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CS2402 - Lecture 1 - In-Class Exercises

Q1. Tom has set a password of 3 characters. Each character can either be a letter or a digit. Suppose the upper case and lower case of a letter are regarded as the same.

(a) Find the number of possible passwords if (i) repeated characters are allowed, and (ii) no repeated characters are allowed.

(b) If repeated characters are allowed, find the number of passwords that contain at least one letter.

(a) $36 \times 36 \times 36 = 46,656$ $26+10$

$36 \times 35 \times 34 = 42,840$

(b) $36^3 - 10^3 = 46,656 - 1000 = 45,656$

Q2. Bag A contains 1 white straw, 2 red straws and 2 green straws. Bag B contains 2 white straws, 2 red straws and 1 green straw. One straw is drawn at random from each bag. Find the number of ways and the frequentist probabilities that

(a) the two straws drawn are of the same color;

(b) one straw is red and the other one is green.

(a) $\frac{1}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{1}{5} = \frac{8}{25}$ 8 ways

(b) $\frac{2}{5} \times \frac{1}{5} + \frac{2}{5} \times \frac{2}{5} = \frac{6}{25}$ 6 ways

Q3. On a chessboard (8 × 8 squares, alternating black and white), you place three chess pieces at random. Find the probability that they are all

(a) in the first row;
(b) on black squares;
(c) in the same row;
(d) in the same row and on the same color.

(a) $\frac{8}{64} \times \frac{7}{63} \times \frac{6}{62} = \frac{8 \times 7 \times 6}{64 \times 63 \times 62} = \frac{1}{744}$ $\frac{C_8^3}{C_{64}^3}$

(b) $\frac{32}{64} \times \frac{31}{63} \times \frac{30}{62} = \frac{1}{2} \times \frac{1}{2} \times \frac{30}{21} = \frac{5}{42}$ $\frac{C_{32}^3}{C_{64}^3} = \frac{36}{63 \times 62} = \frac{6 \times 6}{63 \times 62} \times 3$

(c) $\frac{1}{744} \times 8 = \frac{1}{93}$ $8 \times \frac{C_8^3}{C_{64}^3} = \frac{2}{217}$

(d) $\frac{C_4^3 \times 8 \times 2}{C_{64}^3} = \frac{6 \times 6}{64 \times 63 \times 62}$

Order not important

Q4. In Texas Lotto, you choose five of the numbers 1, . . . , 44 and one bonus ball number, also from 1, . . . , 44. Winning numbers are chosen randomly. Find the probability that you match

write assumptions clearly

(a) four of the first five numbers but not the bonus ball;

(b) three of the first five numbers and the bonus ball.

(a) $\frac{C_4^5}{A_{44}^5} = \frac{5 \times 4 \times 3 \times 2 \times 1 \times 40}{44 \times 43 \times 42 \times 41 \times 40} = \frac{1}{3258024}$

$\frac{C_5^4 \times C_{39}^1 \times 43}{C_{44}^5 \times 44} = \frac{5 \times 39 \times 43}{44 \times 43 \times 42 \times 41} = \frac{13}{890112}$

(b) $\frac{A_{41}^2}{A_{44}^5 \times 44} = \frac{41 \times 40}{44 \times 43 \times 42 \times 41 \times 40 \times 44} = \frac{1}{3496416}$

$\frac{C_5^3 \times C_{39}^2 \times 1}{C_{44}^5 \times 44} = \frac{10 \times 39 \times 38}{44 \times 43 \times 42 \times 41 \times 44} = \frac{13 \times 19}{22 \times 44 \times 43 \times 14} = \frac{247}{1911137408}$

Q5. From the integers 1, . . . , 10, three numbers are chosen at random without replacement.

(a) What is the probability that the smallest number is 4?

(b) What is the probability that the smallest number is 4 and the largest is 8?

(c) If you choose three numbers from 1, . . . , n, what is the probability that the smallest number is j and the largest is k for possible values of j and k?

(a) $C_6^2 = 15$ $\frac{C_6^2}{C_9^2 C_{10}^3} = \frac{6 \times 5}{10 \times 9 \times 8} = \frac{5}{12}$ $\frac{15}{120} = \frac{1}{8}$

(b) $\frac{C_3^1}{C_6^1} = \frac{3}{8}$ $\frac{C_3^1}{C_{10}^3} = \frac{3}{120} = \frac{1}{40}$

(c) $\frac{C_{k-j-1}^1}{C_{n-2}^1} = \frac{k-j-1}{n-2}$ $\frac{C_{k-j-1}^1}{C_n^3} = \frac{(k-j-1) \times 6}{n \times (n-1) \times (n-2)}$