F1 Math ME 20 21 - Mid Exam

Discrete Mathematics (City University of Hong Kong)

Mid-year Examination 2020/21
Mathematics

Form One

Name: _____

Time allowed : 1 hour 45 minutes

Class: _____ ( )

Paper Total: 100 marks

Instructions

1. Answer ALL questions.
2. All answers in Section A should be marked in **pencil** on the MC answer sheet provided or marks will be deducted.
3. Answer Sections B and C with blue/black ball pen in the Answer Book.
4. All diagrams and graphs must be drawn neatly in **pencil**.
5. The diagrams are not necessarily drawn to scale.
6. Unless otherwise specified, numerical answers should either be exact or correct to 1 decimal place.
7. Anything written on the rough work sheet provided **will not be marked**.

Section A Multiple choice (20 marks)

1. What is the product of 16 and the difference of 12 and 8?

   A. -64

   B. 4

   C. 64

   D. 184

2. Find the L.C.M. of 21, 42 and 49.

   A. 7

   B. 294

   C. 882

   D. 6174

3. Arrange $-0.7, -\frac{2}{3}, \frac{3}{8}$ and $-\frac{2}{5}$ in descending order.

   A. $\frac{3}{8} > -\frac{2}{5} > -\frac{2}{3} > -0.7$

   B. $\frac{3}{8} > -0.7 > -\frac{2}{3} > -\frac{2}{5}$

   C. $-0.7 < -\frac{2}{3} < -\frac{2}{5} < \frac{3}{8}$

   D. $-\frac{2}{5} < -\frac{2}{3} < -0.7 < \frac{3}{8}$

4. Which of the following is a prime number?

   A. 63

   B. 65

   C. 67

   D. 69

5. Which of the following has the greatest value?

   A. $4 + (-4)(-5)$

   B. $4 - (-4)(-5)$

   C. $4 + (-4) + (-5)$

   D. $4 - (-4) - (+5)$

6. Kelly first walks 5 m due north, then 25 m due south and finally 10 m due north again. Find the distance between the starting point and her final position.

   A. 10 m

   B. 20 m

   C. 30 m

   D. 40 m

7. "Subtract *a* cubed from *b* squared, then divide *a* by the difference" can be written algebraically as

   A. $\frac{a}{b^2 - a^3}$.

   B. $\frac{a}{a^3 - b^2}$.

   C. $\frac{b^2 - a^3}{a}$.

   D. $\frac{a^3 - b^2}{a}$.

8. Given the formula $S = 2(5a - 6b)$, find the value of *S* if $a = -4$ and $b = -5$.

   A. -20

   B. -100

   C. 12

   D. 20

9. $(-14) + [(-6) - (-8) \div (-2)] =$

   A. -4.

   B. -15.

   C. -24.

   D. 4.

10. For the polynomial $-3xy^2 - x^2 + xy^6$, which of the following is / are correct?

    I. The degree of the polynomial is 6.
    II. The constant term is 0.
    III. The coefficient of x^2 is 1.

    A. I only

    B. II only

    C. III only

    D. II and III only

11. Find the constant term of $4a - 5 - 4(a^2 - 7)$.

    A. -5

    B. -12

    C. -33

    D. 23

12. Simplify $(3q^2 - 4p^2 - 2q^3) - [p^2 - (q^2 - q^3)]$.

    A. $2q^2 - 5p^2 - 3q^3$

    B. $4q^2 - 5p^2 - q^3$

    C. $4q^2 - 5p^2 - 3q^3$

    D. $4q^2 - 3p^2 - 3q^3$

13. Which of the following give a negative value?

    I. $-\frac{(-7)(+35)}{(-4)}$

    II. $[4^2 - (-4)^2](-5)$

    III. $\frac{[-(-6)^3]}{-5^3}$

    A. I and II only

    B. I and III only

    C. II and III only

    D. I, II and III

14. Find the value of $-4^2 \times \frac{5^3}{(-5)^2}$.

    A. -24

    B. -80

    C. 24

    D. 80

15. Suppose that the $(n + 1)^{th}$ term of a sequence is formed by adding 3 to the n^{th} term. If the 6^{th} term of the sequence is 26, find the 12^{th} term of the sequence.

A. 8

B. 29

C. 41

D. 44

16. Simplify $(\frac{2a^4}{b^2})^3 \div (-ab)^4$.

A. $\frac{8a^8}{b^{10}}$

B. $-\frac{8a^8}{b^{10}}$

C. $\frac{6a^8}{b^{10}}$

D. $\frac{8a^{11}}{b^{10}}$

17. What is the remainder when the product of $5123 \times 6414 \times 4583 \times 5412$ is divided by 5?

A. 1

B. 2

C. 3

D. 4

18. Solve the equation $\frac{5x-3}{3} = \frac{10x+3}{7}$.

A. $-\frac{6}{11}$

B. -6

C. $\frac{6}{11}$

D. 6

19. If x, y and z are different positive integers and $xyz = 48$. What is the smallest possible value of $x + y + z$?

A. 11

B. 12

C. 13

D. 15

20. Mary cycles with a speed of 10 km/h from City A to City B and then 15 km/h from City B to City C. The distance from City B to City C is four times the distance from City A to City B. If Mary cycles for 11 hours, find the total distance travelled from City A to City C.

