

# NEW TOPIC



# Camera Models

# Textbook

- If you are interested, this book has it all:

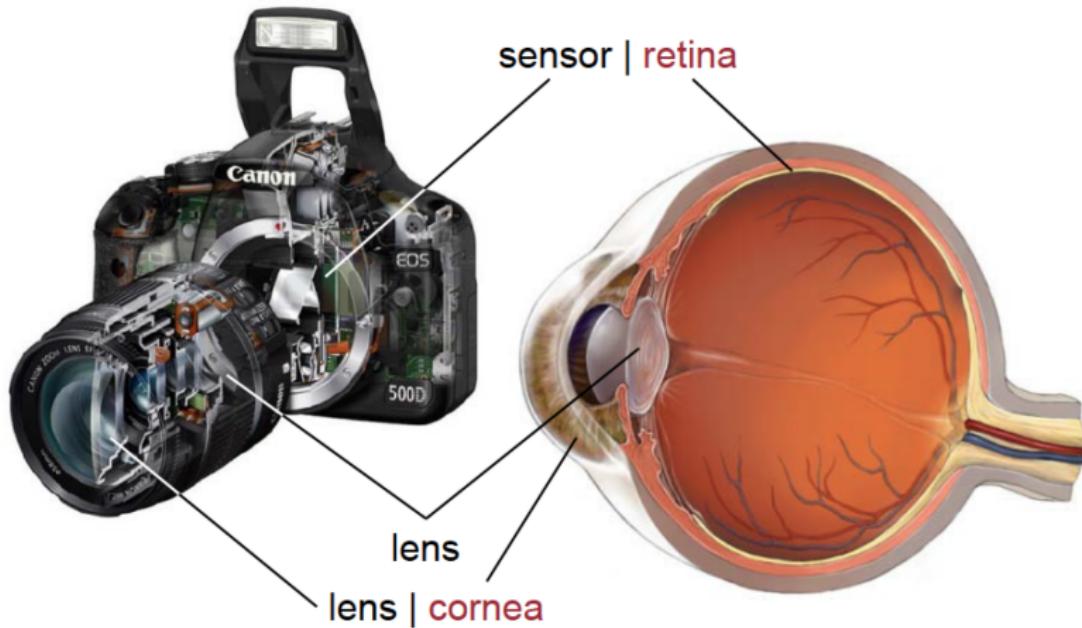
A. Zisserman and R. Hartley

Multiview Geometry

Cambridge University Press, 2003

# Camera

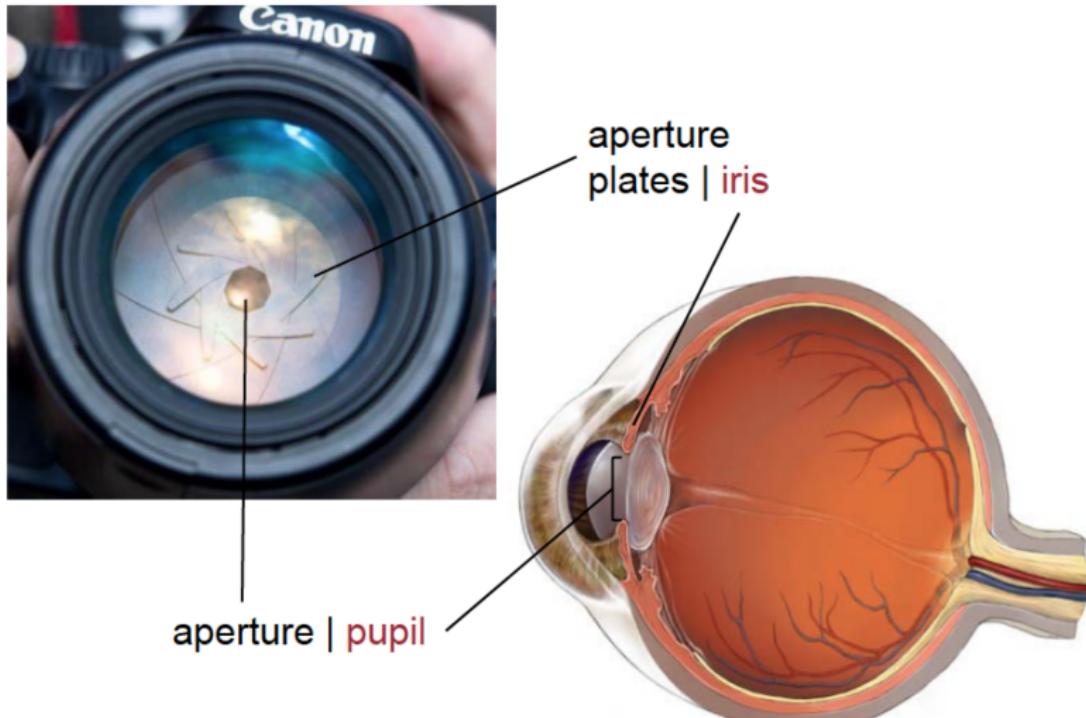
- Camera is structurally similar to the eye



[Source: L.W. Kheng]

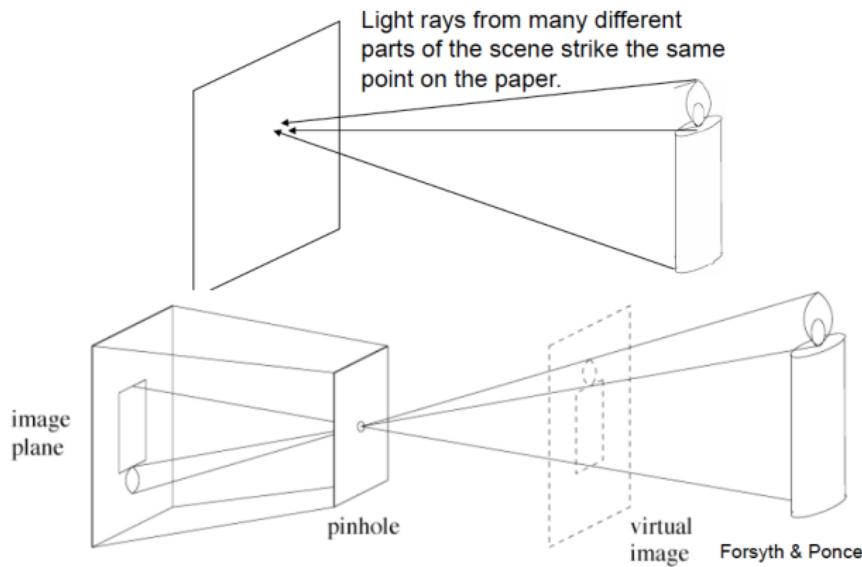
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# Camera

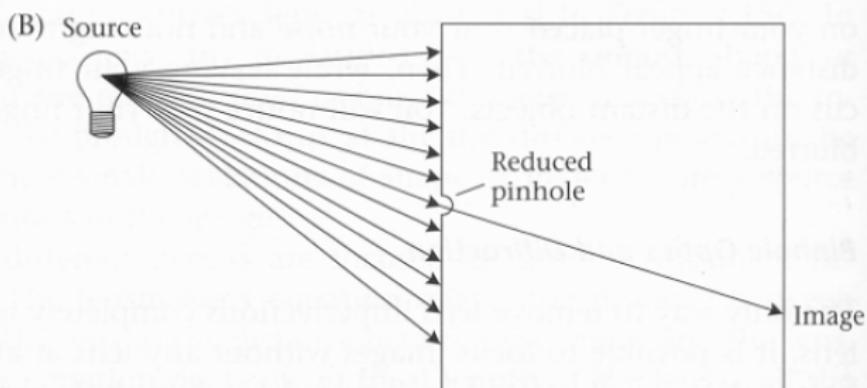
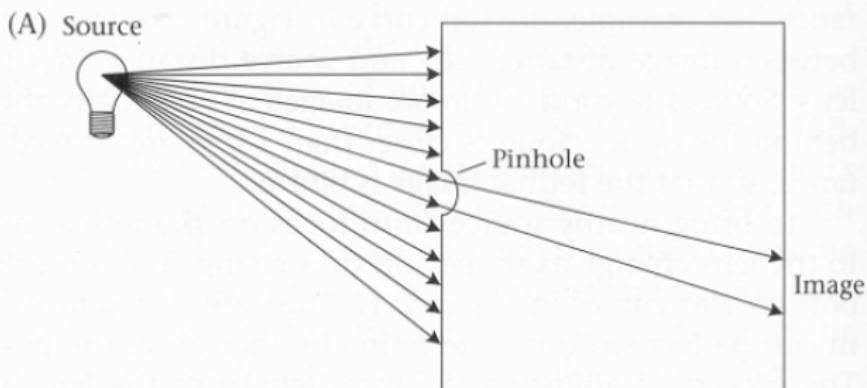
- The Pinhole Camera
- Size of the pinhole is called **aperture**



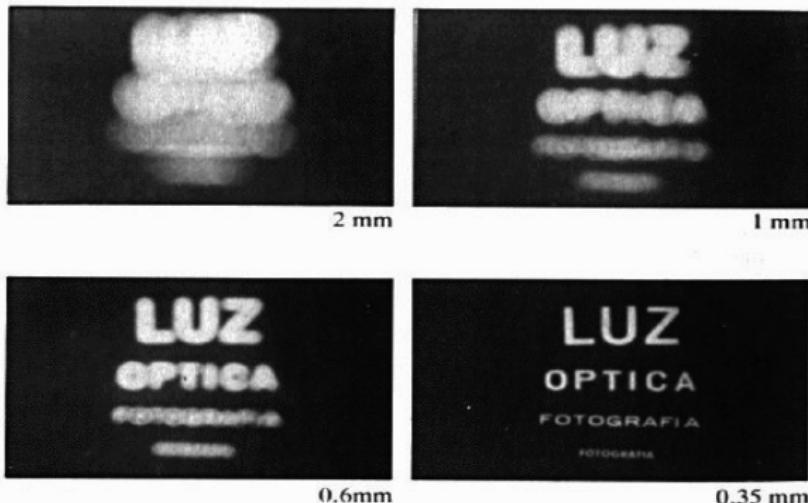
[Source: A. Torralba]

# Pinhole Camera

[Source: A. Torralba]



# Shrinking the Aperture

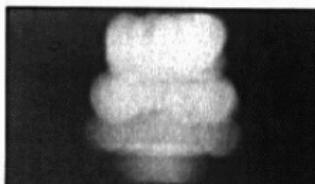


Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects...

[Source: N. Snavely]

# Shrinking the Aperture



2 mm



1 mm



0.6 mm



0.35 mm



0.15 mm

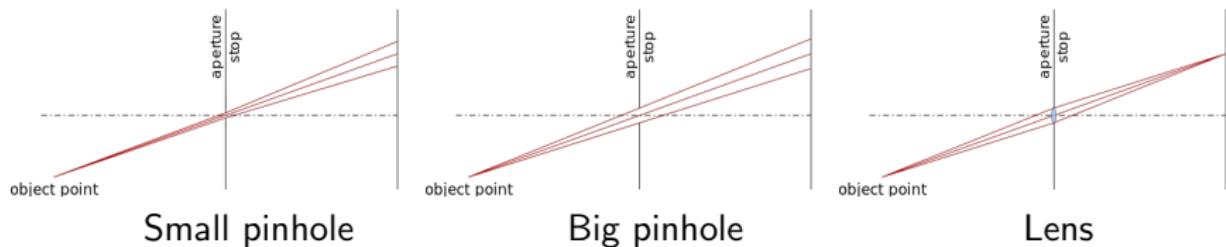


0.07 mm

[Source: N. Snavely]

# Adding a Lens

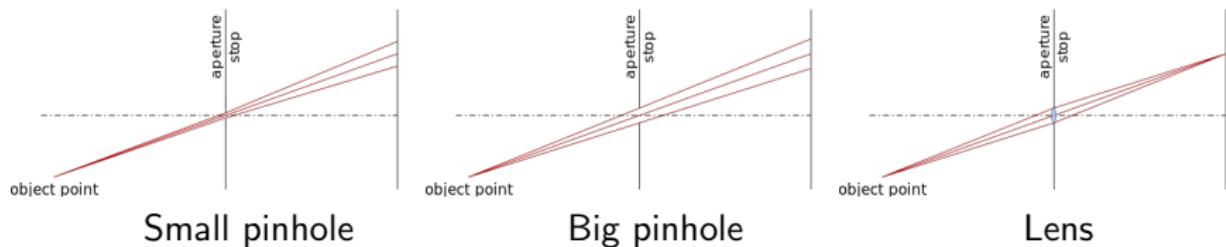
[Pic from Wikipedia]



- A lens focuses light onto the film
- There is a specific distance at which objects are **in focus**

# Adding a Lens

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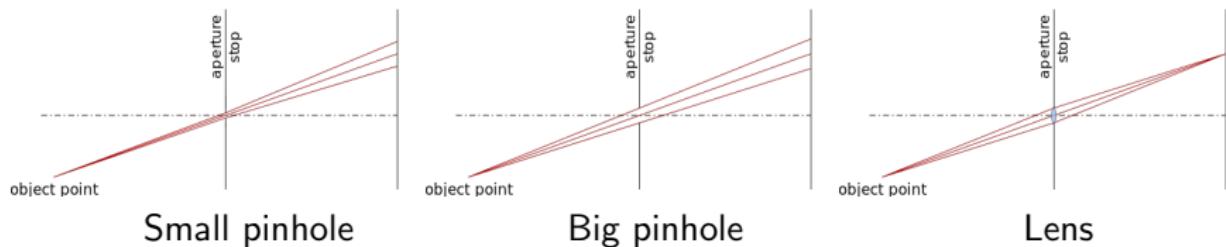


- A lens focuses light onto the film
- There is a specific distance at which objects are **in focus**
- Changing the shape of the lens changes this distance

[Source: N. Snavely]

# Adding a Lens

[Pic from Wikipedia]



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[Source: N. Snavely]

# Imaging

- Images are 2D projections of real world scene
- Images capture two kinds of information:
  - Geometric: positions, points, lines, curves, etc.
  - Photometric: intensity, color
- Complex 3D-2D relationships
- Camera models approximate these relationships

[Source: L.W. Kheng]

# Projection



[Source: N. Snavely]

# Projection



CoolOpticalIllusions.com

[Source: N. Snavely]

# 3D to 2D Projection

- How are 3D primitives projected onto the image plane?
- We can do this using a linear 3D to 2D projection matrix

