第12章 并发编程

线程级并行 Thread-Level Parallelism

100076202： 计算机系统导论

任课教师：

宿红毅   张艳   黎有琦   颜珂

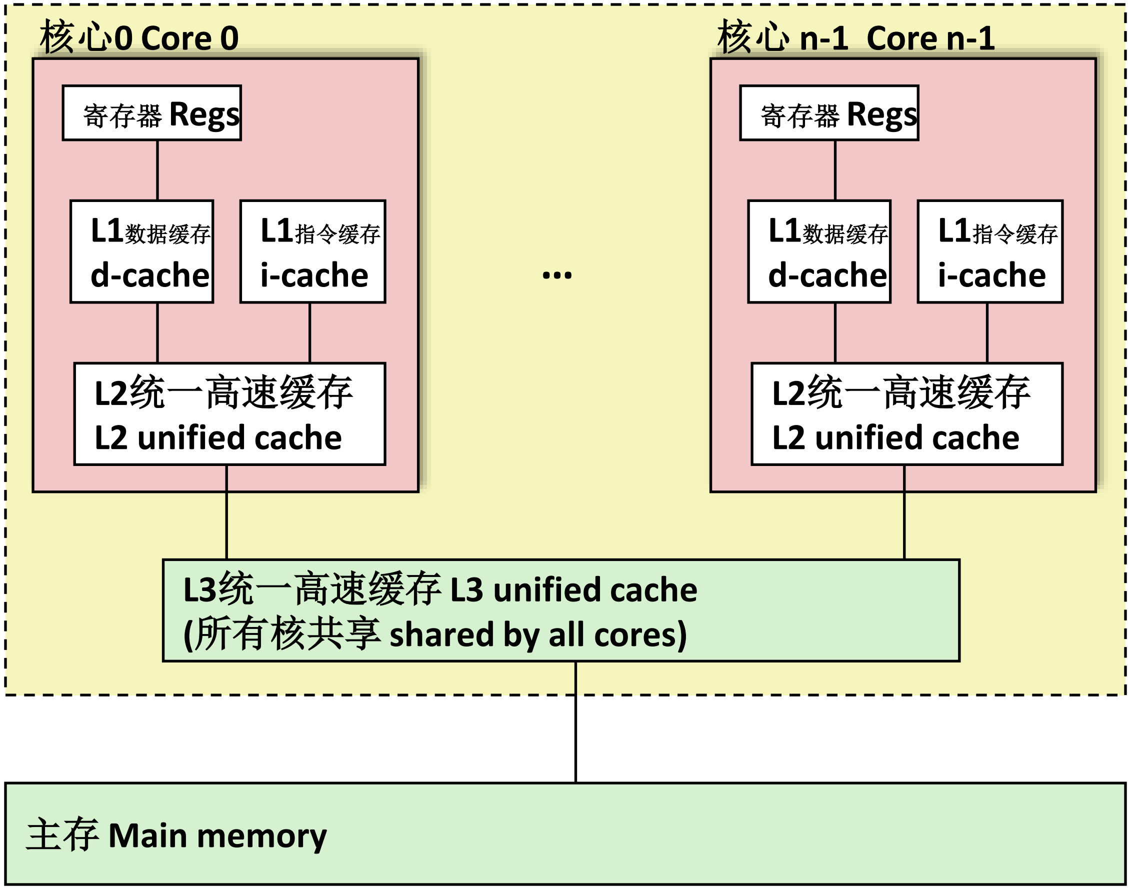
原作者：

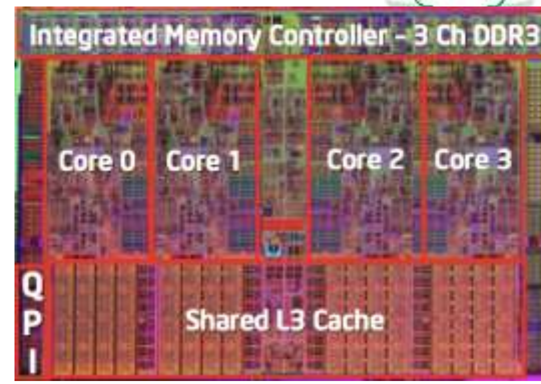
Randal E. **Bryant and** David R. O’Hallaron

议题 Today

- 并行计算硬件 **Parallel Computing Hardware**
  - 多核 Multicore
    - 单个芯片上有多个独立处理器 Multiple separate processors on single chip
  - 超线程化 Hyperthreading
    - 在单核上高效执行多个线程 Efficient execution of multiple threads on single core
- 一致性模型 **Consistency Models**
  - 当多个线程读取和写入共享状态时会发生什么 What happens when multiple threads are reading & writing shared state
- 线程级并行 **Thread-Level Parallelism**
  - 将程序拆分为独立任务 Splitting program into independent tasks
    - 示例：并行求和 Example: Parallel summation
    - 检查一些性能工件 Examine some performance artifacts
  - 分而治之 Divide-and conquer parallelism
    - 示例：并行快速排序 Example: Parallel quicksort

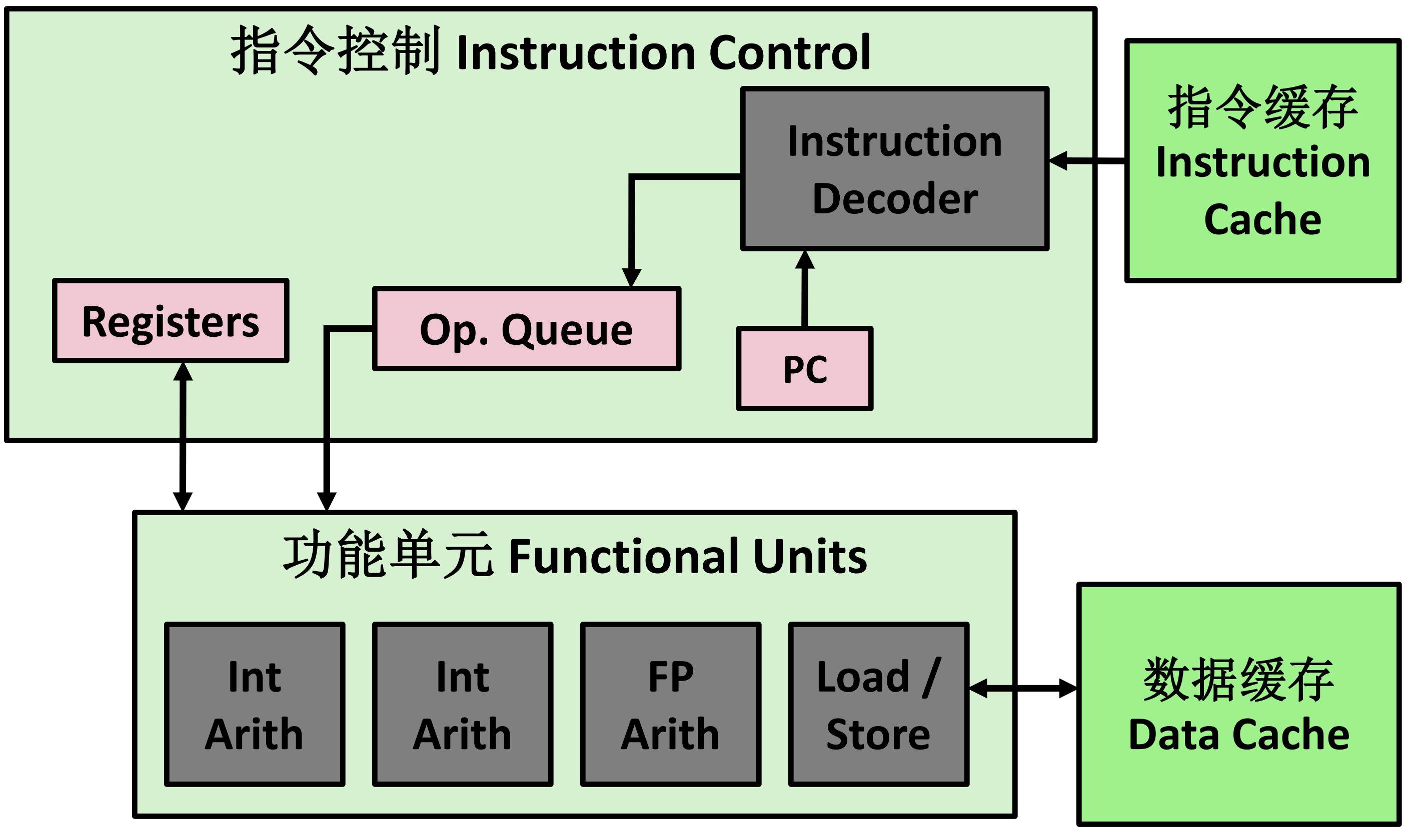
典型的多核处理器 Typical Multicore Processor





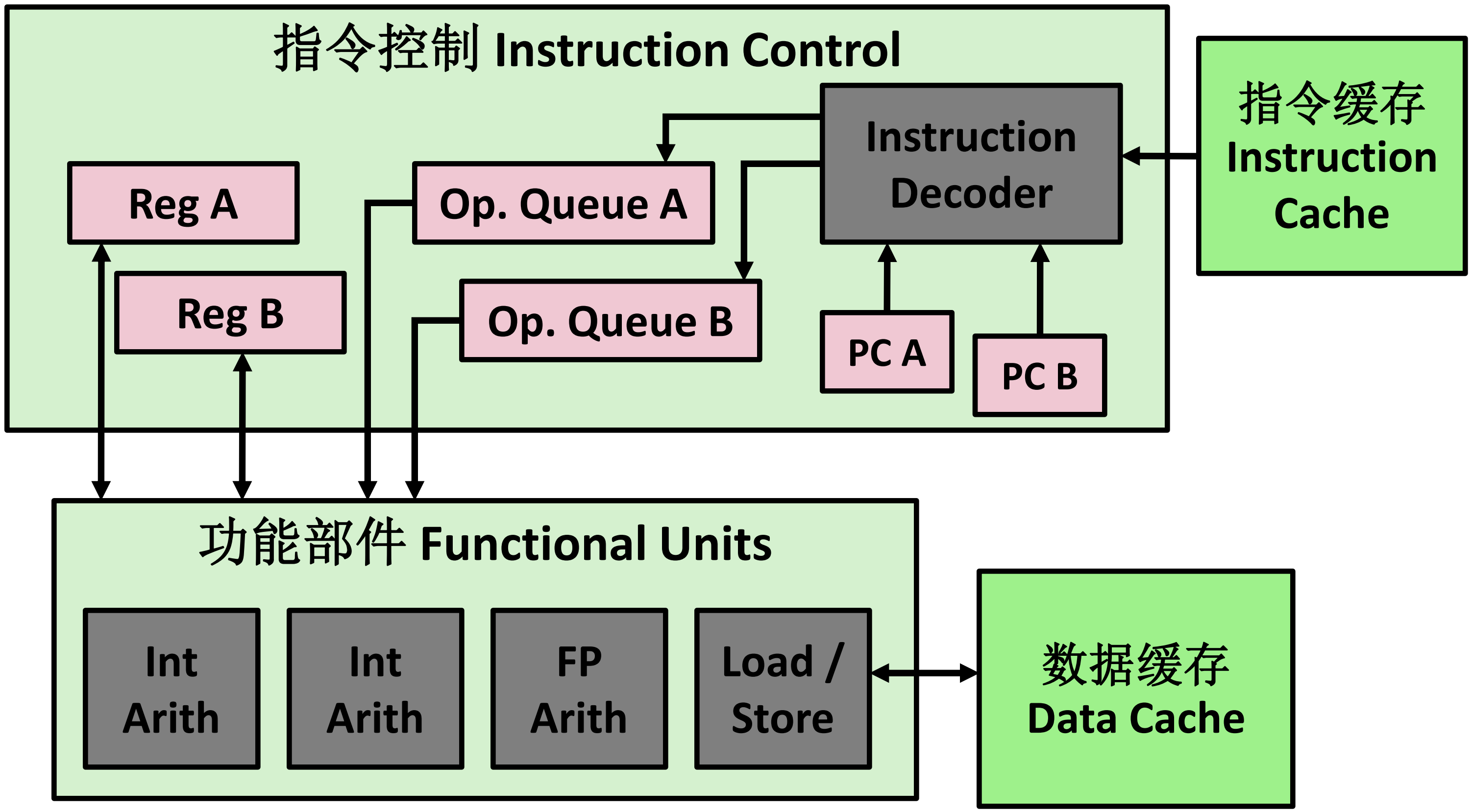
- 多个处理器以一致的内存视图运行 Multiple processors operating with coherent view of memory

乱序处理器结构
Out-of-Order Processor Structure



- 指令控制将程序动态转换为操作流 Instruction control dynamically converts program into stream of operations
- 操作映射到功能单元以并行方式执行 Operations mapped onto functional units to execute in parallel

超线程实现
Hyperthreading Implementation



- 复制指令控制以处理K个指令流 Replicate instruction control to process K instruction streams
- 所有寄存器有K份拷贝 K copies of all registers
- 共享功能单元 Share functional units

