# WuRittSolva for Concrete Geometric Configurations in Elementary Geometry

Using Geometry to Algebra Library, WuRittSmartProver, etc. *On May 16th, 2005*

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## Parallelogram Diagonal Theorem

### The Geometric Figure of the Theorem

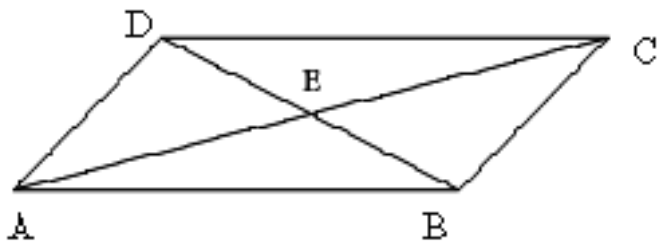


Figure 1

### Setting Coordination for the Geometric Configuration

```
precoord = {A1 → {0, 0}, B1 → {u1, 0}, C1 → {u2, u3}, D1 → {x1, x2}, E1 → {x3, x4}};
```

### Fixing the Concrete Geometric Configuration

```
prethmcfg = {TwoLinesParallel[{A1, B1}, {C1, D1}], TwoLinesParallel[{C1, B1}, {D1, A1}],
    TriplePointsCollinear[A1, C1, E1], TriplePointsCollinear[B1, D1, E1]};
prethmcnd = {TwoLinesEqual[{A1, E1}, {E1, C1}], TwoLinesEqual[{B1, E1}, {E1, D1}]};
```

```
Timing@WuRittSmartProver[precoord, {prethmcfg, prethmcnd}, {x1, x2, x3, x4}, {u1, u2, u3},
  TraceCharacteristicSetOn → True, TraceProverOn → True]
```

## WRSP_Step_I: Now Solving the Characteristic Set

{CS_STEP:1, $\{u_1 (-u_3 + x_2), -x_2 x_3 + u_1 (x_2 - x_4) + x_1 x_4\}$}

{A New Component:1, $u_3 - x_2$}

{CS_STEP:2, $\{-u_1 u_3 (u_1 - u_2 + x_1), u_1 (-u_3 + x_2), u_1 u_3 ((u_2 - x_1) x_3 + u_1 (-u_2 + x_3)), -x_2 x_3 + u_1 (x_2 - x_4) + x_1 x_4\}$}

{A New Component:1, $u_3$}

{A New Component:2, $u_1 - u_2 + x_1$}

{A New Component:1, $u_3 - x_2$}

{A New Component:1, $u_3$}

{A New Component:2, $u_1 u_2 - u_1 x_3 - u_2 x_3 + x_1 x_3$}

{Total 4 Branch(s) of New Component(s) Discovered}

$$\begin{pmatrix} -u_1^2 u_3 + u_1 u_2 u_3 - u_1 u_3 x_1 & \{x_1, 00, 00, 00\} \\ u_1 (-u_3 + x_2) & \{00, x_2, 00, 00\} \\ -u_1^2 u_2 u_3 + u_1^2 u_3 x_3 + u_1 u_2 u_3 x_3 - u_1 u_3 x_1 x_3 & \{x_1, 00, x_3, 00\} \\ -x_2 (-u_1 + x_3) + (-u_1 + x_1) x_4 & \{x_1, x_2, x_3, x_4\} \end{pmatrix}$$

## WRSP_Step_II: Now Proving the Theorem(s)

### WRSP_SubStep_1_I: Now Proving the 1(th) Theorem

$\{$WRP_STEP:1, $u_2^2 x_1 - 2 u_2 x_1 x_3 - u_1 (u_2^2 + u_3 (u_3 - 2 x_2) - 2 u_2 x_3) + u_3 (u_3 x_1 - 2 x_2 x_3)\}$

$\{$WRP_STEP:2, $u_1 u_3 (u_1 - x_1) (-(u_2^2 + u_3^2) (u_2 - x_1) + u_1 (u_2^2 - u_3 (u_3 - 2 x_2)))\}$

$\{$WRP_STEP:3, $u_1 u_3 (u_2^2 + u_3^2) (u_1 - x_1) (u_1 - u_2 + x_1)\}$

{WRP_STEP:4, 0}

### WRSP_SubStep_1_II: The 1(th) Theorem is True

### WRSP_SubStep_2_I: Now Proving the 2(th) Theorem

$\{$WRP_STEP:1, $(u_1^2 - 2 u_1 x_1 + x_1^2 + x_2^2) (u_1 + x_1 - 2 x_3)\}$

$\{$WRP_STEP:2, $u_1 u_3 (u_1 - x_1) (u_1 - u_2 + x_1) (u_1^2 - 2 u_1 x_1 + x_1^2 + x_2^2)\}$

$\{$WRP_STEP:3, $u_1 u_3 (u_1 - x_1) (u_1 - u_2 + x_1) (u_1^2 + u_3^2 - 2 u_1 x_1 + x_1^2)\}$

{WRP_STEP:4, 0}

### WRSP_SubStep_2_II: The 2(th) Theorem is True

## WRSP_Step_III: Now Checking the Initials & the Algebraic Configuration

$\{$**The Initials:** , $\{u_1, -u_1 u_3, u_1 u_3 (u_1 + u_2 - x_1), -u_1 + x_1\}\}$

$\{$**The Algebraic Configuration:** , $\left\{\left\{x_1 \to -u_1 + u_2, x_2 \to u_3, x_3 \to \frac{u_2}{2}, x_4 \to \frac{u_3}{2}\right\}\right\}\}$

```
{1.131 Second, {{0.331 Second, True}, {0.17 Second, True}}}
```

## Pascal's Theorem

### The Geometric  Figure of the Theorem

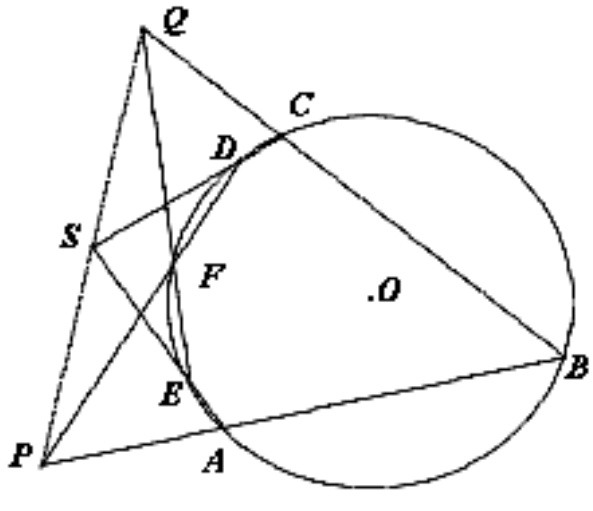


Figure 2

## Setting Coordination for the Geometric Configuration

```
precoord = {A1 → {0, 0}, O1 → {u1, 0}, B1 → {x1, u2}, C1 → {x2, u3}, D1 → {x3, u4}, F1 → {x4, u5},
    E1 → {x5, u6}, P1 → {x7, x6}, Q1 → {x9, x8}, S1 → {x10, x11}};
```

## Fixing the Concrete Geometric Configuration

```
prethmcfg = {TwoLinesEqual[{O1, A1}, {O1, B1}], TwoLinesEqual[{O1, A1}, {O1, C1}],
    TwoLinesEqual[{O1, A1}, {O1, D1}], TwoLinesEqual[{O1, A1}, {O1, E1}], TwoLinesEqual[{O1, A1}, {O1, F1}],
    TriplePointsCollinear[P1, D1, F1], TriplePointsCollinear[P1, A1, B1], TriplePointsCollinear[Q1, F1, E1],
    TriplePointsCollinear[Q1, B1, C1], TriplePointsCollinear[S1, A1, E1],
    TriplePointsCollinear[S1, C1, D1]};
prethmcnd = {TriplePointsCollinear[P1, Q1, S1]};
ord = {x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11};
const = {u1, u2, u3, u4, u5, u6};
```

## Using WuRittSmartProver for the Geometric Configuration

```
Timing@WuRittSmartProver[precoord, {prethmcfg, prethmcnd}, ord, const, TraceCharacteristicSetOn → True,
  TraceProverOn → True]
```

# WRSP_Step_I: Now Solving the Characteristic Set

$$\{\text{CS\_STEP:1}, \{-u_2^2+2u_1x_1-x_1^2,\ u_4(x_2-x_{10})+u_3(-x_3+x_{10})+(-x_2+x_3)x_{11},\ -u_3^2+2u_1x_2-x_2^2,$$
$$-u_4^2+2u_1x_3-x_3^2,\ -u_5^2+2u_1x_4-x_4^2,\ -u_6^2+2u_1x_5-x_5^2,\ x_1x_6-u_2x_7,\ (-x_1+x_2)x_8+u_3(x_1-x_9)+u_2(-x_2+x_9)\}\}$$

$$\{\text{CS\_STEP:2}, \{-u_2^2+2u_1x_1-x_1^2,\ u_4x_5(x_2-x_{10})+u_6(-x_2+x_3)x_{10}+u_3x_5(-x_3+x_{10}),\ u_4(x_2-x_{10})+u_3(-x_3+x_{10})+(-x_2+x_3)x_{11},$$
$$-u_3^2+2u_1x_2-x_2^2,\ -u_4^2+2u_1x_3-x_3^2,\ -u_5^2+2u_1x_4-x_4^2,\ -u_6^2+2u_1x_5-x_5^2,\ (u_4-u_5)x_1x_6+u_2(u_5x_3-u_4x_4-x_3x_6+x_4x_6),$$
$$x_1x_6-u_2x_7,\ (u_5-u_6)(x_1-x_2)x_8+u_3(u_6(x_1-x_4)+u_5(-x_1+x_5)+(x_4-x_5)x_8)+u_2(u_6(-x_2+x_4)+u_5(x_2-x_5)+(-x_4+x_5)x_8),$$
$$(-x_1+x_2)x_8+u_3(x_1-x_9)+u_2(-x_2+x_9)\}\}$$

$$\begin{pmatrix}
u_1^2-u_2^2-(-u_1+x_1)^2 \\
u_1^2-u_3^2-(-u_1+x_2)^2 \\
u_1^2-u_4^2-(-u_1+x_3)^2 \\
u_1^2-u_5^2-(-u_1+x_4)^2 \\
u_1^2-u_6^2-(-u_1+x_5)^2 \\
u_2u_5x_3-u_2u_4x_4+u_4x_1x_6-u_5x_1x_6-u_2x_3x_6+u_2x_4x_6 \\
x_6(x_1-x_7)-(u_2-x_6)x_7 \\
-u_3u_5x_1+u_3u_6x_1+u_2u_5x_2-u_2u_6x_2+u_2u_6x_4-u_3u_6x_4-u_2u_5x_5+u_3u_5x_5+u_5x_1x_8-u_6x_1x_8-u_5x_2x_8+u_6x_2x_8-u_2x_4x_8+ \\
(u_3-x_8)(x_1-x_9)-(u_2-x_8)(x_2-x_9) \\
u_4x_2x_5-u_3x_3x_5-u_6x_2x_{10}+u_6x_3x_{10}+u_3x_5x_{10}-u_4x_5x_{10} \\
-(x_3-x_{10})(u_3-x_{11})+(x_2-x_{10})(u_4-x_{11})
\end{pmatrix}$$

