How Strong are Keynesian Booms?

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Abstract

I study a HANK economy with wealth in the utility function. I show that adding wealth in the utility function lowers the MPCs of households and results in a smaller output response from a deficit shock.

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1 Introduction

Standard Keynesian logic suggests that fiscal deficits stimulate aggregate demand. If so, how large are these Keynesian booms? In a standard Heterogeneous Agent New Keynesian (HANK) economy, liquidity constraints induce non-Ricardian household consumption behavior, so fiscal deficits in HANK have outsized effects on aggregate demand relative to the Representative Agent New Keynesian (RANK) benchmark.

This paper documents the output response of fiscal deficits in a HANK economy with wealth in the utility function. The model is closely related to Angeletos et al. (2024). The key findings of this short paper is that wealth in the utility function 1) lowers the MPCs of households and 2) results in a smaller output response from a deficit shock.

Section 2 introduces a stylized HANK model to derive analytical results. Section 3 constructs a quantitative HANK model to study the effect of a fiscal deficit shock on output and inflation. I conclude in Section 4.

2 A Stylized HANK Economy

I present a partial equilibrium perpetual-youth OLG economy with wealth in the utility function. While this economy lacks certain features of fully-specified HANK models, such as income heterogeneity and liquidity constraints, it still captures the key mechanism that generates non-Ricardian household consumption behavior, namely more discounting of the future.

Model. Time is discrete. There is a measure one continuum of households. Each household has a probability $\omega \in (0, 1]$ of surviving to the next period and is replaced by a new household in the case of death.

Households have access to save and borrow a actuarially fair, risk-free, real annuity $\tilde{A}_{i,t}$ backed by government bonds. If a household survives to the next period, their return on the annuity is R_t/ω, where R_t is the real return on government bonds. Households receive labor income and dividend income $W_t L_{i,t}$ and Q_t and pay taxes $T_{i,t}$. To simplify the analysis, I abstract from heterogeneity in labor supply by assuming that labor unions demand identical hours worked from all households, so $L_{i,t} = L_t$.

Old households contribute to a social fund $S_{i,t}$ that transfers their wealth to newborn households. In total, the date-t budget constraint of household i is given as

$$\frac{\omega}{R_t}\tilde{A}_{i,t+1} = \tilde{A}_{i,t} + Y_{i,t} - C_{i,t} - T_{i,t} + S_{i,t}$$

where $Y_{i,t} = W_t L_{i,t} + Q_t$ is total income.

Letting D^{ss} be the steady-state level of government debt, I set transfers for new households to $S_{i,t} = S^{new} = D^{ss}$ for new households and transfers for old households to $S_{i,t} = S^{old} = -\frac{1-\omega}{\omega}D^{ss}$. The social fund is balanced, i.e. $(1-\omega)S^{new} + \omega S^{old} = 0$. My specification for the social fund guarantees all households have the same wealth and consumption in the steady state. To see why, notice that the steady state values of $\tilde{A}_{i,t}$ for new households and old households is $\tilde{A}^{new} = 0$ and $\tilde{A}^{old} = \frac{D^{ss}}{\omega}$ by the asset market clearing condition. Hence, real financial wealth inclusive of social payments, $A_{i,t} = \tilde{A}_{i,t} + S_{i,t}$, satisfies the following steady-state relation

$$A^{ss} = \tilde{A}^{new} + S^{new} = \tilde{A}^{old} + S^{old}.$$

Using my functional form for $S_{i,t}$, I can rewrite the budget constraint as

$$\frac{\omega}{R_t}(A_{i,t+1} - S_{i,t+1}) = A_{i,t} + Y_{i,t} - C_{i,t} - T_{i,t}.$$

Finally, households value sequences of consumption $\{C_{i,t+k}\}$ and wealth inclusive of social payments $\{A_{i,t+k}\}$ according to expected utility

$$E_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k[u(C_{i,t+k}) + g(A_{i,t+k})]\right]$$

where $u(C) = \frac{C^{1-1/\sigma}-1}{1-\frac{1}{\sigma}}$ and $g(A) = \frac{\omega A^{1+\gamma}}{1+\gamma}$. The γ term corresponds to the inverse of the agent's elasticity of holding wealth. I scale the utility received from wealth by ω to simplify analytical expressions, but it does not qualitatively change the results.

