Homework 8

Deep Learning 2024 Spring

Due on 2024/5/18

1 Q&A

Problem 1. (Gumbel Distribution) Denote $Q(0,1)$ as the Gumbel distribution with $\mu = 0$ and $\beta = 1$. $\epsilon_i \sim Q(0,1)$ are i.i.d. Gumbel random variables with density $p(\epsilon_i) = \exp(-\epsilon_i - \exp(-\epsilon_i))$. x_i-s are known constants. Define random variable $\mathsf{k} = \text{argmax}_i(x_i + \epsilon_i)$ for $0 \leq i \leq K-1$. Prove that k follows the softmax distribution with logits x_i, i.e.,

$$\mathbb{P}(\mathsf{k} = i) = \frac{\exp(x_i)}{\sum_{j=0}^{K-1} \exp(x_j)}.$$

Problem 2. (Cycle Identification) Consider the cycle identification problem: for two undirected graphs shown in Fig. 1, we want to use a GCN to identify whether a given node is located on graph A or graph B. Suppose that all nodes have **the same feature** and the graph is **order-agnostic**. We have learned from the lecture that a vanilla GCN cannot solve this problem.

1. Try to propose a method to make the above two graphs (i.e., 3-cycle and 4-cycle) identifiable.
2. Can your solution be applied to tackle such a cycle identification issue in a more general setting, e.g., to distinguish K-cycles or more general graphs? Discuss the assumption, expressiveness power, and potential limitations of your method.

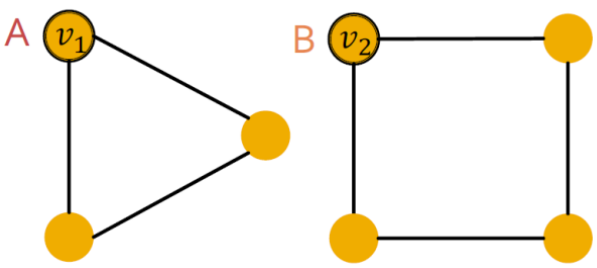

Figure 1: The cycle identification problem.

Problem 3. (Graph Generation) In general, to generate an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, we need to generate a feature matrix $X \in \mathbb{R}^{|\mathcal{V}| \times D}$, where D is the feature dimension, and a symmetric adjacent matrix $A \in \{0,1\}^{|\mathcal{V}| \times |\mathcal{V}|}$. In this problem, we assume a **pre-defined order for any graph**. For the following questions, You should describe (1) how to generate X and A using your approach and (2) how to conduct training on your model.

1. Suppose Alice wants to generate a graph with fixed number of nodes. Developed a VAE-based generative model to help Alice.

2. Design a generative model over graphs **with varying number of nodes**.