CITY UNIVERSITY OF HONG KONG

Department of Mathematics

Course Code & Title	:	MA1300 Enhanced Calculus and Linear Algebra I
Session	:	Semester A, 2015-2016
Time Allowed	:	Three Hours

This paper has **Two** pages. (including this cover page)

Instructions to candidates:

1. Answer **all** questions.
2. Start each main question on a new page.
3. Show all step.

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*This is a **closed-book** examination.*

Candidates are allowed to use the following materials/aids:

Non-programmable portable battery operated calculator.

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

1. Solve the following problems:

   (a) [7 marks] Compute $\lim_{x\to 0+}(x\ln^2 x)$.

   (b) [7 marks] Compute $\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x$.

   (c) [8 marks] Let $f(x)=(x^3+\sin x)^{1/x}$, find $f'(x)$.

   (d) [8 marks] If $g(x)+x\sin g(x)=x^2$, find $g'(0)$.

2. [15 marks] Prove that $\sin x > x-\frac{x^3}{6}$ for all $x>0$.

3. [15 marks] Suppose $f(x)$ is continuous on $[1,5]$ and the only solutions to the equation $f(x)=6$ are $x=1$ and $x=4$. If $f(2)=8$, prove that $f(3)>6$.

4. [15 marks] A sequence $\{a_n\}$ is given by $0<a_1<1$ and $a_{n+1}=a_n(2-a_n)$.

   (a) Apply the Monotonic Sequence Theorem to show that $\lim_{n\to\infty} a_n$ exists.

   (b) Find $\lim_{n\to\infty} a_n$.

5. [15 marks] Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n+1}-\sqrt{n}}{2^n}x^{2n}.$$

6. [10 marks] Let $0<a<b$. If a function f is continuous on the interval $[a,b]$ and differentiable on (a,b), prove that there exists some $\xi\in(a,b)$ such that

$$\frac{af(b)-bf(a)}{b-a}=\xi f'(\xi)-f(\xi).$$

- End -