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Author(s): Kalok Chan, K. C. Chan and G. Andrew Karolyi
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Intraday Volatility in the Stock Index and Stock Index Futures Markets

Kalok Chan
Arizona State University

K. C. Chan and G. Andrew Karolyi
The Ohio State University

We examine the intraday relationship between returns and returns volatility in the stock index and stock index futures markets. Our results indicate a strong intermarket dependence in the volatility of the cash and futures returns. Price innovations that originate in either the stock or futures markets can predict the future volatility in the other market. We show that this relationship persists even during periods in which the dependence in the returns themselves appears to weaken. The findings are robust to controlling for potential market frictions such as asynchronous trading in the stock index. Our results have implications for understanding the pattern of information flows between the two markets.

A growing theoretical and empirical literature has sought to understand the relation between index futures markets and stock markets. The increased research interest in this area no doubt owes itself to

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the heightened public concern that the new trading strategies made possible by the existence of index futures markets have increased stock market volatility [see the Brady Commission Report (1988), the NYSE Report on Market Volatility and Investor Confidence (1990), and some recent studies by Harris (1989b), Bessembinder and Seguin (1989), Furbush (1989), Neal (1990), Harris, Sofianos, and Shapiro (1990), and Froot and Perold (1990)]. But another reason to study the relation between prices of the index futures market and the stock market is to answer a fundamental question often asked in finance: does the same asset trading in two different markets sell at the same price at each point in time? Strictly speaking, the assets traded in the two markets are not identical, but the value of the futures contract should reflect the information about individual stocks impounded in the cash market prices. Similarly, the prices of individual stocks in the cash market should also reflect the information in the value of the futures contract. Further, index arbitrage should reinforce the informational link between the two markets since the value of the index futures contract should not deviate from the cost of buying the individual stocks that make up the index and holding them to maturity, including the transaction costs. Empirical studies, however, report a large number of instances where the index level and the futures price are so different that arbitrage might be profitable [MacKinlay and Ramaswamy (1988) and Chung (1989)]. Furthermore, many studies suggest that price movements in the futures markets systematically lead price movements of the underlying index in the cash markets.[1]

Uncovering lead and lag relations in price changes raises an interesting possibility that the futures and cash markets are not equal in their capacity to discover new information about asset prices. Indeed, studies on this lead and lag relation suggest that the index futures market serves as a primary market of price discovery and argue that new information disseminates in the futures market before the stock market. In this article, we show that a further analysis of the intermarket dependence between the cash and futures prices does not support such a unidirectional relation. Instead, our results suggest that price innovations in either the cash or futures market may be able to predict the arrival of new information in the other market.

Our analysis focuses on the intraday relationship between price changes and price change volatility in the stock index and stock index futures markets. Thus, we extend the current studies of lead and lag relations between stock and futures price changes by allowing the

[1] See Stoll and Whaley (1986, 1990b), Ng (1987), Kawaller, Koch, and Koch (1987a, 1987b, 1990), Cheung and Ng (1990), and Chan (1992). Note that Stoll and Whaley (1990b) and Chan (1992) find a leading relation from cash to futures returns, but it is much weaker than that from futures to cash returns.

volatility of price changes, as well as price changes themselves, to interact across the cash and futures markets. Our evidence from August 1984 to December 1989 for the Standard and Poor's (S&P) 500 stock index and index futures contract offers four major findings. First, intraday volatility patterns in both the cash and futures markets demonstrate strong persistence and predictability. That is, the intraday volatility of stock and futures price changes varies over time and in a predictable manner given the past volatility of price changes. Second, conditional on the persistence and predictability of the intraday volatilities of price changes in their respective markets, the past volatility of futures (cash) price changes is also an important predictor of the future volatility of cash (futures) price changes. In other words, price innovations in either the cash or futures market influence the volatility in the other market. Third, although the lead and lag relations between the price changes of the cash and futures markets appear to diminish over the sample period, the intermarket dependence of volatility grows stronger and comparably in both directions. Finally, these findings are robust to controlling for the potential effects of infrequent trading of the component stocks in the index and other market frictions.

Why focus on the intraday volatility of the cash and futures price changes and not just on the price changes themselves? There are several reasons. First, if the volatility of the price changes in the cash and futures markets vary over time in a related way and if this is ignored in tests of lead and lag relations in the price changes, specification error can lead to incorrect inferences about the relation between futures and cash prices. The lead and lag results reported in previous studies may, therefore, arise from model misspecification. Second, finding a lead or lag relation in price changes may offer only inconclusive evidence on how information flows to the two markets. Time-varying intraday conditional volatility of price changes in the cash and futures markets represents another way in which we can measure how information can flow to those two markets. Exploring this alternative measure is reasonable given studies by Clark (1973), Tauchen and Pitts (1983), and recently Ross (1989) that show that it is the volatility of an asset's price, and not the asset's simple price change, that is related to the rate of flow of information to the market. But, more importantly, this alternative measure may generate new insights. For example, from a number of existing studies that find futures prices systematically lead cash prices, a common interpretation of the result is that traders with marketwide information prefer to go to the futures market before the cash market.[2,3] When we allow

[2] A number of theoretical studies differentiate types of information that flow to the cash and futures markets. For example, Kumar and Seppi (1989), Subrahmanyam (1991), and Chan (1990) show

for time-varying and intermarket-dependent volatility of price changes while simultaneously controlling for the lead or lag effects in price changes themselves, we uncover a strong bidirectional dependence in the intraday volatility of the cash and futures markets. That is, price innovations in either the cash or futures market not only predict the volatility of their own market, but also that of the other market. Our results suggest that new information that hits either market can, in general, predict the arrival rate of information in the other market. This finding is thus inconsistent with the notion posited by some studies that information flows systematically to the futures market before the cash market.

The data are discussed in Section 1. Autocorrelations and cross-correlations of the intraday cash and futures returns are presented in Section 2. These results motivate the econometric model employed, which is discussed in Section 3. The estimation results are presented in Section 4. We focus on important econometric problems relating to asynchronous trading, index reporting delays, and other market frictions that may influence the relation between cash and futures prices in Section 5. We also implement the tests on (a) the Major Market Index contract traded on the Chicago Board of Trade and (b) the S&P 500 futures contract with a single heavily traded stock, IBM. Conclusions follow in Section 6.

1. Data

1.1 Standard and Poor's 500 stock index and stock index futures prices

The data used in the study are from the "Quote Capture" information provided by the Chicago Mercantile Exchange for the period August 1, 1984–December 31, 1989. The first set of data contains the time (to the nearest 10 seconds) and exact price of the S&P futures transactions whenever the price is different from the previous one. The second set of data consists of the S&P 500 index level each time it is

that fixed costs of trading, budget constraints, or different expected profits cause traders in the futures market to collect more marketwide information and traders in the cash market to collect more firm-specific information.

[3] The notion that the futures market incorporates information more rapidly than the cash market is broadly consistent with the results of Kawaller, Koch, and Koch (1987a, 1987b) and Stoll and Whaley (1990b) that futures market prices systematically lead cash market prices, but inconsistent with the new findings of Chan (1992) and theoretical conclusions (proposition 11) of Subrahmanyam (1991). The evidence in Chan (1992) is consistent with the hypothesis that cash market frictions influence the pattern of leads and lags in market price changes. He studies specifically the effects of short-sales constraints, differential trading volume, and marketwide or stock-specific news events.

computed and transmitted to Chicago.[4] Since the nearby contract is usually the most actively traded, only the data for the nearby futures contract are used.

The intraday time series are partitioned into five-minute intervals. During each interval, the last futures price and cash index level quotes are employed. Since the New York Stock Exchange closes at 3:00 P.M. (Central Time), 15 minutes earlier than the futures market, futures price observations after that time are removed. Futures prices recorded before the NYSE opens are also excluded. Before September 30, 1985, the NYSE opened at 9:00 A.M. (Central Time) and there were 72 five-minute intervals during the trading day. After that date, the NYSE began opening at 8:30 A.M. (Central Time) and price observations increased to 78 five-minute quotes. The price observations were used to compute the five-minute intraday and overnight simple returns for the futures and cash index. The overnight return was computed from the 9:05 A.M. (8:35 A.M. after September 30, 1985) price and the 3:00 P.M. price of the previous day for both markets.[5]

1.2 Major market index and index futures prices

In Section 5, we evaluate the impact of mismeasurement of returns due to asynchronous trading of component stocks in the index on the results of the study. One way in which this problem is addressed is by replicating the results with data on the Major Market Index (MMI) and its associated futures contract traded on the Chicago Board of Trade (CBOT). Since the MMI is a price-weighted index of 20 of the largest and most actively traded New York Stock Exchange stocks, the problems of asynchronous trading are likely to be mitigated. A second way in which we deal with the asynchronous problem is by modeling five-minute returns on the S&P 500 index futures contract jointly with five-minute returns computed for only IBM stock.

The MMI futures price data are supplied by the Chicago Board of Trade for the period July 23, 1984–June 30, 1985, which overlaps with the first part of the S&P 500 sample. The data file contains the time and price of futures transactions whenever the price is different from the previous one, as well as the stock index level. For the MMI futures contracts, again, only the nearby futures contract is used.

[4] Prior to June 13, 1986, the stock index was computed approximately once a minute; but, since that time, it has been computed and reported approximately four times per minute.

