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CS2402 - Lecture 4 - In-Class Exercises

false postive paradox

Q1. The probability of sneezing in any 2 second period is 1/40,000. Suppose that Apple has released an iPhone app that detects if a user has sneezed and then says "Gesundheit". The app claims to be 99.9% reliable. In other words, 0.1% of the time the app detects a sneeze even though the user didn't sneeze. If the app says "Gesundheit", is it more likely that the user sneezed or not sneezed? Why? What is the probability that you actually sneezed if the app said "Gesundheit"? Incidence rate: 1/40,000 = 0.0025% False postive rate: 0.1%

sneeze: $99.9\% \times \frac{1}{40{,}000} \times N + 0.1\% \times \frac{39999}{40000} \times N$

$P = \frac{TP}{TP+FR} = \frac{0.0025\ \%}{0.0025\% + (1-0.0025\%) \times 0.1\%}$

Q2. In a particular college class, 60% of the students are female. 50% of all students in the class have long hair. 45% of the students are female and have long hair. Let F be the event that the student is female. Let L be the event that the student has long hair. One student is picked randomly. Are the events of being female and having long hair independent?

$P(F) = 0.6$

$P(L) = 0.5$

$P(FL) = 0.45 \neq P(F) \cdot P(L) = 0.6 \times 0.5 = 0.3$

so $P(FL) \neq P(F) \cdot P(L)$ so not independent.

Q3. Let X be the number of heads in three tosses of a fair coin.

a) Display the distribution of X in a table. b) Find the distribution of $|X-1|$

X	0	1	2	3
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

X	0	1	2	3
$\lvert X-1\rvert$	1	0	1	2
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$\lvert X-1\rvert$	0	1	2
P	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

$X \sim B(n, p)$ $\frac{1}{6}$

$P(X=r) = C_n^r p^r (1-p)^{n-r}$

Q4. Roll a die 10 times. What is the probability of getting a) no 6s, b) at least two 6s, and c) at most three 6s.

(a). $\left(\frac{5}{6}\right)^{10}$

(b). $1 - \left(\frac{5}{6}\right)^{10} - \frac{5^9 \times 10}{6^{10}}$

(c) $\left(\frac{5}{6}\right)^{10} + C_{10}^1 \left(\frac{5}{6}\right)^9 \left(\frac{1}{6}\right) + C_{10}^2 \left(\frac{5}{6}\right)^8 \left(\frac{1}{6}\right)^2 + C_{10}^3 \left(\frac{5}{6}\right)^7 \left(\frac{1}{6}\right)^3$

$X=Y \Leftrightarrow \overset{\frac{1}{n^2}}{(X=1, Y=1)} \cup \overset{\frac{1}{n^2}}{(X=2, Y=2)} \cup \cdots \cup \overset{\frac{1}{n^2}}{(X=n, Y=n)}$ $\quad P(X=Y) = n \cdot \frac{1}{n^2} = \frac{1}{n}$

$X<Y \Leftrightarrow$ $Y=1$, there is no satisfying $X<Y$ $\boxed{0}$

$Y=2$, there is $X=1$ …… $\boxed{\frac{1}{n^2}}$

$Y=3$, there is $X=1,2$ …… $\boxed{\frac{2}{n^2}}$

$Y=n$, there is $X=1,2 \dots n-1$ $\boxed{\frac{n-1}{n^2}}$

$\frac{1+\cdots+(n-1)}{n^2} = \frac{\frac{(1+n-1)(n-1)}{2}}{n^2} = \frac{n-1}{2n}$

$\max(X,Y)=k$ $\quad Y=k, \ X=1,2 \dots k$ $\quad$ when $X=k$, $Y=k$, not disjoint.

$X=k, \ Y=1,2 \dots k$

so $P = \boxed{\frac{2k-1}{n^2}}$ $(X=Y=k)$

$X+Y=k, \ 2 \le k \le 2n$

$k \le n+1$, $\underline{(1, k-1)\ (2, k-2) \cdots (k-1, 1)} \rightarrow \frac{k-1}{n^2}$

$k \ge n+2$, $\underline{(k-n, n) \cdots (n, n-k)} \rightarrow \frac{(n-(k-n)+1)}{n^2} = \frac{2n-k+1}{n^2}$

Q5. Let X and Y be independent, each uniformly distributed on $\{1, 2, \dots, n\}$. Find:

a) $P(X=Y)$; b) $P(X<Y)$; c) $P(max(X,Y)=k)$ for $1 \le k \le n$;

d) $P(X+Y=k)$ for $2 \le k \le 2n$

(a) $P(X=Y) = \frac{1}{n}$

(b) $P(X<Y) = \frac{C_n^2}{C_n^2 \times A_2^2} = \cancel{\frac{1}{2}}$ $\quad \frac{n-1}{2n}$

(c) $P(\max(X,Y)=k) = \frac{1}{n} \times \frac{1}{k-1} \times 2 = \cancel{\frac{2}{n(k-1)}}$ $\quad \frac{2k-1}{n^2}$

(d) $P(X+Y=k) = \frac{k-1}{n} \times A_2^2 = \frac{2(k-1)}{n}$

if $k \le n+1$, $P = \frac{k-1}{n^2}$

if $k \ge n+2$, $P = \frac{2n-k+1}{n^2}$