2402 Assignment1 LIU Hengche 57854329

Q1. (a). Total characters $= 26 \times 2 + 10 = 62$

$$P(\text{at least } 1) = 1 - P(\text{none}) = 1 - \frac{36^5}{62^5} = \frac{26739583}{28629151} \approx 0.934$$

so the probability of containing at least 1 lower case is $\boxed{0.934}$

(b). $$P = 1 - \frac{A_5^5 C_{62}^5}{A_5^5 C_{72}^5} = \frac{3760371}{6995772} \approx 0.5375$$

so the probability of containing at least one special character is $\boxed{0.5375}$

Q2 (a). $$P = \frac{1}{C_{48}^6} = \frac{1}{12271512} \approx 8.1 \times 10^{-8}$$

$\therefore$ probability of winning first prize is $\boxed{8.1 \times 10^{-8}}$

(b) $$P = \frac{C_6^1 \times 42}{C_{48}^6} = \frac{21}{1022626} \approx \boxed{2.0535 \times 10^{-5}}$$

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Q3 ① 1 game : E $\frac{1}{4}$ F $\frac{1}{4}$

② 2 games : E $\frac{1}{2}\times\frac{1}{4}$ F $\frac{1}{2}\times\frac{1}{4}$ G $\frac{1}{4}\times\frac{1}{4}$

③ 3 games : E $3\times\frac{1}{4^3}$ F $3\times\frac{1}{4^3}$ G $2\times\frac{1}{4^3}$ H $\frac{1}{4^3}$

④ 4 games E $3\times\frac{1}{4^4}$ F $3\times\frac{1}{4^4}$ G $3\times\frac{1}{4^4}$ H $3\times\frac{1}{4^4}$

so in total $P(E)=\frac{1}{4}+\frac{1}{8}+\frac{3}{64}+\frac{3}{256}=\frac{111}{256}$

$P(F)=\frac{111}{256}$

$P(G)=\frac{2}{64}+\frac{1}{16}+\frac{3}{256}=\frac{27}{256}$

$P(H)=\frac{4}{256}+\frac{3}{256}=\frac{7}{256}$

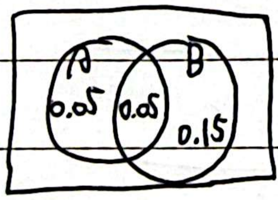
∴ Emily should get $768\times\frac{111}{256}=\boxed{333}$ dollars

Frank should get $768\times\frac{111}{256}=\boxed{333}$ dollars

Grace should get $768\times\frac{27}{256}=\boxed{91}$ dollars

Harry should get $768\times\frac{7}{256}=\boxed{21}$ dollars

Q4



(a). $P(A)+P(B)-P(AB)=\boxed{0.25}$

so P(at lease 1 defect) = 0.25

(b). $P=P(A)-P(AB)=0.05$. so $P(A \text{ not } B)=\boxed{0.05}$

(c). $P(\text{none})=1-P(A\cup B)=1-0.25=\boxed{0.75}$

(d). $P(A)+P(B)-2P(AB)=0.05+0.15=\boxed{0.2}$

∴ P(one defects) = 0.2

Q5 (a). $\frac{1}{2}(\ln 1500 - \ln 700) + \frac{1}{2}(\ln 100 - \ln 700)$

$= \frac{1}{2}\ln\frac{15}{7} + \frac{1}{2}\ln\frac{1}{7} = \frac{1}{2}\ln\frac{15}{49} = -0.592$

so expected utility change is $\boxed{-0.592}$

(b). $\frac{1}{2}(\ln 2000 - \ln 1200) + \frac{1}{2}(\ln 600 - \ln 1200)$

$= \frac{1}{2}\ln\frac{20}{12} + \frac{1}{2}\ln\frac{1}{2} = \frac{1}{2}\ln\frac{5}{6} = -0.0912$

So expected utility change is $\boxed{-0.0912}$

(c). $\frac{1}{2}(\ln 6000 - \ln 5200) + \frac{1}{2}(\ln 4600 - \ln 5200)$

$= \frac{1}{2}\ln\frac{60}{52} + \frac{1}{2}\ln\frac{42}{52} = \frac{1}{2}\ln\frac{315}{338} = -0.0352$

So expected utility change is $\boxed{-0.0352}$

Q6 (a).

