

数据结构与算法 13- 平衡二叉树AVL

笔记本： 我的笔记

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AVL 树

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1962年的论文首次提出

最早的自平衡二分搜索树结构

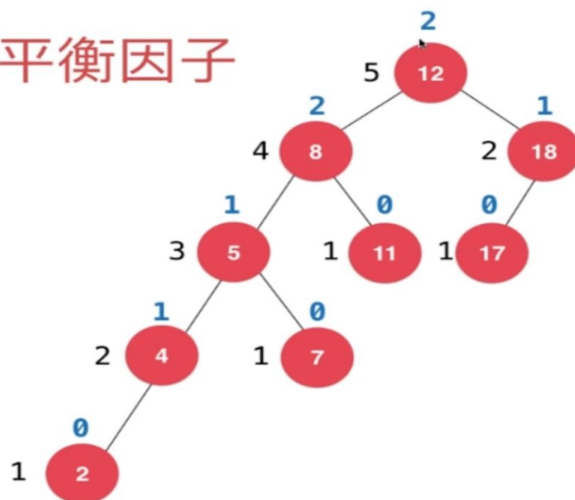
平衡因子

对于任意一个节点，左子树和右子树的高度差不能超过1

平衡二叉树的高度和节点数量之间的关系也是 $O(\log n)$ 的

标注节点的高度

计算平衡因子



1>实现AVL树的各种操作

```
import java.util.ArrayList;

public class AVLTree<K extends Comparable<K>, V> {

    private class Node{
        public K key;
        public V value;
        public Node left, right;
        public int height;

        public Node(K key, V value){
            this.key = key;
            this.value = value;
            left = null;
            right = null;
            height = 1;
        }
    }
}
```

```

    }
}

private Node root;
private int size;

public AVLTree(){
    root = null;
    size = 0;
}

public int getSize(){
    return size;
}

public boolean isEmpty(){
    return size == 0;
}

// 判断该二叉树是否是一棵二分搜索树
public boolean isBST(){

    ArrayList<K> keys = new ArrayList<>();
    inOrder(root, keys);
    for(int i = 1 ; i < keys.size() ; i ++){
        if(keys.get(i - 1).compareTo(keys.get(i)) > 0)
            return false;
    }
    return true;
}

/**
 * 中序遍历
 * @param node
 * @param keys
 */
private void inOrder(Node node, ArrayList<K> keys){

    if(node == null)
        return;

    inOrder(node.left, keys);
    keys.add(node.key);
    inOrder(node.right, keys);
}

// 判断该二叉树是否是一棵平衡二叉树
public boolean isBalanced(){
    return isBalanced(root);
}

// 判断以Node为根的二叉树是否是一棵平衡二叉树，递归算法
private boolean isBalanced(Node node){

    if(node == null)
        return true;

    int balanceFactor = getBalanceFactor(node);
    if(Math.abs(balanceFactor) > 1)
        return false;
    return isBalanced(node.left) && isBalanced(node.right);
}

```

```

// 获得节点node的高度
private int getHeight(Node node){
    if(node == null)
        return 0;
    return node.height;
}

// 获得节点node的平衡因子
private int getBalanceFactor(Node node){
    if(node == null)
        return 0;
    return getHeight(node.left) - getHeight(node.right);
}

// 对节点y进行向右旋转操作，返回旋转后新的根节点x
//      y                      x
//     / \                    /  \
//    x   T4      向右旋转 (y)  /   \
//   / \      - - - - - - -> / \   y
//  z   T3                  T1  T2 T3 T4
// / \
// T1  T2
private Node rightRotate(Node y) {
    Node x = y.left;
    Node T3 = x.right;

    // 向右旋转过程
    x.right = y;
    y.left = T3;

    // 更新height
    y.height = Math.max(getHeight(y.left), getHeight(y.right)) + 1;
    x.height = Math.max(getHeight(x.left), getHeight(x.right)) + 1;

    return x;
}

// 对节点y进行向左旋转操作，返回旋转后新的根节点x
//      y                      x
//     / \                    /  \
//    T1  x      向左旋转 (y)  y   z
//   / \      - - - - - - -> / \   / \
//  T2  z                  T1 T2 T3 T4
//   / \
//  T3 T4
private Node leftRotate(Node y) {
    Node x = y.right;
    Node T2 = x.left;

    // 向左旋转过程
    x.left = y;
    y.right = T2;

    // 更新height
    y.height = Math.max(getHeight(y.left), getHeight(y.right)) + 1;
    x.height = Math.max(getHeight(x.left), getHeight(x.right)) + 1;

    return x;
}

// 向二分搜索树中添加新的元素(key, value)
public void add(K key, V value){
    root = add(root, key, value);
}

