Supplementary Material

A. Comparison with top-rank Algorithms

The proposed PSO-HV and DE-HL are compared with winners of CEC-2013 and CEC-2017 on 50 dimensions. From Tables IV and V, four winners (EBOwithCMAR, JSO, NBIPOP-ACMA-ES and LSHADESPACMA) show better performance than the proposed PSO-HV and DE-HL. The reason is that they integrate many evolutionary algorithms and strategies to adapt to more optimization problems than a single one. Although the proposed algorithms are inferior to the four winners in optimization results, they still have their advantages of a simple structure, ease of its use and competitive performance. Here we use EBOwithCMAR (the top-1 algorithm in CEC-2017) as an example to explain why we claim our proposed method's advantages. It has ten critical parameters that need to be fine-tuned and consists of six algorithms and strategies including Effective Butterfly Optimizer (EBO), Covariance Matrix Adapted Retreat (CMAR), sequential quadratic programming, linear population size reduction, data sharing strategy and multi-swarm strategy. Our proposed PSO-HV has only four parameters and uses only hierarchical learning, variable population and PSO. The proposed algorithms have significantly outperformed some top-rank algorithms.

B. DE-HL

In DE-HL, a linear population size reduction is used to adjust the population size instead of the proposed variable population mechanism. The reason for applying the proposed variable population to PSO to achieve improved performance is that it deals with the problem of PSO's premature convergence in the early search phase. At the beginning, a large population is initialized and the operation of eliminating redundant particles maintains the population diversity. In the later search phase, the population reduces to a minimum and keeps fixed to perform a fine exploitation. However, DE updates particles by crossover, mutation and selection operations, thus maintains a large population diversity and has few redundant particles. A DE based on the proposed adaptive variable population strategy can keep a large population size during the whole search phase and impedes exploitation. Therefore, it is not suitable to apply the proposed variable population strategy to DE. In DE-HL, the population is organized into a min-heap at the beginning. After each generation, the min-Heap is updated according to Algorithm 1. Then, all particles select their leaders by (6). A new $current-to-leader/1$ mutation strategy is proposed as follows:

$$\boldsymbol{v}_i(t+1) = \boldsymbol{x}_i(t) + F_i(\boldsymbol{x}_{Li}(t) - \boldsymbol{x}_i(t)) + F_i(\boldsymbol{x}_{r1}(t) - \boldsymbol{x}_{r2}(t)) \tag{1}$$

where $\boldsymbol{x}_{Li}$ is the leader of particle $\boldsymbol{x}_i$. $\boldsymbol{x}_{r1}$ is chosen randomly from the current population with an index $r1 \in [1, \cdots, N]$. $\boldsymbol{x}_{r2}$ is chosen randomly from the union of the current population and an external archive A that stores the inferior parents recently replaced by the offspring ones. Initially, A is filled with the initial population. At each generation, if the size of A exceeds the population size N, some individuals are eliminated to make space for the newly added individuals. A linear population size reduction is used as follows:

$$N_t = round\left(\hat{N} - (\overset{\vee}{N} - \hat{N}) \times \frac{t}{T}\right), \tag{2}$$

where $\hat{N}$ is the initial population size, $\overset{\vee}{N}$ is the minimum population size, and N_t is the population size at the current generation t. The crossover and selection operations are the same as LSHADE [34].

TABLE S-I
PARAMETERS IN PSO VARIANTS

Algorithm	**Parameter Setting**	**Reerence**	**Year**
TAPSO	$w = 0.7298$, $p_c = 0.5$, $p_m = 0.02$, $M = N/4$, $N = \{20, 40, 60\}$	[46]	2020
DEPSO	$w = 0.9 \rightarrow 0.4$, $k = 20 \rightarrow 10$, $c = 3.2$, $N = 50$	[47]	2019
AWPSO	$w = 0.9 \rightarrow 0.4$, $a = 0.000035$, $b = 0.5$, $c = 0$, $d = 1.5$, $N = 30$	[48]	2021
HCLPSO	$g_1 = 15$, $g_2 = 25$, $c = 3 \rightarrow 1.5$, $c_1 = 2.5 \rightarrow 0.5$, $c_2 = 0.5 \rightarrow 2.5$, $N = 40$	[49]	2015
CJADE	$\rho = 0.988$, $L = 50$, $N = 30$	[50]	2021
LOTFWA	$\mu = 5$, $c_a = 1.2$, $c_r = 0.9$, $\sigma = 0.2$, $\lambda = 300$, $\alpha = 0$	[51]	2018
MAPSO	$w = 0.9 \rightarrow 0.1$, $c_1 = c_2 = 2.05$, $f = 1$, $C_r = 0.5$, $P_m = 0.01$, $gSize = 3$, $gNumber = 25$	[41]	2020
APABC	$SN_{max} = 35$, $SN_{min} = 20$, $T = 20$	[42]	2017
HPSO	w_1=0.99$\rightarrow$0.2, w_2=0.99$\rightarrow$0.1, c_1=3$\rightarrow$1.5, c_2=2.5$\rightarrow$0.5, c_3=0.5$\rightarrow$2.5, N=100, P_{task}={0.01,0.03,0.05,0.2,0.4}	[52]	2020
HGFOA	$Ne = 0.33$, $a = 10$, $N = 200$	[53]	2021

TABLE S-II
RESULTS FOR MAIN EFFECT ON TUNING PARAMETERS.

Source	Sum of squares	Df	Mean square	F-Ratio	p-Value
Main effects					
$\check{N}$	0.006	3	0.002	1.050	0.484
$\hat{N}$	0.000	3	0.000	0.092	0.959
c	0.007	3	0.002	1.232	0.434
ε	0.050	3	0.017	9.389	0.049
Error	0.005	3	0.002		
Total(Corrected)	0.068	15			

TABLE S-III
MULTIPLE COMPARISON ON LEVELS OF ε.

ε	Subset (95%)	
0.1	0.271	-
1	0.296	0.296
10	0.337	0.337
100	-	0.418
Sig.	0.297	0.075

TABLE S-IV
THE FINE TUNE OF FOUR PARAMETERS.

$\check{N}$	47	48	49	50	51	52
ADF	0.1051	0.1084	0.1041	**0.1032**	0.1054	0.1053
$\hat{N}$	167	168	169	170	171	172
ADF	0.1078	0.1067	**0.1034**	0.1040	0.1044	0.1042
c	1.17	1.18	1.19	1.20	1.21	1.22
ADF	0.1041	0.1081	0.1041	**0.1028**	0.1051	0.1051
ε	0.07	0.08	0.09	0.10	0.11	0.12
ADF	0.1088	0.1057	0.1058	**0.1050**	0.1072	0.1057