# WRSP_Step_II: Now Proving the Theorem(s)

## WRSP_SubStep_1_I: Now Proving the 1(th) Theorem

$$\{\text{WRP\_STEP:1}, -u_4(x_7-x_9)(x_2-x_{10})+u_3(x_7-x_9)(x_3-x_{10})+(x_2-x_3)(x_7x_8-x_8x_{10}+x_6(-x_9+x_{10}))\}$$

$$\{\text{WRP\_STEP:2}, -(x_2-x_3)$$
$$(u_6(x_2-x_3)(x_7x_8-x_6x_9)+u_3(x_3(x_5(-x_6+x_8)+u_6(x_7-x_9))+x_5(-x_7x_8+x_6x_9))+u_4(x_5(x_7x_8-x_6x_9)+x_2(x_5(x_6-x_8)+u_6(-x_7+x_9))))\}$$

$$\{\text{WRP\_STEP:3}, -(x_2-x_3)\big((x_1-x_2)(u_6(-x_2+x_3)x_6+u_4(u_6x_2-x_5x_6))x_8+$$
$$u_2(u_4(x_2-x_7)(u_6x_2-x_5x_8)-u_6(x_2-x_3)(x_2x_6-x_7x_8))+u_3^2(u_6x_3(x_1-x_7)+x_5(-x_1x_6+x_3(x_6-x_8)+x_7x_8))+$$
$$u_3(u_6x_1x_2x_6-u_6x_1x_3x_6-u_6x_1x_3x_8+u_6x_2x_3x_8+x_1x_5x_6x_8-x_2x_5x_6x_8-u_6x_2x_7x_8+u_6x_3x_7x_8+$$
$$u_4(u_6x_2(-x_1+x_7)+x_5(x_1x_6-x_7x_8+x_2(-x_6+x_8)))+u_2(u_6x_3(-x_2+x_7)+x_5(x_2x_6-x_7x_8+x_3(-x_6+x_8)))\big)\}$$

$$\{\text{WRP\_STEP:4}, -(u_2-u_3)(x_2-x_3)$$
$$\big(u_4\big(u_5(x_2-x_5)x_5((x_1-x_2)x_6+u_2(x_2-x_7))+u_6((x_1-x_2)((-x_2+x_4)x_5x_6+u_5x_2(x_5-x_7))-u_2x_4(x_2-x_5)(x_2-x_7))-u_6^2(x_1-x_2)x_2(x_4-x_7)\big)+$$
$$u_6(x_2-x_3)(u_5(-x_1+x_2)x_5x_6+u_6(x_1x_4x_6-u_2x_4x_7+x_2(-x_4x_6+u_2x_7))+u_2(u_5x_5x_7+x_2(x_4x_6-x_5x_6-u_5x_7)))+$$
$$u_3^2(x_5((x_1-x_3)(x_4-x_5)x_6+u_5(x_1-x_5)(x_3-x_7))+u_6(-x_4x_5x_7+x_1(-x_3x_4+x_5x_7)+x_3(-x_5x_7+x_4(x_5+x_7))))+$$
$$u_3\big(x_5(-u_5(x_1-x_2)(x_3-x_5)x_6+u_2(-(x_2-x_3)(x_4-x_5)x_6-u_5(x_2-x_5)(x_3-x_7)))+$$
$$u_6^2(-x_3x_4x_7+x_1(x_3x_4-x_2x_7)+x_2(x_4x_7+x_3(-x_4+x_7)))+u_4(x_5(-(x_1-x_2)(x_4-x_5)x_6-u_5(x_1-x_5)(x_2-x_7))+$$
$$u_6(x_4x_5x_7+x_1(x_2x_4-x_5x_7)-x_2(-x_5x_7+x_4(x_5+x_7))))+u_6((x_2(-x_3+x_4)x_5+x_1(x_4(x_3-x_5)+x_2(-x_4+x_5)))x_6+$$
$$u_5(x_3x_5x_7+x_1(-x_3x_5+x_2x_7)+x_2(x_3(x_5-x_7)-x_5x_7))+u_2(x_4x_5x_7+x_2(x_3x_4-x_5x_7)-x_3(-x_5x_7+x_4(x_5+x_7))))\big)\big)\}$$

$$\{\text{WRP\_STEP:5}, -(u_2-u_3)(x_2-x_3)\big(x_1\big(-u_4(u_5-u_6)u_6(x_1-x_2)x_2+u_3^2(u_5x_5(-x_1+x_5)+u_6(x_3(x_4-x_5)+(x_1-x_4)x_5))+$$
$$u_3(u_5(u_4(x_1-x_5)x_5+u_6(x_1x_2+x_3x_5-x_2(x_3+x_5)))+u_6(u_6(-x_1x_2-x_3x_4+x_2(x_3+x_4))+u_4((-x_1+x_4)x_5+x_2(-x_4+x_5))))\big)x_6+$$
$$u_2^2(u_4x_2(x_2-x_5)(-u_6x_4+u_5x_5)+u_6x_2(x_2-x_3)(x_4-x_5)x_6+u_3(u_6x_3x_4(x_2-x_5)+x_5(u_5x_3(-x_2+x_5)-(x_2-x_3)(x_4-x_5)x_6)))+$$
$$u_2\big(u_3^2(u_6x_3x_4(-x_1+x_5)+x_5(u_5x_3(x_1-x_5)+(x_1-x_3)(x_4-x_5)x_6))+$$
$$u_3\big(u_6^2(x_1-x_2)x_3x_4+u_5(x_1(x_2-x_3)+x_2(x_3-x_5))x_5x_6+u_6(u_5(-x_1+x_2)x_3x_5+(x_2(-x_3+x_4)x_5+x_1(-x_2x_4+x_3x_5))x_6)+$$
$$u_4(u_6x_2x_4(x_1-x_5)+x_5(u_5x_2(-x_1+x_5)-(x_1-x_2)(x_4-x_5)x_6))\big)+x_2\big(u_6(x_2-x_3)(u_6(x_1-x_4)+u_5(-x_1+x_5))x_6+$$
$$u_4\big(u_6^2(-x_1+x_2)x_4+u_5x_5(-x_2+x_5)x_6+u_6(u_5(x_1-x_2)x_5+(x_1(x_4-x_5)+(x_2-x_4)x_5)x_6)\big)\big)\big)\big)\}$$

$$\{\text{WRP\_STEP:6},$$
$$-u_2(u_2-u_3)(x_2-x_3)\big(x_2\big(u_2u_5u_6(x_2-x_3)x_3(u_6(-x_1+x_4)+u_5(x_1-x_5)+u_2(-x_4+x_5))+u_4\big(u_2u_6(x_2-x_4)x_4(u_6(x_1-x_3)+u_2(x_3-x_5))-$$
$$u_5(x_3-x_4)\big(u_6^2x_1(x_1-x_2)+u_2^2(x_2-x_5)x_5\big)+u_5^2(u_6x_1(x_1-x_2)(x_3-x_5)-u_2(x_1-x_3)(x_2-x_5)x_5)\big)+$$
$$u_4^2(-u_2u_6(x_2-x_4)x_4(x_1-x_5)+u_5(-u_6x_1(x_1-x_2)(x_4-x_5)+u_2(x_1-x_4)(x_2-x_5)x_5))\big)+$$
$$u_3^2(u_4(-u_6x_1(x_1-x_4)x_4(x_3-x_5)+(u_5x_1(x_3-x_4)(x_1-x_5)+u_2(x_1-x_3)x_4(x_4-x_5))x_5)+$$
$$x_3(u_2u_6(x_3-x_4)x_4(x_1-x_5)+u_5(u_6x_1(x_1-x_3)(x_4-x_5)-u_2(x_1-x_4)(x_3-x_5)x_5)))+$$
$$u_3\big(u_5^2(-u_6x_1(x_1-x_3)x_3(x_2-x_5)+(u_4x_1(x_2-x_3)(x_1-x_5)+u_2(x_1-x_2)x_3(x_3-x_5))x_5)+$$
$$x_4\big(-u_2u_6x_3(x_3-x_4)(u_6(x_1-x_2)+u_2(x_2-x_5))-u_4(x_2-x_3)\big(u_6^2x_1(x_1-x_4)+u_2^2(x_4-x_5)x_5\big)+$$
$$u_4^2(u_6x_1(x_1-x_4)(x_2-x_5)-u_2(x_1-x_2)(x_4-x_5)x_5)\big)+u_5(x_2-x_4)\big(u_6^2x_1(x_1-x_3)x_3+x_5\big(u_2^2x_3(x_3-x_5)+u_4^2x_1(-x_1+x_5)\big)\big)\big)\big)\}$$

