CITY UNIVERSITY OF HONG KONG

Department of Mathematics

Course Code & Title	:	MA1300 Enhanced Calculus and Linear Algebra I
Session	:	Semester A, 2012-2013
Time Allowed	:	Two Hours

This paper has **two** pages. (including this cover page)

Instructions to candidates:

1. Answer **all** questions.
2. Start each main question on a new page.
3. Show all steps.

Materials, aids & instruments which students are permitted to use during examination:

1. No calculator, electronic device, or formula sheet is allowed during exam.

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1. (10 points) Find the limit $\lim_{n\to\infty} n \cdot \sin\left(\frac{3}{n}\right)$.

2. (20 points) Test the series for convergence or divergence.

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ (b) $\sum_{n=1}^{\infty} \ln\left(\frac{2n+1}{2n^3-n^2}\right)$.

3. (24 points) Consider the function f on $\mathbb{R}$ defined by $f(x) = \frac{x^2}{x^2+4}$.

   (a) Find all vertical and horizontal asymptotes.

   (b) Indicate all intervals where the function f is increasing.

   (c) Find all local maxima and minima, if any.

   (d) Indicate all intervals where f is concave downward.

4. (12 points) Consider

$$f(x) = \cos x - 2x - x^3.$$

   (a) Find $f'(x)$.

   (b) Prove that $f(x)$ has its inverse function.

   (c) Compute $(f^{-1})'(1)$.

5. (10 points) Let f be a continuous function on the interval $[0,1]$. If f is differentiable on $(0,1)$, prove that there exists some $\xi \in (0,1)$ such that

$$f(\xi) + f'(\xi) = e^{-\xi}\left[f(1)e - f(0)\right].$$

6. (24 points) Consider the function f on $\mathbb{R}$ defined by $f(x) = \begin{cases} \dfrac{\tan^{-1} x - x}{x^3} & \text{if } x \neq 0 \\ -\dfrac{1}{3} & \text{if } x = 0. \end{cases}$

   (a) Prove that f is continuous everywhere.

   (b) Find the Maclaurine series for f, and its interval of convergence.

   (c) Find the Maclaurine series for f', and its interval of convergence.

   (d) Find $f^{(6)}(0)$.

   Hint: $\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$ in its interval of convergence.

End of the Questions.