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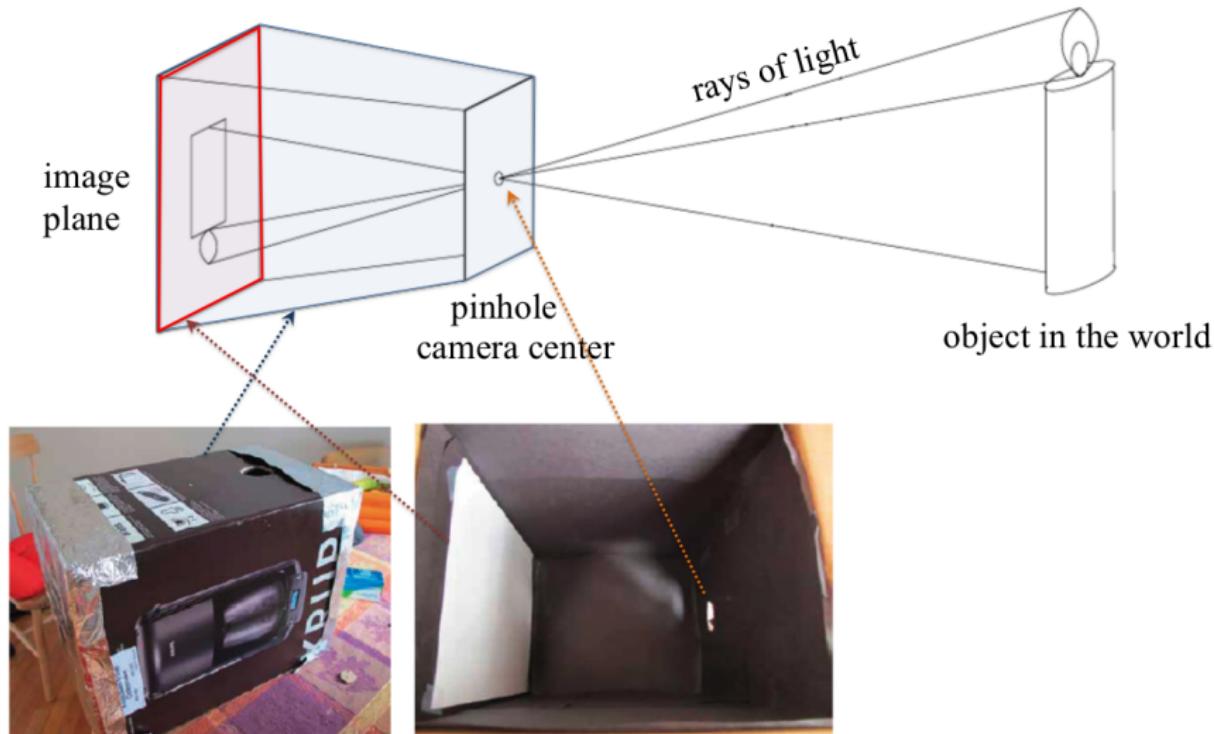
## 3D to 2D Projection

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# Modeling Projection

[Pics from: A. Torralba, Forsyth & Ponce]

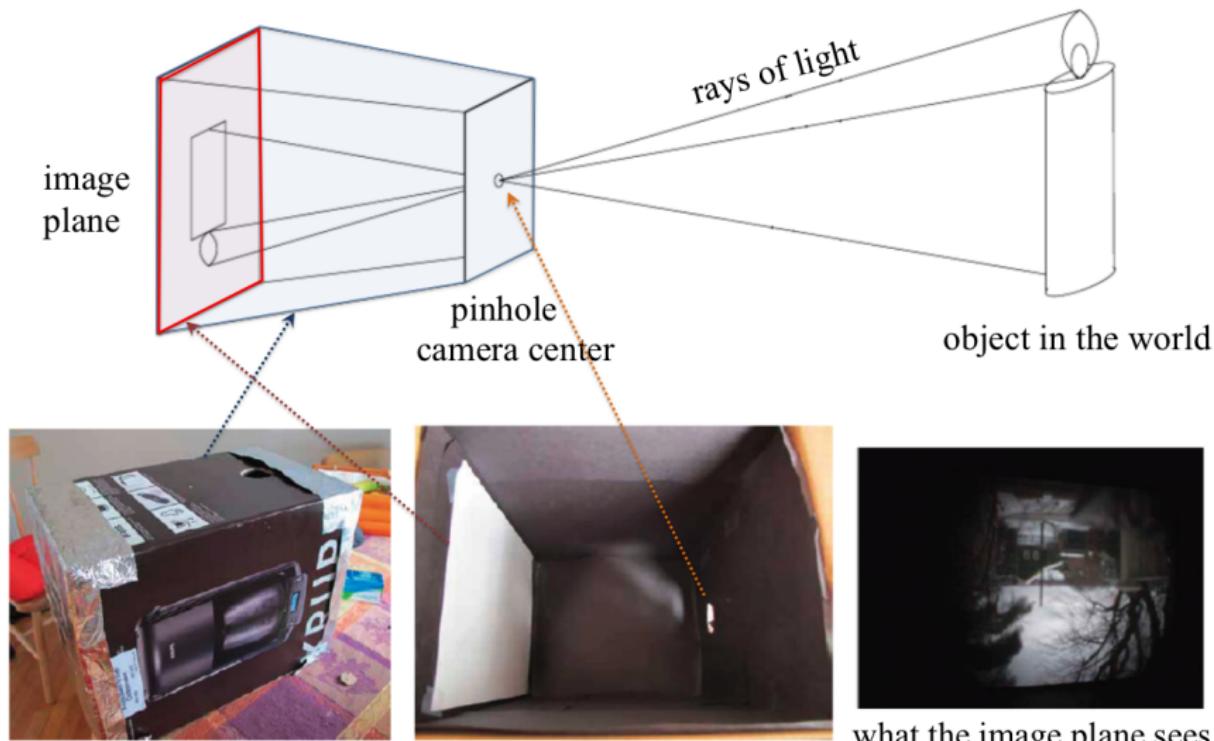
- We will use the pinhole model as an approximation



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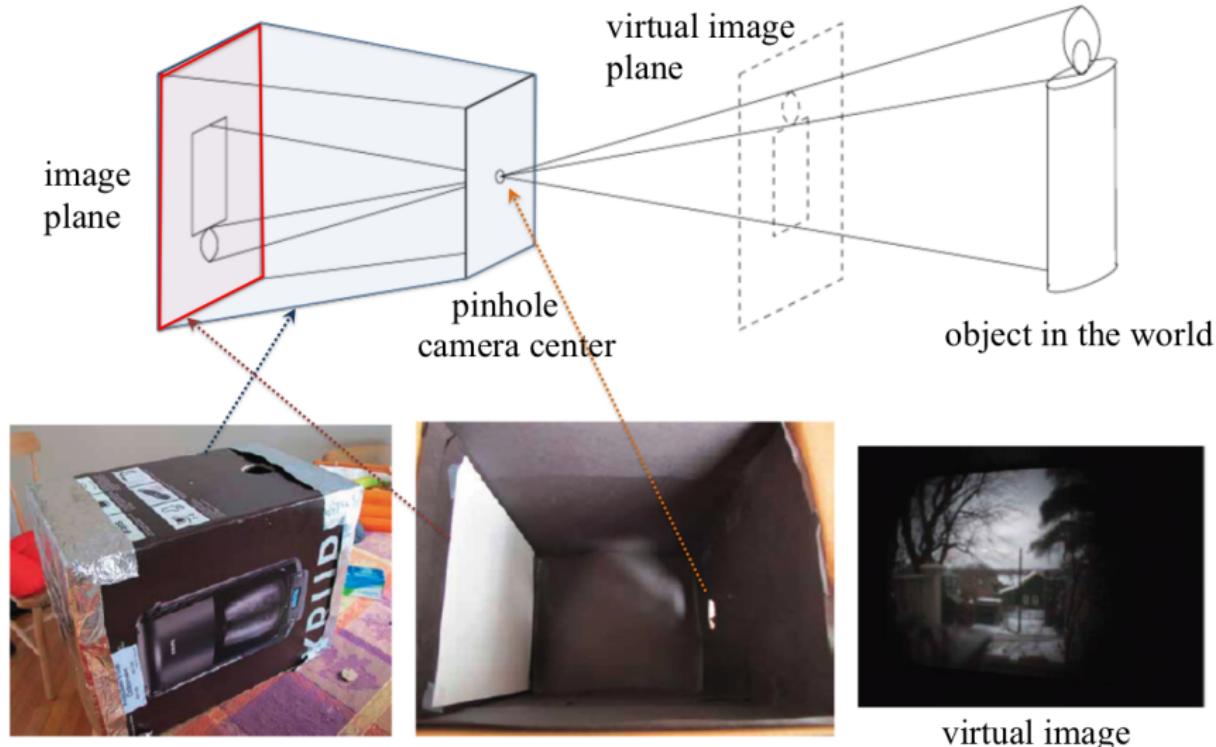
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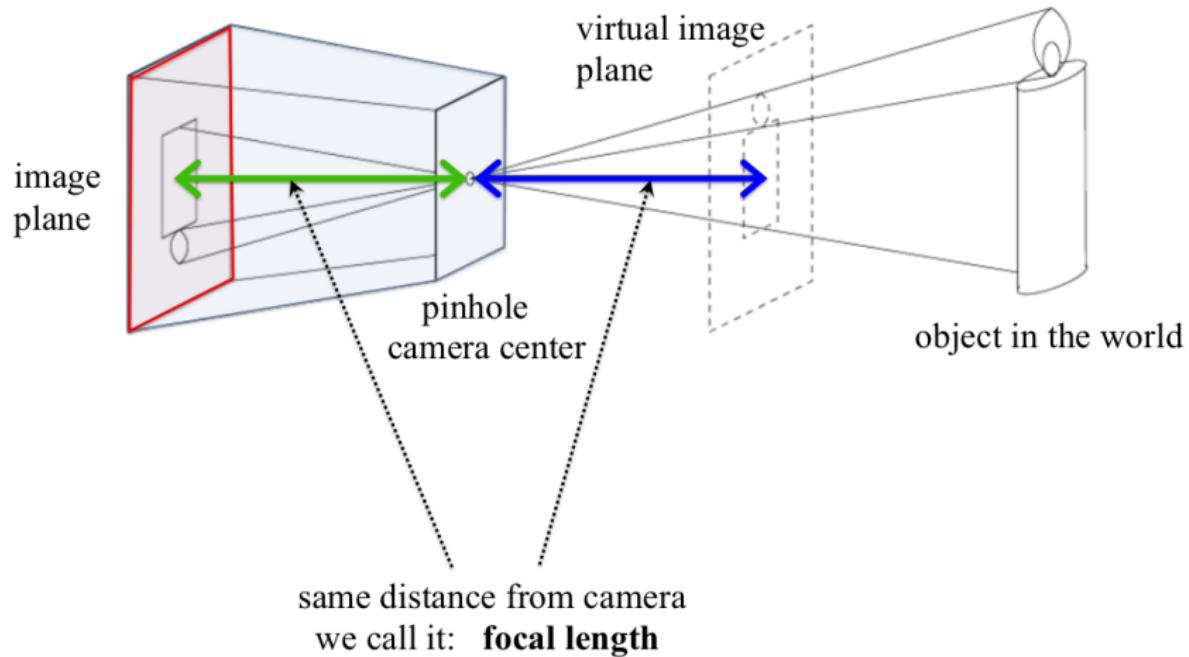
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# Modeling Projection

[Pics from: A. Torralba, Forsyth & Ponce]

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# Focal Length

- Can be thought of as **zoom**
- Larger focal length narrows the **field of view**, more sensors per angle in the scene



24mm



50mm



200mm



800mm

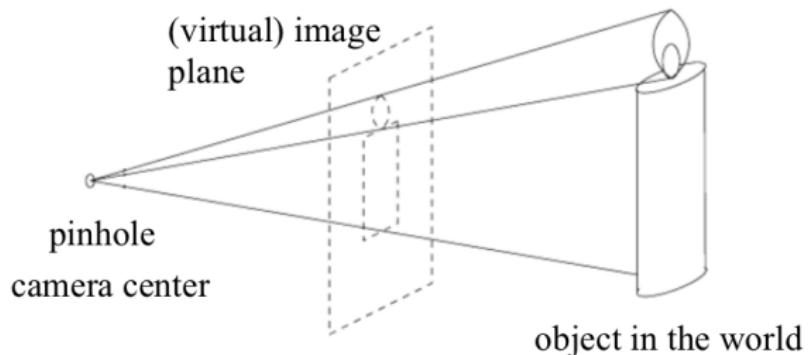


Figure: Image from N. Snavely

# Modeling Projection

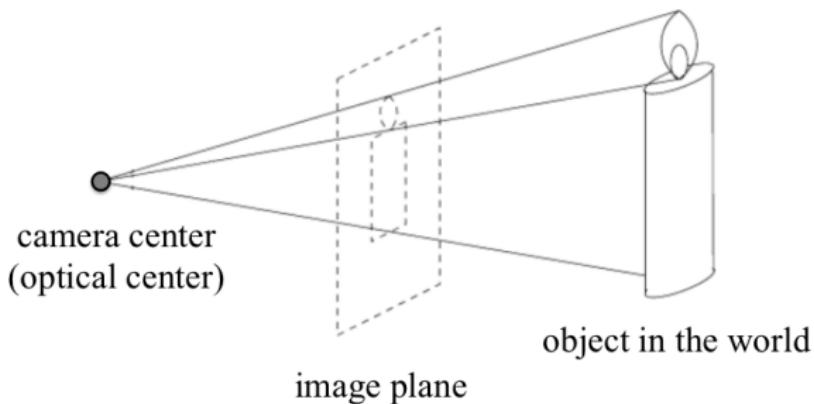
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- Since it's easier to think in a non-upside-down world, we will work with the virtual image plane, and just call it the image plane.
- How do points in 3D project to image plane? If I know a point in 3D, can I compute to which pixel it projects?

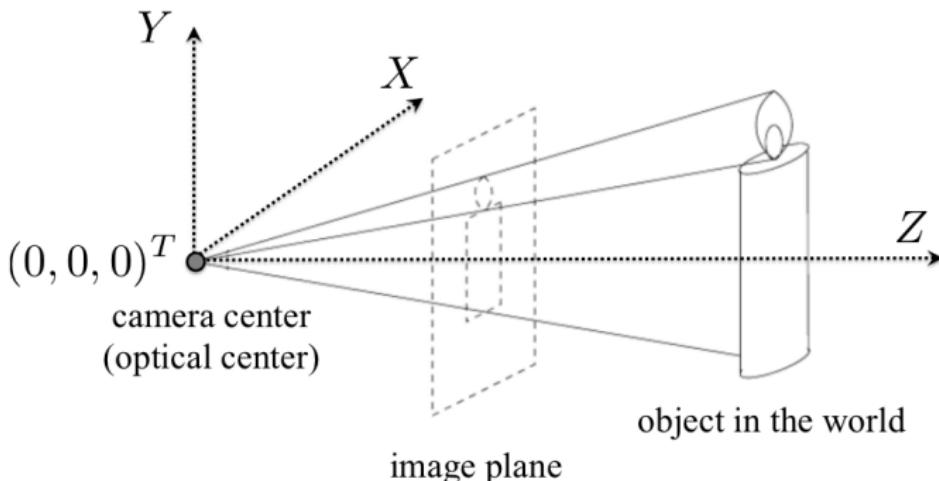
# Modeling Projection



- First some notation which will help us derive the math
- To start with, we need a coordinate system

# Modeling Projection

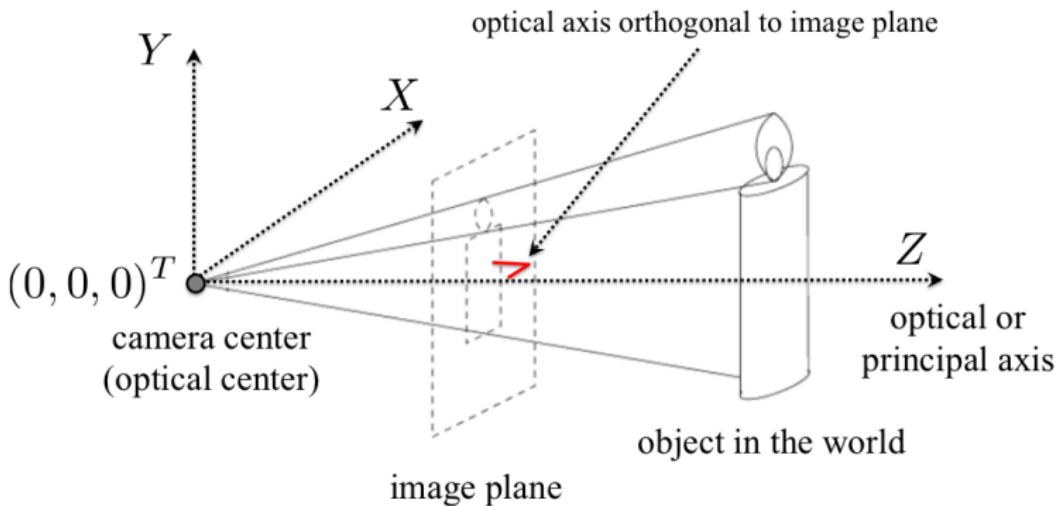
## camera coordinate system in 3D



- We place a coordinate system relative to camera: **optical center** or **camera center C** is thus at origin  $(0, 0, 0)$ .