$$\{\text{WRP\_STEP:7},$$
$$u_2(u_2-u_3)(x_2-x_3)\big(x_1\big(u_4u_5u_6(x_1-x_2)x_2(u_6(x_3-x_4)+u_4(x_4-x_5)+u_5(-x_3+x_5))+u_3\big(u_5u_6(x_1-x_3)x_3(u_6(-x_2+x_4)+u_5(x_2-x_5))-$$
$$u_4(x_2-x_3)\big(u_6^2x_4(-x_1+x_4)+u_5^2\big(u_6^2+(-2u_1+x_1)x_5\big)\big)+u_4^2\big(-u_6(x_1-x_4)x_4(x_2-x_5)+u_5(x_2-x_4)\big(u_6^2+(-2u_1+x_1)x_5\big)\big)\big)+$$
$$u_3^2\big(-u_5u_6(x_1-x_3)x_3(x_4-x_5)+u_4\big(u_6(x_1-x_4)x_4(x_3-x_5)-u_5(x_3-x_4)\big(u_6^2+(-2u_1+x_1)x_5\big)\big)\big)\big)+$$
$$u_2\big(x_3\big(-u_5u_6x_2(x_2-x_3)(u_6(-x_1+x_4)+u_5(x_1-x_5))-u_3(x_1-x_2)\big(u_6^2x_4(-x_3+x_4)+u_5^2\big(u_6^2+(-2u_1+x_3)x_5\big)\big)+$$
$$u_3^2\big(-u_6(x_3-x_4)x_4(x_1-x_5)+u_5(x_1-x_4)\big(u_6^2+(-2u_1+x_3)x_5\big)\big)\big)+$$
$$u_4(x_1-x_3)\big(u_5^2x_2\big(u_6^2+(-2u_1+x_2)x_5\big)-x_4\big(u_6^2x_2(x_2-x_4)+u_3^2\big(u_6^2+(-2u_1+x_4)x_5\big)\big)\big)+$$
$$u_4^2\big(-u_5x_2(x_1-x_4)\big(u_6^2+(-2u_1+x_2)x_5\big)+x_4\big(u_6x_2(x_2-x_4)(x_1-x_5)+u_3(x_1-x_2)\big(u_6^2+(-2u_1+x_4)x_5\big)\big)\big)\big)+$$
$$u_2^2\big(x_2\big(u_5u_6(x_2-x_3)x_3(x_4-x_5)+u_4\big(-u_6(x_2-x_4)x_4(x_3-x_5)+u_5(x_3-x_4)\big(u_6^2+(-2u_1+x_2)x_5\big)\big)\big)+$$
$$u_3\big(-u_5x_3(x_2-x_4)\big(u_6^2+(-2u_1+x_3)x_5\big)+x_4\big(u_6x_3(x_3-x_4)(x_2-x_5)+u_4(x_2-x_3)\big(u_6^2+(-2u_1+x_4)x_5\big)\big)\big)\big)\big)\}$$

$$\{\text{WRP\_STEP:8},$$
$$-u_2(u_2-u_3)(x_2-x_3)\big(x_1\big(u_4u_5u_6(x_1-x_2)x_2(u_6(-x_3+x_4)+u_5(x_3-x_5)+u_4(-x_4+x_5))+u_3\big(-u_5u_6(x_1-x_3)x_3(u_6(-x_2+x_4)+u_5(x_2-x_5))+$$
$$u_4(2u_1-x_1)(x_2-x_3)\big(u_6^2x_4-u_5^2x_5\big)+u_4^2\big(u_5^2u_6(x_2-x_5)-u_6(2u_1-x_1)x_4(x_2-x_5)-u_5(x_2-x_4)\big(u_6^2+(-2u_1+x_1)x_5\big)\big)+$$
$$u_3^2\big(u_5u_6(x_1-x_3)x_3(x_4-x_5)+u_4\big(u_6(2u_1-x_1)x_4(x_3-x_5)+u_5^2u_6(-x_3+x_5)+u_5(x_3-x_4)\big(u_6^2+(-2u_1+x_1)x_5\big)\big)\big)\big)+$$
$$u_2^2\big(u_3\big(u_6x_4(u_4u_6(-x_2+x_3)+(2u_1-x_3)x_3(x_2-x_5))+u_5x_3(x_2-x_4)\big(u_6^2+(-2u_1+x_3)x_5\big)+u_5^2(u_4(x_2-x_3)x_5+u_6x_3(-x_2+x_5))\big)+$$
$$x_2\big(-u_5u_6(x_2-x_3)x_3(x_4-x_5)+u_4\big(u_5^2u_6(x_3-x_5)-u_6(2u_1-x_2)x_4(x_3-x_5)-u_5(x_3-x_4)\big(u_6^2+(-2u_1+x_2)x_5\big)\big)\big)+$$
$$u_2\big(u_4\big(u_3^2+x_2(-2u_1+x_2)\big)(x_1-x_3)\big(u_6^2x_4-u_5^2x_5\big)+u_4^2\big(u_6x_4(u_3u_6(-x_1+x_2)+(2u_1-x_2)x_2(x_1-x_5))+$$
$$u_5x_2(x_1-x_4)\big(u_6^2+(-2u_1+x_2)x_5\big)+u_5^2(u_3(x_1-x_2)x_5+u_6x_2(-x_1+x_5))\big)+x_3\big(u_5u_6x_2(x_2-x_3)(u_6(-x_1+x_4)+u_5(x_1-x_5))+$$
$$u_3(x_1-x_2)(2u_1-x_3)\big(u_6^2x_4-u_5^2x_5\big)+u_3^2\big(u_5^2u_6(x_1-x_5)-u_6(2u_1-x_3)x_4(x_1-x_5)-u_5(x_1-x_4)\big(u_6^2+(-2u_1+x_3)x_5\big)\big)\big)\big)\big)\}$$

$$\{\text{WRP\_STEP:9}, u_2(u_2-u_3)$$
$$\big(u_2\big(u_3^2+x_2(-2u_1+x_2)\big)\big(u_5u_6(2u_1-x_2)x_3(u_6(-x_1+x_4)+u_5(x_1-x_5))+u_4^3\big(u_6^2x_4-u_5^2x_5\big)-u_4(x_1(x_2-x_3)+(2u_1-x_2)x_3)\big(u_6^2x_4-u_5^2x_5\big)+$$
$$u_4^2\big(u_6(x_2-x_3)x_4(x_1-x_5)+u_5^2u_6(-x_1+x_5)+u_5(x_1-x_4)\big(u_6^2+(-x_2+x_3)x_5\big)\big)\big)+$$
$$x_1\big(u_4u_5u_6(x_1-x_2)x_2\big(u_4^2(-u_5+u_6)+2u_1(u_5-u_6)x_3-u_5x_2x_3+u_6x_2x_3-u_6x_2x_4+u_6x_3x_4+u_4(x_2-x_3)(x_4-x_5)+u_5x_2x_5-u_5x_3x_5\big)+$$
$$u_3(2u_1-x_1)\big(u_5u_6(2u_1-x_2)x_3(u_6(-x_2+x_4)+u_5(x_2-x_5))+u_4^3\big(u_6^2x_4-u_5^2x_5\big)-u_4\big(x_2^2+2u_1x_3-2x_2x_3\big)\big(u_6^2x_4-u_5^2x_5\big)+$$
$$u_4^2\big(u_6(x_2-x_3)x_4(x_2-x_5)+u_5^2u_6(-x_2+x_5)+u_5(x_2-x_4)\big(u_6^2+(-x_2+x_3)x_5\big)\big)\big)+u_3^2$$
$$\big(-u_5u_6(2u_1-x_1)(2u_1-x_2)x_3(x_4-x_5)+u_4^2u_5u_6(2u_1-x_1-x_2+x_3)(x_4-x_5)+u_4^3\big(u_5^2u_6+u_6(-2u_1+x_1)x_4-u_5\big(u_6^2+(-2u_1+x_1)x_5\big)\big)+$$
$$u_4\big(4u_1^2x_3(u_6x_4-u_5x_5)-u_5(x_2(x_3-x_4)+x_3x_4)\big(u_6^2+x_1x_5\big)+u_5^2u_6(x_2(x_3-x_5)+x_3x_5)+u_6x_1x_4(x_2(x_3-x_5)+x_3x_5)-$$
$$2u_1\big(u_5^2u_6x_3+u_6x_4(x_1x_3+x_2(x_3-x_5)+x_3x_5)-u_5\big(u_6^2x_3+(x_1x_3+x_2(x_3-x_4)+x_3x_4)x_5\big)\big)\big)\big)+$$
$$u_2^2\big(u_3\big(-u_5u_6(2u_1-x_2)x_3(u_6(-x_2+x_4)+u_5(x_2-x_5))+u_4\big(x_2^2+2u_1x_3-2x_2x_3\big)\big(u_6^2x_4-u_5^2x_5\big)+u_4^3\big(-u_6^2x_4+u_5^2x_5\big)+$$
$$u_4^2\big(u_5^2u_6(x_2-x_5)-u_6(x_2-x_3)x_4(x_2-x_5)-u_5(x_2-x_4)\big(u_6^2+(-x_2+x_3)x_5\big)\big)\big)+$$
$$x_2\big(u_5u_6(-2u_1+x_2)^2x_3(x_4-x_5)-u_4^2u_5u_6(2u_1-2x_2+x_3)(x_4-x_5)+u_4^3\big(-u_5^2u_6+u_6(2u_1-x_2)x_4+u_5\big(u_6^2+(-2u_1+x_2)x_5\big)\big)+$$
$$u_4\big(4u_1^2x_3(-u_6x_4+u_5x_5)+u_5(x_2(x_3-x_4)+x_3x_4)\big(u_6^2+x_2x_5\big)-u_5^2u_6(x_2(x_3-x_5)+x_3x_5)-$$
$$u_6x_2x_4(x_2(x_3-x_5)+x_3x_5)+2u_1\big(u_5^2u_6x_3+u_6x_4(x_2(2x_3-x_5)+x_3x_5)-u_5\big(u_6^2x_3+(x_2(2x_3-x_4)+x_3x_4)x_5\big)\big)\big)\big)\big)\}$$

$$\{\text{WRP\_STEP:10}, -u_2(u_2-u_3)\big(u_2^2+x_1(-2u_1+x_1)\big)$$
$$\big(u_3^3\big(u_4u_6(-u_4+u_6)x_4+u_5^2(u_6x_3-u_4x_5)+u_5\big(-u_6^2x_3+u_4^2x_5\big)\big)+u_4u_5u_6x_2\big(u_4^2(u_5-u_6)-u_4(2u_1-x_3)(x_4-x_5)+(2u_1-x_3)(u_6x_4-u_5x_5)\big)+$$
$$u_3^2\big(2u_4^2u_5u_6(x_4-x_5)-u_5u_6(2u_1-x_2)x_3(x_4-x_5)+u_4^3(-u_6x_4+u_5x_5)+$$
$$u_4\big(u_5^2u_6(-x_3+x_5)+u_5\big(u_6^2(x_3-x_4)+(-2u_1x_3+x_2(x_3-x_4)+x_3x_4)x_5\big)+u_6x_4(2u_1x_3-x_3x_5+x_2(-x_3+x_5))\big)\big)+$$
$$u_3\big(u_5u_6(2u_1-x_2)x_3(u_6x_4-u_5x_5)+u_4^3\big(u_6^2x_4-u_5^2x_5\big)-2u_4(-x_2x_3+u_1(x_2+x_3))\big(u_6^2x_4-u_5^2x_5\big)+$$
$$u_4^2\big(u_5^2u_6(-x_2+x_5)+u_5\big(u_6^2(x_2-x_4)+(-2u_1x_2-x_3x_4+x_2(x_3+x_4))x_5\big)+u_6x_4(2u_1x_2+x_3x_5-x_2(x_3+x_5))\big)\big)\big)\}$$

$$\{\text{WRP\_STEP:11}, 0\}$$

## WRSP_SubStep_1_II: The 1(th) Theorem is True

# WRSP_Step_III: Now Checking the Initials & the Algebraic Configuration

{**The Initials:** , $\{-1, -u_2, u_2 - u_3, -x_2 + x_3, u_4\, x_1 - u_5\, x_1 + u_2\, (-x_3 + x_4),$
$u_5\, (x_1 - x_2) + u_6\, (-x_1 + x_2) - (u_2 - u_3)\, (x_4 - x_5), u_6\, (-x_2 + x_3) + (u_3 - u_4)\, x_5\}\}$

