What if the ice block expedition 1959 happens in 2021?

Zhihao LIU

GEO4432 Lab Report, May. 2022

1 Introduction

A million tons of ice was shipped from Norway each year during the golden age of ice trade to England, India, South America, China and Australia. The business that involved transporting natural ice for commercial purposes, stated at 1806, flourished from the end of 19th-century until World War I (Blain, 2006; Norseng, 2022). A primary concern of the ice transportation is the melt loss. Likewise, in 1959, a three-ton block of ice from Mo i Rana by the Arctic Circle was trucked to Libreville by the Equator, which was a successful advertisement of insulation materials. The event documented that 2714 Kg ice remained with only 11% mass loss ("Ice Block Expedition of 1959," 2022) after 37 day's long journey.

Using the EAR5 climate reanalysis dataset (Muñoz Sabater, 2021), this study simulates the 1959 ice block expedition by surface energy balance equations and heat transfer equations. The numerical model works on two scopes (1) investigating the effect of the different thermal insulation materials (Wang et al., 2020) (2) What if the ice block expedition 1959 happens in 2020 and 2021 again, are we going to have more or less melt loss?

The model parameters and climate reanalysis dataset are described in the first section, following the construction of the 1D energy surface model under various situations. The simulation results are reviewed and contrasted with historical records. Finally, a 2D heat transfer model demonstrates how the ice warmed up and melted in the container. This study provides a fresh viewpoint on the ice block expedition of 1959 and the ice trade.

2 Method and Climate data

2.1 From historical event to models

Information about the size of the ice block, the amount of ice loss, and the route are gathered from the original roadbook, newspapers and documentary (Folkestad, 2009; *Gjenskaper verdens beste reklameidé*, 2009; GLAVA, 2012). The roadbooks detail that 3050 Kg of ice was harvested from Svartisen glaciers, Norway, in 1959 and then was placed in a specially constructed iron container, which was insulated with wood and glass wool. These materials were used due to their known insulating properties.

The journey began at 09:15 a.m. on 22th February 1959. The first segments passed by major European cities like Copenhagen and Paris, arriving in Marseille on 28th February. After crossing the Mediterranean Sea, the expedition traveled over Sahara for 579 hours until arriving in Libreville, Gabon. For this study, we re-planned the routes using Google maps, see Figure 1. Thus, the distance and average speed may be different from the original roadbook.

For this study, we are using a 1D energy balance model and 2D heat transfer model to simulate the ice melt during the described journey. Figure 1.c describes the full model setup. We have a study object in three layers: metal box (thickness 0.02 m), insulation materials (thickness 0.25 m) and ice block (dimension 1.49 x 1.49 m). Table 1 gives the materials properties of each layer.

Energy is added or removed from the top of the box and follows the energy balance equation:

$$\frac{\partial_E}{\partial_t} = LW\downarrow + LW\uparrow + SW\downarrow + SW\uparrow + SF + LF + \ Q_{heatflux}$$

Where the change of energy ∂_E over specific time $\partial_t$is the sum up of LW (longwave radiation), SW (short wave radiation), SF (sensible heat flux), LF (latent heat flux) and Q (heat flux from top surface to ice block).

The link between the energy balance on the box surface and the ice is the heat transfer inside the box: 2D heat transfer model. This is determined by the temperature difference within the box. We can calculate the heat transfer by Fourier's law, given by:

$$\left(\frac{\partial^2{}_T}{\partial_{x^2}} + \frac{\partial^2{}_T}{\partial_{y^2}}\right) + \frac{q}{k} = \frac{\rho \cdot c_p}{k} \cdot \frac{\partial_T}{\partial_t}$$

Where T is temperature,t is time, x, y is the length of dimensions, and k is heat conductivity, ρ is density, c is specific heat capacity, q is the heat generation flux.

We make the following assumptions about the heat transfer through the box:

- when ice melts and the water run off, there is no convection in the air in the box.
- all materials are homogenous in dimension and time.
- the time step of computation in the iteration is 1 second.

As we do not set a control surface on ice skin, we therefore assume the skin temperature of ice is zero degree Celsius, and temperature of box can be solved by the one-dimensional energy balance model. After having calculated the energy balance and the heat transfer, we can then get the mass loss by calculating the melt energy, which is equal to the amount of energy added due to heat transfer (Figure 2). The full physic basis is attached in appendix 1.

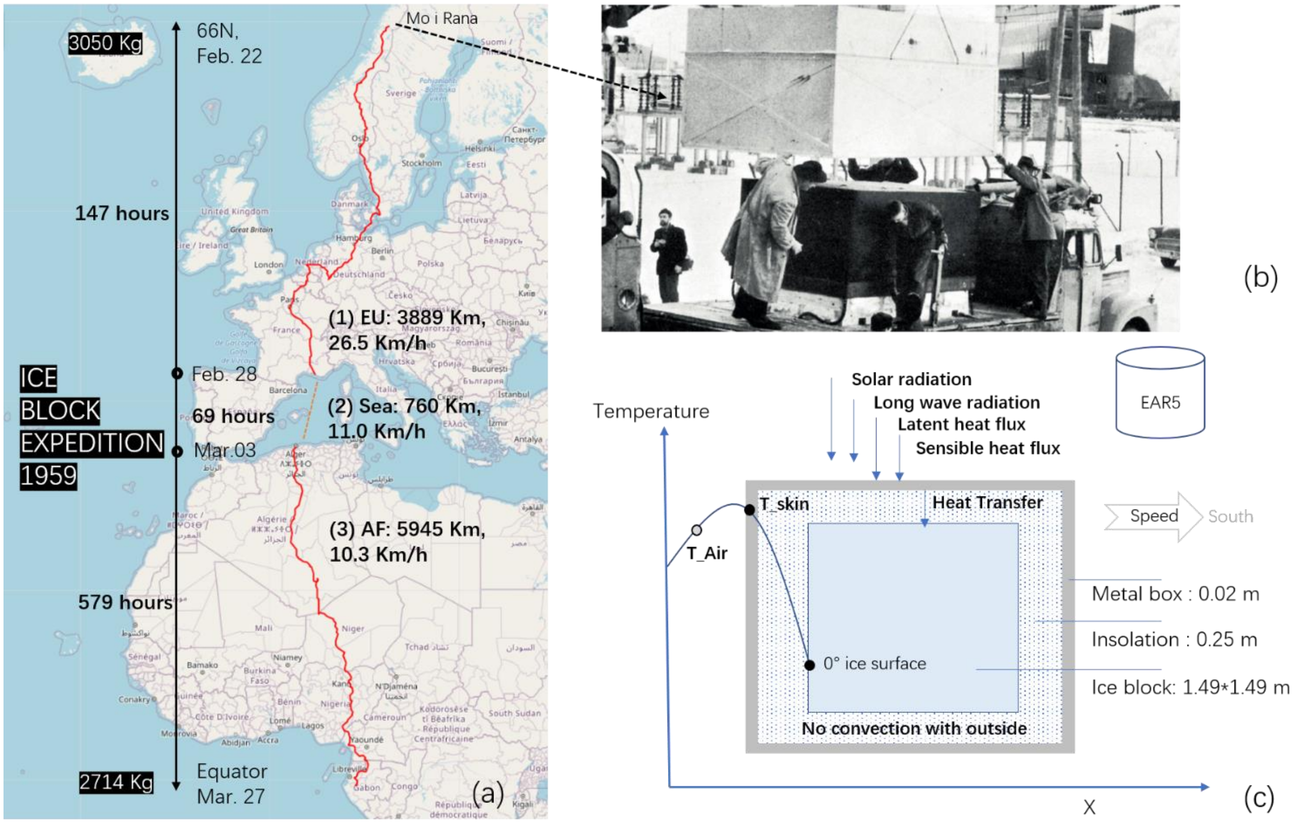


Figure 1. *The concept of ice block modeling. (a) 9834 Km and 726 hours driving. The track is planned by google map, which is not exactly same with the original expedition. The travelling hours and speed are estimated approximately. (b) loading the truck. the photo source: https://www.nrk.no (c) 3 layers models. Note that energy is added or removed from the top of the box and all materials are assumed homogenous and well-sealed (no convection).*

