Final 7 December Autumn 2020, questions

Discrete Mathematics (City University of Hong Kong)

Academic Honesty Pledge

I pledge that the answers in this exam/quiz are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- I will not plagiarize (copy without citation) from any source;
- I will not communicate or attempt to communicate with any other person during the exam/quiz; neither will I give or attempt to give assistance to another student taking the exam/quiz; and
- I will use only approved devices (e.g., calculators) and/or approved device models.

I understand that any act of academic dishonesty can lead to disciplinary action.

Student ID:____________________________________

Signature:____________________________________

Date:____

City University of Hong Kong

Course code & title: MA 2185 Discrete Mathmatics
Session: Semester A, 2020-2021
Time allowed: Two hours

This paper has **three** pages (including this cover page).

1. This paper consists of **5** questions. Full mark is **100**.
2. Attempt ALL questions.
3. Start each question on a **new** page .
4. Show all working and write clearly.
5. The use of pencil is not permitted.

Materials, aids & instruments which students are permitted to use during examination:

Non-programmable portable battery operated calculator

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

Question 1: [10+10+10 marks]

1. Find the negation of the quantification $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$.

2. Consider the following argument:

   If John can solve this problem, then he is clever or lucky. If John is clever, then he understands logic. John gets an A in this course or he is not lucky. Therefore, if John can solve this problem, then he gets an A in this course or he understands logic.

   Determine whether this argument is valid or invalid, and give a proof or counter-example.

3. Prove the following statement: If n is an integer greater than 6,

$$3^n < n!.$$

Question 2: [10+10 marks]

Suppose S and T are two nonempty sets and f is a function from S to T. Let $\tilde{R}$ be an equivalence relation on T. Let R be a binary relation on S such that xRy if and only if $(f(x), f(y)) \in \tilde{R}$.

(a) Show that R is an equivalence relation on S.

(b) Let $S = \{a_1, a_2, a_3\}$, $T = \{b_1, b_2, b_3\}$ and let f and $\tilde{R}$ be defined by the following

$$f(a_1) = b_3, \quad f(a_2) = b_1, \quad f(a_3) = b_2$$

and

$$\tilde{R} = \{(b_i, b_j) \mid i - j \text{ is even}\}.$$

   (a) List the elements of R.

   (b) Draw the directed diagram of R.

Question 3: [8+12 marks]

1. How many elements are in the union of 4 sets if the sets contain $1,000$ elements each, each pair of sets has 100 common elements, each triple of sets has 10 common elements, and there is 1 element in all four sets?

2. A club with 6 men and 4 women needs to form a committee of size four.

   (a) How many committees are possible?

   (b) How many committees are possible if the committee must not contain more men than women?

   (c) Assume there are a man and woman who are both members of the club and are married to each other. How many committees are possible if the committee must contain two men and two women, but may not contain both of the married couple?

Question 4: [20 marks]
Solve the recurrence relation

$$a_n = a_{n-1}a_{n-2}^6$$

for $n \geq 0$, $a_0 = 1$ and $a_1 = 2$. *Hint: Set* $b_n = \ln a_n$.

Question 5: [10 marks]
Try *ONE* of the following two questions.

1. Let A be a set. Let $f : A \to A$ be a surjective function and $f \circ f = f$. Show that $f(a) = a$, for all $a \in A$.

2. Show that
$$\binom{n+r+1}{r} = \sum_{k=0}^{r} \binom{n+k}{k},$$
whenever n and r are positive integers.

END

do not take away this paper