

MA2185 Assignment Three

Question 1

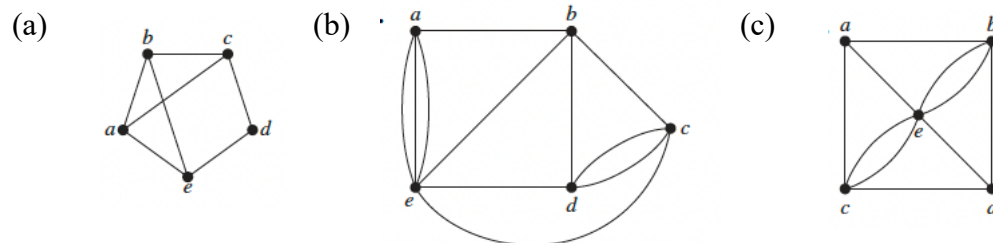
- a) When we randomly select a permutation of $\{1, 2, 3, 4\}$, what is the probability of that 4 precedes 1 and 4 precedes 2?
- b) Suppose that a drug test has a 2% false positive rate and a 5% false negative rate. That is, 2% of people who do not use drug test positive, and 5% of drug users test negative. Furthermore, suppose that 1% of people actually use drug.
- (i) Find the probability that someone who tests negative does not use drug.
- (ii) Find the probability that someone who tests positive actually uses drug.

Question 2

- a) Show that if G is a connected graph, then it is possible to remove vertices to disconnect G if and only if G is not a complete graph. (Remark: here we consider a graph without any vertices and any edges is connect and complete.)
- b) How many subgraphs with at least one vertex does the complete graph K_3 have? Draw them all.

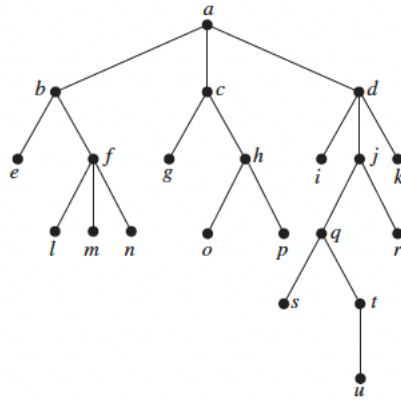
Question 3

Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.



Question 4

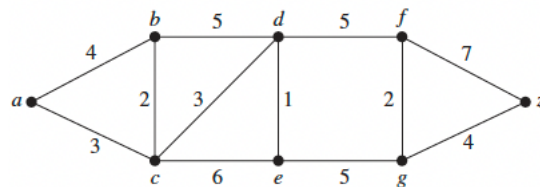
According to the following graph, answer the questions (a) – (i)



- Which vertex is the root?
- Which vertices are internal?
- Which vertices are leaves?
- Which vertices are children of j ?
- Which vertex is the parent of h ?
- Which vertices are siblings of o ?
- Is the rooted tree a full m -ary tree for some positive integer m ?
- What is the level of each vertex of the rooted tree?
- Draw the subtree of the tree that is rooted at a , c , and e .

Question 5

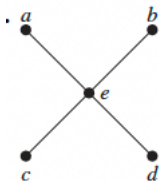
Find the length of a shortest path between a and z in the given weighted graph.



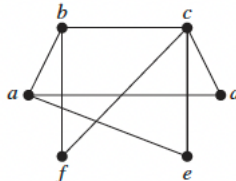
Question 6

Determine whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color. If not, how many colors do we need to color the vertices such that no two adjacent vertices are assigned the same color?

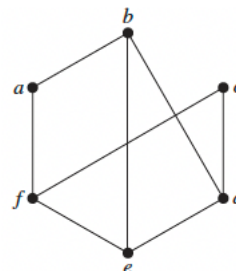
(a)



(b)



(c)



Question 7

a) Suppose we want to send the message “DO NOT PASS”, knowing that it has been enciphered using the affine cipher. In the system we used, it has the 26 letters (A through Z) and 2 notations (total number of letters and notations $n = 28$), and will have the corresponding values found in the following table.

A	B	C	D	E	F	G	H	I	J	K	L	M	N
0	1	2	3	4	5	6	7	8	9	10	11	12	13

O	P	Q	R	S	T	U	V	W	X	Y	Z	!	&
14	15	16	17	18	19	20	21	22	23	24	25	26	27

If the **deciphering function** is

$$D(c) = 3c + 7 \text{ mod } 28,$$

write down the **enciphering function** for this cipher, and write down the ciphertext for the message “DO NOT PASS”. (Please present the function with the form $ap + b$ where a and b are non-negative integers less than 28.)

b) Show that $\text{GCD}(414, 662) = 2$, and express it as a linear combination of 414 and 662 using the Euclidean algorithm.