MA2185 Midterm 2021

Discrete Mathematics (City University of Hong Kong)

CITY UNIVERSITY OF HONG KONG
Department of Mathematics

Course Code & Title : MA2185 Discrete Mathematics

Session : Semester A 2021/22

Time Allowed : 1.5 Hours

This paper has **THREE** pages (including this cover page).

Instructions to candidates:

1. Answer **ALL FOUR** questions.
2. Start each question on a new page.
3. Show **ALL** steps to obtain **FULL** grade.

This is a closed-book examination.

Candidates are allowed to use the following materials/aids:

Approved non-programmable calculator.

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

Special materials (other than standard materials e.g. answer book or supplementary sheets) to be supplied to students:

NIL

Question One (30 marks Total)

Consider the following argument is valid, and give a proof.

If the band could not play jazz music or catering delivery was delayed, then the birthday party would have been cancelled and Kimmy would have been upset. If the party were cancelled, then refunds would have had to be made. However, no refunds were made. Therefore, the band could play jazz music."

a) State the assumptions. **(10 marks)**

b) State the logical statements. **(10 marks)**

c) Provide a proof of its validity. **(10 marks)**

Question Two (10 marks)

Prove by mathematical induction or otherwise. Let n be a positive integer.

$$\sum_{k=1}^{n} (-1)^k k^2 = (-1)^n \frac{n(n+1)}{2}$$

Question Three (30 marks Total)

(a) In how many ways 3 cougars, 4 lions, 3 tigers and 2 monkeys can be arranged at a zoo exhibition in a row on the stage so that all animals of the same species are together? **(10 marks)**

(b) In a concert, four couples are to be seated in a row having eight seats. **(10 marks)**

(i) If couples are to be seated next to each other, in how many ways can they be seated?

(ii) Find the number of ways of their seating if all the gentlemen sit together?

(c) Find the number of permutations of x different things taken y at a time such that two specific things cannot occur together. **(10 marks)**

Question Four (30 marks Total)

(a) The company's profit has greatly declined because of COVID-19. Therefore, a CEO must lay off 10 employees, choosing at least 4 from each of the marketing department and the accounting department. If there are 6 employees in the marketing department and 7 in the accounting department, in how many ways can the CEO choose 10 employees? **(10 marks)**

(b) In the far north, there are 87 polar bear families, of which 52 polar bear families have at most 2 cubs. A philanthropist donated 500 fishes that are enough for 20 polar bear families,

of which at least 18 families must have at most 2 cubs. In how many ways can the choice be made? **(10 marks)**

(c) Each side of a square is divided into 4 equal segments. How many triangles are there with vertices at the Set V = {points of division} $\cup$ {the four corners of the square}.
(10 marks)

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