Midterm-2021 - midterm exam

Introduction to Computational Probability Modeling (City University of Hong Kong)

CITY UNIVERSITY OF HONG KONG

Course code & title : CS2402
Introduction to Computational Probability Modelling

Session : Semester B 2020/21

Time allowed : 2 hours

This paper has 6 pages (including this cover page).

---

1. This paper consists of 21 questions.
2. Answer ALL questions.

---

*This is a **closed-book** examination. You are allowed one two-sided cheat-sheet only.*

Student ID: ____________________________________

Student Name: ____________________________________

Student EID: ____________________________________

Questions	**1-12**	**13**	**14**	**15**	**16**	**17**
Marks						
Max	**48**	**6**	**5**	**5**	**5**	**5**
Questions	**18**	**19**	**20**	**21**		**Total**
Marks						
Max	**6**	**5**	**5**	**10**		**100**

For Q1-Q12, Fill in the answer in the space provided on the left. For multiple-choice questions, choose the most appropriate answer.

_______ **Q1** *Suppose the rule in the unfinished game is that the pot goes to the player who first gets 4 points when playing a fair coin (in each round, they flip a coin and the winner gets one point). If the game stops when Alice has 3 points and Bob has 1 point, in what ratio, the pot should be divided between Alice and Bob?*

_______ **Q2** *What is the variance of the random variable X. Its corresponding probabilities are as follows.*

X	*P(X)*
1	*0.3*
2	*0.2*
3	*0.5*

_______ **Q***3 P(AUB)=0.7, P(A)=0.4, P(B)=0.5. P(AB)=?*

Q4 *Suppose I have two hard-disks on my computer. I set the computer up so that the* 2^{nd} *hard-disk has an exact copy of the* 1^{st} *hard-disk. Assuming the failure rate of each hard-disk is 0.6%, and these two disks are independent of each other. Provide the following probabilities:*

_______ · *I do not lose my data as at least one hard-disk works.*

_______ · *Only one disk fails.*

_______ **Q5** *If you throw a dice twice, which is the probability that the sum of the two obtained numbers is strictly larger than 9.*

_______ **Q6** *In 1980s, the NASA engineers estimated the probability of an accident in the first 25 Shuttle missions as 1/100. What is the probability of any accidents occurring in 25 missions?*

a) $1 - (0.99)^{25}$

---

b) $(1 - 0.99)^{25}$

c) $(0.01)^{25}$

d) $1 - (0.01)^{25}$

e) $(1 - 0.01)^{25}$

Q7 *There is a list of nonnegative numbers with an average of 100. At most how much percentage of these numbers are not smaller than 1000?*

______

Q8 *In a game of Roulette, you decide to bet $100 on the number "6". If the ball lands on "6" the payout is 34 times your bet (you will win $3400). Otherwise, you will lose your bet. There are 38 numbers on the Roulette wheel, and each number has equal probability. What is your expected gain for this bet?*

______

a) *$6.26*
b) *-$6.26*
c) *-$5.26*
d) *$100*
e) *$3400*
f) *others*

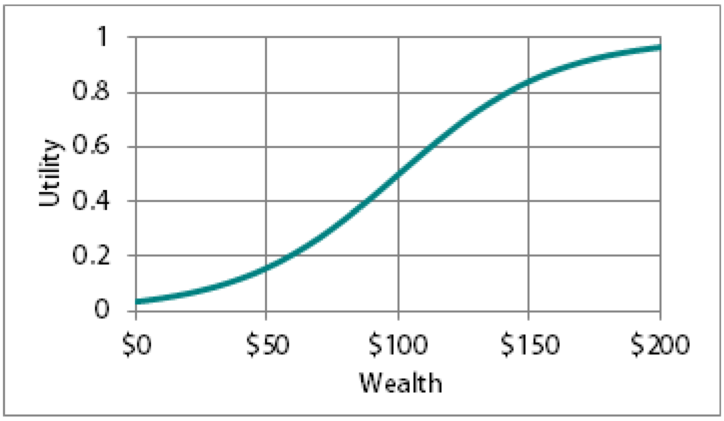
Q9 *Among families with 2 children, one of whom is a* **girl** *born at some time pm, what is the probability that the other child is a* **boy***, and he was born at some time am? (we assume that there are two equal periods: am and pm)*

