Name: LIU Hengchu
(e.g. Chan Siu Ping)

EID: hengchliu2
(e.g. spchan31)

CS2402 - Lecture 2 - In-Class Exercises
Problem of the Points

Q1. Bob feels that Fermat's method is unfair. He argues that the game should terminate after someone has 5 points. Therefore, Alice only has three ways to win ("A", "BA", "BBA"), and Bob has one way to win ("BBB"). Therefore, the pot should be split in 3:1. What do you think about Bob's argument? Is it correct or wrong? Explain it.
$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$

It's wrong, because the possibility of the ways are not equivalent, so we should not split the money according to the ways.

Q2. Suppose that A and B and C are three events, prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC).$$

$= (1+2+5+6) + (2+3+4+5) + (4+5+6+7) - 2 - 5 - 4 - 5 - 6 - 5 + 3$

$= 1 + 6 + 2 + 3 + 4 + 7 + 5 = P(A \cup B \cup C)$

$P(A \cup B \cup C) = P(A \cup B) + P(C) - P((A \cap C) \cup (B \cap C)) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABU)$

Q3. Let A and B and C are three events. Express the following events in terms of A and B and C: **(a)** exactly one of the events occurs. **(b)** None of the events occurs. **(c)** At least one of the events occurs.
Maybe Not Independent

(a). $P(A\bar{B}\bar{C}) + P(\bar{A}B\bar{C}) + P(\bar{A}\bar{B}C)$ $(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$

(b). $P(\bar{A}\bar{B}\bar{C})$ $\overline{A \cup B \cup C}$

(c). $1 - P(\bar{A}\bar{B}\bar{C})$ $A \cup B \cup C$

Q4. (a) Consider the following statement: "I heard on the news that there is a 50% chance of rain on Saturday and a 50% chance of rain on Sunday. Then there must be a 100% chance of rain during the weekend." Is it correct or wrong? Explain it.

wrong.

$\frac{1}{4}$ rain on both, $\frac{1}{4}$ rain on Saturday, $\frac{1}{4}$ rain on Sunday, $\frac{1}{4}$ does not rain.

$P(\text{rain on Saturday}) = 0.5$

$P(\text{rain on Sunday}) = 0.5$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 1 - P(A \cap B)$$

not mutually exclusive.

not 0.

$$P(A) + P(B) = P(A \cup B) + P(A \cap B)$$

(b) People are asked to assign probabilities to the events "rain on Saturday", "rain on Sunday", "rain on both days", and "rain on at least one of the days". Which of the following suggestions are correct: **(i)** 70%, 60%, 40%, and 80% **(ii)** 70%, 60%, 40% and 90 % **(iii)** 70%, 60%, 80%, and 50%, and **(iv)** 70%, 60%, 50%, and 90%?

~~Since independent~~

$P(A) = 0.7$ $P(B) = 0.6$ ~~P(A~~

i). $0.7 + 0.6 \neq 0.8 + 0.4$

ii). $0.7 + 0.6 = 0.4 + 0.9$

iii). $0.7 + 0.6 = 0.8 + 0.5$

iv). $0.7 + 0.6 \neq 0.5 + 0.9$

If $P(A \cap B) = 0.4$ then $P(A \cup B) = 0.9$ so (ii) ✓

if $P(A \cap B) = 80\%$ not possible

If $P(A \cap B) = 50\%$ $P(A \cup B) = 0.8$

$P(A \cup B) > P(A \cap B)$ so (ii) is correct

Q5. Let A and B be two events such that $P(A) = 0.3$, $P(A \cup B) = 0.5$, and $P(A \cap B) = 0.2$.
Find **(a)** $P(B)$, **(b)** $P(A \cap \overline{B})$, **(c)** $P(\overline{A})$, **(d)** $P(\overline{B})$, and **(e)** $P(\overline{A} \cap \overline{B})$.

(a) $P(B) = 0.5 + 0.2 - 0.3 = 0.4$

(b). $P(A \cap \overline{B}) = 0.1$

(c) $P(\overline{A}) = 0.7$

(d). $P(A \cap \overline{B}) = 0.5$