Application of
WuRittSolva in Elementary Geometry

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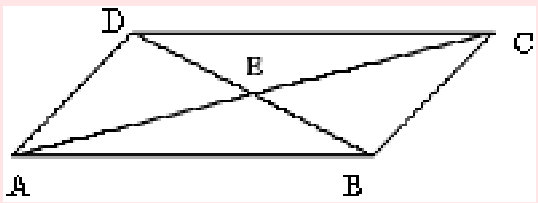
WRS_Example_I: Parallelogram Diagonal Theorem

■ The Geometry Figure of the Theorem



■ Polynomials Corresponding to hypotheses&Conclusion(s)

$$Hyp_1 = x_1 - u_1 - u_2;$$
$$Hyp_2 = x_2 - u_3;$$
$$Hyp_3 = -u_1\,u_3\,x_1 + u_1\,u_3\,x_3 - u_2\,u_3\,x_3 + u_3\,x_1\,x_3;$$
$$Hyp_4 = x_4\,(u_2 - u_1) - (x_3 - u_1)\,u_3;$$
$$HypSet = \{Hyp_1, Hyp_2, Hyp_3, Hyp_4\};$$
$$G_1 = x_1^2 - 2\,x_1\,x_3 - 2\,x_4\,x_2 + x_2^2;$$
$$G_2 = 2\,x_3\,u_1 - 2\,x_3\,u_2 - 2\,x_4\,u_3 - u_1^2 + u_2^2 + u_3^2;$$
$$ord = \{x_1, x_2, x_3, x_4\};$$
$$const = \{u_1, u_2, u_3\};$$

■ Computation & Display of the Concrete Characteristic Set

```
defCS = CharacteristicSet[HypSet, ord, const, TracePrintOn -> True]
```
真

{CS_STEP:1, $\{-u_1 - u_2 + x_1,\ -u_3 + x_2,\ u_3\,((-u_2 + x_1)\,x_3 + u_1\,(-x_1 + x_3)),\ -u_3\,x_3 + u_1\,(u_3 - x_4) + u_2\,x_4\}$}

{A New Component:1, $u_1\,x_1 - u_1\,x_3 + u_2\,x_3 - x_1\,x_3$}

{Total 1 Branch(s) of New Component(s) Discovered}

$$\{-u_1 - u_2 + x_1,\ -u_3 + x_2,\ -u_1\,u_3\,x_1 + u_1\,u_3\,x_3 - u_2\,u_3\,x_3 + u_3\,x_1\,x_3,\ -u_3\,(-u_1 + x_3) + (-u_1 + u_2)\,x_4\}$$

```
CharacteristicForm[Simplify@defCS, ord, const, Padding -> "00"]
```
化简 填充

$$\begin{pmatrix} -u_1 - u_2 + x_1 & \{x_1, 00, 00, 00\} \\ -u_3 + x_2 & \{00, x_2, 00, 00\} \\ u_3\,((-u_2 + x_1)\,x_3 + u_1\,(-x_1 + x_3)) & \{x_1, 00, x_3, 00\} \\ u_3\,(u_1 - x_3) + (-u_1 + u_2)\,x_4 & \{00, 00, x_3, x_4\} \end{pmatrix}$$

Algebraic Configuration of the Characteristic Set

```
WuRittEqnsSolve[defCS, ord, const]
```

$$\left\{\left\{x_1 \to u_1 + u_2,\ x_2 \to u_3,\ x_3 \to \frac{1}{2}(u_1+u_2),\ x_4 \to \frac{u_3}{2}\right\}\right\}$$

Proof Course of the Theorems

```
Timing@WuRittProver[Reverse@defCS, G1, ord, const, TraceProverOn → True]
```

时间 反向 真

$\{\text{WRP\_STEP:1},\ 2 u_3 x_2 x_3 - u_2 (x_1^2 + x_2^2 - 2 x_1 x_3) + u_1 (x_1^2 + x_2 (-2 u_3 + x_2) - 2 x_1 x_3)\}$