TABLE S-V
ABLATION EXPERIMENT OF HIERARCHICAL LEARNING AND VARIABLE POPULATION MECHANISM ON 28 FUNCTIONS WITH 50-D

Func	PSO-HV		PSO-H		PSO-V	
	Mean error	Std.	Mean error (Sig.)	Std.	Mean error (Sig.)	Std.
f_1	6.33E-13	1.53E-13	0.00E+00 $(-)$	3.08E-13	2.29E-12 $(+)$	1.34E-11
f_2	4.83E+06	1.92E+06	8.02E+06 $(+)$	4.61E+06	3.38E+06 $(-)$	9.86E+05
f_3	1.19E+07	2.61E+07	4.95E+07 $(+)$	5.91E+07	9.58E+08 $(+)$	9.45E+08
f_4	4.02E+04	8.72E+03	1.01E+05 $(+)$	9.63E+03	1.05E+03 $(-)$	7.21E+02
f_5	3.21E-11	5.93E-11	1.38E-08 $(+)$	8.47E-08	5.31E-13 $(-)$	1.64E-12
f_6	4.72E+01	7.32E+00	5.15E+01 $(+)$	1.55E+01	5.75E+01 $(\approx)$	2.86E+01
f_7	9.50E-01	5.11E-01	3.77E+00 $(+)$	9.59E+00	1.14E+02 $(+)$	1.93E+01
f_8	2.11E+01	4.61E-02	2.11E+01 $(-)$	4.20E-02	2.11E+01 $(-)$	3.37E-02
f_9	1.40E+01	2.88E+00	1.99E+01 $(+)$	4.80E+00	4.50E+01 $(+)$	5.36E+00
f_{10}	2.75E-01	2.27E-01	3.72E-01 $(+)$	3.24E-01	1.17E-01 $(-)$	7.87E-02
f_{11}	7.67E+00	4.25E+00	1.97E+01 $(+)$	5.44E+00	1.32E+02 $(+)$	2.82E+01
f_{12}	1.70E+01	4.89E+00	3.19E+01 $(+)$	9.95E+00	2.24E+02 $(+)$	6.08E+01
f_{13}	4.09E+01	2.13E+01	9.30E+01 $(+)$	2.83E+01	3.72E+02 $(+)$	7.17E+01
f_{14}	3.14E+02	1.61E+02	4.11E+02 $(+)$	1.95E+02	3.61E+03 $(+)$	6.89E+02
f_{15}	2.19E+03	7.21E+02	3.36E+03 $(+)$	7.86E+02	7.56E+03 $(+)$	9.28E+02
f_{16}	4.52E-01	1.67E-01	6.76E-01 $(+)$	1.84E-01	2.82E+00 $(+)$	5.37E-01
f_{17}	5.88E+01	1.87E+00	7.00E+01 $(+)$	6.13E+00	2.07E+02 $(+)$	4.34E+01
f_{18}	8.97E+01	1.01E+01	1.05E+02 $(+)$	1.22E+01	2.59E+02 $(+)$	7.31E+01
f_{19}	5.15E+00	6.47E-01	5.50E+00 $(\approx)$	6.60E-01	1.22E+01 $(+)$	4.35E+00
f_{20}	2.28E+01	8.04E-01	2.44E+01 $(+)$	3.32E-01	2.13E+01 $(-)$	1.07E+00
f_{21}	8.72E+02	3.02E+02	8.08E+02 $(-)$	3.68E+02	8.63E+02 $(\approx)$	3.53E+02
f_{22}	2.11E+02	8.99E+01	5.85E+02 $(+)$	2.74E+02	4.12E+03 $(+)$	9.61E+02
f_{23}	2.18E+03	7.06E+02	3.37E+03 $(+)$	9.28E+02	9.16E+03 $(+)$	1.53E+03
f_{24}	2.06E+02	9.91E+00	2.22E+02 $(+)$	1.80E+01	3.24E+02 $(+)$	1.32E+01
f_{25}	2.96E+02	9.03E+00	3.09E+02 $(+)$	1.32E+01	3.56E+02 $(+)$	1.47E+01
f_{26}	2.78E+02	4.82E+01	3.20E+02 $(+)$	5.04E+01	4.02E+02 $(+)$	5.25E+01
f_{27}	4.61E+02	1.83E+02	7.36E+02 $(+)$	1.67E+02	1.48E+03 $(+)$	1.25E+02
f_{28}	4.00E+02	3.86E-09	5.75E+02 $(+)$	7.05E+02	1.05E+03 $(+)$	1.32E+03
+/≈/-	PSO-HV VS		24/1/3		20/2/6	
+/≈/-			PSO-H VS		21/2/5	

TABLE S-VI
WILCOXON TEST RESULTS BETWEEN PSO-HV AND RELATED ALGORITHMS ON CEC-2017 FUNCTIONS

	D=100				D=50				D=30			
Func	MAPSO [45]	APABC [46]	HGFOA [57]	HPSO [56]	MAPSO [45]	APABC [46]	HGFOA [57]	HPSO [56]	MAPSO [45]	APABC [46]	HGFOA [57]	HPSO [56]
g_1	≈	+	+	+	≈	+	+	+	≈	−	≈	≈
g_3	−	+	−	−	−	+	−	−	−	+	−	−
g_4	≈	−	≈	+	−	−	≈	+	−	−	≈	−
g_5	+	+	+	+	+	+	+	+	+	+	+	+
g_6	+	−	+	+	+	−	+	+	+	−	+	+
g_7	+	+	+	+	+	+	+	+	+	+	+	+
g_8	+	+	+	+	+	+	+	+	+	+	+	+
g_9	+	+	+	+	+	+	+	+	+	+	+	+
g_{10}	+	+	+	+	+	+	+	+	+	+	+	+
g_{11}	−	+	−	−	−	+	≈	≈	+	+	≈	≈
g_{12}	−	+	+	+	−	+	+	≈	−	+	≈	≈
g_{13}	≈	+	+	≈	+	+	+	+	+	+	−	−
g_{14}	−	+	−	+	−	+	−	≈	−	+	−	−
g_{15}	≈	+	+	≈	≈	+	−	≈	+	+	≈	≈
g_{16}	+	+	+	+	+	+	+	+	+	+	+	≈
g_{17}	+	+	+	+	+	+	+	+	+	+	+	+
g_{18}	−	+	−	≈	−	+	−	−	−	+	−	≈
g_{19}	≈	+	+	≈	≈	≈	−	−	≈	+	−	−
g_{20}	+	+	+	+	+	+	+	+	≈	+	+	−
g_{21}	+	+	+	+	+	+	+	+	+	+	+	+
g_{22}	+	+	+	+	+	+	+	+	+	+	+	+
g_{23}	+	+	+	+	+	+	+	+	+	+	+	+
g_{24}	+	+	+	+	+	+	+	+	+	+	+	+
g_{25}	−	−	≈	+	−	−	≈	+	≈	−	≈	+
g_{26}	+	+	+	+	+	+	≈	+	+	+	≈	≈
g_{27}	−	≈	≈	+	≈	+	−	+	−	≈	−	≈
g_{28}	−	−	−	+	≈	≈	+	+	−	−	≈	+
g_{29}	+	+	+	+	+	+	+	+	+	+	+	+
g_{30}	≈	+	+	+	−	−	+	+	≈	+	+	+
+/≈/−	15/6/8	24/1/4	21/3/5	23/4/2	16/5/8	23/2/4	19/4/6	22/4/3	17/5/7	23/1/5	15/8/6	15/8/6

TABLE S-VII
WILCOXON TEST RESULTS BETWEEN PSO-HV AND RELATED ALGORITHMS ON CEC-2013 FUNCTIONS

	D=100				D=50				D=30			
Func	MAPSO [45]	APABC [46]	HGFOA [57]	HPSO [56]	MAPSO [45]	APABC [46]	HGFOA [57]	HPSO [56]	MAPSO [45]	APABC [46]	HGFOA [57]	HPSO [56]
f_1	−	−	+	+	−	−	+	+	−	−	+	≈
f_2	≈	+	−	+	−	+	−	+	−	+	−	+
f_3	≈	+	+	+	+	+	+	+	+	+	+	+
f_4	−	+	−	−	−	+	−	−	−	+	−	−
f_5	−	−	+	+	+	−	+	≈	+	−	+	+
f_6	≈	−	≈	+	−	−	≈	≈	−	−	≈	≈
f_7	+	+	+	+	+	+	+	+	+	+	+	+
f_8	≈	≈	−	≈	+	+	≈	≈	≈	≈	−	−
f_9	+	+	+	+	+	+	+	+	+	+	+	+
f_{10}	≈	+	+	+	−	+	+	+	−	+	+	+
f_{11}	+	−	+	≈	+	−	−	−	+	−	+	−
f_{12}	+	+	+	+	+	+	+	+	+	+	+	+
f_{13}	+	+	+	+	+	+	+	+	+	+	+	+
f_{14}	−	−	+	−	≈	−	+	−	≈	−	+	−
f_{15}	+	+	+	+	+	+	+	+	+	+	+	+
f_{16}	+	+	+	+	+	+	+	+	+	+	+	+
f_{17}	+	−	+	+	+	−	+	≈	+	−	+	−
f_{18}	+	+	+	+	+	+	+	+	+	+	+	+
f_{19}	+	−	+	+	≈	−	+	≈	≈	−	+	≈
f_{20}	≈	≈	≈	≈	−	+	+	−	−	−	−	−
f_{21}	≈	−	+	≈	+	−	≈	−	≈	−	≈	≈
f_{22}	−	−	+	≈	+	−	+	+	+	−	+	≈
f_{23}	+	+	+	+	+	+	+	+	+	+	+	+
f_{24}	+	+	+	+	+	+	+	+	+	+	+	+
f_{25}	+	+	+	+	+	+	+	+	+	+	+	+
f_{26}	+	−	+	≈	+	−	≈	−	−	−	−	−
f_{27}	+	+	+	+	+	+	+	+	+	+	+	+
f_{28}	≈	≈	+	≈	−	≈	+	+	−	−	+	+
+/≈/−	15/8/5	15/3/10	23/2/3	19/2/7	19/2/7	17/1/10	21/4/3	17/5/6	16/4/8	15/1/12	21/2/5	16/5/7

