

7 xp

Math

Let n be a number with $2d$ digits. Let a be the first d digits of this number and b the last d digits.

For example:

$n = 140400 \quad d = 3 \quad a = 140 \quad b = 400$

This number has an interesting property such that $n = b^2 - a^2$.

How many numbers less than 10^{50} have this interesting property?

Answer format: cnt, $\sum n$

[My timing: ~ 2 s]

Answer 4a7a6cf44e94cf0181dd3e9bb783796

480 Counting modulo pairs**2018-03-12 02:35:13****by liuguangxi****10 xp****Math**

Let $f(m)$ denote the number of integer pairs (x, m) such that there exists a positive integer y satisfies $x^y \equiv 0 \pmod{m}$, where x and m are both positive integers and $1 \leq x \leq m$. For example, $f(12) = 2$, as two pairs $(6, 12)$ and $(12, 12)$ meet the condition.

Define $S(n) = \sum_{m=1}^n f(m)$. You are given $S(10) = 16$ and $S(1000) = 5764$.

Find $S(10^{14})$.

Thanks to **baihacker** for the idea.

Answer e29f327859e90507b3a26a7a57f6d440

481 **Subset of a Farey Sequence****2018-04-13 00:52:42****by Philippe_57721****7 xp****Programming**

Let $\gamma = 0.5772156649\dots$ the Euler-Mascheroni constant.

How many terms in the [Farey sequence](#) of order n are there such as: $\frac{a}{b} < \gamma$ for $n = 10^9$?

You are given 175452524024 for $n = 10^6$.

[My timing: 30 sec]

Answer**417e0c2cde5c9281a937880014d17a4b**

482 Prüfer Code I

2018-04-20 01:03:36

by Philippe_57721

9 xp

Programming

The [Prüfer code](#) is a very compact way to represent a [labeled tree](#).

We can represent a tree as a string with the following rules: (Pseudo BNF)

```
tree = node | node '(' tree [ ',' tree ] ')'
node = an integer
```

For examples:

```
1(2(3(4,5)))
```

```
1(2(3),4,5)
```

Consider a labeled tree with n nodes labeled 1, 2, ..., n in sequential order.

We can see its Prüfer code as the base 10 representation of some integer.

For instance, the tree $1(2(3,4(5)),6(7),8)$ has the following Prüfer code $\{2, 4, 2, 1, 6, 1\} = 242161$.

Find all labeled trees with 10 nodes whose Prüfer code correspond to a prime number. Among these trees find those whose Prüfer code contain distinct values.

Answer format: sum of the values,(/ delimited list of the trees in string representation - lexicographic order)

Example: 11006162,1(2(3(4(5(6)))),7(8))/1(2(3(4(5))),6(7(8))) for 8 nodes.

[My timing: instant]

Answer**2f744e2ab79523f6a11f655e0d1f6376**

483 Prüfer Code II

2018-05-04 00:55:24

by Philippe_57721

7 xp

Programming

For definitions See [problem 482](#).

The file [rc481.txt](#) contains 1000 Prüfer codes.

Which code produces the labeled tree whose string representation is the largest in lexicographic order?

Answer format: the string representation corresponding to that code

Example:

`1(3,9(8(25(28(26(7(12(10(19(4,6),24(11,17))),20(14(21(5,23(13(29))))))),22))),18(15)),30(16(2),27))`

This is the string representation for the 1st record of the file.

[My timing: instant]

Answer **8e1bdb9dce388b5a26ee47512ed66a3d**

484 Period of permutation

2018-03-25 22:18:33

by liuguangxi

10 xp

Math

Let $i_1, i_2, i_3, \dots, i_n$ be a permutation of the set $\{1, 2, 3, \dots, n\}$ where n is a positive integer. A permutation can also be viewed as a function f . We denote this permutation by the 2-by- n array

$$f = \begin{pmatrix} 1 & 2 & \cdots & n \\ i_1 & i_2 & \cdots & i_n \end{pmatrix}$$

It maps number 1 to i_1 , number 2 to i_2 and so on. When this function operates on a permutation of set $1, 2, 3, \dots, n$, that is $x_1, x_2, x_3, \dots, x_n$, a new permutation $y_1, y_2, y_3, \dots, y_n$ can be obtained. Here $y_1 = f(x_1), y_2 = f(x_2), \dots, y_n = f(x_n)$.

For example, when $n = 4$ and there is a permutation $2, 4, 1, 3$. We make this permutation operate on a permutation $1, 2, 3, 4$ continuously, and it will be

$$1, 2, 3, 4 \rightarrow 2, 4, 1, 3 \rightarrow 4, 3, 2, 1 \rightarrow 3, 1, 4, 2 \rightarrow 1, 2, 3, 4 \rightarrow \dots$$

Obviously after 4 operations the result permutation returns to the initial one.

Let $F(l, p)$ be the number of permutations of the set $\{1, 2, 3, \dots, l\}$ which makes the permutation $1, 2, 3, \dots, l$ returns to itself after p operations. For instance, $F(4, 3) = 9$, as there are 9 permutations meet the condition: $(1, 2, 3, 4), (1, 3, 4, 2), (1, 4, 2, 3), (2, 3, 1, 4), (2, 4, 3, 1), (3, 1, 2, 4), (3, 2, 4, 1), (4, 1, 3, 2), (4, 2, 1, 3)$. You are also given $F(5, 5) = 25, F(10, 6) = 625176, F(20, 11) = 609493248001$.

Let $S(L, P) = \sum_{l=1}^L \sum_{p=1}^P F(l, p)$. Find $S(10000, 1000) \bmod 1000000007$.

Thanks to **baihacker** for the idea.

Answer d601163db5ef263ef2c847eeddfdf98e

485 Climbing the mountain

2018-04-27 02:29:55

by Philippe_57721

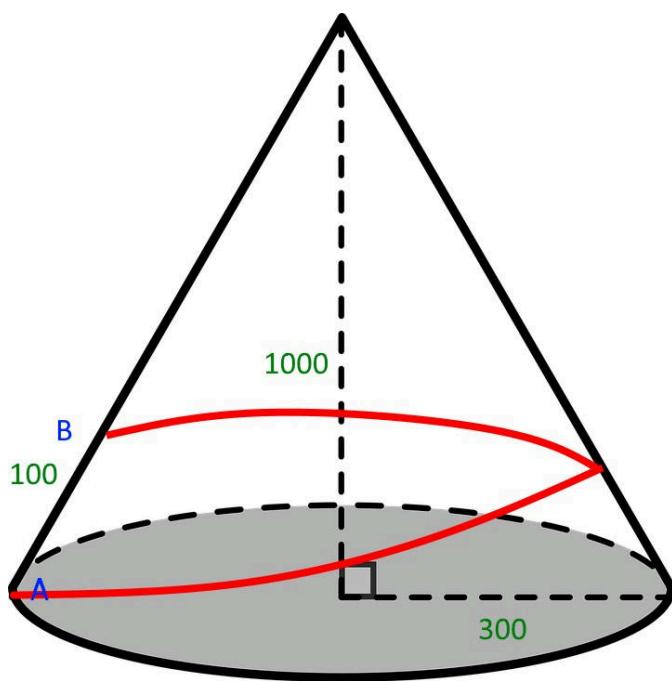
7 xp

Math

A mountain has the shape of a cone. It has a height of 1000 and the radius of the base is 300.

A trail with the shortest length possible goes from point A to point B (A and B are on the same slant at a distance of 100).

We can show that the trail starts to go uphill then downhill.



What are the length of the uphill part (u) and of the downhill part (d)?

Answer format: u, d

rounded to 10 digits after the decimal point

Answer c78068a9140737f81e443204377d5bb2

486 Product of tuple

2018-04-01 23:38:33

by liuguangxi

10 xp

Math

Let $S(p, n)$ denote the number of n -tuples of positive integers whose product is no larger than p . For example, $S(4, 2) = 8$, as there are eight 2-tuples $(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (3, 1), (4, 1)$ meet the condition. You are also given $S(6, 4) = 39, S(10, 10) = 571, S(100, 100) \bmod 1000000007 = 959337187$.

Find $S(10^{12}, 10^{12}) \bmod 1000000007$.

Thanks to **baihacker** for the idea.

This problem was inspired by Problem 452 at Project Euler (<https://projecteuler.net/problem=452>)

Answer

986a23786b940f1dfdd3ca8e6b373aa5

487 Crammed spheres

2018-05-12 00:43:18

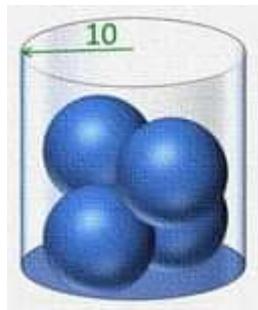
by Philippe_57721

6 xp

Math

A cylinder has a radius of 10.

We place 4 spheres of radius 5 inside the cylinder.



What is the volume of water needed to cover exactly the spheres?

Give your answer rounded to 10 digits after the decimal point.

Answer 18d73f932a250ff94b1c87351b8ef939

488 **Leading digits of a power****2018-05-18 07:23:59****by Philippe_57721****7 xp****Programming**

Let e_n the sequence of integers such as the decimal development of 2^{e_n} starts with the following digits:
66666...

For instance we have:

$$e_1 = 224296 \rightarrow 2^{224296} = 66666468242741993302\cdots$$

$$e_5 = 620220 \rightarrow 2^{620220} = 66666969606590729636\cdots$$

What is $e_{1000000}$?

[My timing: 40 sec]

Answer**4a2302d414aee38953bdb632eb67d0da**

489 Good sequences revisited

2018-05-03 03:54:42

by liuguangxi

10 xp

Programming

Let $F(N, M, R, D)$ be the number of good sequences which meet the following condition:

- (1) The sequence has N elements
- (2) The maximum element in the sequence is equal to M
- (3) The sequence contains R records
- (4) Each number occurs no more than D times in the sequence

The good sequence and records are the same definition in [Problem 476](#).

For example, $F(4, 2, 2, 2)$ is 3. Only three sequences $\{1, 1, 2, 2\}$, $\{1, 2, 1, 2\}$, $\{1, 2, 2, 1\}$ meet the conditions:

- (1) 4 elements
- (2) Maximum element = 2
- (3) 2 records
- (4) Each number occurs no more than 2 times

Notice $\{1, 2, 1, 1\}$ is not in $F(4, 2, 2, 2)$ because number 1 occurs 3 times; $\{2, 1, 2, 2\}$ is not in $F(4, 2, 2, 2)$ because it contains only 1 record and number 2 occurs 3 times; $\{2, 3, 1, 2\}$ is not in $F(4, 2, 2, 2)$ because the maximum element is 3.

You are given $F(11, 8, 3, 2) = 394581$ and $F(20, 13, 7, 4) \bmod 1000000007 = 342894563$.

Find $F(80, 47, 13, 10) \bmod 1000000007$.

Thanks to **C_K_Yang** for the idea.

Answer a324b65544963902c1917142fca1a9df

490 Index of a Farey sequence

2018-04-25 17:30:36

by liuguangxi

8 xp

Math

Let $F_{\text{idx}}(n, k)$ denote the k -th term in the Farey sequence of order n . For example, the Farey sequence of order 5 is $\{0/1, 1/5, 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5, 1/1\}$, and the 7-th term is $3/5$, so $F_{\text{idx}}(5, 7) = 3/5$. You are also given $F_{\text{idx}}(12, 31) = 7/11$, $F_{\text{idx}}(123, 3141) = 82/121$.

Find $F_{\text{idx}}(1234567890, 314159265358979323)$. The answer should be represented as an irreducible fraction.

Thanks to **czp** for the idea.

Answer **673f681e97557a3fddaf4efcdc1f8dae**

491 Generalized Pythagorean Equation**2018-05-26 03:22:08****by Philippe_57721****7 xp****Programming**

Consider the Diophantine equation $x^2 + 13xy + y^2 = z^2$.

