CITY UNIVERSITY OF HONG KONG

Department of Mathematics

Course Code & Title	:	MA1301 Enhanced Calculus and Linear Algebra II
Session	:	Semester B, 2012-2013
Time Allowed	:	Two Hours

This paper has **Three** pages. (including this cover page)

Instructions to candidates:

1. Answer **all** questions.
2. Start each main question on a new page.
3. Show all step.

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*This is a **closed-book** examination.*

Candidates are allowed to use the following materials/aids:

Non-programmable portable battery operated calculator.

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

1. Calculate the following quantities

   (a) [5 marks] $\lim_{n\to\infty} \dfrac{1^p + 2^p + \cdots + n^p}{n^{p+1}}, \qquad p \geq 0.$

   (b) [5 marks] $\lim_{h\to 0} \dfrac{1}{h} \displaystyle\int_2^{2+h} \sqrt{t^3+2}\, dt.$

   (c) [8 marks] $\displaystyle\int_0^{\frac{\pi}{4}} x \tan^2 x \, dx.$

   (d) [8 marks] $\displaystyle\int \frac{dx}{(4x - x^2)^{\frac{3}{2}}}.$

   (e) [9 marks] $\displaystyle\int \frac{1}{\sqrt[3]{x} - x}\, dx.$

2. (i) [5 marks] Find all the roots of $(-8 - 8\sqrt{3}\,i)^{\frac{1}{4}}$.

   (ii) [5 marks] Find the modulus and argument of $(\sqrt{3} - i)^6$.

3. [10 marks] Find the moments M_x and M_y and the center of mass of a thin plate covering the region bounded by the parabola $y = x - x^2$ and the line $y = -x$. Assume the density ρ of the plate is 10.

4. (i) [7 marks] Show that the triangle with vertices $P(4,3,6)$, $Q(-2,0,8)$, $R(1,5,0)$ is a right triangle and find its area.

   (ii) [8 marks] Given arbitrary vectors $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$, $\vec{c} = (c_1, c_2, c_3)$ and $\vec{d} = (d_1, d_2, d_3)$ in R^3, show that these four vectors must be linearly dependent.

5. [15 marks] Let

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}, \quad P = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 3 \\ 0 & -1 & -1 \end{pmatrix}$$

   (i) Find the values of λ such that $|\lambda I - A| = 0$.

   (ii) Compute $P^{-1}AP$.

6. [15 marks] Consider the following linear system

$$\begin{cases} (2-\lambda)x_1 + 2x_2 - 2x_3 = 1, \\ 2x_1 + (5-\lambda)x_2 - 4x_3 = 2, \\ -2x_1 - 4x_2 + (5-\lambda)x_3 = -\lambda - 1. \end{cases}$$

When does this system have (i) no solution, (ii) a unique solution, (iii) infinitely many solutions? Solve the system in terms of λ when it is applicable.

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