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Journal of Econometrics 132 (2006) 363–378
JOURNAL OF Econometrics
www.elsevier.com/locate/jeconom

Testing for short- and long-run causality: A frequency-domain approach

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Available online 18 April 2005

Abstract

The framework of Geweke (1982. Journal of the American Statistical Association 77, 304–324.) and Hosoya (1991. Probability Theory and Related Fields 88, 429–444.) is adopted to construct a simple test for causality in the frequency domain. This test can also be applied to cointegrated systems. To study the large sample properties of the test, we analyze the power against a sequence of local alternatives. The finite sample properties are investigated by means of Monte Carlo simulations. Our methodology is applied to investigate the predictive content of the yield spread for future output growth. Using quarterly US data we observe reasonable leading indicator properties at frequencies around one year and typical business cycle frequencies.

JEL classification: C32; E43

Keywords: Causality; Spectral analysis; Output predictability; Interest rates

1. Introduction

In the special issue of *The Journal of Econometrics* on causality, Granger (1988) emphasized the relevance of the frequency-domain causation decomposition

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doi:10.1016/j.jeconom.2005.02.004

especially in the case of cointegrated systems with causality at the zero frequency (see also Granger and Lin, 1995). Frequency-domain causality measures and test procedures were suggested by Granger (1969), Geweke (1982) and Hosoya (1991). We follow this approach and suggest simple empirical tests to assess the predictive power at some given frequencies.

Geweke (1982) and Hosoya (1991) proposed a causality measure at a particular frequency based on a decomposition of the spectral density. Yao and Hosoya (2000) developed a Wald-type test procedure for causality at some given frequency, which is based on a complicated set of nonlinear restrictions on the autoregressive parameters. To overcome this difficulty, Yao and Hosoya (2000) apply the delta method based on numerical derivatives.

Using a bivariate vector autoregressive (VAR) model, we propose a simple test procedure that is based on a set of linear hypotheses on the autoregressive parameters. This test procedure can easily be generalized to allow for cointegration relationships and higher-dimensional systems.

Our framework can be used to disentangle short- and long-run predictability. Using postwar US data, we found that the yield spread is a powerful predictor for short-run fluctuations of economic growth. We also find predictive power of the term spread at typical business cycle frequencies. No predictive power is observed for cyclical fluctuations between 1 and 2 years.

The rest of the paper is organized as follows. The frequency-domain approach of causality is introduced in Section 2 and the empirical test procedures are considered in Section 3. Section 4 investigates the power properties of the test. Section 5 presents the results of our empirical study and Section 6 offers some conclusions.

2. Causality in the frequency-domain

Our framework is based on the work of Geweke (1982) and Hosoya (1991), who proposed measures of causality in the frequency-domain. First, let $z_t = [x_t, y_t]'$ be a two-dimensional vector of time series observed at $t = 1, \ldots, T$. It is assumed that z_t has a finite-order VAR representation of the form

$$\Theta(L)z_t = \varepsilon_t, \tag{1}$$

where $\Theta(L) = I - \Theta_1 L - \cdots - \Theta_p L^p$ is a 2×2 lag polynomial with $L^k z_t = z_{t-k}$. We assume that the error vector ε_t is white noise with $\mathrm{E}(\varepsilon_t) = 0$ and $\mathrm{E}(\varepsilon_t \varepsilon_t') = \Sigma$, where Σ is positive definite. For ease of exposition we neglect any deterministic terms in (1) although in empirical applications the model typically includes a constant, trend or dummy variables.

Let G be the lower triangular matrix of the Cholesky decomposition $G'G = \Sigma^{-1}$ such that $\mathrm{E}(\eta_t \eta_t') = I$ and $\eta_t = G\varepsilon_t$. If the system is assumed to be stationary, the MA representation of the system is

$$z_t = \Phi(L)\varepsilon_t = \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$
$$= \Psi(L)\eta_t = \begin{bmatrix} \Psi_{11}(L) & \Psi_{12}(L) \\ \Psi_{21}(L) & \Psi_{22}(L) \end{bmatrix} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix}, \tag{2}$$

where $\Phi(L) = \Theta(L)^{-1}$ and $\Psi(L) = \Phi(L)G^{-1}$. Using this representation the spectral density of x_t can be expressed as

$$f_x(\omega) = \frac{1}{2\pi}\{|\Psi_{11}(\mathrm{e}^{-\mathrm{i}\omega})|^2 + |\Psi_{12}(\mathrm{e}^{-\mathrm{i}\omega})|^2\}.$$

The measure of causality suggested by Geweke (1982) and Hosoya (1991) is defined as

$$M_{y\to x}(\omega) = \log\left[\frac{2\pi f_x(\omega)}{|\Psi_{11}(\mathrm{e}^{-\mathrm{i}\omega})|^2}\right] \tag{3}$$
$$= \log\left[1 + \frac{|\Psi_{12}(\mathrm{e}^{-\mathrm{i}\omega})|^2}{|\Psi_{11}(\mathrm{e}^{-\mathrm{i}\omega})|^2}\right]. \tag{4}$$

The measure is zero if $|\Psi_{12}(\mathrm{e}^{-\mathrm{i}\omega})| = 0$, in which case we say that y does not cause x at frequency ω.