基准测试机 Benchmark Machine

- **从/proc/cpuinfo获取有关计算机的数据 Get data about machine from /proc/cpuinfo**
- **Shark机器 Shark Machines**
  - Intel Xeon E5520 @ 2.27 GHz
  - Nehalem, ca. 2010
  - 8核 8 Cores
  - 每个核心可以执行2倍超线程 Each can do 2x hyperthreading

利用并行执行 Exploiting parallel execution

- **到目前为止，我们已经使用线程来处理I/O延迟 So far, we've used threads to deal with I/O delays**
  - 例如每个客户端一个线程，以防止一个线程延迟另一个线程 e.g., one thread per client to prevent one from delaying another
- **多核CPU提供了另一个机会 Multi-core CPUs offer another opportunity**
  - 在N个核心上并行执行的线程上扩展工作 Spread work over threads executing in parallel on N cores
  - 如果有许多独立任务，则自动发生 Happens automatically, if many independent tasks
    - 例如，运行许多应用程序或为许多客户端提供服务 e.g., running many applications or serving many clients
  - 还可以编写代码以加快一项大型任务的执行速度 Can also write code to make one big task go faster
    - 通过将其组织为多个并行子任务 by organizing it as multiple parallel sub-tasks

利用并行执行 Exploiting parallel execution

- **Shark机器可以同时执行16个线程 Shark machines can execute 16 threads at once**
  - 8核心，每个带2路超线程 8 cores, each with 2-way hyperthreading
  - 理论上16倍加速比 Theoretical speedup of 16X
    - 在我们的基准测试中从未达到 never achieved in our benchmarks

内存一致性 Memory Consistency

```
int a = 1;
int b = 100;
```

```
Thread1:
Wa:  a = 2;
Rb:   print(b);
```

```
Thread2:
Wb:  b = 200;
Ra:   print(a);
```

线程一致性约束
Thread consistency constraints

Wa $\longrightarrow$ **Rb**

Wb $\longrightarrow$ **Ra**

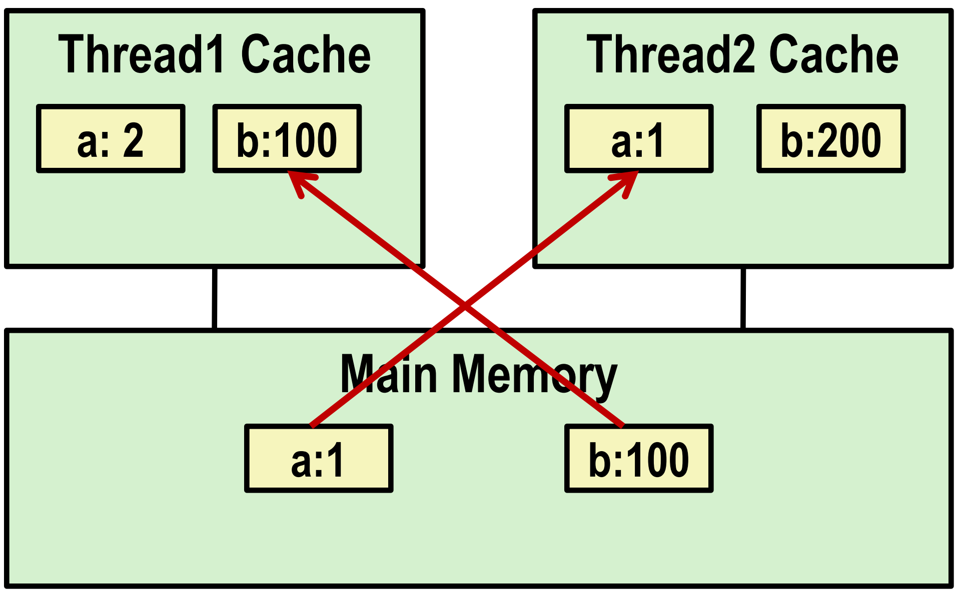
- 打印的可能值是什么？ **What are the possible values printed?**
  - 取决于内存一致性模型 Depends on memory consistency model
  - 硬件如何处理并发访问的抽象模型 Abstract model of how hardware handles concurrent accesses

非一致性高速缓存方案
Non-Coherent Cache Scenario

- 写回高速缓存，线程间没有协作 Write-back caches, without coordination between them

```
int a = 1;
int b = 100;
```

Thread1:	Thread2:
Wa: a = 2;	Wb: b = 200;
Rb: print(b);	Ra: print(a);



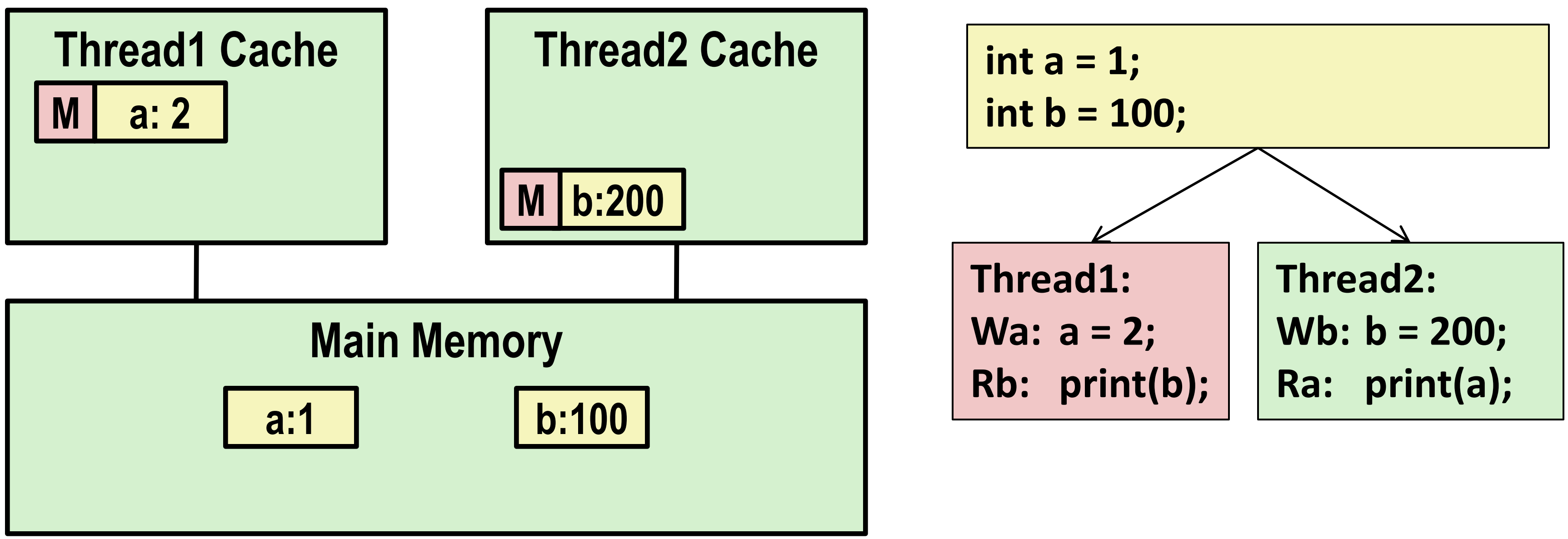
print 1

print 100

稍后，a:2和b:200被写回主存储器
At later points, a:2 and b:200 are written back to main memory

Snoopy缓存
Snoopy Caches

- 用状态标记每个缓存块 **Tag each cache block with state**
  - 无效 Invalid　　不能使用其值 Cannot use value
  - 共享 Shared　　可读拷贝 Readable copy
  - 修改 Modified　　可写拷贝 Writeable copy



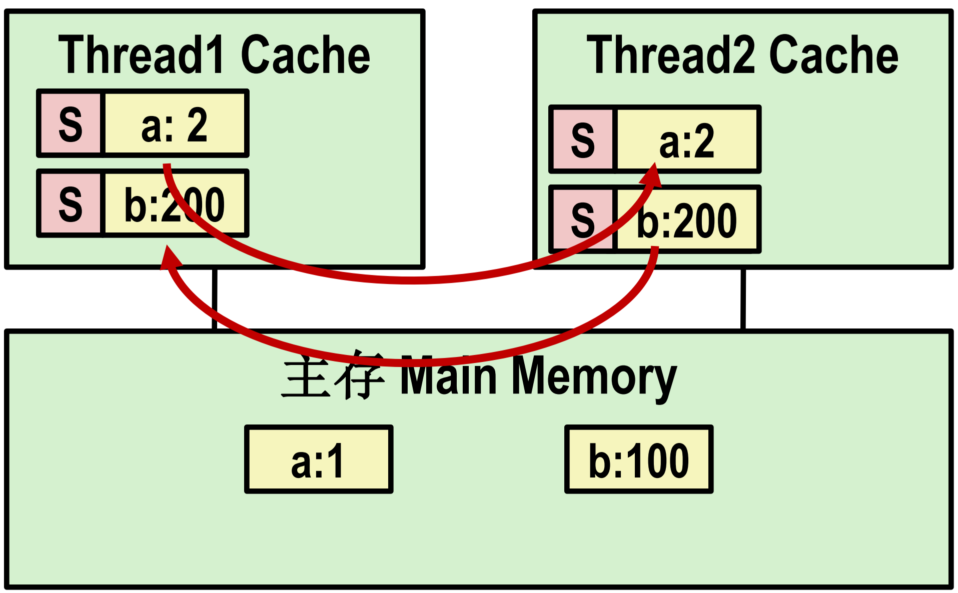
Snoopy缓存
Snoopy Caches

```
int a = 1;
int b = 100;
```

Thread1:	Thread2:
Wa: a = 2;	Wb: b = 200;
Rb: print(b);	Ra: print(a);

- 用状态标记每个缓存块
- **Tag each cache block with state**

无效	Invalid	不能使用其值 Cannot use value
共享	Shared	可读拷贝 Readable copy
修改	Modified	可写拷贝 Writeable copy



print 2

print 200

- 当缓存看到对其M标记块之一的请求时 When cache sees request for one of its M-tagged blocks
  - 从缓存提供值（注意：内存中的值可能已过时） Supply value from cache (Note: value in memory may be stale)
  - 将标记设置为S Set tag to S

内存一致性 Memory Consistency

```
int a = 1;
int b = 100;
```

```
Thread1:
Wa:  a = 2;
Rb:   print(b);
```

```
Thread2:
Wb:  b = 200;
Ra:   print(a);
```

线程一致性约束
Thread consistency constraints

Wa → Rb

Wb → Ra

- **打印的可能值是什么？ What are the possible values printed?**
  - 取决于内存一致性模型 Depends on memory consistency model
  - 硬件如何处理并发访问的抽象模型 Abstract model of how hardware handles concurrent accesses

内存一致性 Memory Consistency

```
int a = 1;
int b = 100;
```

```
Thread1:
Wa:  a = 2;
Rb:  print(b);
```

```
Thread2:
Wb:  b = 200;
Ra:  print(a);
```

线程一致性约束
Thread consistency constraints

Wa → Rb

Wb → Ra

- 打印的可能值是什么？ **What are the possible values printed?**
  - 取决于内存一致性模型 Depends on memory consistency model
  - 硬件如何处理并发访问的抽象模型 Abstract model of how hardware handles concurrent accesses
- 顺序一致性 **Sequential consistency**
  - 就好像一次只有一个操作一样，其顺序与每个线程内的操作顺序一致 As if only one operation at a time, in an order consistent with the order of operations within each thread
  - 因此，总体效果与每个单独的线程一致，但允许任意交错 Thus, overall effect consistent with each individual thread but otherwise allows an arbitrary interleaving

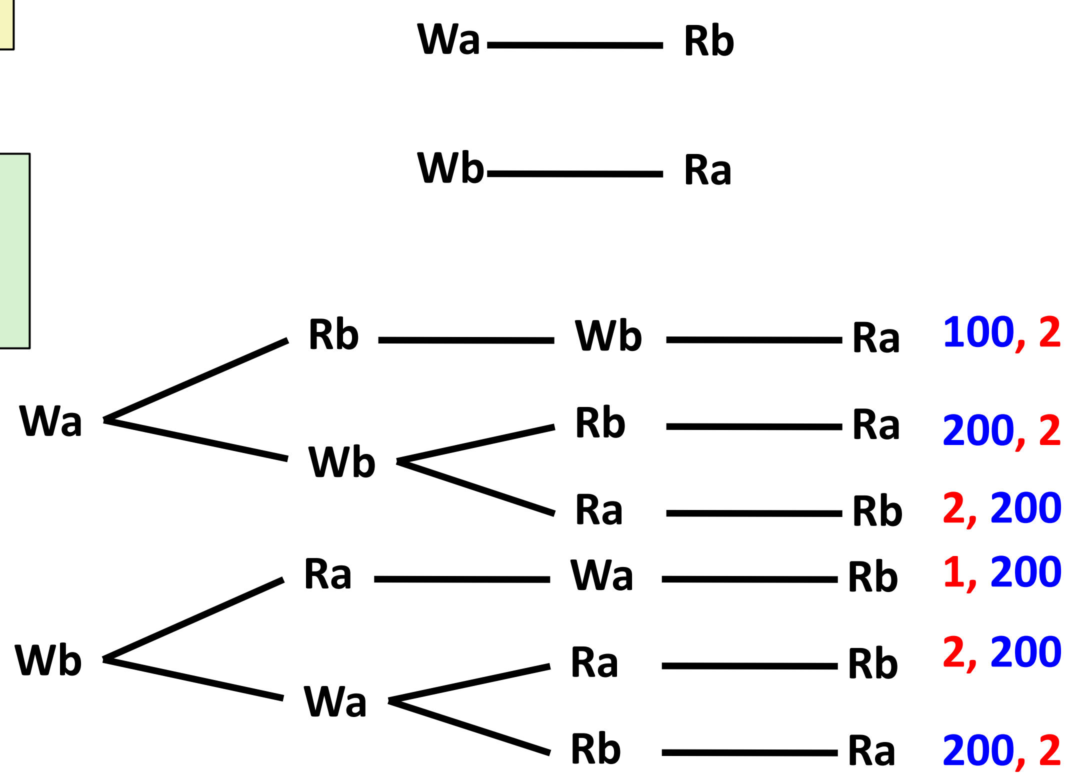
顺序一致性示例
Sequential Consistency Example

```
int a = 1;
int b = 100;
```

```
Thread1:
Wa:  a = 2;
Rb:  print(b);
```

```
Thread2:
Wb:  b = 200;
Ra:  print(a);
```