[5] Stoll and Whaley (1990a) show that greater volatility is observed at market open. This could result from the resolution of uncertainty in the overnight period. They attribute this result to the "one-shot auction" at the open due to the specialists' monopoly powers. Moreover, they find that the average time to first trade for NYSE stocks is 15.48 minutes from the open and only 4.98 minutes for stocks in the largest decile of the NYSE. For this reason, the main results of this paper are replicated with the opening price defined to be that prevailing a half hour after the open. The results are similar.

There are time lags in the index value reported by the exchanges due to computation and subsequently transmission delays.[6] The evidence in Chan (1992) suggests that these reporting lags are nontrivial. To circumvent this problem, in every five-minute interval, the MMI index value is computed using the price-weighted formula from the most recent transaction prices for each of the individual stocks recorded by the "Fitch" data obtained from Francis Emory Fitch, Inc. These data consist of a time-ordered record of every transaction of the 20 component stocks of the MMI. For each transaction, the date, time, price, and number of shares traded are available. Five-minute returns series are computed as for the S&P 500 and MMI reported indexes, except the first observation in the day is arbitrarily taken as at 8:45 A.M. (Central Time) to ensure that all component stocks have opened trading. Finally, the tests using IBM and S&P 500 stock index futures use the Fitch time-stamped prices for the IBM shares.

2. Autocorrelations and Cross-correlations of Intraday Returns

Tables 1 and 2 show a wide range of descriptive statistics for the stock and futures returns series for five roughly evenly divided subperiods over the 1984–1989 sample. Subperiod breaks are, however, specifically chosen to (a) match that for the MMI results shown in Section 5 to allow comparisons and (b) isolate the crash period.[7] In Table 1, the statistics reported include the mean, standard deviation, skewness, excess kurtosis for the intraday cash, and futures returns, respectively. The same statistics for the overnight returns are presented below in brackets.

The sample moments for both stock and futures returns series indicate empirical distributions with heavy tails and sharp peaks at the center compared to the normal distribution. For most subperiods, there is negative skewness in both stock and futures returns, but a more significant effect prevails in the futures returns. Zero excess kurtosis is rejected confidently for both series in all subperiods, but especially in 1988–1989.[8]

[6] Stoll and Whaley (1990b) explain how the cash index value is recorded: "AMEX computes and disseminates the MMI level at 15-second intervals. The time stamps that appear on the CBOT data base are the times at which the prices are received and recorded by the CBOT."

[7] Subperiods 3 and 4 are constructed to exclude the period of October 15, 1987 to November 13, 1987. We do not seek to analyze this highly influential and unusual subperiod. Note that we also exclude the three weeks following the crash because of a number of observed trading suspensions in the futures market during the trading days in that period. For details, see Harris (1989a).

[8] The large excess kurtosis due to the heavy tails of the distribution observed for these intraday returns series are consistent with models of time-varying conditional heteroskedasticity, such as the ARCH and GARCH models of Engle (1982) and Bollerslev (1986), which we employ in Section 3.

Table 1
Summary statistics for intraday five-minute and overnight returns on the S&P 500 stock index and the S&P 500 index futures from July 1984–December 1989

	Intraday stock index returns					Intraday stock index futures returns				
Statistic	1984–1985	1985–1986	1986–1987	1987–1988	1988–1989	1984–1985	1985–1986	1986–1987	1987–1988	1988–1989
Sample size	16,169	20,440	22,925	18,316	22,247	16,169	20,440	22,925	18,316	22,247
	[231]	[274]	[306]	[243]	[294]	[231]	[274]	[306]	[243]	[294]
Mean	0.0010	0.0004	0.0005	0.0003	0.0003	0.0010	0.0004	0.0005	0.0003	0.0003
	[0.0306]	[0.0487]	[0.0665]	[0.0671]	[0.0639]	[0.0306]	[0.0487]	[0.0665]	[0.0671]	[0.0639]
Std. dev.	0.0442	0.0509	0.0721	0.0787	0.0538	0.0442	0.0509	0.0721	0.0787	0.0538
	[0.2234]	[0.1881]	[0.3217]	[0.4599]	[0.3135]	[0.2234]	[0.1881]	[0.3217]	[0.4599]	[0.3135]
Skewness	1.1901	−0.1582	−0.7458	0.2160	−0.3751	1.1901	−0.1582	−0.7458	0.2160	−0.3751
	[0.1672]	[−0.1730]	[0.2559]	[−0.0281]	[0.0074]	[0.1672]	[−0.1730]	[0.2559]	[−0.0281]	[0.0074]
Excess kurtosis	10.771	12.312	15.330	10.763	23.559	10.771	12.312	15.330	10.763	23.559
	[0.4212]	[0.1478]	[0.6226]	[2.1917]	[2.8308]	[0.4212]	[0.1478]	[0.6226]	[2.1917]	[2.8308]
$\rho(r_t, r_{t-k})$										
1	0.45*	0.40*	0.33*	0.30*	0.27*	0.01	−0.01	−0.03*	−0.02*	−0.01
2	0.23*	0.15*	0.05*	0.04*	0.04*	−0.03*	−0.04*	−0.03*	−0.03*	−0.04*
3	0.06*	0.04*	−0.03*	−0.01	−0.02*	−0.05*	−0.03*	−0.02*	−0.01	−0.03*
4	−0.01	−0.00	−0.03*	−0.03*	−0.03*	−0.03*	−0.00	−0.02*	−0.02*	−0.03*
5	−0.04*	−0.01	−0.03*	−0.03*	−0.02*	−0.01	−0.01	−0.02*	−0.02*	−0.01
6	−0.04*	−0.02*	−0.02*	−0.02*	−0.02*	−0.01	−0.01	−0.01	0.00	−0.00
$\rho(r_t^2, r_{t-k}^2)$										
1	0.29*	0.23*	0.19*	0.18*	0.15*	0.04*	0.05*	0.07*	0.07*	0.07*
2	0.09*	0.07*	0.07*	0.06*	0.05*	0.02*	0.03*	0.07*	0.04*	0.04*
3	0.02*	0.04*	0.04*	0.03*	0.03*	0.01	0.02*	0.04*	0.03*	0.03*
4	0.01	0.02*	0.03*	0.02*	0.02*	0.02*	0.02*	0.03*	0.03*	0.01
5	0.00	0.02*	0.02*	0.01	0.02*	−0.00	0.02*	0.03*	0.03*	0.02*
6	0.01	0.02*	0.01	0.02*	0.01	−0.00	0.01	0.02*	0.03*	0.02*

Autocorrelation coefficients $\rho(r_t, r_{t-k})$, for up to k lags are computed from five-minute intraday returns beginning with the 9:05 A.M. (Central Time) price quote and ending with the 3:00 P.M. (Central Time) price quote in the market each day. These reported coefficient estimates are averages computed from the number of days in each subperiod. The kurtosis coefficient is computed in excess of 3. The alignment of futures and cash price quotes required that quotes from the futures market after 3:00 P.M. (Central Time) be removed from the sample. Corresponding statistics computed for overnight returns are given in brackets. Asymptotic standard errors for the autocorrelation coefficients can be approximated as the square root of the reciprocal of the number of observations (e.g., ± 0.0071 for around 20,000 observations) under the null hypothesis of zero autocorrelation. * denotes that the coefficient is at least 2.325 standard errors from zero, which approximates significance at the 1 percent level. The exact calendar days spanned by the subperiods are, respectively, 84/08/01–85/06/30, 85/07/01–86/07/31, 86/08/01–87/10/16, 87/11/13–88/10/31, and 88/11/01–89/12/31.

Table 2
Sample cross-correlation coefficients between intraday S&P 500 stock index and S&P 500 stock index futures returns

Lag	1984–1985	1985–1986	1986–1987	1987–1988	1988–1989
		A. Cross-correlation of returns $\rho(r_{st}, r_{f,t-k})$			
−6	−0.0191*	−0.0153	−0.0179*	−0.0033	−0.0162
−5	−0.0177*	−0.0207*	−0.0248*	−0.0184*	−0.0112
−4	−0.0264*	−0.0162	−0.0206*	−0.0232*	−0.0177
−3	−0.0580*	−0.0234*	−0.0334*	−0.0214*	−0.0302*
−2	−0.0726*	−0.0595*	−0.0527*	−0.0342*	−0.0411*
−1	−0.0404*	−0.0330*	−0.0320*	−0.0323*	−0.0255*
0	0.4055*	0.3641*	0.4131*	0.4894*	0.5083*
1	0.4330*	0.4536*	0.5418*	0.4950*	0.4554*
2	0.3106*	0.2890*	0.1863*	0.1494*	0.1443*
3	0.1487*	0.1199*	0.0431*	0.0389*	0.0345*
4	0.0559*	0.0419*	0.0059	0.0044	−0.0064
5	0.0177*	0.0189*	−0.0073	−0.0173*	−0.0071
6	−0.0097	0.0125	−0.0063	−0.0099	−0.0024
		B. Cross-correlation of squared returns $\rho(r_{st}^2, r_{f,t-k}^2)$			
−6	−0.0164	0.0091	0.0244*	0.0225*	0.0112
−5	−0.0135	0.0152	0.0223*	0.0257*	0.0205*
−4	−0.0060	0.0079	0.0304*	0.0258*	0.0246*
−3	0.0052	0.0157	0.0402*	0.0381*	0.0267*
−2	0.0051	0.0210*	0.0475*	0.0473*	0.0411*
−1	0.0270*	0.0555*	0.0830*	0.0706*	0.0865*
0	0.2977*	0.2507*	0.3300*	0.4680*	0.4963*
1	0.3554*	0.3324*	0.4176*	0.3459*	0.2768*
2	0.1565*	0.1453*	0.1025*	0.0759*	0.0791*
3	0.0553*	0.0562*	0.0648*	0.0456*	0.0294*
4	0.0186*	0.0328*	0.0429*	0.0163	0.0256*
5	0.0154	0.0286*	0.0314*	0.0205*	0.0132
6	0.0129	0.0240*	0.0275*	0.0211*	0.0251*

Cross-correlation coefficients are computed from five-minute intraday returns beginning with the 9:05 A.M. (Central Time) price quote and ending with the 3:00 P.M. (Central Time) price quote in both markets each day. These coefficients are averaged over the number of days in each subperiod. The alignment of futures and cash price quotes requires that futures prices after 3:00 P.M. (Central Time) be removed from the sample. Positive lags (k) indicate cross-correlations, $\rho(r_{st}, r_{f,t-k})$, between past futures returns, $r_{f,t-k}$, and current cash returns, $r_{s,t}$. Negative lags ($k < 0$), or "leads," indicate cross-correlations between future futures returns and current cash returns. Asymptotic standard errors for the correlation coefficients can be approximated as the square root of the reciprocal of the number of observations (e.g., ± 0.0071 for around 20,000 observations) under the null hypothesis of zero autocorrelation. * denotes that the coefficient is at least 2.325 standard errors from zero, which approximates significance at the 1 percent level. The subperiods correspond to those calendar days shown in Table 1.

