CSCI576 Assignment 2 Report

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Part 1

Written Questions

Q 1

Proof:

The chromaticity coordinates (x_c, y_c) are computed by the projective transformation: $x_c = \frac{X}{X+Y+Z}, y_c = \frac{Y}{X+Y+Z}$,
then $\frac{1-x_c-y_c}{y_c} \cdot Y = \frac{Z}{(X+Y+Z)\cdot y_c} \cdot Y = \frac{Z}{Y} \cdot Y = Z$.

Q 2

Solutions:

- Normalized chromaticity coordinates:
  $P_1(x_1, y_1) = (\frac{X_1}{X_1+Y_1+Z_1}, \frac{Y_1}{X_1+Y_1+Z_1})$,$P_2(x_2, y_2) = (\frac{X_2}{X_2+Y_2+Z_2}, \frac{Y_2}{X_2+Y_2+Z_2})$,$P_3(x_3, y_3) = (\frac{X_3}{X_3+Y_3+Z_3}, \frac{Y_3}{X_3+Y_3+Z_3})$.
- Color C XYZ coordinates are: $X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3, Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3, Z = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3$.
  Then, the chromaticity coordinates are: $(\frac{X}{X+Y+Z}, \frac{Y}{X+Y+Z}) = (\frac{\alpha_1 X_1+\alpha_2 X_2+\alpha_3 X_3}{\alpha_1 X_1+\alpha_2 X_2+\alpha_3 X_3+\alpha_1 Y_1+\alpha_2 Y_2+\alpha_3 Y_3+\alpha_1 Z_1+\alpha_2 Z_2+\alpha_3 Z_3}, \frac{\alpha_1 Y_1+\alpha_2 Y_2+\alpha_3 Y_3}{\alpha_1 X_1+\alpha_2 X_2+\alpha_3 X_3+\alpha_1 Y_1+\alpha_2 Y_2+\alpha_3 Y_3+\alpha_1 Z_1+\alpha_2 Z_2+\alpha_3 Z_3}) = (\frac{\alpha_1(X_1+Y_1+Z_1)x_1+\alpha_2(X_2+Y_2+Z_2)x_2+\alpha_3(X_3+Y_3+Z_3)x_3}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}, \frac{\alpha_1(X_1+Y_1+Z_1)y_1+\alpha_2(X_2+Y_2+Z_2)y_2+\alpha_3(X_3+Y_3+Z_3)y_3}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)})$.
- Based on above deduction,
  Chromaticity coordinates of Color C $(\frac{X}{X+Y+Z}, \frac{Y}{X+Y+Z}) = (\frac{\alpha_1(X_1+Y_1+Z_1)x_1+\alpha_2(X_2+Y_2+Z_2)x_2+\alpha_3(X_3+Y_3+Z_3)x_3}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}, \frac{\alpha_1(X_1+Y_1+Z_1)y_1+\alpha_2(X_2+Y_2+Z_2)y_2+\alpha_3(X_3+Y_3+Z_3)y_3}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}) =$

$$(\frac{\alpha_1(X_1+Y_1+Z_1)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}x_1 + \frac{\alpha_2(X_2+Y_2+Z_2)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}x_2 + \frac{\alpha_3(X_3+Y_3+Z_3)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}x_3, \frac{\alpha_1(X_1+Y_1+Z_1)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}y_1 + \frac{\alpha_2(X_2+Y_2+Z_2)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}y_2 + \frac{\alpha_3(X_3+Y_3+Z_3)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}y_3) =$$

$$\frac{\alpha_1(X_1+Y_1+Z_1)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}(x_1, y_1) + \frac{\alpha_2(X_2+Y_2+Z_2)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}(x_2, y_2) + \frac{\alpha_3(X_3+Y_3+Z_3)}{\alpha_1(X_1+Y_1+Z_1)+\alpha_2(X_2+Y_2+Z_2)+\alpha_3(X_3+Y_3+Z_3)}(x_3, y_3) =$$

$$\alpha'_1(x_1, y_1) + \alpha'_2(x_2, y_2) + \alpha'_3(x_3, y_3).$$

Thus, the chromaticity coordinates of any color C can be represented as a linear combination of the chromaticity coordinates of the respective primaries.

Q 3

Solutions:

- $H = -(P(X)\log_2 P(X) + P(Y)\log_2 P(Y)) = -[x^2 \log_2 x^2 + (1-x^2)\log_2(1-x^2)]$

  Matlab codes to plot the function:

```
h=inline('-(x.^2.*log2(x.^2)+(1-x.^2).*log2(1-x.^2))');
x=-1:0.01:1;
y=h(x);
plot(x,y);title('problem 3 entropy function H of x');xlabel('x');ylabel('entropy H');
```

  Below is the function plot:

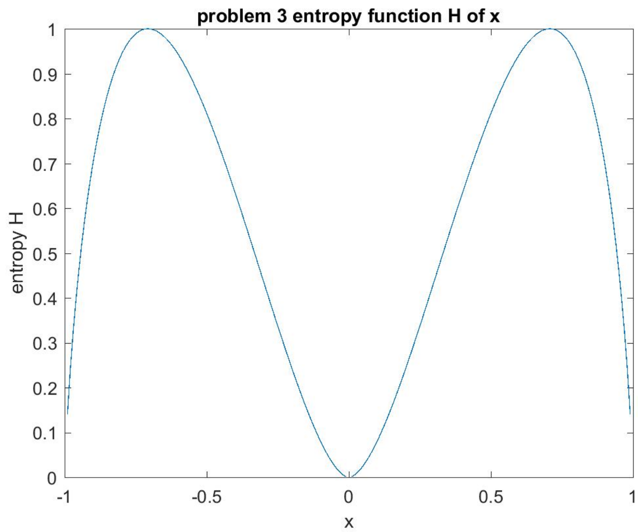

Figure 1.1: Entropy function H of x

- When x equals to -1, 0 or 1, the entropy becomes a minimum value equal to 0. In other words, if only one kind of symbol is transmitted in the system, the entropy is zero. Since x should not be -1, 0 or 1, x can be very closed to these values to get a very small entropy.

- Since the smallest entropy is 0, then let $H = -[x^2 \log_2 x^2 + (1-x^2)\log_2(1-x^2)] = 0$, solutions are $x = -1, 0, 1$.

- Compute the derivative of H in terms of x:
  $\frac{\partial H}{\partial x} = -(2x \log_2 x^2 + x^2 \frac{2x}{x^2 \ln 2} - 2x \log_2(1-x^2) + (1-x^2)\frac{-2x}{(1-x^2)\ln 2}) = 2x[\log_2(1-x^2) - \log_2 x^2] = 2x \log_2(\frac{1}{x^2} - 1)$

  Let $\frac{\partial H}{\partial x} = 0$, solutions are $x = 0, \pm\frac{1}{\sqrt{2}}$.

  Take these values back to H function, and $x = \pm\frac{1}{\sqrt{2}}$ give the maximum entropy 1.