How many solutions are there with $0 < x < y < 10^7$ (x, y, z coprime).

Answer format: count, $\sum x, \sum y, \sum z$

You are given: 28989, 580750261, 1448965526, 3481052101 for a threshold of 10^5 .

[My timing: 20 sec]

Answer **157c3c24ffc82eebc393f394e27b8cd8**

492 A self referential sequence**2018-07-27 00:50:30****by Philippe_57721****8 xp****Programming**

The sequence $G = \{1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, \dots\}$ is defined by: $G_n = \text{number of occurrences of } n \text{ in the sequence.}$

$G_4 = 3 \Rightarrow$ the number 4 occurs 3 times in the sequence.

What is $m = G_{123456789101112}$?

Find the smallest n such as $G_n = 123456654321$.

You are given 1684, 128276792 for 123456, 123321.

Answer format: m,n

[My timing: 1.5 sec]

Answer**21810181572b26acc2d08a5d9d11d22d**

493 Prime permutations

2018-06-01 00:36:41

by Philippe_57721

5 xp

Programming

A permutation P of $\{1, 2, \dots, n\}$ is said prime if:

- $P_1 = 1$
- $P_n = n$
- $\forall i < n, P_i + P_{i+1}$ is a prime number

$\{1, 6, 5, 8, 9, 2, 3, 4, 7, 10\}$ is a prime permutation.

How many prime permutations are there for $n = 21$?

[My timing: 30 sec]

Answer

bcd5fb98fc1a168dc3960a3fa3e5b254

494 High degree Diophantine equation

2018-06-15 02:03:24

by Philippe_57721

5 xp

Math

How many triples (x, y, z) with $0 < x < y < z \leq 123456789$ and (x, y, z) coprime are there such as:
 $(x^4 + y^4 + z^4)^2 = 2 \times (x^8 + y^8 + z^8)$.

Answer format: count

[My timing: 3 sec]

Answer 1e526b820e3747f9338d1f97ae208f59

495 Sum of factorials

2018-06-21 23:29:59

by liuguangxi

9 xp

Math

Let

$$S(n) = \sum_{k=1}^n k!$$

You are given $S(10) = 4037913$, $S(100) \bmod 1000000000039 = 947219250081$.

Find $S(314159265358) \bmod 1000000000039$.

Answer **7d2c11b3facf37df69813c5695521c3b**

496 Fuzzy squaring**2018-06-08 01:22:11****by Philippe_57721****7 xp****Programming**

Let x_0 a rational number > 1 . We define $x_{n+1} = x_n \cdot \lceil x_n \rceil$ where $\lceil x \rceil$ is the smallest integer $\geq x$.

It is conjectured that $\forall x_0$, x_n is eventually an integer for n large enough.

For instance, if $x_0 = \frac{8}{7}$, $x_1 = \frac{16}{7}$, $x_2 = \frac{48}{7}$, $x_3 = 48$. The sequence starting with $\frac{8}{7}$ produces an integer after 3 steps.

How many steps are necessary if we start with $\frac{3012}{3011}$?

You are given 18 steps for $\frac{6}{5}$.

[My timing: 5 sec]

Answer **dac79350b1eb7b44c7b6bc35c14ac824**

497 m-excellent pairs**2018-07-06 02:55:39****by Philippe_57721****6 xp****Math**

Let ν a positive irrational number and m a positive integer.

A pair of integers (a, b) is said m -good if $a \cdot \lceil \nu \cdot b \rceil - b \cdot \lfloor \nu \cdot a \rfloor = m$ ($\lceil x \rceil$ and $\lfloor x \rfloor$ are respectively the smallest integer $\geq x$ and the largest integer $\leq x$).

A pair of positive integers (a, b) is said m -excellent if (a, b) is m -good and neither $(a - b, b)$ nor $(a, b - a)$ are m -good.

How many m -excellent pairs ($m = 123456789101112$) are there if $\nu = 0.5772156649015314\cdots$?

Answer **3a6481020729c7c00ef44da6fe7533ad**

498 Streaks

2018-06-11 02:57:46

by sinan

5 xp

Probability

This problem is about Freecell streaks (in this [site](#) in particular).

A run of wins is called a **streak**.

The standard Freecell game has 8 columns of cards and 4 free cells. This version is commonly referred to as 8×4 . There is a wide range of other variants too. The number of columns of cards may range from 4 – 13, and the number of free cells may range from zero to 10. Thus, a game with 13 columns of cards and no free cells is referred to as a 13×0 .

The game is currently scaled so that when your streak is small, you are automatically given somewhat easier games. As your streak increases, the difficulty level will go up until at a streak of 50 you will be playing complete random deals (difficulty level 10). Here's the progression:

Streak	Difficulty
0–9	5
10–19	6
20–29	7
30–39	8
40–49	9
≥ 50	10 (random)

There is a unique degree of “winnability” for each variant. 12-sum games such as the standard 8×4 are usually pretty winnable. But when you trend down into the < 10 sum games and such, winnability gets pretty dicey. 5×4 games, a 9-sum variant, are only rarely winnable.

At any level there are 2^{15} different games for any variant (numbered 0 to 32767).

What is average streak length for 10×2 variant assuming you can win any “winnable” game dealled randomly? The number of unwinnable games by level for 10×2 given as below:

Level	Count
5	1
6	2
7	5
8	6
9	12
10	18

Input format: streak length rounded to 15 digits after the decimal point

See also: [streaks info](#)

Answer **e8780c0dc0620e401b1f6cd9b82fc96**

499 Ginkgo leaf

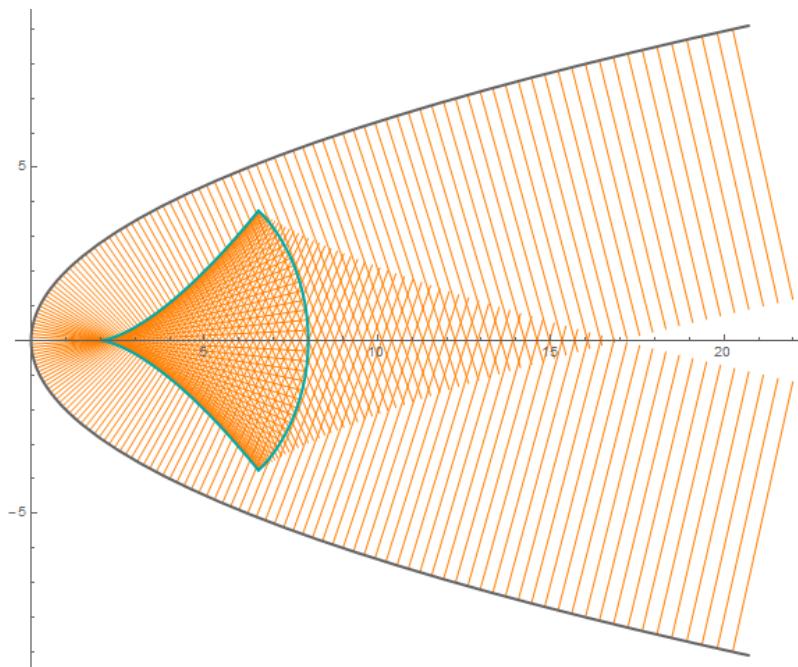
2018-08-01 22:40:43

by liuguangxi

8 xp

Math

There is a parabola with equation $y^2 = 4x$. A line segment of length 8 moves along the parabola, with one end on the curve, and always perpendicular to the curve. The figure below shows the trajectory of the line segment when one end of it moving through the whole curve.



In the whole process of moving the line segment, some points were swept exactly 3 times. Interestingly, these points form a connected region which looks like a ginkgo leaf (blue line around in the figure).

Find the area of the “ginkgo leaf”. Give your answer rounded to 10 digits after the decimal point.

Thanks to **czp** for the idea.

Answer 3e1171b9016f01cb75e9879acf4ef5a2

500 **Binary function****2018-07-09 03:33:48****by liuguangxi****8 xp****Math**

Let binary function $f(x, y) = (xy - 1)/(x + y - 1)$. There are infinite (x, y) pairs of positive integers such that $f(x, y)$ is integer. For example, $f(1, 1) = 0$, $f(3, 5) = 2$, $f(5, 17) = 4$ and so on.

Find all such (x, y) pairs for $1 \leq x \leq y \leq 50000000$.

Answer format: [number of pairs],[sum of x of all pairs], [sum of y of all pairs]

Example: 258,56772,129736 for $1 \leq x \leq y \leq 1000$.

Answer **238cc36edad2557dc0b5a2c14ad1742a**

501 Hyper factorial

2018-07-13 02:46:55

by Philippe_57721

7 xp

Programming

We define the hyper factorial function $!^l$ as $n!^l = 1^1 \times 2^2 \times \dots \times n^n$.

What is $200000000!^l \bmod 2147483647$.

You are given: $60629238 \equiv 1000000!^l \bmod 2147483647$.

[My timing: 60 sec]

Answer

6285e28a9b337838bac4d894a3e399d7

502 Dragon curve

2018-08-03 02:08:54

by Philippe_57721

8 xp

Programming

The [Dragon curve](#) is a fractal curve obtained by repeatedly appending a rotated version of itself.

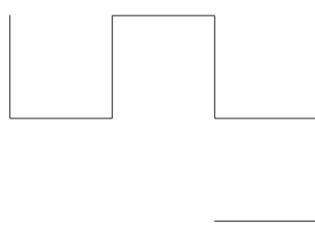
Here are the first 4 iterations of this curve:



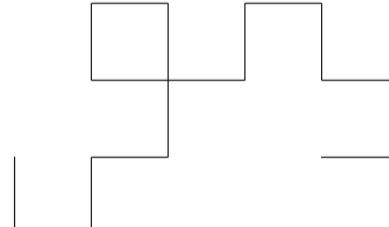
Iteration 1



Iteration 2



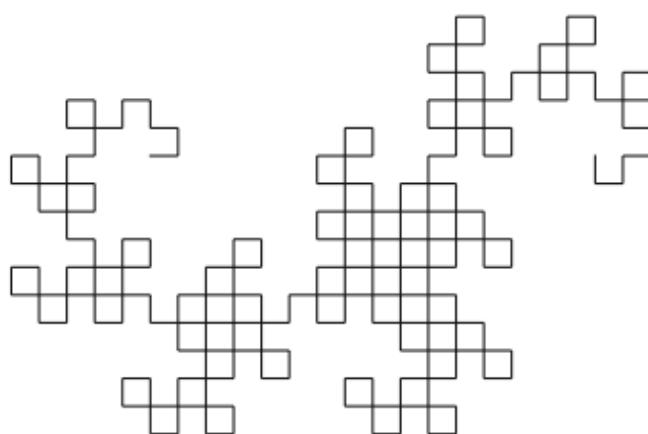
Iteration 3



Iteration 4

At each iteration (starting at 4), this curve contains closed squares grouped in contiguous blocks.

For instance, at iteration 8 we have:



The number of squares in each group starting from the initial point of the curve are: 1, 3, 6, 14, 33, 6, 3, 1.

What is the number of squares in each group at iteration 15?

Answer format: comma delimited list

Example: 1,3,6,14,33,6,3,1 at iteration 8.

[My timing: 30 sec]

Answer e552d4957bb5f32c9aa86f1dd207fb14

503 Product of phi

2018-07-24 19:26:15

by liuguangxi

8 xp

Math

Given two positive integers n and k , define function

$$T(n, k) = \prod_{a_1=1}^k \prod_{a_2=1}^k \cdots \prod_{a_n=1}^k \varphi\left(\prod_{i=1}^n a_i\right)$$

where $\varphi(x)$ is Euler's totient function.

For example, $T(2, 3) = \varphi(1 \times 1) \cdot \varphi(1 \times 2) \cdot \varphi(1 \times 3) \cdot \varphi(2 \times 1) \cdot \varphi(2 \times 2) \cdot \varphi(2 \times 3) \cdot \varphi(3 \times 1) \cdot \varphi(3 \times 2) \cdot \varphi(3 \times 3) = 192$. And you are given $T(10, 10) \bmod 1000000007 = 7103300$.

