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Market depth and order size[1]

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Abstract

In this paper we empirically analyze the permanent price impact of trades by investigating the relation between unexpected net order flow and price changes. We use intraday data on German index futures. Our analysis based on a neural network model suggests that the assumption of a linear impact of orders on prices (which is often used in theoretical papers) is highly questionable. Therefore, empirical studies, comparing the depth of different markets, should be based on the whole price impact function instead of a simple ratio. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

This paper analyzes empirically the relation between order flow and price changes, i.e. the depth of a market. Papers that study block trades, including Kraus and Stoll (1972), Holthausen et al. (1987, 1990), Chan and Lakonishok (1995), Keim and Madhavan (1996) and LaPlante and Muscarella (1997),

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measure market depth, but only around block trades. A second line of research avoids the concentration on block events and measures general market depth (e.g., Hasbrouck, 1991; Algert, 1990).

Here, we follow the latter approach. We use the midquote as the price variable to eliminate the impact of the bid-ask-spread on the results. Midquotes move in response to transactions as market makers want to adjust their inventories and update their beliefs. We concentrate on information induced midquote changes by studying the relation between *unexpected* net order flow and *unexpected* midquote changes. As Madhavan and Smidt (1991) and Madhavan et al. (1997) point out, the revision of beliefs depends on unexpected order flow.

We focus mainly on the following four questions. Does large order flow convey more information than small order flow? Does net buy and net sell volume of the same size convey the same amount of information? Does the information content of order flow increase linearly with its size? Are there alternative measures of trading activity which convey more information than order flow does?

Partial answers to these questions may be found in the literature. The empirical results of Hasbrouck (1988, 1991), Algert (1990), Madhavan and Smidt (1991), and Easley et al. (1997) suggest that large net order flow conveys more information. Empirical work by Karpoff (1988), Madhavan and Smidt (1991), and Chan and Lakonishok (1993) suggests that buy orders are more informative than sell orders, a result consistent with the hypothesis that short selling restrictions prevent insiders from exploiting negative information in the stock market. Hasbrouck (1988, 1991) and Algert (1990) find evidence of a non-linear, concave relation between midquote changes and order flow. Large order flow seems to convey more information than small order flow, but the marginal information content appears to decrease. This result is consistent with the findings of several papers which study transaction price changes (e.g., Marsh and Rock, 1986; Hausmann et al., 1992; deJong et al., 1995) and the shape of the order book (e.g., Biais et al., 1995). Barclay and Warners (1993) find empirical evidence that informed investors tend to use medium size orders, suggesting that insiders exploit signals by trading frequently, a result consistent with Easley and O'Hara (1992). Similarly, Jones et al. (1994) report that it is the occurrence of transactions per se, and not their size, that generates volatility. Therefore, the number of orders might provide superior information in comparison with order flow.

Our study extends previous work in several respects. Firstly, we use neural networks to estimate the relation between order flow and midquote changes, a methodology which combines the advantages of parametric methods (simple parameter inference) and nonparametric methods (functional flexibility) used in previous studies. Secondly, we employ a non-linear forecast model in order to extract unexpected order flow and unexpected midquote changes. Thirdly, we examine the comparative information content of buy and sell orders.

Asymmetric price responses to buy and sell orders might create opportunities for profitable price manipulation (e.g., Allen and Gorton, 1992). Finally, we study the robustness of our results with respect to the information content of different trade variables.

The organization of the paper is as follows. Section 2 includes the data description and a preliminary view on the data. In Section 3 we outline the methodology of our study. We describe the neural network used and briefly contrast it with other approaches found in the literature. We formalize our hypotheses as parameter restrictions within the net and describe our data filtering to obtain unexpected order flow and midquote changes. In Section 4 we present our empirical results including some robustness checks with respect to other non-linear regression techniques and different measures of order flow. Section 5 concludes.

2. Data description

Our empirical study is based on data of German Stock Index futures (DAX futures) from 17 September 1993 to 15 September 1994. DAX futures are screen traded on the fully computerized German Futures and Options Exchange (DTB) between 9.30 a.m. and 4.00 p.m.[2] Liquidity is provided by traders and voluntary market makers who place limit orders into the centralized electronic order book which is open to all market participants. All orders (market and limit orders) are submitted electronically to the market via a trading terminal where orders are automatically matched, based on strict price and time priority. The minimum transaction size is one futures contract, which amounts to DM 100 per index point, and the minimum tick size is half an index point. Bühler and Kempf (1995) provide a more detailed description of DAX futures.

Our data set consists of all time stamped best bid quotes, best ask quotes, transaction prices, and transaction quantities for DAX futures. We concentrate on the contract closest to expiration. There is no information in the data set on whether a trade is initiated by a buyer or a seller. Therefore, we classify trades as buyer or seller initiated using an algorithm similar to Lee and Ready (1991). A trade is classified as buyer-initiated if the transaction price is equal to or higher than the current best ask price. If the transaction price is equal to or lower than the current best bid price, the trade is classified as seller-initiated. The classification procedure leads to a time series which includes bid and ask quotes, transaction prices, transaction sizes and information on whether the trade is buyer or seller initiated.

