Heat and Moisture Transfer in Desiccant Coated Rotary Energy Exchangers: Part I. Numerical Model

Carey J. Simonson & Robert W. Besant

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A numerical model for coupled heat and moisture transfer in rotary energy exchangers is developed. The numerical model is one dimensional, transient, and is formulated using the finite volume method with an implicit time discretization. The model is developed from physical principles with a limited number of simplifying assumptions. This enables the study of several assumptions and their effect on the predicted performance of regenerative energy exchangers. In particular, the diffusion of the energy of phase change is treated in a unique manner which has a significant effect on the performance of rotary energy exchangers with thin desiccant coatings, as shown in Part II of this paper.

LITERATURE REVIEW

The subject of heat transfer in rotary heat exchangers has been studied extensively over a period of many years. Lambertson (1958) numerically solved the temperature distributions in the matrix and air streams using a finite difference method. His results are for an idealized case of no fluid carryover and no axial conduction in the matrix. As an extension to Lambertson's work, Bahnke and Howard (1964) included the effect of axial conduction in the matrix for a range of matrix thermal conductivity and operating conditions. In a thermal stress analysis, Mondt (1964) presented the effect of axial conduction on the temperature distribution in the matrix and fluid. Harper (1957) developed a theory to predict and account for carryover and pressure leakage in rotary regenerators for gas turbines. The transient response of sensible rotary heat exchangers was studied by London et al. (1964). These papers, summarized by Shah (1981), and Kays and London (1984), have formed the basis for the analysis of heat transfer in sensible rotary heat exchangers. These summaries contain many design charts and design equations that allow a designer to predict the performance of sensible rotary heat exchangers under various operating conditions.

The above studies only include sensible heat transfer and therefore can not be directly applied to rotary energy exchangers that are coated with desiccants. Attempts have been made to apply these heat transfer solutions to the coupled heat and mass transfer problem with limited success. Maclaine-Cross and Banks (1972) presented a method to solve the coupled heat and mass transfer problem by analogy with heat transfer for any number of components. The method involves transforming the coupled equations into a number of sets of equations where each set describes only one of the characteristic potentials for heat and mass transfer and is of the same form as the equation for heat transfer alone. This allows the solution for sensible rotary heat exchangers to be used to solve each characteristic potential and hence the temperature and moisture content in the air. Banks (1972), and Close and Banks (1972) used the same analogy for heat and mass

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transfer in fluid flow through a porous medium, assuming thermodynamic equilibrium. They were able to predict the main features of an experiment with a silica-gel dryer. However, as pointed out by Banks (1985a), this method contains two significant errors which fortuitously cancel each other out and give acceptable results for a silica gel desiccant. This canceling of errors cannot be guaranteed for other desiccants and operating conditions. Banks (1985 b) presented a nonlinear analogy that was deemed to be more accurate for other desiccants than the linear analogy of Maclaine-Cross and Banks (1972), but this nonlinear model reduced only one of the two significant errors and had, in fact, higher errors for a silica gel dehumidifier than the linear model.

The main advantage of using the analogy between heat and mass transfer is that it reduces the computation time necessary to find a solution. With the further development of computers and numerical techniques, this advantage has becomes less important. As a result, more recent studies have usually solved the coupled heat and mass transfer equations numerically. This is the approach used in the present paper.

Zheng and Worek (1993) presented an implicit numerical model which solved heat and mass transfer in adiabatic rotary dehumidifiers. They did extensive work on the numerical model to improve the speed of the simulation. Some results obtained with their numerical model are compared with results from other models but not with experimental measurements. In a subsequent paper, Zheng et al. (1993) used this numerical model to show the effect of wheel speed, type and amount of desiccant and size of the dehumidifier on the performance of adiabatic dehumidifiers.

Charoensupaya and Worek (1988) parametrically studied an open-cycle adiabatic desiccant cooling system using an explicit finite difference method. They reported that maximizing the amount of moisture dried from the air does not optimize the dehumidifier-air conditioning system thermal coefficient of performance (COP). COP is the ratio of the cooling provided by the system to the heat added by external sources. They also showed that, for an optimal COP, the optimal mass fraction of desiccant is relatively low (20%) and the optimal isotherm is that of a weakly sorbing, Type I desiccant.

Using a second law analysis, Van den Bulk et al. (1988) studied solid desiccant dehumidifiers. They concluded that parameters which optimize second law efficiency [based on availability (i.e. enthalpy & entropy)] also guarantee nearly optimal first law performance (COP).

The above-mentioned studies developed a theory of regenerative dehumidifiers to the point where this theory has been applied to predict the performance of air-conditioning systems. For example, Nimmo et al. (1993) and Rengarajan and Nimmo (1993) used the desiccant wheel numerical model developed by Collier (1988) to show the advantage of using a desiccant wheel together with conventional air conditioning systems. They compared DEAC (Desiccant Enhanced Air Conditioner) to other air conditioning options and found that for a warm humid climate (e.g., Miami) the DEAC had the best performance considering energy use, comfort and capital cost. The DEAC system provided the same comfort while using 10% less energy than a current high efficiency air conditioner.

Meckler (1995) introduced a desiccant preconditioning module that can supply dry air to two or more conventional air-conditioning units. Simulation results presented in the paper show that preconditioning ventilation air using desiccant dryers can reduce the required installed air chilling capacity by 30% in the United States. This new system results in improved indoor air quality at a lower cost because of the independent temperature and humidity control.

A collection of studies by ASHRAE (1992), shows that the inclusion of desiccant dehumidifiers in HVAC systems for large and small office buildings and special applications such as supermarkets, ice rinks, and storage rooms has proven to be effective. The benefits include reduced capital and operating costs together with improved building performance. The authors state that similar benefits can be realized with the integration of rotary energy exchangers into HVAC systems.

Although the fundamental physics and surface chemistry of desiccant drying and energy exchange are similar, according to Banks (1985a) the appropriate methods for modeling a regenerative dehumidifier and a regenerative energy exchanger may not be the same because the operating conditions are significantly different. Since all models include simplifying assumptions, it is necessary to study the importance of these assumptions for the specific application of the model. Furthermore, experimental validation should be completed before a model is used for other applications. Stiesch et al. (1995) found that the desiccant drying model of Maclaine-Cross (1974) gave good results for a desiccant-coated rotary energy exchanger. Their numerical results agree with the manufacturer's performance data at a certain operating condition, but the effect of individual simplifying assumptions on the predicted performance was not analyzed. In spite of their high performance characteristics, the practical applications of total energy exchangers have been slow to develop, indicating a need for more research.

The major difference between regenerative dehumidifiers and regenerative energy exchangers is that the heat and moisture transfer are of similar importance in total energy exchange applications while heat transfer is typically of secondary importance in dehumidifying applications. Therefore, whether the energy of phase change is transferred to the air or to the matrix can greatly affect the performance of an energy exchanger. The literature surveyed did not discuss this point and, in fact, two recent models (Zheng and Worek, 1993 and Collier et al., 1986) used different assumptions concerning this energy of phase change. Zheng and Worek (1993) assumed that the energy of phase change is delivered to the matrix while Collier et al. (1986) assumed that the energy of phase change is delivered to the air. The correctness and significance of these assumptions needs to be investigated.

The rotational wheel speed of regenerative dehumidifiers is generally an order of magnitude lower than the wheel speed of total energy exchangers. This significant change in the wheel speed will likely change the analysis, design and optimization of total energy exchangers compared to desiccant dehumidifiers. For example, the storage of water vapor and energy in the air and axial conduction through the matrix are expected to be more important in the analysis of rotary energy exchangers. In addition, desiccant dryers, which use high temperatures to desorb moisture from the wheel, are expected to have somewhat different sorption characteristics than energy wheels which operate over a small temperature range.

The purpose of this paper is to develop a numerical model for heat and moisture transfer in rotary energy exchangers, also known as enthalpy wheels or desiccant coated energy wheels. This model is intended to be used to design or optimize rotary energy exchangers, and therefore must include all important parameters that affect the performance of rotary energy exchangers. The model must be complete and accurate, without being overly complex, in order to facilitate design and optimization. In Part I of this paper, the governing equations and assumptions used to develop the numerical model for rotary energy exchangers are presented in detail. In Part II, the model is compared with experiments, and the model is used to show the sensitivity of certain simplifying assumptions on the predicted performance of a total energy exchanger.

