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Information and Volatility: The No-Arbitrage Martingale Approach to Timing and Resolution Irrelevancy

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ABSTRACT

The no-arbitrage martingale analysis is used to study the effect on asset prices of changes in the rate of information flow. The analysis is first used to develop some simple tools for asset pricing in a continuous-time setting. These tools are then applied to determine the effect of information on prices and price volatility, to extend Samuelson's theorem on prices fluctuating randomly, and to study the impact on prices of the resolution of uncertainty. The conditions under which uncertainty resolution is irrelevant for asset pricing are shown to be similar to those which support the MM irrelevance theorems.

SEVERAL YEARS AGO, IN the midst of the New York City fiscal crisis, many newspapers carried nearly daily articles on the behavior of the outstanding New York City bonds. One such article of particular interest described how a certain revenue anticipation issue had rallied on the announcement that the results of a yet to be conducted audit would be released earlier than anticipated. The audit, describing the state of revenue collection, was to have commenced in three months. The audit would be concluded one month later, and it had been traditional to report the results of the audit three months after that, in November. The announcement merely reported the findings of an administrative procedure that had determined that, rather than delaying the release of the audit findings for three months, they would be released as soon as the audit was concluded. Figure 1 illustrates the time line of events in this anecdote.

There are a number of plausible explanations as to why the announcement might have contained some information bearing on the anticipated results of the audit, but accepting the story at face value presents finance scholars with an interesting paradox. To wit, how could the mere announcement of the acceleration of the release of some information concerning its future payoff possibly influence the price of an asset?

A classic exercise for introductory finance classes is to consider the value of a lottery which has a fifty-fifty chance of paying nothing or $1 million one year from now. Suppose that one year from now a fair coin will be flipped to determine whether the lottery pays $1 million or nothing. Assuming that the outcome of the coin flip is unrelated to any other events in the economy, the current value

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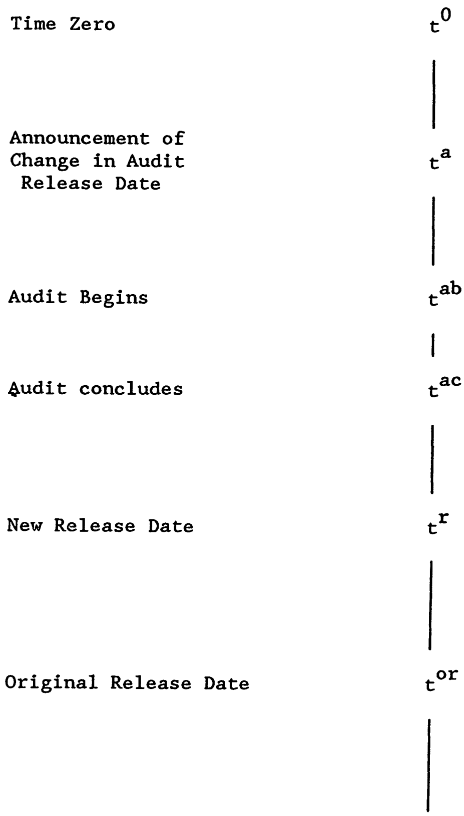

Figure 1. Time Line for New York City Bond Audit

of such a lottery is simply ½ of the discounted value of $1 million. Would this answer change if instead of having to wait a year for the news it was announced that the coin would be flipped tomorrow?

There are two competing intuitions concerning such timing effects. The first of these, articulated by Robichek and Myers [15] and analyzed in detail by Epstein and Turnbull [9], argues that the early resolution of uncertainty helps investors to plan. As a consequence, assets in which the resolution has been accelerated will have their value enhanced. A competing intuition, though, holds that in an efficient market the early release of information cannot influence value when it changes no real cash flows. The point is not whether the release of the information itself will change values—it generally will—but, rather, whether the announcement of an acceleration in the flow of information will change values. The coin flipping example above is used to illustrate the latter line of thought.

