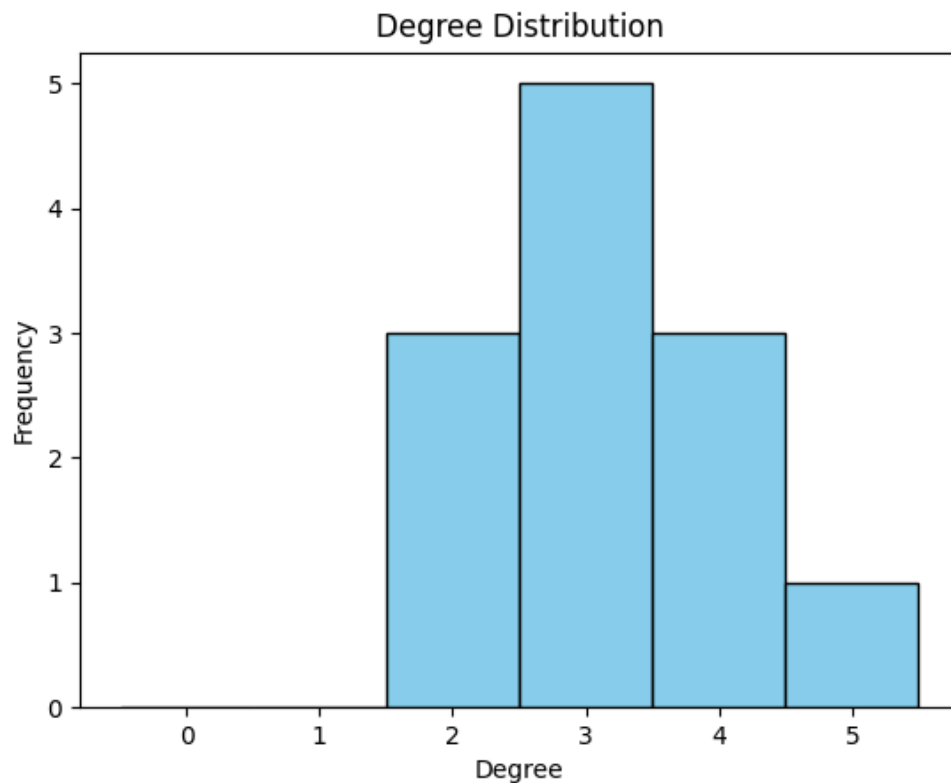


GE2324 Assignment1

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Q1:

- The network contains **12** vertices
- The network contains **19** edges
- The degree distribution:



Q2:

- Degrees of nodes: [3, 4, 5, 3, 3, 4, 3, 2, 2, 3, 4, 2]
- Maximum degree (D_{\max}): 5
- Number of nodes (N): 12
- Centralization numerator ($\sum(D_{\max} - D_i)$): 22
- Freeman's Degree Centralization (C_D) = $22 / (11 * 10) = 0.2$

Q3:

- B:
 - A -> F: 0.5, A -> E: 0.5, C -> D: 0.5, C -> E: 0.5, D -> F: 0.5
 - From D to [G, H, I, J, K, L]: 0.5
 - From E to [G, H, I, J, K, L]: 0.5
 - The answer is $(0.5 * 17) / (11 * 10 / 2) = 0.1545$
- D:

- From A to [H, I, J, K, L]: 1
- From B to [H, I, J, K, L]: 1
- From C to [H, I, J, K, L]: 1
- From D to [H, I, J, K, L]: 1
- From E to [H, I, J, K, L]: 1
- From F to [H, I, J, K, L]: 1
- $H \rightarrow I, H \rightarrow K, I \rightarrow K: 0.5$
- The answer is $(6 * 5 + 0.5 * 3) / (11 * 10 / 2) = \mathbf{0.5727}$

Q4:

- B:
 - Shortest Distances: [1, 0, 1, 1, 1, 1, 2, 3, 3, 4, 3, 4]
 - $11/24 = \mathbf{0.4583}$
- D:
 - Shortest Distances: [2, 2, 1, 3, 3, 2, 0, 1, 1, 2, 1, 2]
 - $11/20 = \mathbf{0.55}$

Q5:

- (i) Betweenness of G:
 - Change: **No change**
 - Reason: Adding an edge between F and G does not reduce the number of shortest paths passing through G. G remains a critical bridge connecting the two main subgraphs (F's subgraph and G's subgraph), as the new edge F-G does not bypass G for most paths between these subgraphs.
- (ii) Closeness of G:
 - Change: **Larger**
 - Reason: Closeness centrality improves when a node's average distance to others decreases. The new edge F-G shortens G's distance to nodes in F's subgraph (e.g., A, B, C, D, E), reducing G's overall average distance and increasing its closeness.
- (iii) Freeman's Centralization Value
 - Change: **Smaller**
 - Reason: Freeman's centralization measures how unevenly centrality is distributed. Adding F-G makes the network more connected and decentralized. Nodes like B and G lose some dominance as the network becomes more balanced, lowering the centralization value.

Q6:

- (i) Betweenness of G:
 - Change: **Smaller**
 - Reason: Before adding the edge, the shortest path between H and I was H-G-I. After adding H-I, the shortest path becomes direct (H-I), bypassing G. This reduces the number of shortest paths passing through G, lowering its betweenness centrality.

- (ii) Closeness of G:
 - Change: **No change**
 - Reason: Closeness centrality depends on the average distance from G to all other nodes. The edge H-I does not affect G's distance to other nodes (e.g., G to H is still 1, G to I is still 1, and other paths remain unchanged). Thus, G's average distance remains the same, leaving its closeness centrality unchanged.
- (iii) Freeman's Centralization Value
 - Change: **Smaller**
 - Reason: Freeman's centralization measures how unevenly centrality is distributed. Adding H-I makes the network more connected and decentralized: G's betweenness decreases (as explained in (i)). H and I become more connected (degree increases), reducing the dominance of nodes like B or G. This leads to a more balanced centrality distribution, lowering the overall centralization value.

Q7:

- Maximal Cliques:
 - Maximal Cliques: [['K', 'L', 'J'], ['K', 'G'], ['G', 'C'], ['G', 'I'], ['G', 'H'], ['H', 'J'], ['J', 'I'], ['B', 'C', 'F'], ['B', 'C', 'A'], ['B', 'E', 'F'], ['B', 'E', 'D'], ['B', 'D', 'A']]
- For the sub - graph {A,B,C,D,E,F}
 - A: 3, B: 5, C: 3, D: 3, E: 3, F: 3. The minimum degree is 3, so **k = 3**
- For the sub - graph {G, H, I, J, K, L}
 - G: 3, H: 2, I: 2, J: 4, K: 3, L: 2. The minimum degree is 2, so **k = 2**.