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CS2402 - Lecture 3 - In-Class Exercises

Q1. Let A and B be events such that P(A) = 0.6 and P(B) = 0.4, and P(AB) = 0.2, compute the probabilities of $a) AUB, b)\ \bar{A}, c)\bar{A}B, d)A\bar{U}B$.

a). $P(A \cup B) = P(A) + P(B) - P(AB) = 0.6 + 0.4 - 0.2 = 0.8$

b). $P(\bar{A}) = 1 - P(A) = 1 - 0.6 = 0.4$

c). $P(\bar{A}B) = P(B) - P(AB) = 0.2$

d) $P(A \cup \bar{B}) = P(\bar{B}) + P(A) - P(A\bar{B}) = 0.4 + 0.6 - 0.2 = 0.8$

Q2. In a high school graduating class of 100 students, 54 studied mathematics, 69 studied history, and 35 studied both mathematics and history. If one of these students is selected at random, find the probability that (a) the student took mathematics or history; (b) the student did not take either of these subjects; (c) the student took history but not mathematics.

(a). $P = \frac{54+69-35}{100} = 0.88$ (c) $P = \frac{69-35}{100} = 0.34$

(b). $P = \frac{1-88}{100} = 0.12$

Q3. A pair of fair dice is tossed. Find the probability of getting (a) a total of 8; (b) at most a total of 5.

(a). {2,6} {3,5} {4,4}

$P = \frac{5}{36}$

(b). {1,1} {1,2} {1,3} {1,4}
{2,2} {2,3}

$P = \frac{1+2+2+3+1+2}{36} = \frac{10}{36} = \frac{5}{18}$

Q4. A six-sided die has four green ($\frac{2}{3}$) and two red ($\frac{1}{3}$) faces and is balanced so that each face is equally likely to come up. The die will be rolled several times and we assume each rolling is independent. You must choose one of the following three sequences of colors; you will win $25 if the first rolls of the die give the sequence that you have chosen.

RGRRR
RGRRRG
GRRRRR

$P(A) = \frac{2}{3^5}$

$P(B) = \frac{4}{3^6}$

$P(C) = \frac{2}{3^6}$

$P(A) > P(B) > P(C)$

So choose {R, G, R, R, R}

Without making any calculations, explain which sequence you choose. Then use some calculations to support your idea. (In a psychological experiment, 63% of 260 students who had not studied probability chose the second sequence.)

Q5. Two sisters maintain that they can communicate telepathically. To test this assertion, you place the sisters in separate rooms and show sister A a series of cards. Each card is equally likely to depict either a circle or a star or a square. For each card presented to sister A, sister B writes down 'circle', or 'star' or 'square', depending on what she believes sister A to be looking at. If ten cards are shown, what is the probability that sister B correctly matches at least one card?

$P = 1 - \left(\frac{2}{3}\right)^{10}$

Q6. Jane has three children, each of which is equally likely to be a boy or a girl independently of the others. Define the events:

A = {all the children are of the same sex},
B = {there is at most one boy},
C = {the family includes a boy and a girl} .

(a) Show that A is independent of B, and that B is independent of C.
(b) Is A independent of C?

(a) $P(A) = \frac{1+1}{2\times2\times2} = \frac{1}{4}$

$P(B) = \frac{4}{2\times2\times2} = \frac{1}{2}$

$P(C) = \frac{6}{2\times2\times2} = \frac{3}{4}$

$P(A \cap B) = \frac{1}{2\times2\times2} = \frac{1}{8} = P(A)\cdot P(B)$

$P(B) = \frac{1+1+1}{2\times2\times2} = \frac{3}{8} = P(B)\cdot P(C)$

(c). $P(A \cap C) = 0$

A and C are mutually exclusive.

So they are dependent