$\{\text{WRP\_STEP:2},\ -(u_1 - u_2) u_3 (u_1 (x_1^2 + (2 u_3 - x_2) x_2) + (u_2 - x_1)(x_1^2 + x_2^2))\}$

$\{\text{WRP\_STEP:3},\ -(u_1 - u_2) u_3 (u_1 + u_2 - x_1)(u_3^2 + x_1^2)\}$

$\{\text{WRP\_STEP:4},\ 0\}$

{0.341 Second, True}

```
Timing@WuRittProver[Reverse@defCS, G2, ord, const, TraceProverOn → True]
```

时间 反向 真

$\{\text{WRP\_STEP:1},\ -(u_1^2 - 2 u_1 u_2 + u_2^2 + u_3^2)(u_1 + u_2 - 2 x_3)\}$

$\{\text{WRP\_STEP:2},\ -(u_1 - u_2) u_3 (u_1^2 - 2 u_1 u_2 + u_2^2 + u_3^2)(u_1 + u_2 - x_1)\}$

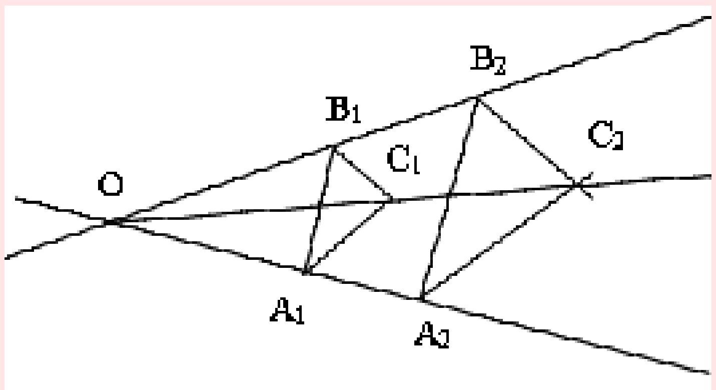
$\{\text{WRP\_STEP:3},\ -(u_1 - u_2) u_3 (u_1^2 - 2 u_1 u_2 + u_2^2 + u_3^2)(u_1 + u_2 - x_1)\}$

$\{\text{WRP\_STEP:4},\ 0\}$

{0.12 Second, True}

WRS_Example_II: Desargus Theorem

The Geometry Figure of the Theorem



Polynomials Corresponding to hypotheses&Conclusion(s)

```
Hyp1 = x1 x6 - x2 x3;
Hyp2 = x4 (x8 - x6) - x7 (x5 - x3);
Hyp3 = (x4 - x1) x8 - x5 (x7 - x2);
HypSet = {Hyp1, Hyp2, Hyp3};
G = x4 x8 - x5 x7;
ord = {x1, x2, x3, x4, x5, x6, x7, x8};
const = {};
```

Computation & Display of the Concrete Characteristic Set

```
defCS = CharacteristicSet[HypSet, ord, const, TracePrintOn -> True]
                                                          真
```

{CS_STEP:1, $\{-x_2 x_3 + x_1 x_6, x_2 x_5 - x_5 x_7 + (-x_1 + x_4) x_8\}$}

{CS_STEP:2, $\{-x_2 x_3 + x_1 x_6, (x_3 x_4 + x_1 (-x_3 + x_5)) (x_2 x_4 - x_1 x_7), x_2 x_5 - x_5 x_7 + (-x_1 + x_4) x_8\}$}

{A New Component:1, $-x_2 x_4 + x_1 x_7$}

{Total 1 Branch(s) of New Component(s) Discovered}

$\{-x_2 x_3 + x_1 x_6, -x_1 x_2 x_3 x_4 + x_2 x_3 x_4^2 + x_1 x_2 x_4 x_5 + x_1^2 x_3 x_7 - x_1 x_3 x_4 x_7 - x_1^2 x_5 x_7, -x_5 (-x_2 + x_7) + (-x_1 + x_4) x_8\}$

```
CharacteristicForm[Simplify@defCS, ord, const, Padding -> "00"]
                   化简                              填充
```

$$\begin{pmatrix} -x_2 x_3 + x_1 x_6 & \{x_1, x_2, x_3, 00, 00, x_6, 00, 00\} \\ (x_3 x_4 + x_1 (-x_3 + x_5)) (x_2 x_4 - x_1 x_7) & \{x_1, x_2, x_3, x_4, x_5, 00, x_7, 00\} \\ x_5 (x_2 - x_7) + (-x_1 + x_4) x_8 & \{x_1, x_2, 00, x_4, x_5, 00, x_7, x_8\} \end{pmatrix}$$