{**The Algebraic Configuration:** ,

$$\left\{\left\{x_1 \to u_1 - \sqrt{u_1^2 - u_2^2}\,,\ x_2 \to u_1 - \sqrt{u_1^2 - u_3^2}\,,\ x_3 \to u_1 - \sqrt{u_1^2 - u_4^2}\,,\ x_4 \to u_1 - \sqrt{u_1^2 - u_5^2}\,,\ x_5 \to u_1 - \sqrt{u_1^2 - u_6^2}\,,\right.\right.$$

$$x_6 \to \frac{u_2 \left(-\sqrt{u_1^2 - u_4^2}\ u_5 + u_1\, (-u_4 + u_5) + u_4 \sqrt{u_1^2 - u_5^2}\right)}{\sqrt{u_1^2 - u_2^2}\ u_4 - u_2 \sqrt{u_1^2 - u_4^2} - \sqrt{u_1^2 - u_2^2}\ u_5 + u_1\, (-u_4 + u_5) + u_2 \sqrt{u_1^2 - u_5^2}},$$

$$x_7 \to \frac{\left(u_1 - \sqrt{u_1^2 - u_2^2}\right)\left(-\sqrt{u_1^2 - u_4^2}\ u_5 + u_1\, (-u_4 + u_5) + u_4 \sqrt{u_1^2 - u_5^2}\right)}{\sqrt{u_1^2 - u_2^2}\ u_4 - u_2 \sqrt{u_1^2 - u_4^2} - \sqrt{u_1^2 - u_2^2}\ u_5 + u_1\, (-u_4 + u_5) + u_2 \sqrt{u_1^2 - u_5^2}},$$

$$x_8 \to \left(u_2 \left(\left(-\sqrt{u_1^2 - u_3^2} + \sqrt{u_1^2 - u_5^2}\right) u_6 + u_5 \left(\sqrt{u_1^2 - u_3^2} - \sqrt{u_1^2 - u_6^2}\right)\right) + \right.$$
$$u_3 \left(\left(\sqrt{u_1^2 - u_2^2} - \sqrt{u_1^2 - u_5^2}\right) u_6 + u_5 \left(-\sqrt{u_1^2 - u_2^2} + \sqrt{u_1^2 - u_6^2}\right)\right)\Big) \Big/ \left(\left(-\sqrt{u_1^2 - u_2^2} + \sqrt{u_1^2 - u_3^2}\right) u_5 - \right.$$
$$\left. u_3 \sqrt{u_1^2 - u_5^2} + \sqrt{u_1^2 - u_2^2}\ u_6 - \sqrt{u_1^2 - u_3^2}\ u_6 + u_3 \sqrt{u_1^2 - u_6^2} + u_2 \left(\sqrt{u_1^2 - u_5^2} - \sqrt{u_1^2 - u_6^2}\right)\right),$$

$$x_9 \to \left(-u_2 \sqrt{u_1^2 - u_3^2}\ \sqrt{u_1^2 - u_5^2} - \sqrt{u_1^2 - u_2^2}\ \sqrt{u_1^2 - u_5^2}\ u_6 + \sqrt{u_1^2 - u_3^2}\ \sqrt{u_1^2 - u_5^2}\ u_6 + u_2 \sqrt{u_1^2 - u_3^2}\ \sqrt{u_1^2 - u_6^2} + \right.$$
$$\sqrt{u_1^2 - u_2^2}\ u_5 \sqrt{u_1^2 - u_6^2} - \sqrt{u_1^2 - u_3^2}\ u_5 \sqrt{u_1^2 - u_6^2} + \sqrt{u_1^2 - u_2^2}\ u_3 \left(\sqrt{u_1^2 - u_5^2} - \sqrt{u_1^2 - u_6^2}\right) +$$
$$u_1 \left(\left(-\sqrt{u_1^2 - u_2^2} + \sqrt{u_1^2 - u_3^2}\right) u_5 - u_3 \sqrt{u_1^2 - u_5^2} + \sqrt{u_1^2 - u_2^2}\ u_6 - \sqrt{u_1^2 - u_3^2}\ u_6 + \right.$$
$$\left. u_3 \sqrt{u_1^2 - u_6^2} + u_2 \left(\sqrt{u_1^2 - u_5^2} - \sqrt{u_1^2 - u_6^2}\right)\right)\Big) \Big/ \left(\left(-\sqrt{u_1^2 - u_2^2} + \sqrt{u_1^2 - u_3^2}\right) u_5 - \right.$$
$$\left. u_3 \sqrt{u_1^2 - u_5^2} + \sqrt{u_1^2 - u_2^2}\ u_6 - \sqrt{u_1^2 - u_3^2}\ u_6 + u_3 \sqrt{u_1^2 - u_6^2} + u_2 \left(\sqrt{u_1^2 - u_5^2} - \sqrt{u_1^2 - u_6^2}\right)\right),$$

$$x_{10} \to \frac{\left(-\sqrt{u_1^2 - u_3^2}\ u_4 + u_1\, (-u_3 + u_4) + u_3 \sqrt{u_1^2 - u_4^2}\right)\left(u_1 - \sqrt{u_1^2 - u_6^2}\right)}{u_1\, (-u_3 + u_4) + \left(-\sqrt{u_1^2 - u_3^2} + \sqrt{u_1^2 - u_4^2}\right) u_6 + (u_3 - u_4) \sqrt{u_1^2 - u_6^2}},$$

$$x_{11} \to \left.\frac{\left(-\sqrt{u_1^2 - u_3^2}\ u_4 + u_1\, (-u_3 + u_4) + u_3 \sqrt{u_1^2 - u_4^2}\right) u_6}{u_1\, (-u_3 + u_4) + \left(-\sqrt{u_1^2 - u_3^2} + \sqrt{u_1^2 - u_4^2}\right) u_6 + (u_3 - u_4) \sqrt{u_1^2 - u_6^2}}\right\},$$

$$\left\{x_1 \to u_1 + \sqrt{u_1^2 - u_2^2}\,,\ x_2 \to u_1 + \sqrt{u_1^2 - u_3^2}\,,\ x_3 \to u_1 + \sqrt{u_1^2 - u_4^2}\,,\right.$$
$$x_4 \to u_1 + \sqrt{u_1^2 - u_5^2}\,,\ x_5 \to u_1 + \sqrt{u_1^2 - u_6^2}\,,$$

$$x_6 \to \frac{u_2 \left(\left(u_1 + \sqrt{u_1^2 - u_4^2}\right) u_5 - u_4 \left(u_1 + \sqrt{u_1^2 - u_5^2}\right)\right)}{-\sqrt{u_1^2 - u_2^2}\ u_4 + u_2 \sqrt{u_1^2 - u_4^2} + \sqrt{u_1^2 - u_2^2}\ u_5 + u_1\, (-u_4 + u_5) - u_2 \sqrt{u_1^2 - u_5^2}},$$

$$x_7 \to \frac{\left(u_1 + \sqrt{u_1^2 - u_2^2}\right)\left(\left(u_1 + \sqrt{u_1^2 - u_4^2}\right) u_5 - u_4 \left(u_1 + \sqrt{u_1^2 - u_5^2}\right)\right)}{-\sqrt{u_1^2 - u_2^2}\ u_4 + u_2 \sqrt{u_1^2 - u_4^2} + \sqrt{u_1^2 - u_2^2}\ u_5 + u_1\, (-u_4 + u_5) - u_2 \sqrt{u_1^2 - u_5^2}},$$

$$x_8 \to \left(u_2 \left(\left(-\sqrt{u_1^2 - u_3^2} + \sqrt{u_1^2 - u_5^2}\right) u_6 + u_5 \left(\sqrt{u_1^2 - u_3^2} - \sqrt{u_1^2 - u_6^2}\right)\right) + \right.$$
$$u_3 \left(\left(\sqrt{u_1^2 - u_2^2} - \sqrt{u_1^2 - u_5^2}\right) u_6 + u_5 \left(-\sqrt{u_1^2 - u_2^2} + \sqrt{u_1^2 - u_6^2}\right)\right)\Big) \Big/ \left(\left(-\sqrt{u_1^2 - u_2^2} + \sqrt{u_1^2 - u_3^2}\right) u_5 - \right.$$
$$\left. u_3 \sqrt{u_1^2 - u_5^2} + \sqrt{u_1^2 - u_2^2}\ u_6 - \sqrt{u_1^2 - u_3^2}\ u_6 + u_3 \sqrt{u_1^2 - u_6^2} + u_2 \left(\sqrt{u_1^2 - u_5^2} - \sqrt{u_1^2 - u_6^2}\right)\right),$$

$$x_9 \to \left(u_2 \sqrt{u_1^2 - u_3^2}\ \sqrt{u_1^2 - u_5^2} + \sqrt{u_1^2 - u_2^2}\ \sqrt{u_1^2 - u_5^2}\ u_6 - \sqrt{u_1^2 - u_3^2}\ \sqrt{u_1^2 - u_5^2}\ u_6 - u_2 \sqrt{u_1^2 - u_3^2}\ \sqrt{u_1^2 - u_6^2} - \sqrt{u_1^2 - u_2^2}\ u_5 \right.$$
$$\sqrt{u_1^2 - u_6^2} + \sqrt{u_1^2 - u_3^2}\ u_5 \sqrt{u_1^2 - u_6^2} + \sqrt{u_1^2 - u_2^2}\ u_3 \left(-\sqrt{u_1^2 - u_5^2} + \sqrt{u_1^2 - u_6^2}\right) + u_1 \left(\left(-\sqrt{u_1^2 - u_2^2} + \sqrt{u_1^2 - u_3^2}\right) u_5 - \right.$$

