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Information Effects on the Bid-Ask Spread

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ABSTRACT

An individual who chooses to serve as a market-maker is assumed to optimize his position by setting a bid-ask spread which maximizes the difference between expected revenues received from liquidity-motivated traders and expected losses to information-motivated traders. By characterizing the cost of supplying quotes, as writing a put and a call option to an information-motivated trader, it is shown that the bid-ask spread is a positive function of the price level and return variance, a negative function of measures of market activity, depth, and continuity, and negatively correlated with the degree of competition. Thus, the theory of information effects on the bid-ask spread proposed in this paper is consistent with the empirical literature.

THIS PAPER IS CONCERNED with the determination of bid-ask spreads in organized financial markets, where the trading is done through economic agents who specialize in market-making for a limited set of securities. The commitment made by dealers to buy or sell at the bid and ask prices, respectively, is analyzed as a combination of put and call options, and empirical results published in previous works are shown to be consistent with the model.

While there have been several papers concerned with the bid-ask spread, no entirely satisfactory theory has yet emerged. Demsetz [10] was the first to formalize the problem. He treated the bid-ask spread as a (transaction) cost to the trader for immediacy, and analyzed it in a static supply and demand framework.

Since Demsetz's seminal paper there have been two main lines of thought about the theory of the bid-ask spread and they are not necessarily antithetical. Several papers focus primarily on the relationship between the bid-ask spread and dealer inventory costs. However, with the exception of Ho and Stoll [19], none of them explains the bid-ask spread with competitive dealers.[1] Some of the studies[2] are based on the assumption that risk-averse specialists are not well diversified, although this is inconsistent with the dealers' common practice of sharing their risks through partnerships and pooling agreements.

The second line of thought follows Bagehot [2].[3] The dealer is assumed to face

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[1] For example, see Stoll [27], Ho and Stoll [18], Garman [16], Cohen et al. [7, 8], Bradfield [6], and Amihud and Mendelson [1].

[2] For example, see Stoll [27], and Ho and Stoll [18, 19].

[3] See Bagehot [2], Logue [21], and Jaffe and Winkler [20].

(at least) two different types of traders, namely those possessing special information and liquidity-motivated traders. Informed traders possess nonpublic information which allows them to have a better estimate of the future security price than either the dealer or liquidity traders. The dealer and liquidity traders are equally well informed vis-à-vis each other but uninformed relative to information-motivated traders. Because traders with special information have the option of not trading with the dealer, he will never gain from them. He can only lose. On the other hand the dealer gains in his transactions with liquidity-motivated traders. They are willing to pay a "fee" in order to obtain immediacy. This paper models the dealer's bid-ask spread as a tradeoff between expected losses to informed traders and expected gains from liquidity traders. The theory will be applied to both monopoly and perfect competition in dealer markets.

Section I of the paper details the assumptions which provide the framework for our analysis of the bid-ask spread. Section II develops a bid-ask model and analyzes the comparative statics of the bid-ask spread for the case of dealer monopoly and for perfect competition. Section III shows how the cost of the dealer's bid-ask spread may be characterized as a combination of a put and a call option (a straddle). Section IV shows via numerical examples that the empirical results of previous studies, namely the positive correlation of the bid-ask spread with the security's price level and its residual risk and the negative correlation with various measures of market activity, depth, and continuity, are consistent with the model proposed. Section V discusses the limitations of the model and some extensions. Section VI concludes the paper.

I. A Simplified Framework for Analyzing the Bid-Ask Spread

We will define S_0 as the current "true" price of the security as perceived by the dealer.[4] The dealer makes a commitment to buy a fixed quantity (for example 100 shares) at the bid price, K_B, or to sell at the ask price, K_A. The commitment is usually very short-lived as it can be terminated with the next transaction or with the arrival of new information.[5]

The assumptions which determine the exogenously given framework for our analysis are listed below and discussed in the paragraphs which follow:

a) There are no taxes and short-selling is unconstrained.
b) The instantaneous risk-free borrowing and lending rate, $r_f \geq 0$, is constant. The "true" underlying asset value, S, follows a stochastic process, $f(S)$, which is known (ex ante) to all market participants.[6]

[4] In accordance with Demsetz [10, p. 36] we define the "true" price as the equilibrium price which would exist in a world without any demand for immediacy and where all market participants are equally well informed. Also, Beja and Goldman [3, p. 599] distinguish between the equilibrium prices which are free of market "imperfections" and follow a "random walk" and the observed prices which are affected by the bid-ask spread, for example.