These considerations have to do with issues of flexibility and liquidity, both of which remain more catchall categories for intuitions than analytical tools. Simple questions such as whether there is a return differential for investments in illiquid instruments are very difficult to answer without some agreement on which intuition is dominant. In principle, the resolution of these two perspectives is not terribly difficult. If there is a real decision that can enhance cash flows and if the early availability of information makes such a change possible, the investment will, indeed, become more valuable. This is somewhat obvious, though, and our principal concern will be with the second intuition. Is it really the case that—in the absence of a cash flow effect—the early resolution of information will not influence values in an efficient market? These questions also bear on the current debates on the volatility of asset prices. As we shall see, market efficiency and the absence of arbitrage have direct implications for volatility.

To explore these issues in as general a setting as is practical, Section I introduces and specializes some of the relevant results from no-arbitrage martingale analysis. Section II derives a simple general result relating the volatility of the information flow to price volatility. Section III expands this result and derives some general theorems on the relations between information, timing, and valuation. Two cases are studied in detail, the first of which illustrates that changes in the intertemporal resolution of the uncertainty that influence cash flows generally alter values. The second case demonstrates that changes in the resolution of uncertainty that do not alter cash flows will not change prices. Section IV relates this work to option pricing theory, and Section V briefly summarizes and concludes the paper.

I. No-Arbitrage Martingale Analysis

We will use the no-arbitrage methodology to explore the timing issues. This methodology was developed in Ross [16] and Cox and Ross [4] and extended in Ross [17], Harrison and Kreps [12], Harrison and Pliska [13], Chamberlain and Rothschild [3], Cox, Ingersoll, and Ross [5], Dybvig, Ingersoll, and Ross [8], and Hansen and Richard [11].

In an arbitrage-free economy there exists a martingale measure and its associated operator, E^*, such that

$$p = E^*(e^{-rt}p_t \mid I_0), \tag{1}$$

where unsubscripted variables are current, subscripts indicate timing, I denotes an information set, p is an asset price, and r is the interest rate. Equation (1) is the valuation equation of an asset with no cash flows on $[0, t]$. We will assume that the interest rate, r, is constant. This assumption is easily generalized, but we will retain it for notational convenience.

In its differential form, equation (1) reveals that assets must be risk-neutrally priced when they are transformed by the martingale measure,

$$E^*\left(\frac{dp}{p}\right) = rdt.$$

Quite generally, we can write equation (1) as

$$p = E(q_t p_t \mid I_0), \tag{2}$$

where q is a pricing standard and the expectation is taken with respect to the actual probabilities. (The martingale measure used in computing E* is absolutely continuous with respect to the actual measure used in taking the ordinary expectation, E.) The pricing standard, q, is interpreted as the price per unit of probability. (See Dybvig, Ingersoll, and Ross [8].)

For concreteness and ease we will assume that all of this is cast in a world where intertemporal variables follow Itô processes. Letting 1_t denote a sure payoff of one unit at time t we have

$$E^*(e^{-rt}1_t) = e^{-rt}, \tag{3}$$

or

$$E^*(1_t) = E^*(e^{-rt}e^{rt}) = E(q_t e^{rt}) = 1. \tag{4}$$

Representing the motion of q as

$$\frac{dq}{q} = \mu_q dt + \sigma_q dz_q \tag{5}$$

and defining

$$v_t \equiv e^{rt}q_t, \tag{6}$$

we have from (4)

$$E(v_t) = 1, \tag{7}$$

which leads to our first result.

LEMMA 1: *The drift of q is given by*

$$\mu_q = -r. \tag{8}$$

Proof:

$$E(v_t) = 1$$

implies that

$$E(dv_t) = 0,$$

or, since

$$\frac{dv}{v} = (r + \mu_q)dt) + \sigma_q dz_q,$$

we have

$$E\left(\frac{dv}{v}\right) = (r + \mu_q)dt = 0. \quad \text{Q.E.D.}$$

Similarly, for any asset we must have

$$p = E(q_t p_t), \tag{9}$$

which implies that

$$E(d(q_t p_t)) = 0. \tag{10}$$

Letting

$$\frac{dp}{p} = \mu_p dt + \sigma_p dz_p,$$

we have from (10) that

$$E\left(\frac{d(qp)}{qp}\right) = E\left(\frac{dp}{p} + \frac{dq}{q} + \left(\frac{dp}{p}\right)\left(\frac{dq}{q}\right)\right)$$
$$= [\mu_p + \mu_q + \mathrm{cov}(p, q)]dt$$
$$= 0,$$

where

$$\mathrm{cov}(x, y) \equiv \mathrm{cov}\left(\frac{dx}{x}, \frac{dy}{y}\right).$$

Hence, we have proven the following.