Table 1. *Characteristics of materials.*

Layer	Properties	density [Kg/m3]	heat capacity [J/(KgK)]	albedo	emissivity	Heat conductivity [W/(mK)]	Diffusivity [mm2/s]	Thickness [m]
Ice block	Ice	917	2108	0.5	0.97	2.3	1.19	-
Ice block	Water	1000	4182	-	0.97	-	-	-
Box	Aluminum	2700	890	0.61	0.25	237	-	0.02
Box	Galvanized steel	7800	470	0.61	0.04	52	-	0.02
Box	Pine wood	510	2301	0.15	0.90	0.11	-	0.02
Insulation	Glass wool	20	840	-	-	0.04	1.79	0.25
Insulation	Clay-Sawdust (10%)	1648	838	-	-	0.63	0.45	0.25
Insulation	Sawdust	210	900	-	-	0.08	0.42	0.25

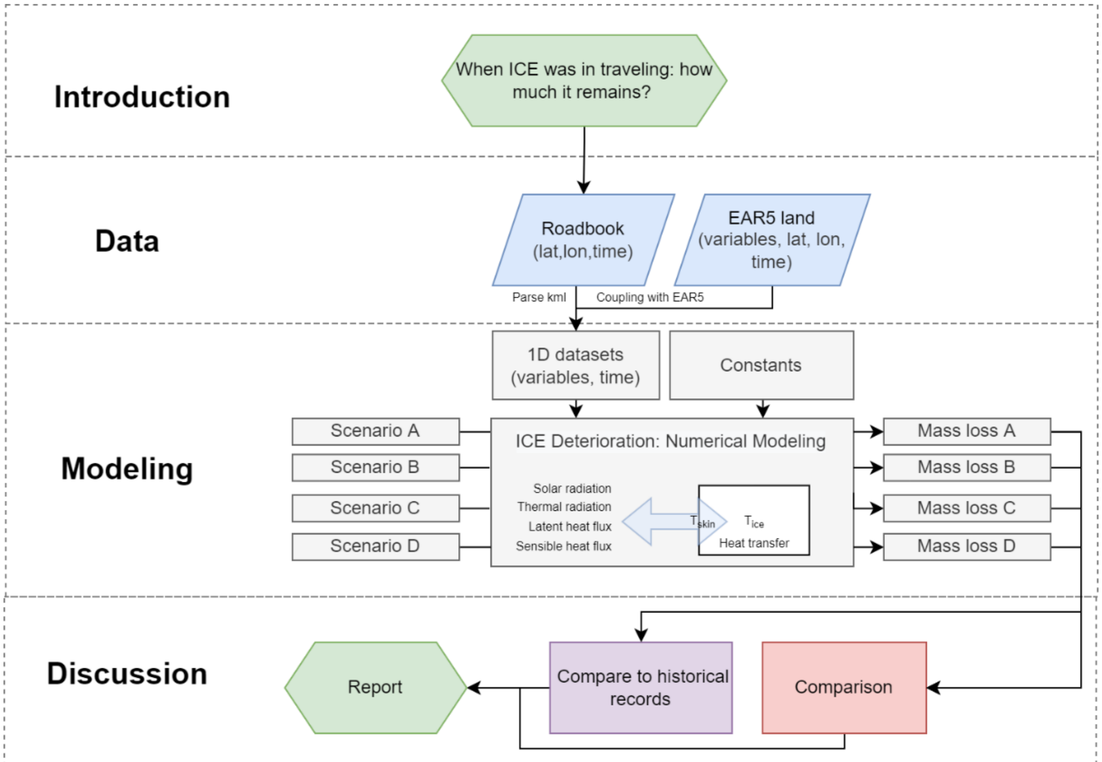


Figure 2. *The workflows.*

2.2 Climate reanalysis dataset

EAR 5 Land hourly dataset offers all environment variables, including (1) Solar radiation (2) Long wave radiation (3) Latent heat flux (4) Sensible heat flux (5) Surface pressure (6) Wind speed in 10 m (7) Surface Skin temperature. (8) Total precipitation. (9) Dew temperature. Note the latent heat is assumed to be as same as bare soil as we do not define the water column on the top surface.

Before the ERA5 land data are used to force the energy balance mode, the first step is to process the raw data. Since the truck transporting the ice is moving, we extract ERA5 data from each location and time to make a one-dimension dataset of the weather conditions during the expedition. Secondly, we had to calculate the wind speed from the ERA-5 land product. For this project, we need the total wind speed at 2 m height. The ERA5 dataset only provides the components of the wind speed (u and v) and at a 10 m height. We therefore first used a logarithmic profile to calculate the wind speed at

2m (see equation in appendix 1 physic basis).Then, the total wind speed is calculated from the aggregation of the u-component, v-component (a positive v wind is from the south) and the moving speed of the van.

Figure 3 displays the meteorological conditions of the trip. The average skin temperature is 292.2 K, slightly higher than air temperature (291.6 K). Skin temperature could be over 325 K, which is 15 K more than air temperature under the solar radiation of Sahara, but lower than air temperature at night. For comparison, Figure 4 shows the weather conditions for the same period in 2021.

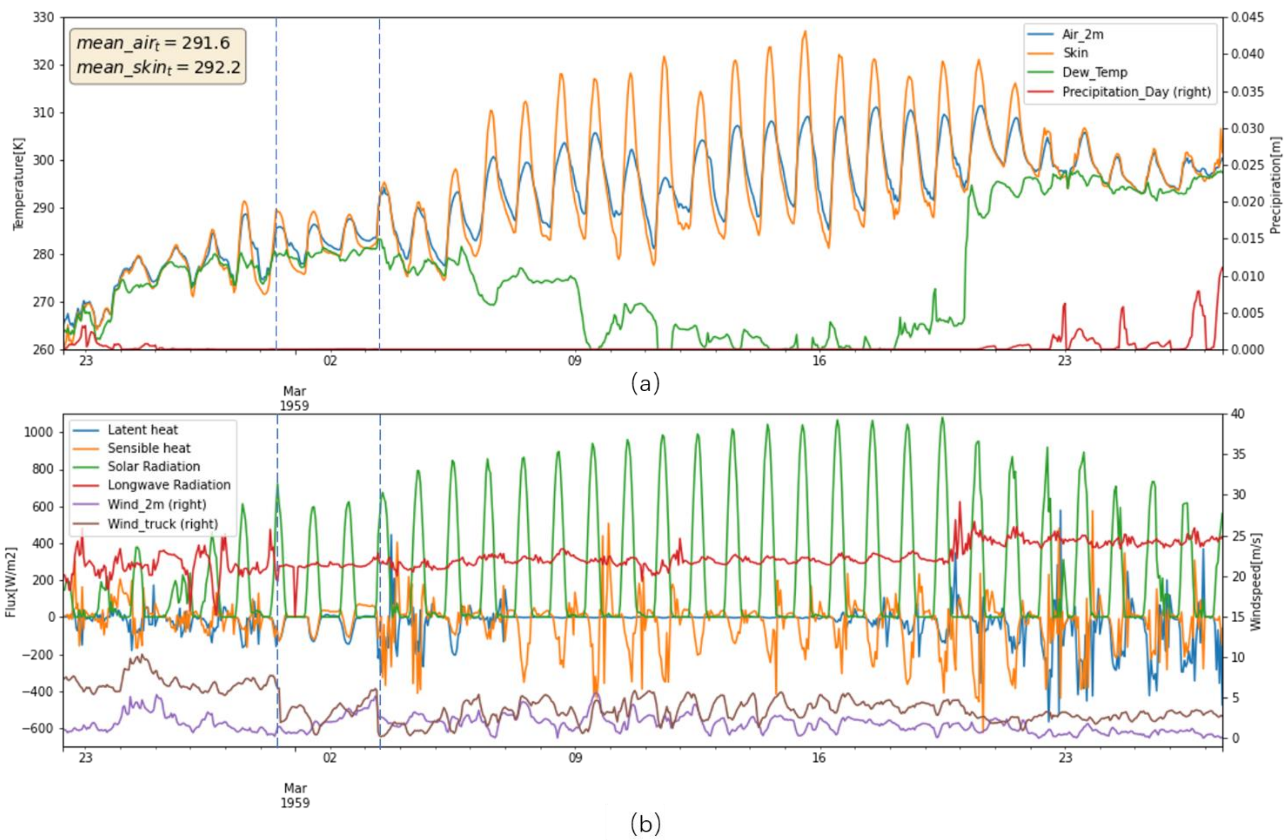


Figure 3. *The hourly weather condition of ice block expedition 1959. All radiation and turbulence flux are defined as downward positive. And the dash lines in blue separate the trip intro Europe/ Mediterranean/ Africa parts.*