2.1 Autocorrelations

The sample autocorrelation functions for the intraday five-minute stock and futures returns are also presented in Table 1. For each day, the autocorrelation coefficients up to the sixth order are computed for the futures and cash returns. The estimates are then averaged over the number of trading days in each subperiod. We approximate the standard error for the correlation coefficients as the square root of the reciprocal of the number of five-minute observations (around 20,000), which is about 0.0071.[9] As found in the previous studies of intraday stock returns by Stoll and Whaley (1990b), MacKinlay and Ramaswamy (1988), and Chan (1992), the autocorrelations in the stock index are positive and more than two standard errors from zero for the first and, possibly, second five-minute interval. Beyond lag 3, a negative serial correlation (although usually less than three standard errors from zero) takes over in the index returns. In contrast, the serial correlation coefficients for the futures returns appear to be negative for the second and third lags. Interestingly, the autocorrelations in the cash returns do diminish over time, as the first-order autocorrelations fall from 0.45 in 1984 to 0.27 in 1989. This finding is consistent with Froot and Perold (1990).

The autocorrelation coefficients for the squared intraday returns are also computed and displayed in Table 1. These are presented as evidence of nonlinear dependence in the returns series.[10] The sample autocorrelations of the squared series for the stock index and stock index futures returns are positive and decay at a slow rate to zero. These results imply that a model for the returns generating processes in the cash and futures markets should account for higher-order dependence in the returns, possibly as a result of changing conditional volatility over time. One family of models that closely approximates second-order nonlinear processes, commonly known as the autoregressive conditional heteroskedastic (ARCH) models, has been developed by Engle (1982). The process allows the first and second moments of the returns to depend on its past values. This paper implements such models to characterize the returns generating process for intraday stock index and futures returns.

2.2 Cross-correlations

Panel A of Table 2 shows the average intraday cross-correlations between the five-minute stock and futures returns for up to six leads and lags for the five subperiods. The average contemporaneous correlation is about 0.4, more than 16 standard errors from zero. For the most part, price changes occur simultaneously in both markets. The lagged futures returns do seem to have some forecast power in explaining current stock index returns as the lags 1 to 3 have correlation coefficients at least four standard errors from zero. On the other hand, the leading cross-correlations (predictability from cash to

[9] Because of the large sample size of this analysis, the appropriate criteria for statistical significance for sample statistics and estimated coefficients are unclear. We highlight throughout the text and tables critical values at the 1 percent significance level or 2.325 asymptotic standard errors from zero, but caution the readers that a more conservative cutoff may be appropriate. For references, see chapter 3.2 of Zellner (1984) or Zellner and Siow (1980).

[10] Generally, if a process x_t is strict white noise, the process for x_t^2 is also strict white noise and intertemporally should exhibit statistical independence.

futures) are negative, but closer to zero. Both lead and lag effects appear to diminish in 1988 and 1989, particularly at the second and third lags. Stoll and Whaley (1990b) also recognize this weakening cross-market dependence in returns over time.

In panel B, the lead and lag correlations are computed for the squared returns for each subperiod. These represent a crude measure of intermarket association of volatility. Most correlations are greater than zero by more than four standard errors contemporaneously and apparently up to three lags in the direction of futures to cash and one lag in the direction of cash to futures. An interesting contrast between panels A and B is that the first and second lead correlations from cash to futures in the squared returns appear to increase over the same period while diminishing in the raw returns. On the other hand, the lead correlations from futures to cash of the squared returns decrease over the sample period similarly to the raw return cross-correlations. These preliminary results indicate that a lead/lag relationship may exist not only for price changes of futures and stocks, but also for the volatility of their price changes. Moreover, these relations appear to be capturing distinctly different phenomena. We utilize these findings for the modeling strategy that follows.[11]

3. The Bivariate GARCH Model

The analysis of autocorrelations and cross-correlations suggests that a model of the dynamics of the intraday returns in the cash and futures markets should seek to capture (a) the time variation in the volatility of intraday stock and futures returns and (b) the intermarket dependence of the returns and volatility of returns between the cash and futures markets. The analysis in this paper uses statistical models based on the autoregressive conditional heteroskedastic (ARCH) family of models developed by Engle (1982) and generalized (GARCH) by Bollerslev (1986). These models have been shown empirically to capture reasonably well the time variation in the volatility of daily and monthly stock returns [Bollerslev (1987), French, Schwert, and Stambaugh (1987), Schwert (1989), Nelson (1991), and Akgiray (1989)]. Moreover, in their generalized multivariate form, such models will allow for intermarket dependence in the returns generating processes of the cash and futures markets.

We posit the following bivariate AR(1)–GARCH(1,3) model for the joint processes governing the stock index and index futures returns:

[11] Examining the cross-correlation of the squared values of filtered 15-minute returns of the S&P index and S&P index futures, Cheung and Ng (1990) also note this bidirectional feedback. They argue that noise in the futures prices feeds into cash prices, which makes it difficult for market participants to establish profitable trading rules to exploit the well known lead–lag relation between futures and cash prices.

$$\mathbf{r}_t = \boldsymbol{\alpha} + \mathbf{d}_1 D_{1t} + \boldsymbol{\beta}\mathbf{r}_{t-1} + \mathbf{d}_3\mathbf{r}_{t-1}D_{2t} + \boldsymbol{\eta}_t,$$

$$\boldsymbol{\eta}_t = (\boldsymbol{\iota} + \mathbf{d}_2 D_{1t})\boldsymbol{\epsilon}_t, \tag{1}$$

$$\boldsymbol{\epsilon}_t | \Phi_{t-1} \sim N(\mathbf{0}, \mathsf{H}_t);$$

$$\begin{bmatrix} h_{ss,t} \\ h_{ff,t} \end{bmatrix} = \mathbf{A} + \mathsf{B}\begin{bmatrix} h_{ss,t-1} \\ h_{ff,t-1} \end{bmatrix} + \sum_{k=1}^{3} \mathsf{C}_k \begin{bmatrix} \epsilon^2_{ss,t-k} \\ \epsilon^2_{ff,t-k} \end{bmatrix}; \tag{2}$$

$$h_{sf,t} = h_{fs,t} = \rho(h_{ss,t}h_{ff,t})^{1/2}; \tag{3}$$

where the returns vector for the stock index and index futures series is given by $\mathbf{r}'_{qt} = [r_{s,t}\ r_{f,t}]$, the residual vectors by $\boldsymbol{\eta}'_t = [\eta_{s,t}\ \eta_{f,t}]$ and $\boldsymbol{\epsilon}'_t = [\epsilon_{s,t}\ \epsilon_{f,t}]$, and the conditional covariance matrix by H_t, where $\{\mathsf{H}_t\}_{ij} = h_{ij,t}$, for $i, j = s, f$. The parameter vectors and matrices are defined as $\boldsymbol{\alpha}' = [\alpha_s\ \alpha_f]$, $\mathbf{d}'_j = [d_{s,j}\ d_{f,j}]$, for $j = 1, 2$, $\mathbf{A}' = \{a_s\ a_f]$, $\boldsymbol{\iota}$, a 2×1 vector of 1's, and $\{\boldsymbol{\beta}\}_{ij} = \beta_{ij,k}$, $\{\mathbf{d}_3\}_{ij} = d_{ij,3}$, $\{\mathsf{B}\}_{ij} = b_{ij}$, $\{\mathsf{C}_k\}_{ij} = c_{ij,k}$, all for $i, j = s, f$, and lags $k = 1, 2$, or 3. Φ_{t-1} is the set of all information available at time $t - 1$, D_{1t} is a dummy variable that is set to unity for overnight returns, and D_{2t} is a dummy variable that is set to unity for the first five-minute trading interval each day.

Our bivariate model is closely related to those employed by Bollerslev, Engle, and Wooldridge (1988), Engle (1987), Baillie and Bollerslev (1987), Schwert and Seguin (1990), Engle, Ito, and Lin (1988), Baba et al. (1989), and Conrad, Gultekin, and Kaul (1991). Equation (1) models the security returns as an AR(1) process. This specification follows from the summary statistics in the previous section. A secondary role is to absorb the potential effects of asynchronous trading in the component stocks of the index, bid–ask bounce in the futures returns, or any other market frictions that may influence the dependence in cash and futures returns.