ASSUMPTIONS

Figure 1 contains a schematic of a rotary energy exchanger operating in a counterflow arrangement and shows one of the flow tubes in detail. The support material shown here is aluminum, but it could be any rigid material. The geometry selected for the core material [Figure 1(b)] changes the details of the model but not the conclusions. The numerical model solves the simultaneous heat and moisture transfer in one tube as it rotates around the axis of the wheel. Under typical operating conditions, warm moist air enters the tube during the hot part of the cycle and transfers energy and moisture to the matrix. This energy and moisture is then transferred from the matrix to the air during the cold part of the cycle. The half plane of the matrix tube is assumed to
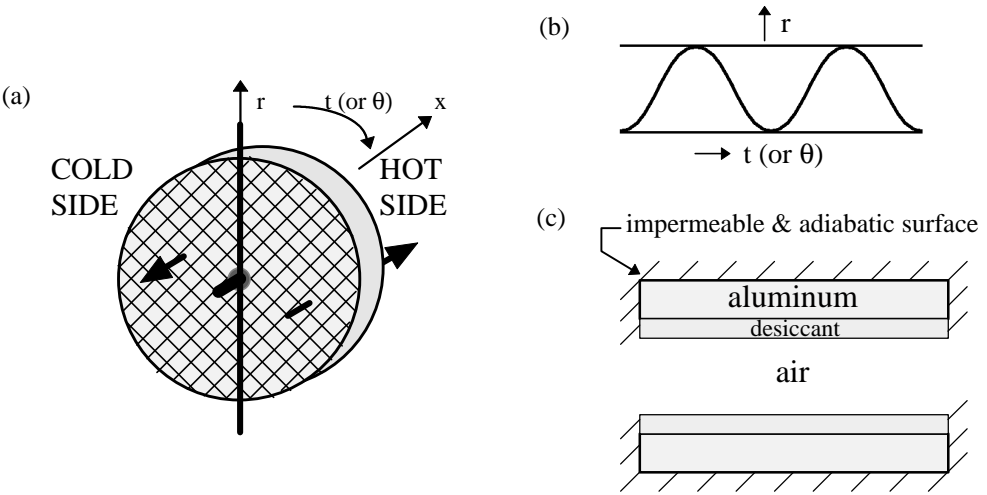
Figure 1. Schematic of the rotary energy exchanger wheel showing (a) entire wheel, (b) tube geometry cross-section, and (c) side view of one of the tubes

be impermeable and adiabatic, and the bulk mean temperatures and moisture concentrations in the air are used in the model. The formulation is therefore one dimensional and transient, with space (x) and time (t) as the independent variables. The model inherently includes the carryover flow and its effects on performance of the rotary energy exchanger. Energy and moisture storage in the air, a variable heat of sorption and the entry length are included in the model.

The assumptions used in the analysis were:

1. The heat and mass transfer process can be modeled using the bulk mean temperatures and moisture concentrations of the air within each tube shown in Figure 1(b) and 1(c)
2. The axial (x direction) heat conduction and water vapor molecular diffusion in the air are negligible
3. The axial molecular diffusion and capillary motion of moisture within the desiccant coating are negligible
4. There are no radial (r direction) temperature or moisture content gradients in the matrix
5. The heat conduction through the matrix in the x direction is dominated by the conduction through the aluminum
6. Hysteresis in the sorption isotherm or the heats of sorption for the desiccant coating are assumed negligible
7. The temperature, velocity and concentration profiles develop simultaneously in each tube
8. The pressure drop across the heat exchanger does not result in a significant change in the absolute pressure or air density
9. The inlet air conditions are uniform radially (r direction), but may vary with time (or rotational angle)
10. The tubes that make up the rotary energy exchanger are identical
11. The matrix properties of the aluminum, desiccant and adsorbed water are constant
12. The dependent variables and properties vary periodically with time with each rotation of the wheel (i.e., within one cycle the dependent variables and properties vary in a cyclic transient manner)
13. Other assumptions concerning boundary conditions will be discussed in conjunction with

the development of the theoretical model

The first two assumptions simplify the analysis in the air. Assumption 1 avoids the problem of determining the radial temperature and moisture concentration distributions in each tube and simplifies the problem to one-dimensional in space for each bulk fluid variable. The second assumption is valid because the flux, or advection of heat and water vapor in air, is much larger than the axial conduction and molecular diffusion for a wheel under normal operating conditions. The Peclet number for typical operating conditions is in the order of 100, whereas, fluid axial heat conduction is generally neglected for Pe > 50, and can be neglected for Pe > 10 in this specific application (Shah and London 1978).

The third assumption is typically valid because water molecules form strong physical bonds with the desiccant and therefore the molecular diffusion within the matrix will be small in the axial direction, yet sufficient in the radial direction to exchange water molecules cyclically. Furthermore, the capillary movement of water in the matrix could only occur for large accumulations of water, which is an operating condition that should be avoided in desiccant-coated wheels. Assumption 4 is valid if the heat and mass transfer Biot numbers are less than 0.1 (see, for example, Incropera and Dewitt, 1996). Both the heat and mass transfer Biot numbers are less than 0.1 for this problem because the extremely thin-walled tubes that make up the wheel are coated with a thin layer of desiccant. This means that the matrix behaves like a heat-exchanger fin with negligible temperature (and moisture content) differences in the r and θ directions, but with significant temperature (and moisture content) differences in the x direction. Assumption 5 arises because the thermal conductivity of typical desiccants (and water) is much lower than the thermal conductivity of aluminum. This, coupled with the manufacturing process where the desiccant is placed in a thin layer on the aluminum, and the operating process where water is adsorbed by the desiccant in a thin layer (often monolayers), results in the desiccant having little effect on the axial (x direction) thermal conduction through the wheel. For non-metallic support materials in place of aluminum foil, such as plastic or paper that have lower thermal conductivity, this assumption will not always be valid. In such cases, the thermal conductivity of the matrix can be easily modified to include conduction through the desiccant.

Assumption 6 is used because molecular sieve desiccants are expected to have a type I isotherm with little or no hysteresis (Gregg and Sing 1982). Similarly, the heats of adsorption and desorption are expected to be equal. Assumption 7 arises because the Prandtl and Lewis numbers for the problem are nearly unity. The pressure drop across the heat exchanger due to skin friction is in the order of 10 Pa. This pressure drop is negligible compared to atmospheric pressure and thus changes in air pressures due to skin friction within the wheel will not influence the other properties. Assumptions 9 and 10 mean that only one of the tubes in the matrix needs to be analyzed and assumption 11 simplifies the analysis with little loss in accuracy. Assumption 9 also implies that air leakage from the hot side to the cold side, or vice versa, is negligible or zero. On the other hand, carryover, due to rotation of the air mass within the wheel matrix, is included in the analysis.

GOVERNING EQUATIONS

The governing equations for coupled heat and moisture transfer in rotary energy exchangers based on the above assumptions are now presented for one flow tube [see Figure 1 (b) and (c)]. The energy equations for the air and matrix include energy storage, convection, conduction and energy associated with phase change and are respectively:

$$\rho_g Cp_g A_g \frac{\partial T_g}{\partial t} + U\rho_g Cp_g A_g \frac{\partial T_g}{\partial x} - \dot{m}' h_{ad}\eta + hp(T_g - T_m) = 0 \tag{1}$$

$$\rho_g Cp_m A_m \frac{\partial T_m}{\partial t} - \dot{m}' h_{ad}(1-\eta) - \dot{m}' Cp_w (T_g - T_m) - hp(T_g - T_m) = \frac{\partial}{\partial x}\left(k_{Al} A_{Al} \frac{\partial T_m}{\partial x}\right) \qquad (2)$$

These energy equations, which are written with temperatures as the dependent variables, are shown in Appendix A to be nearly identical to those in the literature (e.g., Stiesch et al 1995 and Zheng and Worek 1993) that use enthalpies as the dependent variables. A significant difference between Equations (1) and (2) and those in the literature is the term η, where η accounts for the distribution of phase change energy between the desiccant and the air. This new term in the energy equations is discussed in more detail below and in Appendix A.

