CS2402 - Tutorial 1

Task 1. The first 25 missions

Space flight is risky. During the beginning of the Space Shuttle program in the early 1980s, NASA had studied and assessed the risk of an accident resulting in the loss of the Space Shuttle and its crew. This included looking at various risk contributors, such as being hit by small meteorites while in space, catastrophic engine failure, debris hitting the Shuttle during takeoff, exploding gas tanks, and crew error. However, since the Shuttle was new and NASA had little experience flying it, they had greatly underestimated the risk. At the time, NASA managers thought the frequency of an accident was 1-in-100,000 for each mission, while the NASA engineers thought the frequency was more like 1-in-100. In a 2011 internal report, NASA re-estimated the frequency of an accident as 1-in-10.

In this tutorial, we will look at how the probability of an accident occurring accumulates as the number of missions increases.

1. We will first look at the first 25 missions. Fill in this table with the frequencies and probabilities according to various people.

	NASA managers	NASA engineers	2011-report
Frequencies of accident in one mission	1 in 100,000	1 in 100	1 in 10
P(an accident in 1 mission)	$\frac{1}{100,000}$	$\frac{1}{100}$	$\frac{1}{10}$
P(no accident in 1 mission)	$\frac{99,999}{100,000}$	$\frac{99}{100}$	$\frac{9}{10}$
P(no accidents in N missions)	$(1-\frac{1}{100000})^N$	$(1-\frac{1}{100})^N$	$(1-\frac{1}{10})^N$
P(any accidents in N missions)	$1-(1-\frac{1}{100000})^N$	$1-(1-\frac{1}{100})^N$	$1-(1-\frac{1}{10})^N$

Hint: Recall that the probability of an event occurring N times in a row is the probability of it happening once raised to the power of N.

2. According to the *NASA report*, the probability of **no** accidents in 25 missions was: ____.

$\cancel{\frac{9}{10}}$ $\left(\frac{9}{10}\right)^{25}$

3b. $P(ABC) = P(AB) + P(C) - P(AB \cup C)$
$\geq P(AB) + P(C) - 1$
$\geq P(A) + P(B) - 1 + P(C) - 1$

3c. $P(A_1A_2 \cdots A_n) = P(A_1 A_2 \cdots A_{n-1}) + P(A_n) - P((A_1 A_2 \cdots A_{n-1}) \cup C)$
$\geq P(A_1 \cdots A_{n-1}) + P(A_n) - 1$
$\geq P(A_1 \cdots A_{n-2}) + P(A_{n-1}) + P(A_n) - 2$

$\geq P(A) + P(B) - 2$

$\geq P(A_1) + \cdots + P(A_n) - (n-1)$

Task 2. Power plants

There are 4 different power plants. Now suppose that the ith power plant fails independently with probability p_i.

i	1	2	3	4
p_i	0.1	0.05	0.01	0.1

1 Suppose each power plant can produce enough electricity to supply the entire city. What is the probability that the city will experience a black-out?

$0.1 \times 0.05 \times 0.01 \times 0.1 = 0.000005$

2. Now suppose two power plants are necessary to keep the city from a black-out. Find the probability that the city will experience a black-out.

$0.9 \times 0.05 \times 0.01 \times 0.1 = 0.000045$
$0.1 \times 0.95 \times 0.01 \times 0.1 = 0.000095$
$0.1 \times 0.05 \times 0.99 \times 0.1 = 0.000495$
$0.1 \times 0.05 \times 0.01 \times 0.9 = 0.000045$

3a. $P(A) + P(B) - P(AB) = P(A \cup B) \leq 1$

Task 3. Prove

a) $P(AB) \geq P(A) + P(B) - 1$
b) $P(ABC) \geq P(A) + P(B) + P(C) - 2$
c) $P(A_1A_2 \ldots A_n) \geq P(A_1) + P(A_2) + \cdots + P(A_n) - (n-1)$

3b. $P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC) \leq 1$

By M.I.

$P(A) + P(B) + P(C) - P(ABC) \leq 1 + P(AB) + P(AC) + P(BC) - 2P(ABC)$
$\leq 1 + P(AB) + P(AC) + P(BC) - P(AC) - P(BC)$
$\leq 1 + P(AB) \leq 2$.

Task 4 (optional). Let A, B, C be three events, Find expressions for the following probabilities in terms of P(A), P (B), P(C), P(AB), P(AC), P(BC) and P(ABC)

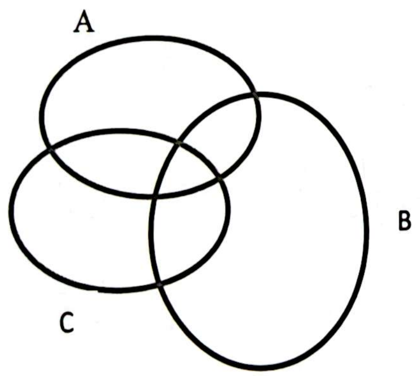
a) The probability that exactly two of A, B, C occur
b) The probability that exactly one of these occur
c) The probability of none of these events occur.

Hint: you can use Venn diagram.

By M.I.

$P(A_1 \cdots A_{n-1}) \geq P(A_1) + P(A_2) + \cdots + P(A_{n-1}) - (n-2)$

$\therefore P(A_1 \cdots A_n) \geq P(A_1 \cdots A_{n-1}) + P(A_n) - 1$
$\geq P(A_1) + \cdots + P(A_{n-1}) - (n-2) + P(A_n) - 1$
$\geq P(A_1) + \cdots + P(A_n) - (n-1)$



4(a) $P(AB) + P(AC) + P(BC) - 3P(ABC)$

(b) $P(A) + P(B) + P(C) - 2P(AB) - 2P(BC) - 2P(AC) + 3P(ABC)$

(c) $1 - (P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC))$

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