[2] Trading hours for DAX futures were expanded to 8.30 a.m.–5 p.m. in 1996.

We next pool all the data within one minute intervals. Pooling produces higher variation in net order flow than single trades, permitting us to study the price effects of a wider range of net order flows.[3] However, the cost is that causality might run not only from order flow to prices, but also from prices to order flow. Investors engaged in program trading, momentum trading and index arbitrage might react to price changes by adjusting their trading quantity. When estimating our models, we implicitly assume that causality within the same interval runs only from order flow to prices. A short interval of one minute helps keep the effects of price changes on net order flow small, while still preserving a reasonable range of values of net order flow. Note also that different functional relationships between order flow and price change might be obtained from single trades or five minute intervals because non-linear functions do not generally aggregate cleanly over time.

For each one minute interval we calculate the net order flow, X, as the difference between the buyer and seller initiated trading volume. Trading volume is measured as the number of futures contracts traded. The price change during each interval, Δp, is calculated as the difference of the log midquotes prevailing at the end of successive intervals. To avoid possible biases at the beginning of a trading session, observations within the first 15 min after the opening of the DTB are excluded. This procedure leaves us with 92,491 observations. Descriptive statistics concerning midquote changes and net order flow are provided in Table 1.

For both, midquote changes and net order flow, the mean values are very small and not significantly different from zero. They exhibit positive autocorrelation at the first few lags, an effect more pronounced for net order flow. As can be seen from the quantiles, both variables are almost symmetric around zero. Very few extremely large observations on absolute net order flow occur and the results are not sensitive to the inclusion or exclusion of them. Thus the whole data set is used in the following analyses.

A plot of X versus Δp is shown in Fig. 1.[4] As can be seen, the relation between net order flow and price changes seems to be non-linear, flattening at very large and very small values of net order flow. A non-linear model is needed to capture this feature. In addition, there is clustering of log price changes induced by the tick size.

[3] Some quantiles of the pooled net order flow are shown in Table 1. On a trade-by-trade basis, trade size is within a much smaller range. For example, less than 0.2% of all buyer initiated trades exceed 50 contracts. The same is true for seller initiated trades.

[4] To achieve a better graphical display, Fig. 1 concentrates on net order flow in the range of -300 to $+300$. This excludes only 30 observations.

Table 1
Descriptive statistics of logarithmic midquote changes and net order flow

	Logarithmic midquote changes in percentage points (Δp)	Net order flow in number of contracts (X)
Mean	– 0.000063	0.0048
Standard deviation	0.0403	35.87
Minimum	– 0.779	– 1817
1%-quantile	– 0.109	– 104
25%-quantile	– 0.023	– 12
50%-quantile	– 0.000	0
75%-quantile	0.023	12
99%-quantile	0.104	105
Maximum	0.735	1824
Autocorrelations:		
Lag 1	0.055	0.185
Lag 2	0.021	0.081
Lag 3	0.003	0.055
Lag 4	0.006	0.043
Lag 5	– 0.002	0.034

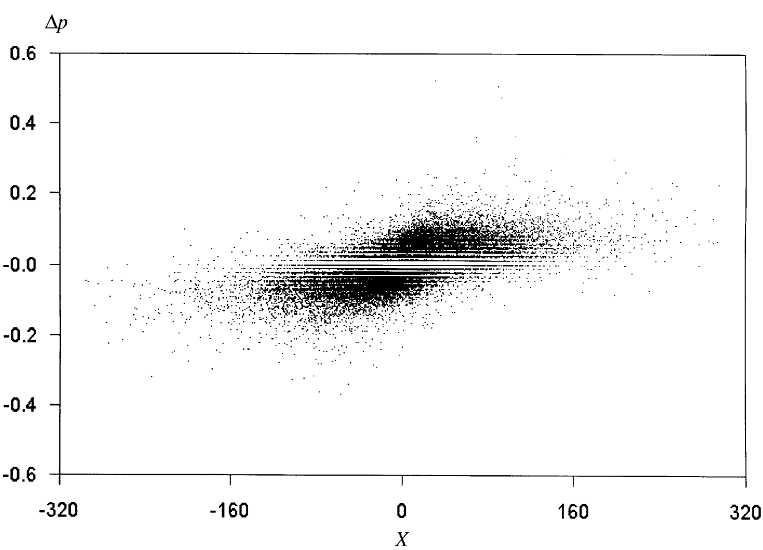


Fig. 1. Relation between net order flow (X) and logarithmic midquote changes (Δp).

3. Methodology

3.1. Neural networks

A flexible and statistically tractable model is needed to capture non-linearities. Neural networks are ideally suited for this purpose. They are flexible enough to approximate virtually any (measurable) function up to an arbitrary degree of accuracy, provided the network complexity increases appropriately with the degree of accuracy required (e.g., Hornik et al., 1989). Chen and White (1997) extend the results of Barron (1993) by showing an improved rate of approximation of neural networks to an unknown target function, i.e., less complex networks are needed to achieve a required accuracy of approximation. As a consequence, we can expect quite accurate estimates of the underlying function even with quite parsimonious networks. In addition, economic hypotheses can be formalized as simple parameter restrictions and checked by statistical test procedures.

