

MA2185 Assignment Two

Question 1

Show that if A, B, and C are sets, then

- a) $A - B = A \cap \bar{B}$.
- b) $(A \cap B) \cup (A \cap \bar{B}) = A$.
- c) $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$.

Question 2

Which of these relations on the set of all functions from Z to Z are equivalence relations? Determine the properties of an equivalence relation that the others lack.

- a) $\{(f, g) \mid f(1) = g(1)\}$
- b) $\{(f, g) \mid f(0) = g(0) \text{ or } f(1) = g(1)\}$
- c) $\{(f, g) \mid f(x) - g(x) = 1 \text{ for all } x \in Z\}$
- d) $\{(f, g) \mid \text{for some } C \in Z, \text{ for all } x \in Z, f(x) - g(x) = C\}$
- e) $\{(f, g) \mid f(0) = g(1) \text{ and } f(1) = g(0)\}$

Question 3

a) Let $g(x) = \lfloor x \rfloor$ (greatest integer less than or equal to x). Find

- i) $g^{-1}(\{0\})$.
- ii) $g^{-1}(\{-1, 0, 1\})$.
- iii) $g^{-1}(\{x \mid 0 < x < 1\})$.

b) Let f be a function from A to B. Let S be a subset of B. Show that $f^{-1}(\bar{S}) = \overline{f^{-1}(S)}$.

Question 4

a) A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes

- i) are there in total?
- ii) contain exactly two heads?
- iii) contain at most three tails?
- iv) contain the same number of heads and tails?

b) How many permutations of the letters ABCDEFG contain

- i) the string BCD?
- ii) the string CFGA?
- iii) the strings BA and GF?
- iv) the strings ABC and DE?
- v) the strings ABC and CDE?
- vi) the strings CBA and BED?

Question 5

- (a) How many ways are there for a horse race with three horses to finish if ties are possible? [Note: Two or three horses may tie.]
- (b) There are six runners in the 100-yard dash. How many ways are there for three medals to be awarded if ties are possible? (The runner or runners who finish with the fastest time receive gold medals, the runner or runners who finish with exactly one runner ahead receive silver medals, and the runner or runners who finish with exactly two runners ahead receive bronze medals.)