# Final 7 December Autumn 2020, questions

Discrete Mathematics (City University of Hong Kong)

# Academic Honesty Pledge

I pledge that the answers in this exam/quiz are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- I will not plagiarize (copy without citation) from any source;
- I will not communicate or attempt to communicate with any other person during the exam/quiz; neither will I give or attempt to give assistance to another student taking the exam/quiz; and
- I will use only approved devices (e.g., calculators) and/or approved device models.

I understand that any act of academic dishonesty can lead to disciplinary action.

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Date: 2020.12.9

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Question 1:

1. Negation: $\neg \forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$

$\equiv \exists x(\neg(\neg P(x) \wedge Q(x)) \rightarrow R(x))$

$\equiv \exists x(\neg\neg(\neg P(x) \wedge Q(x)) \vee R(x))$

$\equiv \exists x((\neg P(x) \wedge Q(x)) \vee R(x))$

2. This argument is valid.

Let a: John can solve this problem
b: John's clever
c: John's ~~is~~ lucky
d: John understands logic
e: John gets an A.

i. John is not lucky

Premises: $\neg c$ ①

$a \rightarrow b \vee c$ ②

$b \rightarrow d$ ③ disjunctive syllogism

①+② $a \rightarrow b$ ④

④+③ $a \rightarrow d$ ⑤ hypothetical syllogism

⑤ $a \rightarrow d \vee e$ addition.

Conclusion: $a \rightarrow e \vee d$.

ii. John gets an A in this course

Premises: $e$ ①

$a \rightarrow b \vee c$ ②

$b \rightarrow d$ ③

If a is true, $a \rightarrow e$ is true. If a is false, $a \rightarrow e$ is also true. As a result, $a \rightarrow e$ is true.

$\therefore a \rightarrow e \vee d$ is true

Conclusion: $a \rightarrow e \vee d$

$\therefore$ This argument is valid.

~~3. (BS) when $n=0$, $0^3 = 0 < 1 = 3^0$. The statement holds true.~~

~~(IS) Assume when $n=k$, the statement is true.~~

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3. $\because$ n is an integer greater than 6

$\therefore$ (BS) When $n=7$, $3^7 = 2187 < 5040 = 7!$. The statement is true.

(IS) Assume when $n=k$, $3^k < k!$ is true.

when $n=k+1$, $3^{k+1} = 3 \cdot 3^k < 3 \cdot k!$

$\because n \geq 7$

$\therefore 3 < k+1$

$\therefore 3 \cdot k! < (k+1)k! = (k+1)!$

$\therefore 3^{k+1} < (k+1)!$

In Conclusion, the statement is proved.

Question 2:

(a) $\because f(x)\tilde{R}f(y) \leftrightarrow xRy$, $\tilde{R}$ is an equivalence relation.

$\therefore$ $f(a)\tilde{R}f(a) \leftrightarrow aRa$ : R is reflexive

$f(y)\tilde{R}f(x) \leftrightarrow yRx$ : R is symmetric

$f(x)\tilde{R}f(y) \wedge f(y)\tilde{R}f(z) \rightarrow f(x)\tilde{R}f(z)$

i.e. $xRy \wedge yRz \rightarrow xRz$ : R is transitive

$\therefore$ R is an equivalence relation on S

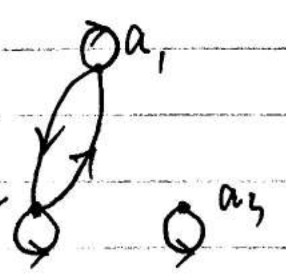
(b)

(a) $\tilde{R} = \{(b_3, b_1), (b_3, b_3), (b_2, b_2), (b_1, b_1), (b_1, b_3)\}$

$\therefore R = \{(a_1, a_2), (a_1, a_1), (a_3, a_3), (a_2, a_2), (a_2, a_1)\}$

(b)

Directed ~~dig~~ diagram:



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Question 3:

1. $|A\cup B\cup C\cup D|$

$= |A|+|B|+|C|+|D| - (|A\cap B|+|A\cap C|+|A\cap D|+|B\cap C|+|B\cap D|+|C\cap D|)$
$+ (|A\cap B\cap C| + |A\cap B\cap D| + |A\cap C\cap D| + |B\cap C\cap D|) - |A\cap B\cap C\cap D|$
$= 1000+1000+1000 - 100\times 6 + 4\times 10 - 1$
$= 2439$

2. (a) $C(6,0)C(4,4)+C(6,1)C(4,3)+C(6,2)C(4,2)$
$+C(6,3)C(4,1)+C(6,4)C(4,0)$
$= 1\times 1 + 6\times 4 + 15\times 6 + 6\times 4 + 15$
$= 154$

(b) $C(6,0)C(4,4)+C(6,1)C(4,3)+C(6,2)C(4,2)$
$= 115$

(c) i. Not to choose either one of the married couple.
$C(3,2)C(5,2)$
ii. Choose the male of the married couple
$C(5,1)C(3,2)$
iii. choose the female of the married couple
$C(5,2)C(3,1)$

There are 75 possible ways.

Question 4:

Set $b_n = \ln a_n$, then $\ln a_n = \ln(a_{n-1}a_{n-2}^6)$
$\ln a_n = \ln a_{n-1} + 6\ln a_{n-2}$
$b_n = b_{n-1} + 6b_{n-2}$

The characteristic equation of $b_n$ is $r^2 - r - 6 = 0$

Solve it we get $r_1 = 3$ $r_2 = -2$

$\therefore b_n = \beta_1 3^n + \beta_2(-2)^n$ $\left\{ \beta_1 = \frac{b_1 - b_0 r_2}{r_1 - r_2}, \beta_2 = \frac{b_0 r_1 - b_1}{r_1 - r_2} \right.$

$b_0 = \ln a_0 = \ln 1 = 0$, $b_1 = \ln a_1 = \ln 2$

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Solve the equation we get $\begin{cases} \beta_1 = \frac{\ln 2}{5} \\ \beta_2 = -\frac{\ln 2}{5} \end{cases}$

$\therefore b_n = \frac{\ln 2}{5} \cdot 3^n - \frac{\ln 2}{5} \cdot (-2)^n$

~~the result~~ $\therefore \ln a_n = \frac{\ln 2}{5} \cdot 3^n - \frac{\ln 2}{5} \cdot (-2)^n$

Question 5: $\therefore a_n = e^{\frac{\ln 2}{5} \cdot 3^n - \frac{\ln 2}{5} \cdot (-2)^n}$

2. Prove by Mathematical induction:

(BS) when $r=1$: $LHS = \binom{n+2}{1} = n+2$ $RHS = \sum_{k=0}^{1} \binom{n+k}{k}$

$= \binom{n}{0} + \binom{n+1}{1} = 1 + n + 1 = n+2$

$LHS = RHS$

(IS) Assume when $r = r_0$ $\binom{n+r_0+1}{r_0} = \sum_{k=0}^{r_0} \binom{n+k}{k}$ is true.

when $r = r_0 + 1$

$RHS = \sum_{k=0}^{r_0+1} \binom{n+k}{k} = \sum_{k=0}^{r_0} \binom{n+k}{k} + \sum_{k=r_0}^{r_0+1} \binom{n+k}{k}$

$= \binom{n+r_0+1}{r_0} + \binom{n+r_0+1}{r_0+1}$

$\overset{\text{Pascal's rule}}{=\!=} \binom{n+(r_0+1)+1}{r_0+1} = LHS$

$\therefore$ The statement holds true.

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