In Equations (1) and (2), η is the fraction of the phase change energy that enters the air directly and is shown schematically in Figure 2.
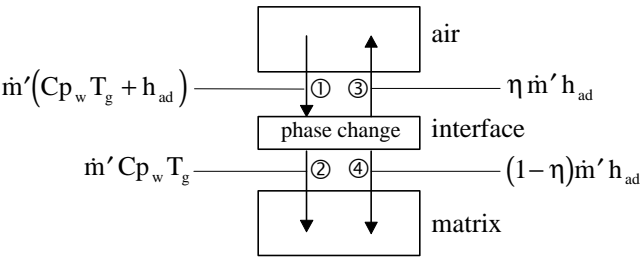

Figure 2. Flow of energy between air and matrix due to phase change at interface

Figure 2 shows that the term η arises from an energy balance at the phase change interface. If the local two-or three-dimensional temperature and moisture concentrations were solved in the air and matrix, η would be intrinsically included in the energy balance at this interface. However, in this case, the bulk mean properties are used for transient fluid flow with heat and mass transfer in the duct, and therefore the authors propose that it is important to define how the energy source terms are distributed between the fluid and solid.

In Figure 2, energy flow 1 represents the flow of energy associated with the water vapor that leaves the air due to lateral diffusion. This water vapor undergoes a phase change at the interface as it attaches to the matrix (i.e., desiccant) as a sorbed liquid, with the energy represented by 2. The water is assumed to be at the bulk mean temperature of the air, which has negligible effect on the results because the energy of phase change is over 100 times greater than the sensible cooling energy. This is further explained by Christianson and Newhouse (1985) for heat and mass transfer in wet-bulb thermometers. Energy flows 3 and 4 represent the fraction of the energy of phase change that is transferred to the air and matrix respectively. To approximate η, the analytical solution for temperatures in two infinite solids with a constant heat source initiated at time $t = 0$ at the interface is used (Carslaw and Jaeger 1959). The analysis is in Appendix A and the result is:

$$\eta = \frac{k_g/\sqrt{\alpha_g}}{k_g/\sqrt{\alpha_g} + k_d/\sqrt{\alpha_d}} \qquad (3)$$

where, for convection, k_g is taken to be,

$$k_g = hD_h \tag{4}$$

and the thermal diffusivity of the air is modified to reflect this increased conduction. Using Equation (3), the calculated value of η for the energy wheel modeled in this paper is $\eta = 0.07$; whereas, the actual value of η is probably slightly smaller because the interface between the desiccant and air is not a smooth surface (see Appendix A for further discussion). A value of $\eta = 0.05$ is estimated for the molecular sieve coating investigated here. This means that 5% of the energy of phase change is assumed to be delivered directly to the air. In Appendix A, the value of η for various applications of heat and mass transfer theory is discussed, the development of Equation (3) is presented and it is suggested that more research is necessary to obtain a more accurate value of η.

The conservation of mass in the air results in two continuity equations, one for the dry air and one for the water vapor. These equations include storage, convection and phase change. The equations are:

$$A_g \frac{\partial \rho_v}{\partial t} + \frac{\partial}{\partial x}(\rho_v U A_g) + \dot{m}' = 0 \tag{5}$$

and

$$\frac{\partial \rho_a}{\partial t} + \frac{\partial}{\partial x}(\rho_a U) = 0 \tag{6}$$

The conservation of mass for the matrix is:

$$\dot{m}' = \rho_{d,dry} A_d \frac{\partial u}{\partial t} \tag{7}$$

where u is the mass fraction of water (vapor or liquid) in the desiccant (i.e. mass of water per mass of dry desiccant). To complete the formulation of the problem, several thermodynamic relations are needed. They are as follows:

$$P_g = P_a + P_v \tag{8}$$

$$P_a = \rho_a R_a T_a \tag{9}$$

$$P_v = \rho_v R_v T_v \tag{10}$$

$$\rho_g = \rho_a + \rho_v \tag{11}$$

$$h_{fg} = 2\,500\,800 - 2\,307(T - 273.15) \tag{12}$$

$$h_{ad} = h^* h_{fg} = \left[1 + (h^*{}_{max} - 1)\left(1 - \frac{u}{Wm} \right)^{\lambda} \right] h_{fg} \tag{13}$$

$$P_{v,sat} = f(T) = e^F \tag{14}$$

where Wm is the maximum moisture capacity of the desiccant, λ describes the shape of heat of sorption as a function of moisture content, and

$$
F = \begin{cases} \dfrac{C_1}{T} + C_2 + C_3 T + C_4 T^2 + C_5 T^3 + C_6 T^4 + C_7 \ln T & 173 \text{ K} < \text{T} < 273 \text{ K} \\[4mm] \dfrac{C_8}{T} + C_9 + C_{10} T + C_{11} T^2 + C_{12} T^3 + C_{13} \ln T & 273 \text{ K} < \text{T} < 473 \text{ K} \end{cases} \tag{15}
$$

The constants in Equation (15) are (ASHRAE 1993):

$C_1 = -5674.5359$, $C_2 = 6.3925247$, $C_3 = -9.677843 \times 10^{-3}$, $C_4 = 6.22115701 \times 10^{-7}$,
$C_5 = 2.0747825 \times 10^{-9}$, $C_6 = -9.484024 \times 10^{-13}$, $C_7 = 4.1635019$, $C_8 = -5800.2206$,
$C_9 = 1.3914993$, $C_{10} = -4.8640239 \times 10^{-2}$, $C_{11} = 4.1764768 \times 10^{-5}$,
$C_{12} = -1.4452093 \times 10^{-8}$, and $C_{13} = 6.5459673$.

The geometry and property relations for the air (g), desiccant (d) and matrix (m) are:

$$
Cp_g = \frac{\rho_a Cp_a + \rho_v Cp_v}{\rho_g} \tag{16}
$$

$$
\rho_d = \rho_{d, dry}(u + 1) \tag{17}
$$

$$
Cp_d = \frac{\rho_{d, dry}(u Cp_w + Cp_{d, dry})}{\rho_d} \tag{18}
$$

$$
\rho_m = \sigma_d \rho_d + \sigma_{Al} \rho_{Al} \tag{19}
$$

$$
Cp_m = \frac{\sigma_d \rho_d Cp_d + \sigma_{Al} \rho_{Al} Cp_{Al}}{\rho_m} \tag{20}
$$

$$
\sigma_d + \sigma_{Al} = 1 \tag{21}
$$

For the fully-developed flow region within the matrix tubes, the convective heat transfer coefficient is taken for laminar flow through a tube with constant heat flux boundary conditions. The geometric shape of each tube in the matrix is assumed to be sinusoidal, as shown in Figure 1(b). The hydrodynamic and thermal entry region in the tubes, which is typically less than 10% of the total matrix length, may be important because the temperature, concentration, and velocity profiles develop in this region. To account for this, the convective heat and mass transfer coefficients are a function of x in the entry region. These heat transfer coefficients are obtained from numerical simulations of simultaneously (thermal and hydrodynamic) developing flow in an equilateral triangular duct (Shah and London 1978).

Moisture Transfer

Modeling the moisture transfer is one of the most important parts of the model. During adsorption and desorption, the moisture transfer can be simply calculated knowing the convective mass transfer coefficient and the difference between the water vapor density in air (ρ_v) and the water vapor density on the surface of the desiccant ($\rho_{v,m}$). That is,

$$\dot{m}' = h_m p(\rho_v - \rho_{v,m}) \tag{22}$$

The wetted perimeter of the tube (p) can be found from geometry and the convective mass transfer coefficient (h_m) can be determined directly or using the analogy between heat and mass transfer. The latter is used in this paper. The only unknown is the water vapor density on the desiccant surface. It is known that the water vapor density on the surface of a desiccant increases as (1) the amount of adsorbed water increases and (2) the temperature of the desiccant increases (ASHRAE 1993).

Therefore, it is logical to propose that the water vapor density on the surface of the desiccant is some function of mass fraction of water in the desiccant (u) and temperature of the desiccant. Assuming that the temperature of the desiccant is the same as the temperature of the matrix (assumption 4) gives,

$$\rho_{v,m} = f(u,T_m) \tag{23}$$

If the function in Equation (23) is known, Equation (23) can be used together with Equation (22) to calculate the rate of phase change. It is proposed to use the sorption isotherms to obtain the function that accurately describes the water vapor density on the desiccant surface as a function of moisture content and temperature.

There are six basic types of isotherms which describe the amount of gas adsorbed or desorbed as a function of the relative pressure of the gas (Gregg and Sing 1982, Kaviany 1991). For a particular desiccant at a given temperature, the sorption isotherm for water vapor is,

$$u = f(\phi) \tag{24}$$

where ϕ is the relative water vapor pressure (or density). By taking the inverse of Equation (24) and knowing the saturation density of the water vapor, the density of the water vapor in the air can be determined as a function of amount of moisture adsorbed. That is,

$$\rho_v = \rho_{v,sat} f^{-1}(u) \tag{25}$$

When taking the inverse of Equation (24) the issues of existence and uniqueness of the solution together with stability of the inverse process must be considered (Beck et al. 1985), especially for desiccants that have complicated sorption isotherms with hysteresis.

