CITY UNIVERSITY OF HONG KONG

Course code & title	:	CS2402
Session	:	Semester B 2023/2024
Time allowed	:	1 hour 30 mins

This paper has 6 pages (including this cover page).

1. This paper consists of 18 questions.
2. Answer ALL questions.
3. Write your answers **in this question paper.**
4. This question paper should **NOT** be taken away.

This is a closed-book examination

Student ID: 57854329

Student Name: LIU Hengche

Student EID: hengchliu2

Questions	1-10	11	12	13	14
Marks		7	7	7	7
Max	40	7	7	7	7
Questions	15	16	17	18	Total
Marks	7	7			
Max	7	7	9	9	100

For Q1-Q10, provide the answers. The problem solving process is NOT required.

$\frac{1}{2}$

Q1. A fair coin is flipped 5 times. What is the probability of getting at least 3 heads?

4

$\frac{1}{10}$

Q2. Each of you and a desired future friend independently selects three out of the five courses our department offers next semester. Find the probability you will study in at least two classes together.

4

0.4

Q3. $P(A \cup B) = 0.8, P(A) = 0.4, P(\overline{B}) = 0.2. P(A \cap B) = ?$

$P(B) = 0.8$

4

$\frac{1}{2}$

Q4. "The midterm exam is held on the ninth of March." Suppose a word is picked at random from the sentence above. What is the probability that the word has at least 4 characters?

4

4 $\frac{5}{12}$

Q5. If you throw a dice twice, what is the probability that the second number is greater than the first one?

Q6. Suppose 10 cards (ace,2,3,4,5,6,7,8,9,10) are shuffled.

(a) If one card is dealt, what is the probability that the card is an ace?

(b) If two cards are dealt sequentially (without replacement), what is the probability that the second card is an ace?

2

(a). $\frac{1}{10}$

(b). $\frac{1}{5}$

6

Q7. The random variable X is sampled from {1,2,10,20} and has the following probability: $P(X = 1) = \frac{1}{3}, P(X = 2) = \frac{1}{3}, P(X = 10) = \frac{1}{6}$. Then $E(X) = ?$

4

1	2	10	20
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{91}{36}$

Q8. Find E(X), where X represents the lesser of two numbers obtained from rolling a pair of dice.

Assume that the lesser of ~~two~~ equal numbers is one of them.

4

1.2

Q9. Let X and Y be two random variables and have the joint distribution in the following table. Calculate E(XY).

P(X, Y)		X			
		0	1	2	3
Y	0	0.150	0.300	0.050	0 0
	1	0.050	0.151	0.052	0.053
	2	0 0	0.052	0.104	0.056

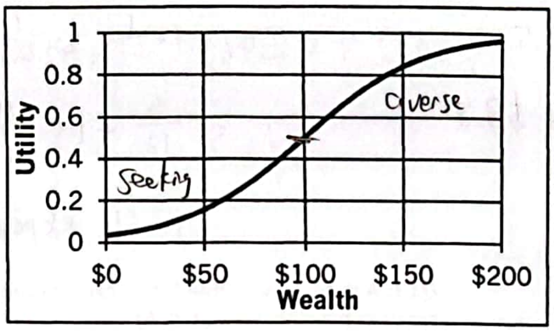
XY	0	1	2	3	4	6
P	0.55	0.15	0.1	0.05	0.1	0.05

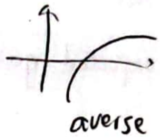
4

C

Q10. What is the behavior of a person with the following utility function?

4





a) Risk-averse.
b) Risk-seeking.
c) Risk-seeking up to \$100 wealth, then risk-averse after \$100. ✓
d) Risk-averse up to \$100 wealth, then risk-seeking after \$100.
e) Risk-neutral.

For Q11-Q18, you need to provide the problem solving process.

Q11: Perform two independent Binomial experiments with p=0.5, and let S be the sum of the two outcomes. For example, if the two outcomes are 0 and 1, then S=1. Find E(S) and Var(S).

X	0	1
P	0.5	0.5

Y	0	1
P	0.5	0.5

$S = X + Y$

Since X, Y are independent, the $E(S) = E(X) + E(Y) = 0.5 + 0.5 = 1$

$Var(X) = Var(Y) = E(X^2) - (EX)^2 = 0.5 - 0.25 = 0.25$.

$Var(S) = Var(X+Y) = Var(X) + Var(Y) = 0.5$.

$\therefore E(S) = 1, \quad Var(S) = 0.5$

7

Q12: Consider a game where you roll a fair die, and after observing its outcome, you have the option to receive the amount it shows and quit, or roll again, and receive the outcome of the second roll. For example, if the first roll is 4, you can choose to receive $4, or to play again and receive the new outcome, if the die turns up, say, 2, then you'll receive $2.
(a) A strategy for this game always takes the amount of the first roll. Find the expected gain of this strategy.
(b) A strategy for this game takes the first outcome and checks if it is 4 or more, and otherwise roll again. For example, if the first roll is 5, you will receive $5; if the first roll is 3, you will roll again and take the second outcome. Find the expected gain of this strategy.

(a).

X	1	2	3	4	5	6
P	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$E(X) = \sum_{ALL} P(X_i) X_i = 3.5.$

So the expected gain is $3.5

(b).

X	1	2	3	4	5	6
P	$\frac{3}{36}$	$\frac{3}{36}$	$\frac{3}{36}$	$\frac{9}{36}$	$\frac{9}{36}$	$\frac{9}{36}$

$E(X) = 1\times\frac{1}{12} + 2\times\frac{1}{12} + 3\times\frac{1}{12} + 4\times\frac{1}{4} + 5\times\frac{1}{4} + 6\times\frac{1}{4}$

$= \frac{6}{12} + 1 + \frac{11}{4} = \frac{17}{4} = \$4.25.$

So the expected gain is $4.25

7

Q13: A random variable X has expectation 8 and standard deviation 2. Find an upper bound as small as possible for P(X ≤ 4).

