## Homework 6

#### Homework 6.1

**Problem 1** We can directly calculate

$$\begin{split} \lim_{k \to \infty} \nabla L_{\text{NCE}}^{(k)}(\theta; h) &= \lim_{k \to \infty} \left[ \sum_{w} \tilde{p}_{w|h}(w|h) \left( \frac{1}{u^{\theta}(w,h)} - \frac{1}{u^{\theta}(w,h) + kq_{\tilde{w}}(w)} \right) \cdot \nabla u^{\theta}(w,h) \right. \\ &+ \sum_{1 \le i \le k, \bar{w}} q_{\tilde{w}}(\bar{w}) \left( -\frac{1}{kq_{\tilde{w}}(\bar{w})} \right) \cdot \nabla u^{\theta}(\bar{w},h) \right] \\ &= \sum_{w} \tilde{p}_{w|h}(w|h) \left( \frac{1}{u^{\theta}(w,h)} \right) \cdot \nabla u^{\theta}(w,h) - \sum_{\bar{w}} \nabla u^{\theta}(\bar{w},h) \\ &\approx \sum_{w} \left( \tilde{p}_{w|h}(w|h) - p_{w|h}^{\theta}(w|h) \right) \nabla \log u^{\theta}(w,h), \end{split}$$

where the final step uses the fact that the partion function Z is approximated to 1.

### Homework 6.2

# Problem 1

- 1. It is the self-attention and cross-attention parts, since the attention calculation has complexity  $O(n^2)$ , where n is the input sequence length.
- 2. The paper proposes to split the sequence into segments. Then, attention is only fully calculated within the segment; however, the hidden states in the previous segment is added in as a context without gradient. The training pseudo-code can be shown as below:
  - 1: Split the sequence into segments
  - 2: for each layer number n do
  - 3: **for** each segment number t **do**
  - 4: Concatenate the hidden states of the previous segment (but without gradient) NoGrad $(h_{t-1}^{(n-1)})$  with the current segment's hidden state  $h_t^{(n-1)}$
  - 5: Calculate the attention query based on solely  $h_t^{n-1}$ , but calculate the key and value based on the concatenated hidden states.

- 6: Use attention mechanism to calculate the output  $h_t^{(n)}$ , which is the hidden state of the current segment at layer n.
- 7: end for
- 8: end for

## Problem 2

For the sentiment analysis task, we should use BERT, since it is a pretrained transformer encoder. BERT can extract features of the text bidirectionaly, so it can perform better on the task. For fine-tuning, we should add a MLP projection head on the top of the output hidden states of BERT and try to learn the sentiment from the output.

For the closed-book question answering task, we should use GPT-2. GPT-2 is a pretrained transformer decoder, which can generate text based on the context. For the fine-tuning, we can use the context as the input and the question as the output, and train the model on the corpus just as training a autoregressive language model.