[5] For example, in the dealer market for the U.S. government securities (the Garban market), the bid-ask quotes are firm for at least two minutes, but may be revised when "hit." The market is organized for broker interdealer trading in currently listed U.S. Treasury issues by providing videoscreen listings of the best bid and offer prices for each issue. For a more complete description of the Garban market see Garbade [14, pp. 433–34].

[6] Examples of stochastic processes are: 1) Brownian motion (continuous price movements), and 2) jump processes (discrete movements), see Cox and Ross [9].

c) Information about the realizations of S is generated by exogenous events (e.g., the weather) and informed traders convey it to the marketplace. Dealers or liquidity traders are uninformed as to the realizations of $f(S)$ until after an informed trade takes place. Liquidity trading is also motivated by exogeneous independent events (e.g., immediate consumption needs), hence all traders arrive at the market trading post (not necessarily a physical location) according to a stationary stochastic process, $g(\tau)$, which is known to all participants and which has calendar time arrival $\tau > 0$ and finite mean $E(\tau)$.[7] The trader arrival process, being exogenously determined, is independent of the price change process, but not necessarily vice versa.

d) $p_I (0 < p_I < 1)$ is the probability, determined exogenously, that the next request for a quote is motivated by superior information regarding the next price realization, and $p_L = 1 - p_I$ is the probability that the quote request is liquidity-motivated.

e) Asset markets are anonymous in the sense that the dealer does not know, ex ante, whether or not the other side of the transaction possesses superior information.

f) The dealer(s) gives a quote limited to a fixed number of shares, n, and only to the first trader to arrive at the trading post. (There may be different quotes for $2n$, $3n$ shares, etc.)

g) The dealer is risk neutral, and hence is an expected profit maximizer.

h) Once at the trading post, the consummation of trades is a function of the bid-ask spread, i.e., both liquidity and information traders have price-elastic demand.[8]

The dealer's objective is to choose a bid-ask spread which maximizes his profits. If he sets the bid-ask spread too wide, he loses expected revenues from liquidity traders but reduces potential losses to informed traders. On the other hand, if he establishes a spread which is too narrow, the probability of losses incurring to informed traders increases, but is offset by potential revenues from liquidity trading. His optimal bid-ask spread is determined by a tradeoff between expected gains from liquidity trading and expected losses to informed trading.

Some of the above assumptions require no further discussion, but others do; for example, a sufficient condition for information to have private value is the anonymity of asset markets. If markets were personal, then traders known to possess superior knowledge could easily be identified and no one would agree to trade with them. One of the services of a broker is to maintain the anonymity of the client who initiates a trade.

The information arrival process, and the dealer's reaction to it, can be characterized as follows. An exogenously determined event (e.g., a thunderstorm over Kansas wheat crops) is revealed to an informed trader who is the first to come to the marketplace. The dealer limits the size of his commitment on each quote in order to limit his potential loss to a better-informed trader. After each trade any private information becomes public, and the dealer may then revise his

[7] For example, think of information events and liquidity needs as being generated by independent Poisson processes. Their convolution is also a Poisson process which can be represented by $g(\tau)$.

[8] The nature of the demand elasticity is specified in the next section of the paper.

estimate of the "true" price. The rate of trader arrival (in this one-period model) is independent of the stochastic process which generates price changes.

The stochastic process which generates information arrival (e.g., see Merton [23]) may follow either a continuous stochastic process, a discontinuous jump process, or some mixture of the two. We will examine the dealer's choice of a bid-ask spread for both types of stochastic process, starting with the jump process. As might be expected, the implications of our model will be the same, regardless of the process chosen.

The dealer knows, ex ante, that p_I is the probability that the next trader is informed. If all traders were better informed than the dealer ($p_I = 1.0$), the dealer could only lose. Therefore, we assume that p_I is strictly less than unity. Moreover, if there were no informed traders ($p_I = 0$), the marginal cost to the dealer would be zero and the competitive bid-ask spread would be driven to zero,[9] which is unrealistic. Consequently, we assume that p_I is strictly positive.

