MA1301 Semester B 2022-23
Midterm Exam

MA1301 **Name:** _____
Semester B 2022-23
Midterm Exam
13/03/2023

This exam contains 3 pages (including this page) and 4 questions. Total of points is 100.

Grade Table (for instructor use only)

Question	Points	Score
1	15	
2	50	
3	20	
4	15	
Total:	100	

1. (15 points) (i) [5pts] Compute

$$\frac{d}{dx}\int_{lnx}^{x^3} e^{\sin t}dt;$$

(ii) [5pts] Compute

$$\lim_{x\to 0}\frac{\int_1^{1+5x}(4-\cos 2\pi t)^3 dt}{x}.$$

(iii) [5pts] Calculate the following limit by integral

$$\lim_{n\to+\infty}\sum_{i=1}^{n}\frac{9i+3n}{3in+2n^2}.$$

2. (50 points) Evaluate the following integrals:
(i)[15pts]

$$\int \frac{1}{x^2(x-1)}dx$$

(ii)[10pts]

$$\int \frac{1}{\sqrt{2x-x^2}}dx$$

(iii)[15pts]

$$\int_0^{\pi} 3x^2\cos\left(\frac{x}{2}\right)dx$$

(iv)[10pts]

$$\int \frac{1}{x^2+x+1}dx$$

3. (20 points) (i) [10pts] Find the volume of the solid generated by rotating the region in the first quadrant bounded to right by $y = \sqrt{4 - x^2}$ and to the left by $y = x$ about the line $x = -1$.
(ii) [10pts] Let R be the region bounded by the four straight lines $y = x$, $x + y = 4$, $y = x - 2$ and $x + y = 2$. Find the surface area of the surface obtained by rotating the region R about x-axis for one complete revolution.;

4. (15 points) Consider the integral

$$\int_0^{+\infty} \frac{\cos x}{\sqrt{x}(1+x)} dx$$

(i)[5pts] What makes this integral improper? (Make sure to state all reasons!)
(ii)[10pts] Determine whether this improper integral converges or not. (No need to compute. Think of some inequalities.)