Algebraic Configuration of the Characteristic Set

```
WuRittEqnsSolve[defCS, ord, const]
```

$$\left\{\left\{x_6 \to \frac{x_2 x_3}{x_1}, x_7 \to \frac{x_2 x_4}{x_1}, x_8 \to \frac{x_2 x_5}{x_1}\right\}\right\}$$

Proof Course of the Theorems

```
Timing@WuRittProver[Reverse@defCS, G, ord, const, TraceProverOn → True]
时间                 反向                                           真
```

{WRP_STEP:1, $x_5 (x_2 x_4 - x_1 x_7)$}

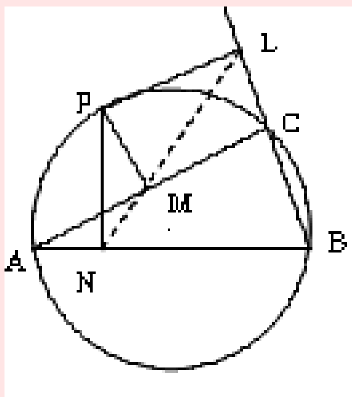
{WRP_STEP:2, 0}

{WRP_STEP:3, 0}

{0.04 Second, True}

WRS_Example_III: Simson Theorem

The Geometry Figure of the Theorem



■ Polynomials Corresponding to hypotheses&Conclusion(s)(s)

$Hyp_1 = (u_1 - x_1)^2 + x_2^2 - x_1^2 - x_2^2;$

$Hyp_2 = (u_2 - x_1)^2 + (u_3 - x_2)^2 - x_1^2 - x_2^2;$

$Hyp_3 = (u_4 - x_1)^2 + (x_3 - x_2)^2 - x_1^2 - x_2^2;$

$Hyp_4 = (x_3 - x_5)\, u_3 + (u_4 - x_4)\, (u_2 - u_1);$

$Hyp_5 = x_5\, (u_2 - u_1) - u_3\, (x_4 - u_1);$

$Hyp_6 = (x_3 - x_7)\, u_3 + (u_4 - x_6)\, u_2;$

$Hyp_7 = x_7\, u_2 - x_6\, u_3;$

$HypSet = \{Hyp_1, Hyp_2, Hyp_3, Hyp_4, Hyp_5, Hyp_6, Hyp_7\};$

$G = x_7\, (x_4 - u_4) - (x_6 - u_4)\, x_5;$

$ord = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\};$

$const = \{u_1, u_2, u_3, u_4\};$

■ Computation & Display of the Concrete Characteristic Set

```
defCS = CharacteristicSet[Expand@HypSet, ord, const, TracePrintOn -> True]
```

展开 真

$\{CS\_STEP{:}1, \{u_1 (u_1 - 2 x_1), u_2^2 - 2 u_2 x_1 + u_3 (u_3 - 2 x_2), u_4^2 - 2 u_4 x_1 + x_3 (-2 x_2 + x_3), -u_3 x_4 + u_1 (u_3 - x_5) + u_2 x_5, -u_3 x_6 + u_2 x_7\}\}$

$\{A\ New\ Component{:}1, u_1 - 2 x_1\}$

$$\{CS\_STEP{:}2, \{u_1 (u_1 - 2 x_1), u_2^2 - 2 u_2 x_1 + u_3 (u_3 - 2 x_2), u_4^2 - 2 u_4 x_1 + x_3 (-2 x_2 + x_3), \\ -u_2 u_3 x_3 + u_3^2 x_4 + u_1^2 (-u_4 + x_4) + u_2^2 (-u_4 + x_4) + u_1 (u_3 (-u_3 + x_3) - 2 u_2 (-u_4 + x_4)), \\ -u_3 x_4 + u_1 (u_3 - x_5) + u_2 x_5, u_2 u_3 x_3 + u_2^2 (u_4 - x_6) - u_3^2 x_6, -u_3 x_6 + u_2 x_7\}\}$$