A. 30 km

B. 100 km

C. 120 km

D. 150 km

End of Section A

Section B Short questions (40 marks)

1. If $N = 437\blacklozenge58$ is a six-digit number divisible by 9, find the largest value of N.
   Show your steps clearly. (2 marks)

2. Calculate the following expressions.

   (a) $-24 - (+16) \div (-8)$ (2 marks)

(b) $\dfrac{-6^2-(-1)}{(-2)(+3)} \div (-2\dfrac{4}{5})$ (3 marks)

(c) $-[4-2(-3.8+6.3)]-\left(\dfrac{1}{3}-\dfrac{3}{2}\right)\times 0.4$ (4 marks)

3. Write down the algebraic expressions of each of the following statements.

(a) Add x to the product of m and n. (2 marks)

(b) Subtract the square of k from –6, then divide the sum of twice of c and y by the result. (2 marks)

4. The general term of a sequence is $T_n = \dfrac{1-3n}{n+2}$. Find $T_5 - T_2$. (3 marks)

5. (a) Simplify the algebraic expression $\dfrac{3a+7a^2 \div a}{2+b\times a-a(b)(4)}$. (3 marks)

(b) Hence, find the value of the algebraic expression in (a) if $a=\dfrac{1}{2}$ and $b=-6$. (2 marks)

6. Solve the following equations.

(a) $4(5k-3)-(7+12k)=-21$ (3 marks)

(b) $\dfrac{4x-7}{6}+\dfrac{x+5}{8}=-\dfrac{2x}{3}$ (4 marks)

7. Simplify the following expressions and express your answers with positive indices.

(a) $-2k^3(-5k^3)$ (1 mark)

(b) $\dfrac{(-a)^{223}}{-a^{224}}$ (1 mark)

(c) $(-2y^3)^4$ (1 mark)

(d) $\dfrac{x^3(2x^5)(-8x)}{6^3x^2}$ (2 marks)

(e) $[(p^2)^4]^8 \div 9p^3 \div 6p$ (2 marks)

(f) $-(3-\dfrac{r^2}{6})-(\dfrac{7}{5}+3r-\dfrac{4r^2}{3})$ (3 marks)

End of Section B

Section C Long questions (40 marks)

8. Consider the polynomial $A = 5xy^n - 2x^3y^3 + 6x^2 - 7$, where n is a positive integral constant.

(a) (i) Write down the constant term of A. (1 mark)

(ii) If the degree of A is 6, write down all possible values of n. (2 marks)

(iii) Write down the coefficient of x^3. (1 mark)

(iv) Arrange the terms of A in ascending powers of x. (1 mark)

(b) Consider another polynomial $B = -4y^3x^3 + y^2 + nx^3y^3 - 6x^2$. It is given that the coefficient of x^3y^3 is -2. Let $C = B - A$.

(i) Find the value of n. (2 marks)

(ii) Using the value of n in (b)(i), simplify C. (2 marks)

(iii) Hence, find the value of C when $x = -\frac{1}{2}$ and $y = 3$. (2 marks)

9. All F.1 students are arranged to attend a workshop in three venues. There are x students in Room A. The number of students in Room B is 39 more than half of that in Room A. The number of students in Room C is 5 less than that in Room B.

(a) Express the number of F.1 students in Room B in terms of x. (2 marks)

(b) Write down a formula for the total number (N) of all F.1 students. (3 marks)

(c) If there are 217 F.1 students in the school, using the result in (b), which room has the least number of students? How many students are there in that room? (5 marks)

10. A piece of wire with a length of 1.6 m is cut into two parts and bent into two different squares. The side length of the smaller square is 6 cm shorter than that of the larger square. Let x cm be the side length of the smaller square.

(a) Express the sum of the perimeters of the two squares in terms of x. (3 marks)

(b) Hence, find the area of smaller square in cm^2. (4 marks)

(c) Someone claims that the difference in areas of the two squares is less than 250 cm^2. Do you agree? Explain your answer. (2 marks)

11. In a rock-paper-scissors game, two players Amy and Joyce take 10 moves. In each move, 5 marks are awarded to the winner and 3 marks are deducted from the loser. If it is a draw, both players get 2 marks.

    (a) If Amy has 5 wins and 4 draws, find the score of Joyce. (3 marks)

    (b) If Amy has k wins, Joyce has 4 more wins than Amy and there are no draws,

        (i) find the value of k. (2 marks)

        (ii) Hence, or otherwise, find the difference between their scores. (3 marks)

    (c) If Joyce has 4 wins and 1 draw in the first 5 moves, is it possible that Amy and Joyce get the same score in the game? Explain your answer. (2 marks)

End of Paper