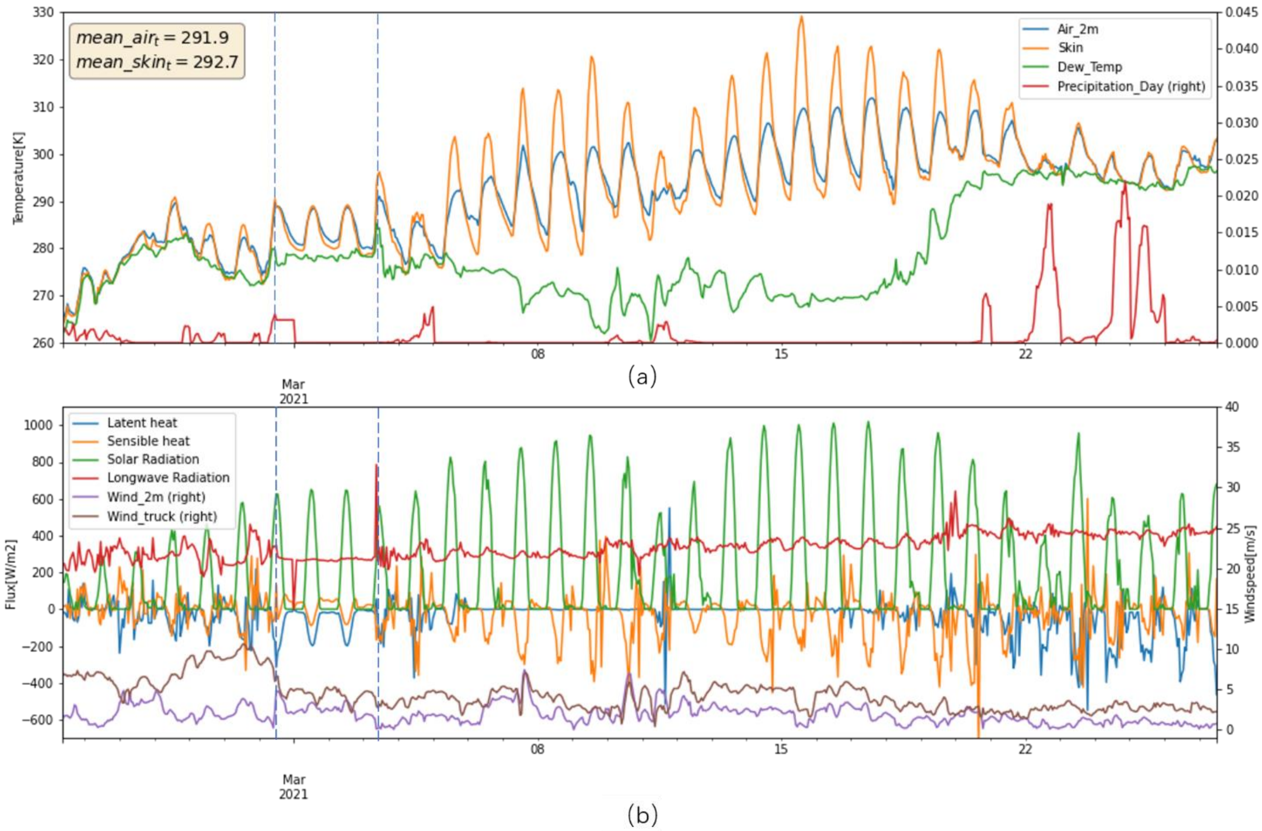


Figure 4. *The hourly weather condition of 2021. All radiation and turbulence flux are defined as downward positive. And the dash lines in blue separate the trip intro Europe/ Mediterranean/ Africa parts.*

3 1D energy balance results

3.1 Ice without any cover, 1959 vs 2021

The scenario is to give control groups to help better understand how the cover and the insulation protect ice from melting and how the weather condition varies from 1959 to 2021. The scenario starts with the simplest assumption that the ice surface is without any cover where the albedo of ice is 0.5 and emissivity is 0.97. Specifically, the actual vapor pressure e_{actual} is calculated by dew temperature, and the specific vapor pressure of ice surface e_{ice} is calculated by surface temperature T_{ice_skin} (equals to 0 degree Celsius). So, the vapor deficit is e_{ice} - e_{actual}, and temperature gradient is T_{air} - T_{ice_skin}.

The simulations show that 5.59 m of ice melted in the scenario of 1959, and 6.50 m in 2021. The Solar radiation contributed more melt energy (18.5%) in 1959. Latent heat flux and longwave radiation contributed more in 2021, which are 20.4% to 15.5%, 3.1% to 1.0%, compared with 1959. This is because even though the air temperature of 2021 is 0.3 K higher than 1959, but the relative windspeed of 2021 (4.3 m/s) is heavier than 1959 (3.9 m/s) (Figure 4). And there was unusual rainy days in Sahara in Mar. 2021, increasing humidity (more latent heat flux) and reducing the solar radiation significantly by cloud.

Furthermore, the 3-ton ice block (1.49×1.49 m) would have melted away on 8th Mar. 1959 or 6th Mar. 2021, if ice block expedition was carried out under the scenarios (Figure 5). At the beginning of the trip, longwave radiation, latent heat flux and sensible heat flux are negative because we assumed ice surface is (Figure 6).

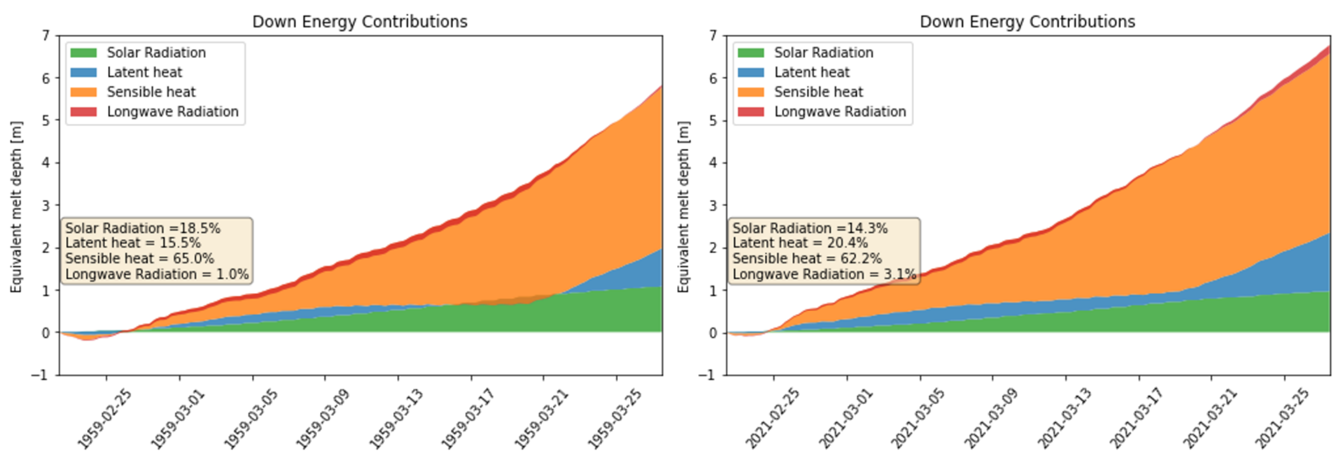


Figure 5. *Melt energy contributions of bare ice on truck (left: 1959, right: 2021).*