THEOREM 1: *Expected returns satisfy the following generalized security market line equation*:

$$\mu_p - r = -\mathrm{cov}(p, q). \tag{11}$$

Proof: See above. Q.E.D.

To make all of this a bit more familiar we will look at two examples drawn from models with a single good and additive state-independent utility.

Example 1.1: Consider a discrete-time one-period model. With complete markets and no arbitrage, the equilibrium is supported by Arrow-Debreu prices and individuals seek

$$\max \textstyle\sum_\theta \pi_\theta u(c_\theta) + u(c_0)$$

subject to

$$\textstyle\sum_\theta p_\theta c_\theta \leq w.$$

With nicely behaved functions, the internal first-order conditions hold:

$$\pi_i \frac{u'(c_i)}{u'(c_0)} = p_i. \tag{12}$$

Given the prices, p_i, the value, v_0, of any asset with payoffs of v_i is given by

$$v_0 = \textstyle\sum_i p_i v_i$$
$$= (\textstyle\sum_i p_i) \sum_i \xi_i v_i$$
$$\equiv \left(\frac{1}{1 + r}\right) E^*(v), \tag{13}$$

where

$$\xi_i \equiv \frac{p_i}{\sum p_i}$$

has the properties of a probability and is the martingale pricing measure in this model and E^* is the martingale expectation.

The individual maximization problem allows us to interpret ξ_i as a marginal rate of substitution distorted time probability, π_i; i.e.,

$$\frac{\xi_i}{\xi_j} = \frac{\pi_i}{\pi_j}\frac{u'(c_i)}{u'(c_j)}.$$

We can also also define a pricing standard, q, in this model:

$$q_i = \frac{u'(c_i)}{u'(c_0)}. \tag{14}$$

In terms of the pricing standard,

$$\begin{aligned} v_0 &= \textstyle\sum_i \pi_i q_i v_i \\ &= E(qv). \end{aligned} \tag{15}$$

Notice, too, that, since p_i is the price in state i and

$$q_i = p_i/\pi_i,$$

q_i is the price per unit of probability. Dybvig [7] has developed an elegant theory of efficient allocations by exploiting the inverse relation between consumption allocations and q.

Example 1.2: In a continuous-time diffusion model some of these relations become much simpler.

Taking

$$q_t = \frac{u'(c_t)}{u'(c_0)}, \tag{16}$$

we have

$$\begin{aligned} \operatorname{cov}(p, q) &= \left(\frac{dp}{p}, \frac{dq}{q}\right) \\ &= \operatorname{cov}\left(\frac{dp}{p}, \left(\frac{cu''}{u}\right)\frac{dc}{c}\right) \\ &= -R \operatorname{cov}\left(\frac{dp}{p}, \frac{dc}{c}\right). \end{aligned} \tag{17}$$

Applying Theorem 1 we can derive the familiar consumption beta-pricing relation:

$$\mu_p - r = R \operatorname{cov}(p, c). \tag{18}$$

We can also learn some other things from this approach that may be a bit less familiar. For example, since

$$\mu_q = -r, \tag{8}$$

we have

$$\mu_q = E\left(\frac{dq}{q}\right)$$
$$= \frac{1}{2}\left(\frac{u'''}{u'}\right)\sigma_c^2 + R\mu_c,$$

or

$$\frac{1}{2}\left(\frac{u'''}{u'}\right)\sigma_c^2 + R\mu_c = -r, \tag{19}$$

which is the equation of movement for consumption, c.

