Demonstration for WuRittSolva

Standard Application Package for Wu-Ritt Process

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Section WRS_I:Class and MainVariable wrt ord

Definition of polynomials

```
Off[General::"spell1"];
```
… 一般系统信息

```
lhspoly = x1^2 x2^3 - x2;
rhspoly = x1^3 x2 - 2;
ord = {x1, x2};
```

Class of polynomials wrt ord

```
Class[lhspoly, ord]
Class[rhspoly]
```

2

2

MainVariable of polynomials wrt ord

```
MainVariable[lhspoly, ord]
MainVariable[lhspoly]
```

x_2

x_2

Initial of polynomials wrt ord

```
Initial[lhspoly]
```

x_1^2

```
Initial[lhspoly, {x1, x2}]
```

x_1^2

Checking polynomials reduced

```
ppoly = u1 u2 x1^6 + 2 u1^2 x1 x2^3 - x1 x2^3 - 3 u2 x2^3 + 1;
qpoly = u2 x1^4 - 3 u1 x2^4 - 2 x1 x2 x3 + 3 u2^2 x2^2 x3 + 4;
qqpoly = x3^4 + u2 x1^2 x2^2 + 3 u1^2 x1 x2 x3 - u1 u2 x2 x3 - 3;
```

```
IsPolyReduced[qpoly, qqpoly, {x1, x2}, {x3}]
```

True

```
{Class[#, {x1, x2}, {u1, u2, x3}], MainVariable[#, {x1, x2}, {u1, u2, x3}], MainVariableExponent[#, {x1, x2}, {u1, u2, x3}]} & /@
 Expand@{ppoly, qpoly, qqpoly}
```
展开

{{2, x_2, 3}, {2, x_2, 4}, {2, x_2, 2}}

Other Properties of polynomials wrt ord

```
PolyVariables[lhspoly]
```

{x_1, x_2}

```
MainVariableExponent[lhspoly]
```

3

```
MainVariableExponent[lhspoly, ord]
```
3

```
LeadCoefficient[lhspoly, x1]
```
x_2^3

Section WRS_II:Pseuduo Division & Resolution

Definition of polynomial sets

```
poly1 = x1^2 - 2 x2^3 x3 + 1;
poly2 = x1 x2 + x3^2;
poly3 = 3 x1^2 - 2 x3^2;
polyset = {poly1, poly2, poly3};
ord = {x1, x2, x3};
```

Poly2Poly Pseudo Division: Remainder & Resolution

```
PseudoRemainder[poly1, poly2]
```
$1 + x_1^2 - 2\, x_2^3\, x_3$

```
PseudoResolution[poly1 * poly3, poly2]
```
$\{1, -2 - 2\, x_1^2 + 4\, x_2^3\, x_3, 3\, x_1^2 + 3\, x_1^4 + 2\, x_1\, x_2 + 2\, x_1^3\, x_2 - 6\, x_1^2\, x_2^3\, x_3 - 4\, x_1\, x_2^4\, x_3\}$

Poly2Polyset Pseudo Division:Remainder & Resolution

```
AuxPseudoRemainder[poly1, polyset]
```
0

```
AuxPseudoResolution[poly1, polyset]
```
{1, 0, 0}

PolySet2PolySet Pseudo Division:Remainder & Resolution

```
PseudoRemainderSet[{lhspoly, rhspoly}, polyset]
```
$\{-x_2 + x_1^2\, x_2^3, -2 + x_1^3\, x_2\}$

```
PseudoResolutionSet[{lhspoly, rhspoly}, polyset]
```
$\{\{1, 0, -x_2 + x_1^2\, x_2^3\}, \{1, 0, -2 + x_1^3\, x_2\}\}$