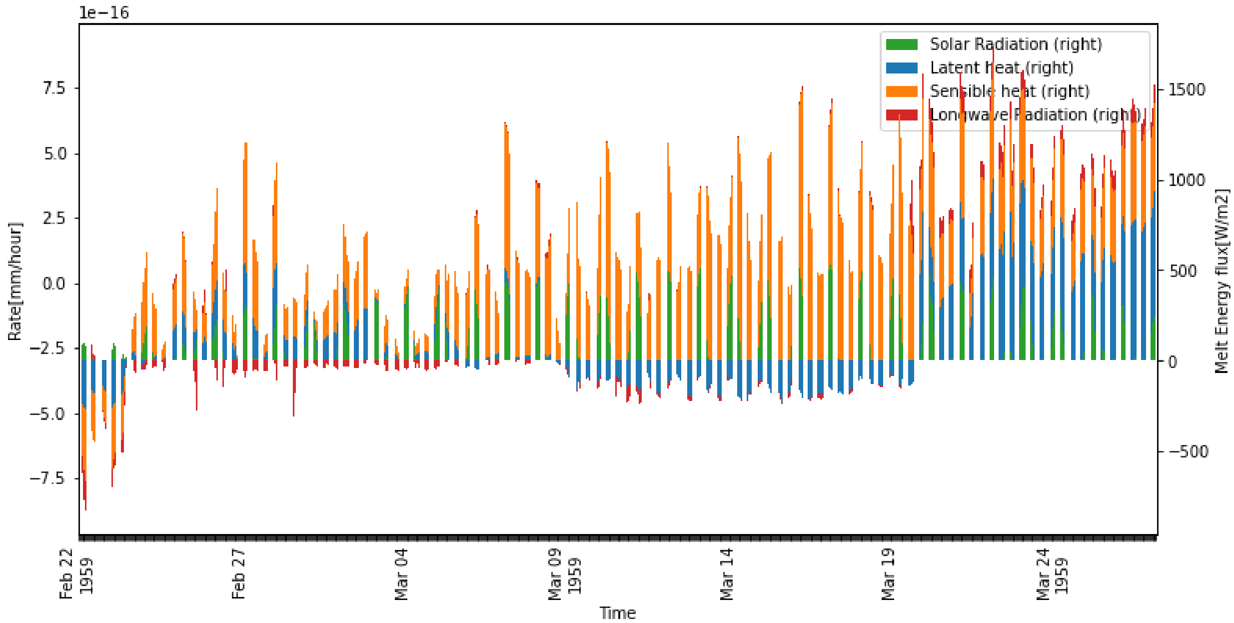


Figure 6. *The energy balance on ice surface (1959). The negative value indicate that the heat is moving away from the surface, and vice versa.*

3.2 Steel box, aluminum box and wooden box

Three types boxes were used in the simulation: (1) Galvanized steel is one of the most popular steel types with a zinc-iron coating, which is also widely used as roof material for high albedo (0.61). (2) Aluminum has the same high albedo (0.61), but with 4.6 times the thermal conductivity of steel and higher emissivity (0.25 to 0.04). (3) Wood is another commonly used box material in ice trade business with ultra-low thermal conductivity but low albedo. albedo or thermal conductivity, which one is more important? The scenario is intended to illustrate the considerations of select good box for ice shipment.

The effect of box material on total thermal resistance is shown in Table 2. The thermal performance between steel-glasswool and aluminum-glasswool is negligible because the insulation layer is thick and well-insulated. The wood box does have some contribution, a 2.83% improvement over the original insulation. Overall, the heat transfer flux for steel, aluminum and wood box with the same insulation layer are 42.62 W/s, 42.63 W/s and 41.42 W/s for a given size and temperature difference.

Table 2. *Overall heat transfer coefficient of different scenarios.*

Box	Box thickness [m]	Insulation	Insulation thickness [m]	**Overall heat transfer coefficient [W/(m2K)]**	Area size	Temperature difference [K]*	**Heat transfer flux [W/s]**
-	-	Glass wool	0.25	0.160000	1.49×1.49×6	20	42.62592
Steel	0.02	Glass wool	0.25	0.159990	1.49×1.49×6	20	42.62335
Aluminum	0.02	Glass wool	0.25	0.159998	1.49×1.49×6	20	42.62592
Wood	0.02	Glass wool	0.25	0.155477	1.49×1.49×6	20	41.42095

In the simulation, the latent heat is assumed to be as same as bare soil, because the evaporation is difficult to estimate when the saturation is unknown. The skin temperature of box is the result of energy balance and heat transfer. Figure 7 and 8 shows the simulation results. As described in Table 2, the heat transfer coefficients of steel box and aluminum box are close, thus, the higher the temperature, the more energy is transferred into ice, resulting in severe melting.

(1) The melting amount of ice in steel-glasswool box is 372.9Kg, which is higher than 352.3 Kg of melt ice in aluminum-glasswool box, because the average skin temperature of steel, 294.3 K, is higher than the latter (293.1 K). Appendix 2 explains the energy partitioning on three types of box surface, where longwave radiation and sensible heat is two important cooling mechanisms, since the metal surfaces are hotter than air temperature most of time. And the same albedo results the same input of solar radiation, but aluminum has a higher emissivity of 0.25 to cool it down, compared with 0.04 of steel (see Table 1).

(2) Wood-glasswool box has the best thermal insulation performance with the similar result (375.9 Kg) to steel-glasswool box. Higher albedo brings it extreme temperature, even though the emissivity is high as well.

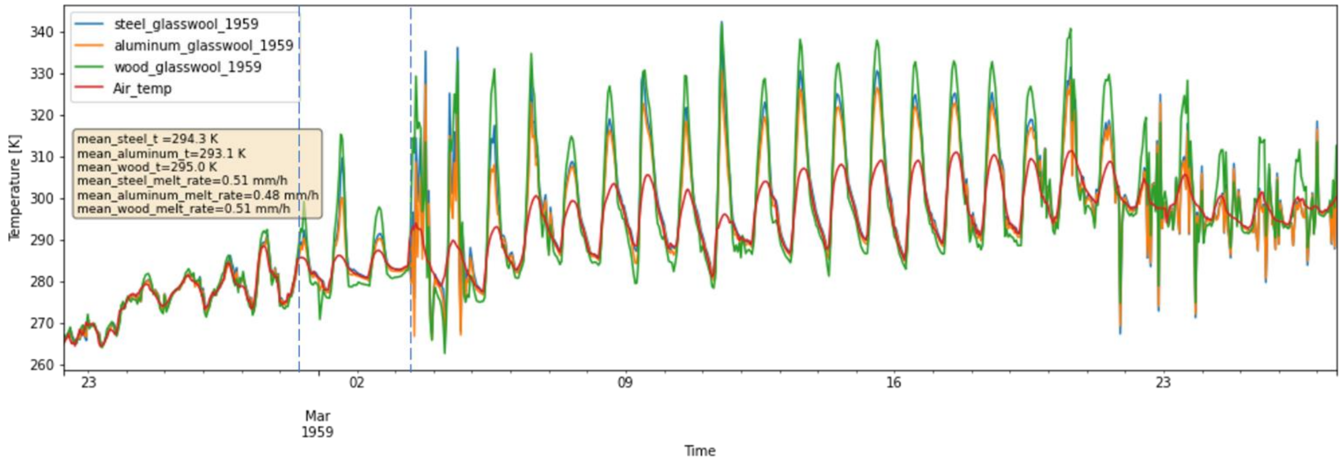


Figure 7. *Melt simulations of box surface temperature.*

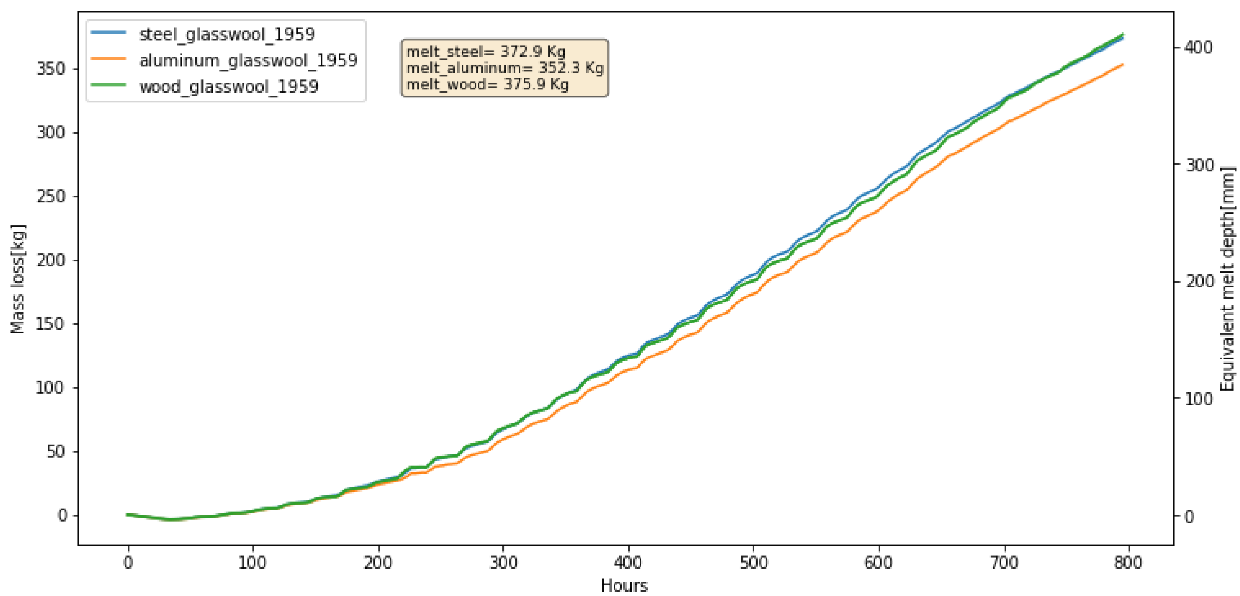


Figure 8. *Melt simulations of mass loss.*

3.3 Glass wool, sawdust and clay-sawdust brick

Glass wool is an excellent insulating product and was widely used after the Second World War. Sawdust is a more common insulation solution in the ice trade. At the time, timber industry considered sawdust to be a problem because of no other use, which was solved by the ice trade. Clay-sawdust brick is a new type of lightweight building brick. The 10% sawdust component can significantly improve thermal performance of the brick, making building more energy efficient (Charai et al., 2020).