线程一致性约束 Thread consistency constraints



- 不可能输出 **Impossible outputs**
  - 100, 1 and 1, 100
  - 需要在Wa或Wb之前达到Ra和Rb Would require reaching *both* Ra and Rb before *either* Wa or Wb

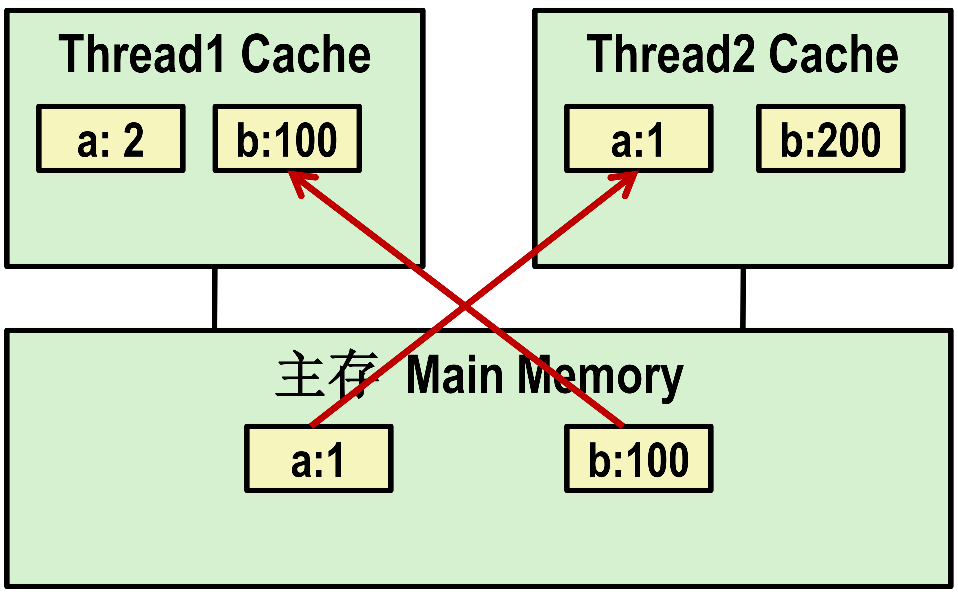
非一致性缓存方案
Non-Coherent Cache Scenario

- 写回缓存，线程间没有协作
  Write-back caches, without coordination between them

```
int a = 1;
int b = 100;
```

```
Thread1:
Wa:  a = 2;
Rb:  print(b);
```

```
Thread2:
Wb:  b = 200;
Ra:  print(a);
```



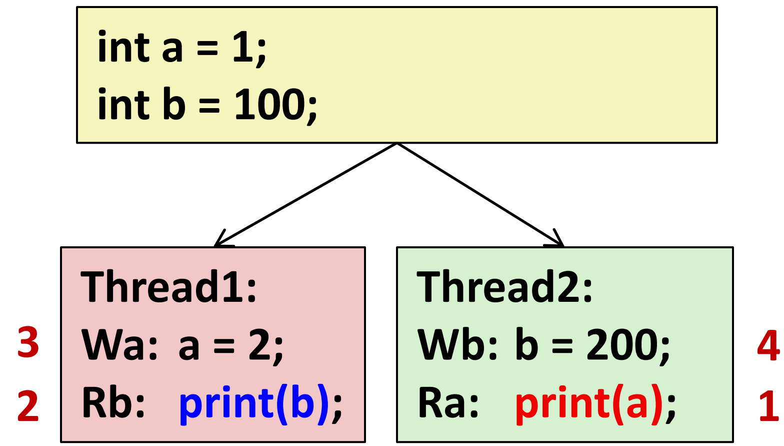
print 1

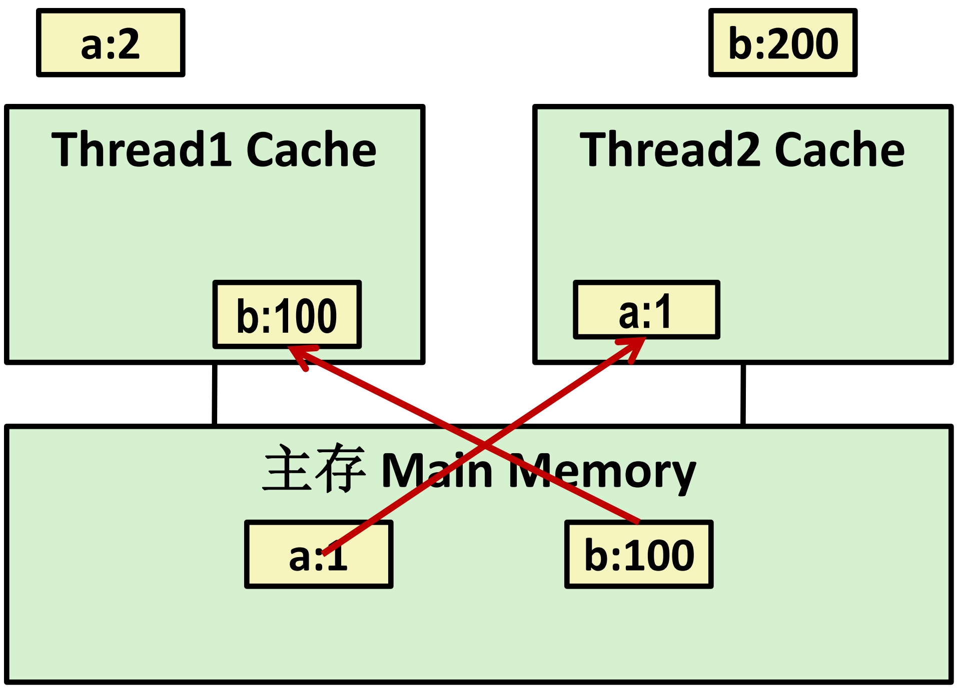
print 100

顺序一致性？ 否！
Sequentially consistent? No!

非顺序一致性方案
Non-Sequentially Consistent Scenario

- 一致性缓存，但由于操作重新排序而违反了线程一致性约束 **Coherent caches, but thread consistency constraints violated due to *operation reordering***



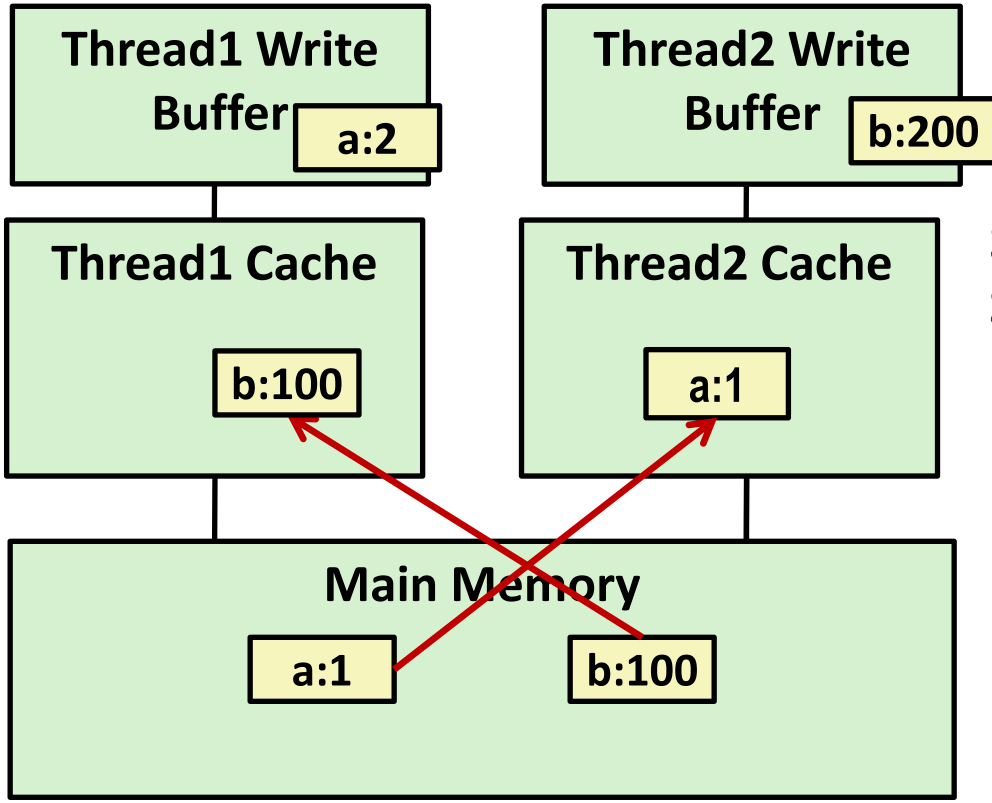


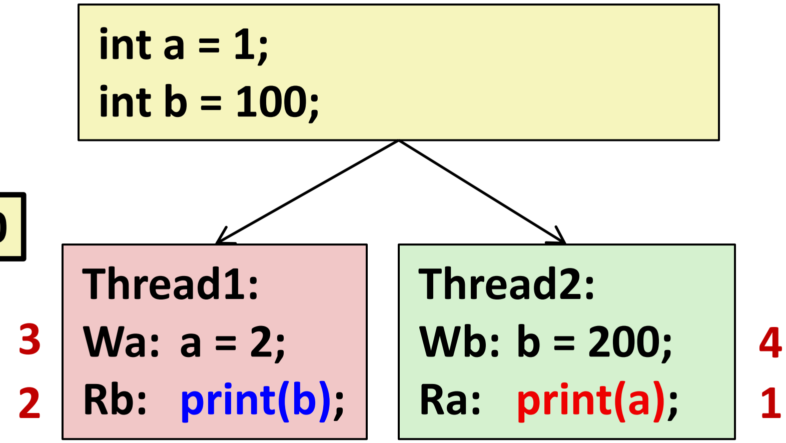
print 1

print 100

- 体系结构允许读取在写入之前完成，因为单个线程访问不同的内存位置 **Architecture lets reads finish before writes because single thread accesses different memory locations**

非顺序一致性方案
Non-Sequentially Consistent Scenario





- 为什么重新排序？写入需要很长时间。缓冲区写入，让读取继续。*指令级并行性* Why Reordered? Writes take long time. Buffer write, let read go ahead. *Instruction-level parallelism*

- 修复：在Wa&Rb和Wb&Ra之间添加SFENCE指令 Fix: Add SFENCE instructions between Wa & Rb and Wb & Ra

内存模型 Memory Models

- 顺序一致： **Sequentially Consistent:**
  - 每个线程以正确的顺序执行，任意交错 Each thread executes in proper order, any interleaving

- 为了确保，需要 **To ensure, requires**
  - 正确的缓存/内存行为 Proper cache/memory behavior
  - 适当的线程内排序约束 Proper intra-thread ordering constraints

- 线程排序约束 **Thread ordering constraints**
  - 使用同步确保程序没有数据竞争 Use synchronization to ensure the program is free of data races

议题 Today

- 并行计算硬件 **Parallel Computing Hardware**
  - 多核 Multicore
    - 单芯片上有多个独立的处理器 Multiple separate processors on single chip
  - 超线程化 Hyperthreading
    - 在单核上高效执行多个线程 Efficient execution of multiple threads on single core
- 一致性模型 **Consistency Models**
  - 当多个线程在读/写共享状态时会发生什么情况 What happens when multiple threads are reading & writing shared state
- 线程级并行 **Thread-Level Parallelism**
  - 将程序分成独立的任务 Splitting program into independent tasks
    - 例如：并行求和 Example: Parallel summation
    - 检查一些性能小工件 Examine some performance artifacts
  - 分而治之 Divide-and conquer parallelism
    - 例如：并行快速排序 Example: Parallel quicksort