Exral Example

This example can be found at P_{17} in *Selected Papers in Symbolic Computation* by Doc.Wang. etc.

```
fpoly = x1 x2^2 + 1;
gpoly = 2 x2^3 - x2^2 + x1^2 x2;

PolyPRemainder[fpoly, gpoly, x2]
```
$1 + x_1\, x_2^2$

```
PseudoRemainder[gpoly, fpoly, {x1, x2}]
```
$1 - 2\, x_2 + x_1^3\, x_2$

```
PseudoResolution[gpoly, fpoly, {x1, x2}]
```
$\{x_1, -1 + 2\, x_2, 1 - 2\, x_2 + x_1^3\, x_2\}$

This example can be found at P_{271} in *Higher Algebra and Analytic Geometry(II)* by *Chen Zhijie*.etc.

```
fpoly = x1^2 x2^3 - x2;
gpoly = x1^2 x2 - 2;

PseudoRemainder[fpoly, gpoly]
```
$8 - 2\, x_1^2$

```
PseudoResolution[fpoly, gpoly]
```
$\{x_1^4, 4 - x_1^2 + 2\, x_1^2\, x_2 + x_1^4\, x_2^2, 8 - 2\, x_1^2\}$

Section WRS_III:Characteristic Set & WuRittProver

Characteristic Set Examples

Test Example

```
h1 = (u1 - x1)^2 + x2^2 - x1^2 - x2^2;
h2 = (u2 - x1)^2 + (u3 - x2)^2 - x1^2 - x2^2;
h3 = (u4 - x1)^2 + (x3 - u2)^2 - x1^2 - x2^2;
h4 = (x3 - x5) u3 + (u4 - x4) (u2 - u1);
h5 = x5 (u2 - u1) - u3 (x4 - u1);
h6 = (x3 - x7) u3 + (u4 - x6) u2;
h7 = x7 u2 - x6 u3;
hset = {h1, h2, h3, h4, h5, h6, h7};
ord = {x1, x2, x3, x4, x5, x6, x7};
const = {u1, u2, u3, u4};

defcs = CharacteristicSet[hset, ord, const, TracePrintOn -> True]
```

真

```
CharacteristicForm[defcs, ord, const]
```

$$\{\text{CS\_STEP:1}, \{u_1 (u_1 - 2 x_1),\ u_2^2 - 2 u_2 x_1 + u_3 (u_3 - 2 x_2),\ -x_1^2 + (-u_4 + x_1)^2 - x_2^2 + (-u_2 + x_3)^2,\ -u_3 x_4 + u_1 (u_3 - x_5) + u_2 x_5,\ -u_3 x_6 + u_2 x_7\}\}$$

$$\{\text{A New Component:1}, u_1 - 2 x_1\}$$

$$\{\text{CS\_STEP:2}, \{u_1 (u_1 - 2 x_1),\ u_2^2 - 2 u_2 x_1 + u_3 (u_3 - 2 x_2),\ -x_1^2 + (-u_4 + x_1)^2 - x_2^2 + (-u_2 + x_3)^2,$$
$$-u_2 u_3 x_3 + u_3^2 x_4 + u_1^2 (-u_4 + x_4) + u_2^2 (-u_4 + x_4) + u_1 (u_3 (-u_3 + x_3) - 2 u_2 (-u_4 + x_4)),\ -u_3 x_4 + u_1 (u_3 - x_5) + u_2 x_5,\ u_2 u_3 x_3 + u_2^2 (u_4 - x_6) - u_3^2 x_6,\ -u_3 x_6 + u_2 x_7\}\}$$

