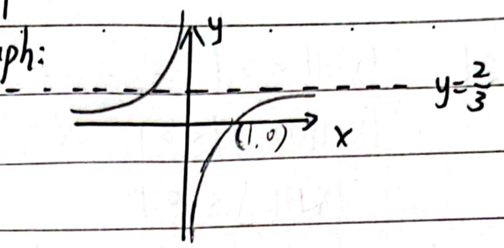
MA1300 Hand-in Assignment 1

1. $f(x) = \begin{cases} \frac{2}{3} - \frac{2}{3x} \\ -\frac{2}{3x} \end{cases}$

Graph:



Domain: $(-\infty, 0) \cup (0, +\infty)$

2. let $f(x) = Ax^2 + Bx + C$

$A(3x+5)^2 + B(3x+5) + C = 3x^2 + 3x + 2$

$\Rightarrow \begin{cases} A = \frac{1}{3} \\ B = -\frac{7}{3} \\ C = \frac{16}{3} \end{cases}$ $\quad \therefore f(x) = \frac{1}{3}x^2 - \frac{7}{3}x + \frac{16}{3}$

3. h is always an odd function

$f \circ g = f(g(x))$

$f(g(-x)) = f(-g(x)) = -f(-g(x))$

$\therefore$ h is an odd function

4. $\lim\limits_{x\to 2} \frac{2-x}{8-x^3} = \lim\limits_{x\to 2} \frac{2-x}{(2-x)(4+2x+x^2)}$

又$\because x \to 2 \therefore x \neq 2$ $\quad \therefore \lim\limits_{x\to 2} \frac{1}{4+2x+x^2} = \frac{1}{12}$

5. $\lim\limits_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1} = \lim\limits_{x\to 2} \frac{6-x-4}{(\sqrt{3-x}-1)(\sqrt{6-x}+2)} = \lim\limits_{x\to 2} \frac{2-x}{(\sqrt{3-x}-1)(\sqrt{6-x}-2)}$

$= \lim\limits_{x\to 2} \frac{\sqrt{3-x}+1}{\sqrt{6-x}+2} = \frac{2}{4} = \frac{1}{2}$

6. $x\cos\frac{\pi}{x-1} + \sin\left(\frac{\pi}{x-1} - \frac{\pi}{2}\right) = (x-1)\cos\frac{\pi}{x-1}$ $\quad \cos\frac{\pi}{x-1} \in [-1, 1]$

① when $x<1$ $\quad x-1 \leq (x-1)\cos\frac{\pi}{x-1} \leq 1-x$

② when $x>1$ $\quad 1-x \leq (x-1)\cos\frac{\pi}{x-1} \leq x-1$

又$\because \lim\limits_{x\to 1}(x-1) = \lim\limits_{x\to 1}(1-x) = 0$

$\therefore \lim\limits_{x\to 1}\left[x\cos\frac{\pi}{x-1} + \sin\left(\frac{\pi}{x-1} - \frac{\pi}{2}\right)\right] = 0$

7. $\varepsilon = 0.1$ $\because |x^2-1| < 0.1$

$\therefore |x-1||x+1| < 0.1$ 又 $\because |x-1| \le \delta$

$\therefore |x+1|\delta \le 0.1$

let $\delta \le 1$, then $|x-1| \le 1$ $0 \le x \le 2$

$\therefore 3\delta \le 0.1$ $\delta \le \frac{1}{30}$

$\therefore \delta = \frac{1}{30}$

8. $\forall \varepsilon > 0$, $\exists \delta = \min\{1, \frac{\varepsilon}{7}\}$, such that if $|x-1| < \delta$, $|f(x)-1| < \varepsilon$

proof: let $0 < \delta \le 1$, then $|x-1| \le 1$, $0 \le x \le 2$

$|x^3-1| = |x-1||x^2+x+1| < \delta|x^2+x+1| \le 7\delta \le \varepsilon$

$\therefore \delta \le \frac{\varepsilon}{7}$

$\therefore \delta = \min\{1, \frac{\varepsilon}{7}\}$

9. ① $\lim_{x\to 0} f(x) = 0$

$|f(x)| = \begin{cases} 0, & \text{if } x \text{ is rational} \\ |x|, & \text{if } x \text{ is irration} \end{cases}$

$\therefore -|x| \le |f(x)| \le |x|$

$\lim_{x\to 0} -|x| \le \lim_{x\to 0} f(x) \le \lim_{x\to 0} |x|$

And $\lim_{x\to 0} -|x| = \lim_{x\to 0} |x| = 0$

$\therefore \lim_{x\to 0} f(x) = 0$

② $\lim_{x\to 1} f(x)$ DNE

Take δ arbitarily small,

x_1 is rational, x_2 is irrational

$x_1, x_2 \in (1, 1+\delta)$ $f(x_1) = 0$ $f(x_2) = x_2$

$\therefore \lim_{x\to 1^+} f(x)$ DNE

similarly $\lim_{x\to 1^-} f(x)$ DNE

$\therefore \lim_{x\to 1} f(x)$ DNE

10. $\forall M > 0$, $\exists \delta > 0$, if $2-\delta < x < 2+\delta$, $\frac{x+1}{(x-2)^2} > M$

proof: let $0 < \delta \le 1$, then $x \in (1, 3)$

$\frac{x+1}{(x-2)^2} > \frac{1}{(x-2)^2} > \frac{1}{\delta^2} \ge M$ $\therefore \delta = \sqrt{\frac{1}{M}}$

$\therefore \delta = \min\{1, \sqrt{\frac{1}{M}}\}$

In conclusion $\forall M > 0$, $\exists \delta = \min\{1, \sqrt{\frac{1}{M}}\}$, such that if $|x-2| < \delta$, $\frac{x+1}{(x-2)^2} > M$

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