If the dealer knows the stochastic process which generates prices, knows the probability that the next trader is informed, and knows the elasticity of demand for liquidity traders, he can establish his equilibrium bid-ask spread by trading off potential losses to information traders against potential gains from liquidity traders. A formal model is discussed in the next section.

II. Bid-Ask Valuation Models

There are two scenarios which capture the mechanics of the trading process and they have very similar implications for the bid-ask spread. The simpler scenario is an instantaneous quote model. It posits that the dealer waits before offering his quote until a trader reaches the trading post. The quote is offered with knowledge that in the next instant the "true" price may jump to a new level (a jump process) if the trader is informed or remain unchanged if the trader is liquidity motivated. No time interval passes between a quote, the trade, and the revelation of a new price (if any). The second scenario, discussed later on, examines an open quote interval. The dealer offers his quote immediately and waits during some open time interval (either fixed in length or stochastic) until either the next trader reaches the market or new information is received via alternate means (whereupon he changes his quote).

A. Instantaneous Quotes

Given that the dealer withholds his quote until requested, we can model the bid-ask spread by first considering the dealer's expected costs and then his expected revenues. His expected losses to informed traders (his expected costs) will depend on p_I, the probability that the next trader is informed; the dealer's knowledge of the stochastic process governing price changes, $f(S)$; and on his choice of ask and bid prices, K_B and K_B. Recall that the quantity traded on any

[9] If the dealer were a monopolist, the bid-ask spread would be positive even though all market participants had homogeneous expectations. This was the case studied by Demsetz [10].

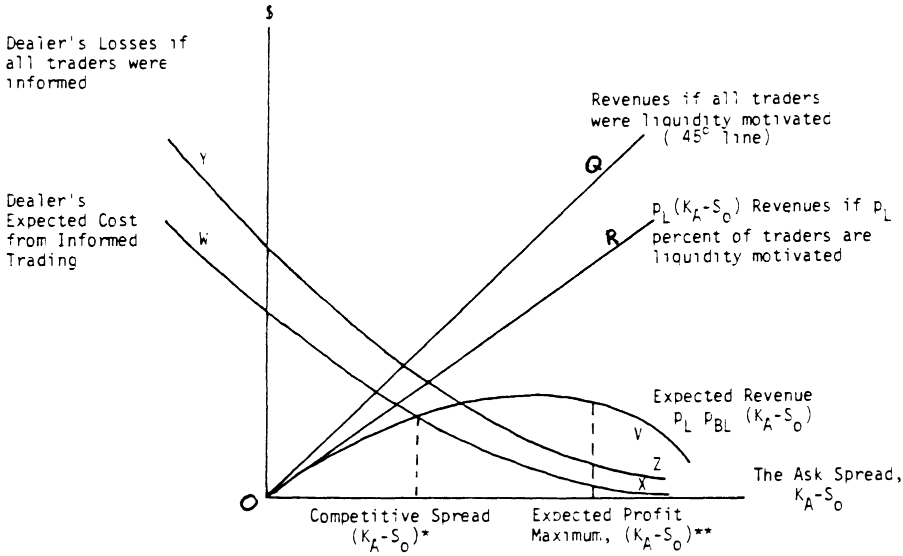

Figure 1. The Competitive and Monopoly Spreads. For each transaction the expected cost (line WX) and expected revenue (line OV) functions are shown as a function of the ask spread, $K_A - S_0$. The competitive ask spread, $(K_A - S_0)^*$, occurs where expected revenue equals expected cost, and the monopoly ask spread, $(K_A - S_0)^{**}$, occurs where expected profit is maximized.

quote is fixed. Without loss of generality, assume it is one share. Then the expected dealer loss to an informed trader is,[10]

$$p_I\left\{\int_{K_A}^{\infty}(S-K_A)f(S)\,dS+\int_{0}^{K_B}(K_B-S)f(S)\,dS\right\} \qquad (1)$$

We have used S to designate the post-trade "true" price of the asset. Not all informed traders who arrive at the marketplace will consummate a trade. Non-traders are informed individuals who believe the post-trade price will fall between K_A and K_B, the ask and bid prices, respectively. Hence, the elasticity of demand by informed traders with respect to the bid-ask interval is implicit in the limits of integration of Equation (1). The dealer's expected cost (i.e., expected losses to informed traders) is graphed as line WX in Figure 1. Note that for convenience we have chosen to graph only the ask spread (only one part of the bid-ask spread) along the horizontal axis. As the ask spread increases, dealer expected losses to better informed traders will decline.