Notation. Before proceeding with the key findings of the model, I introduce some necessary notation. I denote log-deviations (from steady-state) of a variable X_t in lowercase, i.e. $x_t = \log X_t - \log X_{ss}$. However, since I would like for the possibility of zero steady-state debt, $D^{ss} = 0$, I write lower-case fiscal variables (and household wealth) in terms of absolute deviations scaled by steady state output, so $t_t = (T_t - T^{ss})/Y^{ss}$ and $a_t = (A_t - A^{ss})/Y^{ss}$.

Results. I now derive the steady state real interest rate of the economy.

Theorem 1. *The steady-state real interest rate in the economy satisfies*

$$R_{ss} = \frac{1}{\beta\lambda} < \frac{1}{\beta}$$

where $\lambda = 1 + A_{ss}^{\gamma} C_{ss}^{1/\sigma}$.

See Appendix A.1 for a proof. In an economy with no wealth in the utility function, the real interest rate would be $\frac{1}{\beta}$. Theorem 1 shows that wealth in the utility function increases the marginal benefit of saving and drives down the real interest rate by a factor of $\frac{1}{\lambda} = \frac{1}{1+A_{ss}^{\gamma}C_{ss}^{1/\sigma}}$.

My second result establishes the aggregate consumption function of the economy.

Theorem 2. *To a first-order, aggregate consumption satisfies*

$$\begin{aligned} c_t = (1-\beta\omega\mathcal{X})\Bigg(a_t + E_t\Bigg[\sum_{k=0}^{\infty}(\beta\omega)^k(y_{t+k} - t_{t+k})\Bigg]\Bigg) + (1-\beta\omega\mathcal{X})\beta\frac{A^{ss}}{Y^{ss}}E_t\Bigg[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\Bigg] \\ - \frac{\beta\omega}{\mathcal{X}}E_t\Bigg[\sum_{k=0}^{\infty}(\beta\omega)^k(\mathcal{Y}a_{t+k} + \mathcal{Z}r_{t+k})\Bigg] \end{aligned}$$

where $\mathcal{X} = \frac{\mathcal{B}}{\mathcal{A}}, \mathcal{Y} = \frac{\mathcal{C}}{\mathcal{A}}$, *and* $\mathcal{Z} = \frac{\mathcal{D}}{\mathcal{A}}$ *and*

$$\begin{aligned} \mathcal{A} &= \beta R_{ss}\frac{1}{\sigma C_{ss}} \\ \mathcal{B} = \mathcal{B} &= \beta R_{ss}\Big(\frac{1}{\sigma C_{ss}} + \frac{1}{\sigma}A_{ss}^{\gamma}C_{ss}^{\frac{1}{\sigma}-1}\Big) \\ \mathcal{C} &= \beta\gamma R_{ss}A_{ss}^{\gamma-1}C_{ss}^{1/\sigma} \\ \mathcal{D} &= \beta\Big(1 + \frac{A_{ss}^{\gamma}}{C_{ss}^{-1/\sigma}}\Big). \end{aligned}$$

Wealth in the utility function provides an additional motive for agent to save, which is reflected in the last term of the consumption function. This finding suggests that the output response to a fiscal deficit shock will be lower with wealth in the utility function.

Extensions. The derivation of the consumption function heavily relies on being able to aggregate individuals with homogeneous consumption profiles. In Appendix B, I present a static model to show how consumer heterogeneity may lead to depressed output through an "indebted demand" channel similar to Mian et al. (2021).

3 Quantitative HANK Economy

In this section, I consider a general equilibrium quantitative HANK model.

3.1 Setup

Household block. Households are infinitely lived and have a total mass of measure one. Time is discrete with each period corresponding to a year. Households value consumption $C_{i,t}$ and wealth $A_{i,t}$ according to the utility function

$$E_t\left[\sum_{k=0}^{\infty}\beta^t\left(\frac{C_{i,t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}+\psi\frac{A_{i,t}^{1+\gamma}}{1+\gamma}\right)\right].$$

In contrast to the stylized HANK model in the previous section, the utility obtained from wealth is now weighted by ψ. I use ψ to do comparative statics on the consequences of having wealth in the utility function.

Every period, households face idiosyncratic shocks $e_{i,t}$ to their labor income $Y_{i,t} = W_t L_{i,t}$. As in the perpetual-youth OLG economy, I assume that labor is inelastically supplied, so $L_{i,t} = L_t$.