The term 'neural network' refers to a variety of different network types and models. In this study we use the most common type, a single layer perceptron network. A linear regression model can easily be nested in such a network. For our application, this leads to the following specification:

$$\Delta p_t = \alpha_0 + \alpha_1 X_t + \sum_{h=1}^{H} \beta_h g(\gamma_h X_t) + \varepsilon_t. \tag{1}$$

Model (1) combines a linear model with a non-linear network. The network part of (1) consists of H hidden units $\beta_h g(\cdot)$, where $\beta_1, \dots, \beta_H$, and $\gamma_1, \dots, \gamma_H$ are unknown parameters and g is a non-linear transfer function. The number H of hidden units is as yet unspecified. More hidden units are generally needed to adequately approximate a more complex relationship between order flow and midquote changes. The transfer function g is usually chosen to be either the logistic or the hyperbolic tangent function. The tanh function is used here because its symmetry around the origin will be exploited later to test for asymmetic price effects of net buy and sell volume.

Alternative models have been applied to capture non-linearities in the price-order flow relation in the literature. Parametric approaches are used by Hasbrouck (1991), deJong et al. (1995) and Hausmann et al. (1992). Hasbrouck (1991) includes squared trade size as an additional regressor, while deJong et al. (1995) add the reciprocal of trade size to a linear regression equation. Hausmann et al. (1992) focus on the discreteness of price changes due to a minimum tick size and use an ordered probit model. The ordered probit model assumes a linear relationship between the regressors (for example order imbalance) and a latent variable which in turn governs price changes. The relation between this latent variable and the discrete price changes is determined by the breakpoints of the model, which can accomodate non-linearity. Parametric models are easy to estimate and permit straightforward statistical inference and significance tests. However, a drawback lies in the restrictive assumption that the relationship is characterized by a global quadratic function, a global reciprocal function or a linear latent model with specific break points.

Algert (1990) captures non-linearity with a different method. He uses the locally weighted regression of Cleveland and Devlin (1988) to analyze the impact of order imbalances on price revisions. Local fitting permits this method to describe many functional forms. This flexibility, however, comes at a cost. Its main disadvantage is the difficulty of inference regarding aspects of the relations such as the possible symmetry of positive and negative order imbalance effects. Moreover, computational costs are quite high, since a separate regression has to be run for every point of the target function. In addition, the choice of adequate data windows and weighting functions is somewhat arbitrary and may have an impact on the results. Similar pros and cons apply to other nonparametric procedures like kernel regression.

In our view neural networks combine the advantages of parametric models with those of nonparametric local fitting procedures. Like linear regression models, neural networks are parametric, which facilitates statistical inference. Like nonparametric methods, neural networks do not require restrictive assumptions about the form of the target function. From a computational point of view, numerical optimization methods are needed to estimate neural networks. However, they are usually less costly than alternative nonparametric methods. Finally, once the network is estimated, it provides a closed functional form which is available for further analysis.

3.2. *Testing of hypotheses*

The main objective of this study is to characterize the information content of trades by examining the functional form of the relation between midquote changes and net order flow. In particular, we examine the extent to which net demand ($X > 0$) and supply ($X < 0$) lead to price increases and decreases, respectively. In addition, we hope to determine if the information content is identical for net demand and net supply of the same size and if the information content of order flow increases linearly with size. Accordingly, model (1) is extended as follows:

$$\Delta p_t = \alpha_0 + \alpha_1 X_t + \alpha_2 X_t^+ + \sum_{h=1}^{H} \beta_h g(\gamma_{1h} X_t + \gamma_{2h} X_t^+) + \varepsilon_t \tag{2}$$

where $X_t^+ = X_t$, when $X_t > 0$, and $X_t^+ = 0$ otherwise. In the framework of model (2) our hypotheses take the form of the following restrictions on the parameters:

(i) If $\alpha_1 + \alpha_2 > 0$ and $\beta_h(\gamma_{1h} + \gamma_{2h}) > 0, \forall h = 1, \dots, H$, net demand leads to a price increase that is strictly increasing in net demand.[5]
(ii) If $\alpha_1 > 0$ and $\beta_h \gamma_{1h} > 0, \forall h = 1, \dots, H$, net supply causes price declines that are strictly increasing in net supply.
(iii) If $\alpha_2 = 0$ and $\gamma_{2h} = 0, \forall h = 1, \dots, H$, the information content of net demand and net supply is the same holding the magnitude constant.[6]
(iv) If $\beta_1 = \beta_2 = \cdots = \beta_H = 0$, price changes are (piecewise) linear in order flow.

We test the linear model against the alternative of a non-linear neural network (hypothesis (iv)) using tests suggested by White (1989b) and Teräsvirta et al. (1993). Lee et al. (1993) found the first test to be very powerful against a variety of non-linear alternatives. The second test had even better results than the first one in the simulation study of Teräsvirta et al. Once non-linearity is checked and the number of relevant hidden units is specified, hypotheses (i)–(iii) can be tested by standard Wald-tests as proposed by White (1989a).

