Econophysics

Master curve for price-impact function

The price reaction to a single transaction depends on transaction volume, the identity of the stock, and possibly many other factors. Here we show that, by taking into account the differences in liquidity for stocks of different size classes of market capitalization, we can rescale both the average price shift and the transaction volume to obtain a uniform price-impact curve for all size classes of firm for four different years (1995–98). This single-curve collapse of the price-impact function suggests that fluctuations from the supply-and-demand equilibrium for many financial assets, differing in economic sectors of activity and market capitalization, are governed by the same statistical rule.

Our results complement previous efforts[1–9] by using huge amounts of data, by looking at the short-term response to a single trade, and by measuring time in units of transactions rather than in seconds. We used the Trade and Quote database as our data source and studied the 1,000 largest firms on the New York Stock Exchange in 1995–98, by analysing roughly 113 million transactions and 173 million quotes, and investigating the shift in the mid-quote price caused by the most recent transaction.

For each transaction of volume ω made at time t, we observe two cases. First, when the next event is a quote revision, we compare the next quote to the previous quote and compute the difference in the logarithm of the mid-quote price; if the logarithm of the mid-quote price is $p(t)$, we compute the price shift as $\Delta p(t_{i+1}) = p(t_{i+1}) - p(t_i)$, where t_i is the time of the previous quote and t_{i+1} is the time of the immediately following quote.

Second, when the next event is a new transaction, we set the price shift, $\Delta p(t_{i+1})$, to zero. We then investigate the average price shift as a function of the transaction size, ω, doing this separately for buying and selling. The transactions are classified as being initiated by a buyer or seller by using the Lee and Ready algorithm[10]. In order to put all stocks on roughly the same footing, we normalize the transaction size by dividing by the average value for each stock in each year.

To understand how market capitalization affects price impact, we group the 1,000 stocks of our sample into 20 groups. The groups are ordered by market capitalization, and the number of stocks in each group is selected to maintain roughly the same number of transactions in each group. We then bin each transaction on the basis of size, and plot the average price impact against the transaction size for each group.

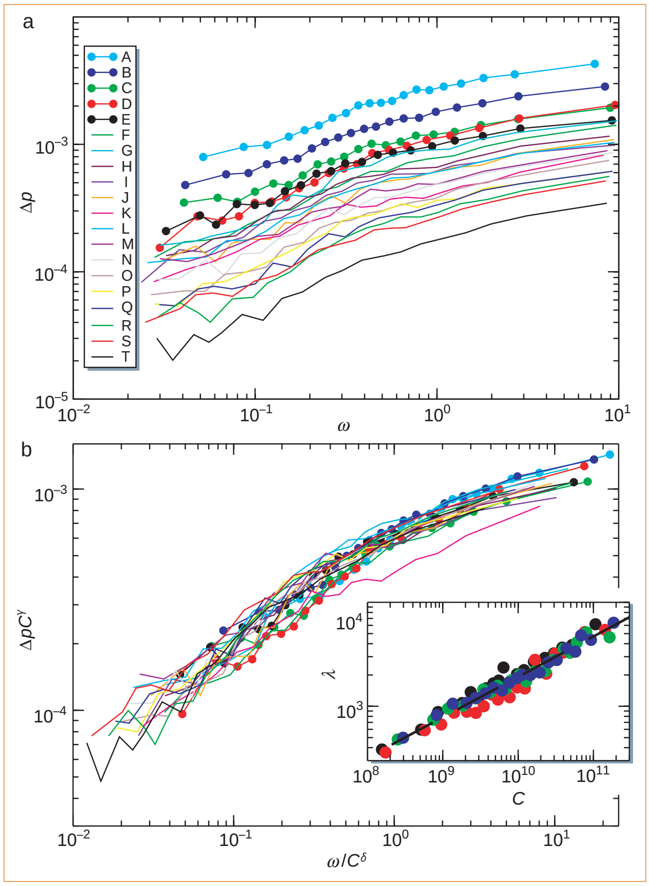

Figure 1 Scaling of the price-impact curves of 1,000 stocks traded on the New York Stock Exchange. **a,** Price shift, Δp, plotted against normalized transaction size, ω, for buyer-initiated trades for 20 groups of stocks, A–T, sorted by market capitalization in 1995. The mean market capitalization increases from group A to group T. **b,** Price shift plotted against transaction size for buy orders in 1995, renormalized as described in the text to make the data collapse roughly onto a single curve. Inset, the liquidity, λ, is shown as a function of mean capitalization, C, of each group of stocks for 1995 (black), 1996 (green), 1997 (blue) and 1998 (red). The black dashed line is the power-law best fit for all points.