The sorption isotherm describes the equilibrium mass fraction of water in the desiccant (i.e. when the net flux of water vapor to the desiccant is zero). That is, at equilibrium, there is no net driving potential for further adsorption or desorption. Since the difference between the partial pressure (or density) of the water vapor on the surface of the desiccant and the partial pressure (or density) of the water vapor in the air is the driving potential for sorption [Equation (22)], it can be assumed that the partial pressure (or density) of water vapor on the surface of the desiccant will be equal to the partial pressure (or density) of the water vapor in the air at equilibrium. Therefore, Equation (25) also gives the density of water vapor on the desiccant surface as a function of the amount of moisture in the desiccant.

Although this analysis concentrates on water vapor sorption on a molecular sieve desiccant, the procedure is applicable to any adsorptive and adsorbent. The rotary energy exchanger, modeled in this analysis, is coated with a molecular sieve desiccant that has a pore size in the order of the diameter of the water molecule (0.3 nm). The surface chemistry of water vapor sorption on these coatings is not fully understood, suggesting the need for more research. This desiccant

is expected to have a Type I isotherm with little or no hysteresis (Gregg and Sing 1982). The isotherm can be described by the Langmuir equation,

$$\frac{u}{Wm} = \frac{C\phi}{1 + C\phi} \tag{26}$$

where u is the mass of water vapor adsorbed per mass of dry desiccant, ϕ is the relative humidity (as a fraction), and C and Wm are constants which are typically determined experimentally. The dimensionless constant C is a function of the heat of sorption and temperature (see Appendix B) and C at one temperature is related to C at another temperature by the following relationship,

$$C_{@T_2} = C_{@T_1} \frac{P_{sat,2}}{P_{sat,1}} \left(\frac{T_2}{T_1}\right)^{\frac{2307h^*}{R_v}} e^{\frac{3130960h^*}{R_v}\left(\frac{T_1 - T_2}{T_1 T_2}\right)} \tag{27}$$

where the reference temperature (T_1) is 293.15 K (20°C) in this paper. Taking the inverse of Equation (26) gives:

$$\phi = \frac{\rho_v}{\rho_{v,\,sat}} = \frac{u}{(Wm - u)C} \tag{28}$$

which is graphed in Figure 3.

Equation (28) and Figure 3 point out two issues that must be considered because of the inversion. One is that the relative humidity cannot be greater than 1, and the other is that the inverted solution is undefined if the denominator in Equation (28) is zero. Therefore, we need to restrict the solution of Equation (28) to include only the range where ϕ is between 0 and 1. By definition u is greater than or equal to zero, therefore it is required that,

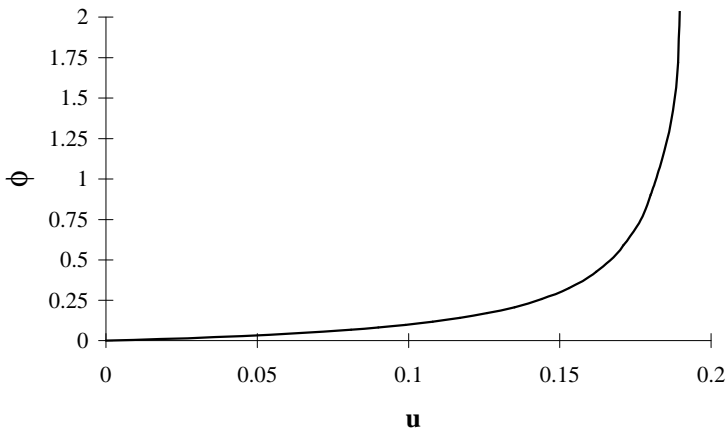$$u \le (Wm - u)C \quad \text{and} \quad (Wm - u)C \ge 0 \tag{29}$$

Figure 3. Inverse of Type I sorption isotherm
Relative humidity (fraction) as a function of the mass fraction of water [Equation (28)].

which simplifies to,

$$u \le \frac{WmC}{1 + C} \quad \text{and} \quad u \le Wm \tag{30}$$

The second condition in Equation (30) is always satisfied if the first condition is satisfied because Wm and C are positive constants, which means that the ratio $C/(1 + C)$ is always less than 1. It should also be noted that the constraints in Equation (30) also ensure that the inversion solution exists by ensuring that the denominator of Equation (28) is never equal to zero. At $u = 0$, $d\phi/du = 1/(WmC)$, suggesting a finite slope at $u = 0$ in the plot of ϕ versus u in Figure 3.

Returning to Equation (28) and using the fact that the water vapor density on the surface of the matrix will be equal to the water vapor density in the air at equilibrium, gives the following:

$$\rho_{v,m} = \frac{P_{v,sat}(T_m)}{R_v T_m} \frac{u}{(Wm - u)C} \qquad 0 \le u \le Wm\frac{C}{1 + C} \tag{31}$$

Equation (31) is used to give the needed function in Equation (22) and completes the formulation of the governing equations during adsorption and desorption.

When the air becomes saturated ($\phi = 1.0$ or 100%), moisture transfer will be in the form of condensation or evaporation (ablimation or sublimation at temperatures below the triple point). This operating condition, although possible, is uncommon because uncontrolled condensation or ablimation will damage the desiccant coating on the wheel. Mathematically and physically the problem becomes more complex when condensation and frosting are included. These conditions require a different numerical solution strategy and are not included in the current model, but are addressed by Simonson et al. (1997).

PERFORMANCE CRITERIA

To assess the overall performance of a rotary energy exchanger, the average inlet and outlet conditions are needed for both the hot and cold air flows. The average inlet and outlet conditions, for any variable or property (χ), are defined as:

$$\chi_{h,i} = \frac{\int_0^{\theta_h} \dot{m}\chi(x = 0)d\theta}{\int_0^{\theta_h} \dot{m}(x = 0)d\theta} \qquad \chi_{c,i} = \frac{\int_0^{\theta_c} \dot{m}\chi(x = L)d\theta}{\int_0^{\theta_c} \dot{m}(x = L)d\theta} \tag{32}$$

$$\chi_{h,o} = \frac{\int_0^{\theta_h} \dot{m}\chi(x = L)d\theta}{\int_0^{\theta_h} \dot{m}(x = L)d\theta} \qquad \chi_{c,o} = \frac{\int_0^{\theta_c} \dot{m}\chi(x = 0)d\theta}{\int_0^{\theta_c} \dot{m}(x = 0)d\theta} \tag{33}$$

where $\dot{m}$ represents $\dot{m}(Cp_a + WCp_v)$ when $\chi = T$. For uniform inlet conditions, as used in this paper, only the average outlet conditions [Equation (33)] are required. These are calculated using numerical integration. The humidity ratio and total enthalpy are given by:

$$W = \rho_v / \rho_a \tag{34}$$

$$H = Cp_a(T - 273.15) + W[h_{fg} + Cp_w(T - 273.15)] \tag{35}$$

Using these definitions, the effectiveness, which is the most important performance criteria of rotary energy exchangers (and any air-to-air heat exchanger), can be calculated. There are three types of effectiveness for total energy recovery devices, namely sensible, latent (or moisture) and total (or enthalpy). These are defined for steady state conditions as follows (ASHRAE *Standard* 84-1991):

• Sensible heat transfer effectiveness:

$$\varepsilon_s = \frac{m_h(T_{h,i} - T_{h,o})}{m_{min}(T_{h,i} - T_{c,i})} = \frac{m_c(T_{c,o} - T_{c,i})}{m_{min}(T_{h,i} - T_{c,i})} \tag{36}$$

• Latent heat transfer effectiveness assuming the enthalpy of phase change is constant (moisture transfer effectiveness):

$$\varepsilon_1 = \frac{m_h(W_{h,i} - W_{h,o})}{m_{min}(W_{h,i} - W_{c,i})} = \frac{m_c(W_{c,o} - W_{c,i})}{m_{min}(W_{h,i} - W_{c,i})} \tag{37}$$

• Total energy (enthalpy) transfer effectiveness:

$$\varepsilon_t = \frac{m_h(H_{h,i} - H_{h,o})}{m_{min}(H_{h,i} - H_{c,i})} = \frac{m_c(H_{c,o} - H_{c,i})}{m_{min}(H_{h,i} - H_{c,i})} \tag{38}$$