If the elements of z_t are $I(1)$ and cointegrated, then the autoregressive polynomial $\Theta(L)$ has a unit root. The remaining roots are outside the unit circle. Subtracting z_{t-1} from both sides of (1) gives

$$\Delta z_t = (\Theta_1 - I)z_{t-1} + \Theta_2 z_{t-2} + \cdots + \Theta_p z_{t-p} + \varepsilon_t$$
$$= \widetilde{\Theta}(L)z_{t-1} + \varepsilon_t, \tag{5}$$

where $\widetilde{\Theta}(L) = \Theta_1 - I + \Theta_2 L + \cdots + \Theta_p L^p$. If y is not a cause of x in the usual Granger sense, then the (1,2)-element of $\Theta(L)$ (or $\widetilde{\Theta}(L)$) is zero (cf. Toda and Phillips, 1993). In the frequency domain the measure of causality can be defined by using the orthogonalized MA representation

$$\Delta z_t = \widetilde{\Phi}(L)\varepsilon_t$$
$$= \widetilde{\Psi}(L)\eta_t, \tag{6}$$

where $\widetilde{\Psi}(L) = \widetilde{\Phi}(L)G^{-1}$, $\eta_t = G\varepsilon_t$, and G is a lower triangular matrix such that $\mathrm{E}(\eta_t\eta_t') = I$. Note that in a bivariate cointegrated system $\beta'\widetilde{\Psi}(1) = 0$, where β is a cointegration vector such that $\beta' z_t$ is stationary (cf. Engle and Granger, 1987). As in the stationary case the resulting causality measure is

$$M_{y\to x}(\omega) = \log\left[1 + \frac{|\widetilde{\Psi}_{12}(\mathrm{e}^{-\mathrm{i}\omega})|^2}{|\widetilde{\Psi}_{11}(\mathrm{e}^{-\mathrm{i}\omega})|^2}\right]. \tag{7}$$

The causality measure can be extended to higher-dimensional systems. Hosoya (2001) approach is based on the bivariate causality measure after "conditioning out" the third variable. Assume that we want to measure the causal effect of y_{1t} on y_{2t} in a three-dimensional system with $y_t = [y_{1t}, y_{2t}, y_{3t}]'$. Let w_t denote the projection residual from a projection of y_{3t} onto the Hilbert space $H(y_{1t}, y_{2t}, y_{t-1}, y_{t-2}, \ldots)$. Furthermore, $u_t(v_t)$ is the projection residual from a projection of $y_{1t}(y_{2t})$ on $H(w_t, w_{t-1}, \ldots)$. From the representation

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \end{bmatrix} = \begin{bmatrix} \Psi_{11}(L) & \Psi_{12}(L) & \Psi_{13}(L) \\ \Psi_{21}(L) & \Psi_{22}(L) & \Psi_{23}(L) \\ \Psi_{31}(L) & \Psi_{32}(L) & \Psi_{33}(L) \end{bmatrix} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \end{bmatrix},$$

it follows that $u_t = \Psi_{11}(L)\eta_{1t} + \Psi_{12}(L)\eta_{2t}$ and $v_t = \Psi_{21}(L)\eta_{1t} + \Psi_{22}(L)\eta_{2t}$. The causality measure suggested by Hosoya (2001) is equivalent to the bivariate causality measure between u_t and v_t

$$M_{y_1 \to y_2 | y_3}(\omega) \equiv M_{u \to v}(\omega),$$

and, therefore, the causality measure in higher-dimensional system can be written as a bivariate causality measure with appropriately transformed variables.

3. Empirical test procedures

To test the hypothesis that y does not cause x at frequency ω we consider the null hypothesis

$$M_{y \to x}(\omega) = 0 \tag{8}$$

within a bivariate framework. Yao and Hosoya (2000) suggest to estimate $M_{y\to x}(\omega)$ by replacing $|\Psi_{11}(\mathrm{e}^{-\mathrm{i}\omega})|$ and $|\Psi_{12}(\mathrm{e}^{-\mathrm{i}\omega})|$ in (4) with estimates obtained from the fitted VAR. Let $\lambda = vec(\Theta_1, \ldots, \Theta_p, \Sigma)$ denote the vector of parameters. Then the delta method gives rise to the expansion

$$\widehat{M}_{y \to x}(\omega) = M_{y \to x}(\omega) + D_\lambda(\lambda)'(\widehat{\lambda} - \lambda) + \mathrm{o_p}(T^{-1/2}), \tag{9}$$

where $\widehat{M}_{y\to x}(\omega)$ denotes the estimated causality measure that is based on estimated VAR parameters and $D_\lambda(\lambda)$ denotes the vector of derivatives of $M_{y\to x}(\omega)$ with respect to λ (cf. Yao and Hosoya, 2000, Section 3). Under suitable regularity conditions the asymptotic distribution of the Wald statistic for (8) is given by

$$W = T\left[\widehat{M}_{y\to x}(\omega)\right]^2 / H(\widehat{\lambda}) \xrightarrow{\mathrm{d}} \chi_1^2,$$

where $H(\widehat{\lambda}) = D_\lambda(\widehat{\lambda})' V(\widehat{\lambda}) D_\lambda(\widehat{\lambda})$ and $V(\widehat{\lambda})$ is the asymptotic covariance matrix of $\widehat{\lambda}$.

