CS2402 - Tutorial 3

Task 1. Let X be the number of heads in three independent tosses of a fair coin.

a) Display the distribution of X in a table,

b) Compute E(|X-1|).

X	0	1	2	3
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$\lvert X-1\rvert$	0	1	2
p	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

Hint: X is a binomial variable? n=? p=?

$E(|X-1|) = \frac{3}{4}$

Let f be a function, f(X) is also a random variable. for example, X^2, $3X+5$, $|X|$

$E[f(X)]=\sum_{all\ x} f(x) * P(X=x)$

Task 2. Let X and Y be the numbers obtained on two independent rolls of fair die. Let Z=max(X,Y).

a) Find the distribution of Z.

b) Find E(Z)

Z	1	2	3	4	5	6
p	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$E(Z) = \frac{161}{36}$

Hint: What values Z can take? Use independence. Hope you can find a simple way to compute P(Z=a value)

Task 3. Suppose all the numbers of 200 numbers are non-negative, and the average of the list is 2. Show that at most 50 numbers in the list are not smaller than 8. $400 = 0\cdot k + (200-k)\cdot 8$

then k is maxium.

Hint: $P(X \geq a) \leq \frac{E(X)}{a}$ $\qquad P \leq \frac{2}{8}$

$\therefore\ n \leq \frac{1}{4}\times 200 \qquad n \leq 50$

Task 4. Let X_1, X_2 and X_3 are three random independent numbers drawn from {0, 1, 2, …, 9}. Let $Y = min\{X_1, X_2, X_3\}$. Find E(Y).

$1+2+\cdots+n = \frac{1}{2}n(n+1)$ $\qquad 1^3+2^3+\cdots+n^3 = \frac{1}{4}n^2(n+1)^2$

Hint: $E(Y) = \sum_{i=1}^{n} P(Y \geq i)$. You need to compute P(Y>=i) first.

$E(Y) = P(Y\geq 1) + P(Y\geq 2) + \cdots + P(Y\geq 9) = (\frac{9}{10})^3 + (\frac{8}{10})^3 + (\frac{7}{10})^3 + \cdots + (\frac{1}{10})^3 = \frac{10^2\times 9^2\times\frac{1}{4}}{10^3} = \frac{81}{40} = 2.025$

Task 5: Suppose your utility function is $u(w) = \ln(w+100)$, where w is the wealth, and you are offered a bet on flipping a coin:

Heads, you win \$500. Tails, you lose \$300.

Please compute your expected utility change when your wealthy is

(a) w=500;

(b) w=1000;

(a) $\frac{1}{2}(\ln(1100)-\ln(600)) + \frac{1}{2}(\ln(300)-\ln(600)) = \frac{1}{2}\ln\frac{11}{6} - \frac{1}{2}\ln 2 = \frac{1}{2}\ln\frac{11}{12}$

(b). $\frac{1}{2}(\ln1600 - \ln1100) + \frac{1}{2}(\ln800 - \ln1100) = \frac{1}{2}\ln\frac{16}{11} - \frac{1}{2}\ln\frac{11}{8} = \frac{1}{2}\ln\frac{128}{121}$

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