CS2402 - Tutorial 8

Task 1: The values of x and their corresponding values of y are shown in the table below

$\bar{x}=0$

$\bar{y}=\frac{13}{3}$

X	-1	0	1
Y	3	4	6

$a=\frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}=\frac{(-1)\times(-\frac{4}{3})+0+1\times\frac{5}{3}}{1+0+1}=\frac{\frac{9}{3}}{2}=1.5$

$\therefore \hat{y}=\frac{3}{2}x+\frac{13}{3}$

1) Find the least square regression line y = a·x + b.
2) Estimate the value of y when x = 2 using the derived parameters in 1)

$\hat{y}=3+\frac{13}{3}=\frac{22}{3}=7.3$

$3\ln p+\ln(1-p)$

① $p=\frac{2}{3}$

$3\ln\frac{2}{3}+\ln\frac{1}{3}$

$\ln(\frac{2}{3})^3\times\frac{1}{3}$

$\therefore p=\frac{2}{3}$

② $p=\frac{1}{3}$

$\ln(\frac{1}{3})^3\times\frac{2}{3}$

Task 2: A digital communication system transmits 0's and 1's. We know that on average, one of the bits is sent twice as often as the other, but we do not know which one. In order to try to decide which, we have the following four observations: 1, 1, 0, 1 (independent transmissions and no transmission error). Find the probability that 1 is sent using maximum likelihood estimation to find the probability that 1 is sent.

p is predefined, $\frac{1}{3}$ or $\frac{2}{3}$

$\log p \to \frac{1}{p}$

$\log(1-p)\to\frac{-1}{1-p}$

~~$p^3(1-p)$~~ ~~$\ln p^3+\ln(1-p)$~~ ~~$3\ln p+\ln(1-p)$~~ $=\frac{3}{p}-\frac{1}{1-p}\Rightarrow\frac{3}{p}=\frac{1}{1-p}$ $\quad 4p=3$ $\quad p=3-3p$ $\quad p=\frac{3}{4}$

Task 3: In task 2, we assume that we do not know anything about p. Using maximum likelihood estimation to find the probability that 1 is sent.

$p^3(1-p)\Rightarrow\ln p^3+\ln(1-p)\Rightarrow\frac{3}{p}-\frac{1}{1-p}=0\Rightarrow p=\frac{3}{4}$

Task 4: 1) Let $\boxed{1,-1,2}$ be 3 random sample from a normal distribution with mean 0 and unknown standard deviation σ. Use the maximum likelihood estimation to estimate σ.

$N(\mu,\sigma^2)$

$N(x;\mu,\sigma^2)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$\sigma^2=\frac{1}{3}(1+1+4)=2 \quad \therefore \sigma=\sqrt{2}$

$N(x;\sigma^2)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}$

2) Let $x_1,x_2,..,x_n$ be n random sample from a normal distribution with mean 0 and unknown standard deviation σ, Use the maximum likelihood estimation to estimate σ.

$\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1^2}{2\sigma^2}}\cdot\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(-1)^2}{2\sigma^2}}\cdot\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{2^2}{2\sigma^2}}$

$3(-\frac{1}{2}\ln 2\pi-\ln\sigma)-\frac{1}{2\sigma^2}-\frac{(-1)^2}{2\sigma^2}-\frac{2^2}{2\sigma^2}$

$-\frac{3}{2}\ln 2\pi-3\ln\sigma-\frac{6}{2\sigma^2}$

$3\ln\sigma+\frac{3}{\sigma^2}$

$\frac{3}{\sigma}-6\frac{1}{\sigma^3}$

$\frac{3\sigma^2-6}{\sigma^3}=0$

$\sigma^2=2$

$\sigma^2=\frac{1}{n}(x_1^2+\cdots+x_n^2)$

$\therefore \sigma=\sqrt{\frac{x_1^2+\cdots+x_n^2}{n}}$

$\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x_1^2}{2\sigma^2}}\cdot\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x_2^2}{2\sigma^2}}\cdots\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x_n^2}{2\sigma^2}}$

$(-\frac{1}{2}\ln 2\pi-\ln\sigma-\frac{x_1^2}{2\sigma^2})+\cdots+(-\frac{1}{2}\ln 2\pi-\ln\sigma-\frac{x_n^2}{2\sigma^2})$

$=-(\frac{n}{2}\ln 2\pi+n\ln\sigma+\frac{1}{2\sigma^2}(x_1^2+\cdots+x_n^2))$

$\frac{n}{2}\ln\sigma^2+\frac{1}{2\sigma^2}(x_1^2+\cdots+x_n^2)$

$\sigma^2=\theta$

$\frac{n}{2}\ln\theta+\frac{1}{2\theta}(x_1^2+\cdots+x_n^2)$

take differentiative

$\frac{n}{2}\frac{1}{\theta}-\frac{1}{2\theta^2}(x_1^2+\cdots+x_n^2)=0$

$\frac{n}{2}\frac{1}{\theta}=\frac{1}{2}\frac{1}{\theta^2}(x_1^2+\cdots+x_n^2)$

$n\theta=(x_1^2+\cdots+x_n^2) \quad \theta=\frac{x_1^2+\cdots+x_n^2}{n}$

Student EID : hengchliu2
(e.g., spchan31)

Student Name : LIU Hengche
(e.g., Chan Siu Pang)

$3\ln\sigma+\frac{3}{\sigma^2}$

$\frac{3}{2}\ln\sigma^2+\frac{3}{\sigma^2}$ (let $\theta=\sigma^2$)

$f(\theta)=\frac{3}{2}\ln\theta+\frac{3}{\theta}$

$f'(\theta)=\frac{3}{2}\frac{1}{\theta}-\frac{3}{\theta^2}$

$\therefore \sigma^2=\frac{x_1^2+\cdots+x_n^2}{n}$

$\sigma=\sqrt{\frac{x_1^2+\cdots+x_n^2}{n}}$