The fiscal authority taxes earnings by a proportional constant τ_y and endows households with transfers $S_{i,t}$. Households may accumulate wealth $A_{i,t}$ with a risk-free bond $A_{i,t}$ that pays interest R_t. In total, the budget constraint of a household can be written as

$$C_{i,t} + A_{i,t+1} = (1-\tau_y)e_{i,t}Y_{i,t} + S_{i,t} + R_t A_{i,t}.$$

Households also face a no-borrowing constraint

$$A_{i,t+1} \geq 0.$$

The combination of liquidity constraints and idiosyncratic income shocks give rise to non-Ricardian household behavior in the model. However, the inclusion of wealth in the utility function increases the marginal value of saving, and dampens the motive to consume out of unexpected fiscal transfers.

Firm block. Firms are the same as the textbook version of firms in the New Keynesian model. In particular, there is a measure one continuum of monopolistically competitive retailers who face Calvo pricing frictions. Since I assume labor is inelastic, the firm block of the model reduces to the New Keynesian Philips curve

$$\pi_t = \kappa y_t + \beta E_t[\pi_{t+1}]$$

where κ denotes the slope of the Philips curve.

Monetary authority. The central bank follows the Taylor-type rule

$$r_t = \phi y_t$$

where ϕ governs the response of monetary authority to fluctuations in output y_t. When $\phi \leq 0$, the monetary authority is "passive" in the sense that it will not raise interest rates in response to a boom in output. On the other hand, the monetary authority is "active" when $\phi > 0$, as the central bank will raise rates in response to an output boom.

Fiscal authority. The law of motion for government debt follows the equation

$$D_t = R_{t-1}D_{t-1} + S_t - \tau_y Y_t.$$

The fiscal authority sets transfers to enforce the fiscal rule

$$S_t = \mathcal{E}_t - \tau_d R_{t-1} D_{t-1}$$

where $\mathcal{E}_t$ are i.i.d. mean-zero fiscal deficit shocks and τ_d parametrizes the speed of fiscal adjustment. Since the monetary rule in the economy is in log-linear terms, it is more convenient to work with the log-linearized equations for the law of motion of debt and transfers. The next theorem derives these expressions.

Theorem 3. *The log-linearized equations for the law of motion of debt and transfers follows*

$$d_t = R_{ss}(d_{t-1} + r_{t-1}) + \frac{S_{ss}}{D_{ss}} s_t - \tau_y \frac{Y_{ss}}{D_{ss}} y_t$$
$$s_t = \epsilon_t - \tau_d R_{ss} \frac{D_{ss}}{S_{ss}} (d_{t-1} + r_{t-1})$$

where $\epsilon_t = \frac{\mathcal{E}_t}{S_{ss}}$.

See Appendix A.3 for a proof.

Self-financing. I now precisely define how I define self-financing in the framework of my HANK model. Angeletos et al. (2024) show that a deficit shock ϵ_0 is financed as follows

$$\epsilon_0 = \underbrace{\tau_d(\epsilon_0 + \sum_{k=0}^{\infty} \beta^k E[d_k]) +}_{\text{fiscal adjustment}} \underbrace{\tau_y(\sum_{k=0}^{\infty} \beta^k E[y_k])}_{\text{self-financing via tax base}} + \underbrace{\frac{D_{ss}}{Y_{ss}} \pi_0}_{\text{self-financing via debt erosion}}$$

The first term of the equation represents the amount of additional taxes the fiscal authority imposes to pay off the deficit shock. Self-financing from the growth of the tax base in response to a deficit shock is encapsulated in the second term. Debt erosion alleviates the fiscal burden of the deficit by the amount shown in the third term. While the first term reflects a partial equilibrium effect of the deficit, the second and third terms represent a general equilibrium response.

Given the expression above, I define the degree of self-financing as

$$\nu = \nu_y + \nu_p = \frac{\tau_y \sum_{k=0}^{\infty} \beta^k E[y_k]}{\epsilon_0} + \frac{1}{\epsilon_0} \frac{D_{ss}}{Y_{ss}} \pi_0$$

where ν_y and ν_p represents the degree of self-financing from the tax base channel and the debt erosion channel respectively.