From the evidence in Table 1, the information shocks from overnight returns are likely to perturb the time series process of intraday returns. The model differentiates the effects of overnight returns from intraday returns for the mean returns and volatility with the dummy variables D_{1t} and D_{2t}. The first dummy variable scales the overnight return for the mean returns via $\mathbf{d}_1$ and the volatility of returns via $\mathbf{d}_2$. This approach is similar to that taken by Glosten, Jagannathan, and Runkle (1989) to deal with seasonality issues. The second dummy variable adjusts the influence of the overnight return as an independent variable in Equation (1) on the returns of the first five-minute trading interval.

Conditional on this dependence in means the cash and futures returns series are assumed to have a bivariate normal distribution with conditional covariance matrix H_t. Equation (2) models the diagonal elements of the conditional covariance matrix H_t as a function of the diagonal elements of the conditional covariance matrix of the past period, as well as the squared return innovations of three past periods. Following Baillie and Bollerslev (1987) and Schwert and Seguin (1990), our model imposes an assumption of a constant correlation matrix of returns over time [Equation (3)]. This parameterization ensures that H_t is positive definite under reasonable conditions and also offers an efficient representation for the tests that follow.

Our model facilitates an analysis of the volatility relations between the stock index and index futures markets in two forms. First, the off-diagonal parameters in matrix B, given by parameters $b_{s,f}$ and $b_{f,s}$, measure the dependence of the conditional return volatility in the cash market on that of the futures market and the dependence of the conditional return volatility in the futures market on that of the cash market in the last period. Second, the absolute size of price shocks originating in one market in previous periods, measured by the squared value of lagged innovations, also transmits to the current period's conditional volatility in the other market by means of the off-diagonal elements of matrices C_k given by parameters $c_{sf,k}$ and $c_{fs,k}$.

Note that these volatility relations are analyzed holding fixed the lead and lag relations in the changes in price levels in the conditional mean returns equation (1), via matrix $\boldsymbol{\beta}$. These predictive relations in price changes can appear because of asynchronous trading, short-sale constraints, and other market frictions that cause information already known in the futures market to be incorporated in the cash index price with a delay [Chan (1992)]. If these market frictions are captured by the lead/lag relation in prices in the conditional mean equation (1), then the conditional volatility of the cash index return should not necessarily be predictable. It is possible, however, that asynchronous trading might be related to volatility so that its effect would not be captured adequately by Equation (1). In Section 5, we deal with this alternative hypothesis in more detail.

Given a sample of T five-minute returns, the parameters of the bivariate system (1)–(3) are estimated by computing the conditional log likelihood function for each time period as

$$L_t(\boldsymbol{\theta}) = -\log 2\pi - \tfrac{1}{2}\log|\mathsf{H}_t(\boldsymbol{\theta})| - \tfrac{1}{2}\epsilon_t'(\boldsymbol{\theta})\mathsf{H}_t^{-1}(\boldsymbol{\theta})\epsilon_t(\boldsymbol{\theta}), \qquad (4)$$

$$L(\boldsymbol{\theta}) = \sum_{t=1}^{T} L_t(\boldsymbol{\theta}), \qquad (5)$$

where $\boldsymbol{\theta}$ is the vector of all parameters. Numerical maximization of (4) and (5) following the Berndt et al. (1974) algorithm yields the maximum likelihood estimates and associated asymptotic standard errors.

4. Results

4.1 Bivariate GARCH results

Table 3 shows the results of fitting the bivariate GARCH model to the intraday index and futures returns. The set of parameters that measure the dependence of the current cash returns on past cash returns, given by β_{ss}, and on past futures returns, given by β_{sf}, are significantly different from zero in all subperiods except 1988–1989. The attenuation in the serial dependence in the cash market for the last subperiod is, as noted earlier, consistent with Froot and Perold (1990). On the other hand, the relation between past futures returns to current cash returns remains strong in all subperiods. The autoregressive coefficient of the futures returns, β_{ff}, as expected, is negative and small for the 1984–1985 period, but larger thereafter. The coefficient for the lagged index returns, β_{fs}, is statistically significant and positive only for the 1984–1985 period and possibly 1987–1988. Overall, our strong evidence of a lead from futures to cash returns and only weak evidence of a lead from cash to futures returns is consistent with many previous studies.

The coefficients for the overnight return dummy in the volatility equation, $\mathbf{d}_2$, are not reported, but are significant in all periods with values ranging from 2.42 in 1985–1986 to 5.54 in 1987–1988. The implied stock index variance of the overnight return is equal to $(1 + d_{s,2})^2$ or, on average, about 20 times the variance of the five-minute return. These ratios of overnight to intraday stock return variances estimated by the GARCH model are comparable to the sample statistics reported in Table 1 and reasonably consistent with French and Roll (1986). The intercept coefficients associated with the overnight dummy for the mean returns, $\mathbf{d}_1$, are significantly positive, but only for the latter subperiods. Finally, the dummy variable for the slope coefficients, $\mathbf{d}_3$, for the first five-minute interval of the day are always jointly significantly different from zero. The coefficient that captures the influence of the overnight cash return on the first cash return of the trading day is consistently negative with values around -0.25. In Figure 1, the intraday volatility patterns are plotted for the cash and futures markets in the last subperiod 1988–1989, although those for the other subperiods are similar. We compute the average conditional variance measure in each five-minute interval during the day over each day in the sample. The figures confirm the familiar U-shaped pattern that a number of earlier studies have uncovered, including Harris (1986).

Estimates from the joint conditional volatility processes for the cash and futures returns indicate that the volatility in each of the two markets is affected by events in its own market as well as the other

Table 3
Estimates from bivariate generalized autoregressive conditional heteroskedastic models (GARCH) of intraday returns on S&P 500 stock index and S&P 500 stock index futures from August 1984–December 1989

α_s α_f	β_{ss} β_{fs}	β_{sf} β_{ff}	a_s a_f	b_{ss} b_{fs}	b_{sf} b_{ff}	$c_{ss,1}$ $c_{fs,1}$	$c_{sf,1}$ $c_{ff,1}$	$c_{ss,2}$ $c_{fs,2}$	$c_{sf,2}$ $c_{ff,2}$	$c_{ss,3}$ $c_{fs,3}$	$c_{sf,3}$ $c_{ff,3}$
Sample period 1984–85											
−0.0005 (−2.12)	0.3822 (50.5)*	0.1226 (30.6)*	0.00004 (19.4)*	0.9044 (159.0)*	−0.0037 (−13.2)*	0.2349 (34.0)*	0.0335 (38.8)*	−0.1811 (−19.9)*	−0.0159 (−13.7)*	−0.0232 (−4.75)*	−0.0077 (−9.91)*
−0.0019 (−3.81)*	−0.0604 (−3.99)*	−0.0021 (−0.245)	0.00010 (12.4)*	−0.1891 (−10.0)*	0.9728 (623.0)*	0.0623 (2.67)*	0.1134 (15.6)*	−0.0308 (−1.11)	−0.0346 (−3.48)*	0.0251 (2.53)*	−0.0421 (−6.69)*
ρ = .5024 (94.4)*; log likelihood = 53,602.869; Wald test A = 2405.2; Wald test B = 330.75											
Sample period 1985–86											
0.0005 (2.52)*	0.3123 (45.0)*	0.1456 (35.4)*	0.00003 (16.9)*	0.8425 (138.0)*	0.0015 (3.55)*	0.1509 (22.7)*	0.0702 (53.5)*	−0.0831 (−10.6)*	−0.0321 (−23.0)*	−0.0146 (−3.49)*	−0.0203 (−25.6)*
0.0011 (2.76)*	−0.0219 (−1.79)	−0.0838 (−12.6)*	0.00004 (15.7)*	−0.2014 (−19.2)*	0.9916 (1116.0)*	0.1770 (8.55)*	0.1187 (18.9)*	−0.1406 (−5.10)*	−0.0237 (−2.65)*	0.0351 (2.02)	−0.0638 (−11.0)*
ρ = .4597 (90.7)*; log likelihood = 64,648.113; Wald test A = 4458.0; Wald test B = 395.34											
Sample period 1986–87											
0.0011 (4.04)*	0.1648 (26.9)*	0.2457 (53.5)*	0.00006 (26.5)*	0.8995 (260.0)*	−0.0081 (−15.0)*	0.0910 (19.7)*	0.1053 (53.7)*	−0.0433 (−8.18)*	−0.0656 (−31.2)*	−0.0094 (−2.91)*	−0.0183 (−17.8)*
0.0016 (3.50)*	−0.0061 (−0.660)	−0.0934 (−15.2)*	0.00034 (22.9)*	−0.1915 (−10.2)*	0.9088 (219.0)*	0.0778 (4.82)*	0.1232 (20.4)*	−0.0337 (−1.75)	0.0428 (4.85)*	0.0246 (2.17)	−0.0654 (−11.3)*
ρ = .5192 (118.0)*; log likelihood = 60,682.251; Wald test A = 4172.3; Wald test B = 457.87											
Sample period 1987–88											
0.0002 (0.621)	0.1301 (16.5)*	0.2550 (45.9)*	0.00008 (21.4)*	0.9058 (185.0)*	−0.0095 (−8.09)*	0.0772 (14.8)*	0.0946 (38.7)*	−0.0443 (−6.87)*	−0.0551 (−20.0)*	−0.0020 (−0.495)	−0.0157 (−11.3)*
−0.0014 (−2.31)	0.0096 (0.78)	−0.0663 (−9.13)*	0.00047 (18.8)*	−0.0895 (−3.50)*	0.8749 (119.0)*	0.0669 (4.12)*	0.1379 (16.6)*	−0.0303 (−1.60)	−0.0286 (−2.66)*	−0.0127 (−1.10)	0.0031 (0.409)
ρ = .5510 (123.0)*; log likelihood = 45,261.163; Wald test A = 2585.5; Wald test B = 122.36											