$$u_3\sqrt{u_1^2-u_5^2}+\sqrt{u_1^2-u_2^2}\,u_6-\sqrt{u_1^2-u_3^2}\,u_6+u_3\sqrt{u_1^2-u_6^2}+u_2\left(\sqrt{u_1^2-u_5^2}-\sqrt{u_1^2-u_6^2}\right)\Big)\Big)\Big/$$

$$\left(\left(-\sqrt{u_1^2-u_2^2}+\sqrt{u_1^2-u_3^2}\right)u_5-u_3\sqrt{u_1^2-u_5^2}+\sqrt{u_1^2-u_2^2}\,u_6-\sqrt{u_1^2-u_3^2}\,u_6+u_3\sqrt{u_1^2-u_6^2}+\right.$$

$$\left.u_2\left(\sqrt{u_1^2-u_5^2}-\sqrt{u_1^2-u_6^2}\right)\right),\ x_{10}\to\frac{\left(u_1(u_3-u_4)-\sqrt{u_1^2-u_3^2}\,u_4+u_3\sqrt{u_1^2-u_4^2}\right)\left(u_1+\sqrt{u_1^2-u_6^2}\right)}{u_1(u_3-u_4)+\left(-\sqrt{u_1^2-u_3^2}+\sqrt{u_1^2-u_4^2}\right)u_6+(u_3-u_4)\sqrt{u_1^2-u_6^2}},$$

$$x_{11}\to\frac{\left(u_1(u_3-u_4)-\sqrt{u_1^2-u_3^2}\,u_4+u_3\sqrt{u_1^2-u_4^2}\right)u_6}{u_1(u_3-u_4)+\left(-\sqrt{u_1^2-u_3^2}+\sqrt{u_1^2-u_4^2}\right)u_6+(u_3-u_4)\sqrt{u_1^2-u_6^2}}\Big\}\Big\}\Big\}$$

```
{82.719 Second, {{70.552 Second, True}}}
```

# Dursargus Theorem

## The Geometric Figure of the Theorem

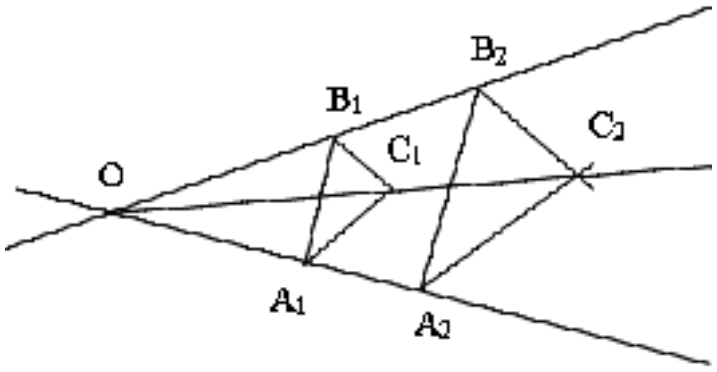


Figure 3

## Setting Coordination for the Geometric Configuration

```
precoord = {OO → {0, 0}, A1 → {u1, 0}, A2 -> {u2, 0}, B1 -> {0, u3}, C1 -> {u4, u5}, B2 → {0, x1}, C2 → {x2, x3}};
```

## Fixing the Concrete Geometric Configuration

### ■ The Geometric Proposition

```
prethmcfg = {TwoLinesParallel[{A1, B1}, {A2, B2}], TwoLinesParallel[{B1, C1}, {B2, C2}],
   TwoLinesParallel[{A1, C1}, {A2, C2}]};
```

### ■ The Geometric Conclusion(s)

```
prethmcnd = {TriplePointsCollinear[OO, C1, C2]};
ord = {x1, x2, x3};
const = {u1, u2, u3, u4, u5};
```

## Using WuRittSmartProver for the Geometric Configuration

```
Timing@WuRittSmartProver[precoord, {prethmcfg, prethmcnd}, ord, const, TraceCharacteristicSetOn → True,
  TraceProverOn → True]
```

### WRSP_Step_I: Now Solving the Characteristic Set

{CS_STEP:1, $\{u_2\,u_3 - u_1\,x_1,\ u_5\,(u_2 - x_2) - u_1\,x_3 + u_4\,x_3\}$}

{CS_STEP:2, $\{u_2\,u_3 - u_1\,x_1,\ (-u_3\,u_4 + u_1\,(u_3 - u_5))\,(-u_2\,u_4 + u_1\,x_2),\ u_5\,(u_2 - x_2) - u_1\,x_3 + u_4\,x_3\}$}

{A New Component:1, $-u_2\,u_4 + u_1\,x_2$}

{Total 1 Branch(s) of New Component(s) Discovered}

$$\begin{pmatrix} u_2\,u_3 - u_1\,x_1 & \{x_1, 00, 00\} \\ -u_1\,u_2\,u_3\,u_4 + u_2\,u_3\,u_4^2 + u_1\,u_2\,u_4\,u_5 + u_1^2\,u_3\,x_2 - u_1\,u_3\,u_4\,x_2 - u_1^2\,u_5\,x_2 & \{00, x_2, 00\} \\ -u_5\,(-u_2 + x_2) + (-u_1 + u_4)\,x_3 & \{00, x_2, x_3\} \end{pmatrix}$$

### WRSP_Step_II: Now Proving the Theorem(s)

#### WRSP_SubStep_1_I: Now Proving the 1(th) Theorem

{WRP_STEP:1, $u_5\,(u_2\,u_4 - u_1\,x_2)$}

{WRP_STEP:2, 0}

{WRP_STEP:3, 0}

#### WRSP_SubStep_1_II: The 1(th) Theorem is True

### WRSP_Step_III: Now Checking the Initials & the Algebraic Configuration

{**The Initials:** , $\{-u_1, -u_1 + u_4, u_1\,(-u_3\,u_4 + u_1\,(u_3 - u_5))\}$}

{**The Algebraic Configuration:** , $\left\{\left\{x_1 \to \frac{u_2\,u_3}{u_1}, x_2 \to \frac{u_2\,u_4}{u_1}, x_3 \to \frac{u_2\,u_5}{u_1}\right\}\right\}$}

```
{0.62 Second, {{0.05 Second, True}}}
```

## Simson Theorem

**The Geometric Figure of the Theorem**

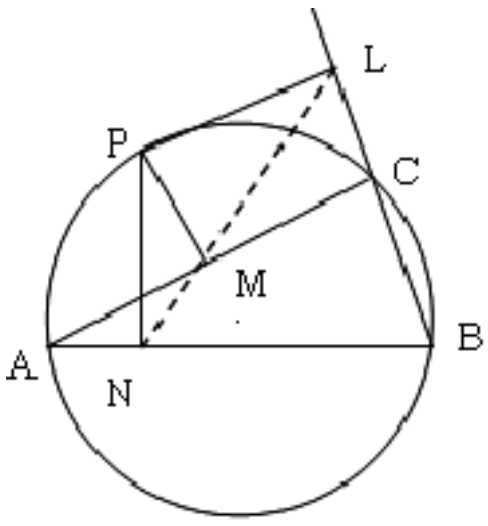


Figure 4

**Setting Coordination for the Geometric Configuration**

```
precoord = {A -> {0, 0}, B → {u1, 0}, C1 -> {u2, u3}, L -> {u4, 0}, O1 -> {x1, x2}, N1 -> {x6, x7},
    P -> {u4, x3}, M1 -> {x4, x5}};
(*precoord={A->{x1,y1},B→{x2,y2},C1->{x3,y3},L->{x4,y4},O1->{x5,y5},N1->{x6,y6},P->{x7,y7},M1->{x8,y8}};*)
```

**Fixing the Concrete Geometric Configuration**

■ **The Geometric Proposition**

```
prethmcfg = {TwoLinesEqual[{O1, B}, {O1, A}], TwoLinesEqual[{O1, C1}, {O1, A}],
    TwoLinesEqual[{O1, P}, {O1, A}], TwoLinesPerpend[{P, M1}, {B, C1}],
    TwoLinesPerpend[{P, N1}, {A, C1}], TriplePointsCollinear[B, C1, M1], TriplePointsCollinear[A, N1, C1]};
```

## ■ The Geometric Conclusion(s)

```
prethmcnd = {TriplePointsCollinear[L, M1, N1]};
```

```
ord = {x1, x2, x3, x4, x5, x6, x7};
const = {u1, u2, u3, u4};
```

### Using WuRittSmartProver for the Geometric Configuration

```
Timing@WuRittSmartProver[precoord, {prethmcfg, prethmcnd}, ord, const, TraceCharacteristicSetOn → True,
  TraceProverOn → True]
```

# WRSP_Step_I: Now Solving the Characteristic Set

$$\{\text{CS\_STEP:1}, \{u_1 (u_1 - 2 x_1),\ u_2^2 - 2 u_2 x_1 + u_3 (u_3 - 2 x_2),\ u_4^2 - 2 u_4 x_1 + x_3 (-2 x_2 + x_3),\ -u_3 x_4 + u_1 (u_3 - x_5) + u_2 x_5,\ u_3 x_6 - u_2 x_7\}\}$$

$$\{\text{A New Component:1}, u_1 - 2 x_1\}$$

$$\{\text{CS\_STEP:2}, \{u_1 (u_1 - 2 x_1),\ u_2^2 - 2 u_2 x_1 + u_3 (u_3 - 2 x_2),\ u_4^2 - 2 u_4 x_1 + x_3 (-2 x_2 + x_3),\ u_2 u_3 x_3 + u_1^2 (u_4 - x_4) + u_2^2 (u_4 - x_4) - u_3^2 x_4 + u_1 (u_3 (u_3 - x_3) + 2 u_2 (-u_4 + x_4)),\ -u_3 x_4 + u_1 (u_3 - x_5) + u_2 x_5,\ -u_2 u_3 x_3 + u_3^2 x_6 + u_2^2 (-u_4 + x_6),\ u_3 x_6 - u_2 x_7\}\}$$

$$\{\text{A New Component:1}, u_1 - 2 x_1\}$$

$$\{\text{Total 2 Branch(s) of New Component(s) Discovered}\}$$

$$\begin{pmatrix} (u_1 - x_1)^2 - x_1^2 & \{x_1, 00, 00, 00, 00, 00, 00\} \\ (u_2 - x_1)^2 - x_1^2 + (u_3 - x_2)^2 - x_2^2 & \{x_1, x_2, 00, 00, 00, 00, 00\} \\ (u_4 - x_1)^2 - x_1^2 - x_2^2 + (-x_2 + x_3)^2 & \{x_1, x_2, x_3, 00, 00, 00, 00\} \\ u_1 u_3^2 + u_1^2 u_4 - 2 u_1 u_2 u_4 + u_2^2 u_4 - u_1 u_3 x_3 + u_2 u_3 x_3 - u_1^2 x_4 + 2 u_1 u_2 x_4 - u_2^2 x_4 - u_3^2 x_4 & \{00, 00, x_3, x_4, 00, 00, 00\} \\ -u_3 (-u_1 + x_4) + (-u_1 + u_2) x_5 & \{00, 00, 00, x_4, x_5, 00, 00\} \\ -u_2^2 u_4 - u_2 u_3 x_3 + u_2^2 x_6 + u_3^2 x_6 & \{00, 00, x_3, 00, 00, x_6, 00\} \\ u_3 x_6 - u_2 x_7 & \{00, 00, 00, 00, 00, x_6, x_7\} \end{pmatrix}$$