$\{A\ New\ Component{:}1, u_1 - 2 x_1\}$

{Total 2 Branch(s) of New Component(s) Discovered}

$$\{u_1^2 - 2 u_1 x_1, u_2^2 + u_3^2 - 2 u_2 x_1 - 2 u_3 x_2, u_4^2 - 2 u_4 x_1 - 2 x_2 x_3 + x_3^2, \\ -u_1 u_3^2 - u_1^2 u_4 + 2 u_1 u_2 u_4 - u_2^2 u_4 + u_1 u_3 x_3 - u_2 u_3 x_3 + u_1^2 x_4 - 2 u_1 u_2 x_4 + u_2^2 x_4 + u_3^2 x_4, \\ u_1 u_3 - u_3 x_4 - u_1 x_5 + u_2 x_5, u_2^2 u_4 + u_2 u_3 x_3 - u_2^2 x_6 - u_3^2 x_6, -u_3 x_6 + u_2 x_7\}$$

```
CharacteristicForm[Simplify@defCS, ord, const, Padding → "00"]
```

化简 填充

$$\begin{pmatrix} u_1 (u_1 - 2 x_1) & \{x_1, 00, 00, 00, 00, 00, 00\} \\ u_2^2 - 2 u_2 x_1 + u_3 (u_3 - 2 x_2) & \{x_1, x_2, 00, 00, 00, 00, 00\} \\ u_4^2 - 2 u_4 x_1 + x_3 (-2 x_2 + x_3) & \{x_1, x_2, x_3, 00, 00, 00, 00\} \\ -u_2 u_3 x_3 + u_1 \left(-u_3^2 + u_3 x_3 + 2 u_2 (u_4 - x_4)\right) + u_3^2 x_4 + u_1^2 (-u_4 + x_4) + u_2^2 (-u_4 + x_4) & \{00, 00, x_3, x_4, 00, 00, 00\} \\ -u_3 x_4 + u_1 (u_3 - x_5) + u_2 x_5 & \{00, 00, 00, x_4, x_5, 00, 00\} \\ u_2 u_3 x_3 + u_2^2 (u_4 - x_6) - u_3^2 x_6 & \{00, 00, x_3, 00, 00, x_6, 00\} \\ -u_3 x_6 + u_2 x_7 & \{00, 00, 00, 00, 00, x_6, x_7\} \end{pmatrix}$$

Algebraic Configuration of the Characteristic Set

```
WuRittEqnsSolve[defCS, ord, const]
```

$$\left\{\left\{x_1\to\frac{u_1}{2},\ x_2\to\frac{-u_1u_2+u_2^2+u_3^2}{2u_3},\ x_3\to\frac{-u_1u_2+u_2^2+u_3^2}{2u_3}-\sqrt{\frac{(-u_1u_2+u_2^2+u_3^2)^2}{4u_3^2}+u_1u_4-u_4^2},\right.\right.$$

$$x_4\to\frac{1}{2(u_1^2-2u_1u_2+u_2^2+u_3^2)}\left(u_1^2u_2-2u_1u_2^2+u_2^3+u_1u_3^2+u_2u_3^2+2u_1^2u_4-4u_1u_2u_4+2u_2^2u_4+\right.$$

$$\left.2u_1u_3\sqrt{\frac{(-u_1u_2+u_2^2+u_3^2)^2}{4u_3^2}+u_1u_4-u_4^2}-2u_2u_3\sqrt{\frac{(-u_1u_2+u_2^2+u_3^2)^2}{4u_3^2}+u_1u_4-u_4^2}\right),$$

$$x_5\to\frac{u_3\left(2u_1^2-3u_1u_2+u_2^2+u_3^2-2u_1u_4+2u_2u_4-2u_3\sqrt{\frac{(-u_1u_2+u_2^2+u_3^2)^2}{4u_3^2}+u_1u_4-u_4^2}\right)}{2(u_1^2-2u_1u_2+u_2^2+u_3^2)},$$