Figure 9 demonstrate the different mass loss and box temperature in each scenario. By improving thermal conductivity from 0.08 W/(mK) (sawdust) to 0.04 W/(mK) (glass wool), the amount of mass loss dropped to 50.4%.

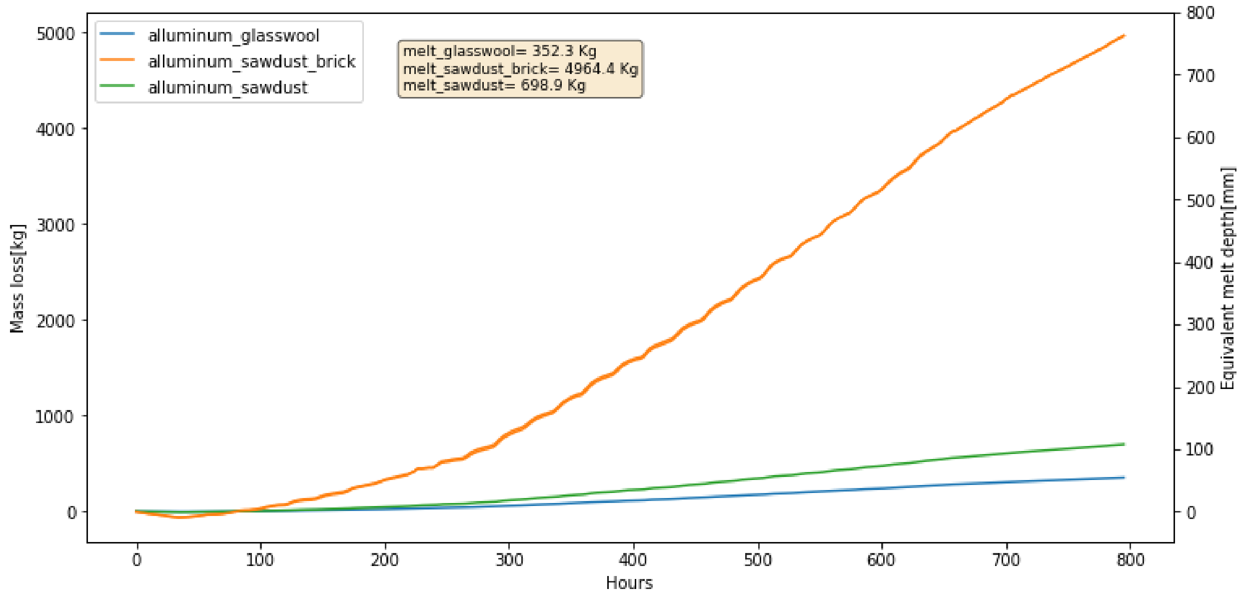


Figure 9. *Melt simulations of mass loss.*

3.4 1959, 2021 and the original records

In original records, only four liters of water had been shed when the truck arrived in Algiers on 3th March 1959. When crossing the Sahara, on average 15 liters melted each day. Finally, only 336 kg ice lost in end of the expedition, which is 11.0% of the total mass.

In the simulations, Figure 9 display that ice mass loss of 352.3 Kg and 348.2 Kg in 1959 and 2021, with 322.8 Kg in 2020 as a reference, which is opposite to the scenario of bare ice. In ice without cover scenario, the bare ice in 2021 melts more than 1959 due to higher air temperature, and more contributions from latent heat and longwave radiation. However, if there is a cover upon ice, the melt mechanism would be different, because the cover does not have phase change. Thus, the longwave radiation and sensible heat actually are going to be the cooling component, not the melt component when it is warmer than air temperature. If water is available, the evaporation would take heat away as well. Figure 11 shows the fluctuation of the surface temperature of the aluminum box, where box in 1959 is warmer than 2021. The detailed energy partitioning on the surface can be found in Appendix 3.

Table 3 shows that, in the scenario of aluminum_glasswool_1959, the mass loss is 26.5 Kg until Algiers, 14.3 Kg per day in desert, 352.3 Kg in the final destination, which is slightly different from the original records.

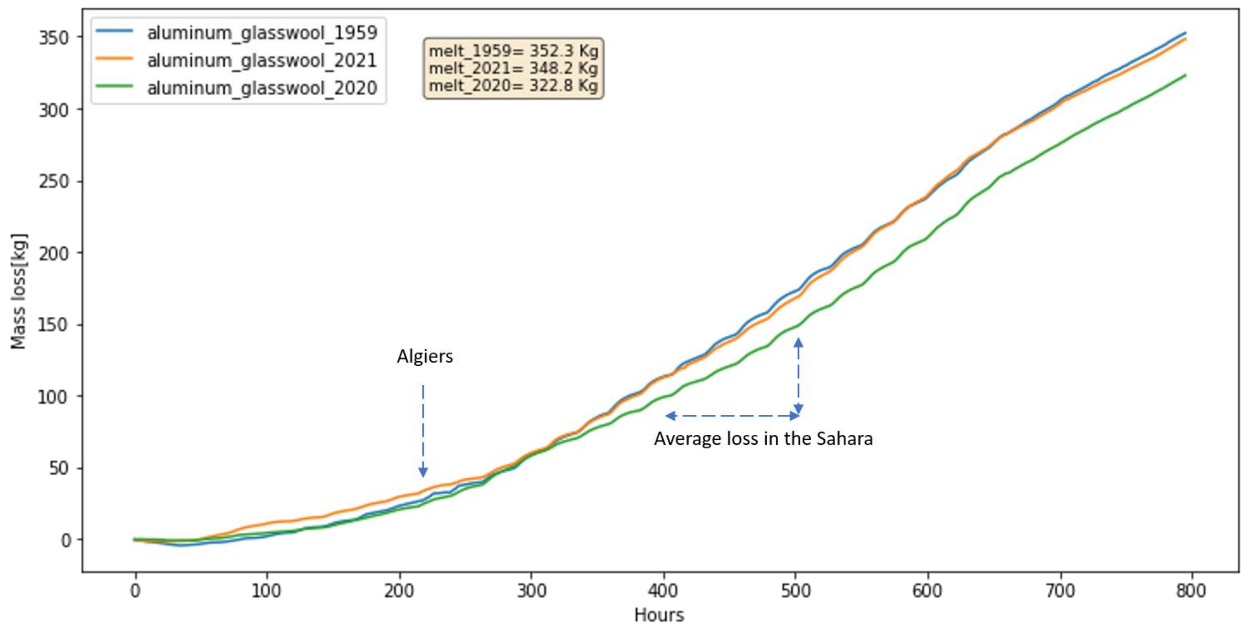


Figure 10. *Melt simulation of total mass loss 1959, 2021 and 2020.*

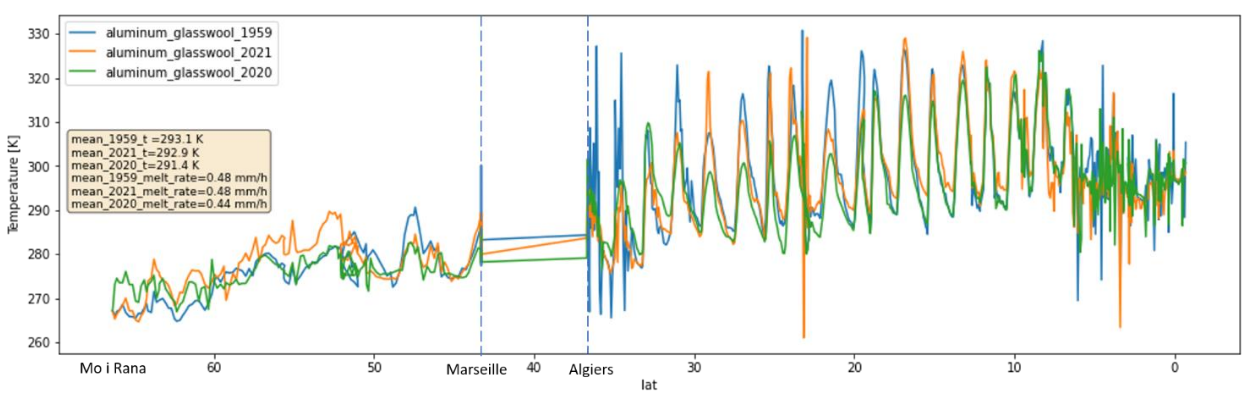


Figure 11. *Box surface temperature of 1959, 2021 and 2020*

Table 3. *Comparison between original record and simulations.*

Event	Initial Mass [Kg]	Insulation	Insulation thickness[m]	Mass lost Algiers [Kg]	Mass loss Sahara [Kg/day]	Mass loss In total [Kg]
Original records	3050	Mainly glass wool	-	About 4	About 15	336
Aluminum_glasswool_1959	3050	Glass wool	0.25	26.5	14.3	352.3
Aluminum_glasswool_2021	3050	Glass wool	0.25	32.0	13.2	348.2
Aluminum_glasswool_2020	3050	Glass wool	0.25	23.0	11.7	322.8

