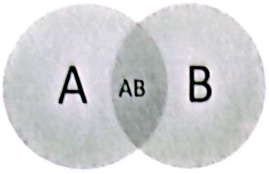
CS2402 - Tutorial 2

Task 1 (Independence)

A) Events A and B are independent of each other. P(A) = 0.1 and P(B) = 0.3. What is the probabilities of

1. Both A and B occur. $P(AB) = 0.1 \times 0.3 = 0.03$
2. A occur and B doesn't occur. $P(A) - P(AB) = 0.1 - 0.03 = 0.07$
3. At least one of events occurs. $P(A) + P(B) - P(AB) = 0.4 - 0.03 = 0.37$
4. Neither of the events occur. $1 - 0.37 = 0.63$
5. Exactly one of the events occurs. $P = 0.07 + 0.27 = 0.34$.

$P(A\bar{B}) = P(A \cup \bar{B}) = 1 - (P(A) + P(\bar{B}) - P(A\bar{B}))$



$P(A\bar{B} \cup \bar{A}B) = P(\bar{A}\bar{B}) + P(\bar{A}B) = P(\bar{A}) - P(A\bar{B}) + P(B) - P(AB)$

B) When a fair coin is tossed twice, let I be the event "heads on the first toss" and J the event "two heads turn up." Check if I and J are independent events.

$P(I) = \frac{1}{2}$ $P(J) = \frac{1}{4}$ $P(IJ) = \frac{1}{4} \neq P(I) \cdot P(J)$ so they are not independent.

C) Suppose that we toss 2 dice. Let E be the event that the sum of the dice is not larger than 6, and F be the event that this first dice is not larger than 3. Check if E and F are independent.

$P(E) = \frac{15}{36}$ $P(F) = \frac{18}{36}$

$P(EF) = \frac{11}{36} \neq P(E) \times P(F)$ so not independent.

Task 2 (Random Variable)

$P(X=r) = C_n^r p^r (1-p)^{n-r}$

Binomial random variable (introduced in L04). For 10 independent trials, with probability 0.4 of success and probability 0.6 of failure on each trial.

$P(X=2) = C_{10}^2 0.4^2 \times 0.6^8$ $(n=10, r=2)$ $(p=0.4, 1-p=0.6)$

1) What is the probability of exactly 2 successes? $P = C_{10}^2 (0.4)^2 (0.6)^8 = 0.121$
2) What is the probability of exactly 8 successes? $P = C_{10}^8 (0.4)^8 (0.6)^2 = 0.011$
3) What is the probability that the number of successes is no more than 2?

$P = C_{10}^0 (0.4)^0 (0.6)^{10} + C_{10}^1 (0.4)^1 (0.6)^9 + C_{10}^2 (0.4)^2 (0.6)^8$
$= 0.006 + 0.04 + 0.121 = 0.167$

Task 3 (Joint Distribution)

A fair coin is tossed 3 times independently, Let X = the number of heads on the first two tosses, and Y = the number of heads on the last two tosses.

1) Make a table showing the joint distribution of X and Y.
2) Compute P(X+Y=3) and P(X-Y=1).

1).

cont	0	1	2
PX	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
PY	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

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2. $P(X+Y=3) = P(X=1)P(Y=2) + P(X=2)P(Y=1)$
$= \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} = \frac{1}{4}$

$P(X-Y=1) = P(X=1)P(Y=0) + P(X=2)P(Y=1)$
$= \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} = \frac{1}{4}$

Y \ X	0	1	2
0	TTT $\frac{1}{8}$	HTT $\frac{1}{8}$	X
1	TTH $\frac{1}{8}$	HTH, THT $\frac{1}{4}$	HHT $\frac{1}{8}$
2	X	THH $\frac{1}{8}$	HHH $\frac{1}{8}$

v2

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