CS2402 - Tutorial 4

a. $100 + 10x - 10(50-x)$

$= 20x - 400$

≥ 0

$x \geq 20$

Task 1. A gambler repeatedly bets 10$ on red at a roulette table, winning 10 dollars with probability 18/38, losing 10 dollars with probability 20/38. He starts with capital 100 dollars, and can borrow money if necessary to keep in the game.

a) Find exact expressions for the probability distribution of his capital after 50 plays, prove the probability that the gambler is not in debt after 50 plays= $\sum_{k=20}^{50} C_{50}^{k}\left(\frac{18}{38}\right)^{k}\left(\frac{20}{38}\right)^{50-k}$

∴ win more than 20 games

∴ $P = \sum_{k=20}^{50} C_{50}^{k}\left(\frac{18}{38}\right)^{k}\left(\frac{20}{38}\right)^{50-k}$

b) Find the mean, variance and standard deviation of the gambler's capital after 50 plays.

(b).

win X games	0	- - -	50
P	$C_{50}^{0}\left(\frac{18}{38}\right)\left(\frac{20}{38}\right)$		
capital	$20X - 400$		

Hint: S= the number of wins among 50 plays.

S=B(n, p). n=?, p=?

Y= his capital after 50 plays.

He won S times, lost 50-S times.

a) Y=100+10*S-10*(50-S).
Not in debt means Y>=0.

b) E(Y)=? Var(Y)=?

$E(20X-400) = 20E(X) - 400$

$= 20 \times 50 \times \frac{18}{38} - 400$

$= 73.6842$

$V(20X-400) = 400\,V(X)$

$= 400 \times 50 \times \frac{18}{38} \times \frac{20}{38} = 4986.15$

Task 2. A doubling cube is a die with faces marked 2, 4, 8, 16, 32, and 64. Suppose two doublings are rolled independently. Let X and Y be the two numbers obtained. Find

a) P(XY<100)
b) P(XY<200)
c) E(XY)
d) Var(XY)
e) SD(XY)

$EX^2 = \frac{1}{6}(4 + \dots + 4096)$

$Var(XY)$

$= E(X^2Y^2) - (E(XY))^2$

$= EX^2\,EY^2 - (EXY)^2$

$P(XY<100) = \frac{5+4+3+2+1}{6\times6} = \frac{6\times5}{36\times2} = \frac{5}{12}$

$P(XY<200) = \frac{21}{6\times6} = \frac{21}{36} = \frac{7}{12}$

Hint: E(X)=1/6*{2+4+6+6+32+64}=?
Var(X)=E(X^2)-[E(X)]^2=?
SD(X)=sqrt(Var(X))
E(Y)=E(X)
Var(Y)=Var(Y)

$E(XY) = E(X)\cdot E(Y) = 21 \times 21 = 441$

$Var(XY) = E((XY)^2) - (EXY)^2 = E(X^2)E(Y^2) - 441^2$

$= 910^2 - 441^2 = 469 \times 1351$

$= 633{,}619$

$SD = \sqrt{633619} = 796$

Task 3. A random variable X has expectation 10 and standard deviation 5.

a) Find an upper bound you can for P(X>=20) lousy bound
b) Could X be a binomial random variable?

$P(X\geq20) = P(X-10\geq10) \leq P(|X-10|\geq10) \leq \frac{1}{4}$

a) $P(X \geq 20) = P(X - 10 \geq 10) \leq P(|X - 10| \geq ?* SD(X))$<=?
b) If X=B(n,p), EX=?, Var(X)=?

(a). $P(X\geq20) \leq P(|X-10|\geq 2.5) \leq \frac{1}{4}$

$\frac{1}{1-p} = \frac{2}{5}$ so not possible

v2

(b). $EX = np = 10$

$Var(X) = np(1-p) = \cancel{5}\ 25$

~~$\frac{1}{1-p} = 2$ so possible~~

~~$\therefore p = \frac{1}{2}$ $n = 20$ $X \sim B(20, \frac{1}{2})$~~

$Var(aX) = a^2 Var(X)$

$Var(X+Y) = Var(X-(-Y)) = var(X) + var(-Y) = var X + (-1)^2 var(Y) = Var X + Var Y$

Task 4: If X and Y are independent, show that Var(X-Y)=Var(X+Y)

$Var(X-Y) = Var(X+Y) = E(X^2) - (EX)^2 + E(Y^2) - (EY)^2 = Var(X) + Var(Y)$

$Var(X-Y) = E((X-Y)^2) - (E(X-Y))^2 = EX^2 - 2EXEY + EY^2 - (EX - EY)^2 = EX^2 - (EX)^2 + EY^2 - (EY)^2$

$Var(X+Y) = E((X+Y)^2) - E(X+Y)^2 = EX^2 + 2EXEY + EY^2 - (EX+EY)^2 = EX^2 - (EX)^2 + EY^2 - (EY)^2$

$\therefore Var(X-Y) = Var(X+Y)$

Task 5: Suppose that there are a list of many numbers, its mean is 10 and standard deviation (SD) is 2. Let X_1 and X_2 be two numbers independently drawn at random with replacement from this list. Find a number c so that

$E(X) = E(X_1) - E(X_2) = 0$

$P(|X - E(X)| < k \cdot SD) \geq 1 - \frac{1}{k^2}$

$$P(|X_1 - X_2| < c) \geq 99\%.$$

$X = X_1 - X_2$

$SD(X) = 2\sqrt{2}$

$E(X) = 0$

$Var(X) = 2Var(X_1) = 2 \times 4 = 8$

Hint: $P(|X - EX| < k * SD(X)) \geq 1 - 1/k^2$
Let $X = X_1 - X_2$, $EX = ?$ $Var(X) = ?$ $SD(X) = ?$

$P(|X - 0| < 2\sqrt{2} \cdot k) \geq 1 - \frac{1}{k^2}$

$1 - \frac{1}{k^2} \geq 0.99$

$0.01 \geq \frac{1}{k^2}$

$k^2 \geq 100$

$\therefore k \geq 10$

$\therefore P(|X - 0| < 20\sqrt{2}) \geq 99\%$

$\therefore c = 20\sqrt{2}$

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