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CS2402 – Lecture6 - In-Class Exercise

Q1. Suppose that X stands for a temperature in degrees Celsius, Y the same temperature in degrees Fahrenheit. So:

$$Y = \frac{9}{5}X + 32$$

Let E(X)=10 and Var(X)=2, Find E(Y) and Var(Y).

$E(Y) = E(\frac{9}{5}X + 32) = 50$

$Var(Y) = Var(\frac{9}{5}X + 32) = (\frac{9}{5})^2 \times 2 = \frac{162}{25}$

Q2. Suppose X and Y are independent of each other. Find a formula of Var(X-Y) in terms of $EX^2, EX, EY^2, and\ EY$.

$Var(X-Y) = Var(X+Y) = Var(X) + Var(Y)$ if X, Y are independent

$= E(X^2 - 2XY + Y^2) - (E(X) - E(Y))^2$

$= E(X^2) - 2E(X)E(Y) + E(Y^2) - (E(X)^2 - 2E(X)\cdot E(Y) + (E(Y))^2)$

$= E(X^2) - (E(X))^2 + E(Y^2) - (E(Y))^2$

Q3. Suppose that 10% of the numbers in a list are 10, 20% of them numbers are 30, and the remaining numbers are 50, what is the variance and the standard deviation of the numbers in the list?

$E(X) = 0.1 \times 10 + 0.2 \times 30 + 0.7 \times 50$

$= 1 + 6 + 35 = 42$

$V(X) = E(X^2) - (EX)^2 = 0.1 \times 100 + 0.2 \times 900 + 0.7 \times 2500 - 42^2$

$= 10 + 180 + 1750 - 1764$

$= 176$

Q4. Suppose that the average temperature in an area is 20 and the variance is 100. Let p=the probability that temperatures are between 0 and 40, find a lower bound for p.

sd = 10

$$P(|X-EX| \geq (2)\cdot 10) \leq \frac{1}{2^2}$$

$$P(|X-EX| < 2\cdot 10) > 1-\frac{1}{4}$$

$$\therefore \frac{3}{4}$$

Q5. Let Y be the number of heads obtained if a fair coin is tossed three times. Find the mean and variance of Y^2

Y	0	1	2	3
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
Y^2	0	1	4	9
Y^4	0	1	16	81

$$E(Y^2) = \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = 3$$

$$\text{var}(Y^2) = E(Y^4) - (E(Y^2))^2$$

$$= \frac{3}{8} + \frac{81}{8} + 6 - 9$$

$$= 10.5 - 3 = 7.5$$

Q6. Suppose three marksmen shoot at a target. The i^{th} marksman fires n_i times, hitting the target each time with probability P_i, independently of his other shots and the shots of the other marksmen. Let X be the total number of times the target is hit.

a) Is the distribution of X binomial?

b) Find E(X) and Var(X).

(a) 1 n_1 times P_1

2 n_2 times. P_2

3 n_3 times P_3

no, because

P_1, P_2, P_3

might not be equal

(b). $E(X) = n_1P_1 + n_2P_2 + n_3P_3$

$\text{Var}(X) = \text{var}(X_1) + \text{var}(X_2) + \text{var}(X_3)$

$= n_1P_1(1-P_1) + n_2P_2(1-P_2) + n_3P_3(1-P_3)$