$$\{\text{A New Component:1}, u_1 - 2 x_1\}$$

$$\{\text{Total 2 Branch(s) of New Component(s) Discovered}\}$$

$$\begin{pmatrix} (u_1 - x_1)^2 - x_1^2 & \{x_1, 00, 00, 00, 00, 00, 00\} \\ (u_2 - x_1)^2 - x_1^2 + (u_3 - x_2)^2 - x_2^2 & \{x_1, x_2, 00, 00, 00, 00, 00\} \\ (u_4 - x_1)^2 - x_1^2 - x_2^2 + (-u_2 + x_3)^2 & \{x_1, x_2, x_3, 00, 00, 00, 00\} \\ -u_1 u_3^2 - u_1^2 u_4 + 2 u_1 u_2 u_4 - u_2^2 u_4 + u_1 u_3 x_3 - u_2 u_3 x_3 + u_1^2 x_4 - 2 u_1 u_2 x_4 + u_2^2 x_4 + u_3^2 x_4 & \{00, 00, x_3, x_4, 00, 00, 00\} \\ -u_3 (-u_1 + x_4) + (-u_1 + u_2) x_5 & \{00, 00, 00, x_4, x_5, 00, 00\} \\ u_2^2 u_4 + u_2 u_3 x_3 - u_2^2 x_6 - u_3^2 x_6 & \{00, 00, x_3, 00, 00, x_6, 00\} \\ -u_3 x_6 + u_2 x_7 & \{00, 00, 00, 00, 00, x_6, x_7\} \end{pmatrix}$$

Test Example 1

```
poly1 = x1^2 - 2 x1 x3 + 1;
poly2 = x1 x2 + x3^2;
poly3 = -3 x2^2 + 2 x3^2;
polyset = {poly1, poly2, poly3};
ord = {x1, x2, x3};
const = {};

CharacteristicForm[CharacteristicSet[polyset, ord, const, TracePrintOn -> True], ord, const]
```

真

$$\{\text{CS\_STEP:1}, \{1 + x_1^2 - 2 x_1 x_3\}\}$$

$$\{\text{CS\_STEP:2}, \{1 + 2 x_1^2 + x_1^4 + 4 x_1^3 x_2,\ 1 + x_1^2 - 2 x_1 x_3\}\}$$

$$\{\text{CS\_STEP:3}, \{(1 + x_1^2)^2 (-3 - 6 x_1^2 + 5 x_1^4),\ 1 + 2 x_1^2 + x_1^4 + 4 x_1^3 x_2,\ 1 + x_1^2 - 2 x_1 x_3\}\}$$

$$\{\text{A New Component:1}, -3 - 6 x_1^2 + 5 x_1^4\}$$

$$\{\text{Total 1 Branch(s) of New Component(s) Discovered}\}$$

$$\begin{pmatrix} -3 - 12 x_1^2 - 10 x_1^4 + 4 x_1^6 + 5 x_1^8 & \{x_1, 00, 00\} \\ 1 + 2 x_1^2 + x_1^4 + 4 x_1^3 x_2 & \{x_1, x_2, 00\} \\ 1 + x_1^2 - 2 x_1 x_3 & \{x_1, 00, x_3\} \end{pmatrix}$$

Test Example 2

```
poly1 = x1^2 - 2 x2 x3 + 1;
poly2 = x1 x2 + x3^2;
poly3 = 3 x1^2 - 2 x3^2;
poly4 = 2 x1 + 2 x2 x1;
poly5 = 2 x1 + 2 x2^2 x1^2;
poly6 = 3 x1^2 + 2 x3^3;
poly7 = 3 x1 x2 + 1;
polyset = {poly1, poly2, poly4, poly3, poly6, poly5, poly7};
ord = {x1, x2, x3};
const = {};
```

```
CharacteristicForm[CharacteristicSet[polyset, ord, const, TracePrintOn -> True], ord, const]
```
真

$\{CS\_STEP:1, \{2 x_1 (1 + x_2), 1 + x_1^2 - 2 x_2 x_3\}\}$

{A New Component:1, $1 + x_2$}

$\{CS\_STEP:2, \{2 - 6 x_1, 2 x_1 (1 + x_2), 1 + x_1^2 - 2 x_2 x_3\}\}$

{A New Component:1, $1 + x_2$}

{Total 2 Branch(s) of New Component(s) Discovered}

$$\begin{pmatrix} 2 - 6 x_1 & \{x_1, 00, 00\} \\ 2 x_1 + 2 x_1 x_2 & \{x_1, x_2, 00\} \\ 1 + x_1^2 - 2 x_2 x_3 & \{x_1, x_2, x_3\} \end{pmatrix}$$