求和示例 Summation Example

- **求数字0，…，N-1的和 Sum numbers 0, …, N-1**
  - 应该加起来得到(N-1)*N/2 Should add up to (N-1)*N/2
- **分区成K个区域 Partition into K ranges**
  - 每个区域有⌊N/K⌋个值 ⌊N/K⌋ values each
  - t个线程每个处理一个区域 Each of the *t* threads processes 1 range
  - 连续累加剩余值 Accumulate leftover values serially
- **方法#1：所有线程更新单个全局变量 Method #1: All threads update single global variable**
  - 1A：无同步 1A: No synchronization
  - 1B：用pthread信号量同步 1B: Synchronize with pthread semaphore
  - 1C：用pthread互斥锁同步 1C: Synchronize with pthread mutex
    - “二元”信号量，仅取值0和1 “Binary” semaphore. Only values 0 & 1

累积在单个全局变量中：声明 Accumulating in Single Global Variable: Declarations

```
typedef unsigned long data_t;
/* Single accumulator */
volatile data_t global_sum;
```

累积在单个全局变量中：声明
Accumulating in Single Global Variable: Declarations

```
typedef unsigned long data_t;
/* Single accumulator */
volatile data_t global_sum;

/* Mutex & semaphore for global sum */
sem_t semaphore;
pthread_mutex_t mutex;
```

累积在单个全局变量中：声明
Accumulating in Single Global Variable: Declarations

```
typedef unsigned long data_t;
/* Single accumulator */
volatile data_t global_sum;

/* Mutex & semaphore for global sum */
sem_t semaphore;
pthread_mutex_t mutex;

/* Number of elements summed by each thread */
size_t nelems_per_thread;

/* Keep track of thread IDs */
pthread_t tid[MAXTHREADS];

/* Identify each thread */
int myid[MAXTHREADS];
```

累积在单个全局变量中：操作 Accumulating in Single Global Variable: Operation

```
    nelems_per_thread = nelems / nthreads;

    /* Set global value */
    global_sum = 0;

    /* Create threads and wait for them to finish */
    for (i = 0; i < nthreads; i++) {
        myid[i] = i;
        Pthread_create(&tid[i], NULL, thread_fun, &myid[i]);
    }
    for (i = 0; i < nthreads; i++)
        Pthread_join(tid[i], NULL);

    result = global_sum;

    /* Add leftover elements */
    for (e = nthreads * nelems_per_thread; e < nelems; e++)
        result += e;
```

线程ID Thread ID

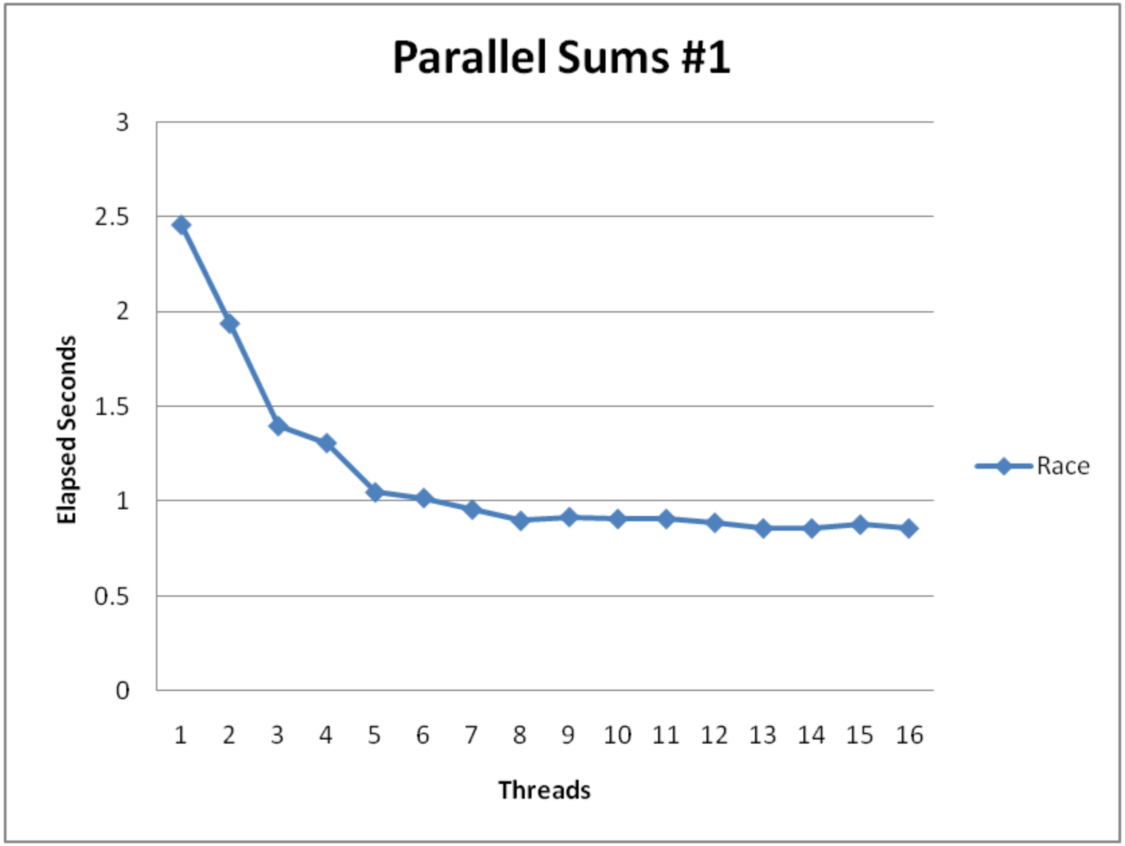
线程例程 Thread routine

线程参数 Thread argum
(void *p)

线程函数：无同步
Thread Function: No Synchronization

```c
void *sum_race(void *vargp)
{
    int myid = *((int *)vargp);
    size_t start = myid * nelems_per_thread;
    size_t end = start + nelems_per_thread;
    size_t i;

    for (i = start; i < end; i++) {
        global_sum += i;
    }
    return NULL;
}
```

无同步的性能 Unsynchronized Performance



- **N = 2^{30}**
- 最佳的加速比 **Best speedup = 2.86X**
- 当大于1个线程时得到错误的答案 **Gets wrong answer when > 1 thread!** 为何？ **Why?**

线程函数：信号量/互斥锁
Thread Function: Semaphore / Mutex

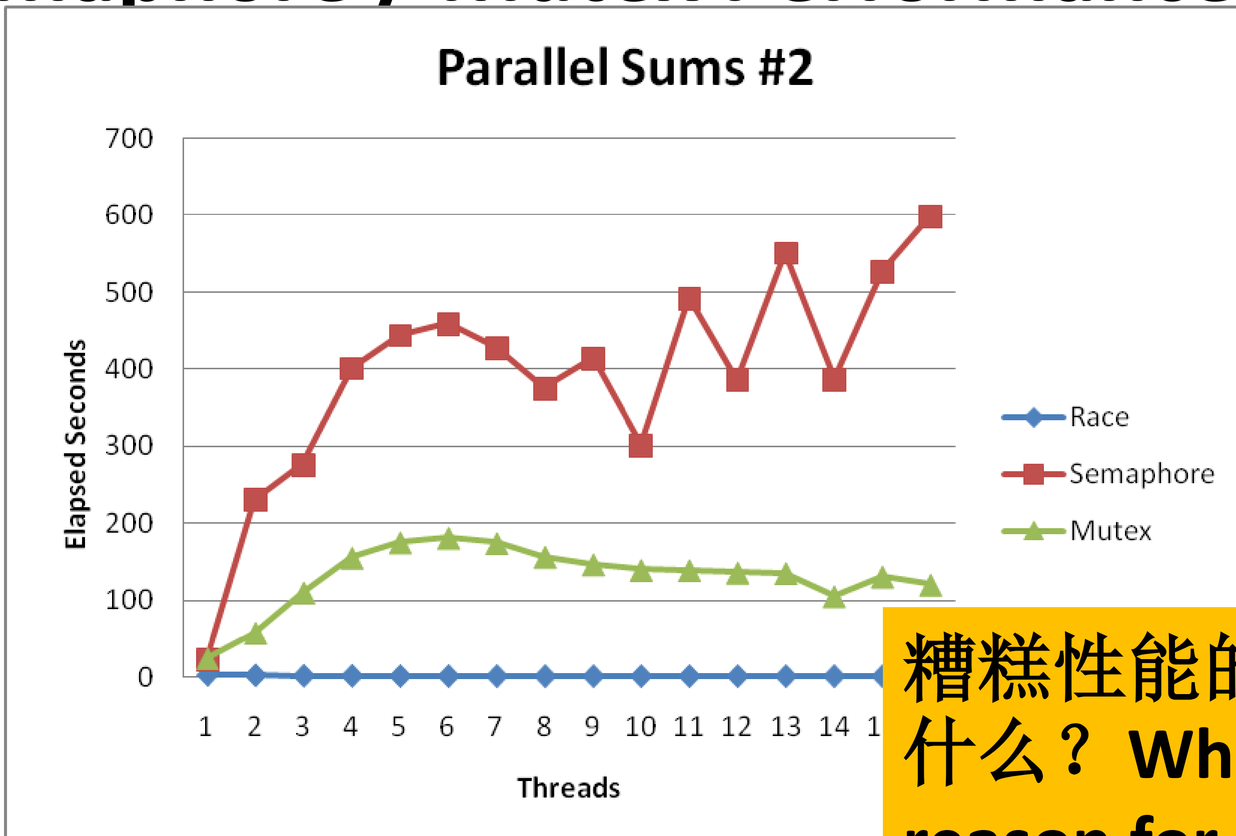
信号量 **Semaphore**

```
void *sum_sem(void *vargp)
{
    int myid = *((int *)vargp);
    size_t start = myid * nelems_per_thread;
    size_t end = start + nelems_per_thread;
    size_t i;

    for (i = start; i < end; i++) {
        sem_wait(&semaphore);
        global_sum += i;
        sem_post(&semaphore);
    }
    return NULL;
}
```

互斥锁 **Mutex**

```
pthread_mutex_lock(&mutex);
global_sum += i;
pthread_mutex_unlock(&mutex);
```

信号量/互斥锁性能 Semaphore / Mutex Performance



糟糕性能的主要原因是什么？ What is main reason for poor performance?

- 糟糕的性能 **Terrible Performance**
  - 2.5 seconds 秒 ➔ ~10 minutes 分钟
- 互斥锁比信号量快3倍 **Mutex 3X faster than semaphore**
- 很明显，这些方法都不成功 **Clearly, neither is successful**

单独累积 Separate Accumulation

- **方法#2：每个线程累积到单独的变量中 Method #2: Each thread accumulates into separate variable**
  - 2A：在相邻数组元素中累加 2A: Accumulate in contiguous array elements
  - 2B：在间隔开的数组元素中累加 2B: Accumulate in spaced-apart array elements
  - 2C：在寄存器中累加 2C: Accumulate in registers

```
/* Partial sum computed by each thread */
data_t psum[MAXTHREADS*MAXSPACING];

/* Spacing between accumulators */
size_t spacing = 1;
```