Unfortunately, the expression $|\Psi_{12}(\mathrm{e}^{-\mathrm{i}\omega})|$ is a complicated nonlinear function of the VAR parameters and, the derivative $D_\lambda(\widehat{\lambda})$ is therefore difficult to evaluate. Yao and Hosoya (2000) suggest using a numerical differentiation instead of the exact analytical expression.

In what follows, a much simpler approach is proposed to test the null hypothesis (8). From (4) it follows that $M_{y\to x}(\omega) = 0$ if $|\Psi_{12}(\mathrm{e}^{-\mathrm{i}\omega})| = 0$. Using $\Psi(L) =$

$\Theta(L)^{-1}G^{-1}$ and

$$\Psi_{12}(L) = -\frac{g^{22}\Theta_{12}(L)}{|\Theta(L)|},$$

where g^{22} is the lower diagonal element of G^{-1} and $|\Theta(L)|$ is the determinant of $\Theta(L)$. It follows that y does not cause x at frequency ω if[1]

$$|\Theta_{12}(\mathrm{e}^{-\mathrm{i}\omega})| = \left|\sum_{k=1}^{p}\theta_{12,k}\cos(k\omega) - \sum_{k=1}^{p}\theta_{12,k}\sin(k\omega)\mathrm{i}\right| = 0,$$

where $\theta_{12,k}$ is the (1,2)-element of Θ_k. Thus, a necessary and sufficient set of conditions for $|\Theta_{12}(\mathrm{e}^{-\mathrm{i}\omega})| = 0$ is

$$\sum_{k=1}^{p}\theta_{12,k}\cos(k\,\omega) = 0, \tag{10}$$

$$\sum_{k=1}^{p}\theta_{12,k}\sin(k\,\omega) = 0, \tag{11}$$

Since $\sin(k\omega) = 0$ for $\omega = 0$ and $\omega = \pi$, restriction (11) can be dropped in these cases.

Our approach is based on the *linear* restrictions (10) and (11). To simplify the notation, we let $\alpha_j = \theta_{11,j}$ and $\beta_j = \theta_{12,j}$, so that the VAR equation for x_t is written as

$$x_t = \alpha_1 x_{t-1} + \cdots + \alpha_p x_{t-p} + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + \varepsilon_{1t}. \tag{12}$$

The hypothesis $M_{y\to x}(\omega) = 0$ is equivalent to the linear restriction

$$\mathrm{H}_0\colon\ R(\omega)\beta = 0, \tag{13}$$

where $\beta = [\beta_1, \ldots, \beta_p]'$ and

$$R(\omega) = \begin{bmatrix} \cos(\omega) & \cos(2\omega) & \cdots & \cos(p\omega) \\ \sin(\omega) & \sin(2\omega) & \cdots & \sin(p\omega) \end{bmatrix}.$$

The ordinary F statistic for (13) is approximately distributed as $F(2, T-2p)$ for $\omega \in (0, \pi)$.

It is interesting to consider the frequency domain causality test within a cointegrating framework. To this end we replace x_t in regression (12) by Δx_t, with the right-hand side of the equation remaining the same. An interesting special case is the test at frequency $\omega = 0$ (see also Granger and Lin, 1995). In this case

$$\widetilde{\Theta}(\mathrm{e}^0) = \Theta_1 - I + \Theta_2 + \cdots + \Theta_p \equiv \Pi,$$

[1]Note that g^{22} is positive due to the assumption that Σ is positive definite.

which is sometimes called the "impact matrix". Using the VECM representation of a cointegrated system

$$\Delta z_t = \Pi z_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta z_{t-j} + \varepsilon_t,$$

the test of causality at frequency zero boils down to a test of the hypothesis $\pi_{12} = 0$ in the regression

$$\Delta x_t = \pi_{11} x_{t-1} + \pi_{12} y_{t-1} + \sum_{k=1}^{p-1} \gamma_{11,k} \Delta x_{t-k} + \sum_{k=1}^{p-1} \gamma_{12,k} \Delta y_{t-k} + \varepsilon_{1t}, \tag{14}$$

where π_{ij} and $\gamma_{ij,k}$ denote the (i,j) element of Π and Γ_k, respectively. In empirical work the test of the hypothesis $\pi_{12} = 0$ is often called a test of long-run causality (e.g. Toda and Phillips, 1993; Caporale and Pittis, 1999).

To see that a zero $(1,2)$-element of the matrix Π implies $|\widetilde{\Psi}_{12}(1)| = 0$, we employ the decomposition $\Pi = \alpha\beta'$, where β is the cointegration vector and α is a vector of loading coefficients. In a bivariate cointegrated system, a zero $(1,2)$-element of the matrix Π implies that the first element of α is zero. Consequently, the second element of the orthogonal complement $\alpha_\perp$ is zero. From Johansen (1991) it is known that

$$\widetilde{\Psi}(1) = \widetilde{\Phi}(1)G^{-1} = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp G^{-1},$$

where $\Gamma = I - \sum_{j=1}^{p-1} \Gamma_j$. Since G is lower triangular and the second element of $\alpha_\perp$ is zero, it follows that $\widetilde{\Psi}_{12}(1) = 0$.

