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CS2402 - Lecture5 - In-Class Exercise

Q1 Suppose that 10% of the numbers in a list are 10, 20% of them are 30, and the remaining numbers are 50, what is the average of the numbers in the list?

$E(x) = 10\% \times 10 + 20\% \times 30 + 70\% \times 50$

$= 1 + 6 + 35 = 42$

Q2. What is the expected (mean) number of sixes appearing in 100 dice rolls? What is the expected number of the odd numbers? What is the expected number of the odd numbers?

$E(X=6) = 100 \times P(X=6) = 16.6$

$E(X=2n+1) = 100 \times P(X=\{1,3,5\}) = 100 \times \frac{1}{2} = 50$

Q3: Suppose the average mark of a class in a test is 85. At most what percentage of the students have got marks not lower than 90?

$90 P_{max} \le 85$

$P_{max} \le 94.4\%$

Q4: Suppose all the number in the list of 100 numbers are non-negative, and the average of the list is 2, At most how many numbers are not smaller than 8?

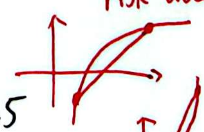
$P(X \ge a) \le \frac{E(X)}{a}$

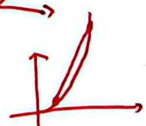
$8P \le 2$

$P \le \frac{1}{4}$

$n \le P \times 100 = 25$ $\therefore n \le 25$

risk averse





risk lover

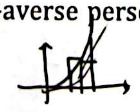
$[(w+x)^2 - w^2]\frac{1}{2}$

$+[(w-x)^2 - w^2]\frac{1}{2}$

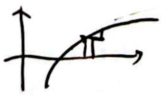
$= \frac{1}{2}(2wx + x^2 - 2wx + x^2)$

$= \frac{1}{2} \times 2x^2 = x^2$

Q5. The utility function "log(wealth)" has diminishing utility for larger amounts of wealth, which can model the behavior of a risk-averse person. If someone's utility function is "(wealth)^2", is he a risk-lover? Why?



$(x+m)^2 - x^2 = \cancel{x^2} + 2mx + m^2 - \cancel{x^2} = m^2 + 2mx$



$x^2 - (x-m)^2 = x^2 - x^2 + 2mx - m^2 = 2mx - m^2$

$m^2 + 2mx > 2mx - m^2$ so risk-lover

$p(X=1) = C_n^1 p^1 (1-p)^n$

$P(X=2) =$

$E(X) = 0 \cdot P(X=0) + 1 \cdot P(X=1) + 2 \cdot P(X=2) + \cdots + n P(X=n)$ $\boxed{E(X) = \sum x P(X)}$

$P(X) = C_n^i p^i (1-p)^i$

$E(X) = \sum C_n^i \, i \, p^i (1-p)^{n-i}$

Q6 (optional) Use the properties of expectation, prove:

$X \sim B(n, p)$

$$\sum_{i=1}^{n} C_n^i \, i \, p^i (1-p)^{n-i} = np$$

$E(X) = 1 \times C_n^1 p^1 (1-p)^{n-1} + \cdots + n \times C_n^n p^n (1-p)^1$

$E(X) = E(X=1) + \cdots + E(X=n) = E(X_1) + E(X_2) + \cdots + E(X_n)$

$\Rightarrow 1 \times C_n^1 p^1 (1-p)^{n-1} + \cdots + n C_n^n p^n (1-p)^0 = [1 \times p + 0 \times (1-p)] + \cdots + [1 \times p + 0 \times (1-p)]$

$\Rightarrow \sum_{i=1}^{n} C_n^i \, i \, p^i (1-p)^{n-i} = np$

Q7. The game of chuck-a-luck is played with three dice, rolled independently. You bet one dollar on one of the numbers 1 through 6 and if exactly k of the dice show your number, you win k dollars k = 1, 2, 3 (and keep your wagered dollar). If no die shows your number, you lose your wagered dollar. What is your expected loss?

~~-6 -5 -4 -3 -2 -1 1 2 3 4 5 6~~

x	-1	1	2	3
p	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

$P(X=1) = C_3^1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2$

$P(X=2) = C_3^2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^1$

$P(X=3) = C_3^3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0$

$E(X) = -\frac{125}{216} + \frac{91}{216} = -\frac{34}{216} = -\frac{17}{108} = -0.157$

$\therefore$ expected loss -0.157

Q8. Suppose the average family income in a particular area is \$10,000. Find an upper bound for the fraction of families in the area with incomes over \$50,000.

$50{,}000 \, P \le 10{,}000$

$P \le \frac{1}{5}$

upper band is 20%