# WRSP_Step_II: Now Proving the Theorem(s)

## WRSP_SubStep_1_I: Now Proving the 1(th) Theorem

$$\{\text{WRP\_STEP:1}, u_2 x_5 (u_4 - x_6) + u_3 (-u_4 + x_4) x_6\}$$

$$\{\text{WRP\_STEP:2}, u_2 u_3 (-u_2 (u_4^2 - u_4 x_4 + x_3 x_5) + u_3 (x_3 x_4 + u_4 (-x_3 + x_5)))\}$$

$$\{\text{WRP\_STEP:3}, u_2 u_3 (u_1 (-u_3^2 u_4 + u_2 u_4 (u_4 - x_4) + u_3 x_3 (u_2 + u_4 - x_4)) + u_4 (-u_2 u_3 x_3 + u_3^2 x_4 + u_2^2 (-u_4 + x_4)))\}$$

$$\{\text{WRP\_STEP:4}, u_1 u_2 u_3^2 (u_1^2 (-u_3 u_4 + u_2 x_3) + u_2 (u_2^2 x_3 + u_3^2 x_3 - u_3 (u_4^2 + x_3^2)) + u_1 (u_2 u_3 u_4 - 2 u_2^2 x_3 + u_3 (u_4^2 - u_3 x_3 + x_3^2)))\}$$

$$\{\text{WRP\_STEP:5}, u_1 (u_1 - u_2) u_2 u_3^2 (-u_2^2 x_3 - u_3^2 x_3 + u_1 (-u_3 u_4 + u_2 x_3) + 2 u_3 (u_4 x_1 + x_2 x_3))\}$$

$$\{\text{WRP\_STEP:6}, u_1 (u_1 - u_2) u_2 u_3^2 (u_1 - 2 x_1) (-u_3 u_4 + u_2 x_3)\}$$

$$\{\text{WRP\_STEP:7}, 0\}$$

## WRSP_SubStep_1_II: The 1(th) Theorem is True

# WRSP_Step_III: Now Checking the Initials & the Algebraic Configuration

$$\{\textbf{The Initials:}, \{1, -2 u_1, -u_2, -u_1 + u_2, -2 u_3, -u_1^2 + 2 u_1 u_2 - u_2^2 - u_3^2, u_2^2 + u_3^2\}\}$$

The Algebraic Configuration: ,

$$\left\{\left\{x_1 \to \frac{u_1}{2},\ x_2 \to \frac{-u_1\,u_2+u_2^2+u_3^2}{2\,u_3},\ x_3 \to \frac{-u_1\,u_2+u_2^2+u_3^2}{2\,u_3}-\sqrt{\frac{(-u_1\,u_2+u_2^2+u_3^2)^2}{4\,u_3^2}+u_1\,u_4-u_4^2}\right.\right.,$$

$$x_4 \to \frac{1}{2\,(u_1^2-2\,u_1\,u_2+u_2^2+u_3^2)}\left(u_1^2\,u_2-2\,u_1\,u_2^2+u_2^3+u_1\,u_3^2+u_2\,u_3^2+2\,u_1^2\,u_4-4\,u_1\,u_2\,u_4+2\,u_2^2\,u_4+\right.$$

$$\left.2\,u_1\,u_3\sqrt{\frac{(-u_1\,u_2+u_2^2+u_3^2)^2}{4\,u_3^2}+u_1\,u_4-u_4^2}-2\,u_2\,u_3\sqrt{\frac{(-u_1\,u_2+u_2^2+u_3^2)^2}{4\,u_3^2}+u_1\,u_4-u_4^2}\right),$$

$$x_5 \to \frac{u_3\left(2\,u_1^2-3\,u_1\,u_2+u_2^2+u_3^2-2\,u_1\,u_4+2\,u_2\,u_4-2\,u_3\sqrt{\frac{(-u_1\,u_2+u_2^2+u_3^2)^2}{4\,u_3^2}+u_1\,u_4-u_4^2}\right)}{2\,(u_1^2-2\,u_1\,u_2+u_2^2+u_3^2)},$$

$$x_6 \to \frac{u_2\left(-u_1\,u_2+u_2^2+u_3^2+2\,u_2\,u_4-2\,u_3\sqrt{\frac{(-u_1\,u_2+u_2^2+u_3^2)^2}{4\,u_3^2}+u_1\,u_4-u_4^2}\right)}{2\,(u_2^2+u_3^2)},$$

$$\left.\left.x_7 \to \frac{u_3\left(-u_1\,u_2+u_2^2+u_3^2+2\,u_2\,u_4-2\,u_3\sqrt{\frac{(-u_1\,u_2+u_2^2+u_3^2)^2}{4\,u_3^2}+u_1\,u_4-u_4^2}\right)}{2\,(u_2^2+u_3^2)}\right\}\right\}$$

```
{5.038 Second, {{0.841 Second, True}}}
```

## Rhombus Theorem

### Setting Coordination for the Geometric Configuration

```
precoord = {A1 -> {x1, y1}, B1 -> {x2, y2}, C1 -> {x3, y3}, D1 -> {x4, y4}};
```

### Fixing the Concrete Geometric Configuration

■ **The Geometric Proposition**

```
prethmcfg = {TwoLinesParallel[{C1, D1}, {A1, B1}], TwoLinesParallel[{A1, D1}, {B1, C1}],
    TwoLinesEqual[{C1, D1}, {A1, D1}]};
```

■ **The Geometric Conclusion(s)**

```
prethmcnd = {TwoLinesPerpend[{A1, C1}, {B1, D1}]};
```

```
ord = {x1, x2, x3, x4, y1, y2, y3, y4};
const = {};
```

```
Timing@WuRittSmartProver[precoord, {prethmcfg, prethmcnd}, ord, const, TraceCharacteristicSetOn → True,
  TraceProverOn → True]
```

## WRSP_Step_I: Now Solving the Characteristic Set

{CS_STEP:1, $\{(x_1-x_4)(y_2-y_3)-(x_2-x_3)(y_1-y_4)\}$}

{CS_STEP:2, $\{-(x_1-x_2+x_3-x_4)(x_3(y_1-y_2)+x_1(y_2-y_3)+x_2(-y_1+y_3)),\ (x_1-x_4)(y_2-y_3)-(x_2-x_3)(y_1-y_4)\}$}

{A New Component:1, $-x_2 y_1+x_3 y_1+x_1 y_2-x_3 y_2-x_1 y_3+x_2 y_3$}

$\{$CS_STEP:3, $\{(x_1-x_3)(x_2-x_3)(x_1+x_3-2x_4)(x_1-x_2+x_3-x_4)(x_1^2-2x_1x_2+x_2^2+(y_1-y_2)^2),$
$-(x_1-x_2+x_3-x_4)(x_3(y_1-y_2)+x_1(y_2-y_3)+x_2(-y_1+y_3)),\ (x_1-x_4)(y_2-y_3)-(x_2-x_3)(y_1-y_4)\}\}$

{A New Component:1, $x_2-x_3$}

{A New Component:2, $x_1+x_3-2x_4$}

{A New Component:3, $x_1-x_2+x_3-x_4$}

$\{$A New Component:4, $x_1^2-2x_1x_2+x_2^2+y_1^2-2y_1y_2+y_2^2\}$

{A New Component:1, $-x_2 y_1+x_3 y_1+x_1 y_2-x_3 y_2-x_1 y_3+x_2 y_3$}

{Total 3 Branch(s) of New Component(s) Discovered}

$\{$ $x_1^5x_2-3x_1^4x_2^2+3x_1^3x_2^3-x_1^2x_2^4-x_1^5x_3+4x_1^4x_2x_3-5x_1^3x_2^2x_3+2x_1^2x_2^3x_3-x_1^4x_3^2+x_1^3x_2x_3^2+2x_1^2x_2^2x_3^2-3x_1x_2^3x_3^2+x_2^4x_3^2+x_1^3x_3^3-$

## WRSP_Step_II: Now Proving the Theorem(s)

### WRSP_SubStep_1_I: Now Proving the 1(th) Theorem

$\{$WRP_STEP:1, $-x_2^2x_3-x_3^2x_4+x_3y_1^2-x_3y_1y_2-x_4y_1y_2+x_2(x_3^2+x_3x_4-(y_1-y_2)(y_1-y_3))+$
$x_1(x_2^2+x_3x_4-x_2(x_3+x_4)+(y_1-y_3)(y_2-y_3))-x_3y_1y_3+x_4y_1y_3+x_3y_2y_3+x_4y_2y_3-x_4y_3^2\}$

$\{$WRP_STEP:2, $(x_1-x_3)(x_2-x_3)(x_2-x_4)(x_1-x_2+x_3-x_4)(x_1^2-2x_1x_2+x_2^2+(y_1-y_2)^2)\}$

{WRP_STEP:3, 0}

### WRSP_SubStep_1_II: The 1(th) Theorem is True

## WRSP_Step_III: Now Checking the Initials & the Algebraic Configuration

$\{$**The Initials:** , $\{x_2-x_3,\ (x_1-x_2)(x_1-x_2+x_3-x_4),\ -(x_1-x_3)(-x_2+x_3)(x_1+x_3-2x_4)(x_1-x_2+x_3-x_4)\}\}$

$\{$**The Algebraic Configuration:** , $\{\{y_2\to -\sqrt{-(x_1-x_2)^2}+y_1,\ y_3\to \frac{\sqrt{-(x_1-x_2)^2}\,x_3-x_2y_1+x_1\left(-\sqrt{-(x_1-x_2)^2}+y_1\right)}{x_1-x_2},$