II. Information and Price Volatility

Now that we have developed the martingale no-arbitrage analysis, we can use it to examine the relation between pricing and the flow of information. Suppose that information is generated by a process of the form

$$\frac{ds}{s} = \mu_s dt + \sigma_s dz_s, \tag{20}$$

where we will use this process to predict the value of s at a future time T, s_T. If s follows a lognormal process, then

$$E(s_T \mid s) = se^{\mu_s(T-t)}; \tag{21}$$

i.e., s is a sufficient statistic for predicting s_T. Since the drift of s is constant and therefore perfectly predictable, it does not affect the resolution of uncertainty and we can set $\mu_s = 0$ without any loss of generality. We will make use of this simplification below. Notice, too, that, since s is not necessarily an asset we do *not* require that

$$\mu_s - r = -\operatorname{cov}(s, q).$$

Now consider an asset the value of which at time T is given by s_T,

$$p_T = s_T, \tag{22}$$

and which has no payouts on $[0, T)$. This is essentially a forward contract with a payout of s_T at time T. (See Black [1] or Cox, Ingersoll, and Ross [5] for a discussion of such contracts.)

From our basic pricing relation, equation (2), we know that

$$p = E(q_T p_T \mid I_0)$$
$$= E(q_T s_T \mid I_0)$$
$$= E(q_T s_T \mid q, s), \tag{23}$$

where we are assuming that (q, s) is sufficient for determining the future movement (path probabilities) of $q_T s_T$. For simplicity we will assume that they are jointly lognormal.

To solve for the current value, p, we observe that, if

$$v \equiv qs,$$

then

$$\frac{dv}{v} = \frac{dq}{q} + \frac{ds}{s} + \left(\frac{dq}{q}\right)\left(\frac{ds}{s}\right)$$
$$= [\mu_s - r + \text{cov}(q, s)]dt + \sigma_q dz_q + \sigma_s dz_s.$$

It follows that

$$p = E(v_T \mid q, s)$$
$$= se^{(\mu_s - r + \text{cov}(q,s))(T-t)}.[1] \tag{24}$$

Notice that by applying Theorem 1 or differentiating the value we obtain

$$\mu_p = r - \text{cov}(p, q). \tag{11}$$

Now we can prove a simple result which is of considerable interest given the recent focus on the relation between information and price volatility. (See, e.g., the work of French and Roll [10].)

THEOREM 2: *The variance of price change equals the rate of information flow*:

$$\sigma_p^2 = \sigma_s^2. \tag{25}$$

Proof: From the pricing relation (24),

$$\frac{dp}{p} = \frac{ds}{s} - [\mu_s - r + \text{cov}(q, s)]dt,$$

or

$$\mu_p dt + \sigma_p dz_p = [r - \text{cov}(q, s)]dt + \sigma_s dz_s,$$

[1] The formula is this rather than

$$p = qse^{(\mu_s - r + \text{cov}(q,s))(T-t)}$$

because q_t is always normalized to one to get prices as of time t. The examples make this clear since, at $t = 0$,

$$q_0 = \frac{u'(c_0)}{u'(c_0)} = 1.$$

If we did not normalize, then assets would be priced as of some fixed reference date, say time 0. In this case we would not expect value to ever change, and, indeed, using the above formula we have

$$\frac{dp}{p} = \sigma_q dz_q + \sigma_s dz_s,$$

or

$$E\left(\frac{dp}{p}\right) = 0.$$

which implies that

$$\sigma_p dz_p = \sigma_s dz_s$$

and

$$\sigma_p^2 = \sigma_s^2.$$

Notice that we have also verified that dp and ds are perfectly correlated. Q.E.D.

In this very limited and parameterized model we have shown that the variance of the price equals the variance of the information flow. Furthermore, this is a consequence solely of the absence of arbitrage. In other words, Theorem 2 implies that, if the volatility of prices is not equal to the rate at which information arrives,

$$\sigma_p^2 \neq \sigma_s^2,$$

then arbitrage is possible. An attraction of this result is that it is independent of the pricing standard, q, which is to say, of the particular asset-pricing model being used. (We are, of course, assuming that $\operatorname{cov}(q, s)$ is not stochastic, i.e., that s has an unchanging relation to other assets.)

