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CSCI 574 Homework #4
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Problem: Structure from Motion (SFM)

I Abstract and Motivation

In this assignment, it requires the reconstruction of 3D object from six images given 8 points. The main part is the SVD decomposition of the 2m*n matrix (m is the number of images, n is the number of points) into camera motion A_0 and scene structure P_0. P_0 will gives the reconstructed 3D points coordinates. This procedure is achieved by affine Tomasi-Kanade factorization algorithm. For checking if the reconstruction is approximately an affine transformation, the expected parallel lines should be approximately parallel to each other. Suppose two 3D lines has equation $ax+by+cz+d=0$ and $a'x+b'y+c'z+d'=0$, and these two lines are parallel if $a/a'=b/b'=c/c'\neq d/d'$. However, with affine distortion and SVD error, the condition is hardly achievable. Relax the condition to $a/a'\approx b/b'\approx c/c'\neq d/d'$ will give approximate parallel lines, then calculate the mean square error to show the distortion error rate.

II Approach and Procedures

1. Construct 2m*n matrix D that includes the image coordinates from given points.
2. Compute the singular value decomposition $D=UWV^T$. U is a 2m*n matrix, W is a diagonal n*n matrix, and V^T is a n*n matrix.
3. From results of step 2, construct U_3, W_3 and V_3^T by the 3 leftmost columns of the matrices U and V, and the corresponding 3 by 3 sub-matrix of W.
4. Define $A_0 = U_3 \sqrt{W_3}$, $P_0 = \sqrt{W_3} V_3^T$. $\sqrt{W_3}$ is calculated by computing the square root of each element of W_3. P_0 is the reconstruction of 3D object.
5. Via weak perspective projection to show the reconstructed 3D points, setting the Z-axis coordinate to 0.
6. Display the reconstructed visualization.
7. Check if expected parallel lines are approximately parallel, and show the mean square distortion error.
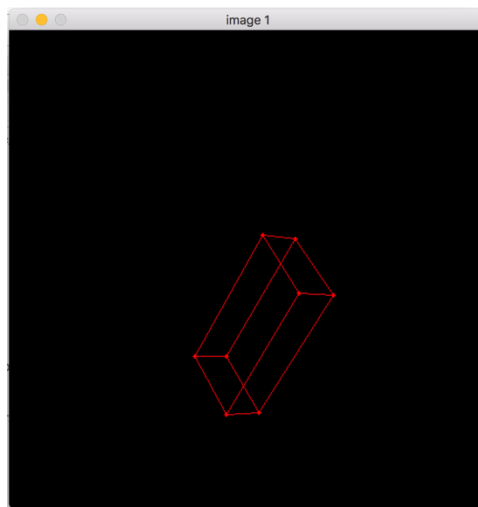
III Experimental Results
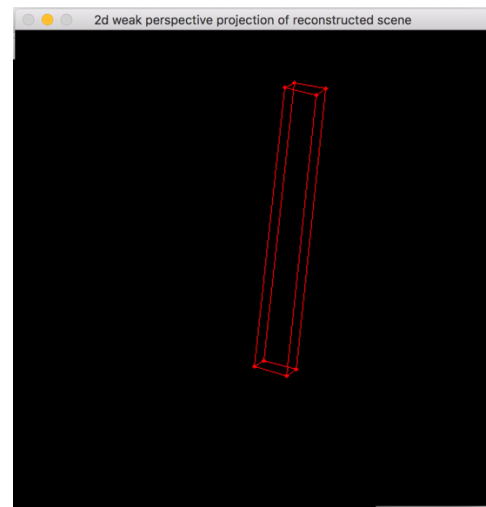
Shown below.

Figure 1 Image 1 (for comparison)

Figure 2 reconstructed 3D object weak perspective projection

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The reconstructed 8 3D points world coordinates:
[14.125732, 15.781492, 14.231357, 12.563315, 14.639466, 16.317501, 14.72503, 13.030081;
 -6.9891868, -6.6037226, 8.1339674, 7.6600528, -7.2584009, -6.9454389, 7.7775898, 7.3688698;
 1.5506141, -5.8854136, -5.8122935, 1.8429245, 5.9587731, -1.4353211, -1.2324338, 6.5138335]
```

Figure 3 Reconstructed 3D points world coordinates

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The reconstructed scaled 8 3D points world coordinates:
[283, -140, 31;
 316, -132, -118;
 285, 163, -116;
 251, 153, 37;
 293, -145, 119;
 326, -139, -29;
 295, 156, -25;
 261, 147, 130]
```

Figure 4 Scaled and transposed 8 3D points world coordinates

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line(1,2) compares with line(3,4):
Ratio of x,y,z axis coefficient: -0.992637 -0.813362 -0.971367.
Mean square error of parrallelism: 0.639529%.

line(1,2) compares with line(5,6):
Ratio of x,y,z axis coefficient: 0.986726 1.23166 1.00567.
Mean square error of parrallelism: 1.23808%.

line(1,2) compares with line(7,8):
Ratio of x,y,z axis coefficient: -0.976879 -0.943101 -0.95995.
Mean square error of parrallelism: 0.0190158%.

line(2,3) compares with line(4,1):
Ratio of x,y,z axis coefficient: -0.992139 -1.00604 -0.250145.
Mean square error of parrallelism: 12.468%.

line(2,3) compares with line(6,7):
Ratio of x,y,z axis coefficient: 0.973415 1.001 0.360398.
Mean square error of parrallelism: 8.74352%.

line(2,3) compares with line(8,5):
Ratio of x,y,z axis coefficient: -0.963185 -1.00755 -0.131734.
Mean square error of parrallelism: 16.2259%.

line(1,5) compares with line(2,6):
Ratio of x,y,z axis coefficient: 0.958443 0.787829 0.990577.
Mean square error of parrallelism: 0.791646%.

line(1,5) compares with line(3,7):
Ratio of x,y,z axis coefficient: 1.04064 0.755418 0.96251.
Mean square error of parrallelism: 1.44821%.

line(1,5) compares with line(4,8):
Ratio of x,y,z axis coefficient: 1.10063 0.924553 0.943748.
Mean square error of parrallelism: 0.622005%.
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Figure 5 Parallelism and corresponding distortion error

IV Discussion

Figure 2 is the reconstructed 3D points weak perspective projection result. The vertical 4 lines preserve the parallelism well. However, some lines have larger distortion. From Figure 5, line (2, 3) and line (8, 5) has the largest distortion rate with over 16%, and the second largest one is 12.47% for line (2, 3) and line (4, 1). There are 3 groups of parallel lines, only the group with lines (2, 3) (4, 1) (6, 7) (8, 5) has great error rate but tolerable, while the other two groups remain fine parallelism with low error. If the error is zero, it means the two lines are truly parallel. So, overall the reconstruction object is approximately the affine transformation of a rectangular solid.