$$x_6\to\frac{u_2\left(-u_1u_2+u_2^2+u_3^2+2u_2u_4-2u_3\sqrt{\frac{(-u_1u_2+u_2^2+u_3^2)^2}{4u_3^2}+u_1u_4-u_4^2}\right)}{2(u_2^2+u_3^2)},$$

$$\left.\left.x_7\to\frac{u_3\left(-u_1u_2+u_2^2+u_3^2+2u_2u_4-2u_3\sqrt{\frac{(-u_1u_2+u_2^2+u_3^2)^2}{4u_3^2}+u_1u_4-u_4^2}\right)}{2(u_2^2+u_3^2)}\right\}\right\}$$

Proof Course of the Theorems

```
Timing@WuRittProver[Reverse@defCS, G, ord, const, TraceProverOn -> True]
```

时间 反向 真

{WRP_STEP:1, $u_2x_5(u_4-x_6)+u_3(-u_4+x_4)x_6$}

{WRP_STEP:2, $u_2u_3\left(-u_2\left(u_4^2-u_4x_4+x_3x_5\right)+u_3(x_3x_4+u_4(-x_3+x_5))\right)$}

{WRP_STEP:3, $u_2u_3\left(u_1\left(-u_3^2u_4+u_2u_4(u_4-x_4)+u_3x_3(u_2+u_4-x_4)\right)+u_4\left(-u_2u_3x_3+u_3^2x_4+u_2^2(-u_4+x_4)\right)\right)$}

{WRP_STEP:4, $u_1u_2u_3^2\left(u_1^2(-u_3u_4+u_2x_3)+u_2\left(u_2^2x_3+u_3^2x_3-u_3\left(u_4^2+x_3^2\right)\right)+u_1\left(u_2u_3u_4-2u_2^2x_3+u_3\left(u_4^2-u_3x_3+x_3^2\right)\right)\right)$}

{WRP_STEP:5, $u_1(u_1-u_2)u_2u_3^2\left(-u_2^2x_3-u_3^2x_3+u_1(-u_3u_4+u_2x_3)+2u_3(u_4x_1+x_2x_3)\right)$}

{WRP_STEP:6, $u_1(u_1-u_2)u_2u_3^2(u_1-2x_1)(-u_3u_4+u_2x_3)$}

{WRP_STEP:7, 0}

{0.861 Second, True}

Construction & Display of the Concrete Characteristic Set, Another Way

```
spCS = {Hyp1, Hyp2, Hyp3, PolyPRemainder[Hyp4, Hyp5, x5], Hyp5, PolyPRemainder[Hyp6, Hyp7, x7], Hyp7} //
  Expand
```

展开

$\{u_1^2-2u_1x_1,\ u_2^2+u_3^2-2u_2x_1-2u_3x_2,\ u_4^2-2u_4x_1-2x_2x_3+x_3^2,$
$u_1u_3^2+u_1^2u_4-2u_1u_2u_4+u_2^2u_4-u_1u_3x_3+u_2u_3x_3-u_1^2x_4+2u_1u_2x_4-u_2^2x_4-u_3^2x_4,$
$u_1u_3-u_3x_4-u_1x_5+u_2x_5,\ u_2^2u_4+u_2u_3x_3-u_2^2x_6-u_3^2x_6,\ -u_3x_6+u_2x_7\}$

```
CharacteristicForm[spCS, ord, const]
```