The effectiveness values presented in this paper are the average of the effectiveness calculated on the hot side and the effectiveness calculated on the cold side. The effectiveness of a rotary energy exchanger typically depends on the number of transfer units (NTU) and the dimensionless wheel capacitance (Cr*). These dimensionless groups are defined for sensible heat transfer as:

$$NTU = \frac{1}{(mCp_a)_{min}} \left[\frac{1}{\frac{1}{(hA_s)_h} + \frac{1}{(hA_s)_c}} \right] \tag{39}$$

$$Cr^* = \frac{MCp_m\omega}{(mCp_a)_{min}} \tag{40}$$

BOUNDARY CONDITIONS

The required boundary conditions for the problem are the hot and cold air stream inlet conditions. These conditions can generally vary with time on both the hot and cold sides of the energy exchanger and are for any integer (n):

Hot side [$n(p_h + p_c) \le t \le n(p_h + p_c) + p_h$]:

$$T_g(t,0) = T_{g,h}(t) \tag{41}$$

$$\rho_v(t,0) = \rho_{v,h}(t) \tag{42}$$

$$\rho_a(t,0) = \rho_{a,h}(t) \tag{43}$$

$$U(t,0) = U_h(t) \tag{44}$$

Cold side $[n(p_h + p_c) + p_h \le t \le (n+1)(p_h + p_c)]$:

$$T_g(t,L) = T_{g,c}(t) \tag{45}$$

$$\rho_v(t,L) = \rho_{v,c}(t) \tag{46}$$

$$\rho_a(t,L) = \rho_{a,c}(t) \tag{47}$$

$$U(t,L) = U_c(t) \tag{48}$$

Because time is a continuous variable from $t = 0$ to $t = (p_c + p_h)$ to $t = n(p_c + p_h)$, the periodic boundary conditions are inherently included in the formulation.

The end surfaces of the matrix at the air flow entrance and exit from the wheel make up less than 0.1% of the total heat and mass transfer area of the matrix. The heat transfer through the ends will therefore be negligible compared to the heat transfer within the tube. Hence, the boundary conditions for the matrix are chosen to be adiabatic. Mathematically this is,

$$\left.\frac{\partial T_m}{\partial x}\right|_{x=0} = \left.\frac{\partial T_m}{\partial x}\right|_{x=L} = 0 \tag{49}$$

It should be noted that the initial conditions for the problem are not critical because the desired solution is the quasi-steady-state solution for an energy wheel rotating at constant speed with constant inlet conditions for both the hot and cold air flows.

NUMERICAL SCHEME

The governing equations were discretized using the using the finite volume method with a staggered grid (Patankar 1980). The upwind difference scheme was used in the air because heat conduction and moisture diffusion are negligible. Central differencing was used in the matrix because there is no air flow. All the dependent variables and properties were solved at the nodes and the velocity solved at the faces. The transient terms were solved using the implicit formulation. The numerical solution scheme is fully implicit for all of the coupled dependent variables in that the most updated value is used in each algebraic equation. The property values and source terms were treated in a semi-implicit manner because they were updated at the end of each iteration, and kept constant during the next iteration. The discretized equations (see Appendix C) were solved using a Gauss-Seidel iteration technique with under relaxation. The energy equation in the matrix was solved using the Tridiagonal Matrix Algorithm. The algorithm used to solve the coupled heat and moisture transfer problem is as follows:

1. Estimate the rate of phase change and the properties needed in the governing equations.
2. Solve the T_m, ρ_v, U, u, and T_g fields in order with Equations (C7), (C17), (C13), (C22), and (C1), respectively (i.e. differential Equations (2), (5), (6), (7), and (1), respectively).
3. Update the property information with Equations (8) through (21) and (31).
4. Update the rate of phase change (m') using Equation (22).
5. Return to step 2 and iterate until a converged solution is reached.
6. Increment time, return to step 2 and iterate until a quasi-steady solution is obtained.

Trial and error established that the order of solution in step 2 seemed to give the most rapid convergence of the equations. This trial and error analysis showed that the most important feature in step two is that T_g be solved last. This reduced the solution time by approximately two-fold compared to the case where T_g is solved first.

The transient solution was solved until quasi-steady state was reached. Even though the most important result is the quasi-steady solution, the model must be time accurate because the quasi-steady solution is a transient solution that varies periodically. The numerical solution is time accurate because time is only incremented when a converged solution is reached. The normalized residuals of the governing equations were low ($\sim 10^{-16}$) because upwinding and the tridiagonal matrix algorithm were used. As a result, the convergence criteria was based on the amount the dependent variables change from iteration to iteration. Time is incremented when the following is satisfied for each dependent variable:

$$\frac{\sum_{i=1}^{n} \left| \chi(i)^{j+1} - \chi(i)^j \right|}{n(\chi_{max} - \chi_{min})} \leq 1 \times 10^{-4} \tag{50}$$

Decreasing the convergence limit from 10^{-4} to 10^{-5} nearly doubles the solution time but changes the predicted effectiveness by less than 0.05%. This justifies the selected convergence limit in Equation (50).

Quasi-steady state can be determined using the periodicity of the energy exchanger or by conservation of energy and moisture across the exchanger. Bahnke and Howard (1964) found that using an energy and mass balance to determine quasi-steady conditions is more stringent than the periodicity conditions at typical operating conditions. Based on energy and mass balances, quasi-steady state is defined as the time when all the energy and moisture that is lost by one air stream is taken up by the other air stream. This means that the cyclic energy or moisture storage in the matrix over one revolution of the energy wheel is zero (or, negligible). The quasi-steady conditions for energy and mass balances are defined when:

$$\left| \frac{m_h(W_{h,i} - W_{h,o}) - m_c(W_{c,o} - W_{c,i})}{m_{min}(W_{h,i} - W_{c,i})} \right| \leq 1 \times 10^{-2} \tag{51}$$

$$\left| \frac{m_h(H_{h,i} - H_{h,o}) - m_c(H_{c,o} - H_{c,i})}{m_{min}(H_{h,i} - H_{c,i})} \right| \leq 1 \times 10^{-2} \tag{52}$$

If the quasi-steady-convergence criteria is set to 1×10^{-3}, the effectiveness changes by 0.1% from that predicted with a convergence criteria of 1×10^{-2}, showing that the convergence limits in Equations (51) and (52) are acceptable. To ensure that Equations (51) and (52) give the quasi-steady solution, the periodicity of the energy exchanger is checked by calculating the effectiveness after each revolution of the wheel. Periodicity is reached when the change in effectiveness is small or negligible, i.e.,

$$\left| \frac{\partial \varepsilon}{\partial t} \right| \approx \frac{\left| \varepsilon - \varepsilon^o \right|}{p_h + p_c} \leq 1 \times 10^{-4} \tag{53}$$

where ε^o is the effectiveness of the previous wheel rotation. Again, decreasing the convergence level in Equation (53) by a factor of 10 has a negligible effect on the calculated effectiveness. Therefore, if Equations (51) to (53) are satisfied, the solution is deemed to be the quasi-steady solution.

CONCLUSIONS

In this paper, a numerical model that predicts the heat and moisture transfer during adsorption and desorption processes in a desiccant-coated, rotary energy exchanger has been developed from physical principles. All the assumptions are explained and justified. The model is one-dimensional and transient and is developed in a general manner to help determine the importance of certain assumptions on the predicted performance of rotary energy exchangers. This model of rotary energy exchangers can been used to predict the energy transfer effectiveness (sensible, latent, and total) for different wheel designs and operating conditions.

The governing energy equations used in the model are developed in a distinct way to include the fact that the energy released during moisture transfer processes can be delivered to the air, the matrix or both. This is expected to have a significant effect on the performance of rotary energy exchangers with thin desiccant coatings because the energy transfer due to phase change in rotary energy exchangers can be up to six times greater than the energy transfer due to advection through the heat exchanger for hot and humid operating conditions. The method used to estimate the fraction of phase change energy that enters the air is based on a boundary condition derived from the analytical solution to heat generation between two solids. This boundary condition needs further research.

The moisture transfer in the model is calculated using the analogy between heat and mass transfer to determine the convective mass transfer coefficient. The driving potential for moisture transfer is the difference between the water vapor density in the air and the water vapor density on the surface of the desiccant. The latter is calculated from the inverse sorption curve.

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APPENDIX A

COMPARISON OF ENERGY EQUATIONS AND INTERFACE BOUNDARY CONDITION BETWEEN AIR AND MATRIX

In this appendix, the energy equations developed in this research are compared with those given in Stiesch et al. (1995). The major difference between these sets of equations is the term η, which accounts for the distribution of phase change between the solid and fluid. A method of estimating the value of η is presented.