The dealer's revenues come from those liquidity traders who are willing to pay $K_A - S_0$ or $S_0 - K_B$ as a price for immediacy. In order to express the price elasticity of the liquidity trader's demand for immediacy, we partition the fraction

[10] We have previously defined p_I as the probability that the next quote request is information-motivated and $p_L = 1 - p_I$ as the fraction who are liquidity traders. These fractions are exogeneously determined and are not a function of the bid-ask spread (assumption d).

of traders who are liquidity motivated, $p_L = (1 - p_I)$, into two parts. Let p_{TL} and p_{NL} be the probabilities of trading and nontrading, given that a trader is a liquidity trader. In addition, decompose p_{TL} into two parts, p_{BL} and p_{SL} (such that $p_{BL} + p_{SL} = p_{TL}$), which give the probability of buying and selling by a liquidity trader.[11] It is assumed that, given S_0, the probability, p_{TL}, that a liquidity trader will transact falls as the dealer spread increases.[12] p_{TL} as a function of the bid-ask spread could have been expressed directly; we have decomposed it into buying and selling components so that the bid and ask spread can be analyzed separately later in the paper,

$$\left.\frac{\partial p_{BL}}{\partial K_A}\right|_{S_0} < 0 \quad \text{and} \quad \left.\frac{\partial p_{SL}}{\partial K_B}\right|_{S_0} > 0 \tag{2}$$

The dealer's expected revenue per transaction from liquidity traders is

$$(1 - p_I)\{p_{BL}(K_A - S_0) + p_{SL}(S_0 - K_B) + p_{NL}\cdot 0\} \tag{3}$$

In Figure 1, the dealer's expected revenue curve is obtained by first multiplying the unconditional gain per transaction (the 45° line OQ) by the percentage of liquidity traders, p_L. The result is line OR. Next, this is multiplied by p_{BL}, the probability of a liquidity trader buying the asset given an ask spread. Elastic demand (Equation (2)) implies that the likelihood that a liquidity trader will consummate a trade declines as the ask spread increases. The resulting expected revenue curve (line OV) will be concave if p_{BL} decreases monotonically as a function of K_A.

The objective of a risk neutral dealer is to choose the bid-ask spread which maximizes his expected profit. Mathematically, this may be expressed as

$$\max_{K_A, K_B}\left\{(1 - p_I)[p_{BL}(K_A - S_0) + p_{SL}(S_0 - K_B)] - p_I\left[\int_{K_A}^{\infty}(S - K_A)f(S)\,dS + \int_0^{K_B}(K_B - S)f(S)\,dS\right]\right\} \geq 0 \tag{4}$$

If the dealer is a monopolist, he will maximize the difference between the expected revenue and cost functions (in Figure 1) by setting the ask price, K_A^{**}.[13] If there is free entry, then the long-run competitive equilibrium will be established where the expected costs and revenues are equal at an asking price of K_A^*. At this point expected long-run profit is zero. If the percentage of informed traders increases, then expected dealer costs increase relative to revenues and the ask price increases.

Although it is difficult to analyze the differences between the monopoly and

[11] Recall that $p_L = 1 - p_I$ is exogenously determined (assumption d), and that K_A and S_0 are known parameters at the time the liquidity trader makes his trading decision: Consequently, the partial derivative in Equation (2) contains no stochastic elements. If p_{BL} were a function of a stochastic parameter, e.g., S, then the problem would be much more complicated.

[12] Because liquidity traders are uninformed, they perceive no price change in the interval between quotes.