The results obtained for 1995 use all of the available data (Fig. 1a). On a double-logarithmic scale, the slope of each curve varies from roughly 0.5 for small transactions in higher-capitalization stocks, to about 0.2 for larger transactions in lower-capitalization stocks. When we repeat this for the years 1996–98, the results are similar, except that the slopes become slightly flatter with time, ranging from roughly 0.4 to 0.1 in 1998.

Higher-capitalization stocks tend to have smaller price responses for the same normalized transaction size. To explain this observation, we carried out a best fit of the impact curves for small values of the normalized transaction size with the functional form $\Delta p = \mathrm{sign}(\omega)|\omega|^{\beta}/\lambda$, where λ is the liquidity and $\mathrm{sign}(\omega)$ is $+1$ or -1 for buying and selling, respectively. For all four years, the liquidity of each group increases as roughly $C^{0.4}$, where C is the average market capitalization of each group (Fig. 1b, inset).

We make use of this apparent scaling to collapse the data shown in Fig. 1a into a single curve. We assume that $\Delta p(\omega, C) = C^{-\gamma} f(\omega C^{\delta})$, where γ and δ are constants, and we rescale the ω and Δp axes of each group according to the transformations $\omega \rightarrow \omega / C^{\delta}$ and $\Delta p \rightarrow \Delta p C^{\gamma}$. We then search for the values of δ and γ that place all of the points most accurately on a single curve. In all the years that we investigated, there is a clear minimum for $\delta \approx \gamma \approx 0.3$. The resulting rescaled price-impact curves for buys in 1995 are shown in Fig. 1b.

We have investigated demand-and-supply fluctuations in a way that is complementary to the traditional approach in economics. The mechanism for making transactions has recently been theoretically modelled by assuming that order placement and cancellation are largely random[11], which results in predictions of price impact that are qualitatively consistent with those made here. Our findings show that it is important to model financial institutions in detail, and that it is may be more useful to model human behaviour as random rather than rational for some purposes.

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Two-phase behaviour of financial markets

Buying and selling in financial markets is driven by demand, which can be quantified by the imbalance in the number of shares transacted by buyers and sellers over a given time interval. Here we analyse the probability distribution of demand, conditioned on its local noise intensity Σ, and discover the surprising existence of a critical threshold, Σ_c. For $\Sigma < \Sigma_c$, the most probable value of demand is roughly zero; we interpret this as an equilibrium phase in which neither buying nor selling predominates. For $\Sigma > \Sigma_c$, two most probable values emerge that are symmetrical around zero demand, corresponding to excess demand and excess supply[1]; we interpret this as an out-of-equilibrium phase in which the market behaviour is mainly buying for half of the time, and mainly selling for the other half.

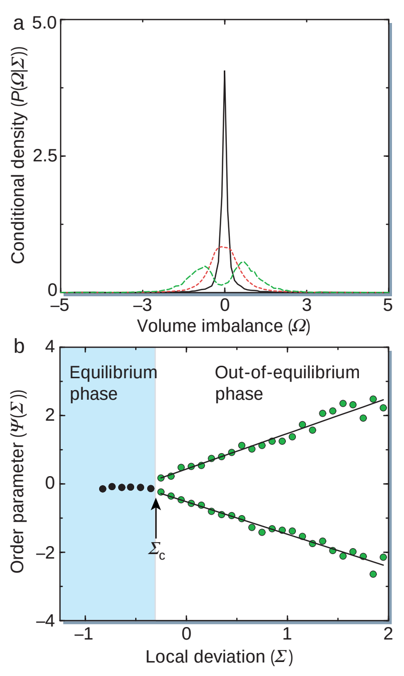

Figure 1 Empirical evidence supporting the existence of two distinct phases in a complex financial market. **a,** Conditional density, $P(\Omega|\Sigma)$, for varying Σ computed using data for all stocks. For each stock, Ω and Σ are normalized to zero mean and unit first centred moment. The distribution has a single peak for $\Sigma < \Sigma_c$ (solid line). For $\Sigma \approx \Sigma_c$ (dotted line), the distribution flattens near to the origin, and for $\Sigma < \Sigma_c$, $P(\Omega|\Sigma)$ displays two peaks (dashed line). **b,** Order parameter Ψ (positions of the maxima of the distribution $P(\Omega|\Sigma)$) as a function of Σ. For small Σ, $P(\Omega|\Sigma)$ displays a single maximum, whereas for large Σ, two maxima are present. To locate the extremes as accurately as possible, we compute all probability densities using the density estimator of ref. 8. Also shown (by shading) is a phase diagram representing the two distinct market phases. Here, $\Delta t = 15$ min; our results hold for Δt ranging from 15 min up to about half a day, beyond which our statistics are insufficient.