It is known (cf. Toda and Phillips, 1993) that in a bivariate cointegrated system with $z_t \sim I(1)$ the least-squares estimator of π_{12} is asymptotically normal and, thus, the Wald test for the hypothesis $\pi_{12} = 0$ has a standard limiting distribution. However, if $x_t \sim I(0)$ and $y_t \sim I(1)$, then the test does no longer have a standard limiting distribution. The reason is, that in this case the coefficient π_{12} is attached to the nonstationary variable y_{t-1}, whereas all other variables in Eq. (15) are stationary. Hence, the estimator of π_{12} possesses a nonstandard limiting distribution (Sims et al., 1990). Similar problems exist in higher-dimensional systems if some block of the matrix Π is singular (Toda and Phillips, 1993).

A convenient way of overcoming this difficulty was suggested by Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996). They showed that the Wald test of restrictions involving nonstationary variables has a standard asymptotic distribution if the VAR model is augmented with a redundant lag, that is, instead of using the VAR(p) model, the restrictions are tested by using a VAR($p+1$) model. This approach can also be used to establish standard inference for the frequency domain causality test.

Finally, our approach can be generalized to test for causality in higher-dimensional systems. A natural way is to include the third variable in the autoregression so that

$$y_{1t} = \sum_{j=1}^{p} \alpha_j y_{1,t-j} + \sum_{j=1}^{p} \beta_j y_{2,t-j} + \sum_{j=1}^{p} \gamma_j y_{3,t-j} + e_t. \tag{15}$$

Such a regression is also used for the usual Granger causality test in three-dimensional systems. To test the hypothesis $M_{y_2 \to y_1 | y_3}(\omega) = 0$, the linear restriction (13) on the parameter vector $\beta = [\beta_1, \ldots, \beta_p]'$ is tested.

Using several examples, Hosoya (2000) argued that it is inappropriate to condition on the past of y_{3t}. He therefore suggested a modified test procedure that is based on the residuals w_t from a regression of y_{3t} on y_{1t}, y_{2t} and $y_{t-1}, \ldots, y_{t-p}$ (see also Section 2). The hypothesis that y_2 is a cause of y_1 at frequency ω in Hosoya's (2000) sense can be tested by an F-test of the restriction given in (13) in the regression

$$y_{1t} = \sum_{j=1}^{p} \alpha_j y_{1,t-j} + \sum_{j=1}^{p} \beta_j y_{2,t-j} + \sum_{j=0}^{p} \gamma_j w_{t-j} + e_t.$$

It is important to note that in Hosoya's approach the information of contemporaneous values of y_{3t} enter the r.h.s. of the regression in form of the variable w_t. This does not fit well to the notion of causality as a measure of the predictive content. On the other hand, leaving out the contemporaneous information of the variable y_{3t} may give spurious inference on causality, as demonstrated by Hosoya (2000).

4. Power

To study the local power of the frequency domain causality test, we consider the simple model

$$x_t = b_\omega(L) y_{t-1} + u_t, \tag{16}$$

where $b_\omega(L) = \alpha\left[1 - 2\cos(\omega)L + L^2\right]$ is a Gegenbauer polynomial, and $\{y_t\}$, $\{u_t\}$ are mutually independent white noise processes. Despite of the simplicity of this model we are able to derive some important features of the test. More general models involving additional parameters do not lead to additional insight.

The gain function of the filter $b_\omega(L)$ is zero at frequency ω so that y does not cause x at frequency ω. We first study the properties of the test for $\omega \neq \omega_0$, where $\omega - \omega_0$ is $O(T^{-1/2})$. Specifically, we consider the sequence

$$\omega_T = \omega_0 + c/\sqrt{T},$$

so that $b_T(L) = \alpha\left[1 - 2\cos(\omega_T)L + L^2\right]$ is used instead of $b_\omega(L)$ in (16). Thus, we study the local power of the test when the frequency being tested converges to the true frequency at a suitable rate. The following proposition gives the distribution of the Wald statistic under the sequence of local alternatives.

Theorem. *Let x_t be generated as $x_t = b_T(L) y_{t-1} + u_t$, where $\{y_t\}$ and $\{u_t\}$ are independent zero-mean white noise processes with $E(y_t^2) = \sigma_y^2$ and $E(u_t^2) = \sigma_u^2$. Under the local alternative $\omega_T = \omega_0 + c/\sqrt{T}$ the Wald statistic is asymptotically distributed* *as noncentral* χ^2 *with noncentrality parameter*

$$\lambda^2 = \frac{\sigma_y^2[2c\alpha\sin(\omega_0)]^2}{\sigma_u^2\left[1+2\cos(\omega_0)^2\right]}.$$