Find $T(10^7, 10^7) \bmod 1000000007$.

Thanks to C_K_Yang for the idea.

Answer

e44dba9d127ad3398c003afa3b2dd304

504 Kostka numbers

2018-08-31 00:43:59

by Philippe_57721

6 xp

Programming

Let λ a sequence of non increasing integers. It can be seen as the size of a [jagged array](#).

Let μ a sequence of $\sum \lambda_i$ consecutive integers starting with 1.

We fill in the array with the elements of μ using the following rules:

- in each row the elements must be in ascending order (equality permitted)
- in each column, the element are strictly in acending order

We defined $K(\lambda, \mu)$ as the number of such arrays. It is called the [Kostka number](#).

If $\lambda = \{3, 2\}$ and $\mu = \{1, 2, 3, 3, 4\}$, there are 3 possible arrays:

1 3 3	1 2 3	1 2 4
2 4	3 4	3 3

Therefore $K(\{3, 2\}, \{1, 2, 3, 3, 4\}) = 3$.

What is $K(\{6, 4, 4, 3\}, \{1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9\})$?

[My timing: 4 sec]

Answer de6180a7b68846f3374ef4e13213c3ab

505 Find the partition

2018-09-14 01:03:23

by Philippe_57721

6 xp

Programming

A partition of an integer n will be represented by a non increasing sequence of integers.

The partitions of 7 in lexicographic order are:

- 1 - 1 1 1 1 1 1
- 2 - 2 1 1 1 1 1
- 3 - 2 2 1 1 1
- 4 - 2 2 2 1
- 5 - 3 1 1 1 1
- 6 - 3 2 1 1
- 7 - 3 2 2
- 8 - 3 3 1
- 9 - 4 1 1 1
- 10 - 4 2 1
- 11 - 4 3
- 12 - 5 1 1
- 13 - 5 2
- 14 - 6 1
- 15 - 7

and in anti-lexicographic order:

- 1 - 7
- 2 - 6 1
- 3 - 5 2
- 4 - 5 1 1
- 5 - 4 3
- 6 - 4 2 1
- 7 - 4 1 1 1
- 8 - 3 3 1
- 9 - 3 2 2
- 10 - 3 2 1 1
- 11 - 3 1 1 1 1
- 12 - 2 2 2 1
- 13 - 2 2 1 1 1
- 14 - 2 1 1 1 1 1
- 15 - 1 1 1 1 1 1 1

The 10th partition is 4, 2, 1 in the 1st case and 3, 2, 1, 1 in the 2nd.

What are the 123456789th partition of 123 in lexicographic and anti-lexicographic order?

Answer format: (comma delimited values)/(comma delimited values)

Example: 4, 2, 1/3, 2, 1, 1

[My timing: 11 sec]

Answer

31d31a067d76e25a7afe513b579110df

506 Special Pythagorean Triples

2018-09-21 03:23:39

by Philippe_57721

7 xp

Programming

(430, 1824, 1874) is a special [Pythagorean triple](#).

Indeed, we have:

$$430^2 + 2133331200 = 46190^2$$

$$1824^2 + 2133331200 = 46224^2$$

$$1874^2 + 2133331200 = 46226^2$$

We call **special** a Pythagorean triple if there exists a positive integer t such as for each element e of the triple $e^2 + t$ is a square.

There can exist multiple integers for a given triple. For instance, the triple (24531, 30600, 39219) is special with the values 133453264 and 119200745664.

Find all special Pythagorean triples (a, b, c) with $a + b + c < 2000000$ (a triple is counted for each distinct t).

Answer format: Count, $\sum t$

You are given: 18,17172343737 for a threshold of 10000.

[My timing: < 1 min.]

Answer 10c63916a11c61133b4ac0e9ad7a6853

507 Side of a square

2018-08-16 08:27:25

by Philippe_57721

4 xp

Math

A point is choosen inside a square.

From that point 4 lines are drawn to the middle of each side of the square, defining 4 regions.

The areas of these regions are 72108967, 65123128, 69011033.

What is the side of the square?

Answer**ef6b2ac6e6235f2e7ab71da8fd0bed73**

508 Yet Another Diophantine Equation

2018-09-28 02:12:26

by Philippe_57721

6 xp

Programming

The Diophantine equation $x^2 - 6xy + y^2 = 1$ has infinitely many solutions in positive integers $x \leq y$.

Find the 10^{100} th.

Answer format: x, y

give the last 10 digits for each variable

Answer

89d9252b5e08dd6f5f1efab9ce48aac6

509 Sum of two squares**2019-03-29 03:36:26****by Philippe_57721****6 xp****Math**

Find all solutions of $x^2 + y^2 = 1234567891011121613 \cdot (x - y)$ in positive integers.

Answer format: $x_1, y_1 / \dots x_n, y_n$
where x_i are in ascending order

Example: 387, 225/850, 225 for the equation $x^2 + y^2 = 1237 \cdot (x - y)$.

[My timing: instant]

Answer **42cc027d449a12e78132078812834e63**

510 Deficiency 128**2018-08-28 03:28:57****by Min_25****11 xp****Math**

Let $S(N)$ be the sum of n ($1 \leq n \leq N$) such that $\sigma(n) - 2n = -128$, where $\sigma(n)$ is the sum of divisors function.

For example, $S(10^7) = 11033992$.

Find $S(10^{15})$.

Answer**cf43ee049a3d772b8a38d73b4e5a8920**

511 Higher order recursive sequence

2018-09-12 20:49:16

by liuguangxi

7 xp

Math

Define a sequence a_n as below: $a_1 = 17$, $a_{n+1} = \frac{a_n^4 + 6a_n^2 + 1}{4a_n(a_n^2 + 1)}$ ($n \geq 1$). Then $a_2 = 10657/2465$, $a_3 = 2134495165562497/1571545212141185$, Obviously all numbers in this sequence are rational.

For a prime p and a fully reduced fraction a/b , define $Q(a/b, p)$ to be the smallest positive q for which $ab \equiv q \pmod{p}$. For example $Q(3/5, 109) = 66$, because $5 \times 66 = 330 \equiv 3 \pmod{109}$ and 66 is the smallest positive such number.

You are given $Q(a_2, 1000000007) = 877890477$, $Q(a_{10}, 1000000007) = 16332768$.

Find the sum of $Q(a_{1234567891011121314}, p)$ over all primes p between 2000000000 and 2000100000.

Thanks to **czp** for the idea.

Answer d85471b108d971c3f9a9c4d7db9d63ea

512 Abundance 128

2018-08-28 03:29:10

by Min_25

11 xp

Math

Let $S(N)$ be the sum of n ($1 \leq n \leq N$) such that $\sigma(n) - 2n = 128$, where $\sigma(n)$ is the sum of divisors function.

For example, $S(10^7) = 25029284$.

Find $S(10^{15})$.

Answer e2f68a37bf705413b53ce73c786f38f5

513 Bell Numbers Modulo Factorial

2018-09-06 06:05:27

by Min_25

8 xp

Math

Let $B(n)$ be the n -th Bell number, which can be computed as $B(0) = 1$ and $B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$ for $n \geq 0$.

For example, $B(5) = 52$ and $B(10) = 115975$.

Let $f(N, M) := B(N) \bmod M$.

You can verify that $f(5, 24) = 4$, $f(10, 8!) = 35335$ and $f(10^4, 12!) = 82759235$.

Find $f(10^7, 30!) \bmod (10^9 + 7)$.

Answer**e7f6268322d32df76ffe47432a0f893d**

514 Trees gracious numberings

2018-10-06 03:27:21

by Philippe_57721

6 xp

Programming

A gracious numbering of a tree with N vertices is defined as follow:

- each vertex is numbered by a number in $[0..N - 1]$
- each edge being numbered by the difference, in absolute value, between the values of its two vertices, the values of the edges contain all integers in $[1..N - 1]$ exactly once.

For the tree $A(B, C(D, E, F))$, here is an example of gracious numbering:

```
[A] 3
+-B 4
+-[C] 5
+-D 2
+-E 1
+-F 0
```

You can verify:

```
[AB] = 1
[AC] = 2
[CD] = 3
[CE] = 4
[CF] = 5
```

It is conjectured that there exists a gracious numbering for all trees.

How many gracious numberings are there for the following tree: $A(B(C, D(E, F)), G(H, I(J, K)))$?

```
[A]
+-[B]
| +-C
| +-[D]
|   +-E
|   +-F
+-[G]
  +-H
  +-[I]
    +-J
    +-K
```

[My timing: 20 sec]

Answer 18b5a74712d19dbd46089a1a6490251d

515 Runs Resistance

2018-09-29 07:22:04

by Min_25

11 xp

Math

Let $A_0(n)$ be the array of the binary representation of positive integer n .

For example, $A_0(1) = [1]$, $A_0(2) = [1, 0]$ and $A_0(11) = [1, 0, 1, 1]$.

Let $R(A)$ be the function that returns the array of run-lengths of the given array A .

For example, $R([1, 0]) = [1, 1]$, $R([1, 0, 1, 1]) = [1, 1, 2]$ and $R([1, 0, 0, 0, 1]) = [1, 3, 1]$.

Let $A_{k(n)} := R(A_{k-1}(n))$ for $k \geq 1$, and $M(n)$ be the minimum integer k (≥ 0) such that $|A_k(n)| = 1$, where $|A|$ denotes the length of A .

Finally, let $S(n) := \sum_{i=1}^n M(i)$.

You can verify that $M(1) = 0$, $M(2) = 2$, $M(3) = 1$, $M(11) = 4$, $S(100) = 346$ and $S(1234567) = 6717538$.

Find $S(1234567654321)$.

Answer**1ae472b6d78f10365f60c339f8b0f652**

516 Proportion of pandigital numbers

2018-10-12 02:15:07

by liuguangxi

10 xp

Math

A pandigital number in base 10 is an integer that contains each of the digits from 0 to 9 (leading zero is not considered). Clearly there cannot be any pandigital numbers below 1023456789. In other words, 1023456789 is the smallest pandigital number. Pandigital numbers become more and more common as number gets bigger. In fact, the least number for which the proportion of pandigital numbers first reaches 1% is 1982653472320.

Given two positive integers m, n and $m > n$, Let $G(m, n)$ be the least number for which the proportion of pandigital numbers is at least n/m . For instance, $G(100, 1) = 1982653472320$, $G(10, 1) = 108552671412935708$.

Find $\sum_{i=2}^{100} G(i, i - 1)$. Since the answer can be very large, give your answer modulo 1000000033.

Thanks to **C_K_Yang** for the idea.

Answer **7580f389e3bb583ff47dc5a9a3dfcd02**

517 **Binary sequences**

2018-10-19 00:45:04

by Philippe_57721

7 xp

Math

Let $F(m, n, r, s)$ be the number of binary sequences containing:

- m times the digit '0'
- n times the digit '1'
- r times the sequence '00'
- s times the sequence '11'

For instance, $F(4, 5, 2, 3) = 24$:

- 0 0 0 1 0 1 1 1 1
- 0 0 0 1 1 0 1 1 1
- 0 0 0 1 1 1 0 1 1
- 0 0 0 1 1 1 1 0 1
- 0 0 1 0 0 1 1 1 1
- 0 0 1 1 0 0 1 1 1
- 0 0 1 1 1 0 0 1 1
- 0 0 1 1 1 1 0 0 1
- 0 1 0 0 0 1 1 1 1
- 0 1 1 0 0 0 1 1 1
- 0 1 1 1 0 0 0 1 1
- 0 1 1 1 1 0 0 0 1
- 1 0 0 0 1 1 1 1 0
- 1 0 0 1 1 1 1 0 0
- 1 0 1 1 1 1 0 0 0
- 1 1 0 0 0 1 1 1 0
- 1 1 0 0 1 1 1 0 0
- 1 1 0 1 1 1 0 0 0
- 1 1 1 0 0 0 1 1 0
- 1 1 1 0 1 1 0 0 0
- 1 1 1 1 0 0 0 1 0
- 1 1 1 1 0 0 1 0 0
- 1 1 1 1 0 1 0 0 0

What is $F(777, 555, 333, 111)$?