[13] By differentiating Equation (4), the necessary condition for maximization in terms of the expected marginal cost and marginal revenue can be presented.

competitive results without knowing more about the demand for immediacy, at least three things can be said. First, the bid-ask spread decreases as we shift from the monopoly to the competitive case (in Figure 1). This result is consistent with the empirical work of Tinic and West [29]. Second, if the percentage of informed traders increases, the difference between the monopoly and competitive spreads expressed as a percentage of the security price, $(K_A^{**} - K_A^*)/S_0$, will decrease; i.e., the monopoly and competitive solutions look more alike as the market becomes "flooded" with informed traders.[14] Third, if the elasticity of demand for liquidity trading decreases, ceteris paribus, then the dealer's expected revenue curve will shift to the left thereby causing the ask price to decrease.

Admittedly, the above model is a simplification, especially in the assumption of dealer risk neutrality. However, an important result is that even with risk neutral dealer behavior the bid-ask spread will be positive so long as there are informed traders. Dealer risk aversion is not necessary in order to explain the existence and behavior of the bid-ask spread. In addition, the above model has intuitively appealing implications regarding the comparative static results which it predicts, mainly the role of the specific risk of the stock as measured by the standard deviation of the rate of return. In Figure 1, as the variance of the stock rate of return increases, ceteris paribus, the dealer expected total cost function (line WX) shifts to the right and the competitive equilibrium ask price is raised (unless there is an offsetting effect on p_{BL}). Another prediction of the model is that an increase in the percentage of traders who are informed will result in an increase in the bid-ask spread. If large price changes result from informed trading, then transactions data should show a positive contemporaneous relationship between large price changes and bid-ask spreads.

The relationship between the bid-ask spread and trading volume may take two forms. First, p_I may be higher for thinly traded stocks (perhaps because they are, on average, more closely held). This implies a negative correlation between the bid-ask spread (or the percentage bid-ask spread) and trading volume *holding the size of the transaction constant.* Second, p_I may increase if more information is associated with the size of the transaction. In this case, there will be a positive correlation between volume and the bid-ask spread (or percentage bid-ask spread) *holding the number of transactions (per unit time) constant.*

B. Open Quote Interval

The instantaneous quote scenario is useful because it provides unambiguous comparative statics results. No discounting is involved. Although more complicated, an open quote interval provides similar comparative static results and provides additional insight into the relationship between the bid-ask spread, information arrival, and trading volume. With an open quote interval, the dealer provides his quote before requested and waits for the next trader to arrive at the trading post. Neither the dealer nor the liquidity traders receive any information

[14] This result can be seen in Figure 1. With a higher p_I the WX curve, i.e., the expected cost from informed trading, will shift upward and to the right thereby increasing both K_A^* and K_A^{**}. Usually, K_A^* will increase by more than K_A^{**} due to the elasticity of the expected revenue function, however even if the difference between K_A^{**} and K_A^* remains constant, the ratio $(K_A^{**} - K_A^*)/S_0$ will decline.

during the interval between quotes, hence both expect the price will be $S_0 e^{r_f \tau}$ at the trade. All participants know the stochastic process which generates trader arrival (assumption c) and know that it is independent of past price changes.

The open quote bid-ask model is complicated by the fact that the dealer's effective quote interval, τ, may be established for a fixed interval or it may be stochastic. The general expression for the dealer's expected losses to informed traders (Equation (1)) must be modified to discount his expected end-of-period losses as follows:

$$p_I \left\{ \int_0^\infty e^{-r_f \tau} \left[\int_{K_A}^\infty (S - K_A) f(S \mid \tau)\, dS + \int_0^{K_B} (K_B - S) f(S \mid \tau)\, dS \right] g(\tau)\, d\tau \right\} \quad (5)$$

If τ is of fixed length, then it is unnecessary to integrate across arrival times, $g(\tau)$.[15] If τ is stochastic, the comparative statics analysis of the dealer's expected cost for any given time of arrival τ, will also hold for the unconditional expectations across all arrival times. This is true because[16]

$$E_\tau[E_s(S \mid \tau)] = E_s(S) \quad (6)$$

The open quote model provides additional insight into the relationship between the expected time between quotes, the bid-ask spread, and trading volume. Empirical evidence indicates that smaller companies have less frequent trading (e.g., see Scholes and Williams [24] or Dimson [11]), i.e., $E(\tau)$ is high. This observation is consistent with other empirical evidence (Demsetz [10], Tinic and West [29], Benston and Hagerman [4], and Hamilton [17]) which has shown the bid-ask spread to be inversely correlated with the volume of trading. Since low trading volume usually means less frequent trading, the open quote model is consistent with this result because the present value of the dealer's expected cost increases with the expected duration of the quote, but the present value of his expected revenue does not.

