

CSCI576 Assignment 2 Report

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Part 1

Written Questions

Q 1

Proof:

The chromaticity coordinates (x_c, y_c) are computed by the projective transformation: $x_c = \frac{X}{X+Y+Z}, y_c = \frac{Y}{X+Y+Z}$,
then $\frac{1-x_c-y_c}{y_c} \cdot Y = \frac{Z}{(X+Y+Z) \cdot y_c} \cdot Y = \frac{Z}{Y} \cdot Y = Z$.

Q 2

Solutions:

- Normalized chromaticity coordinates:

$$P_1(x_1, y_1) = \left(\frac{X_1}{X_1+Y_1+Z_1}, \frac{Y_1}{X_1+Y_1+Z_1} \right), P_2(x_2, y_2) = \left(\frac{X_2}{X_2+Y_2+Z_2}, \frac{Y_2}{X_2+Y_2+Z_2} \right), P_3(x_3, y_3) = \left(\frac{X_3}{X_3+Y_3+Z_3}, \frac{Y_3}{X_3+Y_3+Z_3} \right).$$

- Color C XYZ coordinates are: $X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3, Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3, Z = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3$.

Then, the chromaticity coordinates are: $\left(\frac{X}{X+Y+Z}, \frac{Y}{X+Y+Z} \right) = \left(\frac{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3}{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3 + \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3}, \frac{\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3}{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3 + \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3} \right) = \left(\frac{\alpha_1(X_1+Y_1+Z_1)x_1 + \alpha_2(X_2+Y_2+Z_2)x_2 + \alpha_3(X_3+Y_3+Z_3)x_3}{\alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)}, \frac{\alpha_1(X_1+Y_1+Z_1)y_1 + \alpha_2(X_2+Y_2+Z_2)y_2 + \alpha_3(X_3+Y_3+Z_3)y_3}{\alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)} \right).$

- Based on above deduction,

Chromaticity coordinates of Color C $\left(\frac{X}{X+Y+Z}, \frac{Y}{X+Y+Z} \right) = \left(\frac{\alpha_1(X_1+Y_1+Z_1)x_1 + \alpha_2(X_2+Y_2+Z_2)x_2 + \alpha_3(X_3+Y_3+Z_3)x_3}{\alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)}, \frac{\alpha_1(X_1+Y_1+Z_1)y_1 + \alpha_2(X_2+Y_2+Z_2)y_2 + \alpha_3(X_3+Y_3+Z_3)y_3}{\alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)} \right) =$

$$\begin{aligned}
& \left(\frac{\alpha_1(X_1+Y_1+Z_1)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)} x_1 + \right. \\
& \frac{\alpha_2(X_2+Y_2+Z_2)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)} x_2 + \\
& \frac{\alpha_3(X_3+Y_3+Z_3)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)} x_3, \frac{\alpha_1(X_1+Y_1+Z_1)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)} y_1 + \\
& \frac{\alpha_2(X_2+Y_2+Z_2)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)} y_2 + \\
& \left. \frac{\alpha_3(X_3+Y_3+Z_3)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)} y_3 \right) = \\
& \frac{\alpha_1(X_1+Y_1+Z_1)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)} (x_1, y_1) + \\
& \frac{\alpha_2(X_2+Y_2+Z_2)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)} (x_2, y_2) + \\
& \frac{\alpha_3(X_3+Y_3+Z_3)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)} (x_3, y_3) = \\
& \alpha'_1(x_1, y_1) + \alpha'_2(x_2, y_2) + \alpha'_3(x_3, y_3).
\end{aligned}$$

Thus, the chromaticity coordinates of any color C can be represented as a linear combination of the chromaticity coordinates of the respective primaries.

Q 3

Proof:

- First calculate the perimeter of the wheel, $\pi \times 0.4244 = 1.3333$, then $36 \times 1000 \div 1.3333 \div 3600 = 7.5$ rotations/s, the frame rate 24 fps is larger than twice of this rotation speed, so the true speed is observed as 7.5 rotations/s.

Programming Analysis Questions

Q 1

The distortion curve is generated by the average error percentage for each pixel compared to the original image.

- Distortion curve with Y varying, set U and V to 1:
-0.176407 inf -0.150256 -0.165809 inf -0.133801 -0.173324 inf -0.0869003
-0.179565 inf -0.00607022
- Distortion curve with U varying, set Y and V to 1:
- Distortion curve with V varying, set U and Y to 1:

The pattern is that the error rate increases fast, then slow down up to a limit. First, the average error rates for these three settings are increasing with the sub-sampling factor increasing. Another observation is that Y value is much important compared to the other two values since the error rate increases much faster.

Q 2

The main idea is to subsample differently. For example, instead of sampling with a constant sample interval, for particular image, the histogram is different and the distribution of the texture is way more different. So, with some calculations like gradient, smaller sample interval is applied to complex part of the image.

Another easier way is to increase the total bits for the image.

Some output results are as followed:
optimized one with $Y=4$
original image with $Y=4$.