Midterm Solution CB1 - Refer to title

Discrete Mathematics (City University of Hong Kong)

1- (30)

Define l = "experiment is difficult"

s = "not many students like experiment"

m = "science is easy"

Assumptions 1) $l \vee s$

2) $m \rightarrow \sim l$

a) $\sim s \rightarrow \sim m$

(10) follows from assumptions 3) $\sim s \rightarrow l$ ①

4) $l \rightarrow \sim m$ ②

5) $\sim s \rightarrow \sim m$ ③ ④

b) $\sim m \rightarrow s$

(10) doesn't follow assumption if $m \equiv F$

$l \equiv T$

$s \equiv F$ then

$l \vee s \equiv m \rightarrow \sim l \equiv T$ but $\sim m \rightarrow s \equiv F$

c) $s \rightarrow (\sim m \vee \sim l)$

(10) follows the assumption 6) $\sim m \vee \sim l$ ②

7) $\sim s \vee (\sim m \vee \sim l)$ ⑥

8) $s \rightarrow (\sim m \vee \sim l)$ ⑦

2. (10)

$$\sum_{i=1}^{n} \frac{i^2}{(2i-1)(2i+1)} = \frac{n(n+1)}{4n+2}$$

for $n \geq 1$

Step 1 for $n=1$ $\quad LHS = \frac{1}{(2-1)(2+1)} = \frac{1}{3}$

$$RHS = \frac{1 \times 2}{4+2} = \frac{1}{3}$$

$\therefore P(1)$ is true

step 2 : Assume that $P(n)$ is true for $P(n+1)$

$$LHS = \sum_{i=1}^{n+1} \frac{i^2}{(2i-1)(2i+1)} = \frac{n(n+1)}{4(n+2)} + \frac{(n+1)^2}{(2n+1)(2n+3)}$$

$$= \frac{n+1}{2n+1}\left[\frac{n}{2} + \frac{(n+1)}{2n+3}\right]$$

$$= \frac{n+1}{2n+1}\left[\frac{(2n+3)n + 2(n+1)}{2(2n+3)}\right]$$

$$= RHS$$

$\therefore P(n+1)$ is also true

By M.I $P(n)$ is true for all $n \geq 1$

3. (10)

There are 2 cases

i) the last digit is "1" in row n, then there are f_{n-1} ways to arrange the first $n-1$ digits in the row.

ii) If the last digit in the row is a "0" then the second last digit must be a "1", thus there are f_{n-2} ways to arrange the first $n-2$ digits in the row.

Hence we have $f_n = f_{n-1} + f_{n-2}$, $n \geq 3$

with initial conditions $f_1 = 2$ $f_2 = 3$

4 (10)

(a) Select a hamster among 12 hamsters

(10) Select a lizard among 14 lizards

$12 \times 14 = 168$

(b) A _ _ _ _ _ $= 24$

(10) $\underbrace{\quad}_{4!}$

G _ _ _ _ _ $= \frac{4!}{2!1!1!} = 12$

R _ _ _ _ _ $= \frac{4!}{2!1!1!} = 12$

The 49th word = SHAGR

(c) _ x _ x _ ... _ x _

(10)

${}_xP_x$ (men)

${}_{x+1}P_y$ (women)

$$({}_xP_x)({}_{x+1}P_y) = \frac{x!}{(x-x)!} \cdot \frac{(x+1)!}{(x+1-y)!}$$

$$= \frac{x!(x+1)!}{(x-y+1)!}$$

5. (20)

(a) (10)

From marketing	From Accounting
5	7
or	
6	6
or	
7	5

$\therefore {}_7C_5 \times {}_8C_7 + {}_7C_6 \times {}_8C_6 + {}_7C_7 \times {}_8C_5$

(b) (10)

Case 1 Nemo visits Exhibition B then goes to A

${}_6C_1 = 6$

Case 2 Nemo doesn't visit Exhibition B then

${}_7C_5 = 35$

$\therefore 6 + 35 = 41$