To start, in the energy equation of Stiesch et al. (1995)

$$\frac{\partial i_f}{\partial z} + \frac{M_f}{M_m}\frac{\partial i_f}{\partial t} + \frac{\partial I_m}{\partial \tau} = 0 \tag{A1}$$

The variables can be related to our variables using the equations:

$$z = x/L \tag{A2}$$

$$\tau = t\, m/M_m \tag{A3}$$

$$m = \rho_a A_g U \tag{A4}$$

$$M_m = \rho_{m,dry} A_m L = (\sigma_{Al}\rho_{Al} + \sigma_d\rho_{d,dry})A_m L \tag{A5}$$

$$M_f = \rho_g A_g L \tag{A6}$$

$$f = \frac{\rho_{d,dry}A_d L}{\rho_{m,dry}A_m L} = \sigma_d\frac{\rho_{d,dry}}{\rho_{m,dry}} \tag{A7}$$

$$h_{fg} = h_{fg,o} + (Cp_v - Cp_w)T \tag{A8}$$

$$i_f = Cp_a T_g + W(h_{fg} + Cp_w T_g) = Cp_a T_g + W(h_{fg,o} + Cp_v T_g) \tag{A9}$$

$$I_m = (1-f)Cp_{Al}T_m + f(Cp_{d,dry}T_m + uCp_w T_m) \tag{A10}$$

For simplicity the assumption that $h_{ad} = h_{fg}$ is made. Since $h_{fg,o}$ (the heat of vaporization at a reference temperature) is constant, these relations can be substituted into Equation (A1) giving

$$(Cp_a + WCp_v)L\frac{\partial T_g}{\partial x} + \frac{M_f}{m}(Cp_a + WCp_v)\frac{\partial T_g}{\partial t} + \left(L\frac{\partial W}{\partial x} + \frac{M_f}{m}\frac{\partial W}{\partial t}\right)(h_{fg,o} + Cp_v T_g)$$

$$+ \frac{\rho_{m,dry}A_m L}{m}\left\{\left[\left(1 - \sigma_d\frac{\rho_{d,dry}}{\rho_{m,dry}}\right)Cp_{Al} + \sigma_d\frac{\rho_{d,dry}}{\rho_{m,dry}}(Cp_{d,dry} + uCp_w)\right]\frac{\partial T_m}{\partial t}\right. \tag{A11}$$

$$\left. + \sigma_d\frac{\rho_{d,dry}}{\rho_{m,dry}}Cp_w T_m\frac{\partial u}{\partial t}\right\} = 0$$

It can be shown with the mass conservation equation of Stiesch et al. (1995) that

$$L\frac{\partial W}{\partial x} + \frac{M_f}{m}\frac{\partial W}{\partial t} = \frac{-\partial W_m}{\partial \tau} = -\text{NTU}_w(W_f - W_m) = -m' \tag{A12}$$

Inserting Equation (A12) into Equation (A11) and multiplying by m/L gives

$$A_g U(\rho_a Cp_a + \rho_v Cp_v)\frac{\partial T_g}{\partial x} + A_g\frac{\rho_g}{\rho_a}(\rho_a Cp_a + \rho_v Cp_v)\frac{\partial T_g}{\partial t} - m'(h_{fg,o} + Cp_v T_g)$$

$$+ \rho_{m,dry}A_m\left\{\left[\left(1 - \sigma_d\frac{\rho_{d,dry}}{\rho_{m,dry}}\right)Cp_{Al} + \sigma_d\frac{\rho_{d,dry}}{\rho_{m,dry}}\left(Cp_{d,dry} + uCp_w\right)\right]\frac{\partial T_m}{\partial t}\right. \tag{A13}$$

$$\left. + \sigma_d\frac{\rho_{d,dry}}{\rho_{m,dry}}Cp_w T_m\frac{m'}{\rho_{d,dry}A_d}\right\} = 0$$

where Equation (7) (i.e., $m' = \rho_{d,dry}A_d\partial u/\partial t$) is used. Using the property relations (i.e., Equations (16) to (20), Equation (A13) can be rearranged to give

$$A_g U\rho_g Cp_g\frac{\partial T_g}{\partial x} + A_g\frac{\rho_g}{\rho_a}\rho_g Cp_g\frac{\partial T_g}{\partial t} - m'(h_{fg,o} - Cp_w T_g + Cp_v T_g + Cp_w T_g)$$

$$+ A_m[(\rho_{m,dry} - \sigma_d\rho_{d,dry})Cp_{Al} + \sigma_d\rho_d Cp_d]\frac{\partial T_m}{\partial t} + Cp_w T_m\dot{m}' = 0 \tag{A14}$$

where $A_d/A_m = \sigma_d$. Because $h_{fg} = h_{fg,o} + (Cp_v - Cp_w)T$ and $\rho_{m,dry} = \sigma_{Al}\rho_{Al} + \sigma_d\rho_{d,dry}$, Equation (A14) becomes

$$A_g U\rho_g Cp_g\frac{\partial T_g}{\partial x} + A_g\frac{\rho_g}{\rho_a}\rho_g Cp_g\frac{\partial T_g}{\partial t} - \dot{m}'h_{fg} - \dot{m}'Cp_w(T_g - T_m)$$

$$+ A_m(\sigma_{Al}\rho_{Al}Cp_{Al} + \sigma_d\rho_d Cp_d)\frac{\partial T_m}{\partial t} = 0 \tag{A15}$$

or

$$A_g\frac{\rho_g}{\rho_a}\rho_g Cp_g\frac{\partial T_g}{\partial t} + A_g U\rho_g Cp_g\frac{\partial T_g}{\partial x} - \dot{m}'h_{fg} - \dot{m}'Cp_w(T_g - T_m) + A_m\rho_m Cp_m\frac{\partial T_m}{\partial t} = 0 \tag{A16}$$

Adding energy Equations (1) and (2) gives

$$\rho_g Cp_g A_g\frac{\partial T_g}{\partial t} + U\rho_g Cp_g A_g\frac{\partial T_g}{\partial x} - \dot{m}'h_{fg} - \dot{m}'Cp_w(T_g - T_m) + \rho_m Cp_m A_m\frac{\partial T_m}{\partial t} = 0 \tag{A17}$$

where axial conduction in the matrix has been neglected and it is assumed that $h_{ad} = h_{fg}$. Equations (A17) and (A16) are nearly equal. The only difference between Equations (A17) and (A1) is that the energy storage term in the air (which is a small term) is modified by the ratio ρ_g/ρ_a, which is typically 0.98 to 1.0. If M_f, as defined by Stiesch et al. (1995), represents the mass of dry air rather than the mass of the dry air-water vapor mixture, then the ratio ρ_g/ρ_a would not appear in Equation (A16). In this case, Equations (A17) and (A1) would be identical. This shows that

energy Equations (1) and (2) are consistent with those in the literature and that volume averaged specific heats can be used to account for the moisture content in the air and matrix.

A comparison between the energy equation in the matrix [i.e., Equation (2)] and the second energy equation presented by Stiesch et al. (1995) shows that they are identical except for the term η. η is used in the energy equations because the bulk mean properties are used rather than the local two or three dimensional fields. The energy released by phase change at the interface between the air and matrix cannot be assumed to be completely transferred to the air or the matrix. At this interface, some of this energy will be conducted into the matrix and some will be convected into the air. The main question is: what fraction of the phase change energy is transferred directly to the air or directly to the matrix? This may be similar to the problem of two solids in contact with a uniform heat source along the contact plane, as shown in Figure A1.
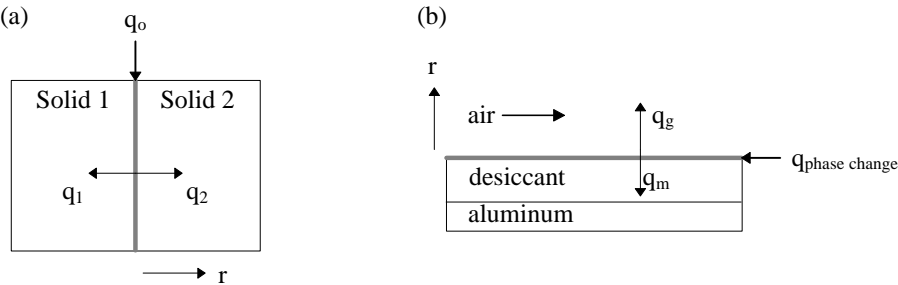