Theorem 2 bears directly on the French and Roll [10] results and efficient market hypotheses. Since price volatility equals information volatility, if prices are more volatile when markets are open for trading, then more information must be released when markets are open than when they are closed. If this is not the case, then arbitrage is possible.

In what form Theorem 2 holds true in more general settings is a topic for further study. Suppose that x is vector of state variables which are sufficient for pricing and which follow a diffusion (see Cox, Ingersoll, and Ross [5]), and let Ω_x denote the covariance matrix for (dx). The pricing relation,

$$p = E(q_T s_T \mid x),$$

now implies that the stochastic portion of dp is given by

$$p_x' dx,$$

and, therefore, the variance of the rate of change in prices is given by

$$\sigma_p^2 = (p_x'/p)\Omega_x(p_x/p).$$

In general, if x influences the rate of growth of s and if it affects the pricing operator, then the simple relation of Theorem 2 will be violated. However, whether the appropriate sense of an information flow equivalence still holds remains to be determined.

III. Resolution Irrelevancy

In this section we will explore the implications of accelerating the flow of information. What happens if we learn about s_T earlier? As we shall see, the answer depends delicately on precisely how we pose the question.

To study these matters, we will make use of the following result. It is the generalization to the martingale analysis in an arbitrary arbitrage-free economy of Samuelson's [18] theorem that properly anticipated prices fluctuate randomly in a risk-neutral economy.

THEOREM 3 (Properly Anticipated Prices Fluctuate Randomly): *If* $(\forall t \leq T)$

$$p_t = E^*(e^{-r(T-t)} s_T \mid I_t), \tag{26}$$

then

$$p_t = E^*(e^{-r(t'-t)} p_{t'} \mid I_t), \quad t < t' < T. \tag{27}$$

Proof: Since $I_t \subset I_{t'}$ (i.e., I_t is a subpartition of $I_{t'}$),

$$\begin{aligned} p_t &= E^*(e^{-r(T-t)} s_T \mid I_t) \\ &= E^*(E^*(e^{-r(T-t)} s_T \mid I_{t'}) \mid I_t) \\ &= E^*(e^{-r(t'-t)} E^*(e^{-r(T-t')} s_T \mid I_{t'}) \mid I_t) \\ &= E^*(e^{-r(t'-t)} p_{t'} \mid I_t). \quad \text{Q.E.D.} \end{aligned}$$

Theorem 3 assures us that (discounted) prices follow a martingale in the martingale measure.

In what follows we will use Theorem 3 together with the single state variable model of Section II to analyze the dynamic influence of changing the rate of information flow. From Section II,

$$\begin{aligned} p_t &= E^*(e^{-r(T-t)} s_T \mid I_t) \\ &= E(s_T \mid s_t) e^{-(r - \mathrm{cov}(q,s))(T-t)}. \end{aligned}$$

Without loss of generality, we set $\mu_s = 0$. This will enable us to focus on the role of the information set, I_t. From equation (24) this implies that

$$p_t = s_t e^{-(r - \mathrm{cov}(q,s))(T-t)}. \tag{28}$$

We begin our analysis with the simplest example of the impact of the acceleration of information. As we will see, somewhat hidden and implicit in this exercise is a change in real cash flows.

Exercise 3.1: Suppose that at time t it is announced that

(i) the uncertainty surrounding the realization of s_T will be resolved by t', $t < t' < T$, rather than at T; i.e., $s_T = s_{t'}$;
(ii) the payoff is actually received at T; and
(iii) the market observes s_τ, $t \leq \tau \leq t'$; i.e., $s_\tau \in I_\tau$.

Figure 2 illustrates the information flow.

Since the uncertainty will be resolved by t', the covariance of s with q is irrelevant for $t \geq t'$. This alters the nature of discounting in the valuation formulas. For $t \geq t'$ we have

$$\begin{aligned} p_t &= E^*(e^{-r(T-t)} s_T \mid I_t) \\ &= E^*(e^{-r(T-t)} s_{t'} \mid s_t) \\ &= e^{-r(T-t)} s_{t'}, \end{aligned}$$

t — Starting time.