Proof. The null hypothesis that y does not cause x at frequency ω_0 is equivalent to $R(\omega_0)\beta = 0$ in the model $x_t = \beta' w_t + u_t$, where $w_t = [y_{t-1}, y_{t-2}, y_{t-3}]'$ and $\beta = [\alpha, -2\alpha\cos(\omega_T), \alpha]'$. Using the matrix

$$Q(\omega_0) = \left[R(\omega_0)', R_\perp(\omega_0)'\right]',$$

where

$$R(\omega_0) = [1 \quad -2\cos(\omega_0) \quad 1],$$

$$R_\perp(\omega_0) = \begin{bmatrix} \cos(\omega_0) & 1 & \cos(\omega_0) \\ -\sin(\omega_0) & 0 & \sin(\omega_0) \end{bmatrix},$$

we can rewrite the model as

$$\begin{aligned} y_t &= \beta' Q(\omega_0)^{-1} Q(\omega_0) w_t + u_t \\ &= \beta^{*\prime} w_t^* + u_t \\ &= \beta_1^* w_{1t}^* + \beta_2^{*\prime} w_{2t}^* + u_t, \end{aligned}$$

where $w_{1t}^* = R(\omega_0)w_t = y_{t-1} - 2\cos(\omega_0)y_{t-2} + y_{t-3}$ and $w_{2t}^* = R_\perp(\omega_0)w_t$. Accordingly, the null hypothesis that y does not cause x at frequency ω_0 is equivalent to $\beta_2^* = 0$.

Using a Taylor expansion around ω_0, the process can be represented as

$$x_t \simeq b^0(L)y_{t-1} - T^{-1/2}2c\alpha\sin(\omega_0)y_{t-2} + u_t,$$

where $b^0(L) = \alpha\left[1 - 2\cos(\omega_0)L + L^2\right]$. By construction w_{1t}^* and w_{2t}^* are orthogonal, so that the Wald statistic is asymptotically equivalent to

$$\xi_W(\omega_0) = \frac{1}{\sigma_u^2}\left(\sum \widetilde{u}_t w_{2t}^{*\prime}\right)\left(\sum w_{2t}^* w_{2t}^{*\prime}\right)^{-1}\left(\sum w_{2t}^*\widetilde{u}_t\right),$$

where

$$\widetilde{u}_t = u_t - T^{-1/2}2c\alpha\sin(\omega_0)y_{t-2},$$

and $\sigma_u^2 = \mathrm{E}(u_t^2)$. Using

$$\mathrm{E}(y_{t-2}w_{2t}^*) = \begin{bmatrix} \sigma_y^2 \\ 0 \end{bmatrix}$$

$$\mathrm{E}(w_{2t}^* w_{2t}^{*\prime}) = \sigma_y^2 \begin{bmatrix} 1 + 2\cos(\omega_0)^2 & 0 \\ 0 & 2\sin(\omega_0)^2 \end{bmatrix},$$

we obtain

$$\lim_{T\to\infty} \mathrm{E}\left[\sigma_u^{-1}\left(\sum w_{2t}^{*} w_{2t}^{*\prime}\right)^{-1/2}\left(\sum y_{t-2} w_{2t}^{*}\right)\right] = \begin{bmatrix} \frac{\sigma_y[2c\alpha\sin(\omega_0)]}{\sigma_u\sqrt{[1+2\cos(\omega_0)^2]}} \\ 0 \end{bmatrix}.$$

Therefore, the noncentrality parameter of the limiting χ^2 distribution of $\xi_W(\omega_0)$ results as

$$\lambda^2 = \frac{\sigma_y^2[2c\alpha\sin(\omega_0)]^2}{\sigma_u^2[1+2\cos(\omega_0)^2]}. \qquad \square$$

From this result two important conclusions can be drawn. Firstly, the test suffers from a ''leakage problem'', that is, the test cannot distinguish causal effects at frequencies that are close to each other. Secondly, the power of the test depends sensitively on the frequency under consideration. The maximal power against local alternatives is at $\omega_0 = \pi/2$. For $\omega_0 \to 0$ and $\omega_0 \to \pi$ the local power against causality at frequencies close to the hypothesized frequency tends to the size. To appreciate this result note that in the Gegenbauer polynomial a different choice of the frequency will change the coefficient attached to the lagged value from $2\alpha\cos(\omega_0)$ to $2\alpha\cos(\omega_0+\delta)$. For small values of δ we have $\cos(\omega_0+\delta)-\cos(\omega_0)\approx\sin(\omega_0)\delta$. It follows that for frequencies around $\pi/2$ the effect on the coefficient is maximal, whereas for frequencies close to 0 or π the value of the coefficient changes only slightly. This should be taken into account when comparing the test results at different frequencies (Table 1).

To investigate the reliability of our asymptotic results we simulated time series according to (16) with $\alpha = 1$, $\sigma_u^2 = \sigma_y^2 = 1$ and $T = 500$. The variable y is not a cause of x at the sequence of frequencies $\omega_T = \omega_0 + c/\sqrt{T}$, whereas the test was performed at frequency ω_0. For causality at $\omega_0 = \pi/2$ the empirical power is very close to the asymptotic power. For $\omega_0 = \pi/4$ and $3\pi/4$ we found that for small values of c the actual power is well approximated by the asymptotic power. The approximation becomes less accurate as c increases.