Example 1 (Paralell Square Theorem)

```
H1 = x1 - u1 - u2;
H2 = x2 - u3;
H3 = -u1 u3 x1 + u1 u3 x3 - u2 u3 x3 + u3 x1 x3;
H4 = x4 (u2 - u1) - (x3 - u1) u3;
HSet = {H1, H2, H3, H4};
G1 = x1^2 - 2 x1 x3 - 2 x4 x2 + x2^2;
G2 = 2 x3 u1 - 2 x3 u2 - 2 x4 u3 - u1^2 + u2^2 + u3^2;
ord = {x1, x2, x3, x4};
const = {u1, u2, u3};

CS = CharacteristicSet[HSet, ord, const, TracePrintOn -> True]
```
真

$\{CS\_STEP:1, \{-u_1 - u_2 + x_1, -u_3 + x_2, u_3 ((-u_2 + x_1) x_3 + u_1 (-x_1 + x_3)), -u_3 x_3 + u_1 (u_3 - x_4) + u_2 x_4\}\}$

{A New Component:1, $u_1 x_1 - u_1 x_3 + u_2 x_3 - x_1 x_3$}

{Total 1 Branch(s) of New Component(s) Discovered}

$\{-u_1 - u_2 + x_1, -u_3 + x_2, -u_1 u_3 x_1 + u_1 u_3 x_3 - u_2 u_3 x_3 + u_3 x_1 x_3, -u_3 (-u_1 + x_3) + (-u_1 + u_2) x_4\}$

```
WuRittProver[Reverse@CS, G1, ord, const, TraceProverOn → True]
```
反向 真

$\{WRP\_STEP:1, 2 u_3 x_2 x_3 - u_2 (x_1^2 + x_2^2 - 2 x_1 x_3) + u_1 (x_1^2 + x_2 (-2 u_3 + x_2) - 2 x_1 x_3)\}$

$\{WRP\_STEP:2, -(u_1 - u_2) u_3 (u_1 (x_1^2 + (2 u_3 - x_2) x_2) + (u_2 - x_1) (x_1^2 + x_2^2))\}$

$\{WRP\_STEP:3, -(u_1 - u_2) u_3 (u_1 + u_2 - x_1) (u_3^2 + x_1^2)\}$

{WRP_STEP:4, 0}

True

```
WuRittProver[Reverse@CS, G2, ord, const, TraceProverOn → True]
```
反向 真

$\{WRP\_STEP:1, -(u_1^2 - 2 u_1 u_2 + u_2^2 + u_3^2) (u_1 + u_2 - 2 x_3)\}$

$\{WRP\_STEP:2, -(u_1 - u_2) u_3 (u_1^2 - 2 u_1 u_2 + u_2^2 + u_3^2) (u_1 + u_2 - x_1)\}$

$\{WRP\_STEP:3, -(u_1 - u_2) u_3 (u_1^2 - 2 u_1 u_2 + u_2^2 + u_3^2) (u_1 + u_2 - x_1)\}$

{WRP_STEP:4, 0}

True

Example 2 (Desargus Theorem)

```
H1 = x1 x6 - x2 x3;
H2 = x4 (x8 - x6) - x7 (x5 - x3);
H3 = (x4 - x1) x8 - x5 (x7 - x2);
HSet = {H1, H2, H3};
G = x4 x8 - x5 x7;
ord = {x1, x2, x3, x4, x5, x6, x7, x8};
const = {};

CS = CharacteristicSet[HSet, ord, const, TracePrintOn -> True]
```
真

$\{CS\_STEP:1, \{-x_2 x_3 + x_1 x_6, x_2 x_5 - x_5 x_7 + (-x_1 + x_4) x_8\}\}$

$\{CS\_STEP:2, \{-x_2 x_3 + x_1 x_6, (x_3 x_4 + x_1 (-x_3 + x_5)) (x_2 x_4 - x_1 x_7), x_2 x_5 - x_5 x_7 + (-x_1 + x_4) x_8\}\}$