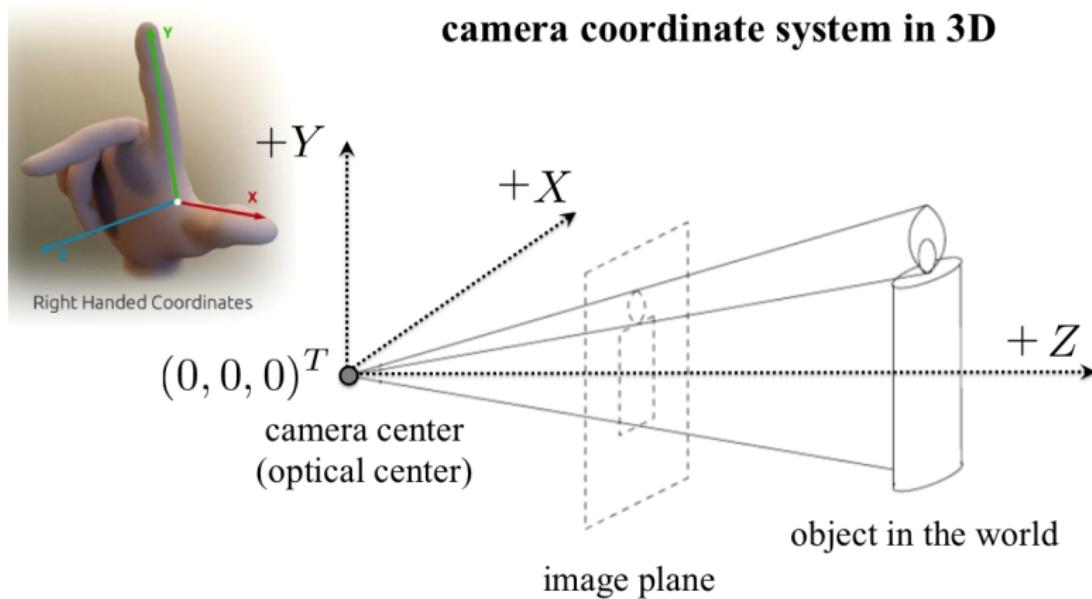
# Modeling Projection

## camera coordinate system in 3D



- The **Z** axis is called the **optical** or **principal axis**. It is orthogonal to the image plane. Axes **X** and **Y** are parallel to the image axes.

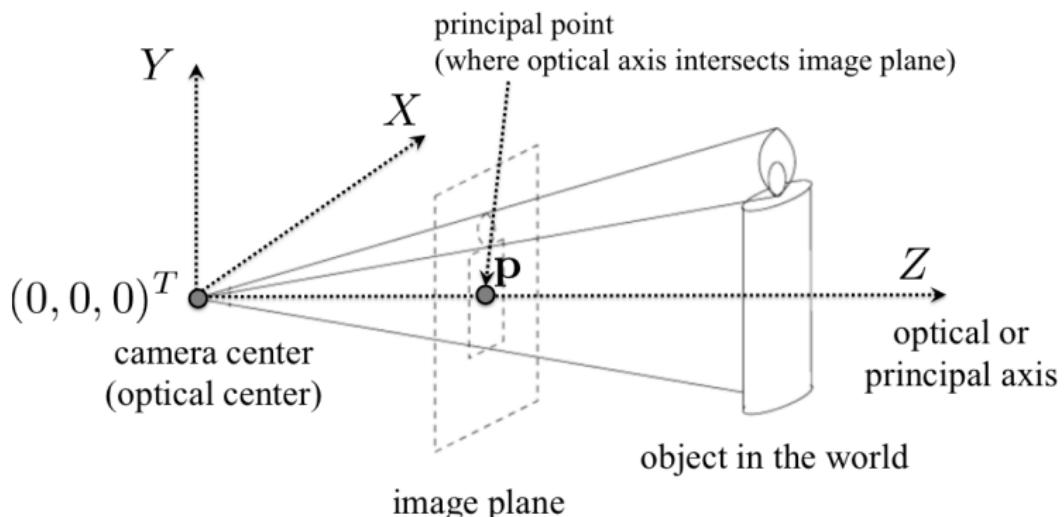
# Modeling Projection



- We will use a **right handed** coordinate system

# Modeling Projection

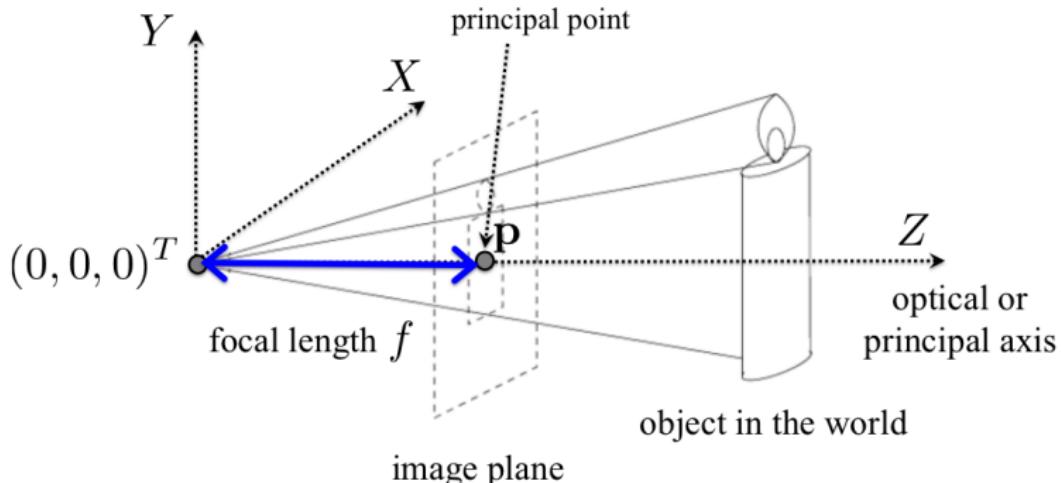
## camera coordinate system in 3D



- The optical axis intersects the image plane in a point, **p**. We call this point a **principal point**. It's worth to remember the principal point since it will appear again later in the math.

# Modeling Projection

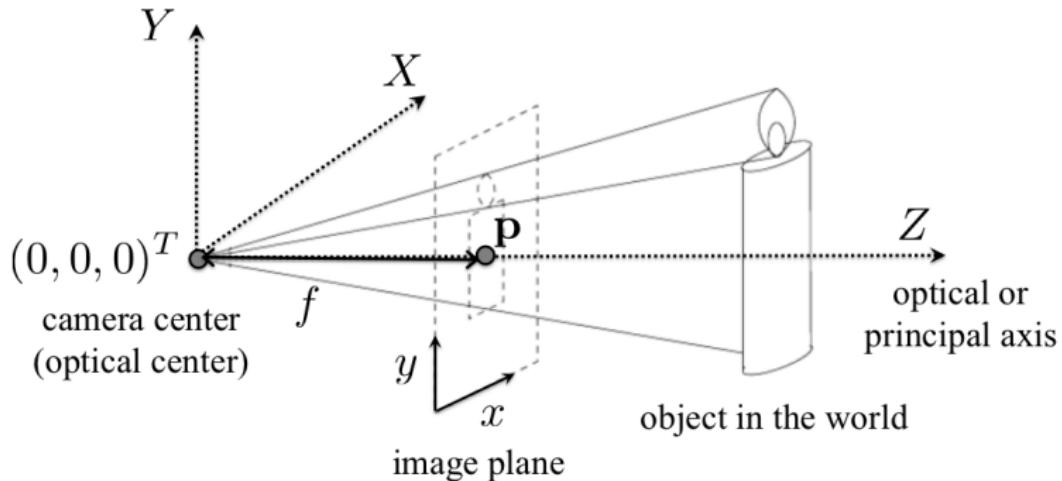
## camera coordinate system in 3D



- The distance from the camera center to the principal point is called **focal length**, we will denote it with  $f$ . It's worth to remember the focal length since it will appear again later in the math.

# Modeling Projection

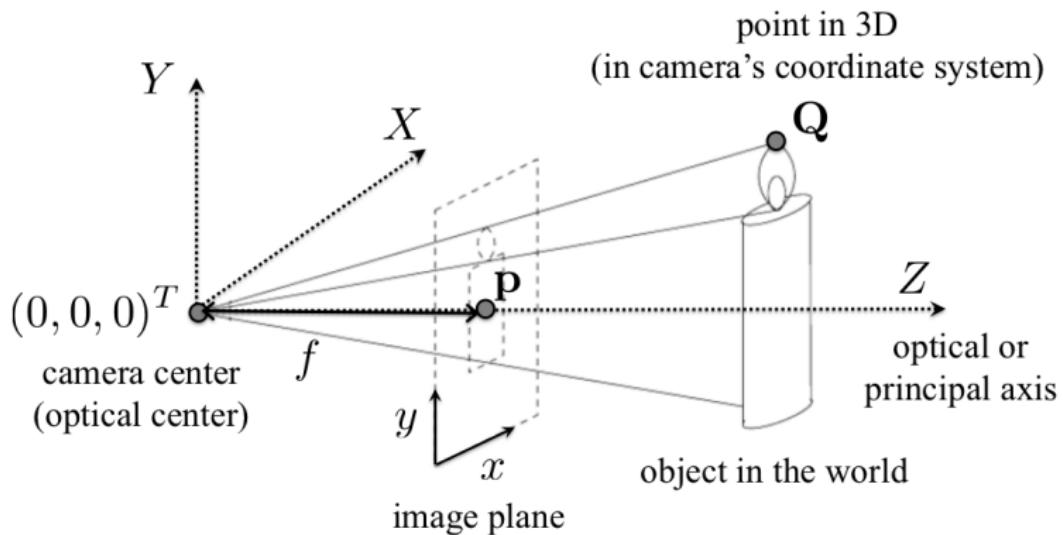
## camera coordinate system in 3D



- We'll denote the image axes with  $x$  and  $y$ . An image we see is of course represented with these axes. We'll call this an **image coordinate system**.
- The tricky part is how to get from the camera's coordinate system (3D) to the image coordinate system (2D).

# Modeling Projection

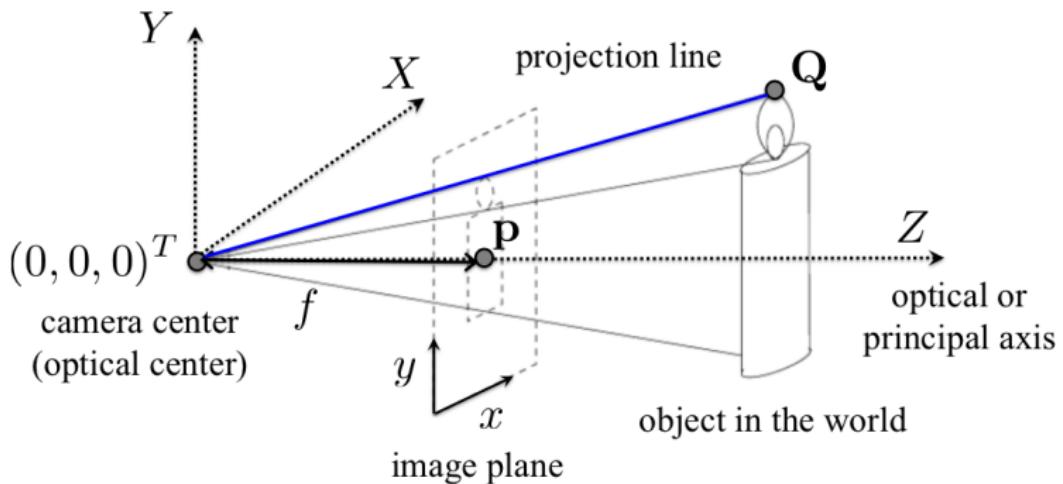
## camera coordinate system in 3D



- Let's take some point  $Q$  in 3D.  $Q$  "lives" relative to the camera; its coordinates are assumed to be in camera's coordinate system.

# Modeling Projection

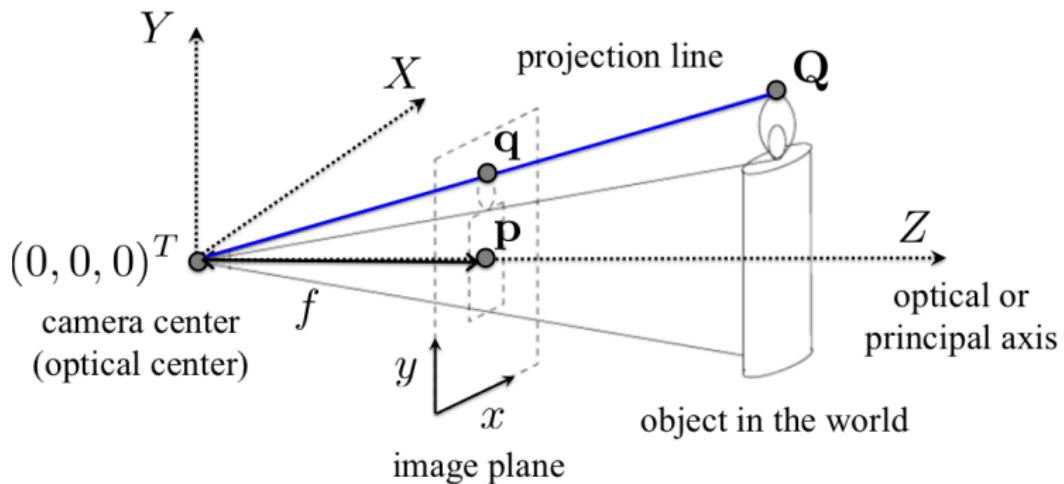
## camera coordinate system in 3D



- We call the line from  $Q$  to camera center a **projection line**.

# Modeling Projection

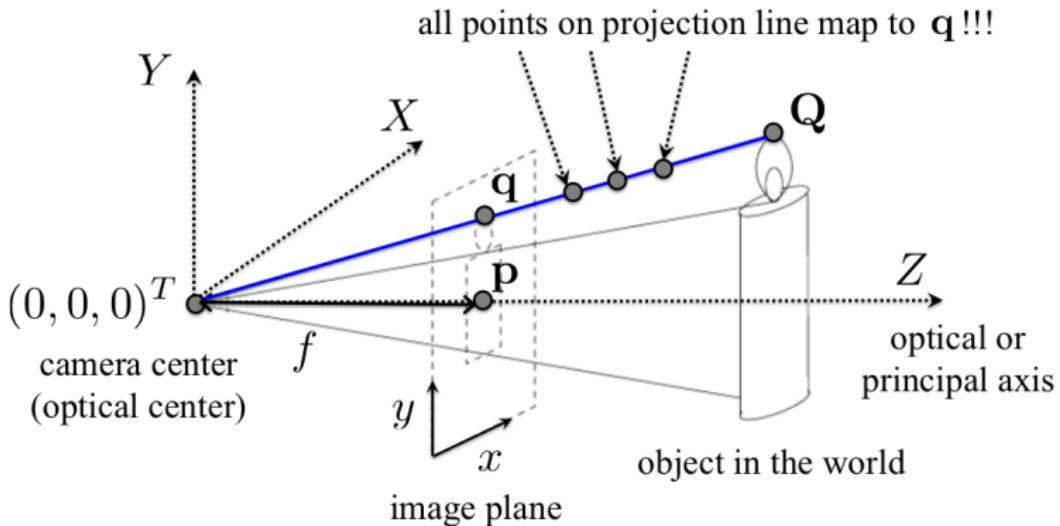
## camera coordinate system in 3D



- The projection line intersects the image plane in a point  $q$ . This is the point we see in our image.