$P(X\le 4) \le P(|X-8| \ge 2\cdot 2) = P(|X-E(X)| \ge 2 SD(X)) \le \frac{1}{4}$

$\therefore$ upper band for $P(X\le 4) \le \frac{1}{4}$

7

Q14: Bob is doing archery for his local club. The probabilities that Bob scores 5/1/0 points for each shot are $\frac{1}{10}, \frac{5}{10}, \frac{4}{10}$, respectively. Let X = the total points scored by Bob among his two independent shots. Find E(X), Var(X), SD(X).

X_i	5	1	0
$P(X_i)$	$\frac{1}{10}$	$\frac{5}{10}$	$\frac{4}{10}$

$\therefore E(X_i) = 5\times\frac{1}{10} + 1\times\frac{5}{10} = 1$

let first shot be X_1, second be X_2

then $X = X_1 + X_2$

$E(X) = E(X_1+X_2) = E(X_1) + E(X_2)$

$= 1 + 1 = 2$

$Var(X_i) = E(X_i^2) - (E X_i)^2 = \frac{30}{10} - 1^2 = 2$

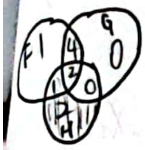
$Var(X) = Var(X_1 + X_2) = Var(X_1) + Var(X_2) = 4$

$SD(X) = \sqrt{Var(X)} = 2$

$\therefore E(X) = 2, \ Var(X) = 4, \ SD(X) = 2$

7

Q15: There are three events F, G and H such that: $P(F) = 0.8$, $P(G) = 0.6$, $P(H) = 0.5$, $P(FG) = 0.6$, $P(FH) = 0.3$, $P(GH) = 0.2$, $P(FGH) = 0.2$. Compute:
(a) $P(F \cup G)$;
(b) $P(F \cup G \cup H)$;
(c) $P(\overline{F}\overline{G}H)$.



(a). $P(F \cup G) = P(F) + P(G) - P(FG) = 0.8 + 0.6 - 0.6 = 0.8$

(b) $P(F \cup G \cup H) = P(F) + P(G) + P(H) - P(FG) - P(FH) - P(GH) + P(FGH)$
$= 0.8 + 0.6 + 0.5 - 0.6 - 0.3 - 0.2 + 0.2 = 1$

(c). $P(\overline{F}\overline{G}H) = P(H) - P(GH) - P(FH) + P(FGH)$
$= 0.5 - 0.3 - 0.2 + 0.2$
$= 0.2$

Q16: Choose a number at random from 1, ..., 100. What is the probability that the chosen number is divisible by at least one number from {2,3,7}.

$P(2) = \frac{50}{100}$ $P(3) = \frac{33}{100}$ $P(7) = \frac{14}{100}$
$P(6) = \frac{16}{100}$ $P(21) = \frac{4}{100}$ $P(14) = \frac{7}{100}$
$P(42) = \frac{2}{100}$

$P = P(2) + P(3) + P(7) - P(6) - P(21) - P(14) + P(42)$
$= \frac{1}{100}(50 + 33 + 14 - 16 - 4 - 7 + 2)$
$= \frac{1}{100}(70 + 2) = \frac{72}{100} = \frac{18}{25}$

Q17: For some constant $c > 0$ the random variable X takes the value $X = j$ with probability $c \cdot j$ (c multiplies j) for $j \in \{1,2,3,4\}$.

(a) What is the probability that X is an even number?

(b) How large would you expect the sum to be of n independent observations $X_1, \dots, X_n$ from this distribution?

(c) What is the probability that n independent observations $X_1, \dots, X_n$ are all equal?

(a).

X	1	2	3	4
p	c	2c	3c	4c

$c+2c+3c+4c=1 \quad \therefore c=0.1$

$\therefore P(\text{even}) = 0.2+0.4=0.6$

(c). $p = P(X_1=\dots=X_n=1) + P(X_1=\dots=X_n=2) + P(X_1=\dots=X_n=3) + P(X_1=\dots=X_n=4)$

$= (0.1)^n + (0.2)^n + (0.3)^n + (0.4)^n$

(b).

X	1	2	3	4
p	0.1	0.2	0.3	0.4

$E(X) = 1\times0.1+2\times0.2+3\times0.3+4\times0.4$

$= 0.1+0.4+0.9+1.6=3$

6

$E(X_1+\dots+X_n) = E(X_1)+\dots+E(X_n) = nE(X) = 3n$

$\therefore$ we expect the sum to be $3n$

Q18: Suppose $E(X^2) = 5$, $E(Y^2) = 10$, and $E(XY) = 6$.

(a) Calculate $E((X-Y)^2)$.

(b) Find the number t that minimize $f(t) = E((X-tY)^2)$.

(a). $E((X-Y)^2) = E(X^2-2XY+Y^2) = E(X^2)-2E(XY)+E(Y^2) = 5+10-12=3$

$\therefore E((X-Y)^2)=3$

(b). $f(t) = E(X^2-2tXY+t^2Y^2) = E(X^2) - 2t\underbrace{E(XY)}_{6} + t^2\underbrace{E(Y^2)}_{10}$

$= 5+10t^2-12t = 10t^2-12t+5$

9

when $t = -\frac{(-12)}{2\times10} = \frac{3}{5}$, $f(t)$ obtains the minimum value.

- END -