TABLE S-VIII
WILCOXON TEST RESULTS BETWEEN PSO-HV AND OTHER COMPARISON ALGORITHMS ON CEC-2017 FUNCTIONS

	D=100						D=50						D=30					
Func	TAPSO [50]	DEPSO [51]	AWPSO [52]	HCLPSO [53]	CJADE [54]	LOTFWA [55]	TAPSO [50]	DEPSO [51]	AWPSO [52]	HCLPSO [53]	CJADE [54]	LOTFWA [55]	TAPSO [50]	DEPSO [51]	AWPSO [52]	HCLPSO [53]	CJADE [54]	LOTFWA [55]
g_1	≈	≈	+	≈	−	−	≈	≈	+	−	−	−	≈	≈	+	−	−	−
g_3	−	+	+	−	−	−	−	+	+	−	−	−	−	−	+	−	−	−
g_4	−	≈	+	≈	−	−	−	≈	+	−	−	−	−	+	+	−	−	−
g_5	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
g_6	+	+	+	≈	−	+	+	≈	+	−	+	+	+	−	+	−	−	+
g_7	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
g_8	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
g_9	+	≈	+	+	+	+	+	−	+	+	+	+	+	+	+	+	+	≈
g_{10}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
g_{11}	−	−	+	−	−	−	−	≈	+	−	−	−	+	+	+	+	+	+
g_{12}	−	≈	+	−	−	+	−	−	+	−	−	+	−	−	+	−	−	+
g_{13}	≈	≈	+	≈	≈	+	+	≈	+	≈	≈	+	≈	≈	+	−	−	+
g_{14}	−	≈	+	≈	−	−	−	−	+	−	−	−	−	≈	+	−	−	−
g_{15}	≈	≈	+	≈	≈	+	+	≈	+	−	−	+	+	≈	+	−	−	+
g_{16}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
g_{17}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
g_{18}	−	−	+	−	−	−	−	≈	+	−	−	−	−	−	+	−	−	−
g_{19}	≈	≈	+	≈	≈	+	≈	≈	+	−	−	+	+	+	+	−	−	+
g_{20}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	≈	≈	+
g_{21}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
g_{22}	+	≈	+	+	+	+	+	+	+	+	+	+	≈	+	+	+	+	+
g_{23}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
g_{24}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
g_{25}	≈	≈	+	≈	≈	−	≈	+	+	−	−	−	≈	+	+	−	−	−
g_{26}	+	≈	+	+	+	+	+	−	+	+	+	+	+	−	+	−	+	≈
g_{27}	+	≈	+	+	+	+	+	+	+	+	+	+	+	+	+	≈	−	+
g_{28}	−	−	+	−	−	−	≈	+	+	≈	+	−	−	−	+	−	−	−
g_{29}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
g_{30}	≈	−	+	−	−	+	−	≈	+	−	−	+	≈	+	+	≈	−	+
+/≈/−	16/6/7	13/12/4	29/0/0	15/8/6	15/4/10	21/0/8	18/4/7	16/9/4	29/0/0	15/2/12	17/1/11	21/0/8	18/5/6	19/4/6	29/0/0	13/3/13	14/1/14	20/2/7

TABLE S-IX
WILCOXON TEST RESULTS BETWEEN PSO-HV AND OTHER COMPARISON ALGORITHMS ON CEC-2013 FUNCTIONS

	D=100						D=50						D=30					
Func	TAPSO [50]	DEPSO [51]	AWPSO [52]	HCLPSO [53]	CJADE [54]	LOTFWA [55]	TAPSO [50]	DEPSO [51]	AWPSO [52]	HCLPSO [53]	CJADE [54]	LOTFWA [55]	TAPSO [50]	DEPSO [51]	AWPSO [52]	HCLPSO [53]	CJADE [54]	LOTFWA [55]
f_1	≈	−	+	≈	−	−	−	−	+	≈	−	−	−	−	+	≈	−	−
f_2	−	−	+	≈	−	−	−	−	+	−	−	−	−	−	+	−	−	−
f_3	+	≈	+	+	+	+	+	+	+	+	+	+	+	+	+	+	−	+
f_4	−	+	+	−	−	−	−	+	+	−	−	−	−	+	+	−	−	−
f_5	−	−	+	−	−	+	−	−	+	−	−	+	−	−	+	−	−	+
f_6	−	−	+	≈	−	−	−	−	+	−	−	−	−	≈	+	−	−	−
f_7	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
f_8	≈	−	−	+	≈	≈	≈	≈	−	+	≈	−	−	+	−	+	+	−
f_9	+	+	−	+	+	+	+	+	−	+	+	+	+	+	−	+	+	+
f_{10}	−	−	+	≈	−	−	−	−	+	≈	−	−	−	≈	+	−	−	−
f_{11}	−	+	+	−	−	+	−	+	+	−	−	+	−	+	+	−	−	+
f_{12}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
f_{13}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
f_{14}	−	+	+	−	−	+	−	+	+	−	−	+	−	+	+	−	−	+
f_{15}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
f_{16}	+	≈	+	+	+	−	+	−	+	+	+	−	+	−	+	+	+	−
f_{17}	−	+	+	−	−	+	−	+	+	−	−	+	−	+	+	−	−	+
f_{18}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
f_{19}	−	≈	+	≈	+	+	−	+	+	−	−	+	−	+	+	−	−	≈
f_{20}	≈	≈	+	≈	≈	≈	−	−	+	−	−	+	−	+	+	−	−	≈
f_{21}	−	≈	+	≈	−	−	≈	≈	+	−	≈	−	≈	+	+	−	≈	−
f_{22}	−	+	+	−	−	+	−	+	+	−	−	+	−	+	+	−	−	+
f_{23}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
f_{24}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
f_{25}	+	+	+	+	+	+	+	+	+	+	+	+	+	≈	+	+	+	+
f_{26}	+	≈	+	≈	+	+	+	+	+	−	+	−	+	+	+	−	≈	−
f_{27}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
f_{28}	≈	−	+	≈	≈	≈	+	−	+	−	+	−	+	−	+	≈	−	≈
+/≈/−	13/4/11	15/6/7	26/0/2	13/9/6	14/3/11	18/3/7	14/2/12	18/2/8	26/0/2	13/2/13	14/2/12	18/0/10	14/1/13	20/3/5	26/0/2	13/2/13	12/2/14	16/3/9

TABLE S-X
COMPARISON RESULTS FOR PSO-HV, DE-HL AND TOP-RANK ALGORITHMS ON CEC 2017 ON 50-D.

