MA2185+Assignment+One+Solution

Discrete Mathematics (City University of Hong Kong)

Q 1.a) $p \to (q \vee r) \Leftrightarrow \sim p \vee (q \vee r)$

$\Leftrightarrow (\sim \vee p) \vee r$

$\Leftrightarrow (p \to q) \vee r$

$\therefore$ they are equivalent

b) $\sim \{\sim [(p \vee q) \wedge r] \vee \sim q\}$

$\Leftrightarrow \sim [\sim (p \vee q) \vee \sim r \vee \sim q]$

$\Leftrightarrow (p \vee q) \wedge r \wedge q$

$\Leftrightarrow [(p \vee q) \wedge (F \vee q)] \wedge r$

$\Leftrightarrow [(p \wedge F) \vee q] \wedge r$

$\Leftrightarrow q \wedge r$

$\therefore$ they are equivalent

c) $(p \to q) \vee (r \wedge s) \Leftrightarrow (\sim p \vee q) \vee (r \wedge s)$

$\Leftrightarrow [\sim p \vee (r \wedge s)] \vee q$

$\Leftrightarrow [(\sim p \vee r) \wedge (\sim p \vee s)] \vee q$

$\Leftrightarrow [(p \to r) \wedge (p \to s)] \vee q$

Q 2

a)

p	q	r	$(p \wedge q)$	$\vee$	$(p \wedge r)$	$p \wedge$	$(q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	F	F	F	F	T
F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	F
				*		*	

... con't

According to colums (∗∗), (∗), observe that $(p \wedge q) \vee (p \wedge r)$ and $p \wedge (q \vee r)$ are equivalent ②

2b)

p	q	$\sim(p \vee q)$		$\sim p \wedge \sim q$		
T	T	F	T	F	F	F
T	F	F	T	F	F	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
		#			∗	

From columns # and ∗, we see that $\sim(p \vee q)$ and $\sim p \wedge \sim q$ are logically equivalent.

3 a) 1. $(P \vee Q) \rightarrow \sim R$

2. $S \rightarrow \sim P$

3. $S \wedge \sim Q$

$\therefore R$

4. $S \equiv T$ 3.

5. $Q \equiv F$ 3

6. $P \equiv F$ 2 & 4

7. $P \vee Q \equiv F$ 5 & 6

8. R can be "T" or "F" s.t premise 1 is true.

$\therefore$ the argument is invalid.

3 b) p = "Josh works hard"

q = "Josh gets the supervisor's position"

r = "Josh gets a promotion"

s = "Josh buys a new dog"

1. $(p \wedge q) \to r$
2. $r \to s$
3. $p \wedge s$

$\therefore q$

4. $p \equiv T$ 3
5. $s \equiv T$ 3
6. $q \to r \equiv T$ 4 & 1

If $r \equiv T$, then all the premises are true no matter what the truth value of q is. Therefore, when $q \equiv F$ the conclusion is false and so the argument is invalid.

4. a) i) $\forall x\, [M(x) \to C(x)]$ (1)

$\exists x\, [M(x) \wedge H(x)]$ (2)

$\forall x\, [E(x) \to H(x)]$ (3)

$\therefore \exists x\, [E(x) \wedge C(x)]$ (4)

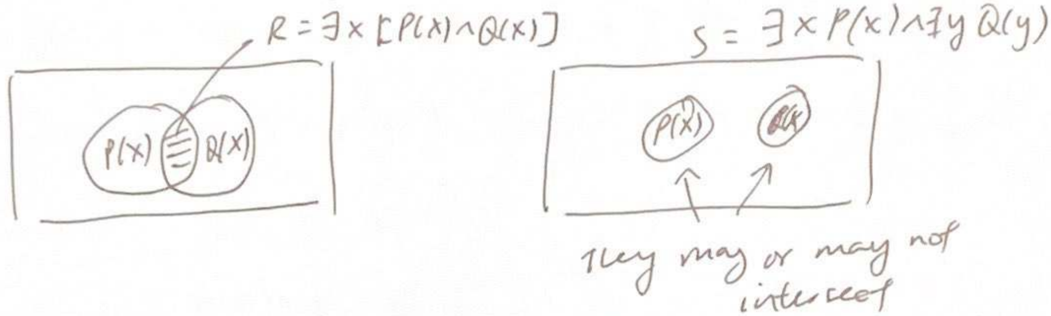
a) ii)

(5)	$M(a) \wedge H(a)$	(2) ei
(6)	$M(a) \equiv T$	(5)
(7)	$H(a) \equiv T$	(5)
(8)	$M(a) \to C(a)$	(1) ui
(9)	$C(a) \equiv T$	(6) and (8)

con't...

If $E(a) \equiv F$, then (3) is still true but (4) becomes false. $\therefore$ the argument is invalid.

4 b)(i) In the venn diagram, there is an intersection of $P(x)$ and $Q(x)$ for R, but there is no construct of $P(x)$ and $Q(x)$ for S.