Table 3
Continued

α_s / α_f	β_{ss} / β_{fs}	β_{sf} / β_{ff}	a_s / a_f	b_{ss} / b_{fs}	b_{sf} / b_{ff}	$c_{ss,1}$ / $c_{fs,1}$	$c_{sf,1}$ / $c_{ff,1}$	$c_{ss,2}$ / $c_{fs,2}$	$c_{sf,2}$ / $c_{ff,2}$	$c_{ss,3}$ / $c_{fs,3}$	$c_{sf,3}$ / $c_{ff,3}$
Sample period 1988–89											
0.0006	0.0838	0.2566	0.00006	0.8696	−0.0015	0.0674	0.1073	−0.0469	−0.0590	0.0166	−0.0248
(2.42)*	(11.3)*	(45.5)*	(25.4)*	(180.0)*	(−2.52)*	(14.9)*	(41.9)*	(−9.56)*	(21.2)*	(6.08)*	(−17.9)*
0.0001	−0.0367	−0.0372	0.00017	−0.1431	0.9371	0.1457	0.1294	−0.0920	−0.0414	−0.0133	−0.0195
(0.333)	(−3.33)*	(−5.08)*	(20.3)*	(−7.79)*	(229.0)*	(11.5)*	(20.2)*	(−5.87)*	(−4.71)*	(−1.31)	(−3.50)*

ρ = .5467 (133.0)*; log likelihood = 71,899.545; Wald test A = 2916.7; Wald test B = 83.723

The model is

$$\mathbf{r}_t = \alpha + \mathbf{d}_1 D_{1t} + \beta \mathbf{r}_{t-1} + \mathbf{d}_3 r_{t-1} D_{2t} + \eta_t, \quad \eta_t = [\iota + \mathbf{d}_2 D_{1t}]\epsilon_t, \quad \epsilon_t|\Phi_{t-1} \sim N(\mathbf{0},\mathbf{H}_t), \quad \begin{bmatrix} h_{ss,t} \\ h_{ff,t} \end{bmatrix} = \mathbf{A} + \mathbf{B}\begin{bmatrix} h_{ss,t-1} \\ h_{ff,t-1} \end{bmatrix} + \sum_{k=1}^{3} \mathbf{C}_k \begin{bmatrix} \epsilon^2_{ss,t-k} \\ \epsilon^2_{ff,t-k} \end{bmatrix}, \quad h_{sf,t} = \rho[h_{ss,t}h_{ff,t}]^{1/2}.$$

The vectors and matrices are defined as follows:

$$\mathbf{r}_t = \begin{bmatrix} r_{st} \\ r_{ft} \end{bmatrix}, \quad \alpha = \begin{bmatrix} \alpha_s \\ \alpha_f \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_{ss} & \beta_{sf} \\ \beta_{fs} & \beta_{ff} \end{bmatrix}, \quad \eta_t = \begin{bmatrix} \eta_{st} \\ \eta_{ft} \end{bmatrix}, \quad \epsilon_t = \begin{bmatrix} \epsilon_{st} \\ \epsilon_{ft} \end{bmatrix}, \quad \mathbf{d}_j = \begin{bmatrix} d_{s,j} \\ d_{f,j} \end{bmatrix},$$

$$\mathbf{H}_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{fs,t} & h_{ff,t} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} a_s \\ a_f \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_{ss} & b_{sf} \\ b_{fs} & b_{ff} \end{bmatrix}, \quad \mathbf{C}_k = \begin{bmatrix} c_{ss,k} & c_{sf,k} \\ c_{fs,k} & c_{ff,k} \end{bmatrix}, \quad \mathbf{d}_3 = \begin{bmatrix} d_{ss,3} & d_{sf,3} \\ d_{fs,3} & d_{ff,3} \end{bmatrix},$$

and r_{it} is the interval t S&P 500 stock index or stock index futures return, Φ_{t-1} is the set of all information at time $t-1$, D_{1t} is the overnight dummy variable set to unity for overnight returns, and zero for intraday returns, and D_{2t} is the dummy set to unity for the first five-minute return of each training day. The model is estimated using the Berndt et al. (1974) maximum likelihood algorithm. The conditional log likelihood function to be maximized is given in Equations (4) and (5). The values in parentheses are asymptotic t-statistics with * denoting coefficient estimates at least 2.325 standard errors from zero. The dummy coefficient estimates are not reported. The number of intraday returns observations used per estimation period are the same as those given in Table 1. The Wald test for independence (zero exclusion test) in the conditional volatility system hypothesizes that

Test A: $H_0 : b_{sf} = 0, \quad b_{fs} = 0$ and $c_{sf,k} = 0, \quad c_{fs,k} = 0 \quad \forall k = 1, 2, 3,$

Test B: $H_0 : b_{sf} = 0, \quad b_{fs} = 0.$

The χ^2 critical values for eight degrees of freedom are 15.5 and 20.1 and for two degrees of freedom are 5.99 and 9.21 for the 5 percent and 1 percent confidence levels, respectively.

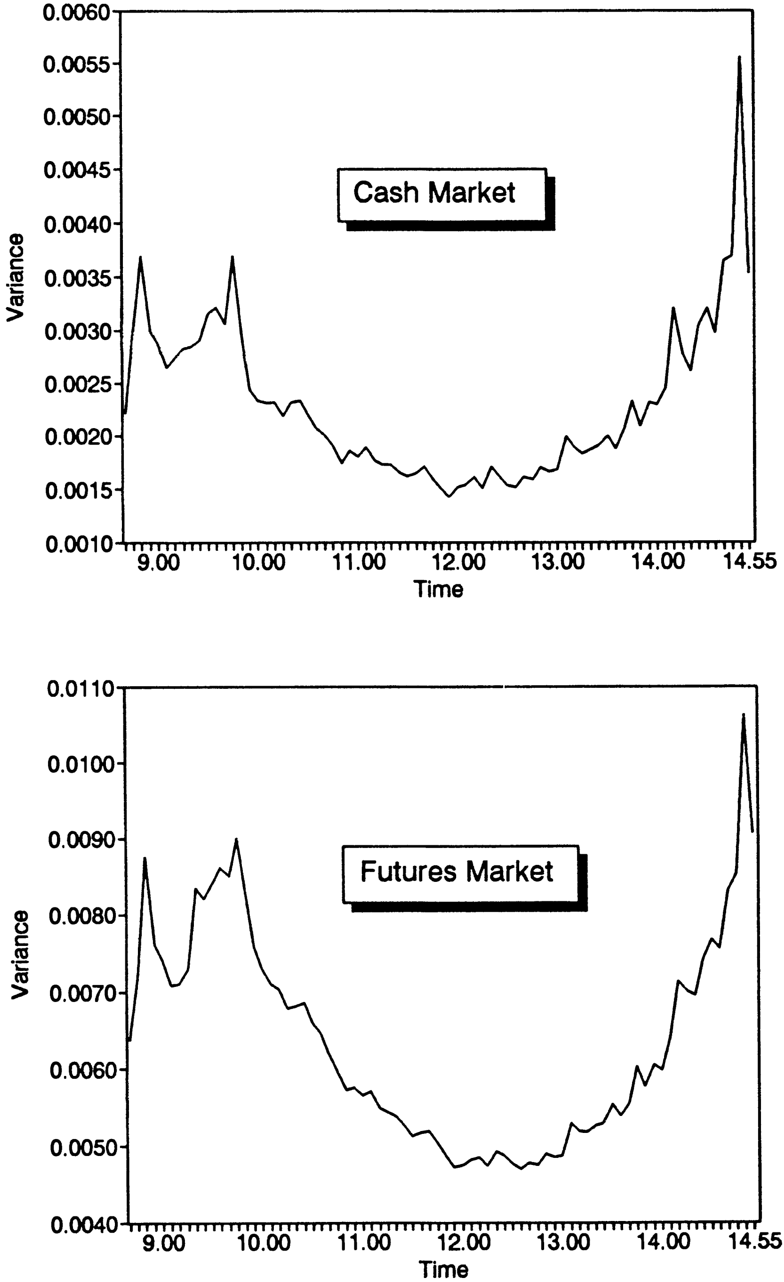

Figure 1
Intraday volatility of the S&P 500 stock index and S&P 500 stock index futures returns for November 1988–December 1989
Subperiod estimates for the bivariate AR(1)–GARCH(1,3) model with five-minute returns data are used to compute the conditional volatility series, which are then averaged over the number of days in the sample. "Time" is aligned to Central Time.

market. The coefficients that measure the cross-market impact of returns shocks on the volatility, $c_{sf,k}$ and $c_{fs,k}$, for all lags k, are significant and usually positive in the first five-minute lag and negative in the second and third five-minute intervals following the return shock. For example, in the 1988–1989 subperiod, the parameters $c_{sf,k}$ (equal to .1072, $-.0590$, and $-.0248$ for the three lags) transmit the past futures return innovations to the conditional variance of the cash market and, similarly, the $c_{fs,k}$ (equal to .1457, $-.0920$, and $-.0133$ for the three lags) transmit the past index return innovations to the conditional variance of the futures market. The parameters that measure the intermarket dependence in the conditional volatilities, b_{sf}, b_{fs}, are also significant and negative in all periods. Note that the generally negative signs of these cross-market coefficients of the $\mathbf{B}$ matrix do not imply that the conditional volatility of one market is correlated negatively with the conditional volatility of the other market in the previous period. Since the latter is itself a function of the past price shocks in both markets, the best way to see how price shocks in one market affect the conditional volatility of the other market is to invert the system so that the conditional volatilities are expressed as a function of all past price shocks originating in both markets. We illustrate the dynamics of the bivariate GARCH system by solving for the impulse response function coefficients in the next subsection.