III. The Cost of the Bid-Ask Spread as a Combination of a Call and a Put Option

Another way of looking at the dealer's problem is to consider the cost of the bid-ask spread as a "free" straddle option.[17] The dealer gives a prospective trader a call option to buy at the asking price, $K_A > S_0$, and also a put option to sell the security at the bid price, $K_B < S_0$. Note that both options are issued out-of-the-

[15] For an example of a market with fixed quote intervals see footnote 5. In the next section the case of fixed time intervals will be discussed and used for illustrative purposes.

[16] For proof see Feller [12, pp. 222–23].

[17] An American call option is the right to buy the underlying asset at a predetermined price (the exercise price) during a certain time period. The American put is the right to sell the underlying asset at a predetermined price during a certain time period. A straddle is a combination of a put and a call on the same underlying security. Usually a straddle has a single exercise price for both put and call. However, our straddle has two different exercise prices. For a characterization of options, see Galai [13].

money (at least from the dealer's perspective). A liquidity trader will be willing to suffer a certain loss by exercising the out-of-the-money option. His loss is the price he is willing to pay for immediacy. The informed trader, on the other hand, will be trading for a gain by using updated information on future states of nature. He trades when $S > K_A$ or when $S < K_B$.

Equation (1), the dealer's expected loss to informed traders, can be transformed into an option-pricing framework if we assume that the quote interval, τ, is of length zero (a jump process). Alternatively, the quote interval may be of fixed length or a random variable, as in Equation (5).[18] The comparative statics results are unchanged because what is true for any time of arrival, τ, will also be true for the unconditional expectation across all intervals (see Equation (5) and the following discussion). Defining $C(K_A)$ as the present value of the call option and $P(K_A)$ as the present value of the put option and given a fixed quote interval, we can rewrite the dealer's expected loss to informed traders as

$$p_I[C(K_A) + P(K_B)] \tag{7}$$

The values of both put and call options increase with the riskiness of the underlying asset. The call, $C(K_A)$, increases in value with the risk free rate, r_f, and with the ratio S_0/K_A. Similarly, the value of the put $P(K_B)$, decreases with an increase in the ratio S_0/K_B.

It is interesting to note, however, that for the above framework, the specific risk of the underlying asset is an important factor which affects the bid-ask spread. This result is consistent with the empirical findings of Benston and Hagerman [4] and eliminates the need to assume costly diversification. It does not require a positive coefficient of risk aversion as in Stoll [27] or Ho and Stoll [18, 19].

Note also that $C(K_A)$ and $P(K_B)$ are homogeneous functions of degree one with respect to S_0 and K. Multiplying both S_0 and K by a constant proportion, λ, affects the cost of the quote by exactly the same proportion. However, if p_{BL} is a declining function of the *difference* between K_A and S_0, then the proportional change in S_0 and K_A will cause the benefit from potential liquidity trading to increase at a slower rate than λ.[19] Hence we can expect that a share with a price of \$100 will have a relatively narrower percentage bid-ask spread than a share whose price is \$50, ceteris paribus.[20]

IV. A Numerical Analysis of the Cost of the Competitive Bid-Ask Spread as a Straddle

A natural question is whether or not the option prices implied by very short-lived dealer quotes can result in realistic estimates of the bid-ask spread. In order to illustrate the potential magnitude, several examples are given below.

[18] Limitations of the option-pricing analogy are discussed in Section V of the paper. An open quote interval is one of them.

[19] Implicit in the assumption that p_{BL} (or p_{SL}) is a decreasing function of the absolute difference $K_A - S_0$ (or $S_0 - K_B$) is that the liquidity trader may have an alternative which requires a fixed cost of transacting. In other words, it is consistent with the assumption of increasing economies of scale in transacting various financial assets.

[20] However, if p_{BL} is a declining function of the ratio K_A/S_0, a proportional change in S_0 and K_A would not affect p_{BL}, and the equilibrium percentage spread will be constant for mere scale changes.

