Final 2021-questions

Introduction to Computational Probability Modeling (City University of Hong Kong)

CITY UNIVERSITY OF HONG KONG

Course code & title : CS2402
Introduction to Computational Probability Modeling

Session : Semester B 2020/21

Time allowed : Two hours This paper has 3 pages (including this cover page). 1. This paper consists of 10 questions.

2. Answer ALL questions. *This is a **closed-book** examination.*
This question paper should NOT be taken away.

Students are allowed to use the following materials/aids:
1. *Approved Calculator*
2. *An A4 paper note, two sides*

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

Academic Honesty

I pledge that the answers in this examination are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,
1) *I will not plagiarize (copy without citation) from any source;*
2) *I will not communicate or attempt to communicate with any other person during the examination; neither will I give or attempt to give assistance to another student taking the examination; and*
3) *I will use only approved devices (e.g., calculators) and/or approved device models.*
4) *I understand that any act of academic dishonesty can lead to disciplinary action.*

I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties.

Student ID:
Name:

Question	1	2	3	4	5	6	7	8	9	10
Marks										
	10	10	10	10	10	10	10	10	10	10

Answer ALL the questions below.

Question 1 (10pts)

A box contains 6 cards numbered 1, 2, 3, 4, 5, and 6. Three cards are drawn from the box **without replacement**. Find the probability that the sum of the numbers on the three cards is equal to or less than 8.

Question 2 (10pts)

A fair coin is tossed 4 times independently. Let X=the number of heads on the first two tosses, and Y=the number of tails on the last two tosses.

1) List a table showing the joint distribution of X and Y.
2) Compute P(X+Y=2) and P(X=Y).

Question 3 (10pts)

Suppose 0.5% of the population is infected with the COVID-19 virus, and an RT-PCR test is developed for COVID-19. The RT-PCR test gives 3% false positives (giving a positive result for a person not infected with COVID-19) and 2% false negatives (giving a negative result for a person infected with COVID-19).

1) What is the probability that Alice (a random person) tests positive?
2) Alice just got the bad news that the test came back positive; what is the probability that Alice has the COVID-19?

Question 4: (10pts)

The values of x and their corresponding values of y are shown in the table below

x	2	3	4	5
y	2.5	3	4	4.5

1) Find the least square regression line $y = a \cdot x + b$.
2) Estimate the value of y when $x = 10$ using the derived parameters in 1).

Question 5 (10pts)

An insurance company is considering the insurance of workers in a new car factory. This factory has only been set up for one year and had six accidents in the past one year. That is, the accident rate is $A = 6$ accidents per year. The insurance company also has accident rates from other 19 car factories (i.e. K=19), with the average accident rate in the past one year being $\bar{A} = 3$ accidents per year. Find the adjusted accident rate $\hat{A}$ for this new car factory.

Question 6: (10pts)

Suppose the average income of the families in a region is $15,000.

1) Find an upper bound (as low as possible) for the percentage of families with income over $40,000 in the region.
2) If the percentage of families with income over $40,000 is no more than 10%. What is the largest standard deviation possible?

Question 7: (10pts)

Suppose we are selling a box of 20 products. The probabilities of the number of defective products being 0, 1, and 2 are 0.8, 0.1, and 0.1, respectively. The probability is 0 for having more than two defective products in the box. A customer randomly selects four products from the box for inspection. If none of the 4 products is defective, he will buy the whole box. Calculate the probability that the customer will buy the whole box.

Question 8 (10pts)

An urn contains 8 white, 4 black, and 2 orange balls. Suppose that we win 2 points for each **black** ball selected and we lose 1 point for each **white** ball selected. Selecting an **orange** ball is 0 points. Let X denote our points after drawing two balls **without placement**. Compute E(X), Var(X).

Question 9: (10pts)

A box contains 7 red pens, 5 black pens, and 3 blue pens. A pen P1 is randomly chosen from the box and put back after recording its color. Subsequently, 5 more pens of the same color are added to the box (so that there are 20 pens in the box). Now, another pen P2 is randomly chosen from the box and put back after recording its color. Subsequently, 2 more pens of the same color are added to the box. Now, another pen P3 is randomly chosen from the box. Given that the third pen (P3) is red, what is the probability that the first pen (P1) is red?

Question 10: (10pts)

Let X_1, X_2, X_3, X_4 be 4 random independent random variables following distributions with the density function:

$$f(x;x_0,\theta)=\theta x_0^{\theta}x^{-\theta-1},\ x\geq x_0,\ \theta>1$$

Assume $x_0>0$ (i.e. x_0 is a constant), find the MLE of θ.

- END -