Our evidence suggests that a "full-feedback" system obtains. Two tests are reported in Table 3 that evaluate the importance of intermarket dependence in the conditional volatility process for the cash and futures returns. The Wald test statistics measure the statistical significance of the off-diagonal coefficient estimates of the $\mathbf{B}$ and $\mathbf{C}_k$ matrices (Wald test A) and, separately, those of the $\mathbf{B}$ matrix alone (Wald test B). The hypothesis of no intermarket dependence for both tests is rejected easily in all periods.

The lead and lag relations found between the intraday volatilities of the cash and futures markets is not the same phenomenon uncovered in the lead and lag relations between the returns of the two markets. While we usually find some predictability from the cash to futures returns, the relation between a cash market shock to futures market volatility is stronger and more robust. In particular, in the two subperiods in which we find no significant lead from cash to futures returns, 1985–1986 and 1988–1989, the impact of cash market return shocks and past cash market volatility on the futures market volatility is even stronger than usual. This result for the volatility relations suggests that the pattern of new information flows to the cash and futures markets may be more symmetric than that inferred from examination of only the cash and futures returns.

Although not reported in the tables, some diagnostic tests of the residuals were performed. No indications of serious model misspecification are observed. The autocorrelations and partial autocorrelations for the squared standardized residuals for the stock index and futures returns series are all insignificantly different from zero for the bivariate AR(1)–GARCH(1,3) model. This suggests that the conditional volatility process captures successfully the intertemporal dependence highlighted in the returns shown in Table 1. The fit of the bivariate model was further evaluated by examining the distributional properties of the standardized residuals. Although the normal hypothesis is rejected by the Kolmogorov–Smirnov goodness-of-fit tests, there is a substantial reduction in the excess kurtosis observed in Table 1 for the raw returns. The results provide evidence of a reasonable fit for the bivariate GARCH(1,3) model.

4.2 Impulse response analysis

To illustrate the dynamics of the bivariate GARCH system estimated for cash and futures returns, we solve for the impulse response functions implied by the system. The impulse response coefficient, say $R_{ij,t+k}$, shows the impact of a unit returns shock (the squared return innovation) originating in market j at time t on the conditional volatility of market i at time $t + k$. These impulse responses can be computed by solving the two recursive equations in the conditional volatility equations (2) and (3) of the bivariate system. This type of analysis is analogous to solving a vector autoregression (VAR) system into a moving average representation [see Engle, Ito, and Lin (1990)].

We plot the impulse response coefficients for the first and last subperiods for which we estimate the model, denoted Figures 2 and 3, respectively. Two important results emerge from the diagrams. First, contrasting the results for the two subperiods shows that the cross-market impulse responses are larger in the more recent subperiods, suggesting that the degree of intermarket dependence in volatility has increased over the sample period. In the first subperiod, it appears that return innovations have a larger immediate impact on their own market volatility than that of the other market, but the latter subperiod shows the opposite effect. Surprisingly, the futures market shock has greater initial impact on cash market volatility and the cash market shock has greater initial impact on futures market volatility. Referring back to Table 3 shows that this trend toward greater intermarket dependence in volatility over the sample period has been gradual. Again, it is useful to contrast this surprising finding with that of previous studies that have shown the lead and lag relations between cash and futures market returns to have attenuated over time [see Stoll and Whaley (1990b)].

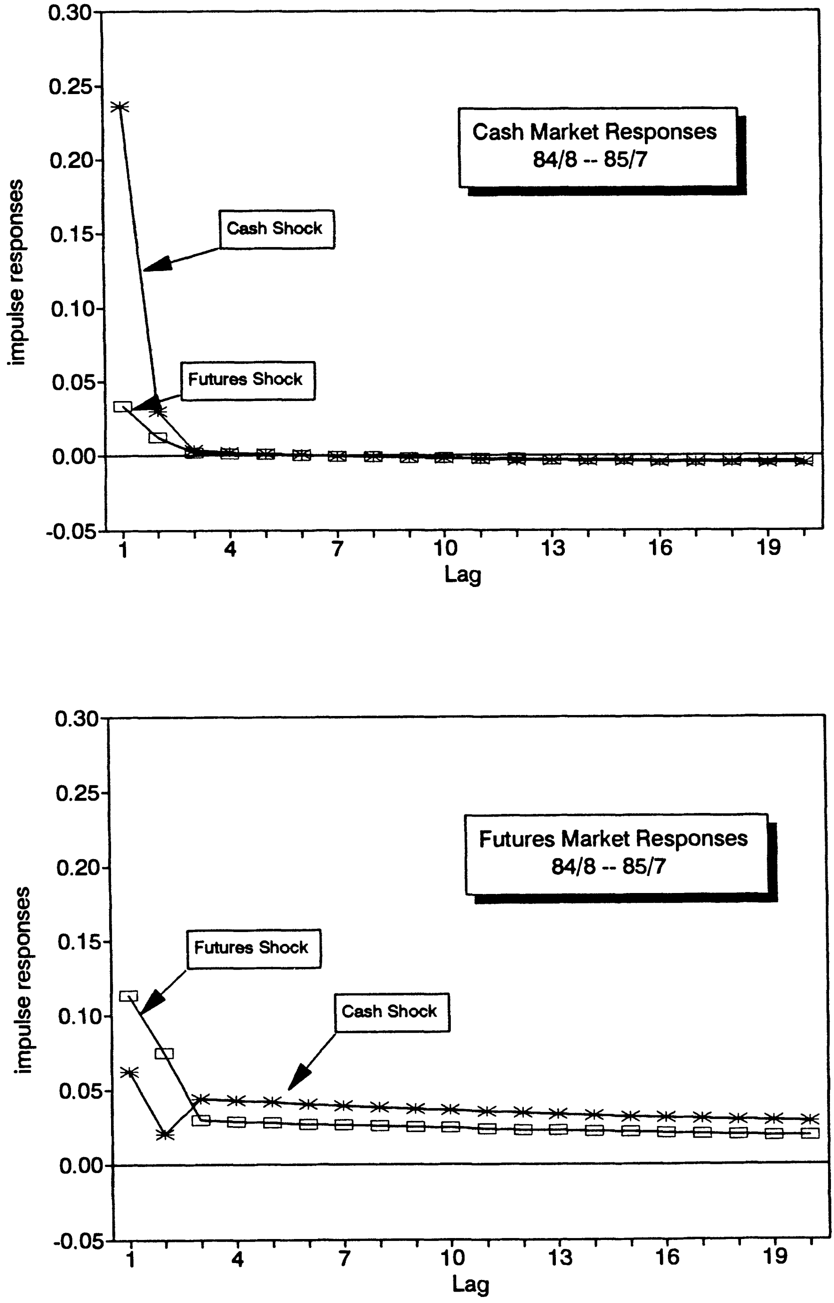

Figure 2
Impulse response functions of intraday conditional volatility of the S&P 500 stock index and S&P 500 stock index futures returns for August 1984–July 1985
Subperiod estimates for the bivariate AR(1)–GARCH(1,3) model with five-minute returns data are used to compute the impulse response coefficients following a unit return shock originating in each market.