X	0	1	2	3	4	5	6	7	8	9
P	0	$\frac{1}{45}$	$\frac{2}{45}$	$\frac{3}{45}$	$\frac{4}{45}$	$\frac{5}{45}$	$\frac{6}{45}$	$\frac{7}{45}$	$\frac{8}{45}$	$\frac{9}{45}$

Y	0	1	2	3	4	5	6	7	8	9
P	$\frac{9}{45}$	$\frac{8}{45}$	$\frac{7}{45}$	$\frac{6}{45}$	$\frac{5}{45}$	$\frac{4}{45}$	$\frac{3}{45}$	$\frac{2}{45}$	$\frac{1}{45}$	$\frac{0}{45}$

(X,Y)	(0,1)	...	(0,9)	(1,2)	...	(1,9)	----	(7,8)	(7,9)	(8,9)
P	$\frac{1}{45}$	...	$\frac{1}{45}$	$\frac{1}{45}$	...	$\frac{1}{45}$	----	$\frac{1}{45}$	$\frac{1}{45}$	$\frac{1}{45}$

(b).

$Z=X-Y$	1	2	3	4	5	6	7	8	9
P	$\frac{9}{45}$	$\frac{8}{45}$	$\frac{7}{45}$	$\frac{6}{45}$	$\frac{5}{45}$	$\frac{4}{45}$	$\frac{3}{45}$	$\frac{2}{45}$	$\frac{1}{45}$

(c). $E(X) = \Sigma X P(X) = \frac{1}{45}\times 1 + \frac{2}{45}\times 2 + \frac{3}{45}\times 3 + \dots + \frac{9}{45}\times 9$

$= \frac{1+2^2+\dots+9^2}{45} = \frac{285}{45} = \frac{57}{9} = \frac{19}{3} = 6.33$

$Var(X) = E(X^2) - (EX)^2 = \frac{1+2^3+\dots+9^3}{45} - (\frac{19}{3})^2$

$= 45 - \frac{361}{9} = \frac{44}{9} = 4.89$

$E(Z) = \frac{9}{45}\times 1 + \frac{8}{45}\times 2 + \frac{7}{45}\times 3 + \dots + \frac{1}{45}\times 9 = \frac{169}{45} = \frac{11}{3} = 3.67$

$E(Z^2) = \frac{9}{45}\times 1^2 + \frac{8}{45}\times 2^2 + \frac{7}{45}\times 3^2 + \dots + \frac{1}{45}\times 9^2 = \frac{825}{45} = \frac{55}{3} = 18.33$

$Var(Z) = E(Z^2) - (EZ)^2 = \frac{55}{3} - \frac{11}{3} = \frac{44}{3} = 14.67$

overall, $E(X) = \frac{19}{3} \approx 6.33$, $Var(X) = \frac{44}{9} \approx 4.89$,

$E(Z) = \frac{11}{3} \approx 3.67$, $var(Z) = \frac{44}{3} \approx 14.67$.

Q7. (a). $S_n \sim B(n, P)$. This is because first n trials are independent and probability is P.

(b). $T_m \sim B(m, P)$ same Reason as (a).

Trials are independent with equal probability

(c) $S_n + T_m$ is the total successful trials. Since $n+m$ are Bernoulli trials, Then $(S_n + T_m) \sim B(n+m, P)$

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(d). $P(X=S_n, Y=T_m) = C_n^{S_n} p^{S_n}(1-p)^{n-S_n} \times C_m^{T_m} p^{T_m}(1-p)^{n-T_m}$.

(since all the trials are independent), this is $P(X=S_n) * P(Y=T_m)$

so $P(X=S_n, Y=T_m) = P(X=S_n) \times P(Y=T_m)$,

so S_n, T_m are independent.

8. since $Z=X+Y$, then $var(Z) = var(X) + var(Y)$,

$E(Z) = E(X) + E(Y)$.

so we have: $m_0p_0(1-p_0) + m_1p_1(1-p_1) \leq (m_0p_0+m_1p_1)\left(1-\frac{m_0p_0+m_1p_1}{m_0+m_1}\right)$

that's to prove $E(Z) - (m_0p_0^2 + m_1p_1^2) \leq E(Z) - \frac{(E(Z))^2}{m_0+m_1}$

we need to prove $m_0p_0^2 + m_1p_1^2 \geqslant \frac{(EZ)^2}{m_0+m_1}$

$\Rightarrow m_0^2p_0^2 + m_0m_1(p_0^2+p_1^2) + m_1^2p_1^2 \geqslant m_0^2p_0^2 + 2m_0m_1p_0p_1 + m_1^2p_1^2$

$\Rightarrow m_0m_1(p_0^2+p_1^2) \geqslant 2m_0m_1(p_0p_1)$

since $(m_0 \neq 0, m_1 \neq 0)$ then $\Rightarrow p_0^2+p_1^2 \geqslant 2p_0p_1$

$\Rightarrow (p_0-p_1)^2 \geqslant 0$ always stands true.

so we have proved the statement. when the equality holds,

$(p_0-p_1)^2 = 0$, then $p_0 = p_1$