

MA2185 Assignment One

There are **5 questions** in total.

Question 1

Let p and q be the propositions

p : It is below freezing. q : It is snowing.

Write these propositions using p and q and logical connectives (including negations).

- a) It is below freezing and snowing.
- b) It is below freezing but not snowing.
- c) It is not below freezing and it is not snowing.
- d) It is either snowing or below freezing (or both).
- e) If it is below freezing, it is also snowing.
- f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
- g) That it is below freezing is necessary and sufficient for it to be snowing.

Question 2

- (a) Show that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.
- (b) Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent.
- (c) Show that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology.

Question 3

For each of these arguments determine whether the argument is correct or incorrect and explain why.

- a) All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.
- b) Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major.
- c) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.
- d) Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

Question 4

- (a) Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.
- (b) Use rules of inference to show that the premises “A student in this class has not read the book,” and “Everyone in this class passed the first exam” imply the conclusion “Someone who passed the first exam has not read the book.”

Question 5

- (a) Prove that $1^2 + 3^2 + 5^2 + \cdots + (2n + 1)^2 = \frac{(n + 1)(2n + 1)(2n + 3)}{3}$ whenever n is a nonnegative integer.
- (b) For which nonnegative integers n is $2n + 3 \leq 2^n$? Prove your answer.