

**Task 1.** Suppose  $P(\text{rain today})=40\%$ ,  $P(\text{rain tomorrow})=50\%$ ,  $P(\text{rain today and tomorrow})=30\%$ . Given that it rains today what is the chance that it will rain tomorrow.

$$P(\text{rain tomorrow} | \text{rain today}) = \frac{0.3}{0.4} = 0.75$$

**Task 2.** Assume that identical twins are always of the same sex, equally likely boys or girls. Assume for non-identical twins the firstborn is equally likely to be a boy or a girl, and so is the secondborn, independently of the first. Assume that 45% of twins are identical, 55% are non-identical. Find the following probabilities:

$$1. 0.45 \times \frac{1}{2} + 0.55 \times \frac{1}{4} = 36.25\%$$

1. Probability of both boys.

2. Probability that the firstborn is a boy and the second is a girl.

3.  $P(\text{secondborn is a girl} | \text{firstborn is a boy})$

4.  $P(\text{secondborn is a girl} | \text{firstborn is a girl})$

Hint: A=Identical twin(45%): {girl, girl}, {boy, boy}

B=Non-identical twin(55%): {girl, girl}, {girl, boy}, {boy, girl}, {boy, boy}

$$3. \frac{0.55 \times \frac{1}{2}}{\frac{1}{2}} = 27.5\%$$

$$4. P = \frac{0.45 \times \frac{1}{2} + 0.55 \times \frac{1}{2}}{\frac{1}{2}} = 0.45 + 0.275 = 0.725$$

**Task 3.** Give a formula for  $P(F|G)$  in terms of  $P(F)$ ,  $P(G)$  and  $P(FG)$  only.

$$P(F|G) = \frac{P(FG)}{P(G)} = \frac{P(F)P(FG)}{1-P(G)}$$

**Task 4.** Let E and H be two events, prove:

$$P(E) = P(E|H)P(H) + P(E|\neg H)P(\neg H).$$

where  $\neg H = \bar{H}$ . (This result will be used in Lecture 10).

$$\frac{P(EH)}{P(H)} \times P(H) + \frac{P(E\bar{H})}{P(\bar{H})} \times P(\bar{H}) = P(EH) + P(E\bar{H}) = P(E)$$

**Task 5.** Prove If  $B_1, B_2, B_3$  are mutually exclusive, and  $B_1 \cup B_2 \cup B_3$  be the outcome space S. Then for any event A:

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)$$

$$\frac{P(AB_1)}{P(B_1)} \times P(B_1) + \dots$$

$$P(AB_1) + P(AB_2) + P(AB_3)$$

$$= P(A \cap (B_1 \cup B_2 \cup B_3))$$

$$= P(A)$$

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