$$\begin{pmatrix} u_1^2 - 2 u_1 x_1 & \{x_1, 00, 00, 00, 00, 00, 00\} \\ u_2^2 + u_3^2 - 2 u_2 x_1 - 2 u_3 x_2 & \{x_1, x_2, 00, 00, 00, 00, 00\} \\ u_4^2 - 2 u_4 x_1 - 2 x_2 x_3 + x_3^2 & \{x_1, x_2, x_3, 00, 00, 00, 00\} \\ u_1 u_3^2 + u_1^2 u_4 - 2 u_1 u_2 u_4 + u_2^2 u_4 - u_1 u_3 x_3 + u_2 u_3 x_3 - u_1^2 x_4 + 2 u_1 u_2 x_4 - u_2^2 x_4 - u_3^2 x_4 & \{00, 00, x_3, x_4, 00, 00, 00\} \\ u_1 u_3 - u_3 x_4 - u_1 x_5 + u_2 x_5 & \{00, 00, 00, x_4, x_5, 00, 00\} \\ u_2^2 u_4 + u_2 u_3 x_3 - u_2^2 x_6 - u_3^2 x_6 & \{00, 00, x_3, 00, 00, x_6, 00\} \\ -u_3 x_6 + u_2 x_7 & \{00, 00, 00, 00, 00, x_6, x_7\} \end{pmatrix}$$

Algebraic Configuration of the Characteristic Set

```
WuRittEqnsSolve[spCS, ord, const]
```

$$\left\{\left\{x_1 \to \frac{u_1}{2},\ x_2 \to \frac{-u_1 u_2 + u_2^2 + u_3^2}{2 u_3},\ x_3 \to \frac{-u_1 u_2 + u_2^2 + u_3^2}{2 u_3} - \sqrt{\frac{(-u_1 u_2 + u_2^2 + u_3^2)^2}{4 u_3^2} + u_1 u_4 - u_4^2},\right.\right.$$

$$x_4 \to \frac{1}{2 (u_1^2 - 2 u_1 u_2 + u_2^2 + u_3^2)} \left(u_1^2 u_2 - 2 u_1 u_2^2 + u_2^3 + u_1 u_3^2 + u_2 u_3^2 + 2 u_1^2 u_4 - 4 u_1 u_2 u_4 + 2 u_2^2 u_4 + \right.$$

$$\left. 2 u_1 u_3 \sqrt{\frac{(-u_1 u_2 + u_2^2 + u_3^2)^2}{4 u_3^2} + u_1 u_4 - u_4^2} - 2 u_2 u_3 \sqrt{\frac{(-u_1 u_2 + u_2^2 + u_3^2)^2}{4 u_3^2} + u_1 u_4 - u_4^2}\right),$$

$$x_5 \to \frac{u_3 \left(2 u_1^2 - 3 u_1 u_2 + u_2^2 + u_3^2 - 2 u_1 u_4 + 2 u_2 u_4 - 2 u_3 \sqrt{\frac{(-u_1 u_2+u_2^2+u_3^2)^2}{4 u_3^2} + u_1 u_4 - u_4^2}\right)}{2 (u_1^2 - 2 u_1 u_2 + u_2^2 + u_3^2)},$$

$$x_6 \to \frac{u_2 \left(-u_1 u_2 + u_2^2 + u_3^2 + 2 u_2 u_4 - 2 u_3 \sqrt{\frac{(-u_1 u_2+u_2^2+u_3^2)^2}{4 u_3^2} + u_1 u_4 - u_4^2}\right)}{2 (u_2^2 + u_3^2)},$$

$$\left.\left. x_7 \to \frac{u_3 \left(-u_1 u_2 + u_2^2 + u_3^2 + 2 u_2 u_4 - 2 u_3 \sqrt{\frac{(-u_1 u_2+u_2^2+u_3^2)^2}{4 u_3^2} + u_1 u_4 - u_4^2}\right)}{2 (u_2^2 + u_3^2)}\right\}\right\}$$

Proof Course of the Theorems

```
Timing@WuRittProver[Reverse@spCS, G, ord, const, TraceProverOn -> True]
时间                      反向                                         真
```

{WRP_STEP:1, $u_2 x_5 (u_4 - x_6) + u_3 (-u_4 + x_4) x_6$}

{WRP_STEP:2, $u_2 u_3 \left(-u_2 (u_4^2 - u_4 x_4 + x_3 x_5) + u_3 (x_3 x_4 + u_4 (-x_3 + x_5))\right)$}