4 2D heat transfer results

The surface energy balance model can only give the amount of heat transfer, which is assumed to be melt energy. However, the true changes are masked by a "black box," as in the case of (1) What is the ice block's temperature distribution? (2) When the temperature rises to 0 degrees Celsius, how long does it take for ice to melt? Is it correct to say that the lock block's core is still below zero degrees, as suggested by several comments on the internet?

Here we use a 2D heat transfer to model the melting process. An ice block consists of 149×149 nodes is heated up from outer nodes to inner nodes. Once the node of ice reaches 0 degree Celsius, and heat accumulates until a phase change, the nodes melt away. The box temperature is from the scenario *aluminum_glasswool_1959* which is the best fit to historical records. And the ice temperature starts at -6° Celsius (the local temperature of Mo i Rana), then is updated in the iteration of heat diffusion.

Figure 12 shows that the warming up of the ice core from -6° to 0° takes approximately 175 hours by heat diffusivity. Then, it requires 334 joules for 1 gram of ice to melt into water, which is a significant amount of energy that can warm the water from 0° to 158.4°. Zoom in on a corner of the ice block, Figure 13 show that the first 1 cm layer takes less 100 hours to reach 0°, and melts away by 400 hours. And then a layer of 1 centimeter dissolves every 100 hours after that.

After 726 hours and arriving at the destination, the first layer, weighing 122.1 kg, and the second, weighing 118.9 kg, melted away. The third layer weighs 115.7 Kg, which is not entirely melted. The total mass loss by 2D heat transfer model is then 348.8 Kg, which is 3.5 Kg less than 1D surface energy model.

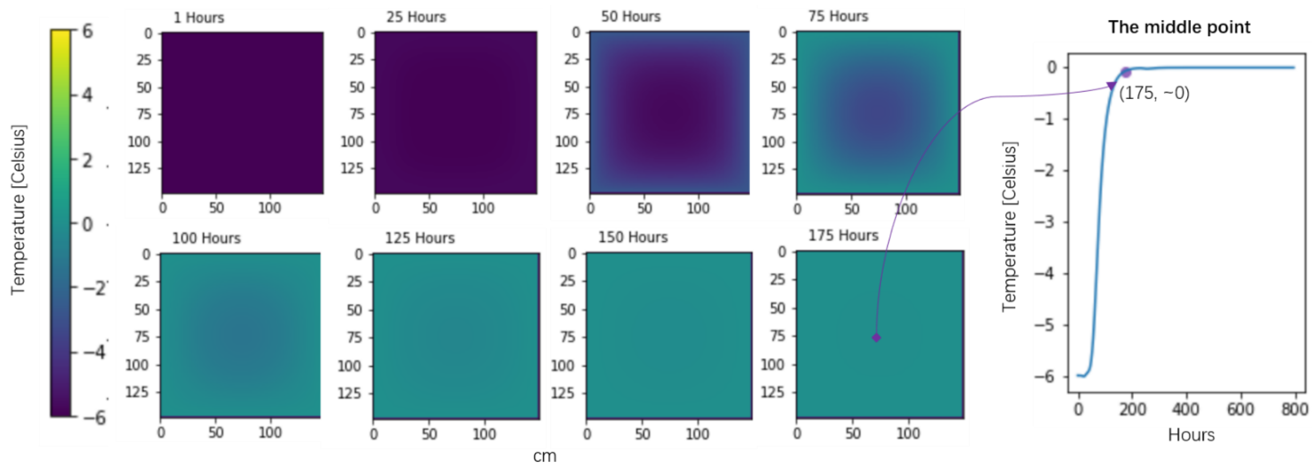


Figure 12. *Warming up process of heat diffusivity. The core of ice lock takes 175 hours warming up to 0° (Scenario* aluminum_glasswool_1959*).*

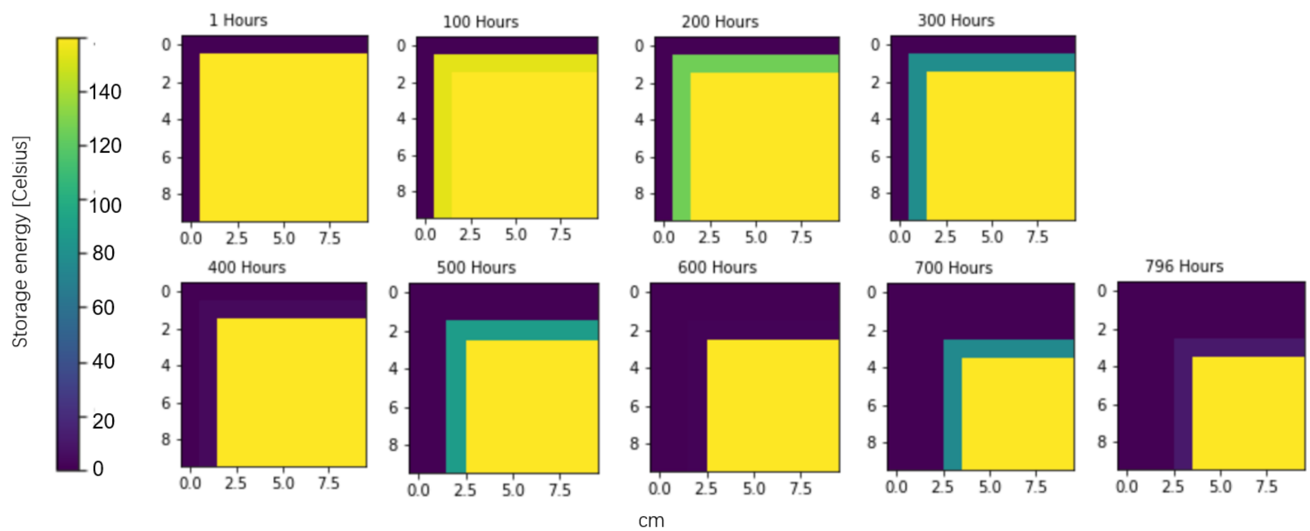


Figure 13. *Ripe process at a corner of the ice block. The storage energy is expressed in equivalent temperature (334* joules equals to heat it up 158.4°*).*

5 Discussions

5.1 How the cover and insulation material prevent ice from melting

Due to the changes in energy input & output (or downward & upward) on the surface, the covering and insulating layers will change the mechanism of ice melt. (1) Net shortwave radiation is always a downward energy. And the higher of albedo, the lower the energy input. (2) Other energy component may cool down the surface. Sensible heat flux contributes to cooling down if the saturated surface temperature is higher than the dew temperature. Thermal radiation and sensible heat flux could also be negative if the surface temperature is higher than air temperature. Likewise, ice surface is seldom to have upward heat fluxes.

The experiment reveals an ideal passive cooling strategy, namely ‘air temperature & dew temperature’. It suggests the surface is capable of:

- following the air temperature as close as possible by using high albedo and smooth surface if there is strong solar radiation.
- Low heat capacity can make cooling easier; and high emissivity helps in most of time;
- moreover, water is a useful strategy. For example, in rainy days, the latent heat flux dominates and resets the surface as low as dew temperature.

  Due to condensation, it is impossible to cool down a surface below the dew point by passive cooling strategy.

