Discrete Math Sample Exm Question 1 (from Internet without solutions)

Discrete Mathematics (City University of Hong Kong)

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Sample Exam Paper

University of Kent

Faculty of bla bla bla ...

Discrete Mathematics and Probability

This paper is divided into **THREE** *sections as follows:*

Section **A**: *Six short questions each marked out of 10.*

Candidates may attempt all **SIX** *questions but are advised that they cannot obtain more than* **FORTY** *marks on this section.*

Section **B**: *Two longer questions each marked out of 30.*

Section **C**: *Two longer questions each marked out of 30.*

Candidates may not attempt more than **ONE** *question from each of the* **TWO** *questions in sections B and C.*

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MA304 Discrete Mathematics

Sample Exam

A1 For a set X, let $|X|$ denote its size and 2^X its powerset.

Match the following formulae (a) - (f) to the correct english descriptions (1) - (6):

(a) $Y = \emptyset$; (b) $Y = X \setminus (A \cup B)$; (c) $Y \in 2^X$;

(d) $A \subseteq X$; (e) $Y = A \cup B$; (f) $Y = A \cap B$.

(1) Y is the intersection of $X \setminus A$ and $X \setminus B$;

(2) every element of A is an element of X;

(3) Y is the set of elements that lie in A or in B;

(4) Y is the set of elements that lie in A and in B;

(5) Y is a set containing no elements;

(6) Y is a subset of X.

marks 10

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MA304 Discrete Mathematics

A2 Construct the truth tables for the following statements:

(i) $p \Rightarrow (q \lor p)$

(ii) $[(p \land q) \lor r] \Rightarrow [(p \lor r) \land q]$

marks 10

A3 Prove the following identity:

$$\sum_{i=0}^{n} \frac{1}{(i+1)(i+2)} = \frac{n+1}{n+2}.$$

marks 10

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MA304 Discrete Mathematics

B1a Give a combinatorial proof of the formula

$$\#\{(d_1, d_2, \cdots, d_n) \in \mathbb{N}_0^n \mid \sum_{i=1}^{n} d_i = k\} = \binom{n+k-1}{k}$$

marks 10

B1b Consider a basket with 5 apples and 4 pears. In how many ways can you collect three apples and two pears?
marks 3

B1c How many different equivalence relations with three equivalence classes are there on the set $\{1, 2, 3, 4, 5\}$?
marks 7

B1d How many surjective functions are there from the set $\{1, 2, 3, 4\}$ to $\{1, 2, 3\}$.
marks 10

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MA304 Discrete Mathematics

B2a Let $F := F(X, Y, Z) = (2X + Y + Z)^4$ and $G := G(X, Y, Z) = (X^2 + Y/2 + Z/3)^3$. Determine the coefficients with which the following terms appear in F and G.
1. X^2YZ; 2. X^2Z^2; 3. X^2YZ^2; 4. XYZ^2; 5. Y^2Z.
marks 10

B2b Consider arithmetic modulo 8. Determine which of the following functions are bijective. If it is, determine the inverse function.
a. $f : \mathbb{Z}_8 \to \mathbb{Z}_8 : f([a]_8) = [a]_8 \cdot [2]_8 + [3]_8$;
b. $f : \mathbb{Z}_8 \to \mathbb{Z}_8 : f([a]_8) = [a]_8 \cdot [3]_8 + [2]_8$;
marks 5

B2c In how many ways can we divide 20 persons into 4 disjoint groups of the same size ?
marks 5

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MA304 Discrete Mathematics

B2d Consider the function $f : \mathbb{R}\backslash\{1\} \to \mathbb{R}$, given by

$$f(x) := \frac{7x-3}{x-1}.$$

What is the image of f? Find $d \in \mathbb{R}$ such that the function

$$\hat{f} : \mathbb{R}\backslash\{1\} \to \mathbb{R}\backslash\{d\}$$

is bijective and construct the inverse function of $\hat{f}$.
marks 10

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