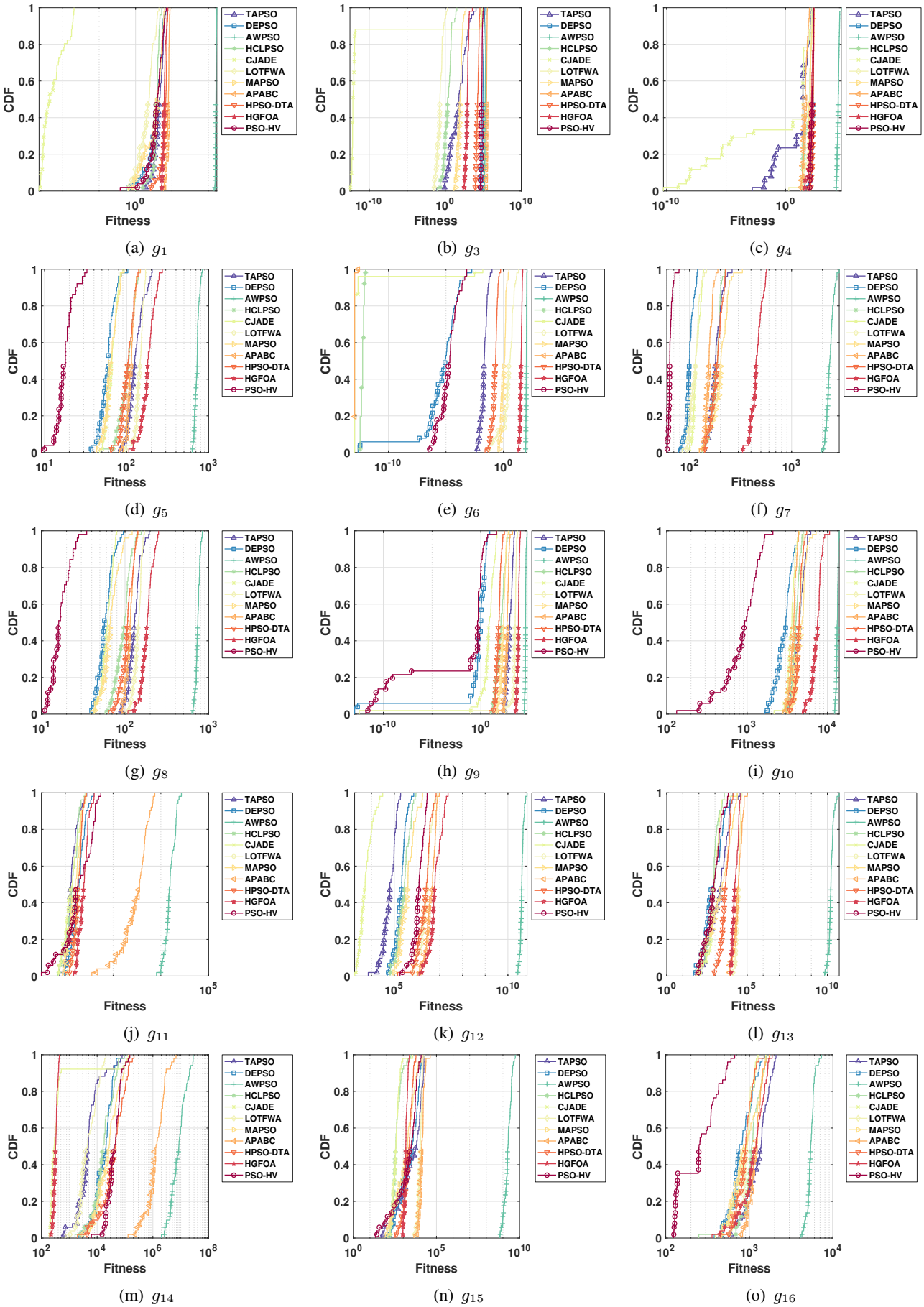
Func		EBOwithCMAR	JSO	RB-IPOP-CMA-ES	LSHADE	LSHADESPACMA	EBLSHADE	PPSO	TLBO-FL	DYYPO	MOS-SOCO	PSO-HV	LSHADE-H
g_1	Mean Err.	0.00E+00 (≈)	0.00E+00 (≈)	1.13E-07 (+)	0.00E+00 (≈)	0.00E+00 (≈)	2.23E-14 (≈)	3.89E+02 (+)	6.06E+05 (+)	6.57E+03 (+)	1.49E+04 (+)	2.67E+03 (+)	0.00E+00
	Std.	0.00E+00	0.00E+00	4.26E-08	0.00E+00	0.00E+00	7.11E-15	2.95E+02	2.21E+06	7.09E+03	1.23E+04	3.66E+03	0.00E+00
g_3	Mean Err.	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	2.03E-13 (≈)	8.65E+02 (+)	2.57E+04 (+)	4.67E+01 (+)	3.36E+04 (+)	6.24E+04 (+)	0.00E+00
	Std.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.15E-14	1.86E+02	4.88E+03	2.30E+02	1.71E+04	1.36E+04	0.00E+00
g_4	Mean Err.	4.29E+01 (-)	5.62E+01 (≈)	2.96E+01 (-)	8.18E+01 (+)	2.94E+01 (-)	7.91E+01 (+)	9.13E+01 (+)	1.90E+02 (+)	1.37E+02 (+)	2.81E+01 (-)	1.72E+02 (+)	5.58E+01
	Std.	3.32E+01	4.88E+01	4.07E+01	4.84E+01	2.98E+01	4.43E+01	3.61E+01	4.58E+01	4.97E+01	2.63E+01	4.11E+01	4.96E+01
g_5	Mean Err.	7.58E+00 (-)	1.64E+01 (-)	2.79E+00 (-)	1.22E+01 (-)	5.99E+00 (-)	1.27E+01 (-)	2.01E+02 (+)	9.67E+01 (+)	1.95E+02 (+)	1.34E+02 (+)	1.83E+01 (+)	2.44E+01
	Std.	2.42E+00	3.46E+00	1.44E+00	2.05E+00	2.02E+00	2.15E+00	1.37E+01	1.73E+01	3.95E+01	2.07E+01	4.78E+00	3.35E+00
g_6	Mean Err.	8.54E-08 (-)	1.09E-06 (-)	1.63E-07 (-)	5.69E-05 (≈)	0.00E+00 (-)	2.11E-05 (-)	3.18E+01 (+)	4.51E+00 (+)	3.84E+00 (+)	0.00E+00 (-)	7.17E-05 (+)	2.67E-05
	Std.	1.14E-07	2.63E-06	1.38E-07	3.71E-04	0.00E+00	1.48E-04	3.91E+00	1.70E+00	2.03E+00	0.00E+00	1.41E-04	2.44E-05
g_7	Mean Err.	5.79E+01 (-)	6.65E+01 (-)	5.66E+01 (-)	6.32E+01 (-)	5.70E+01 (-)	6.28E+01 (-)	2.78E+02 (+)	1.74E+02 (+)	2.61E+02 (+)	1.81E+02 (+)	6.48E+01 (≈)	7.47E+01
	Std.	1.53E+00	3.47E+00	1.39E+00	1.71E+00	1.09E+00	1.67E+00	3.42E+01	4.38E+01	4.27E+01	2.26E+01	3.31E+00	3.72E+00
g_8	Mean Err.	7.91E+00 (-)	1.70E+01 (-)	2.58E+00 (-)	1.20E+01 (-)	5.81E+00 (-)	1.24E+01 (-)	1.99E+02 (+)	9.33E+01 (+)	1.90E+02 (+)	1.38E+02 (+)	1.77E+01 (-)	3.04E+01
	Std.	2.47E+00	3.14E+00	1.79E+00	2.28E+00	1.88E+00	2.22E+00	1.52E+01	1.58E+01	4.70E+01	2.46E+01	4.97E+00	6.55E+00
g_9	Mean Err.	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	1.05E-13 (≈)	6.06E+03 (+)	1.30E+03 (+)	3.51E+03 (+)	1.21E+03 (+)	1.50E+00 (+)	0.00E+00
	Std.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.09E-14	7.28E+02	1.05E+03	1.86E+03	5.08E+02	5.78E+00	0.00E+00
g_{10}	Mean Err.	3.11E+03 (-)	3.14E+03 (-)	1.73E+03 (-)	3.18E+03 (-)	3.49E+03 (≈)	3.03E+03 (+)	5.20E+03 (+)	1.27E+04 (+)	4.80E+03 (+)	4.73E+03 (+)	9.68E+02 (-)	3.37E+03
	Std.	4.01E+02	3.67E+02	9.53E+02	2.55E+02	6.64E+02	3.13E+02	5.51E+02	3.97E+02	6.41E+02	7.01E+02	4.26E+02	3.94E+02
g_{11}	Mean Err.	2.64E+01 (≈)	2.79E+01 (+)	1.83E+02 (+)	4.86E+01 (+)	3.22E+01 (+)	4.40E+01 (+)	1.27E+02 (+)	1.69E+02 (+)	1.90E+02 (+)	1.95E+02 (+)	2.36E+02 (+)	2.24E+01
	Std.	3.36E+00	3.33E+00	5.20E+01	7.92E+00	4.75E+00	8.67E+00	1.45E+01	4.79E+01	5.19E+01	6.05E+01	1.39E+02	1.58E+00
g_{12}	Mean Err.	1.94E+03 (+)	1.68E+03 (+)	2.44E+06 (+)	2.17E+03 (+)	1.56E+03 (+)	2.15E+03 (+)	5.52E+05 (+)	9.16E+05 (+)	7.75E+06 (+)	1.45E+05 (+)	1.38E+06 (+)	6.66E+02
	Std.	8.34E+02	5.23E+02	1.74E+07	4.52E+02	3.99E+02	4.32E+02	2.03E+05	8.94E+05	5.08E+06	8.95E+04	6.97E+05	2.12E+02
g_{13}	Mean Err.	4.14E+01 (≈)	3.06E+01 (≈)	1.65E+03 (+)	6.27E+01 (+)	3.71E+01 (≈)	4.60E+01 (+)	8.47E+02 (+)	8.01E+03 (+)	7.55E+03 (+)	1.09E+04 (+)	1.43E+03 (+)	2.94E+01
	Std.	2.48E+01	2.12E+01	1.15E+03	2.83E+01	2.32E+01	2.24E+01	5.63E+02	5.14E+03	7.38E+03	1.26E+04	1.61E+03	2.42E+01
g_{14}	Mean Err.	3.12E+01 (≈)	2.50E+01 (≈)	2.42E+02 (+)	2.91E+01 (≈)	2.94E+01 (+)	2.69E+01 (+)	1.95E+04 (+)	8.63E+04 (+)	2.89E+04 (+)	3.78E+04 (+)	4.75E+04 (+)	2.57E+01
	Std.	3.52E+00	1.87E+00	7.07E+01	2.92E+00	3.39E+00	2.78E+00	1.00E+04	4.96E+04	2.75E+04	3.23E+04	2.81E+04	9.04E-01
g_{15}	Mean Err.	2.94E+01 (+)	2.39E+01 (≈)	5.29E+02 (+)	4.08E+01 (+)	3.04E+01 (+)	3.44E+01 (+)	1.19E+03 (+)	6.88E+03 (+)	8.24E+03 (+)	1.22E+04 (+)	3.45E+03 (+)	1.99E+01
	Std.	5.20E+00	2.49E+00	1.15E+02	9.92E+00	5.63E+00	7.19E+00	7.80E+02	5.91E+03	7.00E+03	7.87E+03	3.00E+03	1.09E+00
g_{16}	Mean Err.	3.46E+02 (-)	4.51E+02 (+)	8.90E+02 (+)	3.77E+02 (-)	3.35E+02 (-)	3.34E+02 (-)	1.24E+03 (+)	8.47E+02 (+)	1.31E+03 (+)	1.40E+03 (+)	2.87E+02 (-)	4.31E+02