Using a risk neutral valuation model to price the options and assuming that for short intervals the risk free rate is approximately zero, the call and put values become[21]

$$C(K_A) = S_0N(d_1) - K_AN(d_2)$$
$$d_1 = \frac{\ln(S_0/K_A) + \theta^2/2}{\theta}, \qquad d_2 = d_1 - \theta, \qquad \theta^2 = \sigma^2\tau \tag{8}$$

and

$$P(K_B) = -S_0N(-d_3) + K_BN(-d_4)$$
$$d_3 = \frac{\ln(S_0/K_B) + \theta^2/2}{\theta}, \qquad d_4 = d_3 - \theta \tag{9}$$

where $N(\cdot)$ is the cumulative standard normal distribution, and σ is the instantaneous standard deviation of rates of return.

A simplified approach assumes that the quote interval is 2, 5, or 10 minutes and the annual standard deviation is 0.2, 0.4, or 0.6. Table I, Part A, shows estimates of $\theta = \sigma\sqrt{\tau}$, the standard deviation given the quote interval length, τ, and the estimated put and call values for various bid and ask prices for a \$100 stock.[22] Table I, Part B illustrates values of the call and put options for a \$100 stock as a function of the various bid and ask prices, given the time between trades, τ, and the standard deviation, σ.[23]

The following numerical example illustrates the computation of a bid-ask spread. Let

$$p_L = 0.5, \qquad p_I = 0.5, \qquad p_{BL} = 0.5, \qquad \text{and} \quad K_A - S_0 = 0.125$$

For these numbers, the per transaction expected revenue from a liquidity trader is \$0.03125 [i.e., $p_Lp_{BL}(K_A - S_0)$]. Hence the call value at the competitive spread[24] should be $C(100.125) = \$0.0625$. From Table I, Part B, we can see that such a call value is consistent with $\tau = 2$ minutes and $\sigma = 0.6$ or with $\tau = 5$ minutes and $\sigma = 0.4$. Actually it is consistent with all combinations of σ and τ where $\theta = 0.4/\sqrt{82500/5} = 0.003$. If for example, $K_A - S_0 = 0.0625$, the resulting call value should be \$0.0313 (given that $p_Lp_{BL} = 0.25$), and it will be consistent with $\tau = 5$ and $\sigma = 0.2$.

Although these estimates are crude, they serve to demonstrate that the bid-

[21] The risk neutral model (which is equivalent to the Black and Scholes model) is used only for illustrative purposes. No claim is made that it is the ideal closed-form solution for the problem at hand. Other models such as the jump process of the binomal model would serve equally well because they have the same comparative static results. In fact, the jump process yields the same results with no time interval at all.

[22] We assume that σ is annualized on the basis of trading days not calendar days. There were 250 trading days per year, 5.5 trading hours per day, and 60 minutes per hour. This gives 82,500 trading minutes per year. A five minute θ for $\sigma = 0.2$ is $\theta = 0.2/\sqrt{82{,}500/5}$.

[23] Note that short quote intervals, τ, were paired with high standard deviations of return. This was arbitrary. Other values are available from the authors.

[24] If $p_I = 1 - p_L$, S_0 and p_{BL} as function of K_A are given, the value of K_A/S_0 can be found for the competitive spread using one equation of one unknown by means of trial and error in order to get the equality $p_Lp_{BL}(K_A - S_0) = p_IC(K_A)$ (See Figure 1).

Table I

Call and Put Values for Various Bid and Ask Prices in Dollars for S_0 = \$100

Part A: Estimates of θ

Annual σ	2 min	5 min	10 min
0.2	0.0009847	0.0015569	0.0022019
0.4	0.0019694	0.0031139	0.0044038
0.6	0.0029541	0.0046709	0.0066057

Part B: Estimated Option Values
European Call Values

Ask Price	2 min, $\sigma = 0.6$	5 min, $\sigma = 0.4$	10 min, $\sigma = 0.2$
\$100.00	\$0.117355	\$0.123459	\$0.086899
100.05	0.094177	0.100952	0.065002
100.10	0.074570	0.081024	0.046646
100.15	0.058029	0.063568	0.032715
100.20	0.044250	0.049164	0.021744
100.25	0.033020	0.037811	0.014411
100.30	0.024059	0.027985	0.008563
100.35	0.017067	0.020449	0.005337
100.40	0.012185	0.014852	0.003026
100.50	0.008233	0.007310	0.001705
100.60	0.005430	0.003359	0.000895
100.70	0.003620	0.001367	0.000466