$y_4\to \frac{\sqrt{-(x_1-x_2)^2}\,x_4-x_2y_1+x_1\left(-\sqrt{-(x_1-x_2)^2}+y_1\right)}{x_1-x_2}\},\ \{y_2\to \sqrt{-(x_1-x_2)^2}+y_1,$

$y_3\to \frac{-\sqrt{-(x_1-x_2)^2}\,x_3-x_2y_1+x_1\left(\sqrt{-(x_1-x_2)^2}+y_1\right)}{x_1-x_2},\ y_4\to \frac{-\sqrt{-(x_1-x_2)^2}\,x_4-x_2y_1+x_1\left(\sqrt{-(x_1-x_2)^2}+y_1\right)}{x_1-x_2}\}\}\}$

```
{6.369 Second, {{2.604 Second, True}}}
```

---

## Isosceles Trapezoid Theorem

### Setting Coordination for the Geometric Configuration

```
precoord = {A1 -> {x1, y1}, B1 -> {x2, y2}, C1 -> {x3, y3}, D1 -> {x4, y4}};
```

### Fixing the Concrete Geometric Configuration

■ **The Geometric Proposition**

```
prethmcfg = {TwoLinesParallel[{C1, D1}, {A1, B1}], TwoLinesEqual[{A1, D1}, {C1, B1}]};
```

## ■ The Geometric Conclusion(s)

```
prethmcnd = {TwoAnglesEqual[{D1, A1, B1}, {A1, B1, C1}]};
```

```
ord = {x1, x2, x3, x4, y1, y2, y3, y4, y5};
const = {};
```

### Using WuRittSmartProver for the Geometric Configuration

```
Timing@WuRittSmartProver[precoord, {prethmcfg, prethmcnd}, ord, const, TraceCharacteristicSetOn → True,
  TraceProverOn → True, TraceProverOn → True]
```

# WRSP_Step_I: Now Solving the Characteristic Set

$$\{\text{CS\_STEP:1},\ \{(x_3-x_4)(y_1-y_2)-(x_1-x_2)(y_3-y_4)\}\}$$

$$\{\text{CS\_STEP:2},\ \{(x_1-x_2+x_3-x_4)\ (x_1^3+x_2^3-x_2^2(x_3+x_4)-x_1^2(x_2+x_3+x_4)+(x_3-x_4)(y_1-y_2)^2+x_1(-x_2^2+2x_2(x_3+x_4)+(y_1-y_2)(y_1+y_2-2y_3))-x_2(y_1-y_2)(y_1+y_2-2y_3)),\ (x_3-x_4)(y_1-y_2)-(x_1-x_2)(y_3-y_4)\}\}$$

$$\{\text{A New Component:1},\ x_1^3-x_1^2x_2-x_1x_2^2+x_2^3-x_1^2x_3+2x_1x_2x_3-x_2^2x_3-x_1^2x_4+2x_1x_2x_4-x_2^2x_4+x_1y_1^2-x_2y_1^2+x_3y_1^2-x_4y_1^2-2x_3y_1y_2+2x_4y_1y_2-x_1y_2^2+x_2y_2^2+x_3y_2^2-x_4y_2^2-2x_1y_1y_3+2x_2y_1y_3+2x_1y_2y_3-2x_2y_2y_3\}$$

$$\{\text{Total 1 Branch(s) of New Component(s) Discovered}\}$$

$$\{x_1^4-2x_1^3x_2+2x_1x_2^3-x_2^4+2x_1^2x_2x_3-4x_1x_2^2x_3+2x_2^3x_3-x_1^2x_3^2+2x_1x_2x_3^2-x_2^2x_3^2-2x_1^3x_4+4x_1^2x_2x_4-2x_1x_2^2x_4+x_1^2x_4^2-2x_1:$$

# WRSP_Step_II: Now Proving the Theorem(s)

## WRSP_SubStep_1_I: Now Proving the 1(th) Theorem

$$\{\text{WRP\_STEP:1},\ (x_1^3+x_2^3-x_2^2(x_3+x_4)-x_1^2(x_2+x_3+x_4)+(x_3-x_4)(y_1-y_2)^2+x_1(-x_2^2+2x_2(x_3+x_4)+(y_1-y_2)(y_1+y_2-2y_3))-x_2(y_1-y_2)(y_1+y_2-2y_3))\ (x_3(y_1-y_2)+x_1(y_2-y_3)+x_2(-y_1+y_3))\}$$

$$\{\text{WRP\_STEP:2},\ 0\}$$

## WRSP_SubStep_1_II: The 1(th) Theorem is True

# WRSP_Step_III: Now Checking the Initials & the Algebraic Configuration

$$\{\text{The Initials: },\ \{x_1-x_2,\ -2(x_1-x_2)(x_1-x_2+x_3-x_4)(y_1-y_2)\}\}$$

$$\{\text{The Algebraic Configuration: },\ \{\{y_3 \to \frac{1}{2(x_1-x_2)(y_1-y_2)}(x_1^3+x_2^3-x_2^2(x_3+x_4)-x_1^2(x_2+x_3+x_4)+(x_3-x_4)(y_1-y_2)^2+x_1(-x_2^2+2x_2(x_3+x_4)+y_1^2-y_2^2)+x_2(-y_1^2+y_2^2)),\ y_4 \to \frac{1}{2(x_1-x_2)(y_1-y_2)}(x_1^3+x_2^3-x_2^2(x_3+x_4)-x_1^2(x_2+x_3+x_4)-(x_3-x_4)(y_1-y_2)^2+x_1(-x_2^2+2x_2(x_3+x_4)+y_1^2-y_2^2)+x_2(-y_1^2+y_2^2))\}\}\}$$

```
{6.9 Second, {{5.678 Second, True}}}
```

---

## Other Theorems(Unfixed Geometric Configurations)

Problem A

### Setting Coordination for the Geometric Configuration

```
precoord = {A1 -> {x1, y1}, B1 -> {x2, y2}, C1 -> {x3, y2}, M1 -> {x4, y4}, M2 -> {x5, y5}, M3 -> {x6, y6},
    H1 -> {x7, y7}, O1 -> {0, 0}};
```

## ■ The Geometric Proposition

```
prethmcfg = {TwoLinesPerpend[{C1, A1}, {A1, B1}], TwoLinesPerpend[{H1, A1}, {C1, B1}],
    PointOnLineEqual[M1, {A1, B1}], PointOnLineEqual[M2, {C1, A1}], PointOnLineEqual[M3, {B1, C1}]};
```

## ■ The Geometric Conclusion(s)

```
prethmcnd = {FourPointsOnCircle[{M1, M2, M3, H1}, {O1, 1}]};
```

## Using WuRittSmartProver for the Geometric Configuration

```
Timing@WuRittSmartProver[precoord, {prethmcfg, prethmcnd}, {x1, x2, x3, x4, x5, x6, y1, y2, y3, y4, y5, y6},
  {}, TraceCharacteristicSetOn → True, TraceProverOn → True]
```

# WRSP_Step_I: Now Solving the Characteristic Set

$$\{\text{CS\_STEP:1}, \{-(x_1+x_2-2x_4)(-y_1-y_2+2y_4), -(x_1+x_3-2x_5)(-y_1-y_2+2y_5), -2(x_2+x_3-2x_6)(-y_2+y_6), (x_2-x_3)(x_1-x_7), (-x_1+x_2)(x_1-x_3)-(y_1-y_2)^2\}\}$$

$$\{\text{A New Component:1}, y_1+y_2-2y_4\}$$

$$\{\text{A New Component:1}, y_1+y_2-2y_5\}$$

$$\{\text{A New Component:1}, y_2-y_6\}$$

$$\{\text{A New Component:1}, x_1-x_7\}$$

$$\{\text{Total 4 Branch(s) of New Component(s) Discovered}\}$$

$$\begin{pmatrix} (x_2-x_3)(x_1-x_7) & \{x_1, x_2, x_3, 00, 00, 00, x_7, 00, 00, 00, 00, 00\} \\ (-x_1+x_2)(x_1-x_3)+(y_1-y_2)(-y_1+y_2) & \{x_1, x_2, x_3, 00, 00, 00, 00, y_1, y_2, 00, 00, 00\} \\ (-x_1-x_2+2x_4)(-y_1-y_2+2y_4) & \{x_1, x_2, 00, x_4, 00, 00, 00, y_1, y_2, y_4, 00, 00\} \\ (-x_1-x_3+2x_5)(-y_1-y_2+2y_5) & \{x_1, 00, x_3, 00, x_5, 00, 00, y_1, y_2, 00, y_5, 00\} \\ (-x_2-x_3+2x_6)(-2y_2+2y_6) & \{00, x_2, x_3, 00, 00, x_6, 00, 00, y_2, 00, 00, y_6\} \end{pmatrix}$$

# WRSP_Step_II: Now Proving the Theorem(s)

## WRSP_SubStep_1_I: Now Proving the 1(th) Theorem

$$\{\text{WRP\_STEP:1}, 2(x_2+x_3-2x_6)(-1+x_6^2+y_2^2)(-1+x_4^2+y_4^2)(-1+x_5^2+y_5^2)(-1+x_7^2+y_7^2)\}$$

$$\{\text{WRP\_STEP:2}, (x_1+x_3-2x_5)(x_2+x_3-2x_6)(-1+x_6^2+y_2^2)(-4+4x_5^2+y_1^2+2y_1y_2+y_2^2)(-1+x_4^2+y_4^2)(-1+x_7^2+y_7^2)\}$$

$$\{\text{WRP\_STEP:3}, (x_1+x_2-2x_4)(x_1+x_3-2x_5)(x_2+x_3-2x_6)(-1+x_6^2+y_2^2)(-4+4x_4^2+y_1^2+2y_1y_2+y_2^2)(-4+4x_5^2+y_1^2+2y_1y_2+y_2^2)(-1+x_7^2+y_7^2)\}$$