3.2 Calibration

My calibration strategy for the quantitative HANK economy mostly follows Angeletos et al. (2024). I consider three different values for the speed of fiscal adjustment τ_d, .085, .026,.004. These specifications are taken from previous papers that have tried to estimate fiscal rules of this form. I set τ_y to .33, the average labor tax documented in Delong and Summers (2012). The slope of the Philips curve matches the .0062 estimate in Hazell et al. (2022). Since my primary focus is on the effects of fiscal policy, I set $\phi = 0$ so that the monetary policy fixes the real interest rate and maintains a neutral position towards the actions of the fiscal authority. The elasticity of intertemporal substitution (EIS) σ is set to 2 and the elasticity of holding wealth $1/\gamma$ is set to 1.

In the baseline version of the model, I set the weight of receiving utility from wealth ψ to 0, so that wealth disppears from the utility function. Later, I consider a calibration where $\psi = .1$, so agents receive utility from wealth.

In the steady state, I normalize output Y_{ss} to 1. I set the debt to output ratio and transfers in steady state to 1.04 and .06 respectively to match the evidence in Kaplan et al. (2018). I internally calibrate the discount factor β to set the steady state yearly real interest rate R_{ss} to 1.01. I summarize these calibration details for the baseline HANK model in Table 1.

Another key difference from Angeletos et al. (2024) is that I specify a two-state income process for computational tractability. Extending the model by incorporating a more general income process would be a great exercise for future work.

3.3 Results

I solve the model using the sequence-space Jacobian methods in Auclert et al. (2021). Figure 1 shows the impulse responses of the baseline model to a deficit shock equal to one percent

Parameter	Description	Value	Target
Policy			
τ_d	Tax feedback	{.085,.026,.0004}	
τ_y	Tax rate	.33	Average labor tax
ϕ	Real rate feedback	0	
Rest of Model			
κ	NKPC slope	.0062	Hazell et al. (2022)
σ	EIS	2	Standard
$1/\gamma$	Wealth elasticity	2	Standard
ψ	Wealth weight	0	See text
S_{ss}	Transfers	.06	Kaplan et al. (2018)
D_{ss}/Y_{ss}	Debt ratio	1.04	Kaplan et al. (2018)
R_{ss}	Real interest rate	1.01	

Table 1: Calibration strategy for the baseline quantitative HANK model.

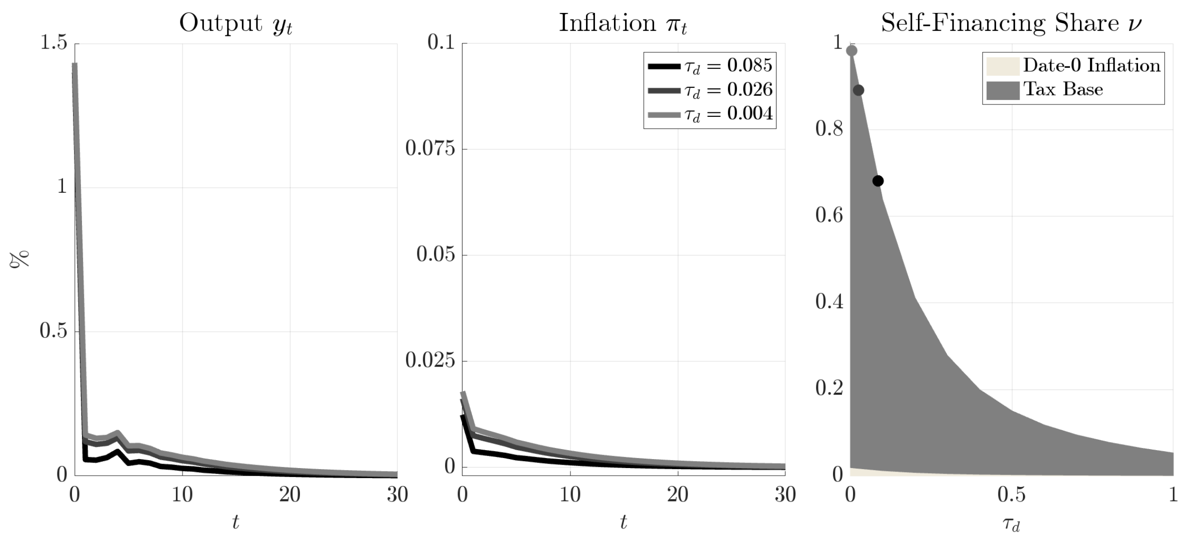

Figure 1: Impulse responses of output, inflation, and self-financing share to a deficit shock equal to one percent of output.

of output.

