CITY UNIVERSITY OF HONG KONG

Course code and title	:	MA2504 Discrete Mathematics
Session	:	Semester B, 2008-2009
Time allowed	:	Three Hours

This paper has FOUR pages (including this page).

Instructions to candidates:

1. This paper has SIX questions.
2. Attempt ALL questions.
3. The paper has 100 marks in total.
4. Start each question on a new page.
5. Show ALL workings.

Materials, aids & instruments which students are permitted to use during examination: Approved calculators

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Do not remove this from exam

Answer ALL Questions

Question 1

(a) Use *Proof by Contradiction* to show the validity of $\begin{array}{l} \sim \exists x[F(x) \wedge G(x)] \\ F(y) \\ \hline \therefore \sim G(y) \end{array}$ (8 marks)

(b) Use (a), or otherwise, to show the validity of $\begin{array}{l} \sim \exists x[F(x) \wedge G(x)] \\ \hline \therefore \forall x[F(x) \rightarrow \sim G(x)] \end{array}$ (4 marks)

(c) There is a mistake in the following derivation, find it and explain.

1.	$\exists x(P(x) \wedge Q(x))$	p
2.	$P(c)$	p
3.	$P(c) \wedge Q(c)$	from $1, ei$
4.	$Q(c)$	from 3

(4 marks)

Question 2

(a) Let A, B and C be sets. Show that

(i) $(A \setminus B) \setminus C \subseteq A \setminus C$ (4 marks)

(ii) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$ (4 marks)

(b) Let R be a binary relation on $A \neq \emptyset$ and R^{-1} be its inverse. Suppose $R \cap R^{-1} = \emptyset$, show that R is not reflexive and R is antisymmetric. (8 marks)

Question 3

(a) Let $f : A \rightarrow B$ and $g : B \rightarrow C$. Suppose that g is injective and $(g \circ f)$ is surjective. Prove that f is surjective and g is surjective. (6 marks)

(b) How many solutions are there to the equation $x_1 + x_2 + x_3 = 90$, where x_1, x_2 and x_3 are nonnegative integer such that $x_1 \geq 10, x_2 \leq 25, x_3 \leq 45$ (Use Inclusion-Exclusion Principle). (7 marks)

(c) A club with fifteen women and ten men needs to form a committee of size five. How many committees are possible if the committee must consist of all women or all men?

(3 marks)

Question 4

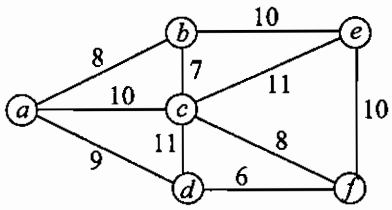
(a) Find a recurrence relation for the number of ways to form a postage of n cents if the post office has only 4-cents stamps and 6-cents stamps. Also, determine the initial condition(s) to solve the recurrence relation. (You are not required to solve the recurrence relation) (4 marks)

(b) Find the solution for the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2} + 4^n$ with initial conditions $a_0 = 0$ and $a_1 = -\frac{9}{5}$. (8 marks)

(c) Let G be a graph containing 14 vertices and 27 edges. Suppose each vertex of G is either of degree 3 or 6. How many vertices of degree 6 does G have? (4 marks)

Question 5

(a) Use Prim's algorithm (in tabular form) to find a minimum spanning tree for the following graph starting at the vertex a (you have to list all steps of the algorithm). What is the weight of a minimum spanning tree? Note that the numbers indicated at edges are the weights of the edges.



(6 marks)

(b) A train travels between pairs of stations A, B, C, D, E, F. The following table indicates in minutes the time it takes to travel from one station to another. The symbol ∞ indicates that no direct route between those stations.

	A	B	C	D	E	F
A	0	6	5	∞	4	∞
B	6	0	∞	5	6	2
C	5	∞	0	7	∞	3
D	∞	5	7	0	4	3
E	4	6	∞	4	0	2
F	∞	2	3	3	2	0

Use Dijkstra's algorithm to find the shortest travel time from A to each station. Also, write down the shortest path from A to F. (10 marks)

Question 6

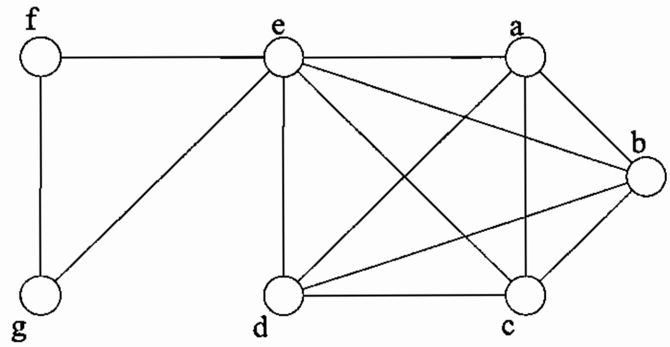
(a) Consider the set of words $W = \{$apple, ape, angry, wash, wind, sky, seat$\}$. Find all equivalence classes with respect with R where R is the equivalence relation defined by:

(i) "has the same number of letters", (3 marks)

(ii) "begins with the same letter". (3 marks)

(b) Draw a graph with six vertices and nine edges which has a Hamilton circuit but no Euler circuit. (Answer with explanation) (7 marks)

(c) Is there an Euler circuit and a Hamilton circuit in the following graph? If so, find such a circuit. If not, explain why no such circuit exists.



(7 marks)

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