单独累积：操作
Separate Accumulation: Operation

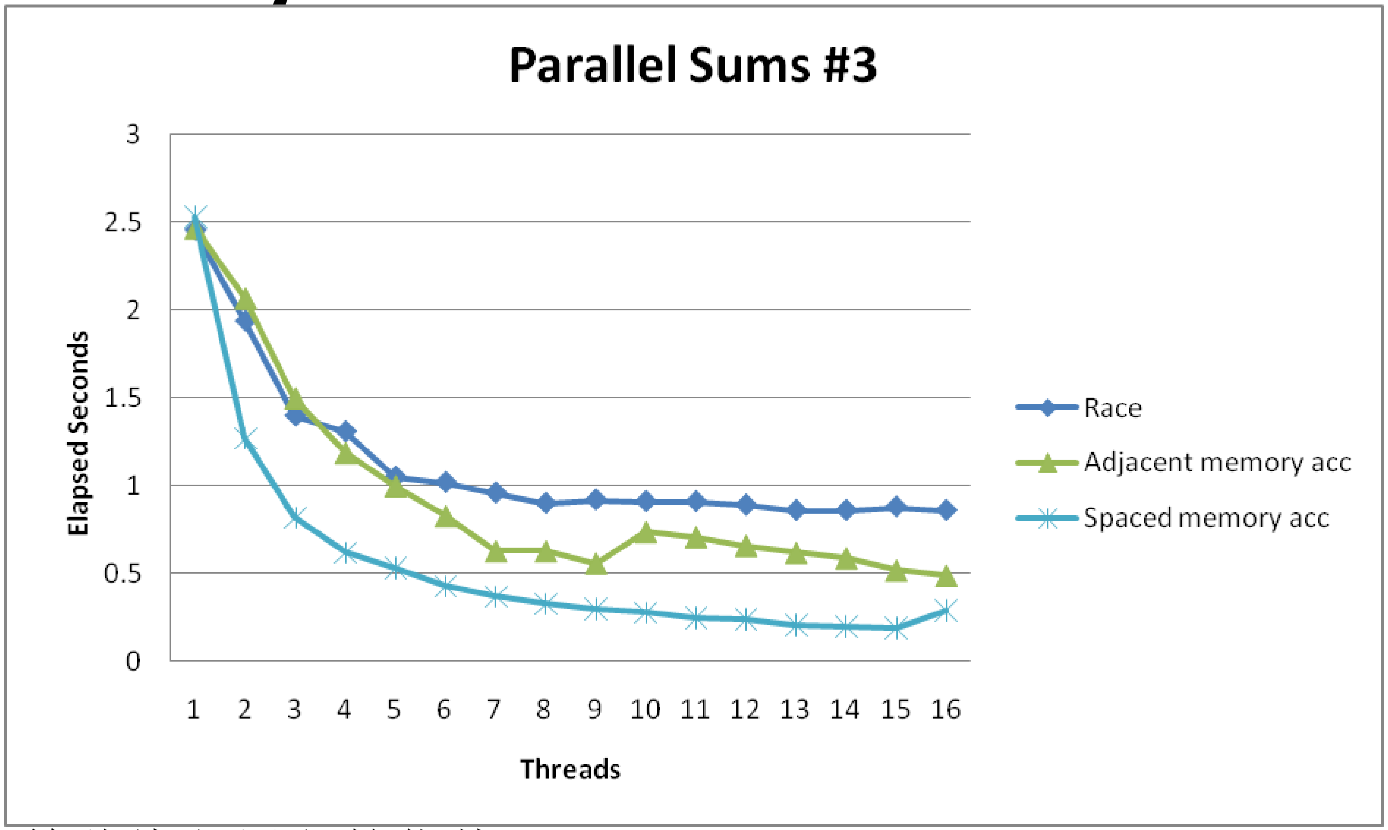
```
    nelems_per_thread = nelems / nthreads;

    /* Create threads and wait for them to finish */
    for (i = 0; i < nthreads; i++) {
        myid[i] = i;
        psum[i*spacing] = 0;
        Pthread_create(&tid[i], NULL, thread_fun, &myid[i]);
    }
    for (i = 0; i < nthreads; i++)
        Pthread_join(tid[i], NULL);

    result = 0;

    /* Add up the partial sums computed by each thread */
    for (i = 0; i < nthreads; i++)
        result += psum[i*spacing];

    /* Add leftover elements */
    for (e = nthreads * nelems_per_thread; e < nelems; e++)
        result += e;
```

线程函数：内存累积
Thread Function: Memory Accumulation

互斥锁在哪？ Where is the mutex?

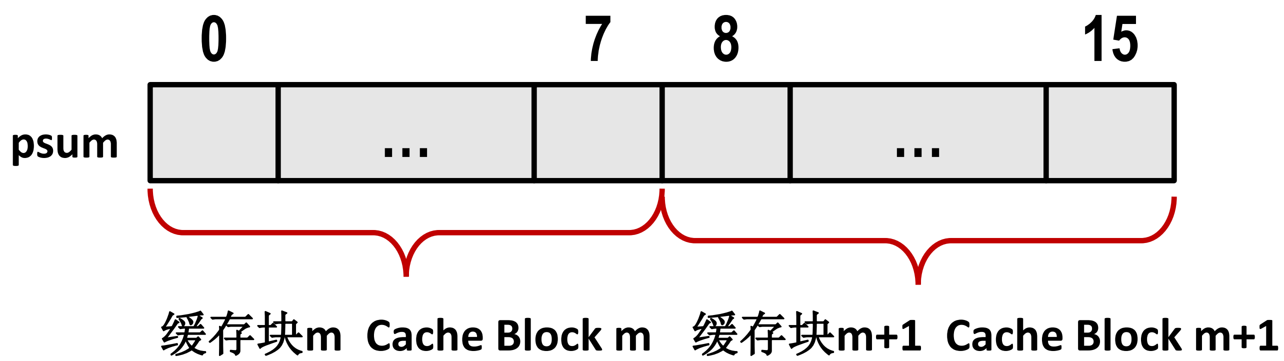
```
void *sum_global(void *vargp)
{
    int myid = *((int *)vargp);
    size_t start = myid * nelems_per_thread;
    size_t end = start + nelems_per_thread;
    size_t i;

    size_t index = myid*spacing;
    psum[index] = 0;
    for (i = start; i < end; i++) {
        psum[index] += i;
    }
    return NULL;
}
```

内存累积性能
Memory Accumulation Performance



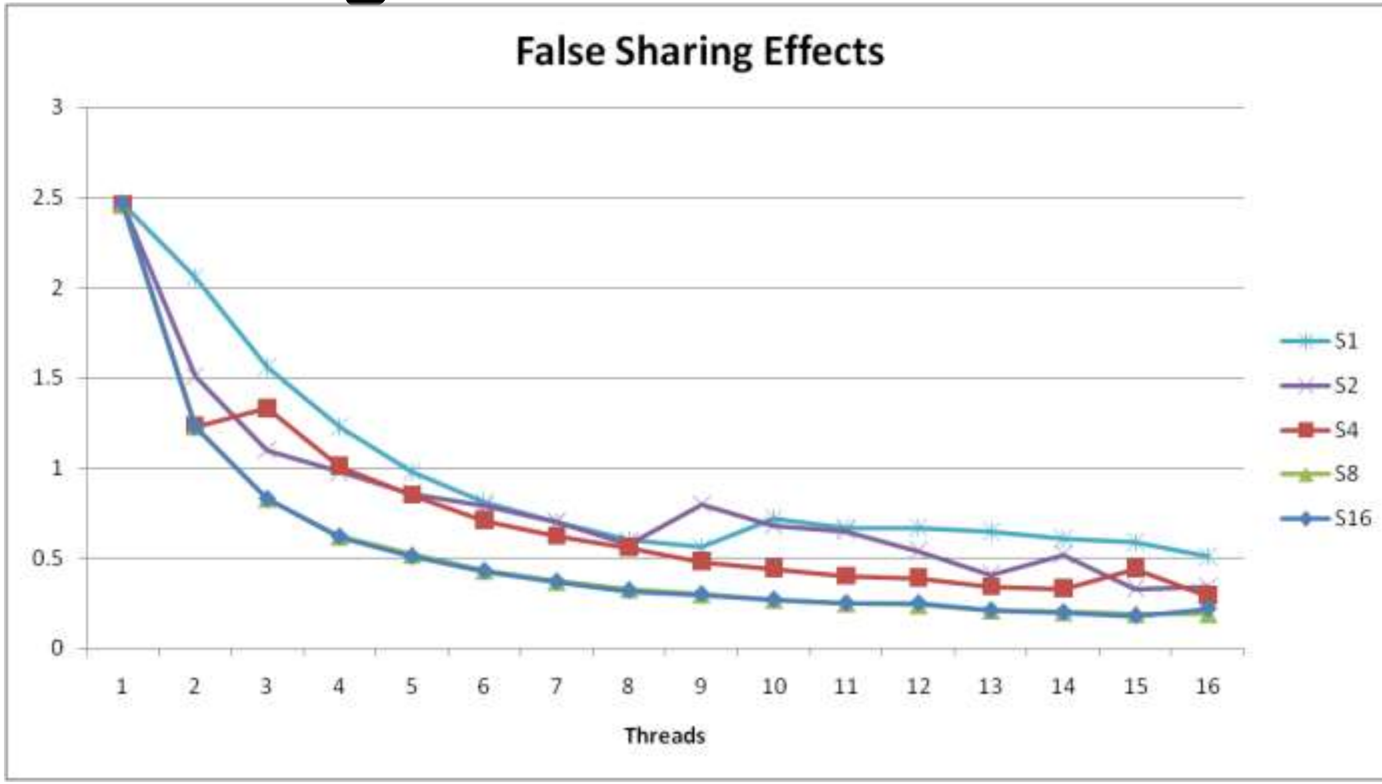
- 单独线程累积的优势 **Clear threading advantage**
  - 连续累积加速比： Adjacent speedup: 5 X
  - 间隔累积加速比： Spaced-apart speedup: 13.3 X (仅观察到加速比大于8 Only observed speedup > 8)
- 为什么进行间隔开累加性能更佳？ **Why does spacing the accumulators apart matter?**

虚假共享 False Sharing



- **缓存块上保持一致性 Coherence maintained on cache blocks**
- **要更新psum[i]，线程i必须具有独占访问权限 To update psum[i], thread i must have exclusive access**
  - 共享公共缓存块的线程将继续为访问块而相互争斗 Threads sharing common cache block will keep fighting each other for access to block

虚假共享的性能
False Sharing Performance



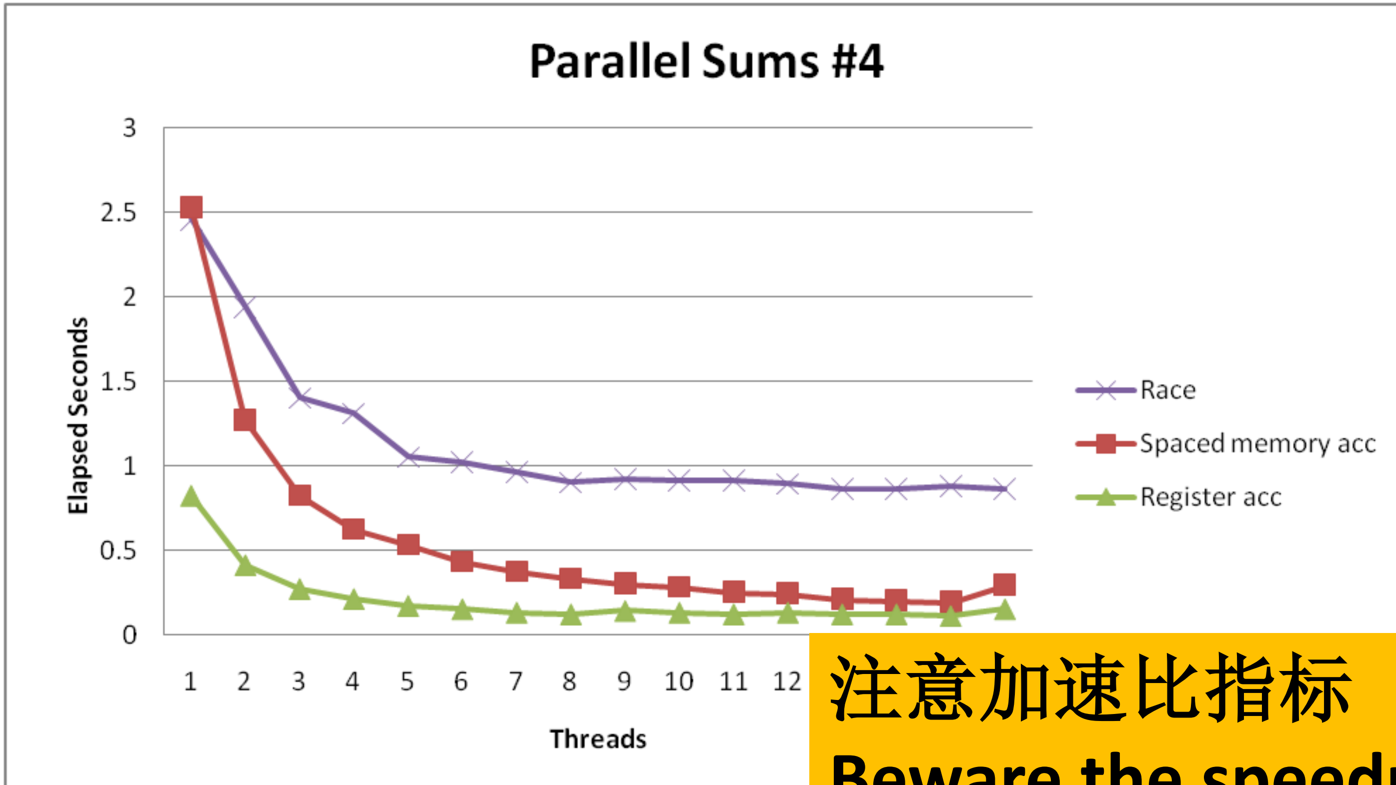
- 最佳间隔性能比最佳相邻性能高2.8倍 Best spaced-apart performance 2.8 X better than best adjacent

- **演示缓存块大小为64 Demonstrates cache block size = 64**
  - 8字节值 8-byte values
  - 将间隔增加到8以上没有性能改善 No benefit increasing spacing beyond 8

线程函数：寄存器累积
Thread Function: Register Accumulation

```
void *sum_local(void *vargp)
{
    int myid = *((int *)vargp);
    size_t start = myid * nelems_per_thread;
    size_t end = start + nelems_per_thread;
    size_t i;
    size_t index = myid*spacing;
    data_t sum = 0;
    for (i = start; i < end; i++) {
        sum += i;
    }
    psum[index] = sum;
    return NULL;
}
```

寄存器累积性能
Register Accumulation Performance



注意加速比指标
Beware the speedup metric!