Under this calibration, the impulse response of output in response to a deficit shock is around one hundred times as large as the impulse response to inflation. Consequently, the degree of self-financing is driven almost entirely through the tax base, regardless of the value for the speed of the fiscal adjustment τ_d.

Steeper Philips curve. One explanation for the outsized response of output compared to inflation is that the slope of the Philips curve in my baseline calibration is rather flat. To test this hypothesis, I solve the model again, but now set the slope of the Philips curve to $\kappa = .31$,

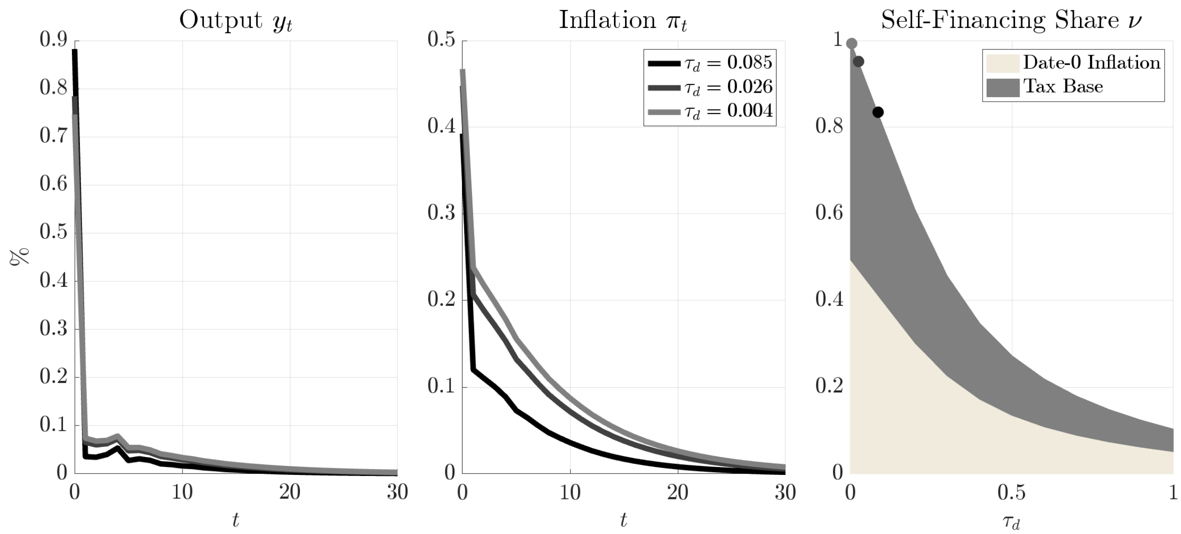

Figure 2: Impulse responses of output, inflation, and self-financing share to a deficit shock equal to one percent of output with $\kappa = .0031$.

which is fifty times larger than my initial calibration. In Figure 2, the inflation response to a deficit shock is now much larger on impact and more persistent, while the impulse response to output is smaller. The self-financing from the debt erosion is stronger and makes up the majority of the total self-financing from the deficit shock. With a steeper Philips curve, the stimulative effects of the deficit shock lead to higher inflation. Higher inflation erodes asset values, so non-Ricardian households consume less and the output response is smaller than the baseline model.

Wealth in the utility function. Finally, we consider the baseline model, but include wealth in the utility by setting $\psi = .1$. Figure 3 presents the impulse responses in this calibration. Again, the self-financing share is largely driven by the tax base, since the Philips curve is flat as mentioned before. The inflation response is largely unchanged as well, which is also consistent with a flat Philips curve. The notable difference in adding wealth in the utility function is visible in the output response, which is roughly half as large as it was before. In the stylized HANK model, we showed that wealth in the utility function lowers the MPCs of consumers in the economy. With lower MPCs, agents consume less in response to the deficit shock, and in equilibrium, output does not rise as much.

