Midterm Solution CA1 (amended)

Discrete Mathematics (City University of Hong Kong)

30

1. Let p = "the band could play jazz music"

(a) q = "catering delievery on time"

r = "the birthday party was cancelled"

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s = "Kimmy was upset"

t = "Refunds had to be made"

The argument above is given below

(b)

1. $(\sim p \vee \sim q) \to (r \wedge s)$

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2. $r \to t$
3. $\sim t$

$\therefore p$

(c)

4. $\sim r$ ② & ③
5. $\sim r \vee \sim s$ ④

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6. $\sim (r \wedge s)$ ⑤
7. $\sim (\sim p \vee \sim q)$ ⑥ & ①
8. $p \wedge q$ ⑦
9. p ⑧

$\therefore$ argument valid.

2. 10

$$\sum_{k=1}^{n} (-1)^k k^2 = (-1)^n \frac{n(n+1)}{2}$$

step 1. Base step. $n=1$

$$\sum_{k=1}^{1} (-1)^k k^2 = (-1)^1 \frac{(1)(1+1)}{2} = -1$$

step 2. Assume that $P(n)$ is true for $P(n+1)$

$$\sum_{k=1}^{n+1} (-1)^k \cdot k^2 = \sum_{k=1}^{n} (-1)^k \cdot k^2 + (-1)^{n+1} \cdot (n+1)^2$$

$$= (-1)^n \cdot \frac{n(n+1)}{2} + (-1)^{n+1} (n+1)^2$$

$$= (-1)^{n+1} (n+1) \left(-\frac{n}{2} + n+1\right)$$

$$= (-1)^{n+1} \frac{(n+1)(n+2)}{2}$$

$\therefore$ $P(n+1)$ also true

$\therefore$ By MI $P(n)$ is true for all n is a positive integer

3. 30

(a) $4! \times 3! \times 4! \times 3! \times 2! = 41472$

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(b) i) $4! \times 2! \times 2! \times 2! \times 2! = 384$

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ii) ${}_4P_4 \times {}_5P_3 = 2880$

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Q3 c) select y objects from x objects with two specific objects A and B (cannot occur together)

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case 1: NO A,B $\quad {}_{x-2}C_y \cdot y!$

case 2: choose A but not B $\quad ({}_{x-2}C_{y-1}) \cdot y!$

case 3: choose B but not A $\quad ({}_{x-2}C_{y-1}) \cdot y!$

Total: $({}_{x-2}C_y) \cdot y! + 2({}_{x-2}C_{y-1})\, y!$

$$= \frac{(x-2)!}{(x-1-y)!}(x+y-1)$$

4. 30

(a) 10

Marketing		Accountant
4		6
	or	
5		5
	or	
6		4

$\therefore {}_6C_4 \times {}_7C_6 + {}_6C_5 \times {}_7C_5 + {}_6C_6 \times {}_7C_4 = 266$

(b) 10

1) ${}_{52}C_{18} \times {}_{35}C_2$

2) ${}_{52}C_{19} \times {}_{35}C_1$

3) ${}_{56}C_{20}$

$\therefore {}_{52}C_{18} * {}_{35}C_2 + {}_{52}C_{19} \times {}_{35}C_1 + {}_{52}C_{20}$

(c) 10

4 cases

case 1 All vertices of the triangle are pts of divisions

i) 3 vertices of triangle must be division pt of 3 consecutive sides clockwise of the square
${}_{12}C_1 \times {}_3C_1 \times {}_9C_1$ such triangles

ii) 2 vertices of the triangle are at the same side of the square. $4 \times {}_3C_2 \times {}_9C_1$

case 2 Exactly one vertex of the triangle is a corner of the square. There are
${}_4C_1 \times {}_{12}C_2 - 2 \times {}_3C_2$ such triangles

Case 3 Exactly 2 vertices of the triangle are corners of the square.

i) The 2 corners are consecutive and there are ${}_4C_1 \times {}_9C_1$ such triangles

ii) The 2 corners are not consecutive and there are $2 \times {}_{12}C_1$ such triangles

Case 4 : All 3 vertices of triangle are corners of the square and there are ${}_4C_3$ such triangles

$\therefore$ total

$${}_{12}C_1 \times {}_3C_1 \times {}_3C_1 + 4 \times {}_3C_2 \times {}_9C_1 + {}_4C_1 \times {}_{12}C_2 - 2 \times {}_3C_2 + {}_4C_1 \times {}_9C_1 + 2 \times {}_{12}C_1 + {}_4C_3$$ triangles