3.3. Data filtering

To obtain unexpected midquote changes and order flow we use a forecast model based on past observations of both variables. The forecast procedure consists of three steps. In the first step the following linear vector autoregressive model is estimated:

$$\Delta p_t = b_{10} + \sum_{i=1}^{I} [b_{1i} \Delta p_{t-i} + c_{1i} X_{t-i}] + \varepsilon_{1t}, \tag{3}$$

$$X_t = b_{20} + \sum_{j=1}^{J} [b_{2j} \Delta p_{t-j} + c_{2j} X_{t-j}] + \varepsilon_{2t}. \tag{4}$$

The number of lags (I and J) are chosen independently for both equations by means of the information criterion of Schwarz (1978). It turns out that the price change is best explained with two lags only ($I = 2$). Observations of up to four periods in the past have noticeable explanatory power for net order flow ($J = 4$). Table 2 provides the OLS regression results for Eqs. (3) and (4).

Both variables enter significantly into either equation, indicating feedback effects. As already seen by the autocorrelations in Table 1, net order flow shows a stronger persistence. Although there are eight significant terms in Eq. (4), the

[5] This follows from the fact that the hyperbolic tangent is a strictly increasing function.

[6] This follows from the symmetry of the hyperbolic tangent function around the origin.

Table 2
Results of the linear forecast model

	Independent variable: Δp [Eq. (3)]		
	Estimated parameter	Standard error	p-Value
b_{10}	– 0.000073	0.000132	0.577
b_{11}	0.01977	0.00915	0.031
b_{12}	0.01738	0.00601	0.004
c_{11}	0.000068	0.000009	0.000
c_{12}	– 0.000016	0.000006	0.005
	Independent variable: X [Eq. (4)]		
	Estimated parameter	Standard error	p-Value
b_{20}	0.0329	0.115	0.775
b_{21}	120.65	5.42	0.000
b_{22}	43.12	4.21	0.000
b_{23}	18.71	4.10	0.000
b_{24}	16.47	3.97	0.000
c_{21}	0.0888	0.0081	0.000
c_{22}	0.0165	0.0047	0.000
c_{23}	0.0211	0.0048	0.000
c_{24}	0.0201	0.0045	0.000
$\bar{R}^2 = 0.0058$ (Eq. (3)) $\bar{R}^2 = 0.0515$ (Eq. (4))	Standard errors are obtained by the heteroskedasticity consistent estimator of White (1980)		

adjusted R-squared of about five percent is small. The explanatory power of the regression for price changes is even a full order of magnitude smaller. This is consistent with the idea of informationally efficient markets.

The second step checks whether past information has a non-linear impact on order flow or price change. The test of Teräsvirta et al. (1993) indicates some non-linear structure in the residuals of models (3) and (4). Several neural networks were fitted to the residual series and compared with the help of the Schwarz Information Criterion (SIC). For both variables, the most adequate network turned out to be a specification with two hidden units and the first two lags of both Δp and X as explanatory variables. The explanatory power of the networks is small, however. In either model the adjusted R-squared is less than 0.01.

Finally, we calculate the residuals of the networks estimated in the second step. These residuals are unpredictable (even by a non-linear model) and, hence, can serve as proxies for unexpected price change and net order flow. They are respectively denoted by Δp^{u} and X^{u}.

4. Results

4.1. Network model of the relation between price changes and order flow

Given the variables, Δp^{u} and X^{u}, we focus on the functional relationship between unexpected net order flow and price changes. The hypotheses formulated in Section 3.2 are examined in turn. The neural network tests of White (1989b)[7] and Teräsvirta et al. (1993) were carried out to test for non-linearity. Both tests strongly reject the null hypothesis of a linear model against the alternative of some neglected hidden units. The corresponding test statistics are shown in Table 3. Thus, hypothesis (iv) is rejected, and a non-linear model such as Eq. (2) describes the relationship between net order flow and price changes more adequately than a pure linear model.

The number of hidden units, H, in Eq. (2) is chosen based on the SIC. The SIC takes its minimal value for a network with one hidden unit. As the choice of an appropriate number of hidden units is important for the power of our subsequent tests, we validate the network by another criterion. The network model (2) with one hidden unit is estimated and tests for additional non-linearity in the data are carried out. The tests of White (1989b) and Teräsvirta et al. (1993) do not detect any further non-linear structure, thus no additional hidden unit is necessary.

Once the number of hidden units is specified we can test hypotheses (i)–(iii) formulated in Section 3.2. Table 3 summarizes the corresponding results:[8] the relationship between unexpected net order flow, X^{u}, and price change, Δp^{u}, is strictly increasing with net demand (hypothesis (i)), strictly decreasing with net supply (hypothesis (ii)) and not significantly asymmetric (iii).

Up to this point, we have no information on how the linear and non-linear parts of model (2) contribute to the overall results. Such information is provided by the estimated parameters shown in Table 4. The slope coefficient for the linear part, α_1, is quite small and insignificant. However, the positive coefficients β_1 and γ_{11} indicate a positive, but non-linear price impact of net order flow. Neither the linear nor the non-linear part of the model induces asymmetry, as both α_2 and γ_{12} are not significantly different from zero.