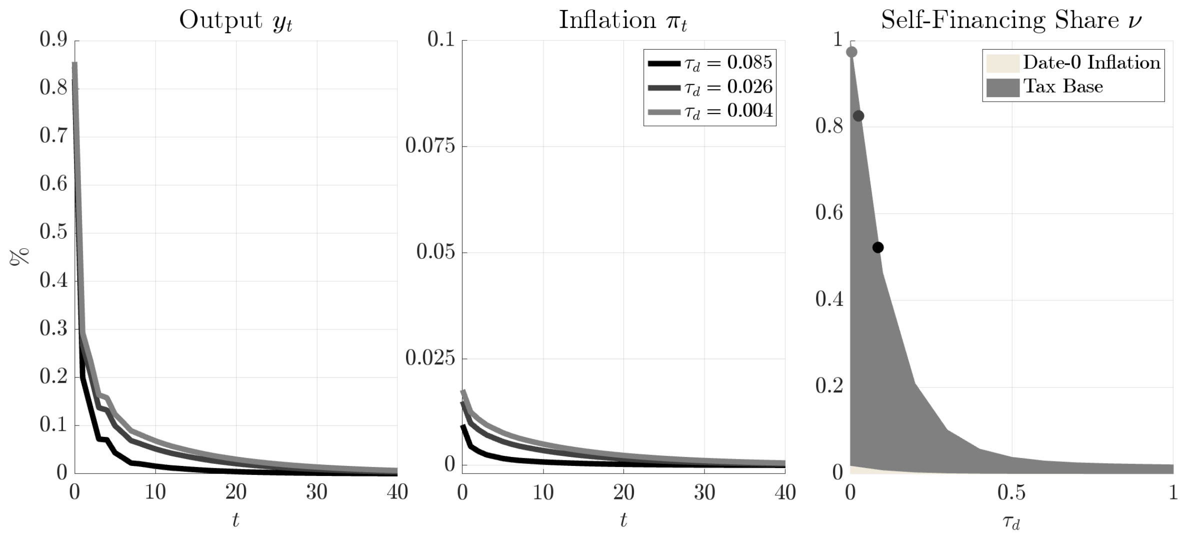

Figure 3: Impulse responses of output, inflation, and self-financing share to a deficit shock equal to one percent of output with wealth in the utility function.

4 Conclusion

In this paper, I study the consequences for fiscal policy when wealth in the utility function is added to a standard HANK model. I show that adding wealth in the utility lowers the MPCs of households and leads to a smaller output response from a deficit shock. Hence, including wealth in the utility function may serve as a tool to discipline MPCs in HANK models.

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Appendix A Proofs

A.1 Proof of Theorem 1

Proof. Taking the first-order condition of the household problem gives

$$-C_t^{-1/\sigma}\left(\frac{\omega}{R_t}\right)+\beta\omega A_{t+1}^{\gamma}+E_t[C_{t+1}^{-1/\sigma}\beta\omega]=0.$$

Move terms to one-side to get

$$1=\beta R_t E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1/\sigma}+\frac{A_{t+1}^{\gamma}}{C_t^{-1/\sigma}}\right].$$

A first-order Taylor approximation implies

$$\begin{aligned}\beta R_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1/\sigma}+\frac{A_{t+1}^{\gamma}}{C_t^{-1/\sigma}}\right] &= \beta R_{ss}\left(1+\frac{A_{ss}^{\gamma}}{C_{ss}^{-1/\sigma}}\right)-\beta R_{ss}\frac{1}{\sigma C_{ss}}(C_{t+1}-C_{ss})\\ &\quad+\beta R_{ss}\left(\frac{1}{\sigma C_{ss}}+\frac{1}{\sigma}A_{ss}^{\gamma}C_{ss}^{\frac{1}{\sigma}-1}\right)(C_t-C_{ss})\\ &\quad+\beta\gamma R_{ss}A_{ss}^{\gamma-1}C_{ss}^{1/\sigma}(A_t-A_{ss})\\ &\quad+\beta\left(1+\frac{A_{ss}^{\gamma}}{C_{ss}^{-1/\sigma}}\right)(R_t-R_{ss})\\ &=\zeta-\mathcal{A}c_{t+1}+\mathcal{B}c_t+\mathcal{C}a_t+\mathcal{D}r_t\end{aligned}$$

where

$$\begin{aligned}\zeta &= \beta R_{ss}(1+\frac{A_{ss}^{\gamma}}{C_{ss}^{-1/\sigma}})\\ \mathcal{A} &= \beta R_{ss}\frac{1}{\sigma C_{ss}}\\ \mathcal{B} &= \mathcal{B}=\beta R_{ss}(\frac{1}{\sigma C_{ss}}+\frac{1}{\sigma}A_{ss}^{\gamma}C_{ss}^{\frac{1}{\sigma}-1})\\ \mathcal{C} &= \beta\gamma R_{ss}A_{ss}^{\gamma-1}C_{ss}^{1/\sigma}\\ \mathcal{D} &= \beta(1+\frac{A_{ss}^{\gamma}}{C_{ss}^{-1/\sigma}}).\end{aligned}$$

By construction, $\zeta=1$. Hence, I have

$$R_{ss}=\frac{1}{\beta\lambda}$$

where $\lambda=1+\frac{A_{ss}^{\gamma}}{C_{ss}^{-1/\sigma}}$.

