CS2402-Final-2022 - final exam

Introduction to Computational Probability Modeling (City University of Hong Kong)

CITY UNIVERSITY OF HONG KONG

Course code & title : CS2402
Introduction to Computational Probability Modeling

Session : Semester B 2021/22

Time allowed : Two hours This paper has 5 pages (including this cover page). 1. This paper consists of 10 questions.

2. Answer ALL questions. *This is a **closed-book** examination.*
This question paper should NOT be taken away.

Students are allowed to use the following materials/aids:
1. *Approved Calculator*
2. *An A4 paper note, two pages*

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

<u>Academic Honesty</u>

I pledge that the answers in this examination are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,
1) *I will not plagiarize (copy without citation) from any source;*
2) *I will not communicate or attempt to communicate with any other person during the examination; neither will I give or attempt to give assistance to another student taking the examination; and*
3) *I will use only approved devices (e.g., calculators) and/or approved device models.*
4) *I understand that any act of academic dishonesty can lead to disciplinary action.*

I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties.

Student ID:

Name:

<u>CS Departmental Hotline (phone, whatsapp, wechat)</u>
+852 6375 3293

Question	1	2	3	4	5	6	7	8	9	10
Marks										
	10	10	10	10	10	10	10	10	10	10

<u>Answer ALL the 10 questions below.</u>

<u>Question 1 (10pts)</u>
A bag contains 10 identical red balls and 20 identical green balls. You draw 3 balls at random **without replacement**.

a) What is the probability that you get no red balls? (3pts)
b) What is the probability that you get exactly k red balls (Give an expression for k, no need to simplify)? (4pts)
c) If we decide to write the letter “A” on 2 balls, “B” on 2 balls, and “C” on 1 ball. Each ball can only have 1 letter. In how many different ways can we write these letters for all 30 balls? (3pts)

Question 2 (10pts)
Suppose we have 2 events, C and D, and we have , .
a) If C and D are mutually exclusive, then ? (3pts)
b) If C and D are independent, then ? (4pts)
c) If C and D are independent, then ? (3pts)

Question 3 (10pts)
Suppose X and Y are **i.i.d. (independent and identically distributed)** random variables and

a) Calculate . (2pts)
b) Derive the joint distribution function of X and Y. (2pts)
c) Calculate . (3pts)
d) Calculate . (3pts)

Question 4 (10pts)
Let be a random variable taking values in , where . Let be a **Uniform** random variable that takes values in . (Uniform means it is equally likely for random variables to take each possible value).
a) Let . Find and . (3pts)
b) Let . and . Find . (You may need to use the fact that .) (3pts)
c) Let and . Assume and are independent. Find . (You may need to use the fact that .) (4pts)

Question 5 (10pts)
One-sided Chebyshev's inequality states that if a random variable has mean 0 and variance , then for any ,

.

A discrete random variable is said to have a Poisson distribution, with parameter , if it has a probability mass function given by

.

If have a Poisson() distribution,

.

Let have Poisson() distribution and let have Poisson(2) distribution. Suppose and are independent, based on one-sided Chebyshev's inequality, find an upper bound for

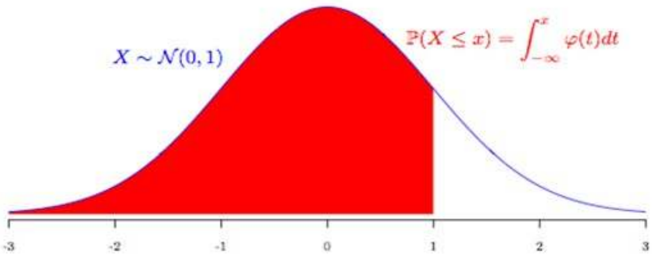
.

Question 6 (10pts)

are **i.i.d.** binomial random variables, where for **.** Given that **, , .**

a) Find the expectation and standard deviation **.** (6pts)

b) Find the approximation of based on the following table. (4pts)



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Question 7 (10pts)

Sally grabs a die at random from a box containing two 4-sided dice, one 8-sided die, and one 12-sided die. All dice are fair. An m-sided dice has m sides with numbers 1, 2, …, m. She rolls the die once and reports that the result is 7.

a) What is the prior predictive probability of rolling a 7? (3pts)

b) What are the posterior odds that the die has 12 sides? (3pts)

c) Given the number of the first roll is 7, what is the probability that the next roll of the same grabbed die will be a 4? (4pts)

Question 8 (10pts)

There are registration forms for 10, 15, and 25 candidates from 3 regions. Among them, there are 3, 7, and 5 registration forms for girls, respectively. A region is **randomly selected with equal probability first**, **then** two registration forms are drawn **without replacement** from the region successively.

a) The probability p that the first one is a girl's registration form. (4pts)

b) It is known that the second drawn one is a boy's registration form, find the probability q that the first drawn one is from a girl. (6pts)

Question 9 (10pts)

A drug dealer Jerry has a 90% chance of going to a bar to drink every day and only goes to a specific 3 bars to drink (**with equal probability** for each bar). One day, FBI agent Tom wanted to catch the drug dealer, so he went to the bar to find him. Tom went to two bars at random and found that Jerry was not there, so he went to the third bar. Find out the probability p that Tom can catch Jerry at the third bar.

Question 10 (10pts)

The values of and their corresponding values of (2, 10), (−3, 3), (-2, 2) are assumed to arise from the model , where is a constant and are random variables.

a) What assumptions are needed on so that it makes sense to do a least squares fit of a curve to the data? (3pts)

b) Given the above data with the assumptions needed on , determine the least squares estimate for . (7pts)

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