Table 1
Actual and asymptotic power

c	$\omega_0 = \pi/4$		$\omega_0 = \pi/2$		$\omega_0 = 3\pi/4$	
	Actual	Asympt.	Actual	Asympt.	Actual	Asympt.
0.5	0.073	0.069	0.131	0.133	0.070	0.069
1.0	0.141	0.133	0.409	0.416	0.126	0.133
1.5	0.282	0.250	0.768	0.771	0.222	0.250
2.0	0.488	0.416	0.954	0.957	0.356	0.416
2.5	0.708	0.603	0.996	0.996	0.501	0.603
3.0	0.877	0.771	1.000	1.000	0.643	0.771

Note: Rejection frequencies of 10,000 Monte Carlo replications based on the model (16) with $\alpha = 1$. The sample size is $T = 500$ and the 0.05 significance level is used.

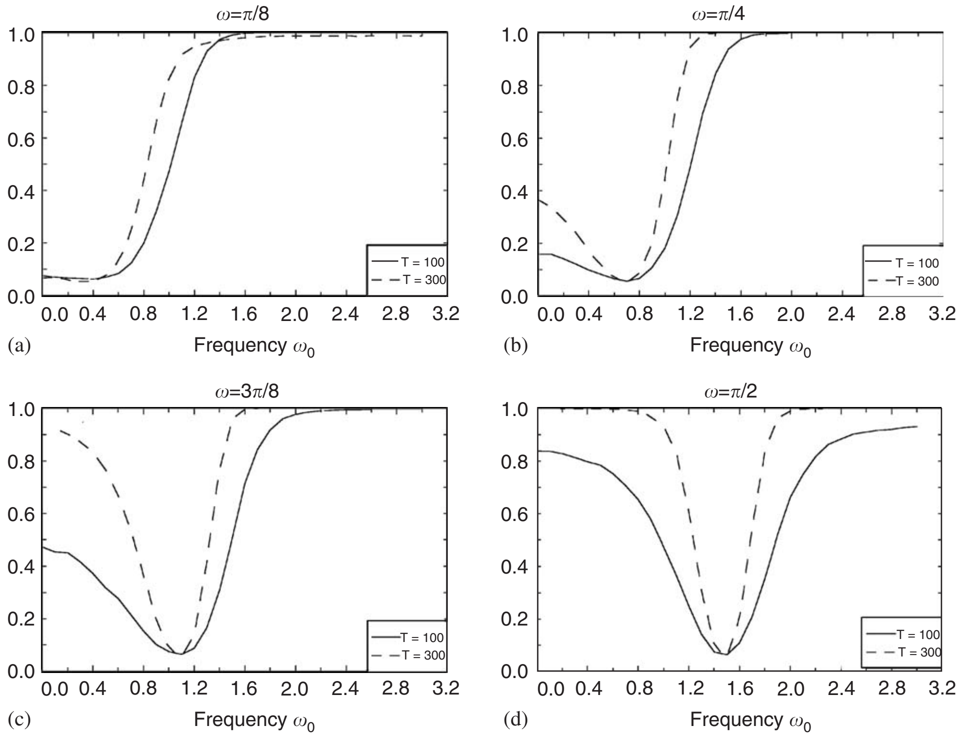


Fig. 1. Empirical power.

To investigate the finite sample properties of the tests for a more complicated data generating process, we generated the data according to the stationary model:

$$x_t = 0.1\,x_{t-1} + 0.3\,b_\omega(L)y_{t-1} + \varepsilon_{1t},$$

$$y_t = -x_{t-1} + 0.1\,y_{t-1} - 0.2\,y_{t-2} + 0.3\,y_{t-3} + \varepsilon_{2t},$$

where

$$\varepsilon_t \sim \mathrm{N}(0, \Sigma), \quad \sum = \begin{bmatrix} 0.5 & 0.2 \\ 0.2 & 0.5 \end{bmatrix}$$

and $b_\omega(L) = 1 - 2\cos(\omega)L + L^2$. At frequency ω the gain function of the polynomial is zero and therefore, y is not a cause of x at this particular frequency.

For the Monte Carlo experiments we computed the rejection frequencies based on 10,000 replications of the process with sample size $T = 100$ and 300. In Fig. 1 the empirical power functions are plotted for the frequencies $\omega = \{j\pi/8;\ j = 1, \ldots, 4\}$. Since the power is (roughly) symmetric around $\omega = \pi/2$, we do not show the results for $\omega > \pi/2$.

As expected, all power functions have a minimum at the typical frequency of the Gegenbauer polynomial $\beta_\omega(L)$. The minimum of all graphs is close to 0.05, verifying that the tests have reasonable size properties. Furthermore, the power of the test increases substantially with the sample size. It is also interesting to note that for ω approaching zero, the power is close to the size for frequencies in the neighborhood ω. This confirms our theoretical findings from the local power analysis.

5. Empirical results

There is already a rich literature that demonstrates the remarkable predictive power of interest rate spreads for real economic growth. Examples include Bernanke (1990), Estrella and Hardouvelis (1991), Estrella and Mishkin (1995), Davis and Fagan (1997), Boulier and Stekler (1999), to name a few. Stock and Watson (1989, 1993) found that interest rate spreads are among the most promising leading variables from the perspective of business cycle forecasting for the US.

To achieve desirable indicator properties it is important that the change in interest rates immediately affects the term structure, whereas monetary policy affects real activity with some delay. The empirical literature demonstrates that changes in the term spread affect output with a time lag of 2 up to 16 quarters. Accordingly, the yield spread is a reliable leading indicator of economic activity up to one year ahead (e.g. Estrella and Hardouvelis, 1991; Plosser and Rouwenhorst, 1994; Bonser-Neal and Morley, 1997; Hamilton and Kim, 2002).