	Std.	1.46E+02	1.38E+02	3.66E+02	1.18E+02	2.01E+02	1.41E+02	2.28E+02	3.05E+02	4.08E+02	2.55E+02	1.51E+02	8.76E+01
g_{17}	Mean Err.	2.75E+02 (-)	2.83E+02 (-)	3.98E+02 (+)	2.55E+02 (-)	2.77E+02 (-)	2.35E+02 (-)	1.03E+03 (+)	8.53E+02 (+)	8.88E+02 (+)	9.49E+02 (+)	2.23E+02 (+)	3.22E+02
	Std.	8.63E+01	8.61E+01	1.58E+02	7.46E+01	9.49E+01	6.83E+01	1.54E+02	4.12E+02	2.65E+02	2.18E+02	1.36E+02	1.02E+02
g_{18}	Mean Err.	3.20E+01 (+)	2.43E+01 (≈)	3.57E+02 (+)	3.93E+01 (+)	3.24E+01 (+)	3.28E+01 (+)	2.09E+05 (+)	1.15E+06 (+)	1.81E+05 (+)	1.88E+05 (+)	7.71E+05 (+)	2.24E+01
	Std.	5.99E+00	2.02E+00	1.56E+02	1.11E+01	6.07E+00	6.05E+00	8.67E+04	5.13E+05	8.14E+04	9.98E+04	4.15E+05	8.44E-01
g_{19}	Mean Err.	2.45E+01 (+)	1.41E+01 (≈)	1.39E+02 (+)	2.46E+01 (+)	2.17E+01 (+)	1.88E+01 (+)	8.67E+03 (+)	1.45E+04 (+)	9.64E+03 (+)	2.42E+04 (+)	1.41E+04 (+)	1.38E+01
	Std.	3.94E+00	2.26E+00	4.77E+01	8.82E+00	3.21E+00	3.29E+00	3.91E+03	9.40E+03	8.85E+03	1.32E+04	4.68E+03	1.70E+00
g_{20}	Mean Err.	1.47E+02 (-)	1.40E+02 (-)	5.47E+02 (+)	1.74E+02 (+)	1.68E+02 (+)	1.50E+02 (+)	7.70E+02 (+)	1.04E+03 (+)	6.54E+02 (+)	7.98E+02 (+)	4.60E+01 (-)	1.58E+02
	Std.	7.44E+01	7.74E+01	2.33E+02	7.93E+01	1.03E+02	5.26E+01	1.92E+02	3.87E+02	2.79E+02	2.31E+02	5.82E+01	5.20E+01
g_{21}	Mean Err.	2.11E+02 (-)	2.19E+02 (≈)	2.06E+02 (-)	2.13E+02 (≈)	2.15E+02 (-)	2.13E+02 (-)	4.33E+02 (+)	2.81E+02 (+)	4.02E+02 (+)	3.46E+02 (+)	2.20E+02 (≈)	2.28E+02
	Std.	4.06E+00	3.77E+00	3.23E+00	1.95E+00	9.31E+00	2.67E+00	2.14E+01	1.52E+01	4.05E+01	2.05E+01	4.63E+00	7.31E+00
g_{22}	Mean Err.	3.65E+02 (-)	1.49E+03 (+)	2.05E+03 (+)	2.50E+03 (+)	1.38E+03 (+)	2.36E+03 (+)	5.97E+03 (+)	6.56E+03 (+)	4.78E+03 (+)	5.07E+03 (+)	1.00E+02 (-)	4.72E+02
	Std.	9.24E+02	1.75E+03	1.76E+03	1.61E+03	1.94E+03	1.71E+03	9.89E+02	6.40E+03	2.04E+03	9.46E+02	6.87E-09	1.16E+03
g_{23}	Mean Err.	4.34E+02 (-)	4.30E+02 (≈)	4.23E+02 (-)	4.30E+02 (-)	4.41E+02 (≈)	4.27E+02 (-)	1.06E+03 (+)	5.66E+02 (+)	6.46E+02 (+)	5.99E+02 (+)	4.28E+02 (-)	4.48E+02
	Std.	8.16E+00	6.24E+00	1.39E+01	5.08E+00	6.45E+00	4.58E+00	7.09E+01	3.40E+01	5.66E+01	2.73E+01	1.25E+01	7.45E+00
g_{24}	Mean Err.	5.06E+02 (≈)	5.07E+02 (≈)	4.91E+02 (-)	5.06E+02 (≈)	5.13E+02 (-)	5.05E+02 (-)	1.08E+03 (+)	6.75E+02 (+)	7.31E+02 (+)	8.22E+02 (+)	4.99E+02 (-)	5.15E+02
	Std.	3.85E+00	4.13E+00	5.73E+00	2.33E+00	6.87E+00	3.46E+00	7.07E+01	4.75E+01	7.36E+01	5.02E+01	8.95E+00	5.95E+00
g_{25}	Mean Err.	4.89E+02 (≈)	4.81E+02 (≈)	4.81E+02 (-)	4.85E+02 (≈)	4.81E+02 (≈)	4.89E+02 (+)	5.41E+02 (+)	6.16E+02 (+)	5.21E+02 (+)	5.12E+02 (+)	5.48E+02 (+)	4.80E+02
	Std.	2.47E+01	2.80E+00	5.18E+00	1.63E+01	2.32E+00	2.04E+01	2.77E+01	2.68E+01	3.03E+01	2.48E+01	2.35E+01	2.49E-02
g_{26}	Mean Err.	7.06E+02 (-)	1.13E+03 (-)	6.55E+02 (-)	1.14E+03 (-)	1.14E+03 (-)	1.12E+03 (-)	5.45E+03 (+)	2.93E+03 (+)	3.42E+03 (+)	2.81E+03 (+)	9.99E+02 (-)	1.33E+03
	Std.	4.06E+02	5.62E+01	3.01E+02	4.50E+01	4.77E+01	5.20E+01	2.59E+03	6.02E+02	7.82E+02	1.88E+02	2.05E+02	9.39E+01
g_{27}	Mean Err.	5.22E+02 (≈)	5.11E+02 (-)	6.08E+02 (+)	5.34E+02 (+)	5.32E+02 (+)	5.24E+02 (+)	1.46E+03 (+)	8.68E+02 (+)	6.72E+02 (+)	6.76E+02 (+)	5.69E+02 (+)	5.21E+02
	Std.	7.75E+00	1.11E+01	5.86E+01	1.92E+01	1.48E+01	1.07E+01	1.70E+02	1.76E+02	7.26E+01	3.85E+01	3.23E+01	7.83E+00
g_{28}	Mean Err.	4.67E+02 (≈)	4.60E+02 (≈)	4.70E+02 (+)	4.73E+02 (+)	4.60E+02 (+)	4.73E+02 (+)	4.89E+02 (+)	6.11E+02 (+)	4.80E+02 (+)	4.86E+02 (+)	4.83E+02 (+)	4.57E+02
	Std.	1.79E+01	6.84E+00	1.94E+01	2.25E+01	6.84E+00	2.25E+01	1.79E+01	4.22E+01	2.49E+01	1.90E+01	2.63E+01	3.20E-01
g_{29}	Mean Err.	3.47E+02 (-)	3.63E+02 (-)	6.69E+02 (+)	3.51E+02 (-)	3.92E+02 (+)	3.53E+02 (+)	1.52E+03 (+)	1.02E+03 (+)	9.82E+02 (+)	9.65E+02 (+)	3.50E+02 (-)	3.77E+02
	Std.	1.97E+01	1.32E+01	1.99E+02	1.04E+01	7.15E+01	1.08E+01	2.09E+02	2.48E+02	3.06E+02	2.05E+02	3.25E+01	9.83E+00
g_{30}	Mean Err.	6.18E+05 (-)	6.01E+05 (-)	6.46E+06 (+)	6.54E+05 (+)	6.68E+05 (+)	6.57E+05 (+)	7.81E+05 (+)	1.16E+06 (+)	1.54E+06 (+)	8.21E+05 (+)	1.02E+06 (+)	6.26E+05
	Std.	3.62E+04	2.99E+04	5.07E+06	7.33E+04	6.85E+04	8.17E+04	4.82E+04	3.15E+05	3.23E+05	1.57E+05	1.07E+05	4.27E+04
	+/≈/−	4/10/15	4/14/11	16/2/11	12/8/9	12/7/10	13/3/13	29/0/0	29/0/0	29/0/0	27/2/0	18/2/9	