□

A.2 Proof of Theorem 2

Proof. By construction, I have that $S_{i,t+1} = -\frac{1-\omega}{\omega} D^{ss} = -\frac{1-\omega}{\omega} A^{ss}$. Since I assumed that there is no heterogeneity in labor supply, agents make the same total income and therefore pay the same taxes. Hence, for all households, I have the budget constraint

$$\frac{\omega}{R_t}(A_{i,t+1} + \frac{1-\omega}{\omega} A^{ss}) = A_{i,t} + Y_{i,t} - C_{i,t} - T_{i,t}.$$

Let $\hat{a}_{t+1} = \frac{A_{t+1} - A^{ss}}{A^{ss}}$. Notice that I can rewrite the left side of the budget constraint as

$$\begin{aligned}
\frac{\omega}{R_t}(A_{t+1} + \frac{1-\omega}{\omega} A^{ss}) &= e^{\log \omega - \log R_t + \log A_{t+1}} + e^{-\log R_t + \log(1-\omega) A^{ss}} \\
&= \frac{\omega}{R^{ss}} A^{ss} + \frac{\omega}{R^{ss}} A^{ss}(\hat{a}_{t+1} - r_t) + \frac{(1-\omega)}{R^{ss}} A^{ss} - \frac{(1-\omega)}{R^{ss}} A^{ss} r_t \\
&= \beta A^{ss} + \beta \omega A^{ss} \hat{a}_{t+1} - \beta A^{ss} r_t
\end{aligned}$$

At a steady state, the equation above reduces to

$$\beta A^{ss} = A^{ss} + Y^{ss} - C^{ss} - T^{ss}$$

Then substitution and dividing both sides by Y^{ss} gives

$$\frac{\beta \omega A^{ss}}{Y^{ss}} \hat{a}_{t+1} = a_t + y_t - c_t - t_t + \frac{\beta A^{ss}}{Y^{ss}} r_t$$

Do some algebraic manipulation to get

$$\beta \omega a_{t+1} = a_t + y_t - c_t - t_t + \frac{\beta A^{ss} r_t}{Y^{ss}}$$

I conclude the proof by iterating on the log-linearized budget constraint and plugging in the log-linearized Euler equation. Iterating the log-linearized budget constraint one period forward gives

$$\beta \omega a_{t+2} = a_{t+1} + y_{t+1} - c_{t+1} - t_{t+1} + \frac{\beta A^{ss} r_{t+1}}{Y^{ss}}$$

Then

$$a_{t+1} = \beta \omega a_{t+2} - y_{t+1} + c_{t+1} + t_{t+1} - \frac{\beta A^{ss} r_{t+1}}{Y^{ss}}$$

Plugging this equation into our original log-linearized budget constraint gives

$$c_t + \beta \omega c_{t+1} = y_t - t_t + \beta \omega (y_{t+1} - t_{t+1}) + a_t + (\beta \omega)^2 a_{t+2} + \frac{\beta A^{ss} r_t}{Y^{ss}} + \frac{\beta(\beta \omega) A^{ss} r_{t+1}}{Y^{ss}}.$$

Iterate infinitely many times to get

$$\sum_{k=0}^{\infty} (\beta \omega)^k c_{t+k} = a_t + \sum_{k=0}^{\infty} (\beta \omega)^k (y_{t+k} - t_{t+k}) + \frac{\beta A^{ss}}{Y^{ss}} \sum_{k=0}^{\infty} (\beta \omega)^k r_{t+k} \tag{1}$$