In this section, we apply causality tests in the frequency domain to assess the predictive content of the term spread for future economic growth. We used quarterly data of real GDP (Y_t), government 10-year bond yield (R_t) and the 3-month bond yield (r_t) for the US economy, as extracted from the Saint-Louis Federal Reserve Bank database. The sample period is 1959q1–1998q4. Since we found a unit root in the autoregressive representation of real GDP, we used first differences of the logged series (i.e. growth rates). The spread (s_t) was computed as the difference of the long-run (R_t) and short-run (r_t) interest rates.[2]

First we specified a bivariate system. According to the AIC criterion, a VAR(6) model was selected.[3]

The results of the causality tests in the frequency domain are presented in Fig. 2. This figure reports the test statistics along with their 5% critical values (broken lines) for all frequencies in the interval $(0, \pi)$. It turns out that the null hypothesis of no predictability is rejected in the range $\omega \in [1.8, 2.4]$ corresponding to a cycle length between 2.5 and 3.5 quarters. This result is in line with the former findings that the spread is a powerful predictor for economic activity at a lag horizon of 2–3 quarters.

[2]Applying unit root tests to the spread series we found that the spread is stationary. The results of the unit root tests are available from the authors upon request.

[3]$AIC(p=0) = -0.0500$, $AIC(p=1) = -2.5933$, $AIC(p=2) = -2.5920$, $AIC(p=3) = -2.6222$, $AIC(p=4) = -2.5600$, $AIC(p=5) = -2.6393$, $AIC(p=6) = -2.6730$, $AIC(p=7) = -2.6000$, $AIC(p=8) = -2.5593$.

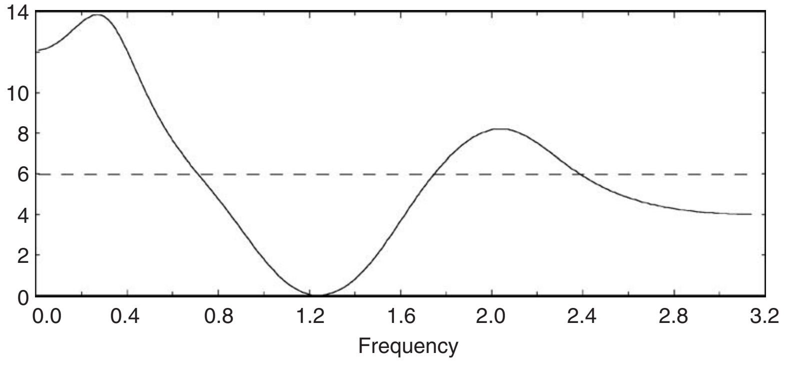


Fig. 2. Causality tests (bivariate system).

Table 2
Cointegration tests

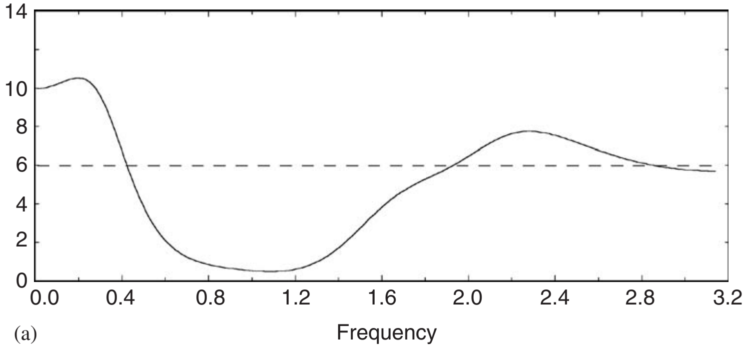
H_0	λ-max	95%	Trace	95%
$r = 0$	20.31	21.0	30.49	29.7
$r \leqslant 1$	9.56	14.1	10.18	15.4
$r \leqslant 2$	0.62	3.8	0.62	3.8

Note: r is the cointegration rank and the column "95%" reports the critical values according to a significance level of 0.05.

We also find predictability at frequencies less than 0.8, which corresponds to (business cycle) frequencies with a wave length of more than 2 years. This result suggests that the behavior at business cycle frequencies is well reproduced in the one-step-ahead forecasts of economic growth. Accordingly, the spread variable is a useful predictor of the business cycle.

Next, we investigate whether there is predictive power of the term structure over and above that provided by other variables reflecting the stance of monetary policy. Following Estrella and Mishkin (1995), Anderson and Vahid (2001) and others, we enhance the VAR system by including the (log of the) real balances (M2/P), as extracted from Saint-Louis Federal Reserve Bank database. From the literature on money demand systems (e.g. Hoffman and Rasche, 1996) it is known that the variables output, interest rates and the monetary base may be characterized by a long-run relationship that is usually interpreted as a money demand function. Therefore, a system that includes the monetary base, interest rates and output has to be tested for a possible cointegration relationship (Table 2).