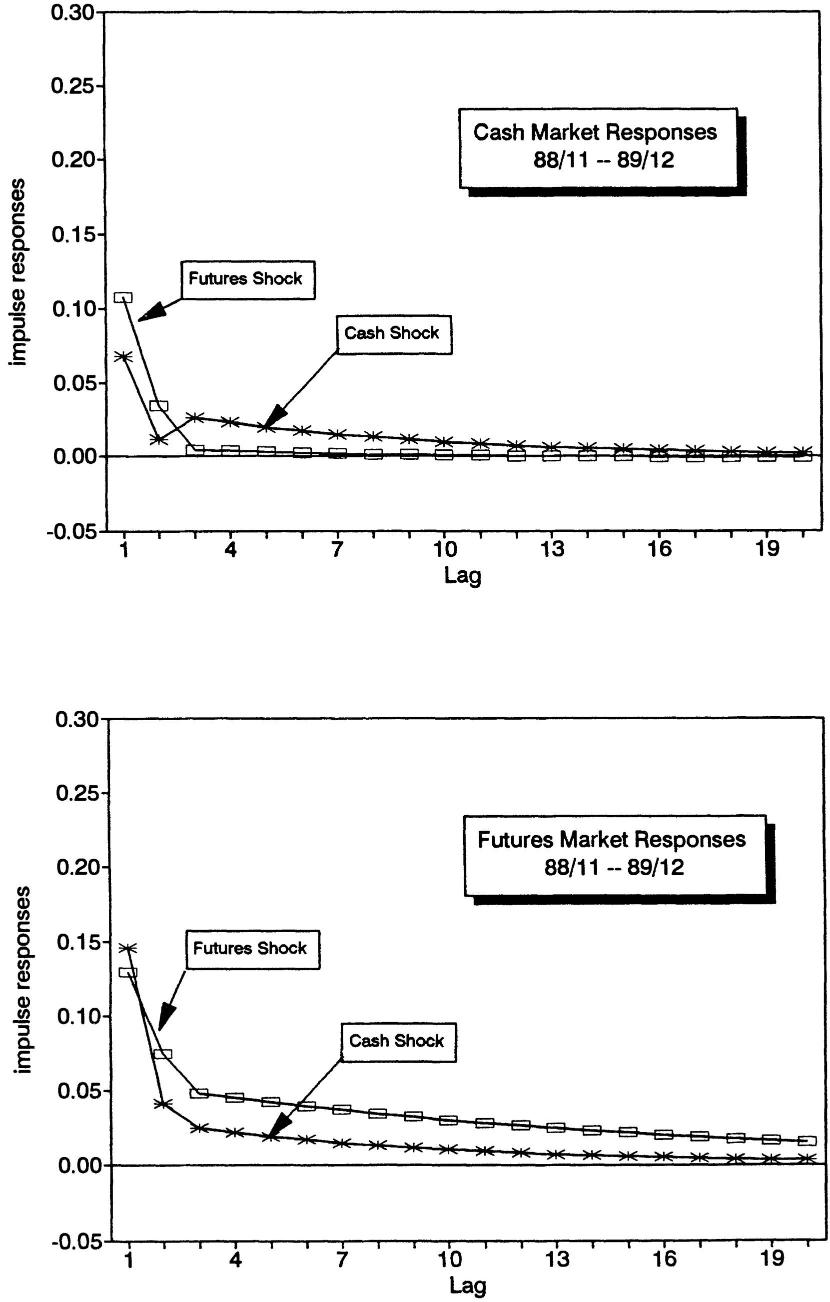

Figure 3
Impulse response functions of intraday conditional volatility of the S&P 500 stock index and S&P 500 stock index futures returns for November 1988–December 1989
Subperiod estimates for the bivariate AR(1)–GARCH(1,3) model with five-minute returns data are used to compute the impulse response coefficients following a unit return shock originating in each market.

The second result shown in the figures is that returns shocks to cash market volatility are less persistent than returns shocks to futures market volatility. We believe this result is somewhat unexpected. Because of the effects of asynchronous trading and other market frictions that are likely to affect the returns process in the cash market, we would expect shocks should take longer to die out in the cash market volatility than in the futures market volatility. We find the opposite effect is pervasive and strong. In the next section, we attempt to control for the effects of asynchronous trading and market frictions more directly.

Overall, the results of this section indicate that there exists a strong intermarket dependence in the volatility processes in the cash and futures markets. This relation is robust and has, in fact, become stronger over the entire 1984–1989 period, unlike the patterns of intermarket dependence captured in the returns alone.

5. Asynchronous Trading

Asynchronous trading of the component stocks in the S&P 500 Stock Index will influence the measurement of the intermarket dependence of intraday returns and volatility of returns. In fact, the magnitude and persistence of the cash and futures market shocks on the volatility of returns in those markets demonstrated by this study may be induced spuriously by the asynchronous trading problem. We evaluate this alternative hypothesis examining results using the Major Market Index with its associated futures contract and IBM stock with the S&P 500 stock index futures contract.

5.1 Tests using the Major Market Index data

The potential dangers of asynchronous trading are likely to be greater for a large index of stocks, like the S&P 500, than for a small and highly actively traded stock index, like the Major Market Index (MMI). The Major Market Index is a price-weighted index of 20 large NYSE traded stocks. The Chicago Board of Trade (CBOT) trades a futures contract on the MMI. All of the results obtained with the S&P 500 data are replicated for the MMI and its futures contract for the 1984–1985 subperiod for which it is available. The data are described in Section 2 and results are reported below.

Table 4 shows the estimates of the bivariate GARCH model on the MMI index and index futures data. Results based on the reported MMI index values and the index futures are shown in panel A; results with the computed MMI index values are in panel B. Recall from Section 2 that the index values reported to the CBOT Exchange floor

Table 4
Estimates from bivariate generalized autoregressive conditional heteroskedastic models (GARCH) of intraday returns on the MMI stock index and MMI stock index futures from July 23, 1984 to June 30, 1985

α_s α_f	β_{ss} β_{fs}	β_{sf} β_{ff}	a_s a_f	b_{ss} b_{fs}	b_{sf} b_{ff}	$c_{ss,1}$ $c_{fs,1}$	$c_{sf,1}$ $c_{ff,1}$	$c_{ss,2}$ $c_{fs,2}$	$c_{sf,2}$ $c_{ff,2}$	$c_{ss,3}$ $c_{fs,3}$	$c_{sf,3}$ $c_{ff,3}$
A. Results based on the MMI stock index values reported to the CBOT exchange floor											
−0.0000	0.0661	0.2718	0.00019	0.8535	0.0035	0.2024	0.0787	−0.1432	−0.0177	−0.0093	−0.0403
(−0.089)	(7.88)*	(39.3)*	(7.64)*	(46.0)*	(1.65)	(31.2)*	(23.4)*	(−15.8)*	(−3.76)*	(−1.63)	(−13.1)*
−0.0008	−0.0237	−0.0186	0.00000	0.0087	0.9832	0.0672	0.1135	−0.0399	−0.0480	−0.0286	−0.0529
(−1.38)	(−2.18)	(−2.01)	(0.014)	(0.857)	(1011.0)*	(5.75)*	(16.3)*	(−2.52)*	(−4.51)*	(−3.04)*	(−7.07)*
ρ = .5093 (95.4)*; log likelihood = 42,747.592											
B. Results based on the MMI stock index values computed from "Fitch" transactions prices for each component stock											
−0.0005	−0.0961	0.2136	0.00021	0.9834	−0.1248	0.0761	0.1226	0.0041	−0.0927	−0.0425	−0.0018
(−0.865)	(−10.7)*	(17.9)*	(8.70)*	(459.0)*	(−8.11)*	(10.6)*	(10.7)*	(0.382)	(−5.96)*	(−5.12)*	(−0.182)
0.0002	0.2219	0.0943	0.00022	−0.0022	0.8543	0.0559	0.1433	−0.0239	−0.0973	−0.0031	−0.0125
(0.457)	(34.1)*	(10.2)*	(11.1)*	(−1.02)	(62.7)*	(14.7)*	(22.1)*	(−5.63)*	(−12.2)*	(−1.21)	(−2.59)*
ρ = .5297 (102.0)*; log likelihood = 41,994.706											

The model is

$$\mathbf{r}_t = \boldsymbol{\alpha} + \mathbf{d}_1 D_{1t} + \boldsymbol{\beta}\mathbf{r}_{t-1} + \mathbf{d}_3 r_{t-1} D_{2t} + \boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t = [\iota + \mathbf{d}_2 D_{1t}]\epsilon_t, \qquad \epsilon_t|\Phi_{t-1} \sim N(\mathbf{0},\mathbf{H}_t), \qquad \begin{bmatrix} h_{ss,t} \\ h_{ff,t} \end{bmatrix} = \mathbf{A} + \mathbf{B}\begin{bmatrix} h_{ss,t-1} \\ h_{ff,t-1} \end{bmatrix} + \sum_{k=1}^{3} \mathbf{C}_k \begin{bmatrix} \epsilon^2_{ss,t-k} \\ \epsilon^2_{ff,t-k} \end{bmatrix}, \qquad h_{sf,t} = \rho[h_{ss,t}h_{ff,t}]^{1/2}.$$

The vectors and matrices are defined as follows:

$$\mathbf{r}_t = \begin{bmatrix} r_{st} \\ r_{ft} \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} \alpha_s \\ \alpha_f \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_{ss} & \beta_{sf} \\ \beta_{fs} & \beta_{ff} \end{bmatrix}, \quad \boldsymbol{\eta}_t = \begin{bmatrix} \eta_{st} \\ \eta_{ft} \end{bmatrix}, \quad \epsilon_t = \begin{bmatrix} \epsilon_{st} \\ \epsilon_{ft} \end{bmatrix}, \quad \mathbf{d}_j = \begin{bmatrix} d_{s,j} \\ d_{f,j} \end{bmatrix},$$

$$\mathbf{H}_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{fs,t} & h_{ff,t} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} a_s \\ a_f \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_{ss} & b_{sf} \\ b_{fs} & b_{ff} \end{bmatrix}, \quad \mathbf{C}_k = \begin{bmatrix} c_{ss,k} & c_{sf,k} \\ c_{fs,k} & c_{ff,k} \end{bmatrix}, \quad \mathbf{d}_3 = \begin{bmatrix} d_{ss,3} & d_{sf,3} \\ d_{fs,3} & d_{ff,3} \end{bmatrix},$$

and r_{it} is the interval t MMI stock index [reported to CBOT exchange floor (panel A) or computed value based on "Fitch" stamped transactions prices for individual stocks from NYSE (panel B)] or stock index futures return, Φ_{t-1} is the set of all information at time $t - 1$, D_{1t} is the overnight dummy variable set to unity for overnight returns and zero for intraday returns, and D_{2t} is the dummy variable set to unity for the first five-minute return of the training day and zero otherwise. The model is estimated using the Berndt et al. (1974) maximum likelihood algorithm. The conditional log likelihood function to be maximized is given in Equations (4) and (5). The values in parentheses are asymptotic t statistics with * denoting the coefficients at least 2.325 standard errors from zero, which approximates significance at the 1 percent level. The dummy coefficient estimates are not reported.