We construct examples according to the above observation

Let $P(x) =$ "x is even" and $Q(x) =$ "x is odd"

Then $\exists x[P(x) \wedge Q(x)] \equiv F$

but $\exists x\, P(x) \wedge \exists y\, Q(y) \equiv T$

b)ii) $R \Rightarrow S$

1. $\exists x\, [P(x) \wedge Q(x)]$

$\therefore \exists x P(x) \wedge \exists y\, Q(y)$

2. $P(a) \wedge Q(a)$ 1 ei

3. $P(a)$ 2

4. $\exists x\, P(x)$ 3 eg

5. $Q(a)$ 2

6. $\exists y\, Q(y)$ 5 eg

7. $\exists x\, P(x) \wedge \exists y\, Q(y)$ 4 & 6

c) 1. $\exists x\, [P(x) \wedge Q(x)] \to \forall y\, [R(y) \to S(y)]$

2. $\exists x\, [R(x) \wedge \sim S(x)]$

$\forall x\, [P(x) \to \sim Q(x)]$

3. $\sim \forall y\, [R(y) \to S(y)] \to \sim \exists x\, [P(x) \wedge Q(x)]$ 1

4. $\exists y \sim [\sim R(y) \vee S(y)] \to \forall x \sim [P(x) \wedge Q(x)]$ 3

5. $\exists y\, [R(y) \wedge \sim S(y)] \to \forall x\, [\sim P(x) \vee \sim Q(x)]$ 4

6. $\forall x\, [\sim p(x) \vee \sim Q(x)]$

7. $\forall x\, [P(x) \to \sim Q(x)]$

$\therefore$ The argument is valid

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5. a) Let $P(n) =$ " $\boxed{1 + \frac{1}{2} + \dots + \frac{1}{2^n}} \geq \boxed{1 + \frac{n}{2}}$ " 10

3 step 1 $P(0)$ is true since $\frac{1}{2^0} \geq 1 + \frac{0}{2} = 1$ base case.

2 step 2 For $n \geq 0$, assume that $P(n)$ is true
To prove that $P(n+1)$ is also true, we have

$$LHS = \boxed{1 + \frac{1}{2} + \dots + \frac{1}{2^n}} + \frac{1}{2^n+1} + \dots + \frac{1}{2^{n+1}}$$

$$\geq \boxed{1 + \frac{n}{2}} + \frac{1}{2^n+1} + \dots + \frac{1}{2^{n+1}} \quad \text{(from assumption)}$$

$$\geq \left(1 + \frac{n}{2}\right) + 2^n \left(\frac{1}{2^{n+1}}\right) \quad \left(\text{since the last } 2^n \text{ terms each} \geq \frac{1}{2^{n+1}}\right)$$

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$$\geq 1 + \frac{n}{2} + \frac{1}{2} = 1 + \frac{(n+1)}{2}$$

From steps 1 + 2 $P(n)$ is true for $n \geq 0$. 1

5b) Let $P(n) =$ " $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} < 2\sqrt{n}$ " ⑥

② Step 1 For $n=1$ LHS $= \frac{1}{\sqrt{1}} < 2\sqrt{1} =$ RHS

$\therefore$ $P(n)$ is true for $n=1$

③ Step 2 Assume that $P(n)$ is true. We are going to prove that $P(n+1)$ is also true, i.e.

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} < 2\sqrt{n+1}$$

Adding $\frac{1}{\sqrt{n+1}}$ to both sides of $P(n)$ we have

⑤

$$\begin{aligned}
\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} &< 2\sqrt{n} + \frac{1}{\sqrt{n+1}} \\
&< \frac{2\sqrt{n(n+1)}+1}{\sqrt{n+1}} \\
&< \frac{\sqrt{4n(n+1)}+1}{\sqrt{n+1}} \\
&< \frac{\sqrt{4n^2+4n+1}+1}{\sqrt{n+1}} \\
&< \frac{(2n+1)+1}{\sqrt{n+1}} \\
&< 2\sqrt{n+1}
\end{aligned}$$

$\therefore$ $P(n+1)$ is also true

From step (1) & (2), $P(n)$ is true for all $n \geq 1$. ①

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5c) let $P(n) = $ " $\frac{3}{1\times 2} + \frac{4}{2\times 3} + \cdots + \frac{n-2}{n(n+1)} = 1 + \cdots + \frac{1}{n} + \frac{n}{n+1}$ "

step 1. To prove that $P(1)$ is true

$$LHS = \frac{3}{1\times 2} = \frac{3}{2}$$

$$RHS = 1 + \frac{1}{1+1} = \frac{3}{2}$$

$\therefore P(1)$ is true

Step 2. Assume that $P(n)$ is true. we are going to prove that $P(n+1)$ is also true

LHS of $P(n+1)$

$$= \frac{3}{1\times 2} + \frac{4}{2\times 3} + \cdots + \frac{n+2}{n(n+1)} + \frac{n+3}{(n+1)(n+2)}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \frac{n}{n+1} + \frac{n+3}{(n+1)(n+2)}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \frac{1}{n+1} + \frac{(n-1)}{(n+1)} + \frac{(n+3)}{(n+1)(n+2)}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n+1} + \frac{(n-1)(n+2)+(n+3)}{(n+1)(n+2)}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n+1} + \frac{n+1}{n+2}$$

$= R.H.S$ of $P(n+1)$

From steps (1) and (2) $P(n)$ is true for all $n \geq 1$.