5.2 The uncertainties of energy balance simulation

The characteristics of the materials are essential for simulation. Additionally, the actual situation involves more than just a three-layer structure. The total heat resistance can be roughly assumed to be equivalent to that of glass wool. However, the simulation did not account for the fact that damp glass wool will have lower insulating performance, which in part explains the difference in mass loss halfway through (Table 3).

The EAR5-Land dataset can provide hourly variables with a resolution of 9 Km. Variables cannot represent the true observations. For example, the air temperature of the Sahara was said to exceed 50° C by the original records. However, the highest record of the ERA 5 dataset was 37° C. Most likely neither is true. Secondly, due to unavailable of data, the 69-hours-cross-sea weather was replaced by the local weather in Marseille. Additionally, relative wind speed and turbulence coefficients are the approximately estimated.

Latent heat flux for bare soil was used to represent the latent heat component on the surface. Because it is impossible to estimate the actual evaporation of the surface, particularly the potential evaporation in the desert is huge and we cannot evaluate the water availability of the surface.

5.3 The uncertainties of 2D heat transfer

The actual situation is rather complicated to simulate because the heat transfer mechanism changes when there is a space between the insulation material and the ice block. When the melting begins. Heat conduction turns into the convection of air and water, as well as thermal radiation. At the same time, the ice block will no longer be a regular cubic. Moreover, in our 2D practice, a 1 cm size node is insufficient to 'see' the mass loss because the 1cm thickness layer with 1.49 m × 1.49 m dimension is equal to 122.1 Kg of ice.

The starting temperature was -6° C. Because one gram of ice would be heated from 0° to 158.4° by 334 joules (the fusion heat). As a result, each 1° C initial temperature error will cause a 0.63% error in final mass loss.

2D heat transfer model has 3.5 Kg of mass loss less than 1D surface energy model. That is because (1) 1D energy balance model assumed the temperature of ice surface is 0° to avoid calculations on multiple surface, but heat equation do not have this assumption. (2) Solving partial differential equation numerically only gives an approximate estimation, the result varies by scale and errors accumulated in iteration.

5.4 The truth of the ice trade

The heat transfer model characterize ice well. (1) Ice has high heat diffusivity, making the warming process well-distributed with a small temperature gradient over the entire ice block. (2) Fusion heat that is far greater than ice's heat capacity, make ice a good agent of the ice trade. Therefore, the ice trade is the transportation of cold content stored in ice, which is a state of energy deficit of phase change.

6 Conclusions

This study reproduced the ice block expedition 1959 in multiple scenarios using 1D surface energy balance models and 2D heat equations. The results include (1) how the ice would melt without the cover and insulation. (2) The influence of box material and insulation material. (3) how long do the warming up and ripe phase take? The study compared simulation case 1959 and 2021. If there are only bare ice, the 2021 would be the first to melt away, and the 1959 would survive 2 more days. If there is covering and insulation, the 1959 lost even more. This is because the melting mechanism behaves differently on the ice bare surface and the cover surface.

There are some results that are very close to the original records. The simulations show that only about 3 cm thick, or 348.8 Kg to 352.3 Kg of the ice block melted. The heat equation simulation revealed the temperature distribution of ice inside the box. The ice block takes 175 hours warming up from -6° to 0° in the expedition.

Transporting ice to the tropics in the 19th century was a totally manageable business. Using a white-covered wooden box filled with double thickness is able to achieve the same insulation effect as glass wool, which is fully expected by simulations. While we set a baseline using the 1959 ice block expedition. The melt loss of transporting ice is not projected to be greater than 1959 if we do another expedition in 2021. Additionally, the simulations do not support attributing the concern to global warming because the ice is covered.

References

Blain, B. B. (2006). Melting markets: The rise and decline of the Anglo-Norwegian ice trade, 1850-1920. *Economic History Working Papers*, Article 22471. https://ideas.repec.org//p/ehl/wpaper/22471.html

Charai, M., Sghiouri, H., Mezrhab, A., Karkri, M., Elhammouti, K., & Nasri, H. (2020). Thermal Performance and Characterization of a Sawdust-Clay Composite Material. *Procedia Manufacturing*, *46*, 690–697. https://doi.org/10.1016/j.promfg.2020.03.098

Folkestad, K. (2009, February 18). *Fra Svartisen til Ekvator*. NRK. https://www.nrk.no/nordland/fra-svartisen-til-ekvator-1.6487241

Gjenskaper verdens beste reklameidé. (2009, February 20). https://www.bygg.no/article/41734!/

GLAVA (Director). (2012, January 2). *GLAVA, Ekspedisjonsleder Sivert Klevan om Isblokkekspedisjonen*. https://www.youtube.com/watch?v=32puW6r2O24

Ice block expedition of 1959. (2022). In *Wikipedia*. https://en.wikipedia.org/w/index.php?title=Ice_block_expedition_of_1959&oldid=1077983606

Muñoz Sabater, J. (2021). *ERA5-Land hourly data from 1950 to 1980, Copernicus Climate Change Service (C3S) Climate Data Store (CDS)[data set]*.

Norseng, P. G. (2022). The 'Last Ice Age' in maritime history: An introduction. *International Journal of Maritime History*, *34*(1), 101–112. https://doi.org/10.1177/08438714221080276

Wang, K., Yang, L., & Kucharek, M. (2020). Investigation of the effect of thermal insulation materials on packaging performance. *Packaging Technology and Science*, *33*(6), 227–236. https://doi.org/10.1002/pts.2500

Appendix 1: Physics basis

For the surface, the change of energy ∂_E over specific time ∂_t could be described by surface energy balance:

$$\frac{\partial_E}{\partial_t} = LW\downarrow + LW\uparrow + SW\downarrow + SW\uparrow + SF + LF + Q_{heatflux}$$

Net longwave radiation and shortwave radiation:

$$LW_{net} = L_{in} - (\varepsilon \cdot \sigma \cdot {T_s}^4 + (1-\varepsilon)\cdot L_{in})$$

$$SW_{net} = SW_{in}(1-\alpha)$$

Where L_{in} is the incoming long radiation, emissivity ε. Further, Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}\ \mathrm{Wm}^{-2}K^{-4}$, T_s is surface temperature in Kelvin. SW_{in} is the incoming short radiation, and α is albedo of the surface.

Sensible heat flux and latent heat flux is given by:

$$Q_{sen} = \rho_a \cdot c_{pa} \cdot A \cdot u \cdot (T_{air} - T_s)$$

$$Q_{lat} = -\rho_a \cdot L_v \cdot A \cdot u \cdot \frac{0.622 \cdot (e_s - e_{actual})}{P}$$

Where $\rho_a$is air density, c_{pa} is air specific heat capacity, A is turbulent exchange coefficient, L_v is latent heat of vaporization, u is the wind speed, P is air pressure, $(T_{air} - T_s)$ is temperature difference, and $(e_s - e_{actual})$ is vapor pressure difference.

Heat flux in one-dimensional is given by Fourier's law:

$$q = -k \cdot \frac{dT}{dx}$$

Or, heat flux could be derived from Fick's second law (heat equation) in 3-D:

$$\left(\frac{\partial^2{}_T}{\partial_{x^2}} + \frac{\partial^2{}_T}{\partial_{y^2}} + \frac{\partial^2{}_T}{\partial_{z^2}}\right) + \frac{q}{k} = \frac{\rho \cdot c_p}{k} \cdot \frac{\partial_T}{\partial_t}$$

Where T is temperature,t is time, x, y, z is the length of dimensions, and k is heat conductivity, ρ is density, c is specific heat capacity, q is the heat generation flux.