# Modeling Projection

## camera coordinate system in 3D



- First thing to notice is that all points from  $\mathbf{Q}$ 's projection line project to the same point  $\mathbf{q}$  in the image!
- Ambiguity:** It's impossible to know how far a 3D point is from the camera along the projection line by looking only at the image (point  $\mathbf{q}$ ).

# Modeling projection



From the movie Bone Collector

- **Ambiguity:** It's impossible to know how far a 3D point is from the camera along the projection line by looking only at the image (point  $q$ ).
- It's impossible to know the real 3D size of objects just from an image
- Why did the detective put a dollar bill next to the footprint?
- How would you compute the shoe's dimensions?

# Modeling projection

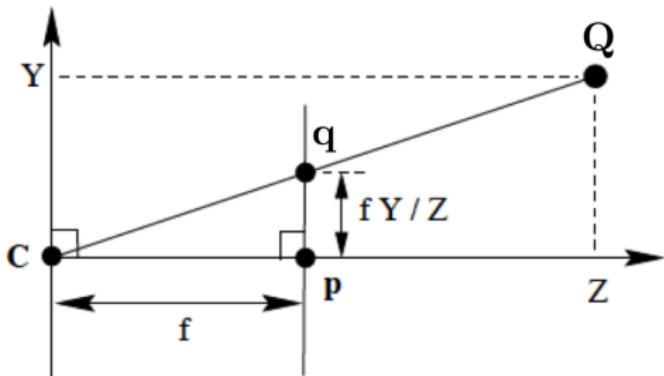


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# Projection: Ready for Math

[Pic from: Zisserman & Hartley]



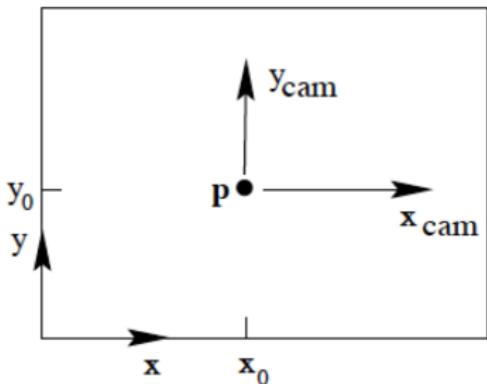
## Projection Equations

- Using similar triangles to preserve ratios:

$$\mathbf{Q} = (X, Y, Z)^T \rightarrow \left( \frac{f \cdot X}{Z}, \frac{f \cdot Y}{Z}, f \right)^T$$

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$$\mathbf{Q} = (X, Y, Z)^T \rightarrow \left( \frac{f \cdot X}{Z}, \frac{f \cdot Y}{Z}, f \right)^T$$

- This is relative to principal point  $\mathbf{p}$ . To move the origin to  $(0, 0)$  in image:

$$\mathbf{q} = (X, Y, Z)^T \rightarrow \left( \frac{f \cdot X}{Z} + p_x, \frac{f \cdot Y}{Z} + p_y, 0 \right)^T$$

where  $\mathbf{p} = (p_x, p_y)$  is the principal point

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- Get the projection by throwing the last coordinate:

$$\mathbf{Q} = (X, Y, Z)^T \rightarrow \mathbf{q} = \left( \frac{f \cdot X}{Z} + p_x, \frac{f \cdot Y}{Z} + p_y \right)^T$$

- This is NOT a linear transformation as a division by  $Z$  is non-linear

# Homogeneous Coordinates!

- We will use homogeneous coordinates, which simply append a 1 to the vector

Homogeneous coordinates to the rescue!

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image  
coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

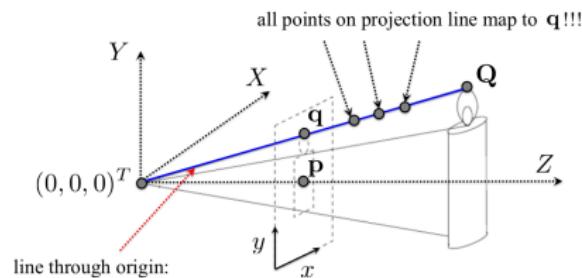
homogeneous scene  
coordinates

[Source: N. Snavely]

# Homogeneous Coordinates!

- We will use homogeneous coordinates, which simply append a 1 to the vector
- In homogeneous coordinates, scaling doesn't affect anything:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} w \cdot x \\ w \cdot y \\ w \end{bmatrix}$$



$$\begin{bmatrix} X_Q \\ Y_Q \\ Z_Q \end{bmatrix} \sim \lambda \cdot \begin{bmatrix} X_Q \\ Y_Q \\ Z_Q \end{bmatrix}$$

- In Projective Geometry, all points are equal under scaling  $\lambda$  along the ray

## Back to Perspective Projection

- We currently have this (the nasty division by  $Z$ ):

$$\mathbf{Q} = (X, Y, Z)^T \rightarrow \mathbf{q} = \begin{bmatrix} \frac{f \cdot X}{Z} + p_x \\ \frac{f \cdot Y}{Z} + p_y \end{bmatrix}$$

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- Write this with homogeneous coordinates:

$$\mathbf{Q} = (X, Y, Z)^T \rightarrow \mathbf{q} = \begin{bmatrix} \frac{f \cdot X}{Z} + p_x \\ \frac{f \cdot Y}{Z} + p_y \\ 1 \end{bmatrix} \sim \begin{bmatrix} f \cdot X + Z \cdot p_x \\ f \cdot Y + Z \cdot p_y \\ Z \end{bmatrix}$$

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- We can now write this as matrix multiplication:

$$\mathbf{Q} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} f \cdot X + Z \cdot p_x \\ f \cdot Y + Z \cdot p_y \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

# Camera Intrinsics

- From previous slide:

$$\mathbf{Q} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} f \cdot X + Z \cdot p_x \\ f \cdot Y + Z \cdot p_y \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- Write:

$$K = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

This is called a **camera calibration matrix** or **intrinsic parameter matrix**.  
The parameters in  $K$  are called **internal camera parameters**.

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This is called a **camera calibration matrix** or **intrinsic parameter matrix**.  
The parameters in  $K$  are called **internal camera parameters**.

- Finally:  $K \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} w \cdot x \\ w \cdot y \\ w \end{bmatrix} \rightarrow \mathbf{q} = \begin{bmatrix} x \\ y \end{bmatrix}$

[Source: Zisserman & Hartley]

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- And there might be a skew angle  $\theta$  between  $x$  and  $y$  image axis:

$$K = \begin{bmatrix} f_x & -f_x \cot \theta & p_x \\ 0 & f_y / \sin \theta & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

[Source: Zisserman & Hartley]

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- Camera calibration matrix:

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We'll work with this one

- It can be a little more complicated. Pixels may not be square:

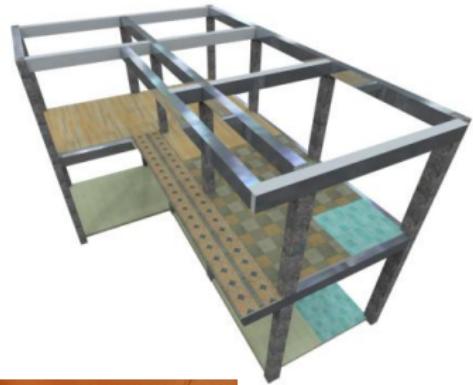
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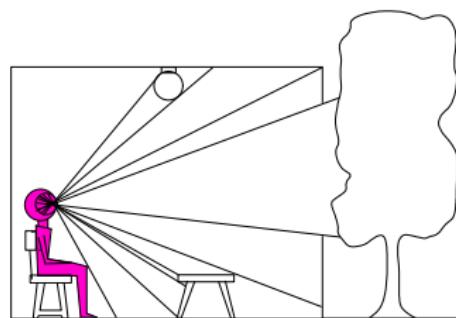
# Perspective Projection



[Source: N. Snavely]

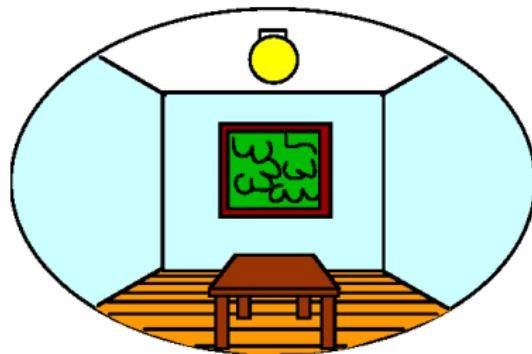
# Dimensionality Reduction Machine (3D to 2D)

*3D world*



Point of observation

*2D image*



## What have we lost?

- Angles
- Distances (lengths)

Slide by A. Efros

Figures © Stephen E. Palmer, 2002

# Projection properties

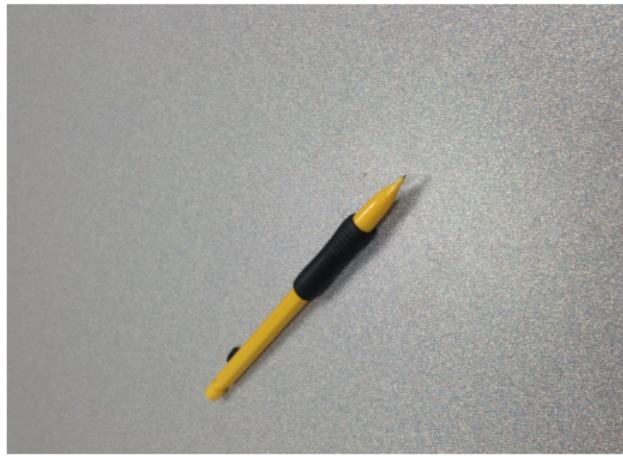
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# Projection properties

- **Many-to-one:** any points along same ray map to same point in image
- Points → points

# Projection properties

- **Many-to-one:** any points along same ray map to same point in image
- Points → points
- Lines → lines.



# Projection properties

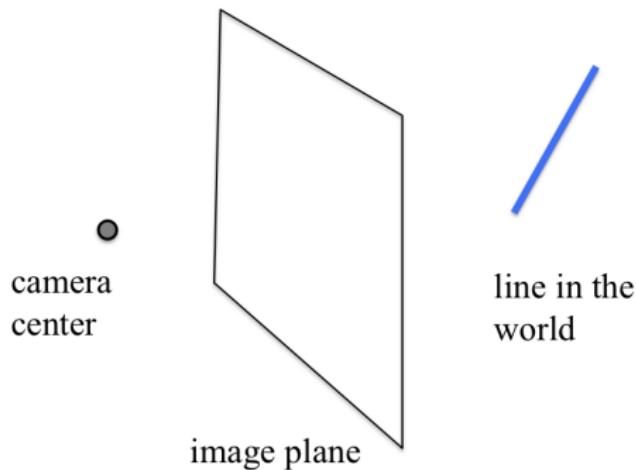
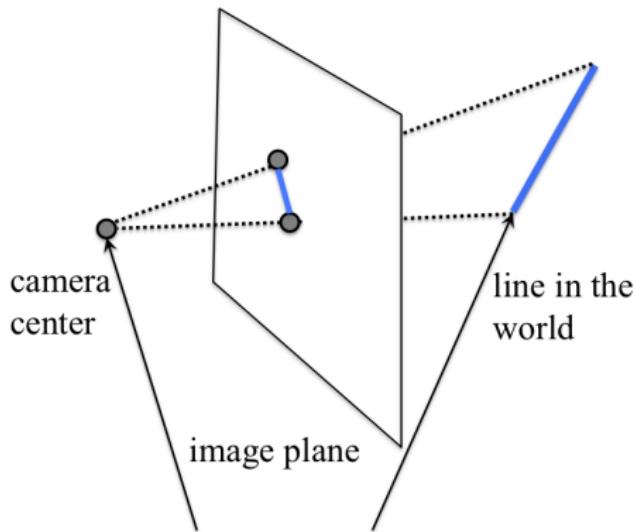


Figure: Proof by drawing

# Projection properties



line and a point make a plane  
→ a plane intersects an (image) plane in a line!

Figure: Proof by drawing

# Projection properties

- **Many-to-one:** any points along same ray map to same point in image
- Points → points
- Lines → lines
- But line through principal point projects to a point.



Figure: Can you tell where is the principal point?

# Projection properties

- **Many-to-one:** any points along same ray map to same point in image
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- Planes → planes



# Projection properties

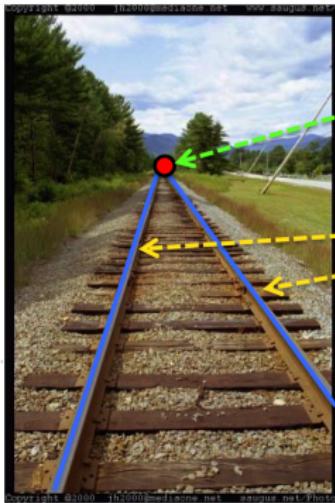
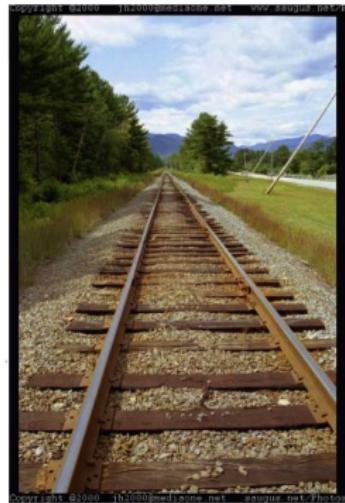
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- Planes → planes
- But plane through principal point projects to line.



# Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- Each different direction in the world **has its own vanishing point**



[Adopted from: N. Snavely]

# Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

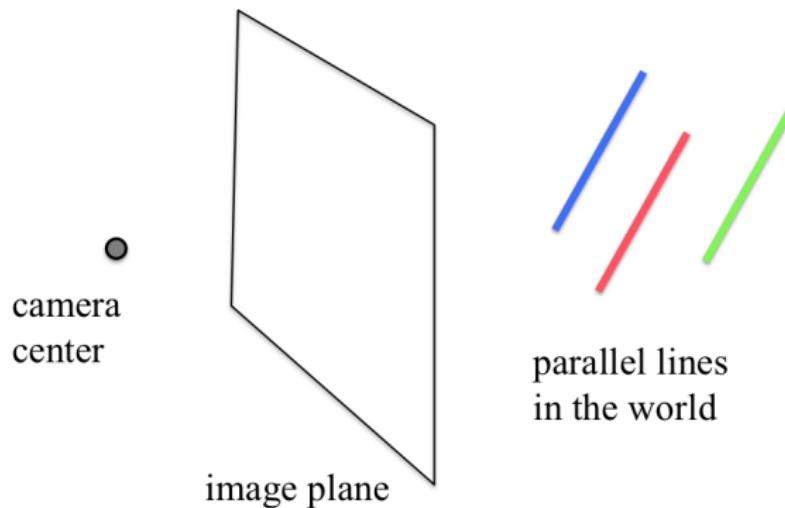
- Each different direction in the world **has its own vanishing point**
- All lines in the same direction in 3D intersect at the **same vanishing point**



[Pic: R. Szeliski]

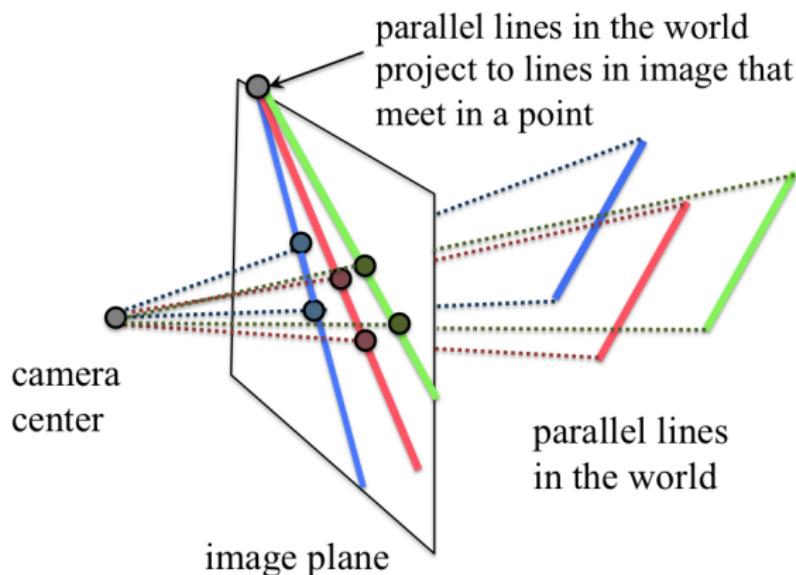
# Projection Properties: Vanishing Point