______

Q10 *Let X and Y be independent, taking on {1,2,3,4,5} with equal probabilities.* $E((X+Y)^2)=?$

______

Q11 *What is the behavior of a person with the following utility function?*



a) *Risk-averse.*

b) *Risk-seeking up to $100 wealth, then risk-averse after $100.*

c) *Risk-seeking.*

d) *Risk-averse up to $100 wealth, then risk-seeking after $100.*

e) *Risk-neutral.*

f) *Risk-averse up to $50 wealth, then risk-seeking after $50.*

Q12 *Let A and B are two independent events with P(A)=0.5 and P (AB)=0.4. P(B)=?*

For Q13-Q22, provide the answers. You need to show the problem solving process.

Q13. Suppose that 3 fair 6-sided dice are rolled independently, each for once. Compute the probability of getting 3 different numbers and the sum is no larger than 10.

Q14: A deck of 52 randomly-shuffled playing cards. Randomly pick 2 cards. What is the probability that the two cards have the same rank (i.e., the same number)? [hint: in a deck of cards we have 13 different ranks]

Q15: The school holds a sports competition. 20% of the students in your class are chosen for participating in ball games (event A), 10% of students are chosen for track and field (event B). (a) If we know that the two events are independent, what fraction of students will participate in at least one of them? (b) If we know that the two events are mutually exclusive, what fraction of students will participate in at least one of them?

Q16: Tom and Jerry play a rock-paper-scissors game. They play a total of 3 independent games, the one who wins at least twice is the winner. Compute the probability of Tom being the winner.

[If you are not familiar with the game, here is the rule: A player who decides to play rock will beat another player who has chosen scissors, but will lose to one who has played paper; a play of paper will lose to a play of scissors. If both players choose the same shape, the game is tied.]

Q17: Let X and Y be the numbers obtained on two independent rolls of fair 3-sided dice. We can compute that the random variable X and Y have expectation 2 and variance.
(E(X)=E(Y)=2 and V(X)=V(Y)=).
Let Z=max(X,Y).

(a) Compute the variance V(X-2Y)

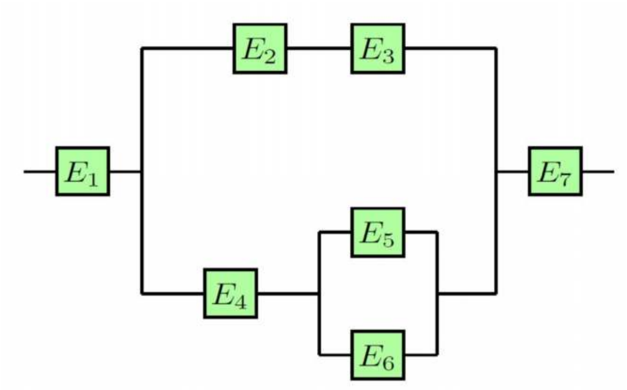
(b) Compute the variance V(X+Z)

Q18: If A and B are independent events of a random experiment such that P(AB)=, and P()=then compute P(A).

Q19: Tom firstly tosses a fair coin 3 times. If none of these 3 results is head, he will toss this coin once more. Then if he still doesn't get one head result, he will try once more. (In this game, Tom tosses a coin at least three times and up to five times.) If the random variable X denotes the number of tosses, what is X's expected value?

Q20: Roll one fair 6-sided dice, let X be the number observed.
(a) Find E(X), Var(X), SD(X).
(b) Let Y be the sum of 10 independent rolls of fair dice. Find E(Y), Var(Y), SD(Y).
(c) Calculate the following quantities: $E(X^2)$, E(3X+2), $E((Y+2)^2)$, Var(-X), SD(2Y).

Q21: The circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is: $P(E_1)=P(E_7)=0.95$, $P(E_2)=P(E_3)=0.9$, $P(E_4)=P(E_5)=P(E_6) = 0.8$. Each device fails independently. Calculate the probability of this circuit function well.



- END -