Fig. 2 depicts the relationship between unexpected net order flow and unexpected price changes. The non-linearity of the relation is apparent. A small net order flow leads to a relatively large absolute price change. A large net order

[7] The test was performed using the first two principal components of the output of originally ten hidden transfer functions. These principal components account for more than 95% of the total variation.

[8] The parameter β_1 is not identified under the null hypothesis in (i) and (ii). As the test statistics are very large, inference is not sensitive to this potential problem.

Table 3
Test results for the hypotheses (i)–(iv)

Hypothesis (H_0)	Test statistic		Distribution under H_0	*p*-Value
(i) $(\alpha_1 + \alpha_2) = 0$ and $\beta_1(\gamma_{11} + \gamma_{12}) = 0$	(Wald) 178.30		$\chi^2(2)$	0.000
(ii) $\alpha_1 = 0$ and $\beta_1\gamma_{11} = 0$	(Wald) 218.59		$\chi^2(2)$	0.000
(iii) $\alpha_2 = 0$ and $\gamma_{12} = 0$	(Wald) 4.18		$\chi^2(2)$	0.124
(iv) $\beta_1 = \beta_2 = \cdots = \beta_H = 0$	(Teräsvirta et al.), 777.78,	(White) 143.12	$\chi^2(2)$, $\chi^2(2)$	0.000, 0.000

Table 4
Regression results for the neural network model (2)

	Estimated parameter	Standard error	*p*-Value
α_0	0.000355	0.00023	0.116
α_1	0.000108	0.00008	0.150
α_2	− 0.000044	0.00003	0.125
β_1	0.056154	0.01044	0.000
γ_{11}	0.017712	0.00232	0.000
γ_{12}	0.000361	0.00127	0.777
$\bar{R}^2 = 0.395$	Estimation is carried out by non-linear least squares. Standard errors are obtained by the heteroskedasticity consistent estimator of White (1980)		

flow leads to a relatively small absolute price change. This is the same characteristic as observed for the raw data in Fig. 1. The most evident difference between the two figures is the diminished clustering of the unexpected price changes due to the forecasting procedure. The estimation and test results do not change qualitatively, however, when the raw data is used instead of the residuals of the forecast model. This is in line with the modest explanatory power of the forecast model.

Fig. 3 provides a further look at the non-linearity between unexpected net order flow and price changes. Part (a) shows the effect of a marginal change in net order flow on the (absolute) price change for different levels of order flow obtained from the estimated network model (2). The dashed lines provide a 95%

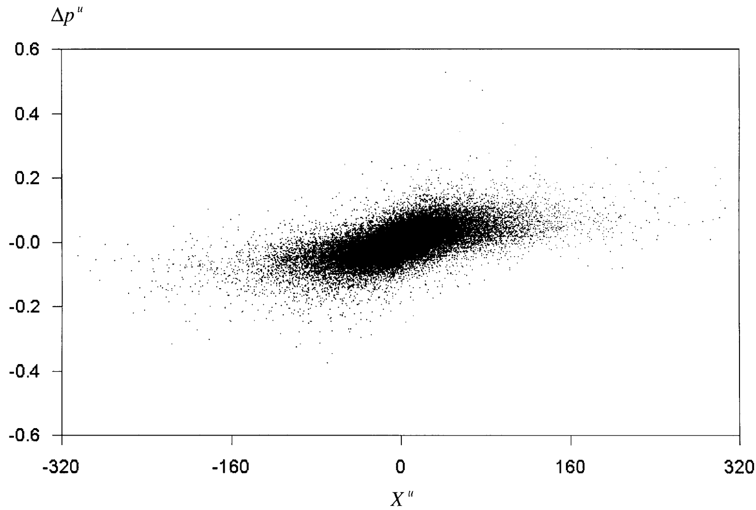


Fig. 2. Relation between unexpected net order flow (X^u) and unexpected logarithmic midquote changes (Δp^u).

confidence band for the estimated values (solid line). The confidence band is obtained via Monte-Carlo simulation from the asymptotic multivariate normal distribution of the network parameters. The price change hardly increases with net order flow for large positive or negative X^u. It is not significantly different from zero for net demand or net supply of more than 130 contracts. For absolute values of net order flow less then about 100 contracts, the non-linearity of the network model is clearly reflected. The impact of an additional unit of net order flow can be ten times as high for small orders than for large ones.