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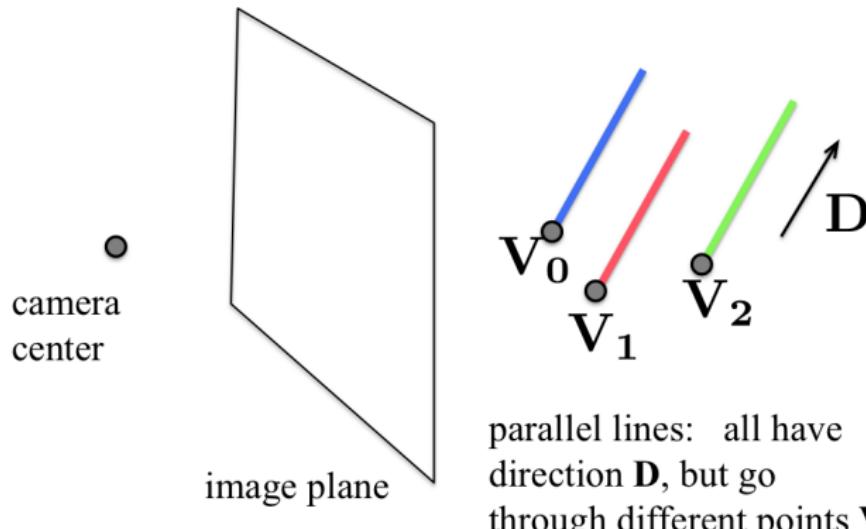
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$$\begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = K\mathbf{X} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x + tD_x \\ V_y + tD_y \\ V_z + tD_z \end{bmatrix} = \begin{bmatrix} fV_x + ftD_x + p_x V_z + tp_x D_z \\ fV_y + ftD_y + p_y V_z + tp_y D_z \\ V_z + tD_z \end{bmatrix}$$

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- Move infinitely far from the camera by taking  $t \rightarrow \infty$  and compute  $x$  and  $y$ :

$$x = \lim_{t \rightarrow \infty} \frac{fV_x + ftD_x + p_x V_z + tp_x D_z}{V_z + tD_z} = \frac{fD_x + p_x D_z}{D_z}$$

$$y = \lim_{t \rightarrow \infty} \frac{fV_y + ftD_y + p_y V_z + tp_y D_z}{V_z + tD_z} = \frac{fD_y + p_y D_z}{D_z}$$

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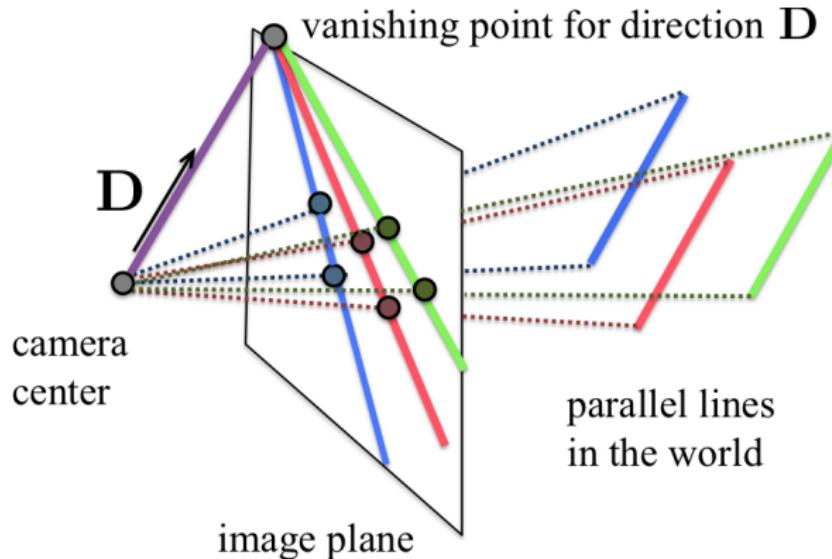
- This doesn't depend on  $\mathbf{V}$ ! So all lines with direction  $\mathbf{D}$  go to this point!

## Projection Properties: Vanishing Point

- All lines with the same 3D direction intersect at the **same vanishing point**.

# Projection Properties: Vanishing Point

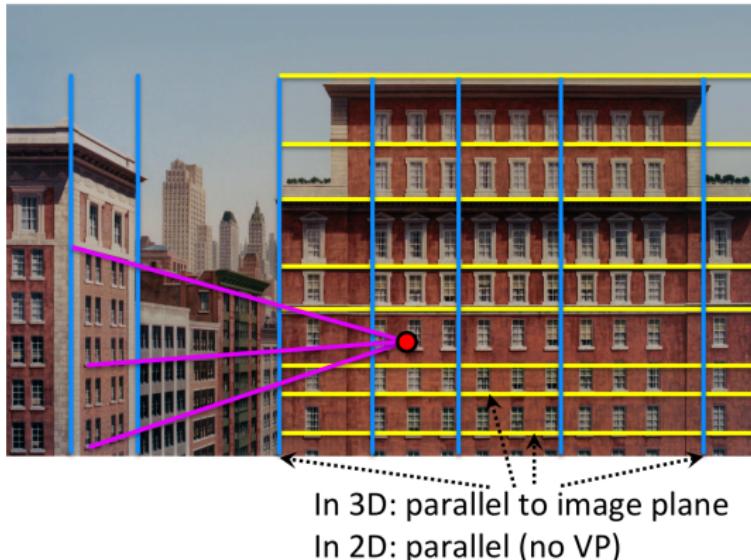
- All lines with the same 3D direction intersect at the **same vanishing point**.
- The easiest way to find this point: Translate line with direction  $\mathbf{D}$  to the camera center. This line intersects the image plane in the vanishing point corresponding to direction  $\mathbf{D}$ !



# Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

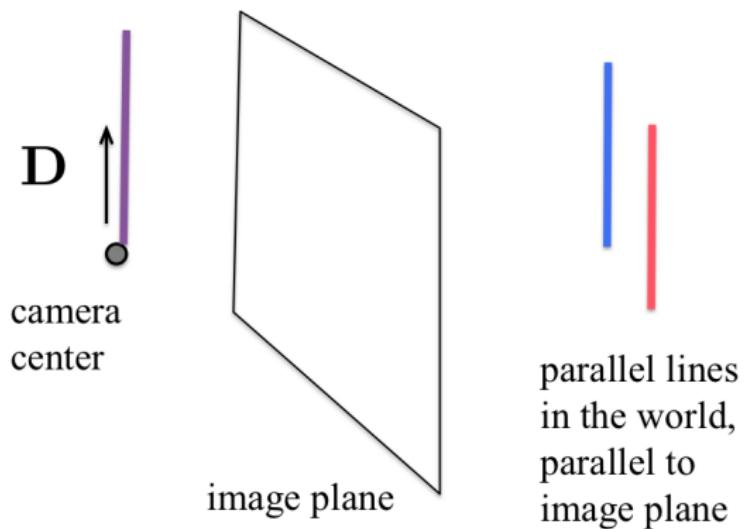
- Each different direction in the world **has its own vanishing point**
- Lines parallel to image plane are also parallel in the image (we say that they intersect at infinity).



## Projection Properties: Cool Facts

- Lines parallel to image plane are also parallel in the image. We say that they intersect at infinity.

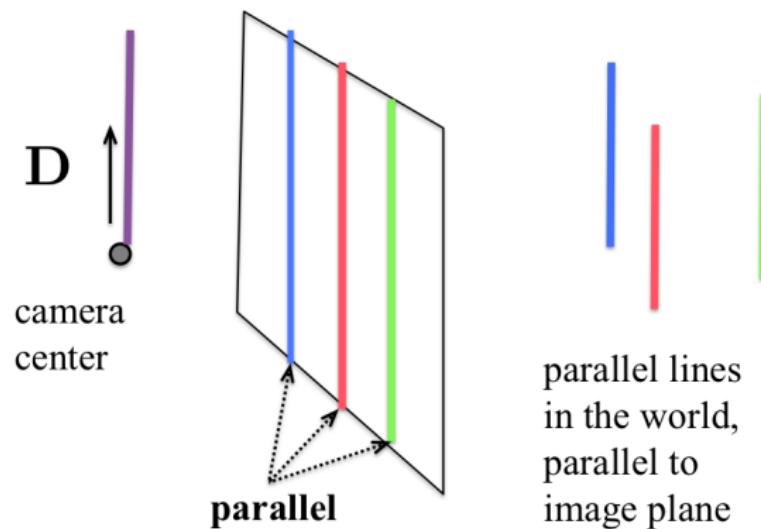
doesn't intersect image plane! So no vanishing point!



# Projection Properties: Cool Facts

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## Projection Properties: Vanishing Point

- Line that passes through  $\mathbf{V}$  with direction  $\mathbf{D}$ :  $\mathbf{X} = \mathbf{V} + t\mathbf{D}$ .
- Project a second line passing through  $R$  and take the distance:

$$d_x = \frac{fV_x + ftD_x + p_x V_z + tp_x D_z}{V_z + tD_z} - \frac{fR_x + ftD_x + p_x R_z + tp_x D_z}{R_z + tD_z}$$

- Simplify with  $D_z = 0$  and  $V_z = R_z = 1$
- Can you prove the above distance remains constant in the image plane regardless of  $t$  i.e., the projected versions of the lines are also parallel.

# Projection Properties: Cool Tricks

- This picture has been recorded from a car with a camera on top. We know the camera intrinsic matrix  $K$ .
- Can we tell the incline of the hill we are driving on?
- How?



# Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

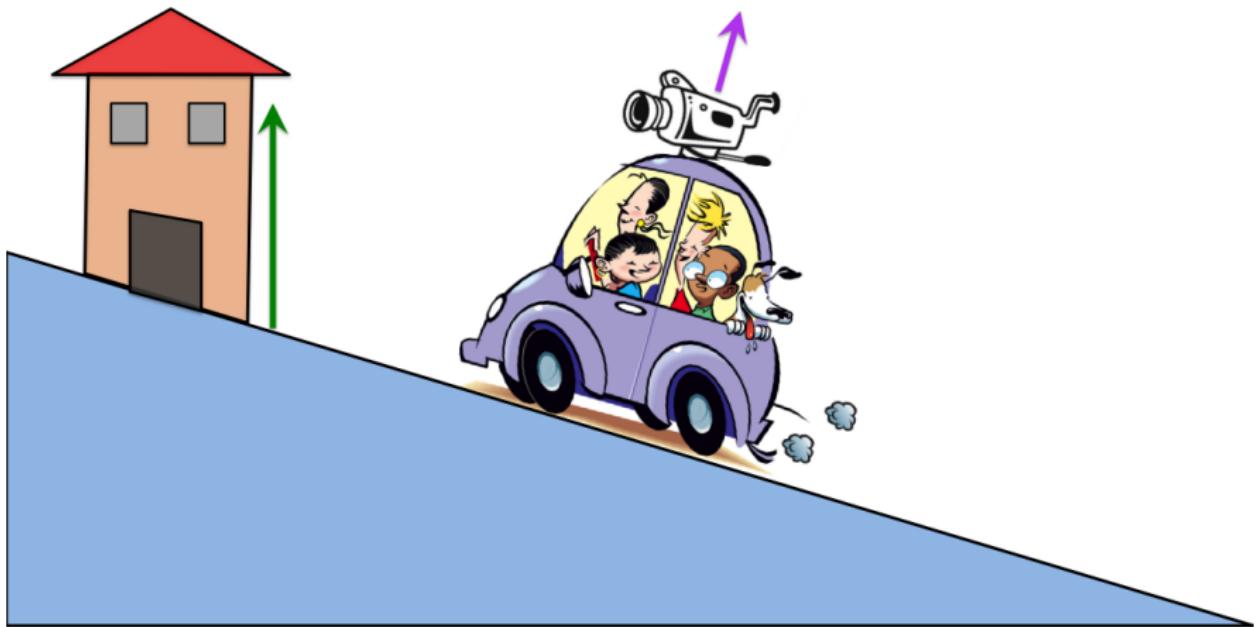


Figure: This is the 3D world behind the picture.

# Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

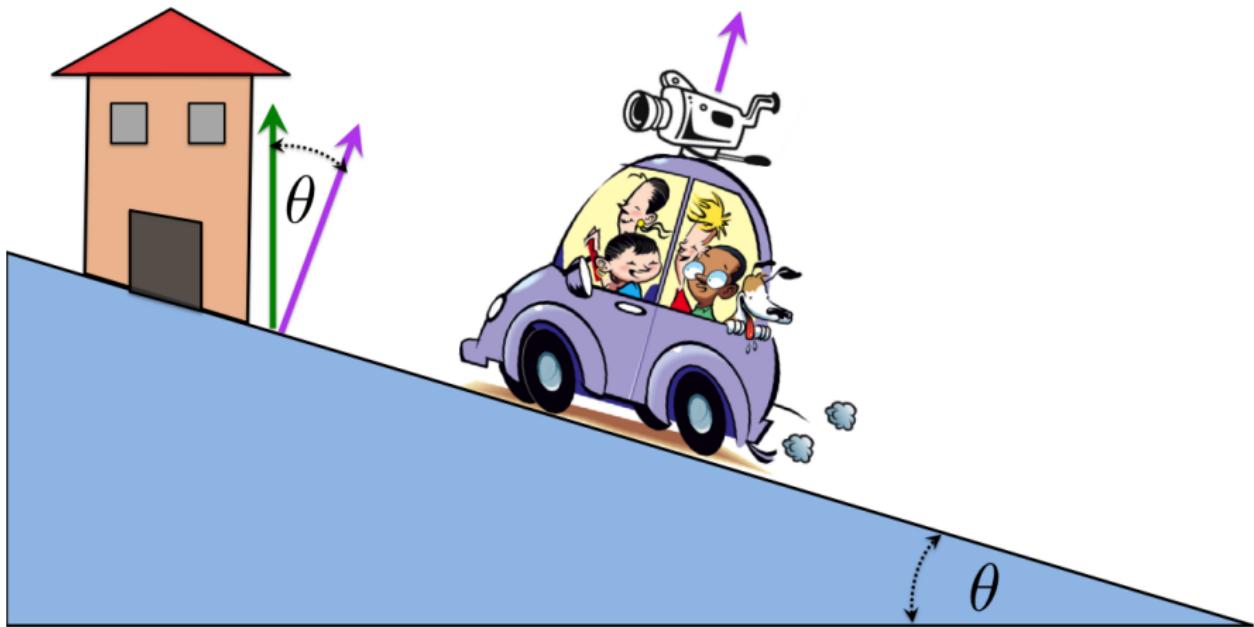


Figure: If we compute the 3D direction of the house's vertical lines relative to camera, we have the incline! How can we do that?

# Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

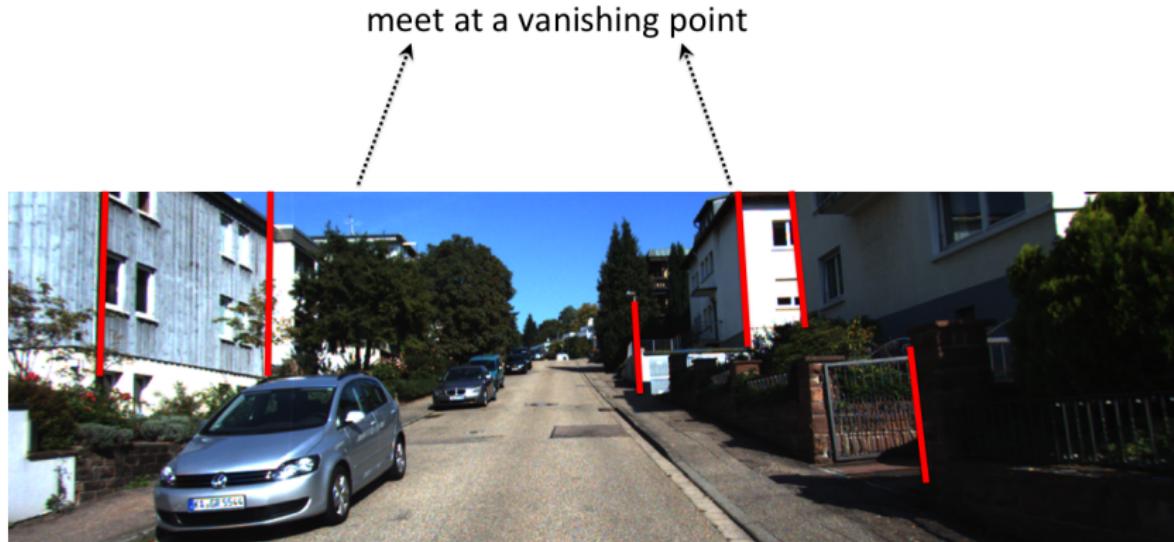


Figure: Extract “vertical” lines and compute vanishing point in 2D via intersections. How can we compute direction in 3D from vanishing point (if we have  $K$ )?

# Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

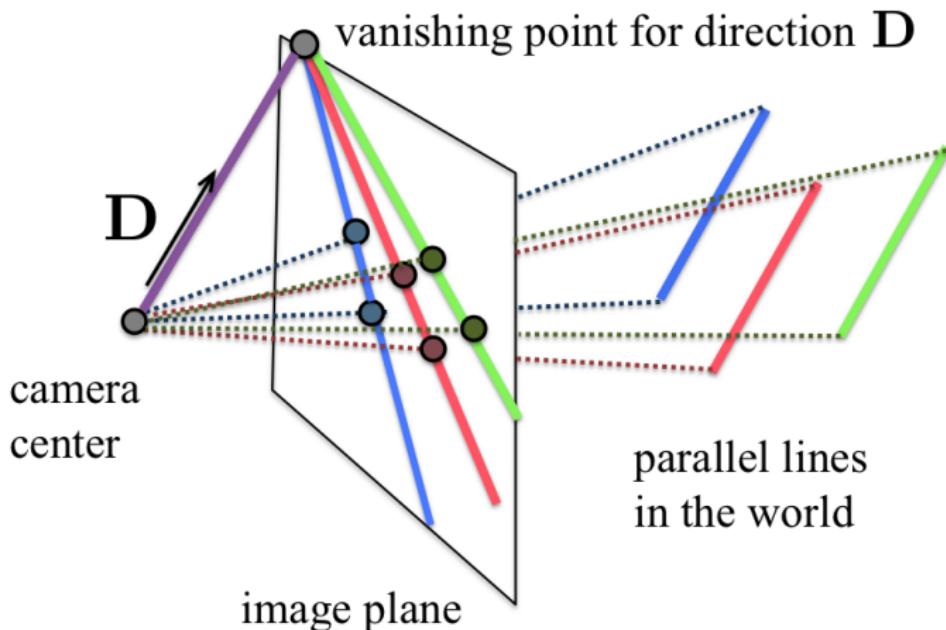
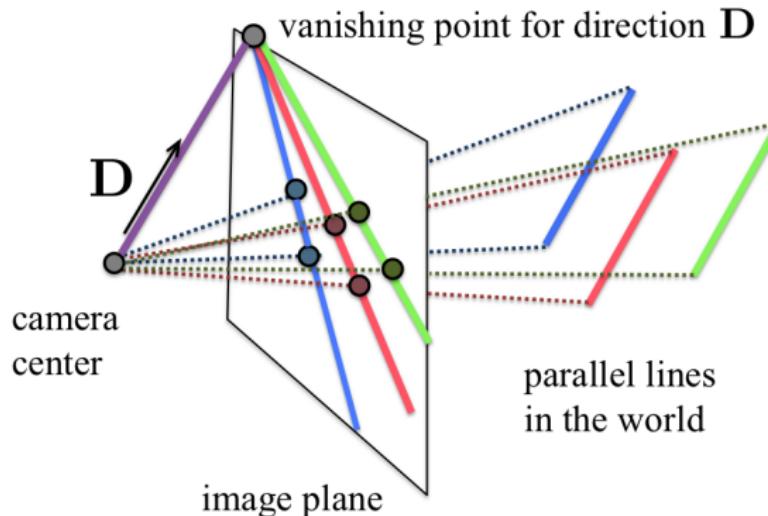


Figure: This picture should help.

# Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

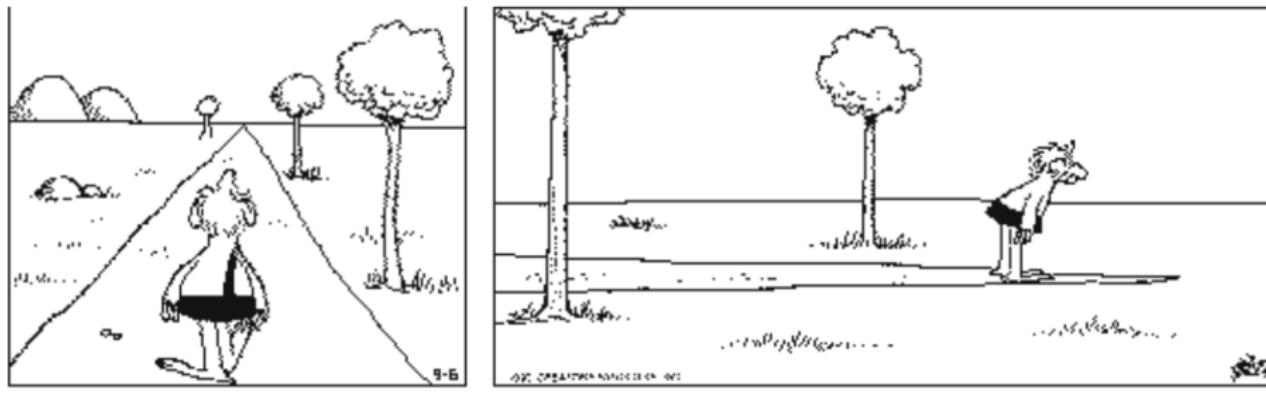


- We have:

$$\begin{bmatrix} w \cdot vp_x \\ w \cdot vp_y \\ w \end{bmatrix} = K\mathbf{D} \rightarrow \mathbf{D} = wK^{-1} \begin{bmatrix} vp_x \\ vp_y \\ 1 \end{bmatrix} \rightarrow \text{normalize } \mathbf{D} \text{ to norm 1}$$

# Vanishing Points Can be Deceiving

- Parallel lines converge at a **vanishing point**.
- But intersecting lines in 2D are not necessary parallel in 3D.

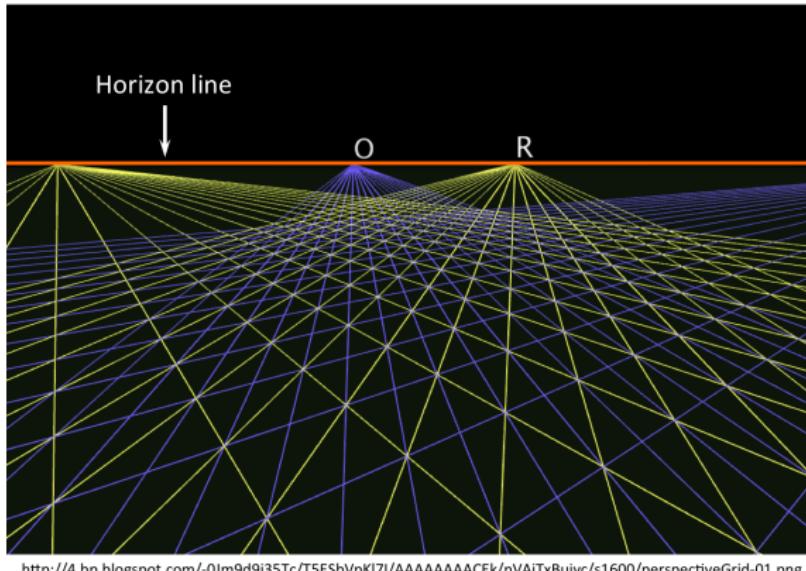


[Source: A. Jepson]

# Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- Each different direction in the world **has its own vanishing point**
- For lines on the same 3D plane, the vanishing points lie on a **line**. We call it a **vanishing line**. Vanishing line for the ground plane is a **horizon line**.



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Parallel lines converge at a **vanishing point**

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- Some horizon lines are nicer than others ;)

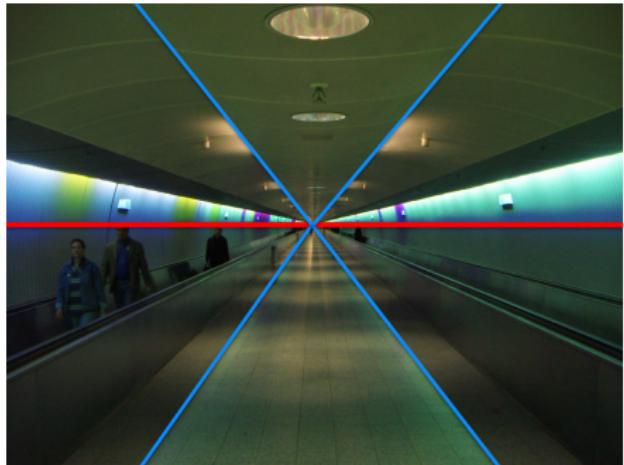


Punta Cana

# Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- For lines on the same 3D plane, the vanishing points lie on a **line**. We call it a **vanishing line** or a **horizon line**.
- Parallel planes in 3D have the **same horizon line** in the image.



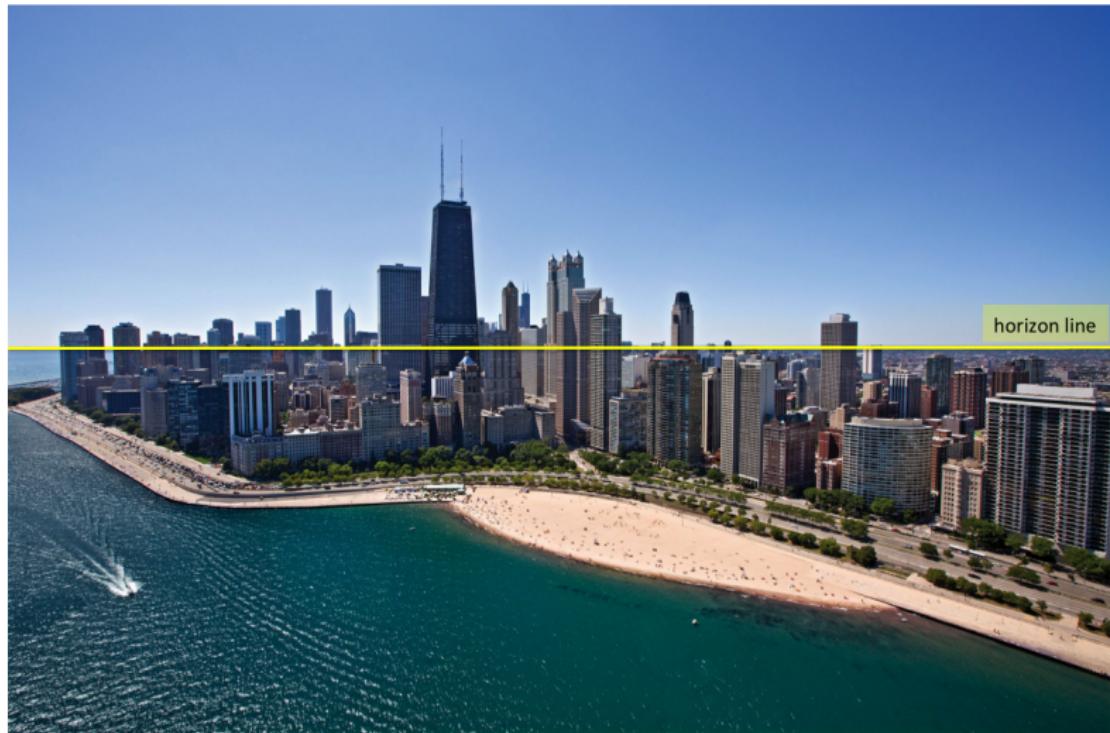
# Projection Properties: Cool Facts

- Can I tell how much above ground this picture was taken?



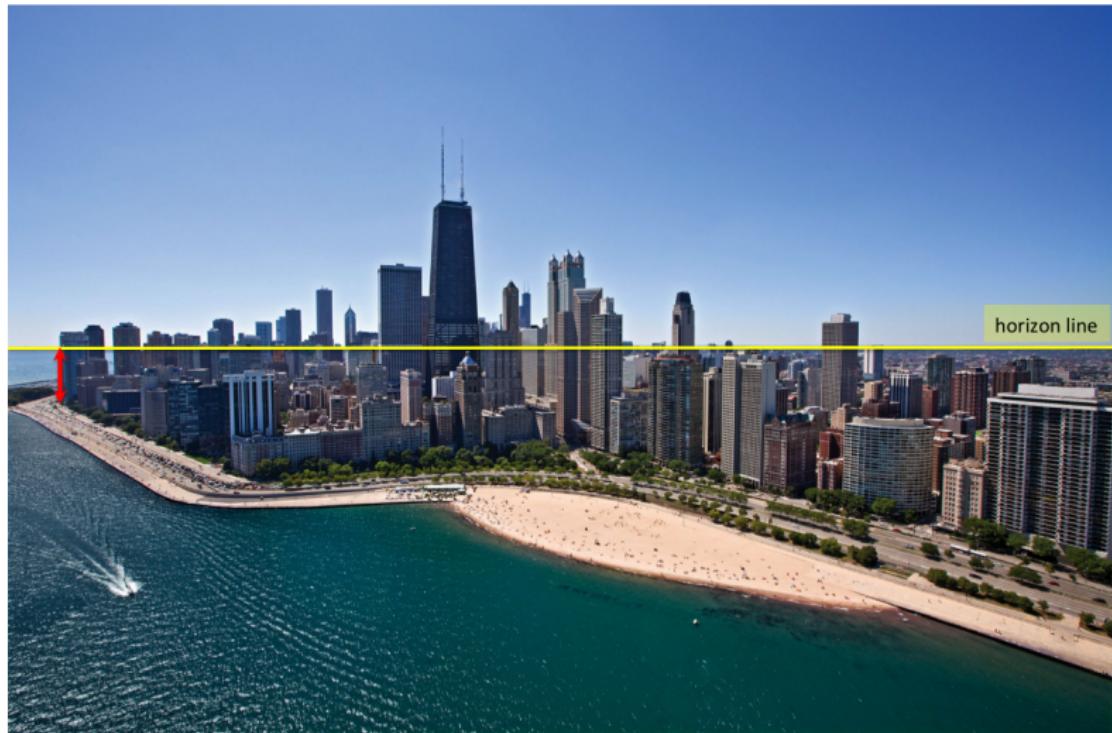
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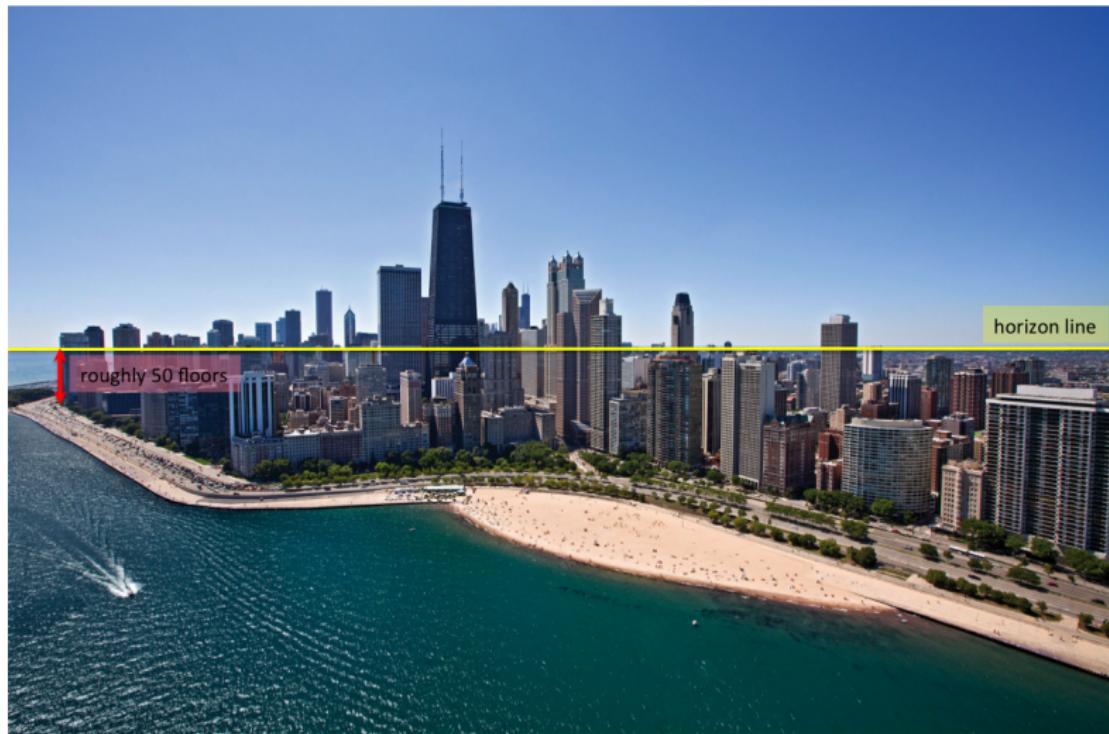
# Projection Properties: Cool Facts