Our Euler equation implies

$$E[c_{t+1}] = \mathcal{X}c_t + \mathcal{Y}a_t + \mathcal{Z}r_t$$

where $\mathcal{X} = \frac{\mathcal{B}}{\mathcal{A}}, \mathcal{Y} = \frac{\mathcal{C}}{\mathcal{A}}$, and $\mathcal{Z} = \frac{\mathcal{D}}{\mathcal{A}}$. Iterate on the Euler equation to get

$$E_t[c_{t+k}] = \mathcal{X}^k c_t + E_t\left[\sum_{j=0}^{k-1} \mathcal{X}^{k-j-1}(\mathcal{Y}a_{t+j} + \mathcal{Z}r_{t+j})\right]$$

Hence, taking expectations of both sides of (2) and substituting in the Euler equation gives

$$E_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(\mathcal{X}^k c_t + \sum_{j=0}^{k-1}\mathcal{X}^{k-j-1}(\mathcal{Y}a_{t+j} + \mathcal{Z}r_{t+j})\right)\right] = a_t + E_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k(y_{t+k} - t_{t+k})\right] + \frac{\beta A^{ss}}{Y^{ss}}E_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right]$$

By the properties of geometric series, notice that

$$E_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\sum_{j=0}^{k-1}\mathcal{X}^{k-j-1}(\mathcal{Y}a_{t+j} + \mathcal{Z}r_{t+j})\right] = \frac{\beta\omega}{(1-\beta\omega\mathcal{X})\mathcal{X}}E_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k(\mathcal{Y}a_{t+k} + \mathcal{Z}r_{t+k})\right].$$

Finally, using the geometric series formula and some algebraic manipulation gives

$$\begin{aligned} c_t = (1-\beta\omega\mathcal{X})\left(a_t + E_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k(y_{t+k} - t_{t+k})\right]\right) + (1-\beta\omega\mathcal{X})\beta\frac{A^{ss}}{Y^{ss}}E_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right] \\ - \frac{\beta\omega}{\mathcal{X}}E_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k(\mathcal{Y}a_{t+k} + \mathcal{Z}r_{t+k})\right] \end{aligned}$$

which is what I wanted to show. □

A.3 Proof of Theorem 3

I begin by proving the law of motion for transfers. A first-order Taylor approximation around the steady state implies

$$R_{t-1}D_{t-1} = R_{ss}D_{ss} + R_{ss}(D_{t-1} - D_{ss}) + D_{ss}(R_{t-1} - R_{ss}).$$

Furthermore, at the steady state

$$S_{ss} = -\tau_d R_{ss} D_{ss}$$

Then subtracting and dividing by S_{ss} on both sides of the law of motion for transfers gives

$$s_t = \frac{\mathcal{E}_t}{S_{ss}} - \tau_d R_{ss}\frac{D_{ss}}{S_{ss}}(d_{t-1} + r_{t-1}).$$

The law of motion for debt is proved in a similar fashion.

Appendix B Static Model

In this section, I present a one-period consumption-saving model to illustrate how adding consumer heterogeneity to my stylized HANK model may generate a "indebted demand" recession. There are two agents in the economy, poor consumers $i = h$, and wealthy consumers $i = l$. Agents have marginal propensities to consume MPC_i and receive labor income y_i. In line with the empirical evidence, I impose the assumption that wealthy consumers have lower MPCs and higher incomes. Hence, $MPC_h > MPC_l$ and $y_l > y_h$.

At the beginning of the period, the government transfers ϵ_h and ϵ_l to poor and rich households respectively. Let ϵ be the cumulative transfer to households, so $\epsilon_h + \epsilon_l = \epsilon$. I assume that consumer demand follows

$$c_i = MPC_i(1-\tau_y)y_i + \epsilon_i$$

Market clearing implies that $c_h + c_l = y$ and $y_h + y_l = y$ where y is the total labor income in the economy. Using the first market clearing condition reveals

$$y = MPC_h(1-\tau_y)y_h + MPC_l(1-\tau_y)y_l + MPC_h\epsilon_h + MPC_l\epsilon_l$$

The second market clearing condition implies

$$y = MPC_h(1-\tau_y)(y-y_l) + MPC_l(1-\tau_y)y_l + MPC_h(\epsilon-\epsilon_l) + MPC_l\epsilon_l$$

Rearrange terms to get

$$y = \frac{(MPC_l - MPC_h)((1-\tau_y)y_l + \epsilon_l) + MPC_h\epsilon}{1 - MPC_h(1-\tau_y)}$$

Since $MPC_l < MPC_h$, transfers to wealthy households ϵ_l depress output. The intuition for this result is that transfers from poor to wealthy households increase savings from the wealthy and consequently debt from the poor.