WuRittSolva User Manual

Documents for Users' Instruction *May 24^{th} 2005*

By Liu Huashan

Department of Applied Mathematics,Tianjin Polytechnic University,P.R.C.

E-mail: liukaitianpidi@sina.com

HomePage: http://magicm.51.net

Instructed by

Associate Prof. Huang Dongwei

Department of Applied Mathematics,Tianjin Polytechnic University,P.R.C.

WUM_I:Introduction to WuRitt Characteristic Set Method

In this section, we introdcue some concepts and notations on the characteristic set method and they are used in the following chapters. Due to implementation in WuRittSolva not only the characteristic set method but also the algebraico-polynomials equations solving , we stress the coefficient filed is the complex field **C**. For more generic details and discussion on the subject can be found in relevant works and papers.

The following contents on charateristic set method are mainly from [14].

Let C be complex field,onsider a set of variables X={x_1, x_2, ..., x_n} and polynomials in X with coeffiecients in C.We shall introduce some partial ordering among all finite systems of such polynomials.For this sake let's arrange the variables{x_k}in natrual asending order or lexical order.Any nonconstant polynomials P in C[X] may be written in a canonical form

$$P = I_d * x_c^{d} + I_{d-1} * x_c^{d-1} + \ ... \ + I_0, \tag{1}$$

in which each I_i are themselves polynomials in varialbes x_1, x_2,..., $x_{n-1}$with $I_d \neq 0$. We call then the c the class of P, denoting as Class(P); d the degree of P, denoting as Degree(P); $x_c$the leading variable of P, denoting as LeadVariable(P); and I_d the leading coeffiecient or initial of P, denoting as LeadCoefficient(P) or Initial(P). To notify that these basic definitions are dependent on the variable order and the constant inderminants. We then introduce a partial ordering of non-zero polynomials first according to class and then to degree, the non-zero constants being considered as polynomials of lowest ordering.

For the partial ordering of polynomial systems let us consider first such polynomial sets well-arranged in the following sense. The polynomials in the set are non-constant ones and may be so arranged with classes c_i steadily increasing:

$$0 < c_1 < . . . < c_r. \tag{2}$$

The leading coefficient or the initial of the i-th polynomial in the set is either a non-zero constant or has a class less than c_i which, if it is of class c_j, $1 \leq j < i$, should have a degree less than that of j-th polynomial in the set. Such a polynomial set is then called an ascending set (abbr. asc-set). Some partial ordering is then introduced among the system of all such asc-sets, with the set consisting of a single non-zero constant considered as a trivial asc-set to be arranged in the lowest ordering.

Consider now arbitrary finite systems of non-zero polynomials. For such a polynomial system, any asc-set of lowest ordering contained wholly in the given system is called a basic set (abbr. bas-set) of the system. A partial ordering is then unambiguously introduced among all non-empty polynomial systems according to the partial ordering of their basic sets. Any polynomial system containing a non-zero constant polynomial will be clearly one of lowest ordering.

After the introduction of partial ordering among all finite polynomial systems let us consider now such a given system PS and consider the scheme (3) shown below:

$$\begin{array}{llllllllll} PS = & PS^0 & PS^1 & \cdots & PS^i & \cdots & PS^m & & \\ & BS^0 & BS^1 & \cdots & BS^i & \cdots & BS^m & = & CS \\ & RS^0 & RS^1 & \cdots & RS^i & \cdots & RS^m & = & \emptyset. \end{array} \tag{3}$$

In the scheme (S) each BS_i is a basic set of PS_i, each RS_i is the set of non-zero remainders, if any, of polynomials in $PS_i \backslash BS_i$ with respect to BS_i, and $PS_{i+1} = PS \bigcup BS^i \bigcup RS^i$if RS^i is non-empty. It is easily proved that the sequence of BS^i is a steadily decreasing sequence:

$$BS^0 \succ BS^1 \succ \cdots \succ BS^r \succ \cdots \tag{4}$$

Such a sequence cannot be an infinite one and should terminate at certain stage m with $RS_m = \Phi$; The corresponding basic set $BS_m = CS$ is then called a characteristic set (abbr.char-set) of the given polynomial system PS. The zero-set of PS; Zero(PS), consisting of all possible complex solutions or zeros of the system of polynomial equations PS = 0, is closely connected with that of CS by the Well-Ordering Principle in the form below

$$Zero(PS) = Zero(CS/IP) \cup Zero(PS \ \cup \ \{IP\}), \tag{5}$$

in which IP is the product of all initials of polynomials in CS and Zero(CS\IP) =Zero(CS)\ Zero(IP).

Now PS $\bigcup${IP} is easily seen to be a polynomial set of lower ordering than PS. If we apply the Well-Ordering Principle to PS $\bigcup${IP} and proceed further and further in the same way we should stopped in a finite number of steps and arrived at the following:

(Zero-Decompostion Theorem) For any finite polynomial system PS there is an algorithm which will give in a finite number of steps a finite set of asc-sets CS^s with initial product IP^s such that

$$\text{Zero (PS)} = \bigcup_s \text{Zero}\left(\text{CS}^2 / \text{IP}^s\right). \tag{6}$$

Now CS^s are all asc-sets. Hence all zero-sets Zero(CS^s) and all Zero(CS^s\IP^s) may be considered as well-determined in some natural sense. The formula (6) gives thus actually an explicit determination of Zero(PS) for all finite polynomial systems PS which serves for the solving of arbitrary systems of polynomial equations.

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WUM_II:User Functions in WuRittSolva

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Names["WuRittSolva`Master`*"]
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```
{AutoClass, AutoMainVariable, Class, FixedClass, FixedMainVariable,
 Initial, intTest, IsCompatibleSet, IsConstantsIn, IsIncompatibleSet,
 IsPolyReduced, IsRankEqual, IsRankGreater, IsRankLess, LeadCoefficient,
 MainVariable, MainVariableExponent, MainVarialbe, Master, MaxElementPos,
 MinElementPos, orderTransform, poly, PolynomialRank, PolyVariables, Separant}
```

```
Names["WuRittSolva`WuRittSolva`*"]
```

```
{AuxPseudoRemainder, AuxPseudoResolution, BasicSet, CharacteristicForm, CharacteristicSet,
 IsAscendingSet, MaxSteps, MiniAscSet, Padding, PolyPRemainder, PseudoRemainder,
 PseudoRemainderSet, PseudoResolution, PseudoResolutionSet, SplitPolySet,
 TraceAll, TracePrintOn, WuRittEqnsSolve, WuRittSolva, ZerosDecomposition}
```

```
Names["WuRittSolva`Geo2AlgLib`*"]
```

```
{FivePointsOnCircle, FourPointsOnCircle, PointInAngle, PointOnCirlce,
 PointOnLineEqual, PointOnLineToRatio, ThreePointsOnCircle, TripleLinesIntersect,
 TriplePointsCollinear, TwoAnglesEqual, TwoLinesEqual, TwoLinesEqualRatio,
 TwoLinesParallel, TwoLinesPerpend, TwoLinesToRatio, TwoPointsOnCircle}
```

```
Names["WuRittSolva`WuRittProver`*"]
```

```
{AuxProverRemainder, IsNewComponent, WuRittProver, WuRittSmartProver}
```

WUM_III:A Collection of User Documents

A Collection of Testing Problems.nb

Details of WuRittSolva.nb

Demonstration of WuRittSolva in Elementary Geometry.nb

WuRittSolva Tools.nb

WuRittSolva UserGuide.nb

WuRittSolva for Concrete Geometric Configurations in Elementary Geometry.nb

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