- Same distance as where the horizon intersects a building



# Projection Properties: Cool Facts

- Same distance as where the horizon intersects a building: 50 floors up



# Projection Properties: Cool Facts

- This is only true when the camera (image plane) is orthogonal to the ground plane. And the ground plane is flat.
- A very nice explanation of this phenomena can be find by Derek Hoiem here:  
[https://courses.engr.illinois.edu/cs543/sp2011/materials/3dscene\\_book\\_svg.pdf](https://courses.engr.illinois.edu/cs543/sp2011/materials/3dscene_book_svg.pdf)



# Camera Parameters

We are not yet done with projection. To fully specify projection, we need to:

- Describe its **internal parameters** (we know this, this is our  $\mathbf{K}$ )

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We are not yet done with projection. To fully specify projection, we need to:

- Describe its **internal parameters** (we know this, this is our  $\mathbf{K}$ )
- Describe its **pose in the world**. Two important coordinate systems:
  - World coordinate system
  - Camera coordinate system



[Source: N. Snavely]



# Camera Parameters

- Why two coordinate systems?



Figure: Imagine this is your room.

# Camera Parameters

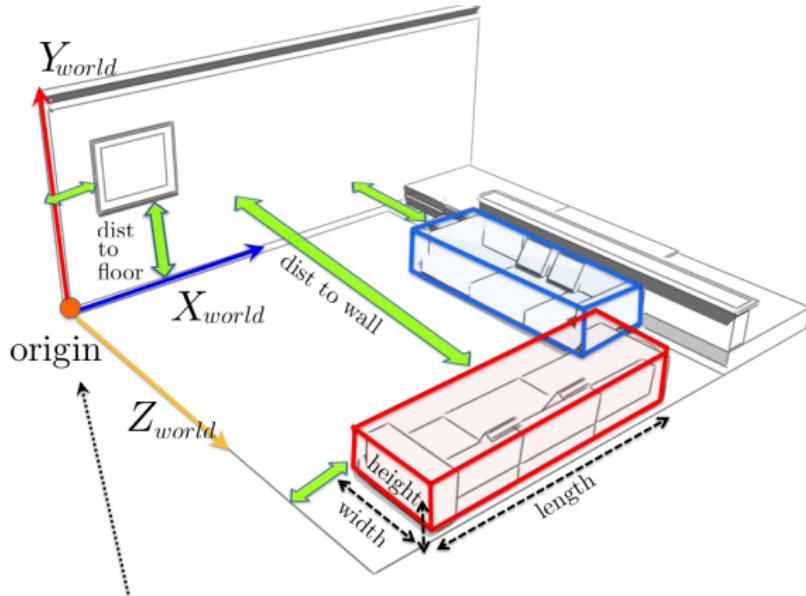
- Why two coordinate systems?



Figure: When you were furnishing you measured everything in detail.

# Camera Parameters

- Why two coordinate systems?

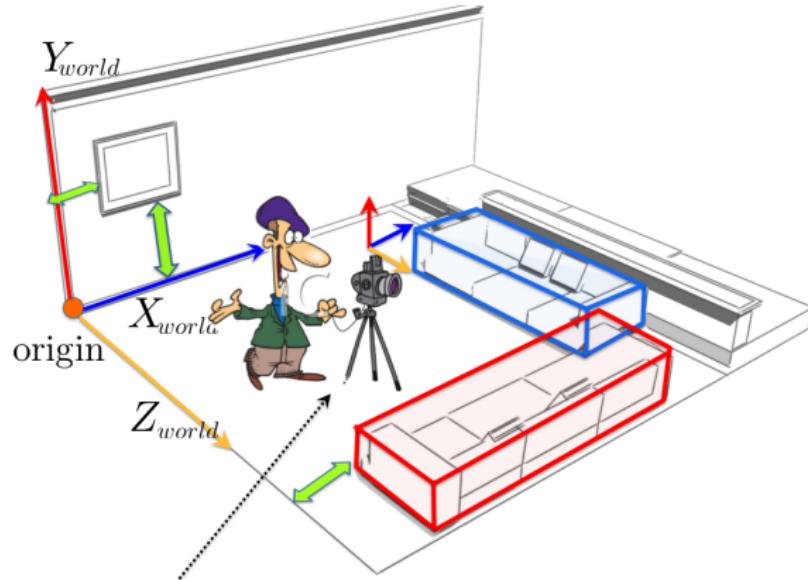


I measured everything in my room relative to this point

**Figure:** Thus you know all coordinates relative to a special point (origin) and coordinate system in the room. This is your **room's (world) coordinate system**.

# Camera Parameters

- Why two coordinate systems?



But to project my room to camera, I need to have the room in the camera coordinate system!

**Figure:** Now you take a picture and you wonder how points project to camera. In order to project, you need all points in **camera's coordinate system**.

# Camera Parameters

- Why two coordinate systems?

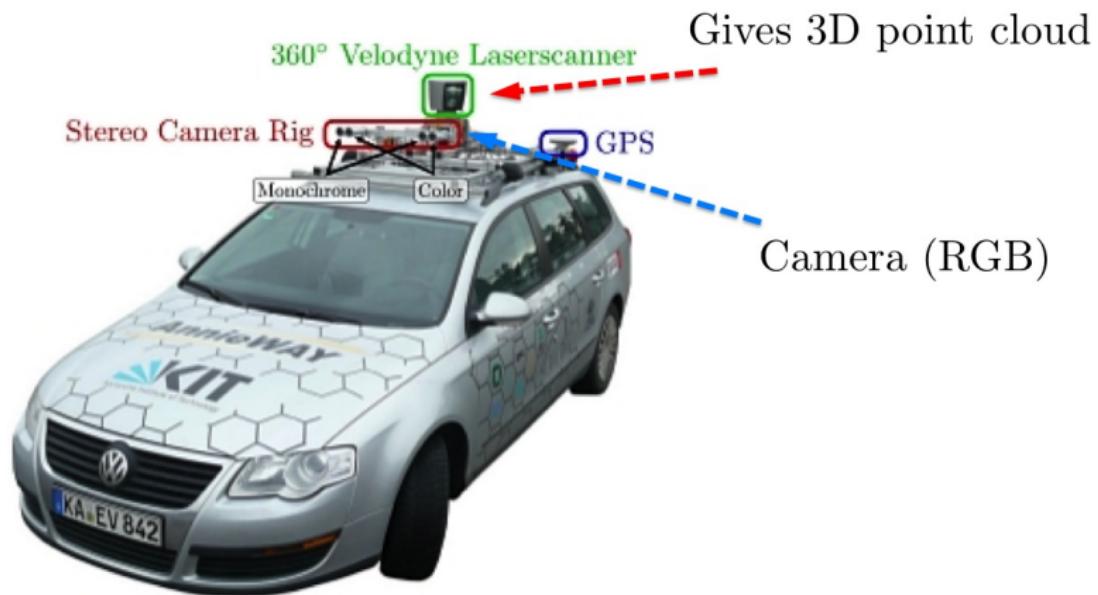
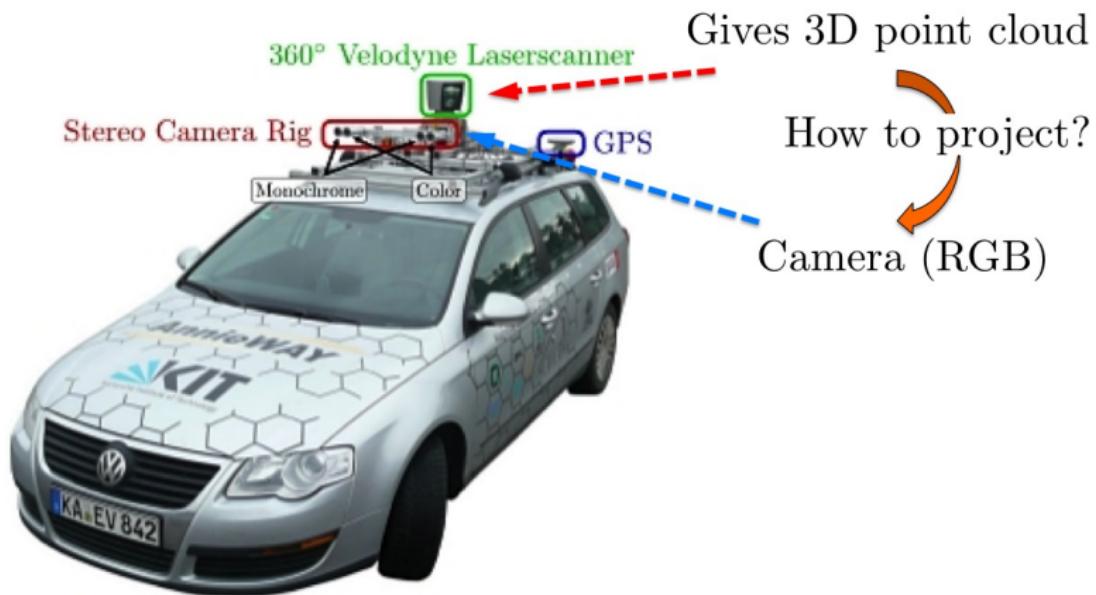


Figure: For e.g. self-driving cars, 3D points are typically measured with Velodyne.

# Camera Parameters

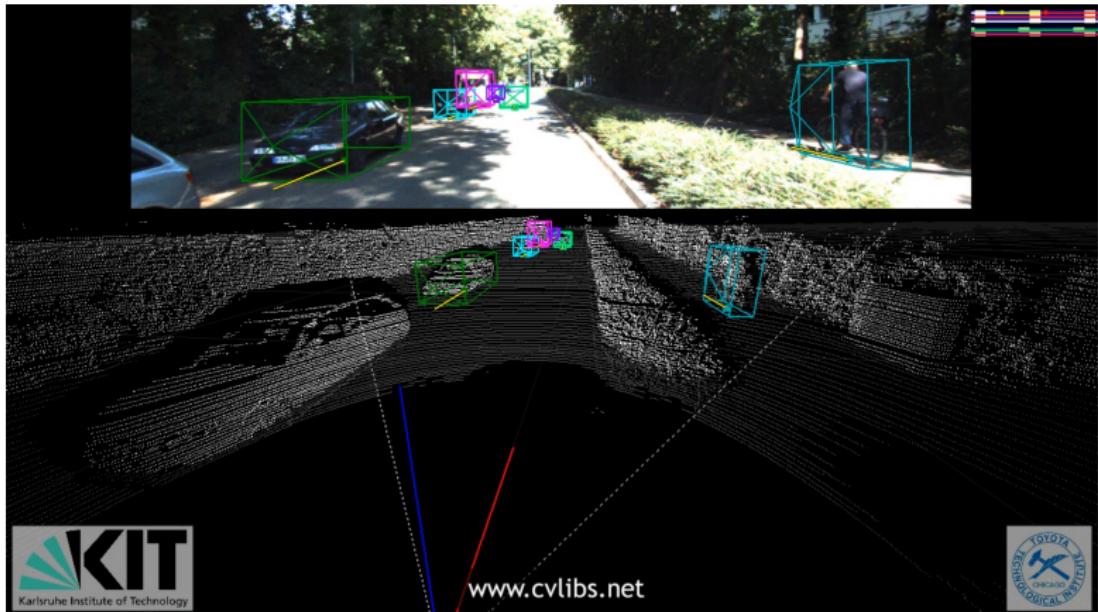
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**Figure:** We want to be able to project the 3D points in Velodyne's coordinate system onto an image captured by a camera.

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  - Need to know **camera intrinsics**

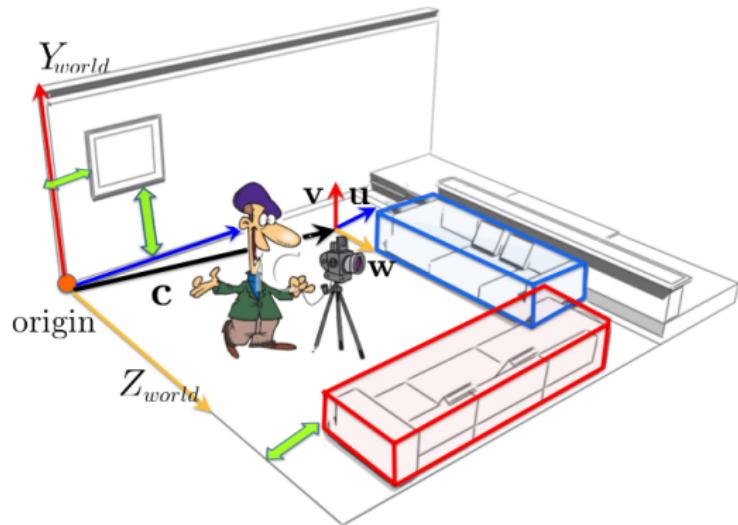
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  - Need to know **camera intrinsics**
- These can all be described with matrices!