{A New Component:1, $-x_2 x_4 + x_1 x_7$}

{Total 1 Branch(s) of New Component(s) Discovered}

$\{-x_2 x_3 + x_1 x_6, -x_1 x_2 x_3 x_4 + x_2 x_3 x_4^2 + x_1 x_2 x_4 x_5 + x_1^2 x_3 x_7 - x_1 x_3 x_4 x_7 - x_1^2 x_5 x_7, -x_5 (-x_2 + x_7) + (-x_1 + x_4) x_8\}$

```
WuRittProver[Reverse@CS, G, ord, const, TraceProverOn → True]
```
反向 真

{WRP_STEP:1, $x_5 (x_2 x_4 - x_1 x_7)$}

{WRP_STEP:2, 0}

{WRP_STEP:3, 0}

True

Example 3 (Simon Theorem)

```
H1 = (u1 - x1)^2 + x2^2 - x1^2 - x2^2;
H2 = (u2 - x1)^2 + (u3 - x2)^2 - x1^2 - x2^2;
H3 = (u4 - x1)^2 + (x3 - x2)^2 - x1^2 - x2^2;
H4 = (x3 - x5) u3 + (u4 - x4) (u2 - u1);
H5 = x5 (u2 - u1) - u3 (x4 - u1);
H6 = (x3 - x7) u3 + (u4 - x6) u2;
H7 = x7 u2 - x6 u3;
HSet = {H1, H2, H3, H4, H5, H6, H7};
G = x7 (x4 - u4) - (x6 - u4) x5;
ord = {x1, x2, x3, x4, x5, x6, x7};
const = {u1, u2, u3, u4};
defCS = CharacteristicSet[Expand@HSet, ord, const, TracePrintOn -> True]
```
展开 真

{CS_STEP:1, $\{u_1 (u_1 - 2 x_1),\ u_2^2 - 2 u_2 x_1 + u_3 (u_3 - 2 x_2),\ u_4^2 - 2 u_4 x_1 + x_3 (-2 x_2 + x_3),\ -u_3 x_4 + u_1 (u_3 - x_5) + u_2 x_5,\ -u_3 x_6 + u_2 x_7\}$}

{A New Component:1, $u_1 - 2 x_1$}

{CS_STEP:2, $\{u_1 (u_1 - 2 x_1),\ u_2^2 - 2 u_2 x_1 + u_3 (u_3 - 2 x_2),\ u_4^2 - 2 u_4 x_1 + x_3 (-2 x_2 + x_3),$
$-u_2 u_3 x_3 + u_3^2 x_4 + u_1^2 (-u_4 + x_4) + u_2^2 (-u_4 + x_4) + u_1 (u_3 (-u_3 + x_3) - 2 u_2 (-u_4 + x_4)),\ -u_3 x_4 + u_1 (u_3 - x_5) + u_2 x_5,\ u_2 u_3 x_3 + u_2^2 (u_4 - x_6) - u_3^2 x_6,\ -u_3 x_6 + u_2 x_7\}$}

{A New Component:1, $u_1 - 2 x_1$}

{Total 2 Branch(s) of New Component(s) Discovered}

$\{u_1^2 - 2 u_1 x_1,\ u_2^2 + u_3^2 - 2 u_2 x_1 - 2 u_3 x_2,\ u_4^2 - 2 u_4 x_1 - 2 x_2 x_3 + x_3^2,\ -u_1 u_3^2 - u_1^2 u_4 + 2 u_1 u_2 u_4 - u_2^2 u_4 + u_1 u_3 x_3 - u_2 u_3 x_3 + u_1^2 x_4 - 2 u_1 u_2 x_4 + u_2^2 x_4 + u_3^2 x_4,$
$u_1 u_3 - u_3 x_4 - u_1 x_5 + u_2 x_5,\ u_2^2 u_4 + u_2 u_3 x_3 - u_2^2 x_6 - u_3^2 x_6,\ -u_3 x_6 + u_2 x_7\}$

```
WuRittProver[Reverse@defCS, G, ord, const, TraceProverOn -> True]
```
反向 真