$$\{\text{WRP\_STEP:4}, -(x_1+x_2-2x_4)(x_1+x_3-2x_5)(x_2+x_3-2x_6)(x_1^6+x_2^3x_3^3-3x_1^5(x_2+x_3)+x_1^4(9+3x_2^2+9x_2x_3+3x_3^2-4x_4^2-4x_5^2-x_6^2-31y_1^2-10y_1y_2)-x_2^2x_3^2(-9+4x_4^2+4x_5^2+x_6^2+31y_1^2+10y_1y_2)-x_1^3(x_2+x_3)(x_2^2+8x_2x_3+x_3^2-2(-9+4x_4^2+4x_5^2+x_6^2+31y_1^2+10y_1y_2))+4x_2x_3(x_5^2(-5+x_6^2+7y_1^2+6y_1y_2)+x_4^2(-5+4x_5^2+x_6^2+7y_1^2+6y_1y_2)+2(3-9y_1^2+2y_1^4-7y_1y_2+11y_1^3y_2+x_6^2(-1+2y_1^2+y_1y_2)))-16(-(-1+y_1^2)((-1+y_1^2)(1+3y_1^2-4y_1y_2)+x_6^2(1+y_1^2-2y_1y_2))+x_5^2(-(-1+y_1^2)(1+2y_1^2-3y_1y_2)+x_6^2(-1+y_1y_2))+x_4^2(-(-1+y_1^2)(1+2y_1^2-3y_1y_2)+x_6^2(-1+y_1y_2)+x_5^2(-1+x_6^2-y_1^2+2y_1y_2)))-x_1(x_2+x_3)(3x_2^2x_3^2-2x_2x_3(-9+4x_4^2+4x_5^2+x_6^2+31y_1^2+10y_1y_2)+4(x_5^2(-5+x_6^2+7y_1^2+6y_1y_2)+x_4^2(-5+4x_5^2+x_6^2+7y_1^2+6y_1y_2)+2(3-9y_1^2+2y_1^4-7y_1y_2+11y_1^3y_2+x_6^2(-1+2y_1^2+y_1y_2))))+x_1^2(3x_2^3x_3+x_2^2(9+9x_3^2-4x_4^2-4x_5^2-x_6^2-31y_1^2-10y_1y_2)-x_3^2(-9+4x_4^2+4x_5^2+x_6^2+31y_1^2+10y_1y_2)+x_2x_3(3x_3^2-4(-9+4x_4^2+4x_5^2+x_6^2+31y_1^2+10y_1y_2))+4(x_5^2(-5+x_6^2+7y_1^2+6y_1y_2)+x_4^2(-5+4x_5^2+x_6^2+7y_1^2+6y_1y_2)+2(3-9y_1^2+2y_1^4-7y_1y_2+11y_1^3y_2+x_6^2(-1+2y_1^2+y_1y_2))))(-1+x_7^2+y_7^2)\}$$

$$\{\text{WRP\_STEP:5}, -2(x_1+x_2-2x_4)(x_1+x_2-2x_5)(x_2-x_6)(x_1-x_7)(x_1^6-6x_1^5x_2+x_2^6-4x_1^3x_2(9+5x_2^2-4x_4^2-4x_5^2-x_6^2-31y_1^2-10y_1y_2)+x_1^4(9+15x_2^2-4x_4^2-4x_5^2-x_6^2-31y_1^2-10y_1y_2)-x_2^4(-9+4x_4^2+4x_5^2+x_6^2+31y_1^2+10y_1y_2)+4x_2^2(x_5^2(-5+x_6^2+7y_1^2+6y_1y_2)+x_4^2(-5+4x_5^2+x_6^2+7y_1^2+6y_1y_2)+2(3-9y_1^2+2y_1^4-7y_1y_2+11y_1^3y_2+x_6^2(-1+2y_1^2+y_1y_2)))-16(-(-1+y_1^2)((-1+y_1^2)(1+3y_1^2-4y_1y_2)+x_6^2(1+y_1^2-2y_1y_2))+x_5^2(-(-1+y_1^2)(1+2y_1^2-3y_1y_2)+x_6^2(-1+y_1y_2))+x_4^2(-(-1+y_1^2)(1+2y_1^2-3y_1y_2)+x_6^2(-1+y_1y_2)+x_5^2(-1+x_6^2-y_1^2+2y_1y_2)))+x_1^2(15x_2^4-6x_2^2(-9+4x_4^2+4x_5^2+x_6^2+31y_1^2+10y_1y_2)+4(x_5^2(-5+x_6^2+7y_1^2+6y_1y_2)+x_4^2(-5+4x_5^2+x_6^2+7y_1^2+6y_1y_2)+2(3-9y_1^2+2y_1^4-7y_1y_2+11y_1^3y_2+x_6^2(-1+2y_1^2+y_1y_2))))-2x_1x_2(3x_2^4-2x_2^2(-9+4x_4^2+4x_5^2+x_6^2+31y_1^2+10y_1y_2)+4(x_5^2(-5+x_6^2+7y_1^2+6y_1y_2)+x_4^2(-5+4x_5^2+x_6^2+7y_1^2+6y_1y_2)+2(3-9y_1^2+2y_1^4-7y_1y_2+11y_1^3y_2+x_6^2(-1+2y_1^2+y_1y_2))))(-1+x_7^2+y_7^2)\}$$

## WRSP_SubStep_1_II: The 1(th) Theorem is False

# WRSP_Step_III: Now Checking the Initials & the Algebraic Configuration

$$\{\text{The Initials:}, \{-1, -2(x_1+x_2-2x_4), -2(x_1+x_3-2x_5), -2(x_2+x_3-2x_6), -x_1+x_7\}\}$$

$$\left\{\text{The Algebraic Configuration:}, \left\{\left\{x_7 \to x_1, y_2 \to -\sqrt{-(x_1-x_2)(x_1-x_3)}+y_1, y_4 \to -\frac{1}{2}\sqrt{-(x_1-x_2)(x_1-x_3)}+y_1, y_5 \to -\frac{1}{2}\sqrt{-(x_1-x_2)(x_1-x_3)}+y_1, y_6 \to -\sqrt{-(x_1-x_2)(x_1-x_3)}+y_1\right\}\right\}\right\}$$

```
{245.653 Second, {{244.431 Second, False}}}
```

Problem B

```
precoord = {A1 -> {x1, u1}, B1 -> {x2, u2}, C1 -> {x3, u3}, D1 -> {x4, u4}, O1 → {x5, u5}};
```

## Fixing the Concrete Geometric Configuration

### ■ The Geometric Proposition

```
prethmcfg = {TwoLinesPerpend[{A1, C1}, {B1, A1}], TwoLinesPerpend[{A1, D1}, {C1, B1}],
    ThreePointsOnCircle[{A1, B1, C1}, {O1, 1}]};
```

### ■ The Geometric Conclusion(s)

```
prethmcnd = {PointOnLineToRatio[D1, {B1, C1}, 1]};
```

## Using WuRittSmartProver for the Geometric Configuration

```
Timing@WuRittSmartProver[precoord, {prethmcfg, prethmcnd}, {x1, x2, x3, x4, x5}, {u1, u2, u3, u4, u5},
  TraceCharacteristicSetOn → True, TraceProverOn → True]
```

# WRSP_Step_I: Now Solving the Characteristic Set

$\{\text{CS\_STEP:1}, \{(u_1-u_2)(-u_1+u_3)+(x_1-x_2)(-x_1+x_3),\ (u_2-u_3)(-u_1+u_4)+(x_2-x_3)(-x_1+x_4),$
$(-1+(u_1-u_5)^2+(x_1-x_5)^2)(-1+(u_2-u_5)^2+(x_2-x_5)^2)(-1+(u_3-u_5)^2+(x_3-x_5)^2)\}\}$

$\{\text{A New Component:1}, -1+u_2^2-2u_2u_5+u_5^2+x_2^2-2x_2x_5+x_5^2\}$

$\{\text{A New Component:2}, -1+u_3^2-2u_3u_5+u_5^2+x_3^2-2x_3x_5+x_5^2\}$

{Total 1 Branch(s) of New Component(s) Discovered}

$$\begin{pmatrix} (u_1-u_2)(-u_1+u_3)+(x_1-x_2)(-x_1+x_3) & \{x_1, x_2, x_3, 00, 00\} \\ (u_2-u_3)(-u_1+u_4)+(x_2-x_3)(-x_1+x_4) & \{x_1, x_2, x_3, x_4, 00\} \\ (-1+(u_1-u_5)^2+(x_1-x_5)^2)(-1+(u_2-u_5)^2+(x_2-x_5)^2)(-1+(u_3-u_5)^2+(x_3-x_5)^2) & \{x_1, x_2, x_3, 00, x_5\} \end{pmatrix}$$

# WRSP_Step_II: Now Proving the Theorem(s)

## WRSP_SubStep_1_I: Now Proving the 1(th) Theorem

$\{\text{WRP\_STEP:1}, (u_2+u_3-2u_4)(x_2+x_3-2x_4)\}$

$\{\text{WRP\_STEP:2}, (u_2+u_3-2u_4)(-2u_1(u_2-u_3)+2u_2u_4-2u_3u_4-2x_1x_2+x_2^2+2x_1x_3-x_3^2)\}$

$\{\text{WRP\_STEP:3}, -u_2^3u_3^2-u_1^4(u_2+u_3-2u_4)+2u_1^3(u_2+u_3)(u_2+u_3-2u_4)-$
$u_1^2(u_2^2+4u_2u_3+u_3^2)(u_2+u_3-2u_4)+2u_1(u_2+u_3-2u_4)(u_2^2u_3+u_2(u_3^2-(x_1-x_2)^2)+u_3(x_1-x_2)^2)+$
$u_2^2(-u_3^3+2u_3^2u_4+2u_4(x_1-x_2)^2)+u_2(-4u_4^2+(x_1-x_2)^2)(x_1-x_2)^2+(-2u_3^2u_4+u_3(4u_4^2+(x_1-x_2)^2)-2u_4(x_1-x_2)^2)(x_1-x_2)^2\}$

## WRSP_SubStep_1_II: The 1(th) Theorem is False

# WRSP_Step_III: Now Checking the Initials & the Algebraic Configuration

$\{\text{The Initials:}, \{1, x_1-x_2, x_2-x_3\}\}$

$$\left\{\text{The Algebraic Configuration:}, \left\{\left\{x_3 \to \frac{u_1^2+u_2u_3-u_1(u_2+u_3)+x_1(x_1-x_2)}{x_1-x_2},\right.\right.\right.$$
$$x_4 \to \frac{u_1^2x_1+u_2(u_3x_1+u_4(x_1-x_2))+(-u_3u_4+x_1(x_1-x_2))(x_1-x_2)+u_1(-u_3x_2+u_2(-2x_1+x_2))}{u_1^2+u_2u_3-u_1(u_2+u_3)+(x_1-x_2)^2},$$
$$\left.\left.\left.x_5 \to -\sqrt{1-u_1^2+2u_1u_5-u_5^2}+x_1\right\}\right\}\right\}$$

```
{12.178 Second, {{1.092 Second, False}}}
```

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