Figure A1. Schematic representation of heat generation at surface between (a) two solids and (b) solid and moving fluid

The analytical solutions for temperatures in two infinite solids with initial temperature of zero and a constant heat flux (due to a heat source initiated at time $t = 0$) at the interface are (Carslaw and Jaeger 1959),

$$T_1 = \frac{2q_o\sqrt{\alpha_1\alpha_2 t}}{k_1\sqrt{\alpha_1} + k_2\sqrt{\alpha_2}} ierfc\left(\frac{r}{2\sqrt{\alpha_1 t}}\right) \tag{A18}$$

$$T_2 = \frac{2q_o\sqrt{\alpha_1\alpha_2 t}}{k_1\sqrt{\alpha_1} + k_2\sqrt{\alpha_2}} ierfc\left(\frac{r}{2\sqrt{\alpha_2 t}}\right) \tag{A19}$$

where

$$ierfc(r) = \int_r^\infty erfc(\xi)\, d\xi = \frac{1}{\sqrt{\pi}}e^{-r^2} - r\, erfc(r) \tag{A20}$$

$$erfc(r) = \frac{2}{\sqrt{\pi}}\int_r^\infty e^{-\xi^2}\, d\xi \tag{A21}$$

From the temperature profile, the fraction of the heat source that is transferred to the air can be determined as

$$\eta \ = \ \frac{q_g}{q_{\text{phase change}}} \ = \ \frac{-k_g \left.\dfrac{\partial T_g}{\partial r}\right|_{r=0}}{q_{\text{phase change}}} \ = \ \frac{k_g / \sqrt{\alpha_g}}{k_g / \sqrt{\alpha_g} + k_d / \sqrt{\alpha_d}} \tag{A22}$$

where, for convection, k_g and α_g will be defined as

$$k_g \ = \ hD_h \ \text{ and } \ \alpha_g \ = \ \frac{k_g}{\rho_g Cp_g} \tag{A23}$$

For steady-state heat transfer, η could be calculated as,

$$\eta \ = \ \frac{\text{Conductance air}}{\text{Conductance air + Conductance matrix}} \ = \ \frac{h}{h + k_d / t_d} \tag{A24}$$

where t_d is the thickness of the desiccant. Using Equation (A22), $\eta = 0.073$ and using Equation (A24), $\eta = 0.035$. Equation (A22) is more appropriate because Equation (A24) does not include thermal storage effects, and because air flows cyclically from hot moist air to cold dry air through the wheel it subjects both the air and the matrix to step-change transient heat sources. If the properties of the entire matrix (i.e. aluminum and desiccant) are used to calculate (instead of the desiccant properties, the value of η will decrease to 3%.

A non-smooth or rough interface between the desiccant and air may, for molecular sieve coatings, tend to decrease the value of η from the 7% calculated with Equation (A22) because some of the water vapor will be deposited in the matrix depressions that are less exposed to air flow. However, the majority of the phase change is expected to occur in the most exposed parts of the desiccant because moisture is desorbed and adsorbed cyclically during energy wheel operation. In addition, the relative humidity of the air is seldom less than 10% during energy exchange applications which means that the water molecules deep within the desiccant will not likely by desorbed during normal operation. This means that η is likely to be more important for energy wheels with thin desiccant coatings than for desiccant dryers that have thicker coatings and use lower relative humidities for regeneration. The scanning electron microscope photographs in Ruthven (1984) show that molecular sieve coatings have some roughness, but no fin-like structures. Fin-like structures may have the opposite effect if most of the water molecules are adsorbed on the fins, causing η to increase beyond 0.07. Considering the above discussion, a value of $\eta = 0.05$ was chosen for the molecular sieve coating in this research. Further research on this boundary condition will be necessary to further quantify the correct value of η. It should be noted that when $\eta = 1$, all the energy of phase change is convected into the air, and when $\eta = 0$, all the energy of phase change is conducted into the matrix. For $0 < \eta < 1$ some of the energy of phase change is convected into the air and the rest is conducted into the matrix. The inclusion of η as a parameter in the model allows the model to be general for any physically possible interface flux conditions and matrix properties.

The above continuum processes need a molecular explanation. The desiccant coating on the metal film of an energy wheel is a porous media into which both dry air and water vapor molecules can penetrate and diffuse energy. Water molecules deposited within and on the porous structure lose their kinetic energy, releasing the heat of sorption but interacting with both the desiccant structure surface and the adjacent air molecules. This complex physical process

within the desiccant coating needs more research to obtain a more accurate estimate of η. This research would probably involve microscopic experimental and numerical studies.

The general approach to considering the energy release at the interface as being delivered partly to the air and desiccant is more general than the assumptions currently in the literature on desiccant drying or energy transfer wheels. This theory has a physical basis and can be used to explain why there is a range of assumptions concerning this issue in the scientific literature. The various assumptions are:

- Condensation theory: The energy of phase change is delivered to the condensate (i.e., $\eta = 0$)
- Solidification theory: Energy of phase change is delivered to both the solid and liquid as calculated by an energy balance at the interface (i.e., η is variable)
- Chemical reaction at an interface theory: Similar to solidification theory
- Phase change in porous media: Energy of phase change is divided between the different components such that local thermal equilibrium is maintained

In the case of film condensation of water vapor, Equation (A22) predicts a very small value of η (less than 0.01) because the thermal conductivity and density of water vapor are orders of magnitude smaller than the thermal conductivity and density of liquid water. Therefore, this theory can be used to explain why the assumption of $\eta = 0$ is acceptable for most condensation problems. This does not mean that $\eta = 0$ can be applied to all simultaneous heat and mass transfer problems. If transient dropwise condensation occurs on a rough surface with liquid droplets forming on top of the roughness elements, the droplets will interact and transfer heat with the air as well as the rough solid surface, in which case $\eta \neq 0$.

In solidification theory, the energy released during solidification cannot be simply assumed to be delivered to the solid (or liquid melt). The amount that is delivered to the solid or liquid must be calculated with heat conduction theory which parallels the ideas that are presented in this paper. Similar statements are applicable for a chemical reaction at an interface.

In the case of porous media, which uses the assumption of local thermal equilibrium, the fraction of the energy of phase change that is delivered to the fluid is such that local thermal equilibrium is maintained. The theory presented here for transient, one-dimensional heat and mass transfer is more general than this and, in fact, could be used to modify porous media theory when there is significant phase change. Furthermore, at the interface between a porous medium and a homogenous fluid, an excess heat exchange term or jump condition, that determines how heat flux is distributed between different phases, is necessary (Ochoa-Tapia and Whitaker 1997). It is important to note that when the local 2 or 3 dimensional temperature and moisture concentrations are solved on both sides of the interface, there is no need to include the parameter η because it will be included intrinsically in the energy balance at the interface. However, when the bulk mean properties are used for transient fluid flow with heat and mass transfer in a duct or tube, it is of fundamental importance to accurately define how the energy source terms are distributed between the fluid and solid.

In conclusion, the idea of energy being delivered to both mediums at the interface during phase change is not entirely new; indeed, it is essential for many similar areas of study as noted above. The authors have raised this issue because this needs to be resolved in the literature, even though there is the need for further research. The correct value of η and its importance in simultaneous heat and mass transfer problems may be developed and exploited by future research.

APPENDIX B

TEMPERATURE DEPENDENCE OF SORPTION ISOTHERM

The sorption isotherm can be described by the following relation,

$$\frac{u}{Wm} = \frac{bP_v}{1 + bP_v} = \frac{C\phi}{1 + C\phi} \tag{B1}$$

where the constants b and C are related by

$$C = bP_{sat} \tag{B2}$$

The constant Wm describes the number of surface sites available for sorption and is independent of temperature. The temperature dependence of constant b is governed by the vant Hoff equation [when the heat of sorption is constant and equal to the heat of vaporization (Ruthven 1984)] which is,

$$b(T) = b_{ref}\, e^{\frac{h_{fg}}{R_v T}} \tag{B3}$$

where b_{ref} is the value of the constant b at some reference temperature. Similarly the temperature dependence of the constant C is,

$$C(T) = C_{ref}\, e^{\frac{h_{fg}}{R_v T}} P_{sat}(T) \tag{B4}$$

where, from the Clapeyron equation,

$$P_{sat}(T) = P_{ref}\, e^{\frac{-h_{fg}}{R_v T_{ref}}}\, e^{\frac{-h_{fg}}{R_v T}} = \text{constant} \left(e^{\frac{-h_{fg}}{R_v T}} \right) \tag{B5}$$

Substituting Equation (B5) into (B4) gives the result that the constant C is independent of temperature when the heat of sorption is constant and equal to the heat of vaporization.