{WRP_STEP:1, $u_2 x_5 (u_4 - x_6) + u_3 (-u_4 + x_4) x_6$}

{WRP_STEP:2, $u_2 u_3 \left(-u_2 \left(u_4^2 - u_4 x_4 + x_3 x_5\right) + u_3 (x_3 x_4 + u_4 (-x_3 + x_5))\right)$}

{WRP_STEP:3, $u_2 u_3 \left(u_1 \left(-u_3^2 u_4 + u_2 u_4 (u_4 - x_4) + u_3 x_3 (u_2 + u_4 - x_4)\right) + u_4 \left(-u_2 u_3 x_3 + u_3^2 x_4 + u_2^2 (-u_4 + x_4)\right)\right)$}

{WRP_STEP:4, $u_1 u_2 u_3^2 \left(u_1^2 (-u_3 u_4 + u_2 x_3) + u_2 \left(u_2^2 x_3 + u_3^2 x_3 - u_3 \left(u_4^2 + x_3^2\right)\right) + u_1 \left(u_2 u_3 u_4 - 2 u_2^2 x_3 + u_3 \left(u_4^2 - u_3 x_3 + x_3^2\right)\right)\right)$}

{WRP_STEP:5, $u_1 (u_1 - u_2) u_2 u_3^2 \left(-u_2^2 x_3 - u_3^2 x_3 + u_1 (-u_3 u_4 + u_2 x_3) + 2 u_3 (u_4 x_1 + x_2 x_3)\right)$}

{WRP_STEP:6, $u_1 (u_1 - u_2) u_2 u_3^2 (u_1 - 2 x_1) (-u_3 u_4 + u_2 x_3)$}

{WRP_STEP:7, 0}

True

```
CS = { H1, H2, H3, PolyPRemainder[H4, H5, x5], H5, PolyPRemainder[H6, H7, x7], H7} // Expand
```
展开

$\{u_1^2 - 2 u_1 x_1,\ u_2^2 + u_3^2 - 2 u_2 x_1 - 2 u_3 x_2,\ u_4^2 - 2 u_4 x_1 - 2 x_2 x_3 + x_3^2,\ u_1 u_3^2 + u_1^2 u_4 - 2 u_1 u_2 u_4 + u_2^2 u_4 - u_1 u_3 x_3 + u_2 u_3 x_3 - u_1^2 x_4 + 2 u_1 u_2 x_4 - u_2^2 x_4 - u_3^2 x_4,$
$u_1 u_3 - u_3 x_4 - u_1 x_5 + u_2 x_5,\ u_2^2 u_4 + u_2 u_3 x_3 - u_2^2 x_6 - u_3^2 x_6,\ -u_3 x_6 + u_2 x_7\}$

```
WuRittProver[Reverse@CS, G, ord, const, TraceProverOn -> True]
```
反向 真

{WRP_STEP:1, $u_2 x_5 (u_4 - x_6) + u_3 (-u_4 + x_4) x_6$}

{WRP_STEP:2, $u_2 u_3 \left(-u_2 \left(u_4^2 - u_4 x_4 + x_3 x_5\right) + u_3 (x_3 x_4 + u_4 (-x_3 + x_5))\right)$}

{WRP_STEP:3, $u_2 u_3 \left(u_1 \left(-u_3^2 u_4 + u_2 u_4 (u_4 - x_4) + u_3 x_3 (u_2 + u_4 - x_4)\right) + u_4 \left(-u_2 u_3 x_3 + u_3^2 x_4 + u_2^2 (-u_4 + x_4)\right)\right)$}