τ — An intermediate pricing date.

t' — Uncertainty about s_T is resolved, i.e., the payoff s_T is determined as $s_T = s_{t'}$.

T — Payoff, $s_T = s_{t'}$ is received.

Figure 2. Time Line for Exercise 3.1; Investors observe s_t throughout.

which is to say that the asset is riskless after s_t' is revealed, $s_{t'} \in I_t$.

Using Theorem 3, for $t < t'$,

$$
\begin{aligned}
p_t &= E^*(p_{t'} e^{-r(t'-t)} \mid I_t) \\
&= E^*(s_{t'} e^{-r(T-t')} e^{-r(t'-t)} \mid I_t) \\
&= e^{-r(T-t)} E^*(s_{t'} \mid I_t) \\
&= e^{-r(T-t')} E(s_{t'} \mid I_t) e^{-[r-\mathrm{cov}(q,s)](t'-t)} \\
&= s_t e^{[\mathrm{cov}(q,s)](t'-t)} e^{-r(T-t)}.
\end{aligned}
\tag{30}
$$

In this exercise the announcement of the early receipt of the information actually has altered the distribution of the terminal flow. Since $s_t = s_{t'}$, the current value rose on the announcement by

$$
\frac{\Delta p}{p} \equiv \frac{p_t(post) - p_t(pre)}{p_t(pre)} = e^{-\mathrm{cov}(q,s)(T-t')} - 1 > 0, \tag{31}
$$

where "*pre*" and "*post*" are, respectively, before and after the announcement and where we are assuming that this is a typical asset with $\mathrm{cov}(q, s) < 0$.

We can use this exercise to make an obvious but important observation. Since, for $t < t'$,

$$
p_t = s_t e^{-r(T-t)} e^{\mathrm{cov}(q,s)(t'-t)}, \tag{32}
$$

if we define β_{sq} as the regression coefficient of s on q, then

$$\sigma_s dz_s = \beta_{sq}\sigma_q dz_q + \sigma_\varepsilon dz_\varepsilon,$$

and, since

$$\mathrm{cov}(q, s) = \beta_{sq}\sigma_q^2, \tag{33}$$

changing the idiosyncratic information flow, $\sigma_\varepsilon dz_\varepsilon$, has no effect on p_t. This implies that we can alter σ_s, subject to

$$\sigma_s^2 = \beta_{sq}^2\sigma_q^2 + \sigma_\varepsilon^2, \tag{34}$$

and, as long as only σ_ε^2 changes, p_t will be unaffected. Information uncorrelated with q may, of course, affect expectations of S_T, but changing the rate of the flow of such information will not affect p_T.

Summarizing, if we break s_T into two parts, one correlated with q and the other depending only on ε, the portion uncorrelated with q, then, as in the introductory coin flipping example, even the announcement that ε will be resolved immediately will not affect p_t.

Why, though, with this exception, did the value change in this exercise? The answer is that stopping the process at t' and letting $p_T = s_{t'}$, rather than s_T, alters the distribution of the terminal payoff. In particular, even though the mean of s is zero, the distribution of s continues to spread from t' to T. As a consequence, stopping the process at t' loses the risk premium deduction that arises from the additional uncertainty over the period from t' to T. This, in turn, results in a rise in value. The next exercise shows how to separate the pure acceleration effect from such changes in real cash flows. It is the relevant model for interpreting our anecdote about the New York City bonds, and it is also more complex.

Exercise 3.2: As in Exercise 3.1, the uncertainty is resolved and the time T payoff is determined at t', but now suppose that the result is revealed at $t'' > t'$. In particular, assume that

(i) $s_T = s_{t'}$ and
(ii) investors learn $s_{t'}$ at $t'' > t'$.

Figure 3 illustrates the information flow for this exercise.