Applying Johansen's (1988) cointegration tests for a trivariate system including the log of output, the log of real money base and interest rates yields ambiguous results: the λ-max test accepts the hypothesis of no cointegration, whereas the trace test rejects at a 5% significance level. All tests are performed by using an unrestricted



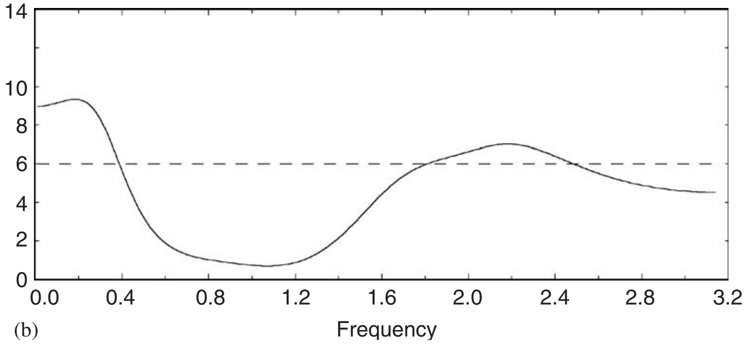


Fig. 3. Causality tests (trivariate, no cointegration): (a) According to Hosoya's measure and (b) According to Geweke's measure.

constant in the VAR representation.[4] Therefore, we consider both cases. The results of the trivariate system, without cointegration, displayed in Fig. 3, are roughly similar to the bivariate case. Fig. 3(a) presents the results of the tests based on Hosoya's (2001) measure of causality. For this test the contemporary information in the additional variable is included whereas in Fig. 3(b) the tests using Geweke's causality measure based on the past of the variables alone. It turns out that the different information sets do not affect the results to a great extent. Moreover, the results are qualitatively similar to the results of the bivariate model; the hypothesis of no causality is rejected for frequencies in the intervals $[0, 0.4]$ and around $\omega = 2$.

Finally, Geweke's variant of the causality measure is used to test for causality, where it is assumed that the system is cointegrated. Since the results of Hosoya's approach are very similar, we do not present the respective results. Overall, the findings are quite similar to the ones obtained when assuming a stationary system.

[4]If a restricted trend is included which enters only in the error correction terms we do not find any cointegration relation at a 5% nominal size. Nevertheless, the trace test for $r = 0$ has a value of 41.14 just below the 5% significance level.

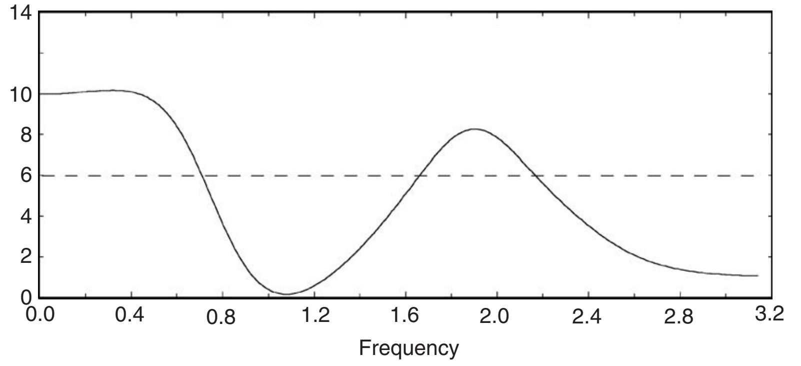


Fig. 4. Causality tests (trivariate, cointegrated system).

This is expected for frequencies larger than zero. However, this similarity can also be observed at frequencies close to zero. This might be due to the fairly weak evidence for cointegration presented above (Fig. 4).

6. Discussion

Based on the work of Geweke (1982) and Hosoya (1991) we suggest a simple test procedure that allows us to test for predictability at some pre-specified frequency. It is shown that the test procedure can also be adapted to test for causality in cointegrated systems. In cointegrated systems the definition of causality at frequency zero is equivalent to the concept of "long-run causality" as considered by Toda and Phillips (1993), for example.

In stationary systems the concept of long-run causality is not as obvious. Assume that the x_t is predicted using only the past of the series $x_{t-1}, x_{t-2}, \ldots$. If the spectral density of the resulting forecast error at low frequencies can be explained by the additional past information of y_t, then y_t is said to be a long-run cause for x_t. Although, in a stationary framework there exists no long-run relationship between time series, a series may nevertheless explain future low frequency variation of another time series. Consequently, our concept does not postulate that a variable y_t affects another variable x_t at a infinite time horizon. Rather, causality at low frequencies implies that the additional variable is able to forecast the low frequency component of the variable of interest *one period ahead*. This is an important conceptual difference to the approach suggested by Dufour and Renault (1998, 2005).

The new frequency domain causality tests are applied to investigate the predictive power of the yield spread for real economic growth. Our empirical results demonstrate once again the good leading indicator properties of the yield spread at typical business cycle frequencies.

Acknowledgements

The authors thank the participants of the EC^2 meeting in Louvain-la-Neuve, in particular Peter Boswijk, Catherine Bruneau, Jean-Marie Dufour, Jean-Pierre Urbain, two anonymous referees and the editors. Furthermore, we have benefited from discussions with Clemens Kool, Carsten Trenkler and Walter Zucchini. The research for this paper was carried out within the SFB 373 at the Humboldt University Berlin and the METEOR research project "Macroeconomic Consequences of Financial Crises" at the University of Maastricht.

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