- 单独线程累积优势 **Clear threading advantage**
  - 加速比/Speedup = 7.5 X
- 比最快的内存累积好2倍 **2X better than fastest memory accumulation**

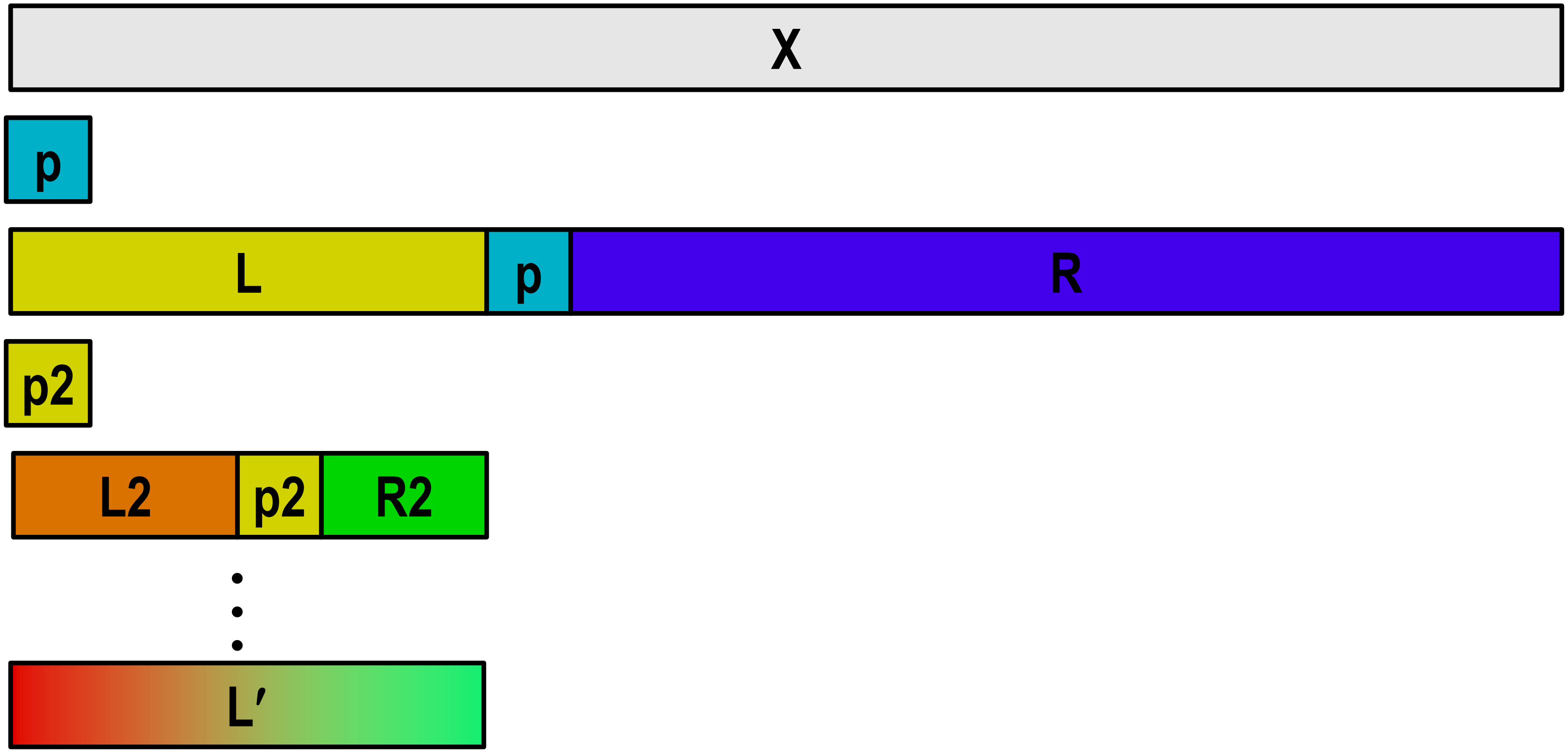
经验教训 Lessons learned

- 共享内存可能开销很高 **Sharing memory can be expensive**
  - 关注真实共享 Pay attention to true sharing
  - 注意虚假共享 Pay attention to false sharing
- 尽可能使用寄存器 **Use registers whenever possible**
  - (记住cachelab Remember cachelab)
  - 尽可能使用本地缓存 Use local cache whenever possible
- 处理剩余的数据 **Deal with leftovers**
- 在检查性能时，与最佳顺序实现进行比较 **When examining performance, compare to best possible sequential implementation**

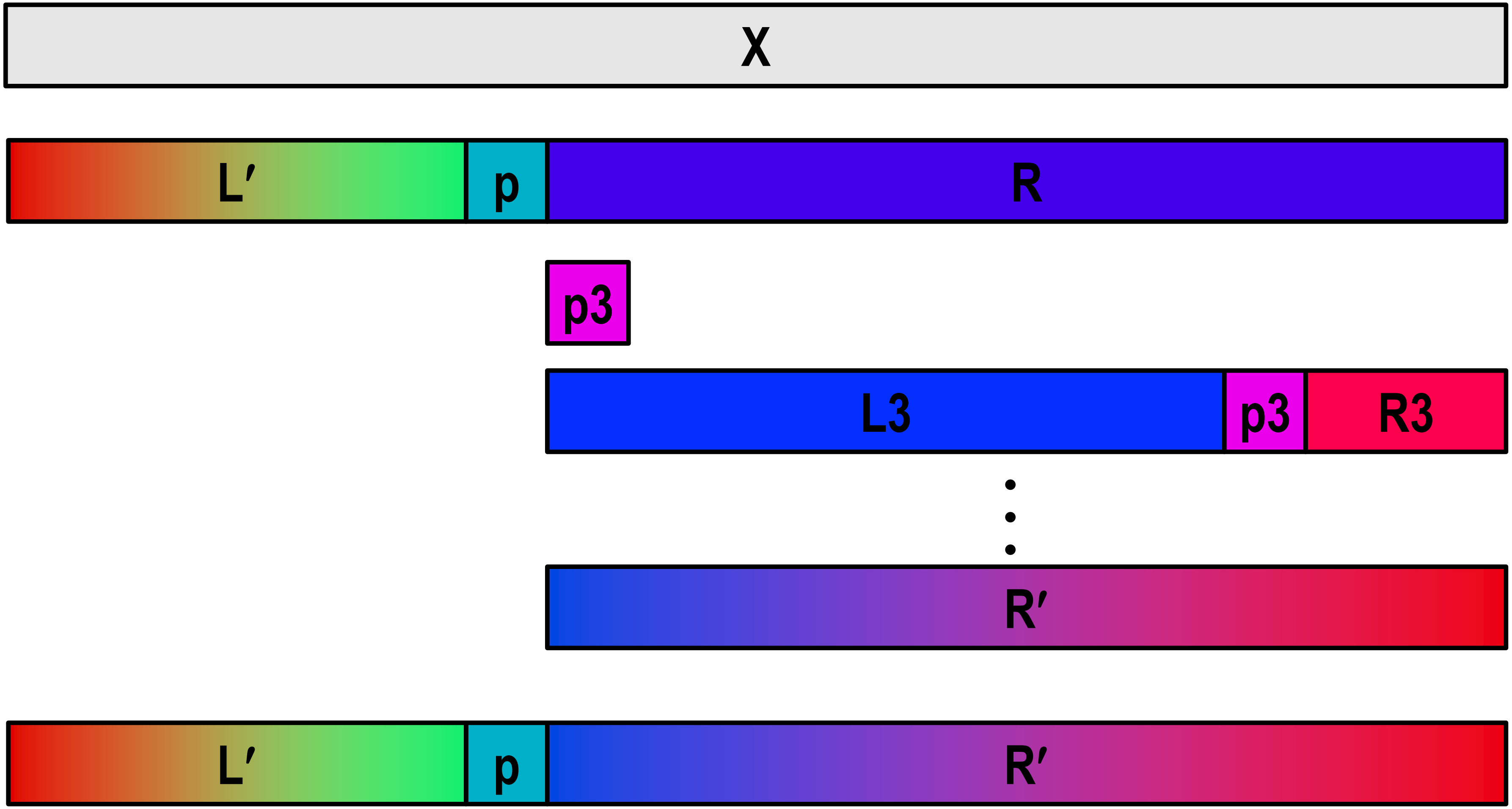
更重要的示例：排序
A More Substantial Example: Sort

- **N个随机数集合排序 Sort set of N random numbers**
- 多种可能的算法 **Multiple possible algorithms**
  - 使用并行版本的快速排序 Use parallel version of quicksort
- 对**X**集合进行顺序快速排序 **Sequential quicksort of set of values X**
  - 从X选择“中心点”p　Choose “pivot” p from X
  - 重新排列X　Rearrange X into
    - 左边集合：值小于等于p　L: Values $\leq$ p
    - 右边集合：值大于等于p　R: Values $\geq$ p
  - 对左边集合进行递归排序得到L′　Recursively sort L to get L′
  - 对右边集合进行递归排序得到R′　Recursively sort R to get R′
  - 返回　Return L′ : p : R′

顺序快速排序可视化
Sequential Quicksort Visualized



顺序快速排序可视化
Sequential Quicksort Visualized



顺序快速排序代码
Sequential Quicksort Code

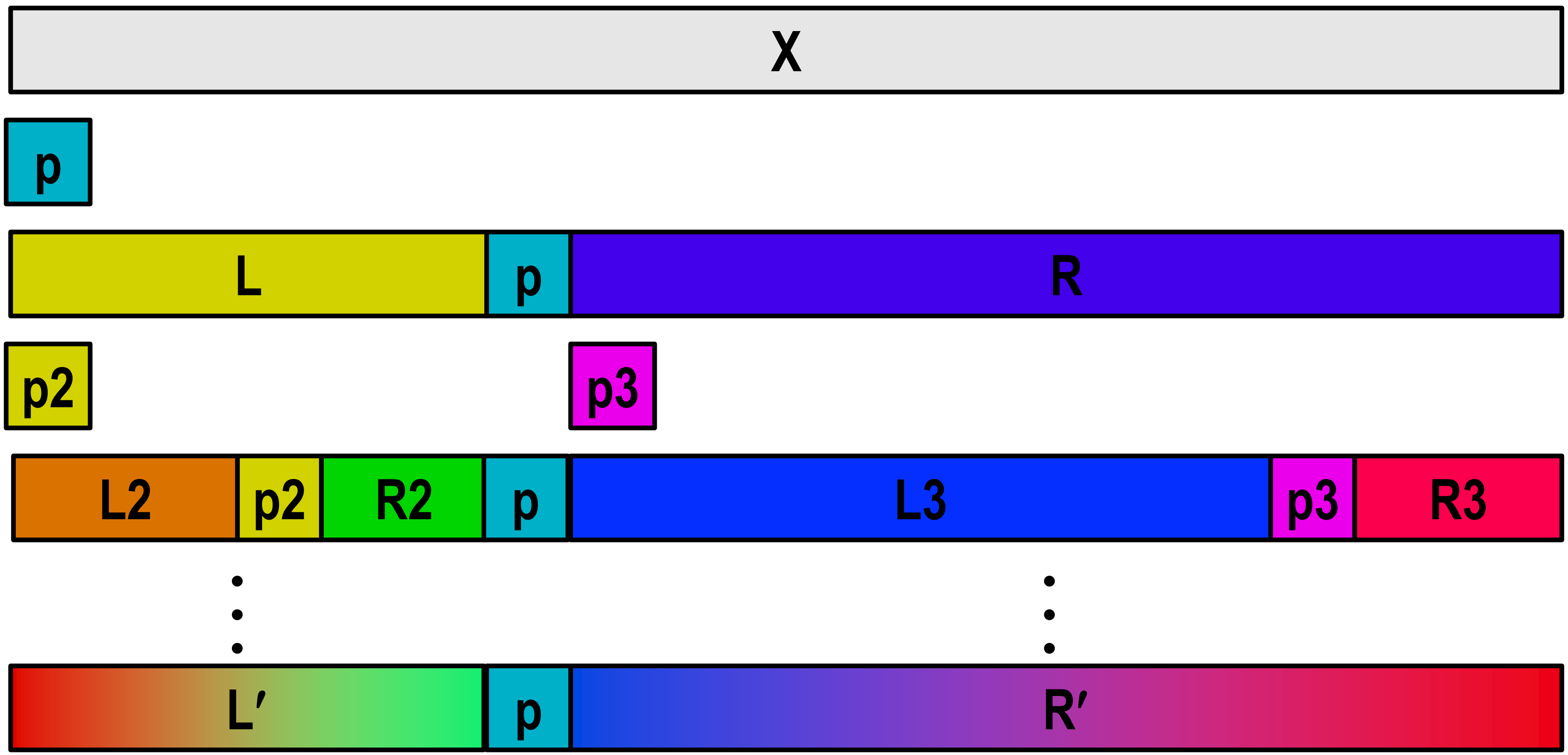
```
void qsort_serial(data_t *base, size_t nele) {
  if (nele <= 1)
    return;
  if (nele == 2) {
    if (base[0] > base[1])
      swap(base, base+1);
    return;
  }

  /* Partition returns index of pivot */
  size_t m = partition(base, nele);
  if (m > 1)
    qsort_serial(base, m);
  if (nele-1 > m+1)
    qsort_serial(base+m+1, nele-m-1);
}
```