While investors do not learn $s_{t'}$ until t'', they can make inferences about it from the other variables they do observe. In particular, they will observe the motion of the pricing standard, q, and in general this will enable them to learn something about $s_{t'}$. Interestingly, this will not affect our main result, and we do not have to become involved with the questions of what inferences concerning $s_{t'}$ investors can make.[2]

[2] To make this explicit we could introduce an information variable, x, which is a sufficient statistic for s when s is unobserved. Specifically, then, we would add the assumptions:

(iii) the variable s is last observed at $t^0 < t'$; i.e., investors know s_{t^0} but do not observe s_t until t'', when they learn $s_{t'}$;
(iv) investors continuously observe (x, q), where (x, q, s) follows a joint lognormal diffusion, where

t^0	Starting time and time of last observation of s_t until t''.
τ	An intermediate pricing date.
t'	Uncertainty about s_T is resolved, i.e., the payoff s_T is determined as $s_T = s_{t'}$.
t''	Time at which $s_{t'}$ is revealed.
T	Payoff, $s_T = s_{t'}$, is received.

Figure 3. Time Line for Exercise 3.2

To value the asset at any time t we proceed backwards in steps from the payoff date, T.

$t \in [t'', T]$. For $t \in [t'', T]$, the uncertainty has been both resolved and revealed, and it is known that the payoff will be $s_T = s_{t'}$. Hence, $s_{t'} \in I_t$ implies that

$$\begin{aligned} p_t &= E^*(e^{-r(T-t)} s_T \mid I_t) \\ &= E^*(e^{-r(T-t)} s_{t'} \mid I_t) \\ &= e^{-r(T-t)} s_{t'}. \end{aligned} \tag{35}$$

$t \in (t', t'')$. On (t', t'') the uncertainty has been resolved but not revealed. To value the asset in this interval we make use of the observation that the development of the pricing standard on $(t', t'']$ is independent of s_t, which is resolved by $t' \leq t$.

each variable is a sufficient statistic for itself, and where x is sufficient for s when s is not observed.

Assumption (iv) tells us that investors use their knowledge of x to make inferences about $s_{t'}$ during the period from t^0 to t''. This assumption implies that the residual component of s that is not predictable from the development of x is also unrelated to information about q. The uncertainty in its evolution that is pertinent for pricing, then, is that portion which is dependent on x and on the pricing of x by q.

It follows from Theorem 3 that

$$\begin{aligned} p_t &= E(q_{t''}p_{t''} \mid I_t) \\ &= E(q_{t''}e^{-r(T-t'')}s_{t'} \mid I_t) \\ &= e^{-r(T-t'')}E(q_{t''}s_{t'}I_t) \\ &= e^{-r(T-t'')}E(q_{t''} \mid I_t)E(s_{t'} \mid I_t) \\ &= e^{-r(T-t'')}e^{-r(t''-t)}E(s_{t'} \mid I_t) \\ &= e^{-r(T-t)}E(s_{t'} \mid I_t). \end{aligned} \tag{36}$$

Since the time at which the uncertainty is revealed, t'', does not appear in this formula, it follows that p_t is independent of t''. In other words, changing the future time at which investors will learn the payoff leaves the current asset value unchanged. We will state this more precisely in the following theorem.[3]

THEOREM 4 (Resolution Irrelevancy): *Consider an asset with a random value of s_T at a future date T and no cash payouts between now and T. Suppose that the random payoff is independent of events that occur on the interval $(t', T]$. It follows that the value of the asset at all times $t \leq t'$ will be independent of the date $t'' \in (t', T]$ at which the payoff at T is learned. In particular, then, accelerating the receipt of this information will not affect the current value of the asset.*

Proof: From the above analysis, we know that the value of the asset at date t' is independent of t'', and, by Theorem 3, the value at any earlier date will also be unaffected. Q.E.D.

Theorem 4 establishes in a quite general setting that, when changes in the resolution of uncertainty preserve the distribution of cash flows, they preserve current values. A direct application of Theorem 4 to the introductory anecdote on the New York City bonds shows that the strict interpretation of the market response is not compatible with an efficient market hypothesis.