TABLE S-XI
COMPARISON RESULTS FOR PSO-HV, DE-HL AND TOP-RANK ALGORITHMS ON CEC 2013 ON 50-D.

Func		NBIPOP-ACMA-ES	DRMA-LSCh-CMA	SHADE	EAGDE	MVMO-SH	TLBSaDE	SPSOABC	TPC-GA	PSO-HV	LSHADE-H
f_1	Mean Err.	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	6.33E-13 (≈)	0.00E+00
	Std.	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.53E-13	0.00E+00
f_2	Mean Err.	0.00E+00 (-)	0.00E+00 (-)	2.66E+04 (+)	3.10E+05 (+)	9.13E-04 (-)	1.68E+05 (+)	4.95E+05 (+)	4.76E+05 (+)	4.83E+06 (+)	8.81E-03
	Std.	0.00E+00	0.00E+00	1.13E+04	0.00E+00	6.81E-04	2.85E+04	1.48E+05	2.14E+05	1.92E+06	1.38E-02
f_3	Mean Err.	1.82E+01 (-)	9.95E+03 (≈)	8.80E+05 (+)	5.36E+05 (+)	3.64E-04 (-)	7.08E+05 (+)	1.21E+08 (+)	1.06E+08 (+)	1.19E+07 (+)	8.16E+01
	Std.	1.21E+02	3.97E+04	1.96E+06	3.97E+04	1.62E-03	3.59E+05	1.18E+08	1.49E+08	2.61E+07	1.28E+02
f_4	Mean Err.	0.00E+00 (-)	1.21E+02 (≈)	1.61E-03 (+)	1.70E+01 (+)	2.14E-06 (≈)	6.58E+02 (+)	4.88E+03 (+)	3.33E+00 (+)	4.02E+04 (+)	9.77E-07
	Std.	0.00E+00	5.34E+02	1.41E-03	5.34E+02	1.17E-06	1.76E+02	1.20E+03	4.88E+00	8.72E+03	9.83E-07
f_5	Mean Err.	0.00E+00 (≈)	4.95E-04 (+)	0.00E+00 (≈)	0.00E+00 (≈)	1.98E-08 (+)	0.00E+00 (≈)	0.00E+00 (≈)	0.00E+00 (≈)	3.21E-11 (≈)	0.00E+00
	Std.	0.00E+00	2.31E-04	0.00E+00	2.31E-04	1.09E-08	0.00E+00	0.00E+00	0.00E+00	5.93E-11	0.00E+00
f_6	Mean Err.	0.00E+00 (-)	4.34E+01 (+)	4.28E+01 (-)	4.34E+01 (+)	3.76E+01 (≈)	4.07E+01 (-)	4.05E+01 (-)	4.72E+01 (≈)	4.72E+01 (+)	4.34E+01
	Std.	0.00E+00	3.51E-09	5.52E+00	3.51E-09	1.49E+01	6.96E+00	2.28E+01	1.40E+01	7.32E+00	1.96E-02
f_7	Mean Err.	4.97E+00 (+)	1.54E+01 (+)	2.33E+01 (+)	8.51E+00 (+)	4.32E+01 (+)	4.96E+01 (+)	7.34E+01 (+)	4.17E+01 (+)	9.50E-01 (+)	2.18E-01
	Std.	5.72E+00	1.37E+01	9.32E+00	1.37E+01	8.24E+00	5.31E+00	1.17E+01	1.83E+01	5.11E-01	1.17E-01
f_8	Mean Err.	2.11E+01 (≈)	2.11E+01 (≈)	2.09E+01 (-)	2.11E+01 (≈)	2.10E+01 (≈)	2.11E+01 (-)	2.11E+01 (+)	2.12E+01 (≈)	2.11E+01 (≈)	2.11E+01
	Std.	4.52E-02	3.45E-02	1.68E-01	3.45E-02	2.64E-01	2.71E-02	3.42E-02	3.84E-02	4.61E-02	7.57E-02
f_9	Mean Err.	7.22E+00 (-)	1.76E+01 (-)	5.54E+01 (+)	5.35E+01 (+)	3.28E+01 (-)	6.09E+01 (+)	5.84E+01 (+)	7.30E+01 (+)	1.40E+01 (-)	5.02E+01
	Std.	2.29E+00	2.81E+00	1.98E+00	2.81E+00	4.57E+00	1.66E+00	3.92E+00	3.82E+00	2.88E+00	2.28E+00
f_{10}	Mean Err.	0.00E+00 (≈)	1.89E-03 (≈)	7.37E-02 (+)	3.97E-02 (+)	2.90E-04 (≈)	1.76E-02 (+)	1.55E-01 (+)	1.05E-01 (+)	2.75E-01 (+)	0.00E+00
	Std.	0.00E+00	3.26E-03	3.67E-02	3.26E-03	1.45E-03	6.85E-03	8.10E-02	7.09E-02	2.27E-01	0.00E+00
f_{11}	Mean Err.	5.52E+00 (+)	6.13E+00 (+)	0.00E+00 (-)	0.00E+00 (-)	3.61E+01 (+)	0.00E+00 (-)	6.65E-02 (+)	5.57E+01 (+)	7.67E+00 (+)	5.16E-07
	Std.	2.97E+00	2.29E+00	0.00E+00	2.29E+00	1.00E+01	0.00E+00	2.39E-01	2.23E+01	4.25E+00	7.56E-07
f_{12}	Mean Err.	5.37E+00 (-)	3.33E+01 (≈)	5.86E+01 (+)	2.22E+02 (+)	8.96E+01 (+)	1.20E+02 (+)	1.73E+02 (+)	9.62E+01 (+)	1.70E+01 (-)	2.63E+01
	Std.	2.54E+00	7.06E+00	1.11E+01	7.06E+00	2.17E+01	8.18E+00	3.23E+01	2.15E+01	4.89E+00	2.41E+00
f_{13}	Mean Err.	7.60E+00 (-)	8.29E+01 (+)	1.45E+02 (+)	2.59E+02 (+)	1.81E+02 (+)	2.19E+02 (+)	2.87E+02 (≈)	1.92E+02 (+)	4.09E+01 (≈)	4.91E+01
	Std.	5.47E+00	1.91E+01	1.95E+01	1.91E+01	3.36E+01	1.74E+01	3.94E+01	5.04E+01	2.13E+01	1.42E+01
f_{14}	Mean Err.	1.38E+03 (+)	5.08E+02 (+)	3.45E-02 (-)	2.24E+02 (+)	2.40E+03 (+)	8.25E+02 (+)	2.64E+01 (+)	2.55E+03 (+)	3.14E+02 (+)	1.43E+01
	Std.	5.67E+02	2.19E+02	1.93E-02	2.19E+02	7.20E+02	1.07E+02	8.03E+00	1.14E+03	1.61E+02	4.66E+00