Answer format: the last 10 digits before the trailing 0 of this large number

You are given 7741694224 for $F(77, 55, 33, 11)$.

Answer 3852fccf67d025c68cf038bd197026e3

518 Hypergraph Intro

2019-02-06 03:11:52

by elasolova

3 xp

Math

In mathematics, [hypergraph](#) is a generalization of a graph in which an edge can join any number of vertices (denoted as m), therefore introducing hyperedges.

In that case, the number of hyperedges of size m that a complete hypergraph with n many vertices has a more general formula, which boils down to well-known formula $\frac{n(n-1)}{2}$ for $m = 2$.

Given that a complete hypergraph with $n = 11$ number of vertices, it can be shown that the number of hyperedges of size $m = 3$ is 165. Then, what is the answer for $n = 1007$ and $m = 35$?

Note: This is rather an easy problem with an immediate solution, but serves as a conceptual introduction to prospective problems related to hypergraphs.

Answer

36d725a03c77bda265a490cd93dc9648

519 Largest root of a polynomial**2018-10-26 02:28:44****by Philippe_57721****5 xp****Math**

Let $P_n(X) = X^5 + F_{2n}X^4 + 2(F_{2n} - 2F_{n+1}^2)X^3 + 2F_{2n}(F_{2n} - 2F_{n+1}^2)X^2 + F_{2n}^2X + F_{2n}^3$ (F_n is the n -th Fibonacci number).

Example: $P_4(X) = X^5 + 21X^4 - 58X^3 - 1218X^2 + 441X + 9261$.

It can be proved that equation $P_{n(X)} = 0$ has only integer roots.

Find the largest positive root of $P_{1000}(X) = 0$ (Give your answer mod 10^{10}).

Answer **0b6f76c165c5585eac51b291882e509b**

520 q-Factorial

2018-11-06 20:49:04

by liuguangxi

8 xp

Math

For n a positive integer, the q -factorial is defined by

$$\begin{aligned}[n]_q ! &= \prod_{k=1}^n [k]_q = \prod_{k=1}^n \sum_{i=0}^{k-1} q^i \\ &= 1(1+q)(1+q+q^2)\cdots(1+q+\cdots+q^{n-1})\end{aligned}$$

Let $S(n, q) = [n]_q !$. You are given $S(5, 2) = 9765$, $S(100, 7) \bmod 1000000000039 = 850054509459$.

Find $S(123456789876, 7) \bmod 1000000000039$.

Answer**8aa1b38d42e7f6e041cc9ba33bf03d6f**

521 Factorial-like Product

2018-11-14 02:17:52

by Min_25

9 xp

Math

Let $R(p, k) := \left(\prod_{i=1}^{p-1} (i^k + 1) \right) \bmod p$.

Find $\sum_{3 \leq p \leq 10^{10}} R(p, 20181026)$, where p is prime.

Answer

3d637b69fab3d2a487116091991246fd

522 Fibonacci Partitions Revisited

2018-11-08 06:25:20

by Philippe_57721

0 xp

Programming

Let $F = \{1, 2, 3, 5, 8, 13, 21, 34, 55, \dots\}$ the sequence of distinct Fibonacci numbers.

The number 1000 can be decomposed in 15 ways as a sum of elements of F

- 5, 14
- 3, 4, 14
- 5, 12, 13
- 1, 2, 4, 14
- 3, 4, 12, 13
- 5, 10, 11, 13
- 1, 2, 4, 12, 13
- 3, 4, 10, 11, 13
- 5, 8, 9, 11, 13
- 1, 2, 4, 10, 11, 13
- 3, 4, 8, 9, 11, 13
- 5, 6, 7, 9, 11, 13
- 1, 2, 4, 8, 9, 11, 13
- 3, 4, 6, 7, 9, 11, 13
- 1, 2, 4, 6, 7, 9, 11, 13

For each decomposition, we give the indexes in F (0-origin):

$$\{5, 12, 13\} \Rightarrow 1000 = F_5 + F_{12} + F_{13} = 13 + 377 + 610$$

Find the number of decompositions of 1234568 and give the middle one (if there is n decompositions, give the decomposition $\lfloor n/2 \rfloor$). The 1st one has index 0. The decompositions are sorted by length, then lexicographically.

Answer format: count/(comma delimited list of indexes)

You are given: 15/3,4,10,11,13 for $n = 1000$.

[My timing: 50 sec]

Answer**3b236df9fead529dfe3ab682b478ce38**

523 Sum of bigomega

2018-11-30 19:42:32

by liuguangxi

8 xp

Math

Arithmetic function $\Omega(n)$ is the total number of prime factors of n , counting prime factors with multiplicity. For example, $\Omega(3) = 1$, $\Omega(6) = 2$, $\Omega(72) = 5$. Let $S(n) = \sum_{x=1}^n \Omega(x)$. You are given $S(10) = 15$, $S(1000) = 2877$.

Find $S(10^{13})$.

Thanks to **baihacker** for the idea.

Answer

51dac4bff56799427878789ccdbc8a66

524 Number of Distinct Unordered Factorization

2018-11-21 03:45:56

by Min_25

7 xp

Math

Let $C(n)$ denote the number of factorizations of n into distinct factors greater than 1.

For example, $C(24) = 5$ since $24 = 2 \times 12 = 2 \times 3 \times 4 = 3 \times 8 = 4 \times 6$.

Let $S(n) := \sum_{i=2}^n C(n)$. You are given $S(10) = 12$ and $S(10^4) = 99733$.

Find $S(10^{12})$.

Answer**2d44a12d4dfadedb8b04ebd1ba65c761**

525 Sum of largest prime factors revisited

2018-12-28 17:42:42

by liuguangxi

9 xp

Math

Let $f(n)$ be the largest prime factor of n . For example $f(1) = 0$, $f(6) = 3$ and $f(30) = 5$.

$$\text{Define } S(k, n) = \sum_{x_1=1}^n \sum_{x_2=1}^n \cdots \sum_{x_k=1}^n f\left(\prod_{i=1}^k x_i\right).$$

You are given $S(2, 3) = f(1 \times 1) + f(1 \times 2) + f(1 \times 3) + f(2 \times 1) + f(2 \times 2) + f(2 \times 3) + f(3 \times 1) + f(3 \times 2) + f(3 \times 3) = 21$, $S(3, 10) = 4790$, $S(6, 10) = 5697722$.

Find $S(9, 10^9) \bmod 10^9$.

Thanks to **baihacker** for the idea.

This problem was inspired by Problem 642 at Project Euler (<https://projecteuler.net/problem=642>)

Answer c5ee640c7f942386d5d29b532f41ad31

526 Mysterious Asymptotic Behavior

2018-11-18 04:15:34

by Min_25

0 xp

Math

Let $C(N)$ be the number of positive integers $n \geq 2$ such that $n \cdot (\text{largest prime factor of } n) \leq N$.

You are given that $C(10) = 3$, $C(10^6) = 9107$, $C(10^{11}) = 34113192$ and $C(10^{12}) = 188014194$.

Find $C(10^{16})$.

Answer `5cd6eb297de9398d79515fe95d0af868`

527 Mysterious Asymptotic Behavior 2

2018-11-19 05:21:03

by Min_25

0 xp

Math

Let $C(N)$ be the number of positive integers $n \geq 2$ such that $n \cdot (\text{largest prime factor of } n)^2 \leq N$.

You are given that $C(10) = 1$, $C(10^6) = 1228$, $C(10^{11}) = 888908$ and $C(10^{12}) = 3613513$.

Find $C(10^{18})$.

Answer **bf733aabf4f4294f7c73e96483896e21**

528 One chord and two segments

2018-12-06 08:52:43

by Philippe_57721

7 xp

Math

Three segments, $[AB]$ of length a , $[BC]$ of length b and $[CD]$ of length c are perpendicular to each other.

The circle passing by the points A, B, D has a radius r .

How many triples (a, b, c) such as $1 \leq a, b, c \leq 1000$ and $\text{GCD}(a, b, c) = 1$ are there such as r is an integer?

Answer format: count, $\sum a, \sum b, \sum c$

You are given: 4, 30, 18, 10 for a threshold of 10.

Answer 0c70e5c1a13f2f0de1a2be8986a3c639

529 1-2-3 type Pythagorean triangles

2018-12-03 22:45:08

by C_K_Yang

8 xp

Math

An integer-sided triangle is a 1-2-3 type Pythagorean triangle if its three sides a, b, c satisfy the condition: $1 \times a^2 + 2 \times b^2 = 3 \times c^2$.

For example, A triangle with three sides $\{5, 13, 11\}$ is a 1-2-3 type Pythagorean triangle because $1 \times 5^2 + 2 \times 13^2 = 3 \times 11^2$.

Find all distinct 1-2-3 type Pythagorean triangles with perimeter ≤ 25000000 .

Answer format: count, \sum perimeter

You are given: 595, 315428 for a threshold of 1000.

Answer**acbbb9361d2f4d1a78f736cc738418ee**

530 Range for random sequence

2019-01-15 05:20:26

by liuguangxi

8 xp

Probability

There are n ($n > 1$) independent and identically distributed random variables whose cumulative distribution function is

$$F(x) = \begin{cases} 0 & : x < 0 \\ x^2 & : 0 \leq x \leq 1 \\ 1 & : x > 1 \end{cases}$$

Let R be the range for this random variables (difference between maximum and minimum). Let $E(n)$ be the expected value of R . Obviously $E(n)$ is between 0 and 1, furthermore it is rational number. You are given $E(2) = 4/15$ and $E(3) = 2/5$.

Find $E(20)$. Give your answer rounded to 10 digits after the decimal point.

Thanks to **baihacker** for the idea.

Answer 82a1a7fc40d7c005226824d06d9b40a1

531 Sum for random sequence

2019-01-23 22:12:49

by liuguangxi

8 xp

Probability

A sequence generator start generating independent and identically distributed random variables whose cumulative distribution function is

$$F(x) = \begin{cases} 0 & : x < 0 \\ x^2 & : 0 \leq x \leq 1 \\ 1 & : x > 1 \end{cases}$$

The cumulative sum is updated for every new generated number. When the sum is larger than 1, the generator stops. Let C denotes the count of generated numbers when it stops. For example, the probability for $C = 3$ is $7/45$.

Find the expected value of C . Give your answer rounded to 10 digits after the decimal point.

Thanks to **baihacker** for the idea.

Answer 0d55f2da2739bf119492802d335b0675

532 Throwing dice

2018-12-29 02:05:11

by Philippe_57721

6 xp

Programming

With 10 6-faces dice (numbered 1 to 6), we can obtain all sums between 10 and 60.

The sum 35 is the *champion*. It can be obtain in 4395456 ways. it's actually the sum that can be obtained in the largest number of ways.

What is the champion if we use $n = 100$ dice ?

Answer format: Champion, Number of ways

Example: 35, 4395456

Answer b924370091b8da38346340216b30031d

533 Every day is a holiday revisited

2019-02-22 01:23:50

by liuguangxi

9 xp

Math

On planet J, a year lasts for D days. Holidays are defined by the two following rules.

1. At the beginning of the reign of the current Emperor, his birthday is declared a holiday from that year onwards.
2. If both the day before and after a day d are holidays, then d also becomes a holiday.

Initially there are no holidays. Let $f(n, d)$ be the number of methods such that all the days of the year are holidays after exactly d days, with the year lasts for n days.

You are given $f(4, 2) = 4$, $f(10, 5) = 240$, $f(50, 100) \bmod 1000000007 = 263154905$.

Find $f(1000, 10^{12}) \bmod 1000000007$.