Implicit in this analysis, though, has been the tacit assumption that changing the resolution of uncertainty does not alter the no-arbitrage martingale pricing operator. This seems reasonable enough at the microlevel, and it is really the same assumption that is made in a variety of no-arbitrage financial arguments such as the familiar Modigliani and Miller irrelevance results. In all of these we assume that the financial change being considered does not alter the existing state space spanning or, more generally, the pricing operator itself.

Continuing with the maintained assumption that market pricing is unaffected,

[3] Using the material in the previous footnote, we could proceed to obtain an exact solution for the value of the asset at all times as a function of the "betas" of s on x and on q. For example, at time t, I_t includes knowledge of how the state variable, x, developed from t^0 to t' and what relation it had to s in that period. Since the variables are all Markov, this permits us to write

$$E(s_{t'} \mid I_t) = E(s_{t'} \mid x_{t'}, s_{t^0}).$$

The exact evaluation of this pricing relation is straightforward but tedious and is somewhat tangential to our main point.

in the next section we will verify resolution irrelevancy using a more limited but more familiar approach.

IV. Option Prices and Resolution Irrelevancy

In Section I we described the relation between the state space model and the no-arbitrage martingale analysis. In this section we use the option pricing approach to analyze timing problems.

Suppose that p is an asset price the value of which depends on an information process, s, and that the other assumptions of Section II are satisfied. With the assumptions we have made, p must satisfy the option-pricing relation,

$$\tfrac{1}{2}\sigma_s^2 s^2 p_{ss} + (\mu_s - \lambda\sigma_s)sp_s - rp = p_\tau, \tag{37}$$

with

$$p(s, 0) = s, \tag{38}$$

where τ is the time remaining to maturity and λ is a coefficient describing the price, as a rate of return, for bearing type s risk. If s is an asset, then $\mu_s - \lambda\sigma_s = r$, and equation (37) is the familiar Black-Scholes option pricing equation with a deterministically time-varying variance. This is the same equation first examined by Merton [14] for pricing a call option, and our analysis follows his with a change in the terminal value of the derivative security.

The solution to the differential pricing equation (37) with boundary condition (38) is, as we found in Section II,

$$p(s, \tau) = se^{(\mu_s - r - \lambda\sigma_s)\tau}. \tag{39}$$

The following simple generalization,

$$\sigma_s = \sigma_s(\tau), \tag{40}$$

permits us to study a variety of timing problems.

The resulting solution to the valuation problem is

$$p(s, \tau) = se^{(\mu_s - r)\tau - \lambda \int_0^\tau \sigma_s(y)\,dy}. \tag{41}$$

Notice that the standard deviation enters as an integral of future standard deviations. This lets us examine timing issues in a very straightforward fashion. Changes in the time path of the variance that leave the average standard deviation constant leave the martingale distribution of the future price and, therefore, the current value unchanged.

This is equivalent to observing that alterations in the intertemporal resolution of uncertainty which preserve the average standard deviation also leave the current price unaltered. For example, all paths of standard deviation of the form

$$\sigma_s(y) = \begin{cases} \sigma_s\tau(b - a)^{-1} & \text{for } y \in [a, b], \\ 0 & \text{for } y \notin [a, b], \end{cases}$$

with $0 < a < b < \tau$, lead to the same current price, p. The change in Exercise 3.2 was of this form. Exercise 3.1, by contrast, lowered the variance of the log of the future value by $\sigma^2(T - t')$.

V. Summary and Conclusion

In an arbitrage-free economy, the volatility of prices is directly related to the rate of flow of information to the market. In a simple model the two were found to be identical. This result links volatility tests such as those initiated by French and Roll [10] to efficient market hypotheses which specify the information set which the market uses for pricing.

We have also seen that, in an arbitrage-free economy, resolution irrelevancy holds, which means that changes in the timing of the resolution of uncertainty that leave cash flows unaltered do not change current prices. This result was derived independently of any particular parameterization of the stochastic information flow process.

Resolution irrelevancy and the relation between price volatility and the rate of information flow are important consequences of arbitrage-free economies. Together with efficient market hypotheses, they can form the basis for a class of efficient market tests that are robust to the precise choice of an asset-pricing model.

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