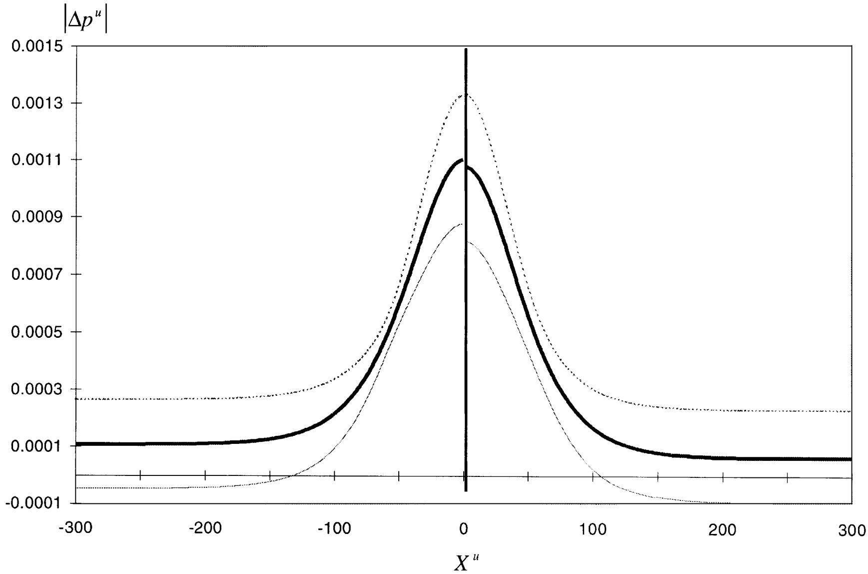
Part (b) of Fig. 3 shows the estimated average absolute unexpected price change caused by one unit of unexpected net order flow for different order sizes together with a 95% confidence band. The average price impact decreases monotonically with order flow. This indicates that the information content per trade unit is smaller, the larger the order size. This result is consistent with Algert (1990) and Hasbrouck (1991).

Our results suggest that measuring market depth by a single number may lead to erroneous conclusions. Instead, a meaningful analysis should be based on the whole price impact function.

4.2. Alternative non-linear models

As mentioned in Section 3.1 above, alternative non-linear methods have been used for similar studies, most notably in Hasbrouck (1991) and Algert (1990).

Part a): Marginal Price Impact



Part b): Average Price Impact

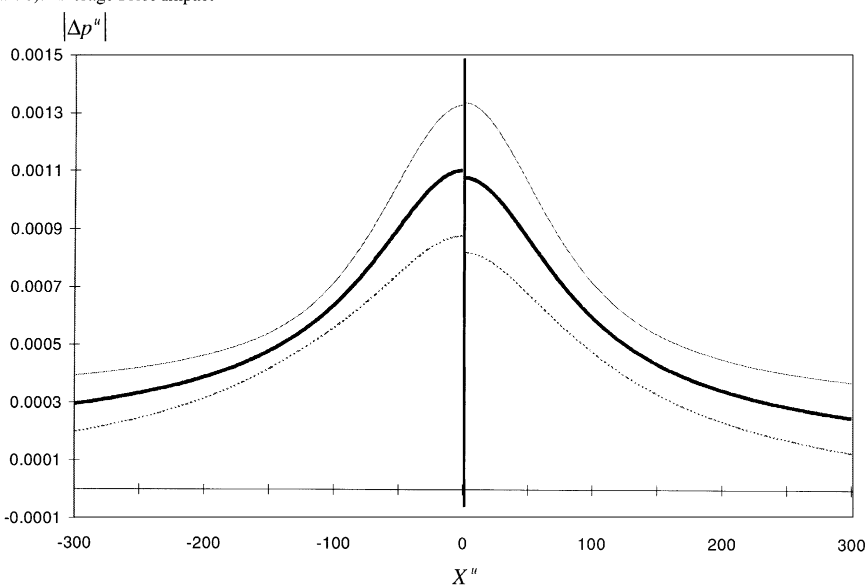


Fig. 3. Price impact implied by the estimated network model (2).

Therefore, it is instructive to check the robustness of our results with respect to the non-linear model chosen. Similar to Hasbrouck we run a quadratic regression of the following form:

$$\Delta p_t^{\mathrm{u}} = \alpha_0 + \alpha_1 X_t^{\mathrm{u}} + \beta X_t^{\mathrm{u}}|X_t^{\mathrm{u}}| + \gamma\, \mathrm{sign}(X_t^{\mathrm{u}}) + \varepsilon_t \quad \text{with}$$

$$\mathrm{sign}(X_t^{\mathrm{u}}) \equiv \begin{cases} +1 & \text{when } X_t^{\mathrm{u}} > 0, \\ -1 & \text{when } X_t^{\mathrm{u}} < 0, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

To make the specification more flexible, $\mathrm{sign}(X_t^{\mathrm{u}})$ is included as an additional regressor. This allows a jump to occur at zero net order flow. Estimation results are provided in Table 5.

As expected, the quadratic term is highly significant and negative, indicating a concave relationship. Thus, we obtain qualitatively the same result as for the neural network model (2). Compared to the neural network, however, the adjusted R-squared has dropped.

Algert (1990) employs the locally weighted regression procedure of Cleveland and Devlin (1988). The method estimates the value of a regression function $y = f(x)$ at a particular point x in p-dimensional space, where p is the number of regressors. At the outset, one has to specify a fraction k from a total of N observations and identify the kN points with the least (euclidean) distance to x. This subset of observations is used to perform a weighted linear regression on y, where the weights are given by

$$w_i(x) = \left(1 - \left(\frac{d(x, x_i)}{d_{\max}}\right)^3\right)^3 \quad \text{for } i = 1, \ldots, Nk. \tag{6}$$

In Eq. (6), $d(x, x_i)$ denotes the euclidean distance between x and the ith observation x_i in the selected subset and $d_{\max}$ is the distance between x and the furthest

Table 5
Results for the quadratic regression model (5)

	Estimated parameter	Standard error	p-Value
α_0	0.0000258	0.000103	0.802
α_1	0.0006424	0.0000092	0.000
β	− 0.0000004	0.00000004	0.000
γ	0.0062121	0.0001753	0.000
$\bar{R}^2 = 0.384$	Estimation is carried out by ordinary least squares. Standard errors are obtained by the heteroskedasticity consistent estimator of White (1980)		

observation in the subset. The estimate $\hat{f}(x)$ can finally be obtained from the regression function as the fitted value $\hat{y}$ with the regressors set equal to x.