f_{15}	Mean Err.	1.55E+03 (-)	3.32E+03 (-)	6.82E+03 (+)	1.25E+04 (+)	6.16E+03 (≈)	7.69E+03 (+)	7.42E+03 (+)	9.40E+03 (+)	2.19E+03 (-)	6.16E+03
	Std.	5.48E+02	7.58E+02	4.41E+02	7.58E+02	5.62E+02	2.86E+02	5.46E+02	2.73E+03	7.21E+02	6.24E+02
f_{16}	Mean Err.	8.78E-01 (≈)	1.06E-02 (-)	1.28E+00 (≈)	3.18E+00 (+)	1.14E+00 (≈)	1.80E+00 (+)	1.37E+00 (+)	3.38E+00 (+)	4.52E-01 (-)	1.32E+00
	Std.	1.44E+00	3.32E-03	2.07E-01	3.32E-03	2.05E-01	2.00E-01	1.88E-01	3.89E-01	1.67E-01	2.01E-01
f_{17}	Mean Err.	5.74E+01 (+)	6.63E+01 (+)	5.08E+01 (-)	6.41E+01 (+)	1.04E+02 (+)	7.95E+01 (+)	5.20E+01 (+)	1.15E+02 (+)	5.88E+01 (+)	5.09E+01
	Std.	2.73E+00	4.76E+00	4.57E-14	4.76E+00	1.20E+01	1.81E+00	2.40E-01	2.00E+01	1.87E+00	5.55E-02
f_{18}	Mean Err.	1.34E+02 (≈)	7.83E+01 (-)	1.37E+02 (≈)	3.74E+02 (+)	1.05E+02 (-)	1.81E+02 (+)	2.16E+02 (+)	1.68E+02 (≈)	8.97E+01 (-)	1.32E+02
	Std.	1.00E+02	4.64E+00	1.29E+01	4.64E+00	1.57E+01	7.64E+00	2.63E+01	1.02E+02	1.01E+01	8.36E+00
f_{19}	Mean Err.	4.46E+00 (+)	3.39E+00 (≈)	2.64E+00 (-)	7.90E+00 (+)	3.93E+00 (+)	7.57E+00 (+)	5.19E+00 (+)	8.92E+00 (+)	5.15E+00 (+)	3.02E+00
	Std.	5.93E-01	3.80E-01	2.83E-01	3.80E-01	7.34E-01	4.78E-01	1.41E+00	3.17E+00	6.47E-01	1.25E-01
f_{20}	Mean Err.	2.25E+01 (+)	1.87E+01 (-)	1.93E+01 (-)	2.16E+01 (+)	1.96E+01 (≈)	1.93E+01 (-)	1.98E+01 (-)	2.34E+01 (+)	2.28E+01 (+)	2.06E+01
	Std.	1.18E+00	8.18E-01	7.70E-01	8.18E-01	6.21E-01	3.55E-01	6.78E-01	7.93E-01	8.04E-01	1.65E+00
f_{21}	Mean Err.	1.98E+02 (≈)	6.95E+02 (+)	8.45E+02 (+)	3.58E+02 (+)	2.49E+02 (≈)	3.12E+02 (≈)	8.96E+02 (+)	7.93E+02 (+)	8.72E+02 (+)	2.58E+02
	Std.	1.40E+01	4.01E+02	3.63E+02	4.01E+02	1.91E+02	2.45E+02	2.90E+02	3.63E+02	3.02E+02	1.92E+02
f_{22}	Mean Err.	1.45E+03 (+)	2.53E+02 (+)	1.33E+01 (≈)	3.27E+01 (+)	2.76E+03 (+)	2.59E+03 (+)	5.11E+01 (+)	3.51E+03 (+)	2.11E+02 (+)	2.67E+01
	Std.	6.01E+02	1.44E+02	7.12E+00	1.44E+02	8.38E+02	3.82E+02	1.40E+01	1.90E+03	8.99E+01	3.91E+00
f_{23}	Mean Err.	1.71E+03 (-)	3.30E+03 (-)	7.63E+03 (+)	1.25E+04 (+)	6.78E+03 (≈)	9.68E+03 (+)	9.04E+03 (+)	9.93E+03 (+)	2.18E+03 (-)	6.10E+03
	Std.	8.09E+02	7.24E+02	6.58E+02	7.24E+02	5.53E+02	3.99E+02	7.81E+02	3.15E+03	7.06E+02	4.68E+02
f_{24}	Mean Err.	2.40E+02 (+)	2.21E+02 (+)	2.34E+02 (+)	2.27E+02 (+)	2.43E+02 (+)	3.98E+02 (+)	3.08E+02 (+)	3.77E+02 (+)	2.06E+02 (+)	2.01E+02
	Std.	2.04E+01	1.41E+01	1.01E+01	1.41E+01	7.77E+00	2.27E+00	2.13E+01	1.98E+01	9.91E+00	2.61E-01
f_{25}	Mean Err.	2.48E+02 (-)	2.94E+02 (≈)	3.40E+02 (+)	3.69E+02 (+)	3.24E+02 (+)	3.79E+02 (+)	3.65E+02 (+)	3.86E+02 (+)	2.96E+02 (≈)	2.84E+02
	Std.	5.06E+00	8.71E+00	3.09E+01	8.71E+00	1.18E+01	2.88E+00	1.86E+01	4.06E+00	9.03E+00	1.66E+01
f_{26}	Mean Err.	1.96E+02 (-)	2.91E+02 (+)	2.58E+02 (+)	2.49E+02 (+)	2.03E+02 (+)	2.01E+02 (+)	3.97E+02 (+)	4.22E+02 (+)	2.78E+02 (+)	2.00E+02
	Std.	1.43E+01	6.28E+01	8.08E+01	6.28E+01	2.49E+01	1.72E+00	9.87E+01	3.51E+01	4.82E+01	8.84E-06
f_{27}	Mean Err.	7.28E+02 (+)	6.39E+02 (+)	9.36E+02 (+)	1.37E+03 (+)	1.03E+03 (+)	2.17E+03 (+)	1.63E+03 (+)	2.03E+03 (+)	4.61E+02 (≈)	3.47E+02
	Std.	1.44E+02	1.56E+02	3.07E+02	1.56E+02	1.33E+02	3.22E+01	1.99E+02	2.29E+02	1.83E+02	2.26E+01
f_{28}	Mean Err.	4.00E+02 (≈)	4.59E+02 (≈)	4.58E+02 (≈)	4.00E+02 (≈)	4.59E+02 (≈)	4.00E+02 (≈)	8.52E+02 (+)	4.59E+02 (≈)	4.00E+02 (+)	4.00E+02
	Std.	0.00E+00	4.20E+02	4.13E+02	4.20E+02	4.21E+02	0.00E+00	1.15E+03	4.24E+02	3.86E-09	7.19E-14
	+/≈/−	9/8/11	12/9/7	15/6/7	23/4/1	13/11/4	20/4/4	23/3/2	22/6/0	16/6/6	