Thanks to **baihacker** for the idea.

This problem was inspired by Problem 645 at Project Euler (<https://projecteuler.net/problem=645>)

Answer

5b4d7b5648d1d38485d0850de6f75d69

534 The number of pandigital number

2018-12-18 01:42:47

by C_K_Yang

7 xp

Math

In mathematics, a pandigital number is an integer that in a given base B has each digit $\{0, 1, 2, 3, \dots, B - 1\}$ at least once.

Given two positive integers B and L , let $C(B, L)$ be the number of all the L -digit pandigital numbers in base B . For example, $C(3, 3) = 4$, there are only four 3-digit pandigital numbers in base 3, namely $(102)_3, (120)_3, (201)_3$ and $(210)_3$.

Now we define a new function $D(B, N) = \sum_{L=1}^N C(B, L)$.

You are given $D(5, 10) = 4865472$ and $D(123, 321) \bmod 1000000007 = 844902472$.

Find $D(12345678, 87654321) \bmod 1000000007$.

Note: leading zeroes are not allowed.

Answer**fe3110eb5af8f132db9b643d63a75c27**

535 The sum of pandigital number

2018-12-18 01:43:17

by C_K_Yang

9 xp

Math

In mathematics, a pandigital number is an integer that in a given base B has each digit $\{0, 1, 2, 3, \dots, B - 1\}$ at least once.

Given two positive integers B and L , let $S(B, L)$ denote the sum of all the L -digit pandigital numbers in base B . For example, $S(3, 3) = 66$, the sum of four 3-digit pandigital numbers in base 3 is 66 since $(102)_3 + (120)_3 + (201)_3 + (210)_3 = (2110)_3 = (66)_{10}$.

Now we define a new function $H(B, N) = \sum_{L=1}^N S(B, L)$.

You are given $H(5, 10) = 24678960400080$ and $H(123, 321) \bmod 1000000007 = 372652728$.

Find $H(12345678, 87654321) \bmod 1000000007$.

Note: leading zeroes are not allowed.

Answer

ed49378173eec010291909883428227f

536 Zumkeller Numbers

2018-12-27 02:04:55

by Philippe_57721

0 xp

Programming

A *Zumkeller* number is a number such as the set of its divisors can be partitioned in two subsets of equal sum.

For instance, $\text{divisors}(108) = E = \{1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108\}$ and $E = A \cup B$ with $A = \{2, 12, 18, 108\}$ and $B = \{1, 3, 4, 6, 9, 27, 36, 54\}$, $\sum_{a \in A} a = \sum_{b \in B} b = 140$.

Find all Zumkeller numbers in the range [439050, 439100].

Answer format: count,sum

[My timing: 6 sec]

Answer

968786c262221a2fdf72e2b85271c1b0

537 Skipping squares revisited

2019-04-08 00:19:48

by liuguangxi

7 xp

Math

For positive integer k ($k > 1$), we begin a sum s at 0 and repeatedly apply a process: we add one integer to s from 1 to k with equal probability. The process ends when s is a perfect square. For example, if s goes through 0, 2, 3, 5, 7, 9, the process ends at $s = 9$, and two squares 1 and 4 were skipped over.

Let $E(k)$ be the expected number of perfect squares skipped over when the process finishes for parameter k . For example, $E(2) = 0.6832612735$ and $E(10) = 4.9658004390$, both rounded to 10 digits after the decimal point.

Find $E(100)$. Give your answer rounded to 10 digits after the decimal point.

This problem was inspired by Problem 648 at Project Euler (<https://projecteuler.net/problem=648>)

Answer 8ffb7a6f011b7da4ebcf8eef241379bf

538 Counting unlock patterns

2019-10-08 20:25:33

by liuguangxi

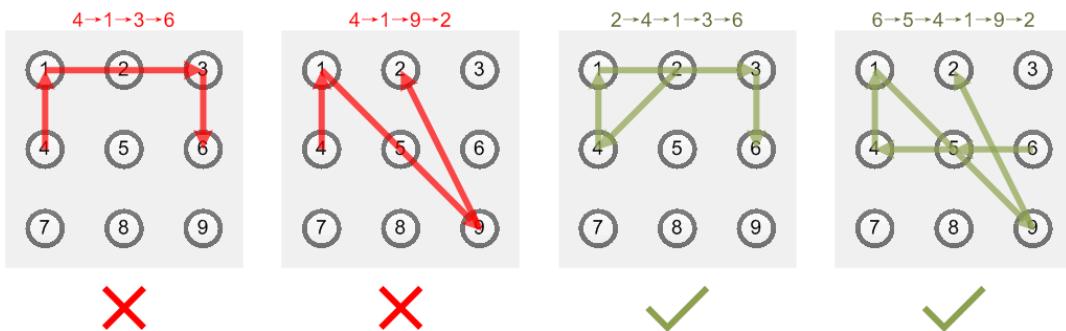
9 xp

Math

Given a square tiled $n \times n$ key lock screen and two integers k_1 and k_2 , where $1 \leq k_1 \leq k_2 \leq n^2$. Here are the rules for a valid pattern:

- Each pattern must connect at least k_1 keys and at most k_2 keys.
- All the keys must be distinct.
- If the line connecting two consecutive keys in the pattern passes through any other keys, the other keys must have previously selected in the pattern. No jumps through non selected key are allowed.
- The order of keys used matters.

For $n = 3$, here are some invalid and valid patterns:



- Invalid move: $4 \rightarrow 1 \rightarrow 3 \rightarrow 6$. Line $1 \rightarrow 3$ passes through key 2 which had not been selected in the pattern.
- Invalid move: $4 \rightarrow 1 \rightarrow 9 \rightarrow 2$. Line $1 \rightarrow 9$ passes through key 5 which had not been selected in the pattern.
- Valid move: $2 \rightarrow 4 \rightarrow 1 \rightarrow 3 \rightarrow 6$. Line $1 \rightarrow 3$ is valid because it passes through key 2, which had been selected in the pattern.
- Valid move: $6 \rightarrow 5 \rightarrow 4 \rightarrow 1 \rightarrow 9 \rightarrow 2$. Line $1 \rightarrow 9$ is valid because it passes through key 5, which had been selected in the pattern.

Define $S(n, k_1, k_2)$ be the total number of unlock patterns of the $n \times n$ key lock screen, which consist of minimum of k_1 keys and maximum k_2 keys. You are given $S(3, 4, 9) = 389112$, which is the well-known total number of possible Android unlock patterns.

Find $S(5, 4, 25)$.

Thanks to **baihacker** for the idea.

Answer 747917eb3bb56efca7e81ad5aefd191d

539 Egyptian fractions

2019-01-05 21:55:38

by C_K_Yang

10 xp

Math

Let $A(n)$ be the number of Egyptian fractions for which $\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_k} = 3$ (for any $k > 0$) with $0 < x_1 < x_2 < \dots < x_k \leq n$.

For example, $A(24) = 1$, there is only one Egyptian fraction which is equal to 3.

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{15} + \frac{1}{18} + \frac{1}{20} + \frac{1}{24} = 3$$

You are given $A(30) = 25$.

Find $A(200)$.

Answer

c9bd875dd8c102d452e8a34fa28820ff

540 Tesseract Pentatope Picking

2019-02-04 18:00:38

by elasolova

0 xp

Probability

First, some definitions are given:

- Simplex is the generalization of line-segments (1d) or triangles (2d) to n -dimensions. Tetrahedron is the well-known 3-simplex. [Pentatope](#) then is the 4-simplex.
- Similarly, hypercube is the generalization of squares (2d) or cubes (3d) to n -dimensions. [Tesseract](#) is the 4-dimensional hypercube.

Then the question builds up as follows:

In [line line picking](#), the expected length of picked line-segment is shown to be 13.

Up one dimension, namely in [square triangle picking](#), the expected area of picked triangle is analytically given as $\frac{11}{144}$.

Up one more, namely in [cube tetrahedron picking](#) it gets a bit complicated, but there is again an analytic answer for the expected volume of picked tetrahedron, $\frac{3977}{216000} - \frac{\pi^2}{2160}$.

What about going one more up, namely tesseract pentatope picking? Since an analytic answer is hard to find, devise a numerical solution that gives the expected content of picked pentatope rounded to 6 decimal places.

There might be some inaccuracy in my calculation for the time being ...

Answer**066f5c7650cb359a42a98a6df6356cd4**

541 Lattice points on the trace 1

2019-11-19 20:46:18

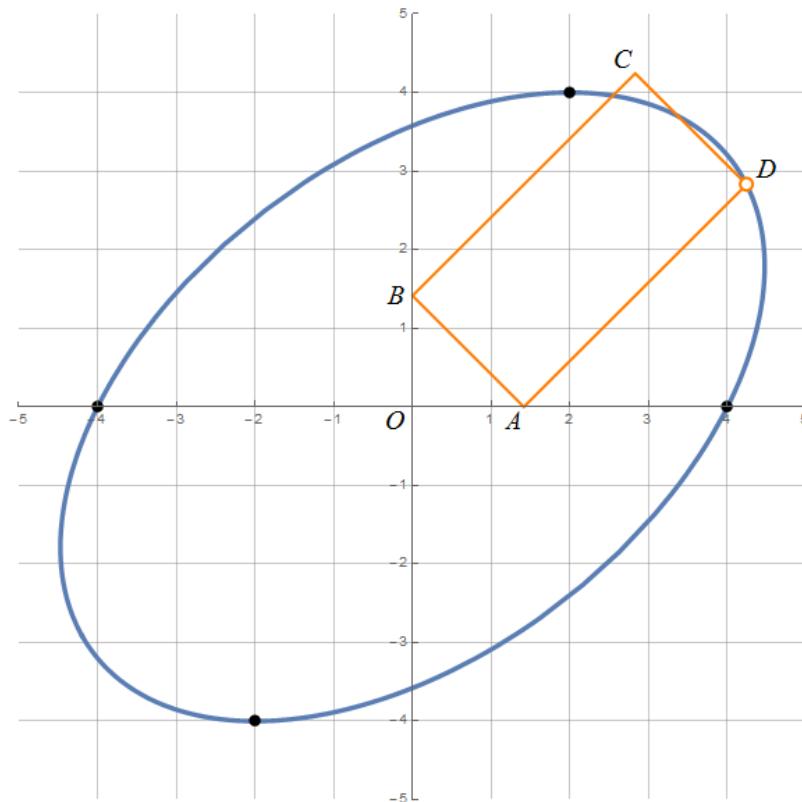
by liuguangxi

8 xp

Math

Given a rectangle $ABCD$ with side $AB = CD = a$ and side $BC = AD = b$. A curve is traced by point D of the rectangle as point A slides along the x -axis and point B slides along the y -axis.

The curve with $a = 2$ and $b = 4$ is shown below.



There are 4 lattice points $(-4, 0), (-2, -4), (4, 0), (2, 4)$ on the curve above. Let $C(a, b)$ be the number of distinct lattice points on the curve with the rectangle side a and b . For example, $C(2, 4) = 4$, $C(5, 5) = 12$ and $C(6, 3) = 4$.

Define $S(n) = \sum_{a=1}^n \sum_{b=1}^n C(a, b)$. You are given $S(10) = 432$, $S(50) = 10960$ and $S(100) = 44128$.

Find $S(10^8)$.