{WRP_STEP:3, $u_2 u_3 \left(u_1 \left(-u_3^2 u_4 + u_2 u_4 (u_4 - x_4) + u_3 x_3 (u_2 + u_4 - x_4)\right) + u_4 \left(-u_2 u_3 x_3 + u_3^2 x_4 + u_2^2 (-u_4 + x_4)\right)\right)$}

{WRP_STEP:4, $u_1 u_2 u_3^2 \left(u_1^2 (-u_3 u_4 + u_2 x_3) + u_2 \left(u_2^2 x_3 + u_3^2 x_3 - u_3 (u_4^2 + x_3^2)\right) + u_1 \left(u_2 u_3 u_4 - 2 u_2^2 x_3 + u_3 (u_4^2 - u_3 x_3 + x_3^2)\right)\right)$}

{WRP_STEP:5, $u_1 (u_1 - u_2) u_2 u_3^2 \left(-u_2^2 x_3 - u_3^2 x_3 + u_1 (-u_3 u_4 + u_2 x_3) + 2 u_3 (u_4 x_1 + x_2 x_3)\right)$}

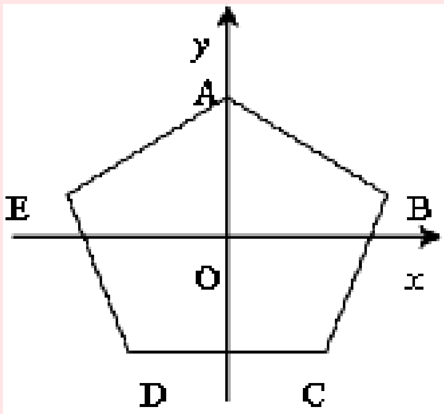
{WRP_STEP:6, $u_1 (u_1 - u_2) u_2 u_3^2 (u_1 - 2 x_1) (-u_3 u_4 + u_2 x_3)$}

{WRP_STEP:7, 0}

{0.16 Second, True}

WRS_Example_IV: Algebraic Relations Automatic Discovery

■ The Geometry Figure of the Theorem



■ Polynomials Corresponding to hypotheses

```
Hyp1 = x1^2 + x3^2 - x5^2;
Hyp2 = x2^2 + x4^2 - x5^2;
Hyp3 = x1^2 + (x3 - x5)^2 - (x2 - x1)^2 - (x3 - x4)^2;
Hyp4 = x1^2 + (x3 - x5)^2 - 4 x2^2;
HypSet = {Hyp1, Hyp2, Hyp3, Hyp4};
ord = {x1, x2, x3, x4, x5};
const = {};
```

■ Computation & Display of the Concrete Characteristic Set

```
defCS = CharacteristicSet[HypSet, ord, const, TracePrintOn -> True] // Simplify
                                                              真          化简
```

$\{\mathrm{CS\_STEP}:1,\ \{x_1^2-4x_2^2+(x_3-x_5)^2\}\}$

$\{\mathrm{CS\_STEP}:2,\ \{-x_1^2+2x_1x_2+3x_2^2-(x_3-x_4)^2,\ 2(x_1^2-2x_2^2+x_3(x_3-x_5))\}\}$

$\{\mathrm{CS\_STEP}:3,\ \{2(x_1^4+4x_2^2(x_2^2-x_3^2)+x_1^2(-4x_2^2+x_3^2)),\ -2(x_1^2-x_1x_2-2x_2^2+x_3(x_3-x_4)),\ 2(x_1^2-2x_2^2+x_3(x_3-x_5))\}\}$

$\{\mathrm{CS\_STEP}:4,$
$\{8x_2^3(-x_1^3+2x_1x_2^2+x_2^3),\ 2(x_1^4+4x_2^2(x_2^2-x_3^2)+x_1^2(-4x_2^2+x_3^2)),\ -2(x_1^2-x_1x_2-2x_2^2+x_3(x_3-x_4)),\ 2(x_1^2-2x_2^2+x_3(x_3-x_5))\}\}$