```

```

// 向以node为根的二分搜索树中插入元素(key, value)，递归算法
// 返回插入新节点后二分搜索树的根
private Node add(Node node, K key, V value){

    if(node == null){
        size ++;
        return new Node(key, value);
    }

    if(key.compareTo(node.key) < 0)
        node.left = add(node.left, key, value);
    else if(key.compareTo(node.key) > 0)
        node.right = add(node.right, key, value);
    else // key.compareTo(node.key) == 0
        node.value = value;

    // 更新height
    node.height = 1 + Math.max(getHeight(node.left),
getHeight(node.right));

    // 计算平衡因子
    int balanceFactor = getBalanceFactor(node);

    // 平衡维护
    // LL
    if (balanceFactor > 1 && getBalanceFactor(node.left) >= 0)
        return rightRotate(node);

    // RR
    if (balanceFactor < -1 && getBalanceFactor(node.right) <= 0)
        return leftRotate(node);

    // LR
    if (balanceFactor > 1 && getBalanceFactor(node.left) < 0) {
        node.left = leftRotate(node.left);
        return rightRotate(node);
    }

    // RL
    if (balanceFactor < -1 && getBalanceFactor(node.right) > 0) {
        node.right = rightRotate(node.right);
        return leftRotate(node);
    }

    return node;
}

// 返回以node为根节点的二分搜索树中，key所在的节点
private Node getNode(Node node, K key){

    if(node == null)
        return null;

    if(key.equals(node.key))
        return node;
    else if(key.compareTo(node.key) < 0)
        return getNode(node.left, key);
    else // if(key.compareTo(node.key) > 0)
        return getNode(node.right, key);
}

```

```

public boolean contains(K key){
    return getNode(root, key) != null;
}

public V get(K key){

    Node node = getNode(root, key);
    return node == null ? null : node.value;
}

public void set(K key, V newValue){
    Node node = getNode(root, key);
    if(node == null)
        throw new IllegalArgumentException(key + " doesn't exist!");

    node.value = newValue;
}

// 返回以node为根的二分搜索树的最小值所在的节点
private Node minimum(Node node){
    if(node.left == null)
        return node;
    return minimum(node.left);
}

// 从二分搜索树中删除键为key的节点
public V remove(K key){

    Node node = getNode(root, key);
    if(node != null){
        root = remove(root, key);
        return node.value;
    }
    return null;
}

private Node remove(Node node, K key){

    if( node == null )
        return null;

    Node retNode;
    if( key.compareTo(node.key) < 0 ){
        node.left = remove(node.left, key);
        // return node;
        retNode = node;
    }
    else if(key.compareTo(node.key) > 0 ){
        node.right = remove(node.right, key);
        // return node;
        retNode = node;
    }
    else{ // key.compareTo(node.key) == 0

        // 待删除节点左子树为空的情况
        if(node.left == null){
            Node rightNode = node.right;
            node.right = null;
            size--;
            // return rightNode;
            retNode = rightNode;
        }
    }
}

```

```

        // 待删除节点右子树为空的情况
        else if (node.right == null) {
            Node leftNode = node.left;
            node.left = null;
            size--;
            // return leftNode;
            retNode = leftNode;
        }

        // 待删除节点左右子树均不为空的情况
        else {
            // 找到比待删除节点大的最小节点，即待删除节点右子树的最小节点
            // 用这个节点顶替待删除节点的位置
            Node successor = minimum(node.right);
            // successor.right = removeMin(node.right);
            successor.right = remove(node.right, successor.key);
            successor.left = node.left;

            node.left = node.right = null;

            // return successor;
            retNode = successor;
        }
    }

    if (retNode == null)
        return null;

    // 更新height
    retNode.height = 1 + Math.max(getHeight(retNode.left),
        getHeight(retNode.right));

    // 计算平衡因子
    int balanceFactor = getBalanceFactor(retNode);

    // 平衡维护
    // LL
    if (balanceFactor > 1 && getBalanceFactor(retNode.left) >= 0)
        return rightRotate(retNode);

    // RR
    if (balanceFactor < -1 && getBalanceFactor(retNode.right) <= 0)
        return leftRotate(retNode);

    // LR
    if (balanceFactor > 1 && getBalanceFactor(retNode.left) < 0) {
        retNode.left = leftRotate(retNode.left);
        return rightRotate(retNode);
    }

    // RL
    if (balanceFactor < -1 && getBalanceFactor(retNode.right) > 0) {
        retNode.right = rightRotate(retNode.right);
        return leftRotate(retNode);
    }

    return retNode;
}

}

```

2>用avl树实现set集合和map映射

```
public class AVLSet<E extends Comparable<E>> implements Set<E> {

    private AVLTree<E, Object> avl;

    public AVLSet(){
        avl = new AVLTree<>();
    }

    @Override
    public int getSize(){
        return avl.getSize();
    }

    @Override
    public boolean isEmpty(){
        return avl.isEmpty();
    }

    @Override
    public void add(E e){
        avl.add(e, null);
    }

    @Override
    public boolean contains(E e){
        return avl.contains(e);
    }

    @Override
    public void remove(E e){
        avl.remove(e);
    }
}
```

```
public class AVLMap<K extends Comparable<K>, V> implements Map<K, V> {

    private AVLTree<K, V> avl;

    public AVLMap(){
        avl = new AVLTree<>();
    }

    @Override
    public int getSize(){
        return avl.getSize();
    }

    @Override
    public boolean isEmpty(){
        return avl.isEmpty();
    }

    @Override
    public void add(K key, V value){
        avl.add(key, value);
    }
}
```

```
@Override
public boolean contains(K key){
    return avl.contains(key);
}

@Override
public V get(K key){
    return avl.get(key);
}

@Override
public void set(K key, V newValue){
    avl.set(key, newValue);
}

@Override
public V remove(K key){
    return avl.remove(key);
}
}
```