Answer d9b87ea258ffcba5bd940ffff25ea101

542 Lattice points on the trace 2

2019-11-19 20:52:20

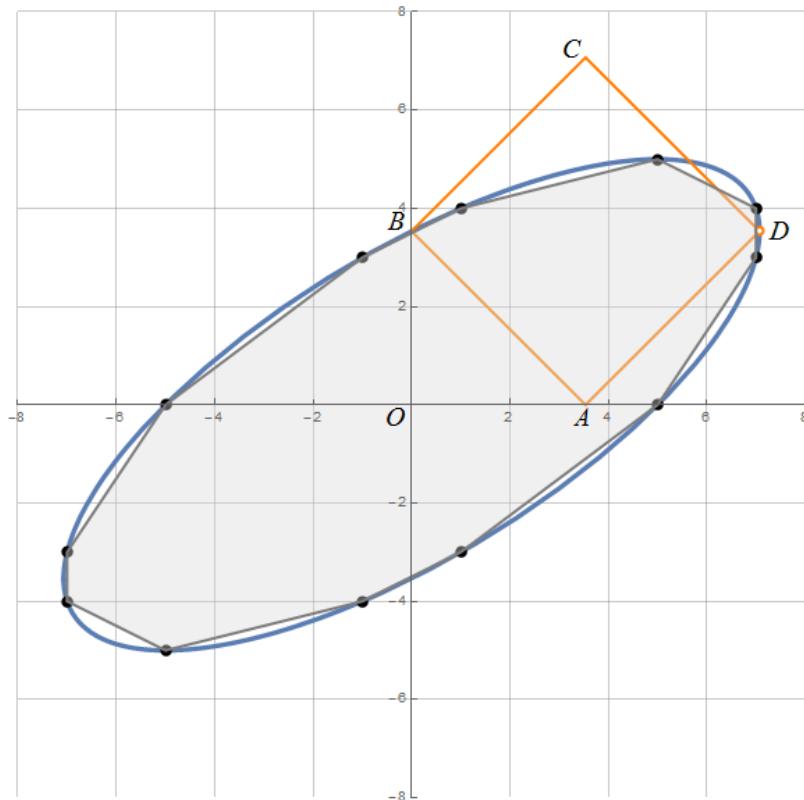
by liuguangxi

8 xp

Math

Given a square $ABCD$ with side length a . A curve is traced by point D of the square as point A slides along the x -axis and point B slides along the y -axis.

The curve with $a = 5$ is shown below.



For positive integer a , it can be shown that at least 4 lattice points are on the curve. An inscribed polygon can be formed by the lattice points on the curve. Let $R(a)$ be the ratio of the area of the inscribed polygon to the area of the closed curve with parameter a . Then $R(5) = 0.942197263104$.

For $a \leq 100$, the maximum value of $R(a)$ is 0.992825486421, with $a = 65$.

Find the maximum value of $R(a)$ and corresponding value a for $a \leq 10^{15}$. If there are two or more values of a taking the maximum value R , give the smallest a .

Answer format: [value of a],[value of $R(a)$ (rounded to 12 digits after the decimal point)]

Example: 65, 0.992825486421 for $a \leq 100$.

Answer 8526c1ed3352028c2f729ac24953c9f6

543 Number divisible by a range of numbers

2019-03-04 16:15:27

by C_K_Yang

8 xp

Programming

Given two positive integers a and b with $a \leq b$, let $P(N, a, b)$ denote the number of numbers not greater than N and divisible by at least one number between a and b (inclusive).

For example, $P(15, 3, 5) = 9$

- 3 can divide 3, 6, 9, 12, 15
- 4 can divide 4, 8, 12
- 5 can divide 5, 10, 15

There are 9 numbers not greater than 15 and divisible by at least one number between 3 and 5, namely 3, 4, 5, 6, 8, 9, 10, 12, 15.

You are given $P(10^8, 21, 40) = 41982923$.

Find $P(10^{17}, 61, 120)$.

Answer

2e2eea4cf1963d05b3809165febfb6f2c

544 Spatial Awareness

2019-08-12 02:55:29

by elasolova

6 xp

Math

Usually, in data processing multidimensional data (i.e. a multidimensional array) is vectorized and then processed. However, because of vectorization the spatial information in the data is lost.

For this problem, assume that you receive 1000 distinct values after a vectorization of a 4D array and you have no other information whatsoever. Then, how many possible spatial states are there for the original data? Give your answer in mod 100000009.

Note: Assume that an array of size $1 \times 1 \times 1 \times 1000$ or any other combination involving 1, will also count as a 4D array.

Answer **9f287d589c478f59ff659fbcd85b1c74**

545 Distinct powers revisited

2019-03-16 08:15:18

by C_K_Yang

10 xp

Programming

Consider all integer combinations of a^b for $2 \leq a \leq 5$ and $1 \leq b \leq 5$:

$$2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32$$

$$3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243$$

$$4^1 = 4, 4^2 = 16, 4^3 = 64, 4^4 = 256, 4^5 = 1024$$

$$5^1 = 5, 5^2 = 25, 5^3 = 125, 5^4 = 625, 5^5 = 3125$$

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 18 distinct terms:

$$2, 3, 4, 5, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125$$

Given two integers $N > 1$ and $M > 0$, let $R(N, M)$ be the number of distinct terms in the sequence generated by a^b for $2 \leq a \leq N$ and $1 \leq b \leq M$. For example, $R(5, 5) = 18$, $R(10^2, 10^3) = 92715$ and $R(10^5, 10^6) = 99813359409$.

Find the last 20 digits of $R(10^{21}, 11^{21})$.

This problem is a harder version of Project Euler Problem29 (<https://projecteuler.net/problem=29>)

Answer**b68b986b18c38afdb425510782b820d7**

546 Pandigital representation of a number**2019-04-12 01:10:10****by Philippe_57721****7 xp****Programming**

Using the 9 digits 1, 2, ..., 9 in that order and the 5 operations $[+ - * / ^]$ (digits can be concatenated too) it is possible to express some integers.

For instance, we have:

$38706 = -12 + 3^4 * (567 - 89)$

Can you find this representation for 88205? (The solution is unique)

Answer format: as in the above example, without spaces and the minimum parentheses

Answer**6ab4562e5bdbd849250997df08e1201c**

547 Collisions

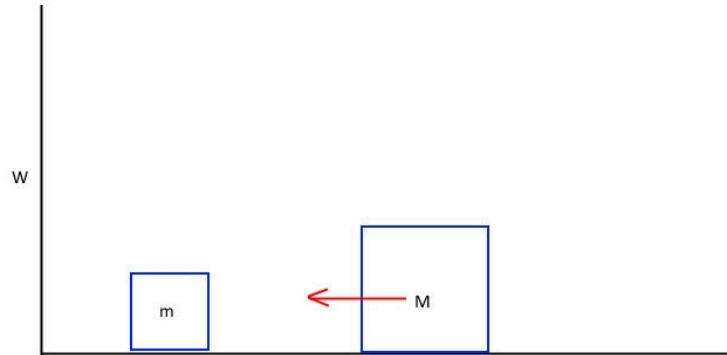
2019-04-26 01:03:10

by Philippe_57721

8 xp

Math

Two blocks of mass m and M ($m < M$) are initially still.



We push the block M in the direction of the block m . The block m will hit the wall and bounces back toward M . After a certain number of collisions of m between M and the wall, M will move to the right indefinitely.

Given that $m = 1$, $M = 123456789$, how many collisions are there?

It is assumed that the collisions are perfect: no friction, no energy loss.

Answer 3885e7d745f7a780484eb1182532bbf7

548 Intersecting chords

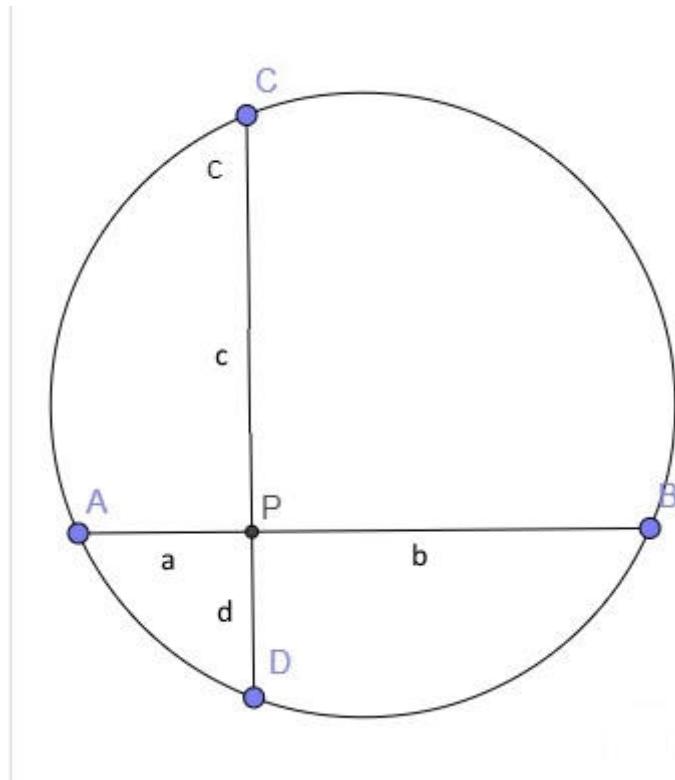
2019-05-17 01:12:39

by Philippe_57721

6 xp

Math

In a circle, two perpendicular chords AB and CD intersect in P .



We have the following lengths:

- $\|AP\| = a$
- $\|PB\| = b$
- $\|CP\| = c$
- $\|PD\| = d$

and $a < b, c < d, a < c$.

Given that a, b, c, d are positive integers ≤ 5000 , how many distinct circles (distinct radius) can we find?

You are given: 1913 for a threshold of 100.

[My timing: 60 sec]

Answer **b0236506212d871c03a1580f5bafb61d**

549 Stone game

2019-05-10 20:47:06

by C_K_Yang

9 xp

Math

Alice and Bob play a game with some piles of stones denoted by an array $[a_1, a_2, a_3, \dots]$ where a_i is the number of stones in the i -th pile. They take stones by turns and Alice always goes first. In each turn they must take **at least one stone** and at most a certain number of stones from a single pile. The game begins with taking stones from the first pile. If the first pile becomes empty, they are permitted to take stones from the second pile. If the second pile becomes empty, they are permitted to take stones from the third pile, and so on. The player who can not make a valid move loses.

We define an array $[a_1, a_2, a_3, \dots]$ as the winning configuration if the first player (Alice) can force a win no matter what the second player (Bob) does, or the losing configuration if Bob can force a win no matter what Alice does. We also assume that Alice and Bob always play perfectly.

For example, $[2, 1]$ is a winning configuration if they can take at most 2 stones in his/her turn. The strategy is explained below.

- (1) Alice takes one stone in the first pile.
- (2) Bob takes one stone in the first pile.
- (3) Alice takes one stone in the second pile.
- (4) Bob can not make a valid move, Alice wins!!!

However, $[1, 2]$ is a losing configuration if they are allowed to take at most 2 stones in a single move. The strategy is described below.

- (1) Alice takes one stone in the first pile.
- (2) Bob takes two stones in the second pile.
- (3) Alice can not make a valid move, Alice loses!!!

We define the function $S(K, M, N)$ as the number of winning configurations $[a_1, a_2, a_3, \dots]$ that satisfy the following conditions:

- (1) There are K elements in the array(K piles of stones).
- (2) $1 \leq a_i \leq M$ for $1 \leq i \leq K$.
- (3) Alice and Bob can take no more than N stones in a single move.

You are given $S(3, 4, 2) = 39$, $S(5, 5, 3) = 2084$ and $S(25, 25, 8) \bmod 1000000007 = 313496033$.

Find $S(10^{18}, 11^{18}, 1234567890) \bmod 1000000007$.

Thanks to **liuguangxi** for promoting this problem to larger K, M, N .

Answer **b182fa6de7fc957c81dd2c705a1538b8**

550 Sum of the numbers with a fixed digit sum**2019-05-29 19:44:44****by C_K_Yang****9 xp****Programming**

Let $F(B, L, D)$ be the sum of all the L -digit numbers in base B , the digit sum of which is D .

For example, $F(3, 3, 3) = 79$. In base 3, there are five 3-digit numbers the digit sum of which is 3. The sum of these five numbers is 79: $(102)_3 + (111)_3 + (120)_3 + (201)_3 + (210)_3 = (2221)_3 = (79)_{10}$.

