Name: LIU Hengche (e.g., Chan Siu Pang)   Student Number: 57854329 (e.g., 12345678)

CS2402- Lecture 9 - In-Class Exercises

Q1. Let the pair of random variables (X, Y) take values (1, 0), (0, 1), (−1, 0), and (0,−1), each with equal probability (1/4).

(a) Find P(X) and P(Y).

Y \ X	-1	0	1	P(X=x)
-1	0	1/4	0	$\frac{1}{4}$
0	1/4	0	1/4	$\frac{1}{2}$
1	0	1/4	0	$\frac{1}{4}$
P(Y=y)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1

(b) When Y=0, compute P(X|Y).
Hint: When P(Y)>0, P(X=x|Y=0)=P(X=x, Y=0)/P(Y=0).

X	-1	0	1
P(X=x|Y=0)	$\frac{1}{4}$	0	$\frac{1}{4}$

(c) Are X and Y independent of each other?

Q2. Alice is studying whether Android and iPhone users stay up late playing games. After a survey, she compiled the following joint probability table:

		Bed time		
		early	late	
Phone user	Android	12%	28%	40%
	iPhone	24%	36%	60%
		36%	64%	

Who is more likely to go to bed late, an Android user or an iPhone user? Calculate the probability that an Android user will go to bed late, i.e., p(late | Android), and the probability that an iPhone user will go to bed late, i.e. p(late | iPhone).

$P(late|Android) = \frac{28\%}{40\%} = \frac{7}{10} = 70\%$

android.

$P(late|iphone) = \frac{36\%}{60\%} = \frac{6}{10} = 60\%$

Q3. If someone goes to bed late, are they more likely to be an Android or an iPhone user? Calculate the probability that someone who goes to bed late is an Android user, i.e. p(Android | late), and an iPhone user, i.e., p(iPhone | late).

$p(Android|late) = \frac{28\%}{64\%} = \frac{7}{16}$

iphone user

$P(iphone|late) = \frac{36\%}{64\%} = \frac{9}{16}$

$(3p^2(1-p)+p^3)(p^3+(1-p)^3) = p^3$

$(3-3p+p)(\cancel{p^3}+1-3p+3p^2\cancel{-p^3}) = p$

$(3-2p)(3p^2-3p+1) = p$

$9p^2-9p+3-6p^3+6p^2-2p = p$

$15p^2-6p^3-12p+3=0$

Q4. Suppose you are given a box of 100 balls. Half of the balls are red, and the other half are black. Now you are asked to take one ball out of the box. After recording its color, you put the ball back into the box. What is the probability of seeing 10 red balls?

$(p^2-p)(-6p+9)-3p+3$

Given that you saw 10 red balls, is it more likely to see a black ball next? What is the probability of picking a black ball next?

10 red balls : $(\frac{1}{2})^{10}$

$P(A) = 3p^2-2p^3$ $\quad p(p-1)(-6p+9)-3(p-1)$

$P(B) = 3p^2-3p+1$

$P(A)\cdot P(B) = (3p^2-2p^3)(3p^2-3p+1) = p^3$

No. still $(\frac{1}{2})$ independent.

$9p^4-9p^3+3p^2-6p^5+6p^4-2p^3-p^3=$

$15p^4-6p^5-11p^3+3p^2-p^3$

$15p^2-6p^3-12p+3=0$

$$\begin{array}{r} -6p+9 \\ p^2-p \overline{)\,-6p^3+15p^2-12p+3} \\ -6p^3+6p^2 \\ \hline 9p^2-12p \\ 9p^2-9p \\ \hline -3p+3 \end{array}$$

$(p-1)(-6p^2+9p-3)$

$-3(p-1)(2p^2-3p+1)$

Q5. A coin has probability p of showing heads. Flip it three times and consider the events A={at most one tails} and B={all flips are the same}. For which values of p are A and B independent?

Hint: $(1-p)^3 = 1-3p+3p^2-p^3$

000
001
010
011
100
101
110
111

$P(A) = C_3^3 p^3 + C_3^1 (1-p)^1 p^2 = p^3+3p^2-3p^3$

$P(B) = C_3^3 p^3 + C_3^3 (1-p)^3 = \cancel{p^3}+1-3p+3p^2\cancel{-p^3}$

$= 3p^2-3p+1$

$P(AB) = p^3$

$\therefore$ ~~$p^3 = p^3+3p^2(1-p)+p^3+(1-p)^3$~~

~~$0 = 0+3p^2-3p^3-6p^3+1-3p+3p^2-p^3$~~

~~$6p^2-3p^3-3p+1=0$~~

~~$p(-6p+3p^2+3)+1=2$~~

~~$1=3p(p^2-2p+1)$~~

~~$1=3p(p-1)^2$~~

~~$p(1-p)^2=$~~ $\frac{1}{3}$

Q6. There are three boxes, each with two drawers. Box 1 has a gold coin in each drawer, and box 2 has a silver coin in each drawer. Box 3 has a silver coin in one drawer and a gold coin in the other. One box is chosen at random, then a drawer is chosen at random from the box. Find the probability that box 1 is chosen, given that the chosen drawer yields a gold coin.

Box 1	Box 2	Box 3
gold coin	silver coin	silver coin
gold coin	silver coin	gold coin

$$\begin{array}{r} -6p^2+9p-3 \\ p-1 \overline{)\,-6p^3+15p^2-12p+3} \\ -6p^3+6p^2 \\ \hline 9p^2-12p+3 \\ 9p^2-9p \\ \hline -3p+3 \end{array}$$

$P(\text{Gold}) = \frac{1}{3}+\frac{1}{3}\times\frac{1}{2} = \frac{1}{2}$

$P(\text{Gold} \,\&\, \text{Box}1) = \frac{1}{3}$

$-3(p-1)(2p^2-3p+1)$

$\therefore P(\text{Gold} \mid \text{Box}1) = \dfrac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$.

$-3(p-1)(2p-1)(p-1)$

CS2402

$-3(p-1)^2(2p-1)$

$\therefore p=1$ or $p=\frac{1}{2}$

$p=\frac{1}{2}$