

CITY UNIVERSITY OF HONG KONG

Course code & title : MA2144 Discrete Mathematics

Session : Semester B, 2006–2007

Time Allowed : Two Hours

This paper has **THREE** pages. (Including this page)

Instructions to candidates:

1. This paper consists of three questions.
 2. Answer ALL questions.
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Materials, aids and instruments permitted to be used during examination:

Approved calculator

Question 1

- (a) Determine whether the following two propositions are equivalent

$$(p \vee q) \wedge [(p \wedge q) \wedge r] \quad \text{and} \quad (p \wedge q) \wedge r. \quad [5 \text{ marks}]$$

- (b) Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements “ x is a hummingbird”, “ x is large”, “ x lives on honey”, and “ x is richly coloured”, respectively. Suppose that the universe of discourse consists of all birds. Express each of the following statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$ and $S(x)$.

- (i) All hummingbirds are richly coloured.
- (ii) No large birds live on honey.
- (iii) Birds that do not live on honey are not richly coloured.
- (iv) Hummingbirds are small.
- (v) Does (iv) follow from (i), (ii) and (iii)? Please explain your answer. If you do not think so, is there a correct conclusion?

[13 marks]

- (c) Prove by mathematical induction that

$$\frac{1}{2} \cdot \frac{3}{4} \cdots \frac{2n-1}{2n} < \frac{1}{\sqrt{3n+1}}.$$

whenever n is a positive integer greater than 1.

[15 marks]

Question 2

- (a) Let A , B and C be sets. Prove or disprove:

$$\text{if } A \cup B = B \cup C \text{ and } A \cap C = B \cap C, \text{ then } A = B. \quad [8 \text{ marks}]$$

- (b) Let $A = \{1, 2, 3, \dots, 15\}$. Define a relation R on the set A by

$$(x, y) \in R \text{ for } x, y \in A \Leftrightarrow \text{The difference between } x \text{ and } y \text{ is a multiple of either 3 or 5.}$$

- (i) Is x related to itself? Give reason for your answer.
- (ii) Find the values of x in A such that x is not related to $2x$.
- (iii) If $(x, y) \in R$ and $(y, z) \in R$, is it true that $(x, z) \in R$? Give reason for your answer.
- (iv) Find all the elements in A which are related to 4.

[12 marks]

- (c) Let R be the set of real numbers and $f: R \times R \rightarrow R \times R$ be defined by

$$f(x, y) = (x + 3y + 1, -2x - 6y + 2).$$

The inverse of f is defined as

$$f^{-1}(x_1, y_1) = \{(x, y) \in R \times R \mid f(x, y) = (x_1, y_1)\}$$

- (i) What is the image of $(1, -2)$?
- (ii) Find $f^{-1}(2, 0)$ and $f^{-1}(3, 6)$.
- (iii) Is f an injective function? Give reason for your answer.
- (iv) Is f a surjective function? Give reason for your answer.
- (v) Describe the range of f .

[13 marks]

Question 3

- (a)
 - (i) How many strings of length 8 can be formed from the word *BARBARAS* if no two *B*'s are consecutive?
 - (ii) How many strings of length 7 can be formed from the word *BARBARAS*?
[10 marks]
- (b) How many ways can the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 be arranged so that no even digit is in its original position?
[12 marks]
- (c) A university graduate joined a company in 2007 with a starting salary of \$10,000. Every year this university graduate receives a raise of \$1,000 plus 5% of the salary of the previous year.
 - (i) Set up a recurrence relation for the salary of this university graduate n years after 2007.
 - (ii) Find an explicit formula for the salary of this university graduate n years after 2007.
 - (iii) When will the salary of this university graduate be doubled?

[12 marks]

*** End ***