Midterm 7 December Autumn 2020, questions

Discrete Mathematics (City University of Hong Kong)

MIDTERM: 2 NOVEMBER 2020 (3:30PM-5:30PM)

Question 1. **(15')** Determine the truth value of following statements if the domain consists of all real numbers.

(1) (5') $\exists x(x^3 = -1)$
(2) (5') $\neg\forall x(x^2 > 0)$
(3) (5') $\forall x(x^2 \neq x)$.

Question 2. **(15')** Show the validity of

$$\begin{array}{ll} & p \to q \\ & r \to s \\ & p \vee r \\ \hline \therefore & q \vee s \end{array}$$

Question 3. **(15')** Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

Question 4. **(15')** Let $A = \{1, 2, 3, 4\}$. R is a relation on A with

$$R = \{(1, 1), (1, 3), (3, 1), (2, 2), (2, 4), (3, 3), (3, 4), (4, 4)\}.$$

(1) (6') Draw the directed diagram for R.
(2) (9') Determine and explain whether R is reflexive, symmetric, antisymmetric, or transitive.

Question 5. **(18')** Let A be the set $\{a, b\}$.

(1) (5') Find the power set $\mathcal{P}(A)$ of A.
(2) (5') Find $A \times A \times A$.
(3) (8') Determine whether the function $f : B \to \overline{B}$ is a bijection, where the domain is $\mathcal{P}(A)$.

Question 6. **(12')** Show that the relation $\{(a, b)|a = \pm b (\text{mod } 7)\}$ is an equivalence relation on the set of integers.

Question 7. **(6')** The logical operator NAND, written as $|$ is defined by

$$p|q = \neg(p \vee q).$$

Using the NAND operator only, rewrite

(1) $\neg p$
(2) $p \wedge q$
(3) $p \to q$.

Question 8. **(4')** Let A and B be sets. Show that $A \subset B$ if and only if $A \cap B = A$.