We use the method of Cleveland and Devlin (1988) to obtain an alternative estimate of the relationship between unexpected net order flow and quote changes. The regression function is estimated at 300 equally spaced points in the range -300 and 300 with k varying between 0.3 and 0.9. The results are quite insensitive to the particular choice of k. In Fig. 4 the regression lines for $k = 0.4$ and $k = 0.8$, together with the fitted network and the quadratic regression function, are depicted.

As Fig. 4 shows, all estimation methods point in the same direction: the information content of trades grows less than linearly with the trade size.[9] Especially in the range between -100 and $+100$ contracts, which covers the great majority of observations, at least three of the curves are very close together. The two curves obtained from the locally weighted regression lie almost everywhere within a 95% confidence band of the estimated network function.[10]

In comparison with the other methods, non-linearity is less pronounced for the quadratic regression. This is observed even in the range between -100 and 100 contracts, but shows up strongly for large order flow. The quadratic regression attributes a larger price impact to high order flow than the other methods.[11] For more than 100 and less than -100 contracts the estimated function is outside the 95% confidence band of the network model. Such a divergence from the other methods may indicate that a quadratic model does not suffice to describe the underlying relationship.

4.3. *Alternative measures of order imbalance*

The fourth and final question raised in the introduction is whether alternative measures of order imbalance exist which convey more information than the difference between buyer initiated and seller initiated trade volume. If the net number of trades within an interval and not the size of these trades better measures information, the number of buy trades in excess to sell trades should lead to a better explanation of price changes than net volume. To see if this is the case we estimate a non-linear forecast model for the net number of trades within a one minute interval. The residuals of this model serve as a proxy for unexpected order imbalance, while the unexpected price changes are obtained from

[9] As mentioned in Section 2, nonlinear functions do not aggregate over time and different results may be obtained for intervals other than one minute. As a robustness check the analysis is redone for five minute intervals. Results do not change substantially.

[10] The confidence band is not shown in Fig. 4 to better display the estimated functions.

[11] This effect is even stronger for a quadratic model which does not allow for a jump at zero, i.e. a model without sign(X_t^u) as additional regressor.

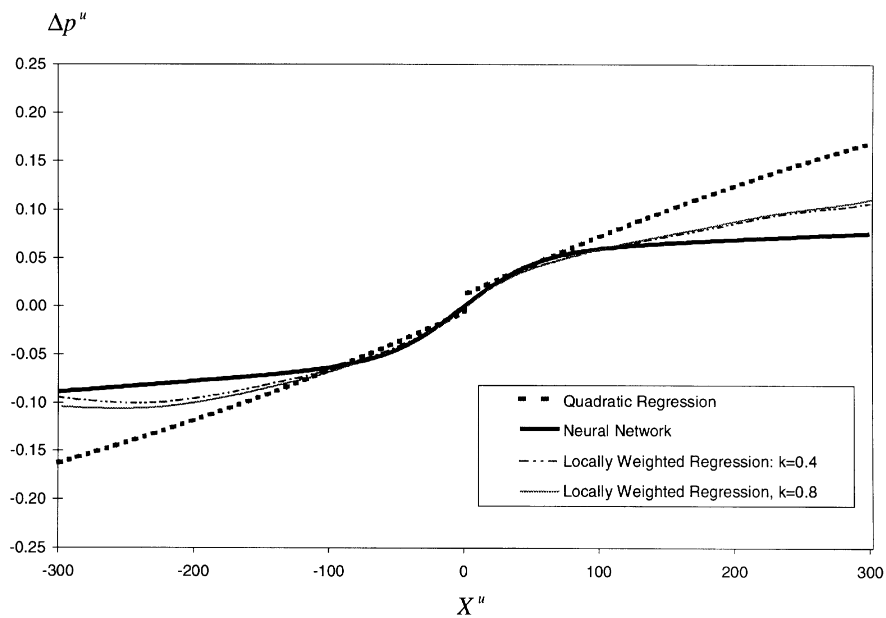


Fig. 4. Fitted regression lines for the relation between unexpected net order flow (X^{u}) and unexpected logarithmic midquote changes (Δp^{u}). Different non-linear models: neural network, quadratic regression, locally weighted regression.

the forecast model of Section 3.3. We also estimate a forecast model for Δp with lagged values of the net number of trades as additional explanatory variables. The residuals in both models are almost identical, making the choice of forecast model irrelevant.