Now we define a new function $S(B, L, N) = \sum_{p \leq N, p \text{ is prime}} F(B, L, p)$.

For example, $S(5, 5, 11) = 964140$ and $S(56, 20, 101) \equiv 913761394 \pmod{1000004321}$.

Find $S(556678, 999, 12345701) \bmod 1000004321$.

Note: Leading zeroes are not allowed for a L -digit number in base B .

Answer **6a9047dd2c1f5d7c5f61539b408b2ec0**

551 A specific sum over all subsets of a set

2019-06-09 18:07:22

by C_K_Yang

9 xp

Math

Given two positive integers N and M with $0 < N \leq M$, let $I(N, M)$ be the set which contains all natural numbers between N and M (inclusive). For example, $I(3, 6) = \{3, 4, 5, 6\}$.

We define the product of a set as the product of all elements in that set, written as $P(\{\dots\})$ where $\{\dots\}$ is a specific set. For example, $P(I(3, 6)) = 3 \times 4 \times 5 \times 6 = 360$.

Finally, We define a function $C(N, M, K)$ as the sum of product of sets over all $I(N, M)$'s subsets which contain exactly K elements.

In other words, $C(N, M, K) = \sum_{S \in I(N, M), \text{len}(S)=K} P(S)$.

You are given $C(10, 20, 3) = 549450$ and $C(100, 200, 10) \bmod (10^{18} + 3) = 491824643147614094$.

Find $\sum_{i=1}^{500} C(10^{20}, 11^{20}, i)$. Give your answer modulo $(10^{18} + 3)$.

[My timing: 4 s (Python)]

Answer

b76f1f2b5792d11379f3d67e9af92700

552 Bonus for p550

2019-06-18 20:15:56

by C_K_Yang

8 xp

Math

Given two natural numbers a and b with $1 < a \leq b$, let $J(a, b, N)$ be the set which contains all natural numbers not greater than N and not divisible by any number between a and b (inclusive). For example, $J(4, 6, 10) = \{1, 2, 3, 7, 9\}$

- 4 can divide 4 and 8
- 5 can divide 5 and 10
- 6 can divide 6

Hence 4, 5, 6, 8 and 10 will not appear in the set which $J(4, 6, 10)$ represents. Although they are all not greater than 10.

We define the product of a set as the product of all elements in that set, written as $P(\{\dots\})$ where $\{\dots\}$ is a specific set. For example, $P(J(4, 6, 10)) = 1 \times 2 \times 3 \times 7 \times 9 = 378$.

Finally, We define a function $D(a, b, N, K)$ as the sum of product of sets over all $J(a, b, N)$'s subsets which contain exactly K elements.

In other words, $D(a, b, N, K) = \sum_{S \in J(a, b, N), \text{len}(S)=K} P(S).$

You are given $D(5, 8, 20, 3) = 46035$ and $D(7, 12, 3000, 10) \bmod (10^{18} + 3) = 736056499082019203$.

Find $\sum_{i=1}^{150} D(11, 20, 11^{20}, i)$. Give your answer modulo $(10^{18} + 3)$.

[My timing: 18 s (Python)]

Answer

4b78d38d24c4b94ece4c59a68c7d51ca

553 Kostka Numbers 2

2019-10-23 23:39:53

by sinan

8 xp

Math

By a partition of m , we mean an n -tuple: $p = [a_1, a_2, \dots, a_n]$ of positive integers with $a_1 \geq a_2 \geq \dots \geq a_n$ and $a_1 + a_2 + \dots + a_n = m$.

So to any partition p of m , a Ferrer's diagram consisting of m boxes is associated, arranged in n rows in such a way that i -th row contains a_i boxes.

Let c be another partition of m with k -tuple: $c = [b_1, b_2, \dots, b_k]$ of positive integers with $b_1 \geq b_2 \geq \dots \geq b_k$ and $b_1 + b_2 + \dots + b_k = m$.

If $p = [a_1, a_2, \dots, a_n]$ and $c = [b_1, b_2, \dots, b_k]$ are two arbitrary partitions of m , then by a semi-standard Young tableau of shape p and content c , we mean any distribution of the numbers $1, 2, \dots, m$ in the boxes of the associated Ferrer's diagram of p in such a way that

1. every row is non-decreasing;
 2. every column is (strictly) increasing; and
 3. for any $1 \leq i \leq m$, the multiplicity of i in the distribution is b_i .

For example if $m = 6$: $p = [3, 2, 1]$ and $c = [2, 2, 2]$ (which means we have 2 of 1's, 2 of 2's and 2 of 3's, totally 6 of them).

1 1 2
2 3
3

1 1 3
2 2
3

The number of all semi-standard Young tableaux of shape p and content c is denoted by $K(p, c)$ here and it is called the Kostka coefficient or the Kostka number.

Let $p = [21, 13, 8, 5, 3]$. Find the following:

$K(p, c_3)$ where $c_3 = [3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 2]$

$K(p, c_4)$ where $c_4 = [4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 2]$

$K(p, c_5)$ where $c_5 = [5, 5, 5, 5, 5, 5, 5, 5, 5, 5]$

Answer format: $K(p, c_1), K(p, c_2), K(p, c_3), K(p, c_4), K(p, c_5)$

Answer 12e8e84459709a220834363aca0152df

554 Primorial**2019-09-26 20:40:58****by liuguangxi****12 xp****Math**

For a positive integer n , the primorial $n\#$ is defined as the product of all primes $\leq n$: $2\# = 2$, $3\# = 6$, $6\# = 30$,

$$\text{Define } F(n, p) = \begin{cases} n\# \bmod p & , \text{ if } n < p, \\ \frac{1}{p} \times (n\#) \bmod p & , \text{ if } n \geq p. \end{cases}$$

For example, $F(3, 7) = 2 \times 3 \bmod 7 = 6$ and $F(7, 3) = 2 \times 5 \times 7 \bmod 3 = 1$.

For $P = 998244353$, You are given $F(143, P) = 675572163$ and $F(10^9, P) = 574463500$.

Find $F(10^{13}, P)$.

[My timing: ~ 20 s for 10^{13} , ~ 65 s for 10^{14}]

Answer fe807eface99f4ac172bcc7b898b5600

555 Distinct trees I

2019-08-17 00:03:46

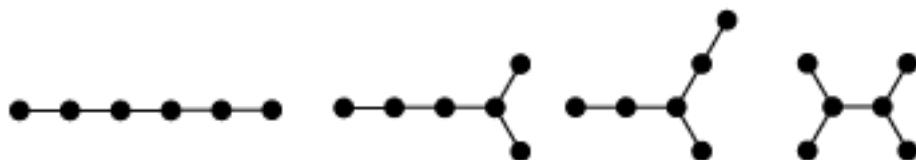
by C_K_Yang

9 xp

Math

Let $Q(n, m)$ be the number of distinct trees with n nodes where each node can have no more than m edges connected to it. Two trees are not considered distinct if they are [isomorphic](#).

For example, $Q(6, 3) = 4$. Four distinct trees are shown below.



You are given $Q(15, 4) = 4347$ and $Q(50, 7) \equiv 646618404 \pmod{1000000007}$.

Find $Q(1000, 15) \pmod{1000000007}$.

Answer dd0edc085ebcca1cafa9ca39a2930df5

556 A Simpler Goodstein Sequence

2019-08-13 10:25:58

by Philippe_57721

0 xp

Math

Given a positive integer n , let the following process:

- 1) We start with base 2
- 2) Express n in the current base
- 3) Increment the base by 1
- 4) Take the number n in this new base and decrement it by 1
- 5) Go to step 2 until $n = 0$

Here an example for $n = 4$

Step	Number	Base	Digits	Comments
1	4	2	1, 0, 0	
2	8	3	2, 2	$8 = 1 \times 3^2 + 0 \times 3^1 + 0 - 1$
3	9	4	2, 1	$9 = 2 \times 4^1 + 2 - 1$
4	10	5	2, 0	
5	11	6	1, 5	
6	11	7	1, 4	
7	11	8	1, 3	
8	11	9	1, 2	
9	11	10	1, 1	
10	11	11	1, 0	
11	11	12	11	
12	10	13	10	
13	9	14	9	
14	8	15	8	
15	7	16	7	
16	6	17	6	
17	5	18	5	
18	4	19	4	
19	3	20	3	
20	2	21	2	
21	1	22	1	
22	0	23	0	Done!

We reach the value 0 in 22 steps. Let write $G(4) = 22$.

What is $G(1024)$? Give your answer modulo 1000000007.

PS: This is a variant of Problem 396.¹

¹ Actually Problem 398.

Answer

549864c4aa8cc916404162feb6984073

557 **The Two Cities****2019-09-01 09:28:55****by Philippe_57721****7 xp****Math**

A traveler goes from city A to city B .

When traveling uphill, he moves at speed u km/h

When traveling on flat road, he moves at speed f km/h

When traveling downhill, he moves at speed d km/h (of course $0 < u < f < d$)

To go from A to B , it takes T_1 hours.

To go from B to A , it takes T_2 hours.

For some triples (u, f, d) these information are sufficient to determinate the distance between A and B (for instance $(30, 35, 42)$).

How many such triples of positive integers (u, f, d) with $\text{GCD}(u, f, d) = 1$ are there if $d < 10^8$?

[My timing: 40 sec]

All values in this problem (speed or distance) are assumed to be integers.

Answer**de0686b9d7ced3405a599e8b90202016**

558 Number of solutions to a Diophantine Equation I

2019-08-30 18:47:55

by C_K_Yang

8 xp

Math

Consider the following Diophantine Equation: $x^2 - y^2 - z^2 - xy - yz - zx = 0$.

We call an integer triple (x, y, z) a primitive solution if it is a solution to the Diophantine Equation with $\gcd(x, y, z) = 1$.

For example, $(3, 1, 1)$ is a primitive solution to the above Diophantine Equation:

$$3^2 - 1^2 - 1^2 - 3 \times 1 - 1 \times 1 - 1 \times 3 = 0$$

In fact, there are only five primitive solutions for $0 < z \leq y \leq x \leq 100$, namely $(3, 1, 1)$, $(19, 11, 1)$, $(37, 19, 5)$, $(61, 29, 11)$ and $(91, 41, 19)$. The sum of these five primitive solutions is 349, i.e. $\sum x + y + z$.

How many primitive solutions are there to the above Diophantine Equation for $0 < z \leq y \leq x \leq 10^{13}$? What is the sum of all these primitive solutions?

Answer format: count, $(\sum x + y + z) \bmod 10^9$

You are given: 479,3971343 for a threshold of 10000.

Answer

47469eb26d8ca25d155c9f5de61d8505

559 Number of solutions to a Diophantine Equation II

2019-09-09 20:51:07

by C_K_Yang

8 xp

Math

Consider the following Diophantine Equation: $x^2 + y^2 + z^2 - xy - yz - zx - x - y - z = 0$.

If x, y, z are all integers, then it is a solution to the Diophantine Equation.

For example, $x = 4, y = 6, z = 9$ is one solution to the above Diophantine Equation:
 $4^2 + 6^2 + 9^2 - 4 \times 6 - 6 \times 9 - 9 \times 4 - 4 - 6 - 9 = 0$.

How many solutions are there to the above Diophantine Equation for $0 < x \leq y \leq z \leq 10^{14}$? What is the sum of all these solutions?

Answer format: count, $(\sum x + y + z) \bmod 10^9$

You are given: 12082, 178899731 for a threshold of 10000.

Answer a769cf65af91e968827a77f46f1c6c26

560 **Balanced numbers revisited****2019-09-21 07:39:49****by C_K_Yang****9 xp****Programming**

A positive integer with k (decimal) digits is called balanced if its first $\lceil \frac{k}{2} \rceil$ digits sum to the same value as its last $\lceil \frac{k}{2} \rceil$ digits, where $\lceil x \rceil$, pronounced ceiling of x , is the smallest integer $\geq x$, thus $\lceil \pi \rceil = 4$ and $\lceil 5 \rceil = 5$.

