Midterm-2020Question

Introduction to Computational Probability Modeling (City University of Hong Kong)

CITY UNIVERSITY OF HONG KONG

Course code & title	:	CS2402 Introduction to Computational Probability Modeling
Session	:	Semester B 2019/20
Time allowed	:	1 hour 50 min

This paper has 4 pages (including this cover page).

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1. This paper consists of 21 questions.
2. Answer ALL questions.

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*This is a **closed-book** examination*

Student ID: ________________________________

Student Name: ________________________________

Student EID: ________________________________

Questions	**1-12**	**13**	**14**	**15**	**16**	**17**
Marks						
Max	**48**	**6**	**5**	**5**	**5**	**5**
Questions	**18**	**19**	**20**	**21**		**Total**
Marks						
Max	**6**	**5**	**5**	**10**		**100**

For Q1-Q12, Fill in the answer in the space provided on the left. For multiple choice questions, choose the most appropriate answer.

Alice gets 75% Bob gets 25%_	**Q1** *Suppose the rule in the unfinished game is that the pot goes to the player who first gets 5 points when playing a fair coin (in each round, they flip a coin and the winner gets one point). If the game stops when Alice has 4 points and Bob has 3 points, in what ratio, the pot should be divided between Alice and Bob?*
__SD(X) =	**Q2** *What is the standard deviation of the random variable X. Its corresponding probabilities are as follows.*
P(AB) = 0.1	**Q3** *P(AUB)=0.8, P(A)=P(B)=0.45. P(AB)=?*

X	*P(x)*
1	*0.3*
2	*0.2*
3	*0.1*
4	*0.4*

Q4 *Suppose I have two hard-disks on my computer. I set the computer up so that the 2^{nd} hard-disk has an exact copy of the 1^{st} hard-disk. Assuming the failure rate of each hard-disk is 0.5%, and these two disks are independent of each other. Provide the following probabilities:*

0.0025%	• *I lose my data as both hard-disks fail.*
0.0025%	• *Only one disk fails.*
1/12	**Q5** *If you throw a dice twice, which is the probability that the sum of the two obtained numbers is smaller than 4.*
A or C	**Q6** *In 1980s, the NASA engineers estimated the odds of an accident in the first 25 Shuttle missions as 1-in-100. What is the probability of any accidents occurring in 25 missions?*

a) $(1 - 0.99)^{25}$
b) $1 - (0.99)^{25}$
c) $(0.01)^{25}$
d) $1 - (0.01)^{25}$
e) $(1 - 0.01)^{25}$

20 percent at most	**Q7 :** *There are a list of nonnegative numbers with average 100. At most how much percentage of these numbers are not smaller than 500?*
D	**Q8** *In a game of Roulette, you decide to bet $100 on number "0". If the ball lands on "0" the payout is 35 times your bet (you will win 3500). Otherwise, you will lose your bet. There are 38 numbers on the Roulette wheel, and each number has equal probability. What is your expected gain for this*

bet?

a) *$100*
b) *($3500×1/2) – ($100×1/2)*
c) *($3500×37/38) – ($100×1/38)*
d) *($3500×1/38) – ($100×37/38)*
e) *$0*

Q9 *Among families with 2 children, one of whom is a girl born at some time pm, what is the probability that the other child is a* **boy**? *(we assume that there are two equal periods: am and pm)*

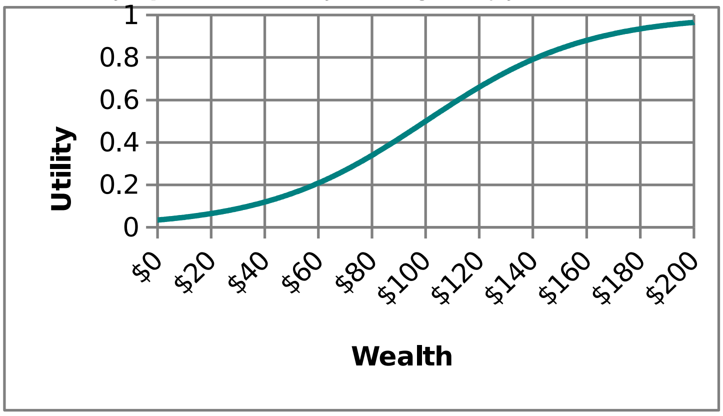
1/4

Q10 *Let X and Y be independent, each uniformly distributed on {1,2,3,4,5}. E(XY)=?*

*E(X) * E (Y) =*
*3 * 3 = 9*

Q11 *What is the behavior of a person with the following utility function?*

C



a) *Risk-averse.*
b) *Risk-seeking.*
c) *Risk-seeking up to $100 wealth, then risk-averse after $100.*
d) *Risk-averse up to $100 wealth, then risk-seeking after $100.*
e) *Risk-neutral.*

Q12 *Let A and B are two independent events with and .2. P(B)=?*

P(AB) = 0.5

<u>For Q13-Q22, provide the answers. You need to provide the problem solving process.</u>

Q13. *Suppose that four fair dices are rolled independently, compute the probability that the results are two different pairs.*

Answer : 5/36

Q14: *Suppose we roll a red dice and a green dice independently, compute the probability that the number on the red dice is larger than the number on the green dice.*

Answer: 15/36

Q15: *In a group of professors, 25% smoke cigarettes (event C), 60% drink alcohol (event A), and 15% do both, (a) what fraction of professors have at least one of these bad habits? (b) Are A and C independent? Why?*
Answer: P(AUC) = 0.7
They are independent.

Q16: *In the World Series, two teams play until one team has won four games. Suppose that the outcome of each game is determined by flipping a coin. What is the probability that the World Series will last (a) four games, (b) five games, (c) six games, (d) seven games?*

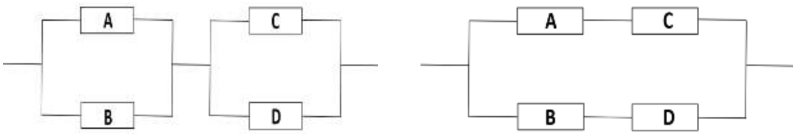
Q17: *A random variable have expectation 100 and standard deviation 2. Find a upper bound as small as possible for .*

Q18: *Roll a die twice and consider the events* ***A****={first gives at least 4},* ***B*** *= {second roll gives at most 4} and* ***C****={the sum of the roll is 10}.*
(a) Find ***P(A), P(B), P(C)*** *and* ***P(A∩B∩C)****.*
(b) Are ***B*** *are* ***C*** *independent?*

Q19: *A fair die is rolled two times independently. Let* $\mathbf{X_1}$ **and** $\mathbf{X_2}$ *are the results of each rolling. Let* $\mathbf{Y=\min\{ X_1 , X_2 \}}$. *Find* **E(Y)**.

Q20: *Bob is playing basketball for his local club. The probability that Bob scores a goal is p=3/5 for each shoot. Let X= the total number of the goals among his 60 independent shots. Find E(X), Var(X), SD(X).*

Q21: *Compute the reliability of the two systems below given each component functioning independently with probability* ***p*** *(P(A)=P(B)=P(C)=P(D)=****p****).*



(a) *(b)*

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