A plot of the unexpected net number of trades against the unexpected price change is provided in Fig. 5. While there is a positive relation, no noticeable non-linearity is apparent. A neural network test of Teräsvirta et al. (1993) for neglected non-linearity in the residuals of the piecewise linear model in Eq. (7) is carried out, where NT^{u} denotes the unexpected net number of trades.

$$\Delta p_t = \alpha_0 + \alpha_1\, NT_t^{\mathrm{u}} + \alpha_2\, NT_t^{\mathrm{u}+} + \varepsilon_t. \tag{7}$$

The test results confirm the visual impression. The test statistic of 3.76 is not significant at a five percent level, when compared with the relevant critical value of the $\chi^2(2)$ distribution. Thus, the linear model (7) provides an adequate characterization of this relation.

Estimation results for model (7) are provided in Table 6. When compared with the most suitable model for the net order flow (Table 3), the explanatory power of the net number of trades is similar but slightly smaller. The adjusted

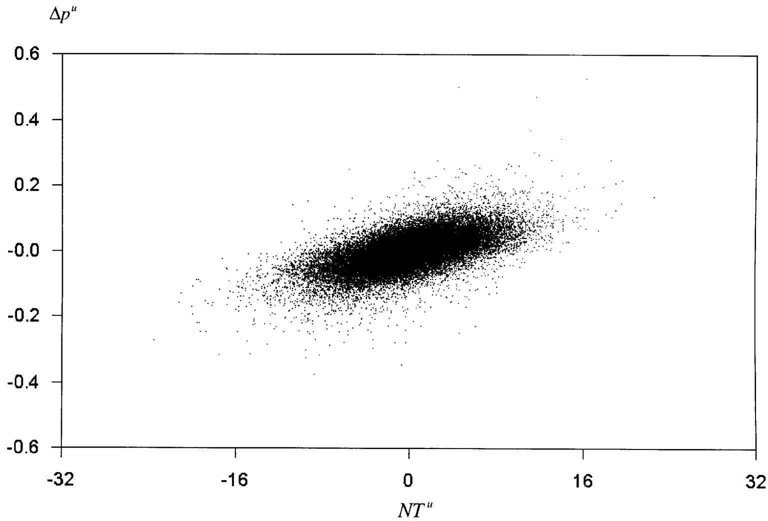


Fig. 5. Relation between unexpected net number of trades (NT^{u}) and unexpected logarithmic midquote changes (Δp^{u}).

Table 6
Regression results for the piecewise linear model (7): net order flow measured by the unexpected net number of trades

	Estimated parameter	Standard error	p-Value
α_0	0.000230	0.000175	0.188
α_1	0.007788	0.000081	0.000
α_2	− 0.000199	0.000149	0.183
$\bar{R}^2 = 0.374$	Estimation is carried out by ordinary least squares Standard errors are obtained by the heteroskedasticity consistent estimator of White (1980)		

R-squared decreases from 0.395 to 0.374. In addition to the number of trades, the trade sizes seem to contain at least some information.

So far the analysis concentrated on measures of order imbalance which do not directly relate to overall market activity. It is possible that market participants interpret the information content of a given order flow differently in periods of large total volume and in periods with low trading activity. Therefore, more information may be contained in a relative measure of order imbalance than in an absolute one. To check this, we construct a series of net volume divided by total volume. This variable just equals the difference between the proportions of buyer initiated and seller initiated volume. As in Section 3.3, a non-linear time series model is built to extract unexpected changes in the order flow variable. The relation between the resulting residuals and the unexpected price changes is positive. Tests indicate some non-linearity and an asymmetric impact of positive and negative order flow. The explanatory power of the relative measure of order imbalance is, however, comparatively weak. The most adequate network model shows only an adjusted R-squared of about 0.28. This indicates that more information is contained in the net order flow variable used in Section 4.1 than just in the proportions of buyer and seller initiated trades.

5. Conclusion

In this paper we analyze the informational price impact of trades by investigating the relation between unexpected net order flow and price changes. We mainly focus on four questions. Does large order flow convey more information than small order flow? Does net buy and net sell volume convey the same amount of information? Does the information content of order flow increase linearly with its size? Are there alternative measures of trading activity which convey more information than order flow?

We use intraday data on German index futures. Our analysis based on a neural network model provides us with the following results. Firstly, the information content of order flow increases with its size. Secondly, buyer initiated trades and seller initiated trades do not differ with respect to their information content. Thirdly, the relation between net order flow and price changes is strongly non-linear. Large orders lead to relatively small price changes whereas small orders lead to relatively large price changes. Finally, net order flow – measured by the number of contracts traded – offers the best explanation for price changes. Net number of trades explains price changes almost as well. However, the relative net order flow, i.e., net order flow divided by volume, does not provide the same level of explanation. The results are found to be quite robust with respect to the estimation procedure.

Overall, market depth cannot be described sufficiently by a single number. Therefore, empirical studies comparing the depth of different markets should be based on the whole price impact function instead of a simple ratio. Since our paper focuses solely on the informational price effect, our results do not imply that large orders should never be broken up. Non-informational costs of trading might lead to order splitting being a good strategy. However, the results do imply that the assumption of a linear total (informational and non-informational) price impact of orders (which is often used in theoretical papers) might not be a reasonable approximation of reality.

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