For example, 77, 101, 13722 and 585774 are balanced numbers.

Find the number and the sum of all balanced numbers n for $0 < n \leq 11^{1111}$ and give your answer modulo 1000000007.

Answer format: count,sum
(both modulo 1000000007)

You are given: 634521,744405873 for a threshold of 12345678.

[My timing: 1 s]

Answer **52924012a87c12822a1faf7c734303b3**

561 Grid painting

2019-10-14 00:17:18

by sinan

8 xp

Programming

Let there be a grid with M rows and N columns. The cells are numbered starting from 0 (topmost left) to $M \times N - 1$ (bottommost right) and some cells are painted to yellow randomly using the following code:

```
uint8 grid[NROWS*NCOLS] = {0};

void FillGrid(const int NROWS, const int NCOLS, const int NCELLS)
{
    int NCellsPainted = (NCELLS+1)>>1;
    uint painted = 0;
    uint32 seed = 10001;
    while (painted < NCellsPainted)
    {
        seed = seed * 10001 + 1001;
        uint rowcol = seed % NCELLS;
        if (grid[rowcol] == 0)
        {
            grid[rowcol] = 1; // [row,col] painted
            ++painted;
        }
    }
}
```

Consider as an example a 6×7 grid filled with 20 painted cells by generating random numbers in the range $[0, 40)$:

0	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	32	33	34
35	36	37	38	39	40	41

Notice that some cells are painted contiguously (on the same row the cells to the right or to the left of a painted cell or on the same column upper/lower cells are also painted). Let's find such randomly painted blocks with more than 1 cell and treat them as some sets. As for the example we would have the following two sets:

$$S_1 = \{0, 1, 2, 3, 4, 5, 7, 8, 11, 15\}$$

$$S_2 = \{17, 23, 24, 25, 28, 29, 30\}$$

S_1 has 10 elements that sum to 56 and S_2 has 7 elements that sum to 176.

How many such sets would we have if we have a 9991×1001 grid filled with 5000000 painted cells by generating random integers in the range $[0, 10000000)$?

What would be the set number of the set with the maximal number of elements?

What would be the set number of the set with the maximal sum of elements?

Note that the sets are sorted by their smallest elements and numbered accordingly.

Answer format: a, b, c, d, e, f, g

where

a is the number of sets

b is the set number of the set with the maximal number of elements

c is the number of elements of the set S_b

d is the sum of elements of the set S_b

e is the set number of the set with the maximal sum of elements

f is the number of elements of the set S_e

g is the sum of elements of the set S_e

Example: 2, 1, 10, 56, 2, 7, 176 for the 6×7 grid above.

[My timing: < 10 s]

Answer df57cac5372dbc5a6ab1bb24894f760e

562 Least number divisible by each number of a sequence

2019-10-19 18:49:10

by C_K_Yang

8 xp

Math

Given two integers $m > 0$ and $n > 0$, let $L(m, n)$ be the number of sequences which satisfy the following properties:

- Each number in the sequence is a natural number.
- The least number divisible by each number of the sequence is m .
- The length of the sequence is n .
- Any two adjacent terms in the sequence are co-prime.

For example, $L(4, 3) = 6$, there are only 6 co-prime sequences: {1, 1, 4}, {1, 4, 1}, {2, 1, 4}, {4, 1, 1}, {4, 1, 2} and {4, 1, 4}.

You are given $L(10!, 10) \equiv 800548088 \pmod{1000000007}$ and
 $L(100!, 100) \equiv 73129372 \pmod{1000000007}$.

Find $L(10^{12}!, 10^{12}) \pmod{1000000007}$.

Answer

17395b280e332232578d56da7a2c3ca4

563 The number with digit sum 10

2019-10-31 22:30:05

by C_K_Yang

10 xp

Math

Writing down the numbers which have a digit sum of 10 in ascending order, we get:

19, 28, 37, 46, 55, 64, 73, 82, 91, 109, 118, ...

Let $f(n)$ be the n -th occurrence of the digit sum 10. For example, $f(1) = 19$, $f(10) = 109$ and $f(100) = 1423$.

Let $S(k) = \sum_{n=1}^k f(n)$. For example $S(100) = 67582$ and $S(10^5) = 178986850167799$.

Find $\sum_{i=1}^{100} S(11^i)$.

Give your answer modulo $(10^{18} + 3)$.

Answer

54dc20c76c6e9fb602e07cedb72d7323

565 Distinct trees II

2019-11-29 17:01:50

by C_K_Yang

8 xp

Math

Let $T(n, m)$ be the number of distinct trees with n nodes where the distance between any two nodes is no more than m . The distance between two vertices p and q is the number of edges we will come across as we travel from vertex p to vertex q in the shortest path. Two trees are not considered distinct if they are [isomorphic](#).

For example, $T(6, 3) = 3$. Three distinct trees are shown below.



You are given $T(15, 4) = 128$ and $T(50, 7) \equiv 270742753 \pmod{1000000007}$.

Find $T(1000, 15) \pmod{1000000007}$.

Answer 6e620c59fd033b2bdःa3415ae0292a12

566 **Tetration**

2019-12-09 17:08:54

by C_K_Yang

7 xp

Math

The [tetration](#) of a number a by a positive integer b , denoted by $a \uparrow\uparrow b$ or ${}^b a$, is recursively defined by:

$$a \uparrow\uparrow 1 = a$$

$$a \uparrow\uparrow (k+1) = a^{a \uparrow\uparrow k}$$

Thus we have e.g. $2 \uparrow\uparrow 2 = 4$ and $3 \uparrow\uparrow 3 = 7625597484987$.

Let $P_{n,i}$ be the i -th prime number among all prime numbers $> n$ such that $P_{10,1} = 11$, $P_{30,2} = 37$ and $P_{100,3} = 107$.

Let $\text{MOD}(n, m) = n$ modulo m .

Further define $f(k) = \sum_{i=1}^k \text{MOD}(i \uparrow\uparrow i, P_{k,i})$.

For example $f(3) = \text{MOD}(1 \uparrow\uparrow 1, 5) + \text{MOD}(2 \uparrow\uparrow 2, 7) + \text{MOD}(3 \uparrow\uparrow 3, 11) = 14$.

You are given $f(100) = 20245$.

Find $f(10^6)$.

Answer**e818d3161bd84ffe7c21552c6b937a09**

567 Minimum logarithmic pair

2019-12-19 17:14:59

by C_K_Yang

10 xp

Math

For any two integers $m > 1$ and $n > 1$, we call an integer pair (m, n) a minimum logarithmic pair if there does **not** exist two integers a and b with $1 < a < m$ and $1 < b < n$ such that $\log_a(b) = \log_m(n)$.

For example, $(4, 9)$ is not a minimum logarithmic pair because $\log_4(9) = \log_2(3)$. Also $(3, 9)$ is not a minimum logarithmic pair since $\log_3(9) = \log_2(4)$. However $(2, 3)$, $(3, 4)$ and $(2, 4)$ are all minimum logarithmic pairs.

Let $D(N)$ be the sum of $m + n$ over all minimum logarithmic pairs (m, n) for $1 < m, n \leq N$.

You are given $D(10) = 818$, $D(100) = 982052$ and $D(10000) = 999830459302$.

Find $D(11^{111})$ and give your answer modulo $10^{18} + 3$.

Answer 9248079081914d2cfaf8f21fdc53ce4

568 Shuffling Cards revisited

2019-12-29 22:40:34

by C_K_Yang

9 xp

Probability

A standard deck of 52 playing cards, which consists of thirteen ranks (Ace, Two, ..., Ten, King, Queen and Jack) each in four suits (Clubs, Diamonds, Hearts and Spades), is randomly shuffled. A *connection* is defined as two cards with the same rank appearing next to each other. Let us call a rank *perfect-k* if k connections occur in that rank after the shuffle.

For examples, the following card arrangement has one perfect-0, one perfect-1, and one perfect-2 (where S = spade, H = Heart, D = diamond, and C = club).

```
[7D][5C][7C][2S][7H][2D][2H][7S][2C][5D][5S][5H]
```

Rank 2 is perfect-1 because there is only one connection between 2D and 2H, Rank 5 is perfect-2 because one connection occurs between 5D and 5S, another connection occurs between 5S and 5H. Rank 7 is perfect-0 since no connections occur.

Let $E(m, n)$ be the probability that the number of perfect- i ranks is always prime for all non-zero number of perfect- i ranks after a random shuffle if we use a deck of playing cards which consists of m ranks, each in n suits.

You are given $E(5, 3) \approx 0.55272$ and $E(13, 4) \approx 0.26393$ (A standard deck of 52 playing cards).

Find $E(10, 8)$ and give your answer rounded to 12 places after the decimal point.

Answer **a5c7e7cd7411c0d328f65af353d70104**

569 Maximal Unity

2020-03-02 00:51:05

by sinan

6 xp

Math

Let S_N be the set of positive numbers less or equal to N . For example $S_5 = \{1, 2, 3, 4, 5\}$.

Find the subset of S_{100} with maximal number of elements (MN) such that $\sum(1/e_i) = 1$ for $i = 1$ to MN and the sum $T = \sum(i \cdot e_i)$ for $i = 1$ to MN is maximal.

Answer format: MN, T

Example: 3, 26 for the subset $S = \{e_1 = 2, e_2 = 3, e_3 = 6\}$ of S_6 .

Answer

4bca322860c89e701c10b91726a83ff0

570 The number of divisors of cube-free numbers

2020-04-09 18:47:25

by C_K_Yang

8 xp

Math

A cube-free number is a positive integer which is divisible by no [cubic number](#) other than 1. Given a number $n > 0$, $f(n)$ is defined as follows:

$$f(n) = \begin{cases} \sigma_0(n) & \text{if } n \text{ is cube-free} \\ b(n) & \text{otherwise} \end{cases}$$

where $\sigma_0(n)$ is the number of divisors of n , $b(n) \equiv \sigma_0(c(n)^2)$ and $c(n)$ is the maximum number p such that p^3 divides n .

For example, $f(28) = 6$ and $f(432) = 9$.

Explanation: 28 is a cube-free number, $f(28) = \sigma_0(28) = 6$. However, 432 is not a cube-free number and 216 is the maximum cubic number which divides 432. $c(432) = 6$ and $b(432) = \sigma_0(6^2) = 9$.

Let $S(N) = \sum_{n=1}^N f(n)$. You are given $S(10^8) = 1223779154$.

Find $S(12345678987654321)$.

Answer**1f85c2826ca6bd51bd0f0b19152a8c9d**

571 Balls in the boxes

2020-05-19 22:30:01

by C_K_Yang

9 xp

Math

We have several blue balls and red balls and we want to put them into boxes.

The rules for placing the balls into boxes are described below.

- Each box contains at least 2 balls and at most 6 balls.
- Each box contains at least 1 red ball and at most 4 blue balls.
- Balls are different from each other, even they have the same color.
- The order of the box doesn't matter. The order of each ball in the box doesn't matter.

For example, if we have 3 blue balls and 2 red balls. There are 7 different ways to put them into boxes.

- (1) $\{B1, R1\}, \{B2, B3, R2\}$
- (2) $\{B1, R2\}, \{B2, B3, R1\}$
- (3) $\{B2, R1\}, \{B1, B3, R2\}$
- (4) $\{B2, R2\}, \{B1, B3, R1\}$
- (5) $\{B3, R1\}, \{B1, B2, R2\}$
- (6) $\{B3, R2\}, \{B1, B2, R1\}$
- (7) $\{B1, B2, B3, R1, R2\}$

Let $W(B, R)$ be the number of different ways to place B blue balls and R red balls into the boxes.

You are given $W(3, 2) = 7$, $W(5, 4) = 1510$ and $W(31, 27) \equiv 336072473 \pmod{1000000007}$.

Find $W(3141, 2718) \pmod{1000000007}$.

Answer **6beb14c4c8c769c366b9e278f056bec2**