may be subject to significant time delays.[12] As a result, we seek to control for this reporting delay by implementing our analysis with MMI index values computed in each five-minute interval directly from the most recent transactions prices for each of the component stocks in the index recorded on the "Fitch" data. Though we must restrict our analysis to the 1984–1985 subperiod because of data limitations with the MMI index futures, some useful comparisons with the first subperiod of the S&P 500 results of Table 3 arise. As expected, the MMI index returns derived from the reported index have much smaller serial correlation than that found in the S&P 500 index returns. The value for β_{ss} is only 0.066 for the MMI results in contrast with a value in excess of 0.382 for the S&P 500 results in the same 1984–1985 subperiod. The magnitude and significance of the coefficients of the conditional volatility equations (2) and (3) are very similar to those reported in Table 3. For example, the impact of lagged shocks from the cash market to futures market volatility, $c_{sf,j}$, is significant at all lags with a large positive initial impact and subsequent negative values for the second and third lags. For the reverse direction, the effect of a futures market shock to cash market volatility, $c_{sf,j}$, is similar for the S&P 500 results but changes sign at the third lag for the MMI results. Finally, a comparison of the **B** matrix in Tables 3 and 4 shows less intermarket dependence in the MMI results where neither coefficient b_{sf} nor b_{fs} is significant. Overall, these results offer further supportive evidence that the primary results are not sensitive to the presence of asynchronous trading in the S&P 500 stock index.

The results based on the computed MMI index and the index futures are also reported in Table 4. Compared with the results on the reported index, this panel suggests that the futures returns have weaker predictive power for the MMI index returns. On the other hand, there is evidence that the index return can predict the futures return, as shown by the statistical significance of the β_{fs} coefficient estimates. These results are consistent with the findings of Chan (1992). The parameters in the conditional volatility equation are largely similar to those shown for the reported MMI index. They do show a stronger link between the past cash index price shocks and the current conditional volatility of the futures market, but a slightly weaker link between the past futures price shocks and the current conditional volatility of the cash market.

In sum, the MMI results also indicate that unexpected price changes in both the cash and futures markets impact the conditional volatility of price changes in the same and the other markets. Although asyn-

[12] Studies have shown that reporting lags have exacerbated the extent of the leading relation in price changes from futures to cash. See, in particular, Chan (1992).

Table 5
Estimates from bivariate generalized autoregressive conditional heteroskedastic models (GARCH) of intraday returns on IBM and the S&P 500 stock index futures from August 1984 to July 1986

α_s	β_{ss}	β_{sf}	a_s	b_{ss}	b_{sf}	$c_{ss,1}$	$c_{sf,1}$	$c_{ss,2}$	$c_{sf,2}$	$c_{ss,3}$	$c_{sf,3}$
α_f	β_{fs}	β_{ff}	a_f	b_{fs}	b_{ff}	$c_{fs,1}$	$c_{ff,1}$	$c_{fs,2}$	$c_{ff,2}$	$c_{fs,3}$	$c_{ff,3}$
0.0007	0.4443	−0.2033	0.00073	0.8335	0.0258	0.1438	0.1225	−0.0645	−0.0359	−0.0128	−0.0192
(1.32)	(53.5)*	(−34.3)*	(18.9)*	(126.0)*	(5.87)*	(34.9)*	(18.9)*	(−11.4)*	(−3.58)*	(−3.39)*	(−3.03)*
0.0005	−0.0546	0.0789	0.00016	−0.0334	0.9952	0.0199	0.1057	−0.0142	−0.0269	0.0077	−0.0568
(1.41)	(−8.68)*	(19.6)*	(13.4)*	(−13.8)*	(827.0)*	(11.9)*	(22.9)*	(−6.72)*	(−3.94)*	(5.59)*	(−12.4)*

$\rho = .4983$ (144.0)*; log likelihood = 73,015.051

The model is

$$\mathbf{r}_t = \boldsymbol{\alpha} + \mathbf{d}_1 D_{1t} + \boldsymbol{\beta}\mathbf{r}_{t-1} + \mathbf{d}_3 r_{t-1} D_{2t} + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t = [\iota + \mathbf{d}_2 D_{1t}]\boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t | \Phi_{t-1} \sim N(\mathbf{0}, \mathbf{H}_t), \quad \begin{bmatrix} h_{ss,t} \\ h_{ff,t} \end{bmatrix} = \mathbf{A} + \mathbf{B}\begin{bmatrix} h_{ss,t-1} \\ h_{ff,t-1} \end{bmatrix} + \sum_{k=1}^{3} \mathbf{C}_k \begin{bmatrix} \epsilon^2_{ss,t-k} \\ \epsilon^2_{ff,t-k} \end{bmatrix}, \quad h_{sf,t} = \rho[h_{ss,t} h_{ff,t}]^{1/2}.$$

The vectors and matrices are defined as follows:

$$\mathbf{r}_t = \begin{bmatrix} r_{st} \\ r_{ft} \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} \alpha_s \\ \alpha_f \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_{ss} & \beta_{sf} \\ \beta_{fs} & \beta_{ff} \end{bmatrix}, \quad \boldsymbol{\eta}_t = \begin{bmatrix} \eta_{st} \\ \eta_{ft} \end{bmatrix}, \quad \boldsymbol{\epsilon}_t = \begin{bmatrix} \epsilon_{st} \\ \epsilon_{ft} \end{bmatrix}, \quad \mathbf{d}_j = \begin{bmatrix} d_{s,j} \\ d_{f,j} \end{bmatrix},$$

$$\mathbf{H}_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{fs,t} & h_{ff,t} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} a_s \\ a_f \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_{ss} & b_{sf} \\ b_{fs} & b_{ff} \end{bmatrix}, \quad \mathbf{C}_k = \begin{bmatrix} c_{ss,k} & c_{sf,k} \\ c_{fs,k} & c_{ff,k} \end{bmatrix}, \quad \mathbf{d}_3 = \begin{bmatrix} d_{ss,3} & d_{sf,3} \\ d_{fs,3} & d_{ff,3} \end{bmatrix},$$

and r_{st} is the interval-t IBM stock return, r_{ft} is the interval-t S&P 500 stock index futures return, Φ_{t-1} is the set of all information at time $t-1$, D_{1t} is the overnight dummy variable set to unity for overnight returns and zero for intraday returns, and D_{2t} is the dummy set to unity for the first five-minute return of each training day. The model is estimated using the Berndt et al. (1974) maximum likelihood algorithm. The conditional log likelihood function to be maximized is given in Equations (4) and (5). The values in parentheses are asymptotic t statistics with * denoting coefficient estimates at least 2.325 standard errors from zero, which approximates significance at the 1 percent level. The dummy coefficient estimates are not reported.

chronous trading and reporting lags may account for the transmission lag between S&P 500 index price changes and index futures price changes, these additional tests employing two different sets of MMI index data corroborate the results found for the cash and futures conditional volatility processes using the S&P 500 data.

5.2 Additional tests using IBM and S&P 500 stock index futures
One way to confront the asynchronous trading problem even more directly is to implement the model with intraday returns of the S&P 500 index futures and IBM stock. Clearly, just as infrequent trading is less of a problem for the MMI than for the S&P 500 stocks, it is essentially not a problem for IBM. Over the 1984 to 1986 period, IBM averages 7.83 trades per five-minute interval and there is only a 0.5 percent chance that it will not trade in any given five-minute interval. Again, we focus on the earlier subperiods because the asynchronous trading problem is likely to be most serious.

Table 5 shows the results for the 1984–1986 subperiod for which the "Fitch" data are available. In the conditional mean equation, predictability of price changes from S&P 500 futures to IBM and vice versa are both statistically significant, but the coefficient value for β_{fs} is lower than that of Table 4 for the computed MMI index. Similarly, the cross-market effects in the conditional volatility processes are significant, yet weaker than for the indexes.

On the whole, the results of Table 5 lend support to the claim that the main results related to the intraday patterns in conditional volatility between the futures and cash markets are not likely to be driven by either asynchronous trading of component stocks in the stock index or significant delays in reporting the stock index levels.

6. Conclusions

We examine the intraday relationship between price changes and price change volatility in the stock index and stock index futures markets. Our evidence for the S&P 500 stock index and stock index futures markets from 1984 to 1989 indicates that (i) the intraday volatility patterns in both markets demonstrate strong persistence and predictability; (ii) there exists strong and pervasive intermarket dependence in the volatility of their price changes, even in subperiods in which the dependence in the price changes themselves appears to be diminished; and (iii) these findings are robust even when the analysis controls for potential market frictions such as asynchronous trading in the stock index.

Previous studies that have found intraday futures market price changes systematically leading cash market price changes conclude that index futures markets serve as the primary market for price discovery. They argue that new information appears to disseminate in the futures market first and subsequently in the cash market. Our article extends and generalizes the findings of these existing studies by examining the intraday relations of the volatility of these markets while at the same time controlling for lead and lag relations in their price changes. We show much stronger dependence in both directions in the volatility of price changes between the cash and futures markets than that observed in the price changes alone. Information in price innovations that originate in the cash market is transmitted to the volatility of the futures market and information in price innovations that originate in the futures market is transmitted to the volatility of the cash market. Our evidence is thus consistent with the hypothesis that new market information disseminates in both the futures and stock markets and that both markets serve important price discovery roles.

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