{WRP_STEP:4, $u_1 u_2 u_3^2 \left(u_1^2 (-u_3 u_4 + u_2 x_3) + u_2 \left(u_2^2 x_3 + u_3^2 x_3 - u_3 \left(u_4^2 + x_3^2\right)\right) + u_1 \left(u_2 u_3 u_4 - 2 u_2^2 x_3 + u_3 \left(u_4^2 - u_3 x_3 + x_3^2\right)\right)\right)$}

{WRP_STEP:5, $u_1 (u_1 - u_2) u_2 u_3^2 \left(-u_2^2 x_3 - u_3^2 x_3 + u_1 (-u_3 u_4 + u_2 x_3) + 2 u_3 (u_4 x_1 + x_2 x_3)\right)$}

{WRP_STEP:6, $u_1 (u_1 - u_2) u_2 u_3^2 (u_1 - 2 x_1) (-u_3 u_4 + u_2 x_3)$}

{WRP_STEP:7, 0}

True

Example 4 (Algebra Relations Discovery)

```
H1 = x1^2 + x3^2 - x5^2;
H2 = x2^2 + x4^2 - x5^2;
H3 = x1^2 + (x3 - x5)^2 - (x2 - x1)^2 - (x3 - x4)^2;
H4 = x1^2 + (x3 - x5)^2 - 4 x2^2;
HSet = {H1, H2, H3, H4};
ord = {x1, x2, x3, x4, x5};
const = {};

CharacteristicSet[HSet, ord, const, TracePrintOn -> True] // Simplify
```

真 化简

$$\{\text{CS\_STEP:1}, \{x_1^2 - 4x_2^2 + (x_3 - x_5)^2\}\}$$

$$\{\text{CS\_STEP:2}, \{-x_1^2 + 2x_1x_2 + 3x_2^2 - (x_3 - x_4)^2, 2(x_1^2 - 2x_2^2 + x_3(x_3 - x_5))\}\}$$

$$\{\text{CS\_STEP:3}, \{2(x_1^4 + 4x_2^2(x_2^2 - x_3^2) + x_1^2(-4x_2^2 + x_3^2)), -2(x_1^2 - x_1x_2 - 2x_2^2 + x_3(x_3 - x_4)), 2(x_1^2 - 2x_2^2 + x_3(x_3 - x_5))\}\}$$

$$\{\text{CS\_STEP:4}, \{8x_2^3(-x_1^3 + 2x_1x_2^2 + x_2^3), 2(x_1^4 + 4x_2^2(x_2^2 - x_3^2) + x_1^2(-4x_2^2 + x_3^2)), -2(x_1^2 - x_1x_2 - 2x_2^2 + x_3(x_3 - x_4)), 2(x_1^2 - 2x_2^2 + x_3(x_3 - x_5))\}\}$$

$$\{\text{A New Component:1}, x_1 + x_2\}$$

$$\{\text{A New Component:2}, x_1^2 - x_1x_2 - x_2^2\}$$

$$\{\text{Total 1 Branch(s) of New Component(s) Discovered}\}$$

$$\{8x_2^3(-x_1^3 + 2x_1x_2^2 + x_2^3), 2(x_1^4 + 4x_2^2(x_2^2 - x_3^2) + x_1^2(-4x_2^2 + x_3^2)), -2(x_1^2 - x_1x_2 - 2x_2^2 + x_3(x_3 - x_4)), 2(x_1^2 - 2x_2^2 + x_3(x_3 - x_5))\}$$

Relevent Resources

Some resources are available for developing the WuRittSolva Tools,and these coresponding notebooks are listed below:

[1]. Details of WuRittSolva.nb
[2]. WuRittSolva User Guide.nb
[3]. Demonstration of WuRittSolva in Elementary Geometry.nb
[4]. A Collection of Testing Problems.nb
[5]. WuRittSolva for Concrete Geometric Configurations in Elementary Geometry.nb
[6]. WuRittSolva User Manual.nb