MA2185 Discrete Math Midterm 2021/22 Semester A

Discrete Mathematics (City University of Hong Kong)

CITY UNIVERSITY OF HONG KONG
Department of Mathematics

Course Code & Title : MA2185 Discrete Mathematics

Session : Semester A 2021/22

Time Allowed : 1.5 Hours

This paper has **THREE** pages (including this cover page).

Instructions to candidates:

1. Answer **ALL FIVE** questions.
2. Start each question on a new page.
3. Show **ALL** steps to obtain **FULL** grade.

This is a closed-book examination.

Candidates are allowed to use the following materials/aids:

Approved non-programmable calculator.

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

Special materials (other than standard materials e.g. answer book or supplementary sheets) to be supplied to students:

NIL

Question One (30 marks Total)

We assume the following two statements:

1. "Experiment is difficult **or** not many students like experiment"

2. "If science is easy, then experiment is not difficult."

Formalize using logical inference the two assumptions (in the above) as well as other statements (below (a) to (c)) as propositions. Determine which of the statements below (a) to (c) follow from the two assumptions (in the above) and which do or does not. Give a proof or counter-example.

(a) Science is not easy, if many students like experiment. **(10 marks)**

(b) Not many students like experiment, if science is not easy. **(10 marks)**

(c) If not many students like experiment, then science is not easy or experiment is not difficult. **(10 marks)**

Question Two (10 marks)

Prove by mathematical induction or otherwise that, for $n \geq 1$,

$$\sum_{i=1}^{n} \frac{i^2}{(2i-1)(2i+1)} = \frac{n(n+1)}{4n+2}$$

Question Three (10 marks Total)

Let f_n be the number of ways to arrange n binary digits in a row so that no two 0s are placed together. Show that the numbers f_n, $n = 1, 2, \cdots$, are Fibonacci numbers, that is, show that

$$f_n = f_{n-1} + f_{n-2},\ n \geq 3 = 1, 2, \cdots \text{and } f_1 = 2, f_2 = 3.$$

Question Four (30 marks Total)

(a) In a pet store, there are 12 hamsters and 14 lizards. The store manager wants to select 1 hamster and 1 lizard for advertisement. In how many ways can the store manager make this selection? **(10 marks)**

(b) This is a permutation of the letters of the word **AGARS**. The letters are arranged in the order as appeared in a dictionary, what is the 49th word? **(10 marks)**

(c) Suppose x boys and y girls are to be seated in a row so that no two girls sit together. If $x > y$, show that the number of ways in which they can be seated is **(10 marks)**

$$\frac{x!(x+1)!}{(x-y+1)!}$$

Question Five (20 marks Total)

(a) The company's profit has greatly inclined because COVID-19 is over! Therefore, a CEO must hire 12 employees, choosing at least 5 from each of the marketing department and the accounting department. If there are 7 employees in the marketing department and 8 in the accounting department, in how many ways can the CEO choose 12 employees? **(10 marks)**

(b) Nemo has 3 museum tickets and 8 exhibitions of his interest in the museum. Of these 8 exhibitions, he does not want to visit Exhibition A, unless he can visit Exhibition B. In how many ways can he choose the three exhibitions to be visited? **(10 marks)**

exhibitions are within the museum

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