CITY UNIVERSITY OF HONG KONG

Course code and title : MA2184 Discrete Mathematics for Computing

Session : Semester B, 2007-2008

Time allowed : Two Hours

This paper has FIVE pages (including this page).

Instructions to candidates:

1. This paper has FIVE questions.
2. Attempt ALL questions.
3. Each question carries 22 marks and the paper has 110 marks in total.
4. The maximum obtainable mark is 100 marks.
5. Start each question on a new page.
6. Show ALL workings.

Materials, aids & instruments which students are permitted to use during examination: Approved calculators

Do not remove this from exam

Answer ALL Questions

Question 1

(a) Use *Proof by Contradiction* to show the validity of $\dfrac{\forall x(P(x) \rightarrow \sim Q(x))}{\therefore \quad \sim \exists x(P(x) \wedge Q(x))}$.

(10 marks)

(b) The following derivation is to show the validity of $\dfrac{\forall x(P(x) \rightarrow Q(x))}{\therefore \exists x P(x) \rightarrow \exists x Q(x)}$. However, there is a mistake in the derivation, find it and provide a correct proof using *cp rule*.

1	$\forall x(P(x) \rightarrow Q(x))$	p
2	$P(c) \rightarrow Q(c)$	$1, ui$
3	$\exists x P(x)$	add p
4	$P(c)$	$3, ci$
5	$Q(c)$	$2, 4$
6	$\exists x Q(x)$	$5, eg$
7	$\exists x P(x) \rightarrow \exists x Q(x)$	3,6 cp rule

(6 marks)

(c) Does there exist a simple graph with five vertices of the following degrees? If yes, please draw the graph. If no, state your reason.

(i) $(0, 1, 2, 2, 3)$

(3 marks)

(ii) $(1, 1, 2, 2, 3)$

(3 marks)

Question 2

(a) Simplify $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)) \cap \mathcal{P}(\emptyset))$. ($\mathcal{P}(A)$ is the power set of A)

(5 marks)

(b) Let A be a non-empty set, R and S are binary relations on $A \times A$. State whether the following statements true or not. If yes, give a proof, if no, give a counter example.

(i) If R and S are reflexive, then $R \cup S$ is reflexive.

(3 marks)

(ii) If R and S are transitive, then $R \cup S$ is transitive.

(3 marks)

(iii) If R and S are antisymmetric, then $R \cap S$ is antisymmetric.

(3 marks)

(c) Let $f : A \to B$ and $g : B \to C$ be functions.

(i) Prove that if $g \circ f$ is surjective, then g is surjective.

(4 marks)

(ii) Prove that if $g \circ f$ is injective, then f is injective.

(4 marks)

Question 3

(a) It is given 13 integers $c_1, c_2, \ldots, c_{13}$ (some of them may be the same). Use pigeonhole principle to prove that there exist i and j with $0 \leq i < j \leq 13$ such that $c_{i+1} + c_{i+2} + \cdots + c_j$ is divisible by 13, for example, $c_4 + c_5 + c_6 + c_7$ is divisible by 13. (Hint: consider the following 13 integers

$$\begin{aligned} n_1 &= c_1 \\ n_2 &= c_1 + c_2 \\ &\cdot \\ &\cdot \\ n_{13} &= c_1 + c_2 + \cdots + c_{13} \end{aligned}$$

and their remainder when divided by 13)

(14 marks)

(b) How many solutions are there to the equation $x_1 + x_2 + x_3 = 30$, where x_1, x_2 and x_3 are integers such that $x_1 \geq 3, x_2 \geq 5$ and $3 \leq x_3 \leq 14$?

(8 marks)

Question 4

(a) Find a recurrence relation for the number of ways to climb n stairs if stairs can be climbed two or three at a time. Also, determine the initial condition(s) to solve the recurrence relation. (You are not required to solve the recurrence relation)

(6 marks)

(b) Find the solution for the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 2^n$ with initial conditions $a_0 = 0$ and $a_1 = 3$.

(11 marks)

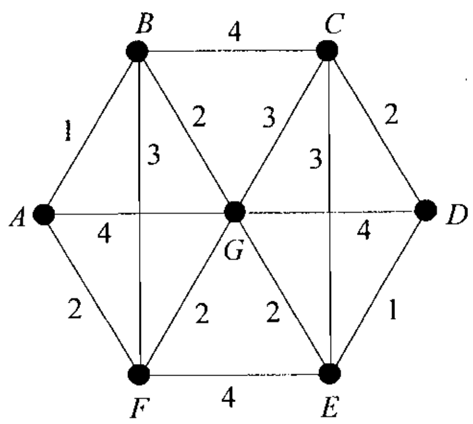
(c) A graph is called *self-complementary* if it is isomorphic to its complement. State, with reason, whether a graph with 22 vertices is self-complementary or not.

(5 marks)

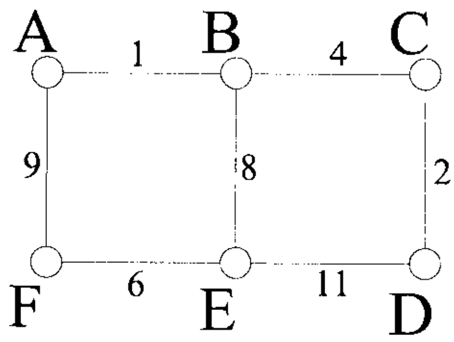
Question 5

(a) Use Dijkstra's algorithm to find the shortest path from A to D in the following weighted graph.

(10 marks)



(b) Consider the weighted graph G



(i) Write down the adjacency matrix of G.

(3 marks)

(ii) Use Prim's algorithm with starting at D to find a minimal spanning tree.

(9 marks)

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