MA2185+Assignment+one+

Discrete Mathematics (City University of Hong Kong)

MA2185 Assignment One
Name and Student ID:
Due Date: **September 24, 2021 11:59pm Hong Kong Time Zone**
There are **5 questions** in total.
Instruction: Please **SHOW ALL YOUR WORK TO RECEIVE FULL MARKS**

Question 1

Are the following pairs of compound propositions logically equivalent to each other? Give reason to your answer.

(a) $p \to (q \vee r)$, $(p \to q) \vee r$
(b) $\sim\{\sim[(p \vee q) \wedge r] \vee \sim q\}$, $q \wedge r$
(c) $(p \to q) \vee (r \wedge s)$, $[(p \to r) \wedge (p \to s)] \vee q$?

$\Leftrightarrow \sim\{\sim[(p \vee q) \wedge r] \vee \sim q\}$
$\Leftrightarrow \sim\{\sim(p \vee q) \vee \sim r \vee \sim q\}$
$\Leftrightarrow (p \vee q) \wedge r \vee q$
$\Leftrightarrow [(p \vee q) \wedge (F \vee q)] \wedge r$
$\Leftrightarrow [(p \wedge F) \vee q] \wedge r$
$\Leftrightarrow q \wedge r$

Question 2

Use truth tables to verify the following laws:
(a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.
(b) $\sim(p \vee q) \equiv \sim p \wedge \sim q$.

Question 3

(a) Let P = "there is a chance of rain", Q = "Sandra's blue headband is missing", R = "Sandra will mow her lawn" and S = "the temperature is over 30°C".

Determine the validity of the following argument by using logical inference:

"*If there is a chance of rain or her blue headband is missing, then Sandra will not mow her lawn. Whenever the temperature is over 30℃, there is no chance for rain. Today the temperature is over 30°C and Sandra is wearing her blue headband. Therefore (sometime today) Sandra will mow her lawn.*"

(b) Determine the validity of the following argument.

"*If Josh works hard and gets the supervisor's position, then he will get a promotion. If he gets a promotion, he will buy a new dog. In fact, Josh worked very hard and has already purchased a new dog. Therefore, he got the supervisor's position.*"

1. $(p \wedge q) \to r$
2. $r \to s$
3. $p \wedge s$

$\therefore q$

4. $p \equiv T$ ③
5. $s \equiv T$ ③
6. $q \to r \equiv T$ ④①

p = "Josh works hard"
q = "Josh gets the supervisor's position"
r = "Josh gets a promotion"
s = "Josh buys a new dog"

Question 4

(a) Consider the following argument:

If $r \equiv T$ then all the premises are true no matter what the truth value of q is. $\therefore q \equiv F$ the conclusion is false and argument is invalid.

1. $\forall x[M(x) \to C(x)]$
2. $\exists x[M(x) \wedge H(x)]$ ←
3. $\forall x[E(x) \to H(x)]$
4. $\therefore \exists x[E(x) \wedge C(x)]$

"All the students who have got a pass in discrete mathematics are clever. Some of the students who have got a pass in discrete mathematics work very hard. All the students who are good at English also work very hard. Therefore, some of the students who are good at English are clever."

Let U be the set of all students, $M(x)$ = "x has got a pass in mathematics", $C(x)$ = "x is clever", $H(x)$ = "x works very hard", $E(x)$ = "x is good at English".

(i) Rewrite the above argument using predicates and quantifiers.
(ii) Determine the validity of the above argument by using logical inference.

(b) Let $P(x)$ and $Q(x)$ be predicates in the variable x where the universe of discourse is the set of integers. Let R and S be propositions defined as

$$R = \exists x[P(x) \wedge Q(x)] \text{ and } S = \exists x P(x) \wedge \exists y Q(y).$$

(i) Give an example that R and S have different truth values.
(ii) What is the relation between R and S? Give reason to your answer.

(c) Determine whether the following argument is valid.

$$\begin{array}{l} \exists x[P(x) \wedge Q(x)] \to \forall y[R(y) \to S(y)] \\ \exists x[R(x) \wedge \sim S(x)] \\ \hline \forall x[P(x) \to \sim Q(x)] \end{array}$$

5. $M(a) \wedge H(a)$ (2) ei
6. $M(a) \equiv T$ (5)
7. $H(a) \equiv T$ (5)
8. $M(a) \to C(a)$ (1) ui
9. $C(a) \equiv T$ (6)(8)

If $E(a) \equiv F$ then (3) R still true but (4) becomes F
$\therefore$ invalid.

Question 5

(a) Show that

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2^n - 1} + \frac{1}{2^n} \geq 1 + \frac{n}{2},$$

whenever n is a nonnegative integer.

(b) Prove that for all integers $n \geq 1$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} < 2\sqrt{n}.$$

(c) Prove that for all integers $n \geq 1$,

$$\frac{3}{1\times 2} + \frac{4}{2\times 3} + \frac{5}{3\times 4} + \cdots + \frac{n+1}{(n-1)n} + \frac{n+2}{n(n+1)} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} + \frac{1}{n} + \frac{n}{n+1}.$$

-End-