We use the Trade and Quote database to analyse each and every transaction of the 116 most actively traded stocks in the two-year period 1994–95. We quantify demand by computing the volume imbalance, $\Omega(t)$, defined as the difference between the number of shares, Q_B, traded in buyer-initiated transactions and the number, Q_S, traded in seller-initiated transactions in a short time interval, Δt (refs 2, 3).

$$\Omega(t) \equiv Q_B - Q_S = \sum_{i=1}^{N} q_i a_i$$

where $i = 1,\ldots,N$ labels each of the N transactions in the time interval Δt, q_i denotes the number of shares traded in transaction i, and $a_i = \pm 1$ denotes buyer-initiated and seller-initiated trades, respectively[2].

We also calculate, for the same sequence of intervals, the local noise intensity, $\Sigma(t) \equiv \langle |q_i a_i - \langle q_i a_i \rangle| \rangle$, where $\langle \ldots \rangle$ denotes the local expectation value, computed from all transactions of that stock during the time interval Δt.

We find (Fig. 1a) that for small Σ, the conditional distribution, $P(\Omega|\Sigma)$, is single-peaked, displaying a maximum at zero demand, $\Omega = 0$. For Σ larger than a critical threshold, Σ_c, the behaviour of $P(\Omega|\Sigma)$ undergoes a qualitative change, becoming double-peaked with a pair of new maxima appearing at non-zero values of demand, $\Omega = \Omega_+$, and $\Omega = \Omega_-$, which are symmetrical around $\Omega = 0$.

Our findings for the financial-market problem are identical to what is known to occur in all phase-transition phenomena, wherein the behaviour of a system undergoes a qualitative change at a critical threshold, K_c, of some control parameter K. The change in behaviour at K_c can be quantified by an order parameter $\Psi(K)$, where $\Psi(K) = 0$ for $K < K_c$, and $\Psi(K) \neq 0$ for $K > K_c$.

For the financial-market problem, we find that the order parameter $\Psi = \Psi(\Sigma)$ is given by the values of the maxima of $\Omega_\pm$ of $P(\Omega)$. Figure 1b shows that the change in $\Psi(\Sigma)$ as a function of Σ is described by

$$\Psi(\Sigma) = \begin{cases} 0 & [\Sigma < \Sigma_c] \\ \Sigma - \Sigma_c & [\Sigma >> \Sigma_c] \end{cases}$$

We interpret these two market phases as corresponding to the following two distinct conditions of the financial market.

First is the '$\Sigma < \Sigma_c$' market phase, in which the distribution of demand, Ω, is single-peaked, with the most probable value being zero; we interpret this to be the market equilibrium phase, because the price of the stock is such that the probability of a transaction being buyer-initiated is equal to the probability of a transaction being seller-initiated[4]. In the equilibrium phase, there is statistically no net demand, and prices fluctuate around their 'equilibrium' values, suggesting that most of the trading is due to 'noise' traders who trade from misperceived information or for idiosyncratic reasons[5–7].

Second is the '$\Sigma > \Sigma_c$' market phase, in which the distribution of demand is bimodal. We interpret this to be the out-of-equilibrium phase, because the price of the stock is such that there is an excess of either buyers or sellers and there is a non-zero net demand for stock. Thus, in the out-of-equilibrium phase, the prevalent 'equilibrium' price has changed, so the stock price is now being driven to the market's new evaluation of a fair value, which is consistent with the possibility that most of the trading arises from informed traders who possess superior information[5–7].

Our findings suggest that there is a link between the dynamics of a human system with many interacting participants (the financial market) and the ubiquitous phenomenon of phase transitions that occur in physical systems with many interacting units. Physical observables associated with phase transitions undergo large fluctuations that display power-law behaviour, so our results raise the possibility that volatile market movements and their empirically identified power-law behaviour are related to general aspects of phase transitions.

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