TABLE S-XII
COMPARISON RESULTS FOR PSO-HV, DE-HL AND TOP-RANK ALGORITHMS ON CEC 2013 ON 50-D.

	D=100					D=50					D=30				
Algorithms	TAPSO	DEPSO	HCLPSO	CJADE	LOTFWA	TAPSO	DEPSO	HCLPSO	CJADE	LOTFWA	TAPSO	DEPSO	HCLPSO	CJADE	LOTFWA
Simple Functions (+/≈/−)	1/2/4	2/2/3	1/3/3	1/0/6	2/0/5	1/2/4	3/1/3	1/1/5	1/0/6	2/0/5	1/1/5	2/1/4	1/1/5	0/0/7	2/0/5
Complex Functions (+/≈/−)	28/8/14	28/16/8	27/14/9	28/7/15	37/3/10	31/4/15	31/10/9	27/3/20	30/2/17	37/0/13	31/5/14	37/6/7	24/5/21	25/4/21	34/5/11



(a) g_1 (b) g_3 (c) g_4

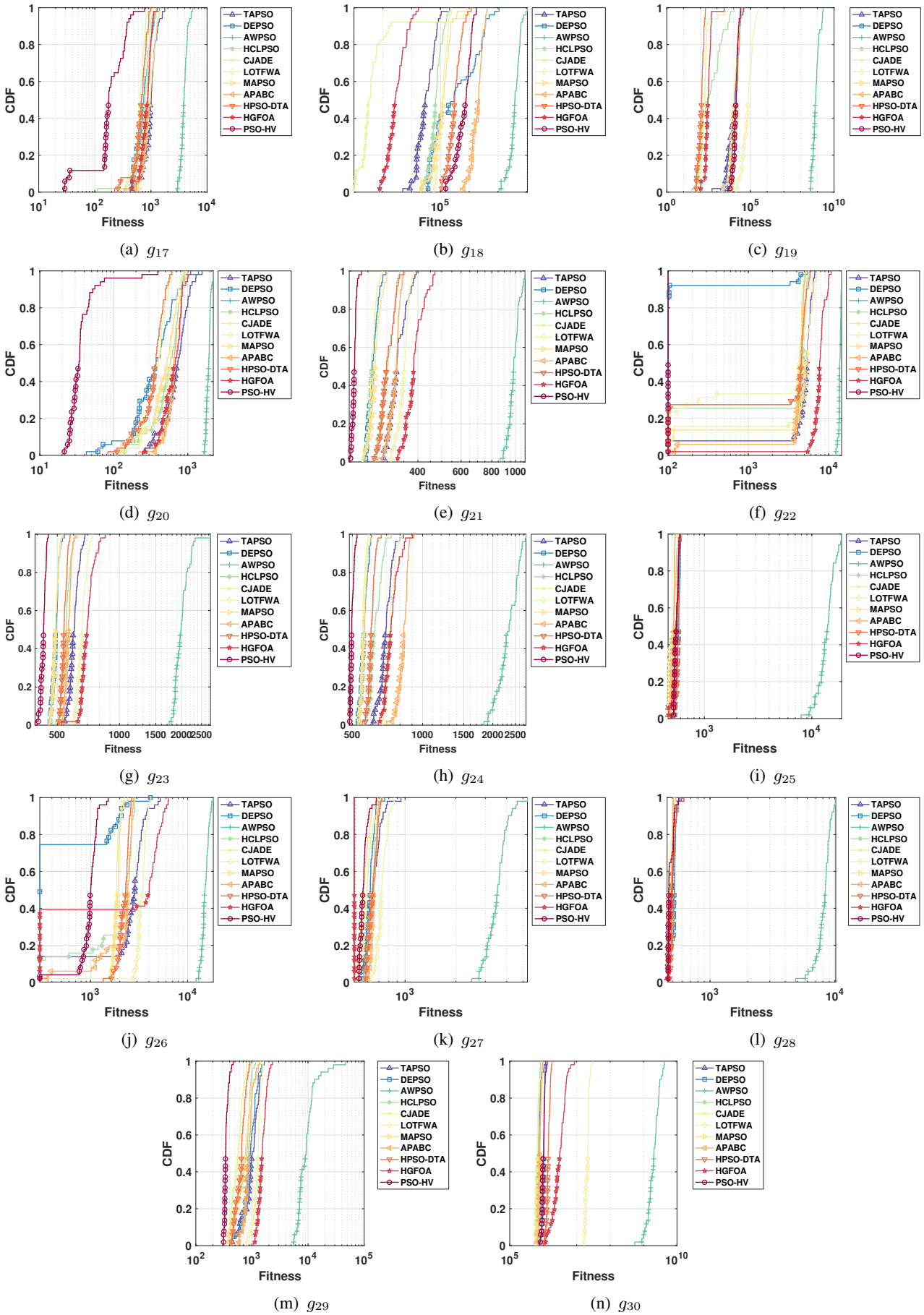
(d) g_5 (e) g_6 (f) g_7

(g) g_8 (h) g_9 (i) g_{10}

(j) g_{11} (k) g_{12} (l) g_{13}

(m) g_{14} (n) g_{15} (o) g_{16}

Fig. S-1. The cumulative distribution curves of comparison algorithms on g_1, g_3-g_{16}.



(a) g_{17} (b) g_{18} (c) g_{19}

(d) g_{20} (e) g_{21} (f) g_{22}

(g) g_{23} (h) g_{24} (i) g_{25}

(j) g_{26} (k) g_{27} (l) g_{28}

(m) g_{29} (n) g_{30}

Fig. S-2. The cumulative distribution curves of comparison algorithms on g_{17}-g_{30}.