[Source: N. Snavely]

# Camera Extrinsic

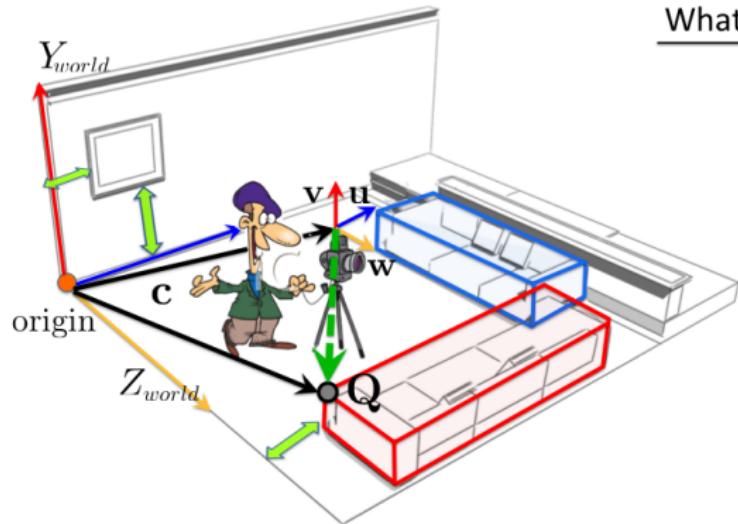


**c** ... camera position in room  
coordinate system

**u, v, w** ... 3 orthogonal directions of camera  
in room coordinate system

**Figure:** We first need our camera position and orientation in the room's world.

# Camera Extrinsic

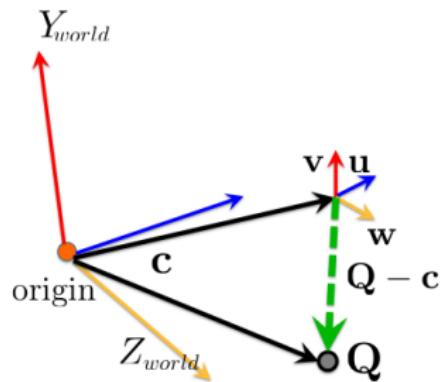


What is  $\mathbf{Q}$  in camera's coordinate system??

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# Camera Extrinsic



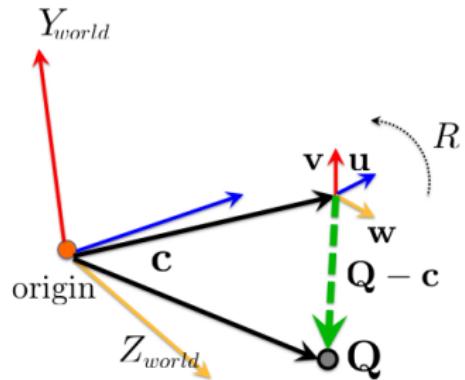
What is  $Q$  in camera's coordinate system??

$Q - c$  ... makes **position** relative to camera

$c$  ... camera position in room coordinate system

$u, v, w$  ... 3 orthogonal directions of camera in room coordinate system

# Camera Extrinsic



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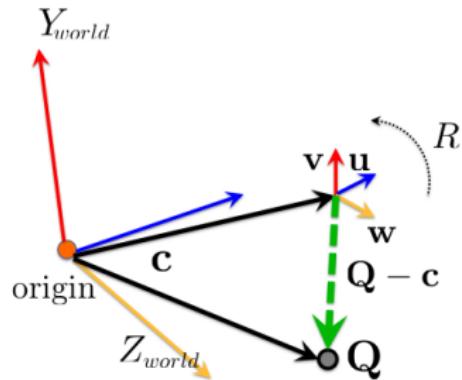
$Q - c$  ... makes **position** relative to camera

$R [u \ v \ w] = I$  (looking for rotation to canonical orientation)

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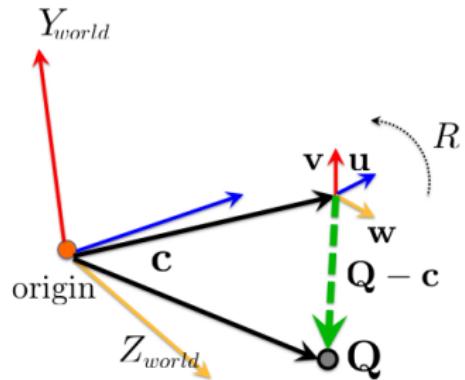
$R [\mathbf{u} \ \mathbf{v} \ \mathbf{w}] = I$  (looking for rotation to canonical orientation)

$R \cdot R^T = I$  (since orientation is orthogonal matrix)

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# Camera Extrinsic



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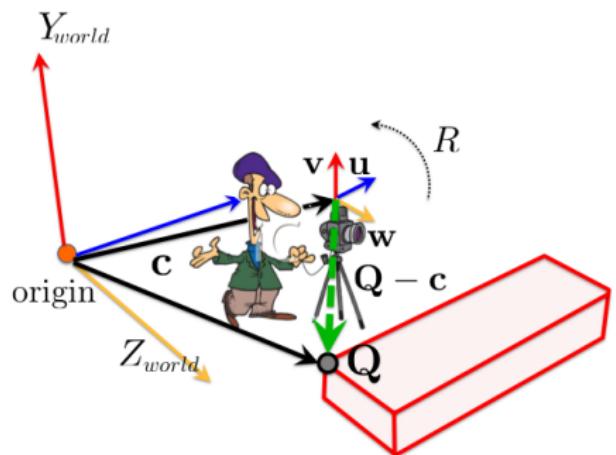
$R \cdot R^T = I$  (since orientation is orthogonal matrix)

$$R = [u^T \ v^T \ w^T]$$

**c** ... camera position in room coordinate system

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## Camera Extrinsics



**c** ... camera position in room coordinate system

**u, v, w** ... 3 orthogonal directions of camera  
in room coordinate system

What is Q in camera's coordinate system??

$\mathbf{Q} - \mathbf{c}$  ... makes **position** relative to camera

$$R \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} \end{bmatrix} = I \quad (\text{looking for rotation to canonical orientation})$$

~~$R \cdot R^T = I$~~       (since orientation is orthogonal matrix)

## Figure: Final Transformation

# Projection Equations

- **Projection matrix  $P$**  models the cumulative effect of all intrinsic and extrinsic parameters. We use homogeneous coordinates for 2D and 3D:

$$\mathbf{q} = \begin{bmatrix} ax \\ ay \\ a \end{bmatrix} = \mathbf{P} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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- It can be computed as

$$\mathbf{P} = \underbrace{\begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics } K} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3 \times 3} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{T}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}}_{\text{translation}}$$

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- To get a point  $\mathbf{q}$  in the image plane, I need to compute  $\mathbf{P}(X, Y, Z, 1)^T$ , where  $\mathbf{P}$  is a  $3 \times 4$  matrix. This gives me a  $3 \times 1$  vector. Now I divide all coordinates with the third coordinate (making the third coordinate equal to 1), and then drop the last coordinate. As simple as that.

# The Projection Matrix

- The projection matrix is defined as

$$\mathbf{P} = \underbrace{\mathbf{K}_{\text{intrinsics}}}_{\text{projection}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{R}_{3 \times 3} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}}_{\text{translation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{T}_{3 \times 3} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}}_{[\mathbf{R} \quad \mathbf{t}]}$$

- More compactly

$$\mathbf{P} = \mathbf{K} [\mathbf{R} \quad \mathbf{t}]$$

- Sometimes you will see notation:

$$\mathbf{P} = \mathbf{K} [\mathbf{R} \mid \mathbf{t}]$$

It's the same thing.

# The Projection Matrix

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- More compactly

$$\mathbf{P} = \mathbf{K} [\mathbf{R} \quad \mathbf{t}]$$

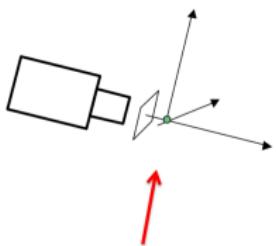
- Sometimes you will see notation:

$$\mathbf{P} = \mathbf{K} [\mathbf{R} \mid \mathbf{t}]$$

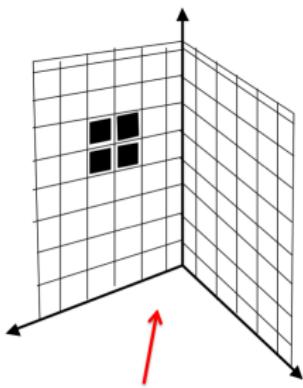
It's the same thing.

- This might look complicated. Truth is, in most cases you don't have  $\mathbf{P}$  at all, so you can't really compute any projections. When you have a calibrated camera, then someone typically gives you  $\mathbf{P}$ . And then projection is easy.

# A Short Note on Camera Calibration



Detect corners in image and figure out which corner corresponds to which point in the 3D pattern.



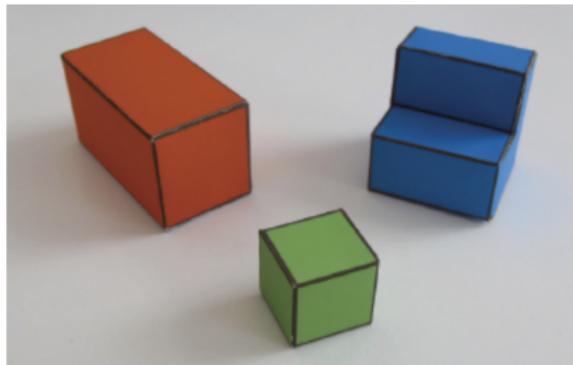
You measured all distances for this pattern.

The general procedure:

- Place a 3D pattern (for which you know all distances) in front of camera.
- Take a picture. Detect corners in image and find correspondences with the points in the pattern.
- Go to the internet and check out the math that tells you how to compute  $K$  from these 2D-3D correspondences. ;) We won't cover in class.

# Camera Calibration: Interesting Fact

- Let's say you have an image but you don't know **anything** about the camera (for example, image downloaded from the web).
- For images where you see lines corresponding to 3 orthogonal directions, like cubes or rooms, you can compute the camera matrix  $K$  as well as  $R$  and  $t$ !



- How to do this is explained in the Zisserman & Hartley book.

# Projection Properties: Cool Facts

- As a consequence, for scenes with lots of lines (e.g. man-made scenes) one can reconstruct the scene in 3D from a **single image!**



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## Projection Properties: Cool Facts

- As a consequence, for scenes with lots of lines (e.g. man-made scenes) one can reconstruct the scene in 3D from a **single image**!
- For those interested, check out the math here:

A. Criminisi, I. Reid, and A. Zisserman

*Single View Metrology*

International Journal of Computer Vision, vol 40, num 2, 2000

<http://www.cs.cmu.edu/~ph/869/papers/Criminisi99.pdf>

# Exercise (Not Very Easy, But Fun)

- We want to render (project) a 3D CAD model of a car to this image in a realistic way
- How?



# Exercise

- First get a CAD model. There are tones of them, e.g. 3D Warehouse (free)

<https://3dwarehouse.sketchup.com/search.html?q=car&backendClass=entity>

**3D Warehouse**

car  Sign In

54,155 Results  Results Per Page

The screenshot shows a search results page for 'car' on the 3D Warehouse website. The top navigation bar includes the site logo, a search bar with the word 'car', and a 'Search' button. Below the search bar, it displays '54,155 Results'. There are three dropdown menus: 'ALL', 'Results Per Page', and 'Sort by Relevance'. The main content area is a grid of 3D car models. Each model has a thumbnail image, its name, and the user who uploaded it. Some thumbnails also show download icons. The models include various types of cars like sedans, SUVs, and sports cars in different colors and styles.

Model Name	Uploader
Aerodynamic Car Coupe 3.3i C...	Dwiz6
Concept Car	BILMENT
CAR-TOONY 5 / Fun Little Car	COL 1
B-type race spec car concept	Peter B.
Super-Duper Drivey Thing	MLS16E
Fururistic Yellow car	Small Po
LL Laurend GT 1972	IIILe-09-III
Custom Car - COL 1 Edition	COL 1
CAR-TOONY 3 / Fun Little Car	COL 1
google race car 3MB all done =...	*A44444444*
CAR-TOONY 6 / Fun Little Car	COL 1
Sports Car	watty188
Crystal Shine Car Wash -Fully F...	Cordelle
barosso 2.2 concept car	Peter B.
barosso b-type concept car	Peter B.
Yellow Car	(Dodge33) - AJ

## Exercise

- We downloaded this model. Now what?



**Figure:** A CAD model is a collection of 3D vertices and faces that connect the vertices. Each face represents a small triangle. It typically has color.

# Exercise

- Our image was collected with a car on the road:
  - A camera was on top of the car, approximately 1.7m above ground
  - Image plane is orthogonal to the ground
  - We have the internal parameters of the camera,  $\mathbf{K}$ .



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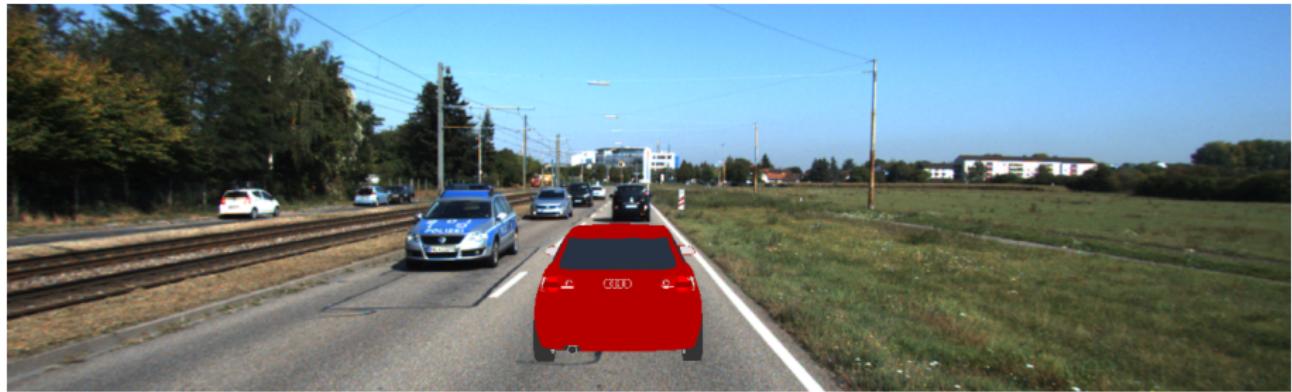
### How?

- We can now “place” our CAD model to this point (compute  $R$  and  $\mathbf{t}$ )
- Rendering:

Compute  $[ax, ay, a]^T = K[R \mid \mathbf{t}][X, Y, Z, 1]^T$  for each CAD vertex  $[X, Y, Z]^T$ . Divide  $[ax, ay, a]^T$  with third coord and drop it.

# Exercise

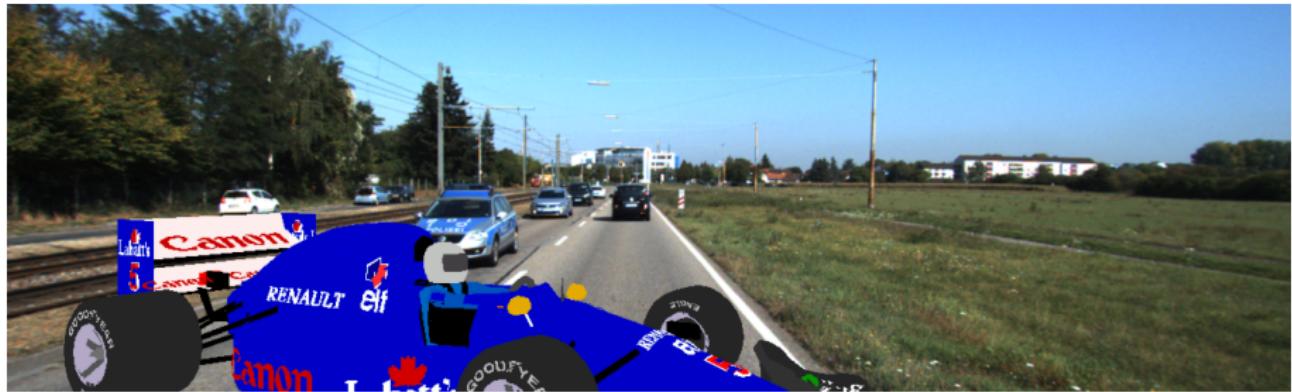
- That's it. Make a video for more fun



(click on image to play video)

# Exercise

- That's it. Make a video for more fun



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(click on image to play video)

# A Little More on Camera Models