CITY UNIVERSITY OF HONG KONG

Department of Mathematics

Course Code & Title	:	MA1301 Enhanced Calculus and Linear Algebra II
Session	:	Semester B, 2015-2016
Time Allowed	:	Three hours

This paper has **Three** pages. (including this cover page)

Instructions to candidates:

1. This paper has **EIGHT** questions.
2. Answer **ALL** questions.
3. Start each main question on a new page.
4. Show all step.

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*This is a **closed-book** examination.*

Candidates are allowed to use the following materials/aids:

Non-programmable portable battery operated calculator.

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

1. (a) [7 marks] Express the following integral as a limit of Riemann sum and use the limit to evaluate the integral:

$$\int_{-1}^{0} (x^2 + x)dx.$$

(**Hint**: $\sum_{k=1}^{n} k = n(n+1)/2$ and $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$.)

(b) [5 marks] Prove or disprove (provide a counterexample) the following statement

If f is continuous on $(-\infty, \infty)$, then $\int_{-\infty}^{\infty} f(x)dx = \lim_{t\to\infty} \int_{-t}^{t} f(x)dx$.

(c) [5 marks] Prove or disprove (provide a counterexample) the following statement

If $f(x) \leq g(x)$ and $\int_0^{\infty} f(x)dx$ diverges, then $\int_0^{\infty} g(x)dx$ also diverges.

2. Calculate the following quantities

(a) [9 marks] $$\int \frac{e^{2x}}{4e^{2x} + 4e^x + 5} dx.$$

(b) [9 marks] $$\int te^{\sqrt{t}}dt.$$

3. (a) [6 marks] Find the arc length of $y = \ln(\sec x)$ between $0 \leq x \leq \frac{\pi}{4}$.

(b) [6 marks] Find the area of the surface obtained by rotating the following curve about the y-axis,

$$y = \frac{x^4}{16} + \frac{1}{2x^2}, \quad 1 \leq x \leq 2.$$

4. (a) [8 marks] Let L_1 be a line passing through $A(1,0,1)$ and $B(1,2,3)$, and L_2 be a line passing through $C(-1,-2,-3)$ and $D(3,2,1)$. Use the concept of vectors to find the shortest distance between L_1 and L_2.
(**Hint**: Find a vector perpendicular to both lines.)

(b) [6 marks] Use the concept of vectors to prove that the points $E(1,1,1)$, $F(2,3,4)$, $G(3,5,7)$, $H(3,4,3)$ are located in the same plane.

5. (a) [6 marks] Solve the equation $z^5 + 4\sqrt{3}\,i = 4$ in the set of all complex numbers and express your answer in **Euler's form** with **principal value of argument**.

(b) [8 marks] Find the modulus and argument of

$$\left(\frac{1-\cos\theta - i\sin\theta}{1+\cos\theta + i\sin\theta}\right)^3, \quad \text{with } 3\pi/2 < \theta < 2\pi.$$

Express your answers in terms of θ.

6. [10 marks] Consider the following linear system

$$\begin{cases} \alpha x_1 + \alpha x_2 + x_3 & = 2+\alpha, \\ 2\alpha x_1 + \alpha x_2 + x_3 & = 2, \\ x_1 + x_2 + (2-\alpha)x_3 & = 2. \end{cases}$$

Find all possible values of α such that the system has

(a) no solution.

(b) infinitely many solutions.

(c) a unique solution.

7. Let

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$

(a) [3 marks] Verify that $A^3 - 3A^2 + 3A = I$.

(b) [3 marks] Hence or otherwise find A^{-1}.

8. (a) [5 marks] Let M be an $n \times n$ matrix and $M^{100} = 0$, which is a zero matrix. Show that $I + M$ is invertible.

(b) [4 marks] Let N be an 4×4 matrix and the determinant $\det(N) = 5$. Compute the determinants: $\det(N^T N^2)$ and $\det(2N^{-1})$.

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