European Put Values

Bid Price	2 min, $\sigma = 0.6$	5 min, $\sigma = 0.4$	10 min, $\sigma = 0.2$
\$100.00	\$0.117355	\$0.123458	\$0.086899
99.95	0.095398	0.100861	0.064926
99.90	0.074326	0.080902	0.047028
99.85	0.057114	0.062927	0.032227
99.80	0.043304	0.048462	0.021882
99.75	0.032303	0.037094	0.014023
99.70	0.023398	0.027756	0.008607
99.65	0.017083	0.020391	0.005210
99.60	0.011967	0.014464	0.002953
99.50	0.008121	0.006987	0.001635
99.40	0.005523	0.003105	0.000859
99.30	0.003598	0.001291	0.000423

ask spread represented by a short-lived option is of the same order of magnitude as bid-ask spreads observed on a stock exchange.[25] To further illustrate this point, we employed an estimation of the transaction's standard deviation, $\theta = \sigma\sqrt{\tau}$, measured by Garbade and Lieber [15] for Potlatch, Inc. They estimated θ to be equal to 0.002, which is consistent with an annualized standard deviation of 0.4 and a time interval of 2 minutes per transaction. If we assume that $p_{BL} = 0.5$ and $p_I = 1 - p_L = 0.5$, then the implied ratio of the ask price to the "true" value, K_A/S_0, is 1.00085. This ratio implies a bid-ask spread of approximately ⅟16

[25] Some of the difference in magnitude might be attributed to clerical costs, clearing costs, etc.

for the stock, which had a price of around \$37 on December 2, 1980.[26] However, the observed spread for the stock was 3/8, which is higher than the competitive spread under the assumed set of parameters.

V. Limitations and Extensions

Throughout the paper we have maintained the assumption that the stochastic process governing price changes is either discrete or continuous but not mixed. The dealer's response to a mixed stochastic process will depend on what information he has. For example, if public information generates the Brownian motion component of price changes, the dealer can continuously adjust his quote. The only cost he bears is related to price jumps which come as surprises to him. If the Brownian motion component contains nonpublic information, an open quote interval is an undesirable assumption because competitive dealers could reduce their costs by shortening their quote interval. Ultimately, the quote duration would diminish to zero in a competitive equilibrium.

A logical extension of the model is to make the quote interval an endogenous variable which is determined simultaneously with the bid-ask spread. This may be done by introducing transaction costs which are associated with revising a quote. More frequent revisions will reduce the cost associated with giving a combination of options, but also may result in higher transaction costs per unit time. An equilibrium quote interval can be found for a given bid-ask quote and a specific transaction cost function. It can be expected that higher volatility of the stock's return will lead to a shorter quote interval, ceteris paribus.

VI. Summary and Conclusions

Given the behavior of liquidity traders and informed traders, the dealer is assumed to offer an out-of-the-money straddle option for a fixed number of shares during a fixed time interval. The exercise prices of the straddle determine the bid-ask spread. The dealer establishes his profit maximizing spread by balancing the expected total revenues from liquidity trading against the expected total losses from informed trading. A monopolistic dealer will establish a wider bid-ask spread than will perfectly competitive dealers.

The positive implications of the model are consistent with empirically observed phenomena. The bid-ask spread increases with greater price volatility in the asset being traded, with a higher asset price level, and with lower volume. These predictions are verified by the empirical work of Demsetz [10], Tinic and West [29], Tinic [28], Benston and Hagerman [4], and Hamilton [17]. In addition, Stoll [26] finds that NASDAQ dealers tend to acquire shares when prices fall and sell when prices rise; also that dealer inventories tend to increase on days prior to price declines and decrease prior to price rises. This type of dealer behavior is consistent with our supposition that dealers suffer losses to informed traders. Within the context of our model a rational dealer will always set an ask price higher and a bid price lower than what he believes the "true" market price to be.

[26] For $S_0 = \$37$ the ask price, K_A, should be $1.00085 \cdot S_0$ or \$37.031. In a similar way $S_0 - K_A = 0.031$ for a total spread of \$0.062 or 1/16.

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