- 从base开始对nele个元素排序 **Sort nele elements starting at base**
  - 如果有多于一个元素，则递归排序L或R Recursively sort L or R if has more than one element

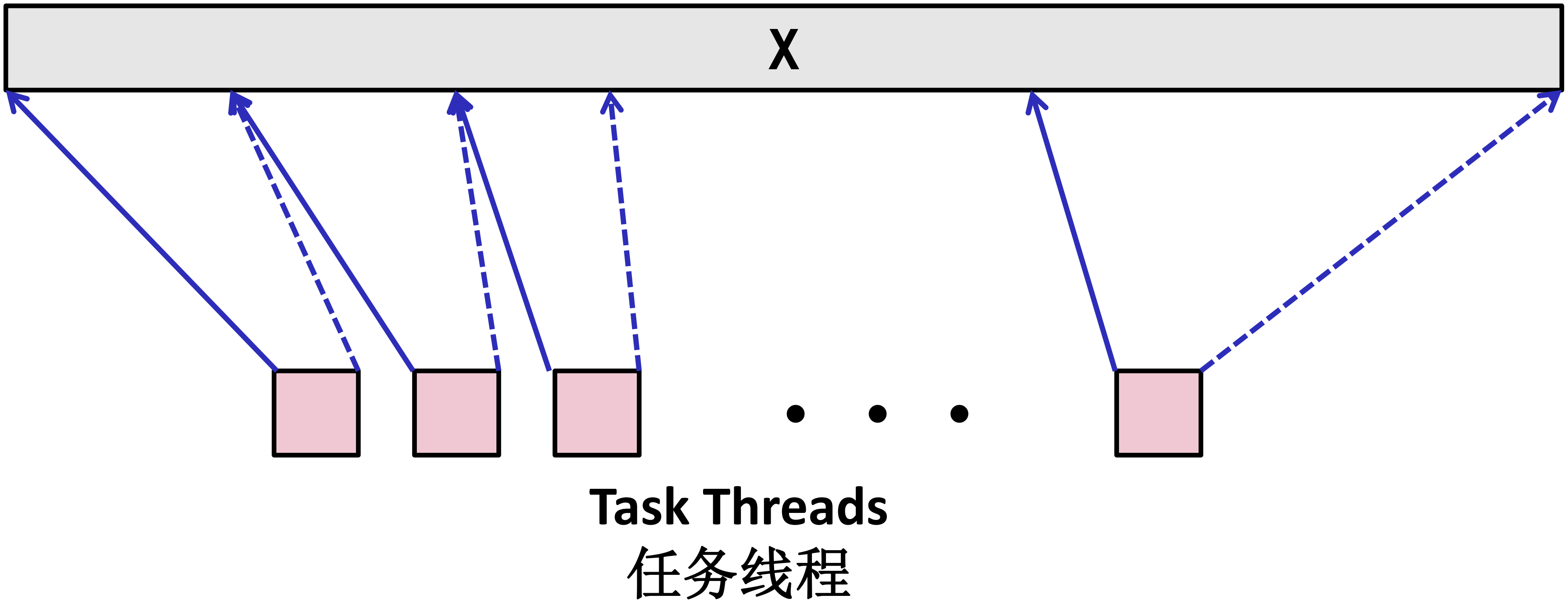
并行快速排序 Parallel Quicksort

- 集合X的并行快速排序 **Parallel quicksort of set of values X**
  - 如果N小于等于Nthresh，执行顺序快速排序 If N ≤ Nthresh, do sequential quicksort
  - 否则 Else
    - 从X选择“中心点”p Choose “pivot” p from X
    - 重新排列X Rearrange X into
      - 左集合：值小于等于p L: Values ≤ p
      - 右集合：值大于等于p R: Values ≥ p
    - 递归生成单独的线程 Recursively spawn separate threads
      - 排序L以获得L′ Sort L to get L′
      - 排序R以获得R′ Sort R to get R′
    - 返回 Return L′ : p : R′

并行快速排序可视化
Parallel Quicksort Visualized

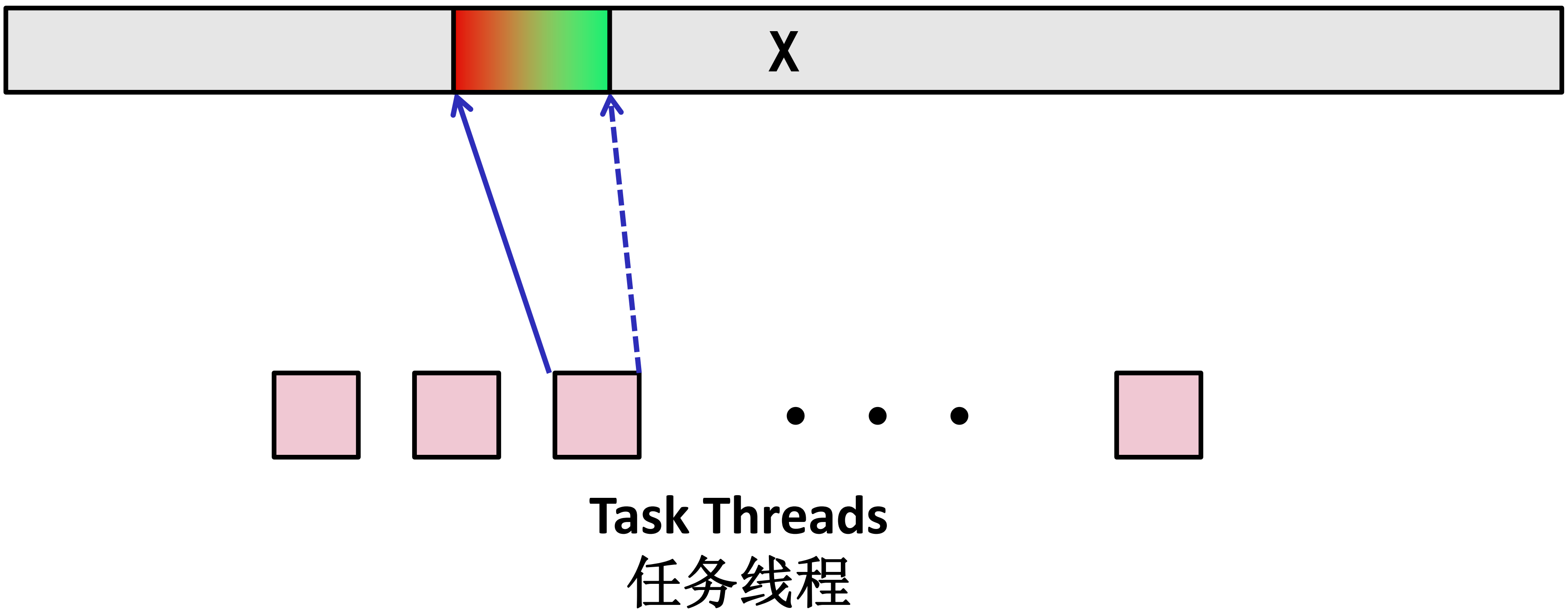


线程结构：排序任务
Thread Structure: Sorting Tasks



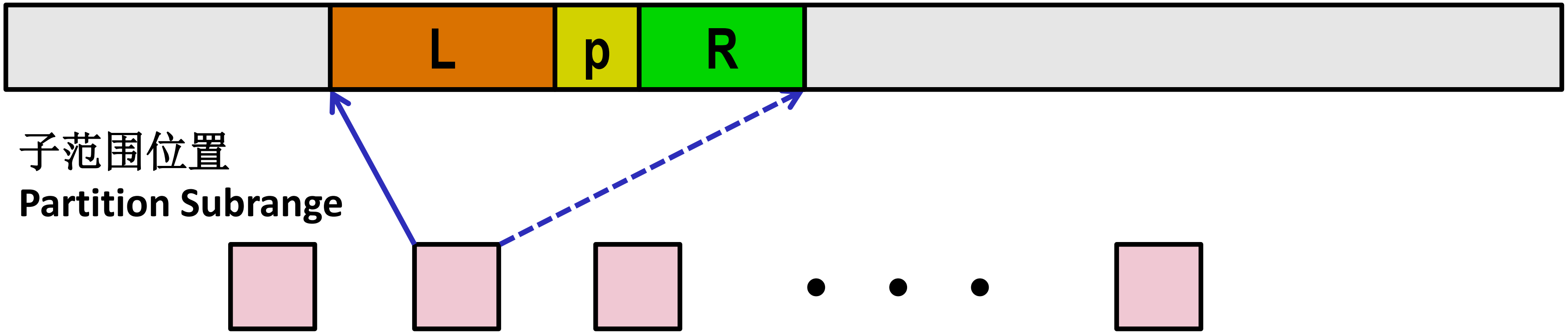
- 任务：排序子范围数据　**Task: Sort subrange of data**
  - 指定为：　Specify as:
    - base：起始地址　`base`: Starting address
    - nele：子范围中的元素数　`nele`: Number of elements in subrange
- 作为单独线程运行 **Run as separate thread**

小排序任务操作 Small Sort Task Operation

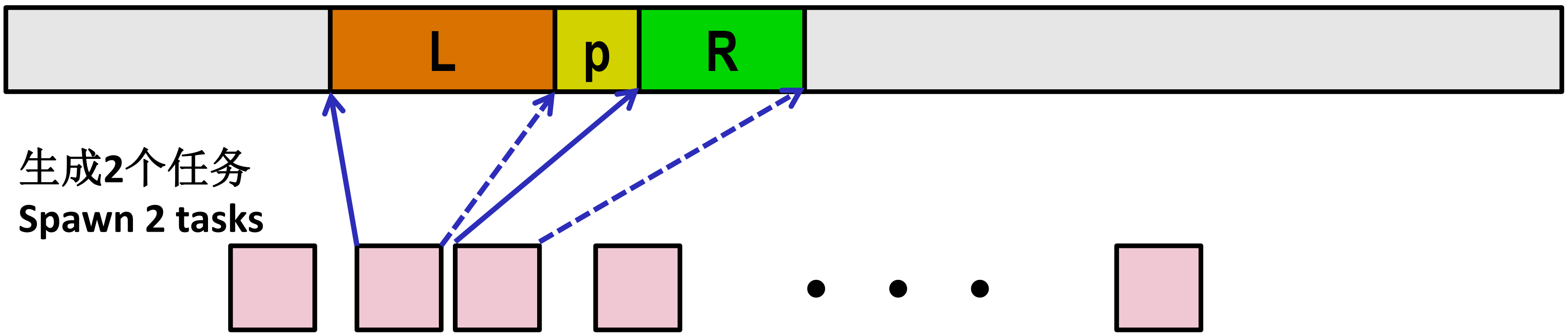


- 排序子范围数据使用串行快速排序 Sort subrange using serial quicksort

大排序任务操作
Large Sort Task Operation



子范围位置
Partition Subrange



生成2个任务
Spawn 2 tasks

顶层函数（简化）
Top-Level Function (Simplified)

```
void tqsort(data_t *base, size_t nele) {
    init_task(nele);
    global_base = base;
    global_end = global_base + nele - 1;
    task_queue_ptr tq = new_task_queue();
    tqsort_helper(base, nele, tq);
    join_tasks(tq);
    free_task_queue(tq);
}
```

- 初始化数据结构 **Sets up data structures**
- 调用递归排序例程 **Calls recursive sort routine**
- 保持加入线程，直到没有剩余 **Keeps joining threads until none left**
- 释放数据结构 **Frees data structures**

递归排序例程（简化）
Recursive sort routine (Simplified)

```
/* Multi-threaded quicksort */
static void tqsort_helper(data_t *base, size_t nele,
                          task_queue_ptr tq) {
    if (nele <= nele_max_sort_serial) {
        /* Use sequential sort */
        qsort_serial(base, nele);
       return;
    }
    sort_task_t *t = new_task(base, nele, tq);
    spawn_task(tq, sort_thread, (void *) t);
}
```