$\{\text{A New Component}:1,\ x_1+x_2\}$

$\{\text{A New Component}:2,\ x_1^2-x_1x_2-x_2^2\}$

{Total 1 Branch(s) of New Component(s) Discovered}

$$\{8x_2^3(-x_1^3+2x_1x_2^2+x_2^3),\ 2(x_1^4+4x_2^2(x_2^2-x_3^2)+x_1^2(-4x_2^2+x_3^2)),$$
$$-2(x_1^2-x_1x_2-2x_2^2+x_3(x_3-x_4)),\ 2(x_1^2-2x_2^2+x_3(x_3-x_5))\}$$

```
CharacteristicForm[defCS, ord, const]
```

$$\begin{pmatrix} 8x_2^3(-x_1^3+2x_1x_2^2+x_2^3) & \{x_1, x_2, 00, 00, 00\} \\ 2(x_1^4+4x_2^2(x_2^2-x_3^2)+x_1^2(-4x_2^2+x_3^2)) & \{x_1, x_2, x_3, 00, 00\} \\ -2(x_1^2-x_1x_2-2x_2^2+x_3(x_3-x_4)) & \{x_1, x_2, x_3, x_4, 00\} \\ 2(x_1^2-2x_2^2+x_3(x_3-x_5)) & \{x_1, x_2, x_3, 00, x_5\} \end{pmatrix}$$

Algebraic Configuration of the Characteristic Set

```
WuRittEqnsSolve[defCS, ord, const]
```

$$\left\{\left\{x_2\to 0,\ x_3\to -\frac{\sqrt{-x_1^4}}{\sqrt{x_1^2}},\ x_4\to 0,\ x_5\to 0\right\},\ \left\{x_2\to 0,\ x_3\to \frac{\sqrt{-x_1^4}}{\sqrt{x_1^2}},\ x_4\to 0,\ x_5\to 0\right\},\right.$$

$$\left\{x_2\to 0,\ x_3\to -\frac{\sqrt{-x_1^4}}{\sqrt{x_1^2}},\ x_4\to 0,\ x_5\to 0\right\},\ \left\{x_2\to -x_1,\ x_3\to \frac{\sqrt{-x_1^4}}{\sqrt{x_1^2}},\ x_4\to -\frac{\left(x_1^2\right)^{3/2}}{\sqrt{-x_1^4}},\ x_5\to -\frac{2\left(x_1^2\right)^{3/2}}{\sqrt{-x_1^4}}\right\},$$

$$\left\{x_2\to \frac{1}{2}\left(-x_1-\sqrt{5}\,x_1\right),\ x_3\to -\frac{\sqrt{-x_1^4}}{\sqrt{x_1^2}},\ x_4\to \frac{\left(5+\sqrt{5}\right)\left(x_1^2\right)^{3/2}}{2\sqrt{-x_1^4}},\ x_5\to \frac{\left(3+\sqrt{5}\right)\left(x_1^2\right)^{3/2}}{\sqrt{-x_1^4}}\right\},$$

$$\left.\left\{x_2\to \frac{1}{2}\left(-x_1+\sqrt{5}\,x_1\right),\ x_3\to \frac{\sqrt{-x_1^4}}{\sqrt{x_1^2}},\ x_4\to \frac{\left(-5+\sqrt{5}\right)\left(x_1^2\right)^{3/2}}{2\sqrt{-x_1^4}},\ x_5\to \frac{\left(-3+\sqrt{5}\right)\left(x_1^2\right)^{3/2}}{\sqrt{-x_1^4}}\right\}\right\}$$

WRS_References

[1]. Original algorithm of the general process is out of Academician Wu Wentsun's works(like ON MATHEMATICS MECHANNIZATION) and some others' books such as SELECTED PAPERS IN SIMBOLIC COMPUTATION(By Doc.Wang DongMing etc.), ARITHMETIC ALGEBRA(By Bhubaneswar Mishra), etc..

[2]. Referential realization in any computer algebra language is helpful. An available package is WSOLVE in Maple format at present.

[3]. Stephen Wolfram,The Mathematica Book,4th ed.(Wolfram Media/Cambridge University Press,1999).

Note:

WSOLVE of Maple V realease 3 is developped by Doctor Wang DingKang

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