The saturation pressure equation used in this paper is an empirical equation which should be nearly the same as the Clapeyron equation. Therefore, it is expected that the constant C will be nearly temperature independent when the empirical Equation (14) is used to calculated $P_{sat}(T)$. This is in agreement with the results of the Heat of Sorption and Sorption Isotherm section of part II of this report.

When the heat of sorption is a function of the moisture content of the desiccant, the Clausius-Clapeyron equation can be used to determine the temperature dependence of the constant C. The Clausius-Clapeyron equation in the differential form is

$$h_{ad} = R_v T^2 \frac{d(\ln P_v)}{dT}\bigg|_u \tag{B6}$$

Integrating Equation (B6) from T_1 to T_2 with the temperature dependence of h_{ad} defined as

$$h_{ad} = h^*(a - mT) \tag{B7}$$

gives the following,

$$\frac{P_{v,2}}{P_{v,1}} = e^{\frac{-h^*a}{R_v}\left(\frac{T_1 - T_2}{T_1 T_2}\right)}\left(\frac{T_2}{T_1}\right)^{\frac{-h^*m}{R_v}} \tag{B8}$$

Substituting the inverse of Equation (B1) into Equation (B8) and arranging allows the calculation of constant C at temperature T_2 knowing the value of C at temperature T_1. The relation is,

$$C_{@T_2} = C_{@T_1}\frac{P_{sat,2}}{P_{sat,1}}\left(\frac{T_2}{T_1}\right)^{\frac{h^*m}{R_v}} e^{\frac{h^*a}{R_v}\left(\frac{T_1 - T_2}{T_1 T_2}\right)} \tag{B9}$$

where temperatures are in units of K.

APPENDIX C

DISCRETIZED GOVERNING EQUATIONS

The discretization of the governing equations is performed based on the control volume shown in Figure C1.
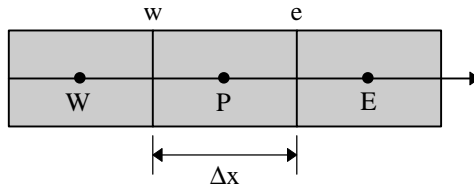

Figure C1. Control volume used to discretize governing equations

The discretized energy equation for air on the hot side $[U(+)]$ is:

$$a_p T_{g,P} = a_W T_{g,W} + a_m T_{m,P} + b \tag{C1}$$

where

$$a_P = a_p^o + a_W + a_m \tag{C2}$$

$$a_p^o = \frac{A_g(\rho_g C p_g)_P \Delta x}{\Delta t} \tag{C3}$$

$$a_W = A_g(U\rho_g C p_g)_w \tag{C4}$$

$$a_m = (hp)_P \Delta x \tag{C5}$$

$$b = a_P^o T_{g,P}^o + \eta(m' h_{ad})_P \Delta x \tag{C6}$$

The discretized energy equation for air on the cold side $[U(-)]$ is the same as on the hot side $[U(+)]$ except that the west node (subscript W) is replaced with the east node (subscript E), the east face (subscript e) and west face (subscript w) are interchanged and U retains its positive sign.

The discretized energy equation for the matrix is

$$a_P T_{m,P} = a_E T_{m,E} + a_W T_{m,W} + a_g T_{g,P} + b \tag{C7}$$

where:

$$a_P = a_p^o + a_E + a_W + a_g \tag{C8}$$

$$a_P^o = \frac{A_m (\rho_m Cp_m)_P \Delta x}{\Delta t} \tag{C9}$$

$$a_E = a_W = \frac{k_{Al} A_{Al}}{\Delta x} \tag{C10}$$

$$a_g = (hp)_P \Delta x + m'_P Cp_w \Delta x \tag{C11}$$

$$b = a_P^o T_{m,P}^o + (1 - \eta)(m' h_{ad})_P \Delta x \tag{C12}$$

The discretized conservation of mass for dry air (when U is positive) is:

$$a_e U_e = a_w U_w + b \tag{C13}$$

where:

$$a_e = \rho_{a,e} \tag{C14}$$

$$a_w = \rho_{a,w} \tag{C15}$$

$$b = \frac{(\rho_a^o - \rho_a)_P \Delta x}{\Delta t} \tag{C16}$$

When U is negative the equation is the same except the east and west faces are interchanged and $U = -U$ (i.e. U is positive).

The discretized conservation of mass for water vapor (when U is positive) is:

$$a_P \rho_{v,P} = a_W \rho_{v,W} + b \tag{C17}$$

where:

$$a_P = a_P^o + A_g U_e + (h_m p)_P \Delta x \tag{C18}$$

$$a_P^o = \frac{A_g \Delta x}{\Delta t} \tag{C19}$$

$$a_W = A_g U_w \tag{C20}$$

$$b = a_P^o \rho_{v,P}^o + (h_m p \rho_{v,m})_P \Delta x \tag{C21}$$

When U is negative the equation is the same except the east node replaces the west node, the east and west faces are interchanged, and $U(+)$ is used (i.e. $U = -U$).

The discretized conservation of water vapor in the matrix is:

$$a_P u_P = b \tag{C22}$$

where:

$$a_P = a_p^o \tag{C23}$$

$$a_P^o = \frac{\rho_{d,dry} A_d}{\Delta t} \tag{C24}$$

$$b = a_P^o u_P^o + m_P' \tag{C25}$$

NOMENCLATURE PARTS I AND 2

Symbols

A	cross-sectional area, m^2	m	constant
A_s	heat transfer surface area, m^2	n	integer
a	coefficient in the discretization equation	NTU	number of transfer units
b	constant term in the discretization equation	Nu	Nusselt number
C	constant	P	pressure, Pa
Cp	specific heat, J/(kg·K)	p	perimeter of each tube, m
Cr*	dimensionless matrix capacity	p_c	cold gas flow period, s
D_h	hydraulic diameter of one tube in the energy exchanger	p_h	hot gas flow period, s
F	function used in saturation pressure equation	R	specific gas constant, J/(kg·K)
f	general function	r	radial coordinate, m
f^{-1}	inverse of function f	T	bulk temperature, K
H	total enthalpy per mass of dry air, J/kg	t	time, s
h	convective heat transfer coefficient, W/(m^2·K)	U	mean airflow velocity in tube, m/s
h_m	convective mass transfer coefficient, m/s	u	mass fraction of water in desiccant, kg (water)/kg (desiccant)
h_{fg}	heat of vaporization, J/kg	W	humidity ratio, kg (water vapor)/kg (dry air)
h_{ad}	heat of sorption (adsorption and desorption), J/kg	Wm	empirical coefficient used in sorption isotherm describing maximum moisture capacity of desiccant, kg (water)/kg (desiccant)
h^*	ratio of the heat of sorption to the heat of vaporization	x	axial coordinate, m
j	iteration number	α	thermal diffusivity, m^2/s
k	thermal conductivity, W/(m·K)	χ	general variable or property
L	length of the heat exchanger, m	ε_s	sensible heat transfer effectiveness
M	total mass of the heat exchanger, kg	ε_l	latent heat transfer (or moisture transfer) effectiveness
$\dot{m}$	mass flow rate of dry air, kg/s, or $\dot{m}Cp$ when $\chi = T$ in Equations (32) and (33)	ε_t	total energy (enthalpy) effectiveness
$\dot{m}'$	rate of phase change per unit length, kg/(s·m)	ϕ	relative humidity or relative partial pressure of water vapor with respect to saturated air

η fraction of the phase change energy that enters the air

λ coefficient that describes how, as a function of moisture content (u) the heat of sorption approaches the heat of vaporization, see Equation (13)

θ rotational angle, °

ρ density, kg/m^3

σ volume fraction

ω rotational speed, cycles/s

ζ dummy variable

Subscripts

Al   aluminum

a   air

ave   average

c   cold side

c,i   average cold air stream inlet conditions

c,o   average cold air stream outlet conditions

d   desiccant

g   total gas phase (air and water vapor)

h   hot side

h,i   average hot air stream inlet conditions

h,o   average hot air stream outlet conditions

m   matrix (including aluminum, desiccant and moisture)

max   maximum

min   minimum

sat   saturation properties

v   water vapor

w   liquid water

Superscripts

o   old value (value of a variable at the previous time step or the previous cycle)