The equation to estimate the mean wind speed u_z at height z (meters) above the ground is:

$$u_z = \frac{u_*}{k}[\ln(\frac{z-d}{z_0})]$$

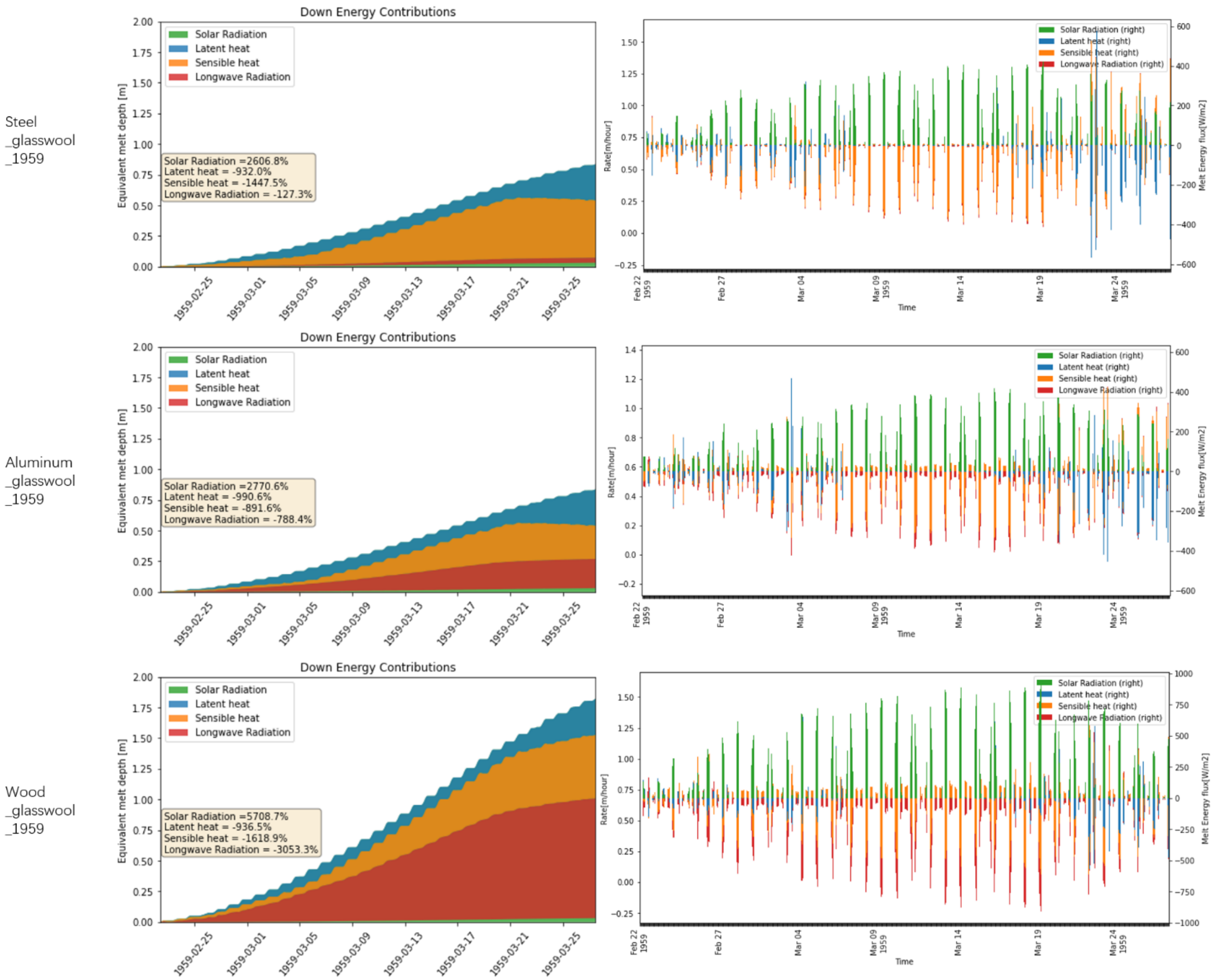
where u_* is the friction velocity (m/s), k is the Von Kármán constant (~0.41). Zero-plane displacement (d) is the height in meters above the ground at which zero wind speed is achieved as a result of flow obstacles such as trees or buildings. It can be approximated as 2/3 to 3/4 of the average height of the obstacles. Z_0 is the surface roughness.

Overall heat transfer coefficient is:

$$U = \frac{1}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + .. + \frac{L_i}{k_i}}$$

Where U is overall heat coefficient [W /(m^2k)], k is the material heat conductivity [W /(mk)], L is the thickness [m].

Appendix 2: Energy Partitioning of Three Types Box Surface 1959



The stackplot looks so strange when the variables become negative and subtract each other. Here is a full explanation. For a thin layer of surface:

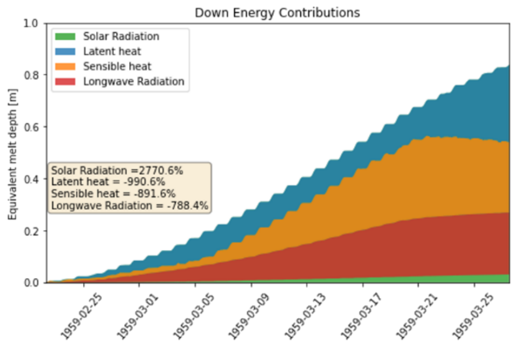
$$E = (positive + \ negetive) * t$$

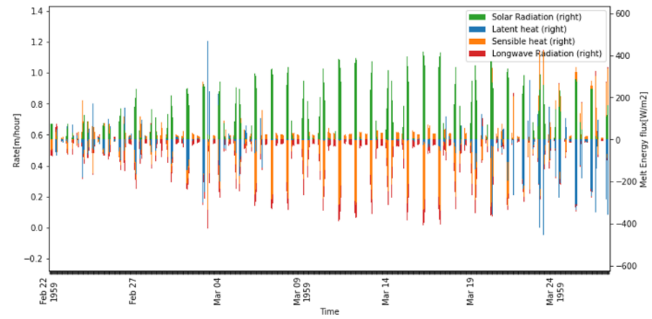
In these scenarios, the positive is SW (solar radiation), and negative are the rest of energy partition except heat transfer flux. E.g., SW×t/E = 2606.8%. If sum with the rest of energy partition, the result is E (100%). The y-axes indicator the amount of E, which is equivalent to the energy of melting 0.0299 m or 26.45 Kg ice (in 0 degree Celsius). However, these plots can only show it with a green line.

If most of components are positive (like Figure 5), the plot does not have this issue.

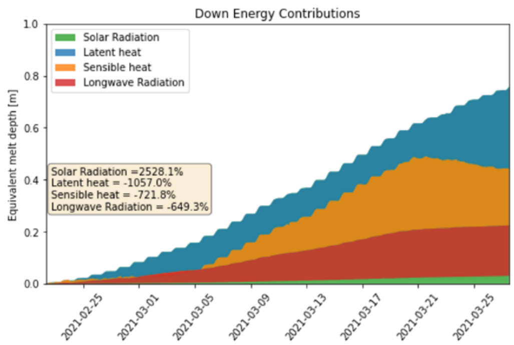
Appendix 3: Energy Partitioning of Box Surface in 1959, 2021 and 2020

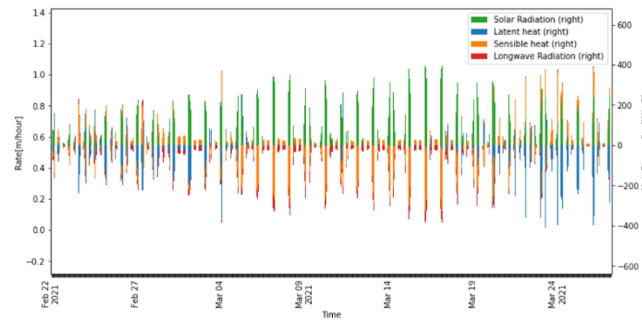
Aluminum _glasswool _1959



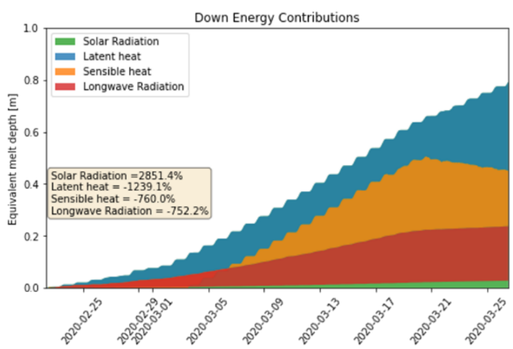


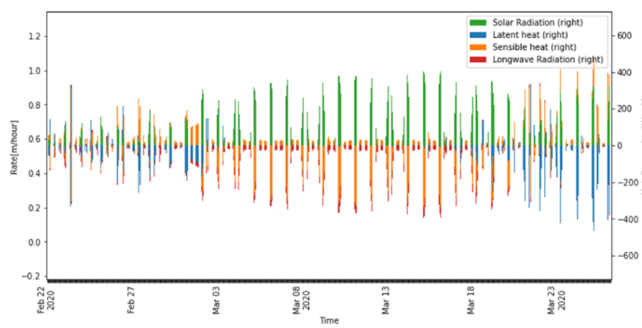
Aluminum _glasswool _2021





Aluminum _glasswool _2020





Appendix 4: Code

https://github.com/liuh886/the_ice_block_expedition_1959