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CS2402 – Lecture8 - In-Class Exercise

Q1. A gambler repeatedly bets 2$ on red at a roulette table, winning 2 dollars with probability 18/38, losing 2 dollars with probability 20/38. He starts with capital 100 dollars, and can borrow money if necessary to keep in the game. Estimate the probability that gambler is not in debt after 100 plays.

X	+2	−2
P	$\frac{18}{38}$	$\frac{20}{38}$

$E(X) = \frac{-4}{38}$ $\quad E(S_{100}) = -\frac{4}{38} \times 100$

$Var(X) = E(X^2) - (EX)^2 = 4 - \frac{4^2}{38^2} = \frac{5760}{38^2}$ $\quad \sigma(S_{100}) = \sqrt{100} \times \frac{\sqrt{5760}}{38}$

$M = 100 + S_{100}$

$P(M>0) = P\left(M > \frac{100 + \frac{4}{38}\times 100}{10\times\frac{\sqrt{5760}}{38}}\right) = P\left(M > \frac{3800 + 4\times 100}{10\times\sqrt{5760}}\right)$

$= P\left(M > \frac{4200}{10\times\sqrt{5760}}\right) = P\left(M > \frac{4200}{758.9}\right)$

$= P(M > 5.53)$

≈ 0

So the gambler would be in debt.

Q2: For the random walk problem in lecture note, estimate the probability that after 20000 steps, the particle ends up less than 100 meters to the right of its starting point.

$E(S) = 0 \quad \sigma(S) = 100\sqrt{2} \times \sqrt{\frac{2}{3}} = \frac{200}{\sqrt{3}}$

$P\left(S < \frac{100-0}{\frac{200}{\sqrt{3}}}\right) = P\left(S > \frac{\sqrt{3}}{2}\right) = 0.8078$

$P\left(S < \frac{0-0}{\frac{200}{\sqrt{3}}}\right) = 0.5$

$\therefore P = 0.8078 - 0.5 = 0.3078$

Q3. Let X equal the weight in grams of a miniature candy bar. Assume that $\mu = E(X) = 24.43$ and $\sigma^2 = Var(X) = 2.20$. Let $\bar{X}$ be the sample mean of a random sample of n = 30 candy bars. Find

(a) $E(\bar{X})$. (b) $Var(\bar{X})$. (c) $P(24.17 \leqslant \bar{X} \leqslant 24.82)$, approximately.

(a). $E(\bar{X}) = \frac{30 \times E(X)}{30} = 24.43$

(b). $Var(\bar{X}) = \frac{1}{30} \times 2.20 = 0.073$

(c) $\frac{24.17 - 24.43}{0.270} = -0.962$ $\quad P = 0.9251 - (1 - 0.8315)$

$= 0.7566$

$\frac{24.82 - 24.43}{0.270} = 1.44$

Q4: (Normal distribution) Let $\bar{X}$ be the mean of a random sample of n = 25 currents (in milliamperes) in a strip of wire in which each measurement has a mean of 15 and a variance of 4. Then $\bar{X}$ has an approximate N(15, 4/25) distribution. Then $P(14.4 < \bar{X} < 15.6)$=?

$P(14.4 < \bar{X} < 15.6)$

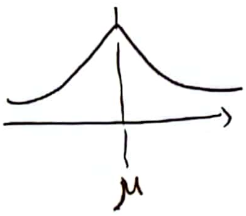
$P(25 * 14.4 < \bar{X} < 25 \times 15.6)$

$\frac{14.4 - 15}{\frac{2}{5}} = -1.5$ $\quad P = (0.9332 - 0.5) \times 2 = 0.8664$

$\frac{15.6 - 15}{\frac{2}{5}} = 1.5$

$P\left(\frac{25 * 14.4 - 25 \times 15}{\sqrt{25 \times 4}} < S^* < \frac{15.6 * 25 - 25 \times 15}{\sqrt{25 \times 4}}\right)$

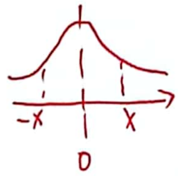
Q5. For the Normal distribution N(μ, σ²), prove E(ξ)= μ



$X \sim N(\mu, \sigma^2)$

$E(X) = \mu$

Integration Needed.



Symmetric

so $E(X) = 0$.

$\int_{-\infty}^{\infty} x f(x)\, dx$