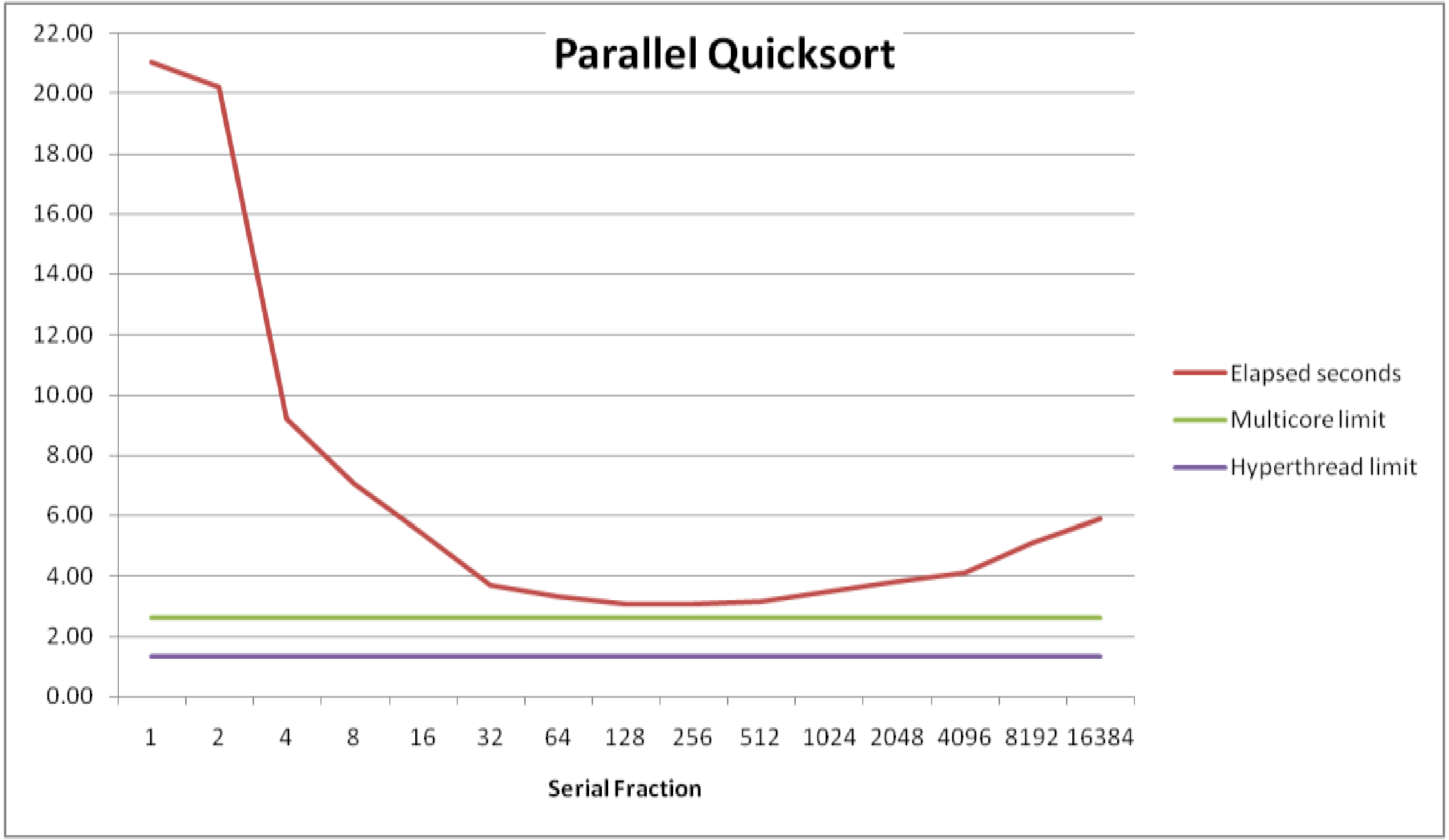
- 小分区：按顺序排序 **Small partition: Sort serially**
- 大分区：生成新的排序任务 **Large partition: Spawn new sort task**

排序任务线程（简化）
Sort task thread (Simplified)

```
/* Thread routine for many-threaded quicksort */
static void *sort_thread(void *vargp) {
    sort_task_t *t = (sort_task_t *) vargp;
    data_t *base = t->base;
    size_t nele = t->nele;
    task_queue_ptr tq = t->tq;
    free(vargp);
    size_t m = partition(base, nele);
    if (m > 1)
        tqsort_helper(base, m, tq);
    if (nele-1 > m+1)
        tqsort_helper(base+m+1, nele-m-1, tq);
    return NULL;
}
```

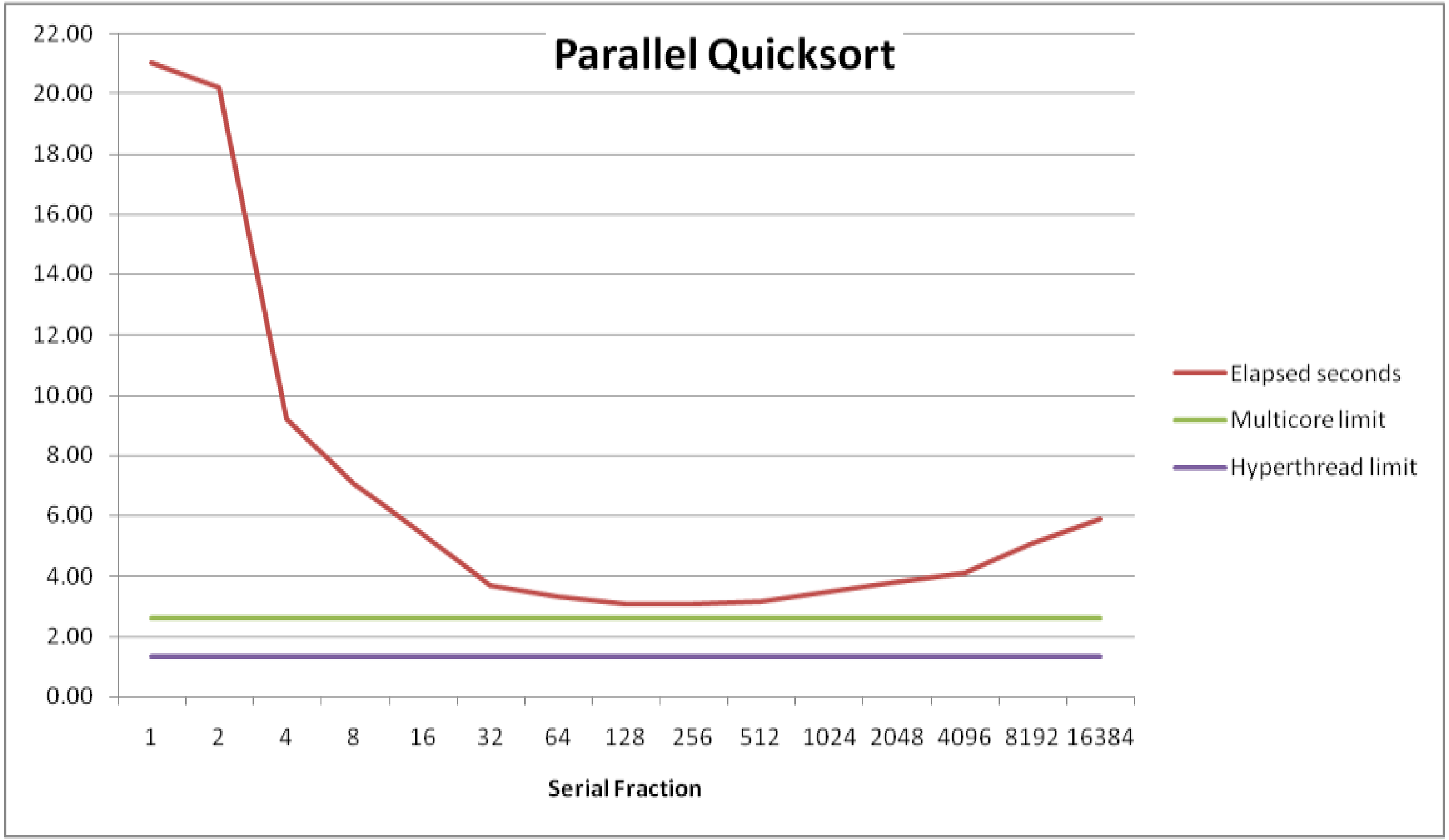
- 获取任务参数 **Get task parameters**
- 执行分区步骤 **Perform partitioning step**
- 在每个分区上调用递归排序例程（如果部分大小大于1）
  Call recursive sort routine on each partition (if size of part > 1)

并行快速排序性能
Parallel Quicksort Performance



- 串行比例：进行串行排序的输入比例 **Serial fraction: Fraction of input at which do serial sort**
- **排序128M随机值 Sort 2^{27} (134,217,728) random values**
- **最佳加速比 Best speedup = 6.84X**

并行快速排序性能
Parallel Quicksort Performance



- 在广泛的串行比例范围内表现良好 **Good performance over wide range of fraction values**
  - F太小：并行度不够 F too small: Not enough parallelism
  - F太大：线程开销太高 F too large: Thread overhead too high

阿姆达尔定律（旅行模拟）
Amdahl's Law (Travel Analogy)

		加速比 Speed-Up
从PIT直飞LHR Flying jet non-stop from PIT -> LHR:	7.5 Hours	1
或者，老式SST方式： Or, old fashioned SST way:		
Fly jet from PIT -> JFK: 1.5 Hours		
Fly SST from JFK -> LHR: 3.5 Hours	5 Hours	1.5x
或者，使用FTL Or, Using FTL:		
Fly jet from PIT -> JFK: 1.5 Hours		
Fly FTL from JFK -> LHR: .01 Hours	1.51 Hours	~5x

- 最好的加速比是5倍，即使是FTL，因为必须到达纽约 **Best possible speed up is 5X, even with FTL because have to get to New York.**
- **PIT**：匹兹堡 **LHR**：伦敦 **JFK**：纽约
- **SST**：超音速客机 **FTL**：超光速

阿姆达尔定律 Amdahl's Law

- **总体问题 Overall problem**
  - T所需的总顺序执行时间 T Total sequential time required
  - p可加速的总比例 p Fraction of total that can be sped up ($0 \leq p \leq 1$)
  - k加速系数 k Speedup factor
- **最终性能 Resulting Performance**
  - $T_k = pT/k + (1-p)T$
    - 可以加速的部分速度快k倍 Portion which can be sped up runs k times faster
    - 无法加速的部分保持不变 Portion which cannot be sped up stays the same
  - 最大可能加速比 Maximum possible speedup
    - $k = \infty$
    - $T_\infty = (1-p)T$

阿姆达尔定律（旅行模拟）
Amdahl’s Law (Travel Analogy)

		加速比 Speed-Up
■ 从PIT直飞LHR **Flying jet non-stop from PIT -> LHR:**	**7.5 Hours**	**1**
■ **或者，老式SST方式： Or, old fashioned SST way:**		
▪ Fly jet from PIT -> JFK: 1.5 Hours		
▪ Fly SST from JFK -> LHR: 3.5 Hours	**5 Hours**	**1.5x**
■ **或者，使用FTL Or, Using FTL:**		
▪ Fly jet from PIT -> JFK: 1.5 Hours		
▪ Fly FTL from JFK -> LHR: .01 Hours	**1.51 Hours**	**~5x**

- **最好的加速比是5倍，即使是FTL，因为必须到达纽约 Best possible speed up is 5X, even with FTL because have to get to New York.**
  - $T=7.5$, $p=6/7.5=.8$, $k=\infty \Rightarrow T_\infty = (1-p)T=1.5$ 最大加速比 max speed-up =5x

阿姆达尔定律的示例
Amdahl's Law Example

- 总体问题 **Overall problem**
  - T = 10 Total time required 所需总时间
  - p = 0.9 Fraction of total which can be sped up 可加速的总比例
  - k = 9 Speedup factor 加速系数
- 最终性能 **Resulting Performance**
  - $T_9 = 0.9 * 10/9 + 0.1 * 10 = 1.0 + 1.0 = 2.0$ (5倍加速比 a 5x speedup)
- 最大可能加速比 **Maximum possible speedup**
  - $T_\infty = 0.1 * 10.0 = 1.0$ (10倍加速比 a 10x speedup)
    - 拥有**无限的**并行计算资源！ With **infinite** parallel computing resources!
  - 极限加速比显示**算法**极限 Limit speedup shows **algorithmic** limitation

阿姆达尔定律和并行快速排序
Amdahl’s Law & Parallel Quicksort

- 顺序程序瓶颈 **Sequential bottleneck**
  - 顶层分区：无加速 Top-level partition: No speedup
  - 第二级：小于等于2倍加速比 Second level: ≤ 2X speedup
  - 第k级：小于等于2^{k-1}加速比 k^{th} level: ≤ 2^{k-1}X speedup
- 启示 **Implications**
  - 小规模并行的良好性能 Good performance for small-scale parallelism
  - 需要并行化分区步骤以获得大规模并行性 Would need to parallelize partitioning step to get large-scale parallelism
    - 基于规则抽样的并行排序 Parallel Sorting by Regular Sampling
      - “并行与分布式计算” H. Shi & J. Schaeffer, J. Parallel & Distributed Computing, 1992

经验教训 Lessons Learned

- **必须具有并行化策略 Must have parallelization strategy**
  - 划分为K个独立部分 Partition into K independent parts
  - 分而治之 Divide-and-conquer
- **内部循环必须无同步 Inner loops must be synchronization free**
  - 同步操作非常耗时 Synchronization operations very expensive
- **当心硬件瑕疵 Watch out for hardware artifacts**
  - 需要了解处理器和内存结构 Need to understand processor & memory structure
  - 共享和虚假共享全局数据 Sharing and false sharing of global data
- **当心阿姆达尔定律 Beware of Amdahl's Law**
  - 串行代码可能成为瓶颈 Serial code can become bottleneck
- **你能行！ You can do it!**
  - 实现适度的并行性并不困难 Achieving modest levels of parallelism is not difficult
  - 建立实验框架并测试多种策略 Set up experimental framework and test multiple strategies