机器人技术与实践

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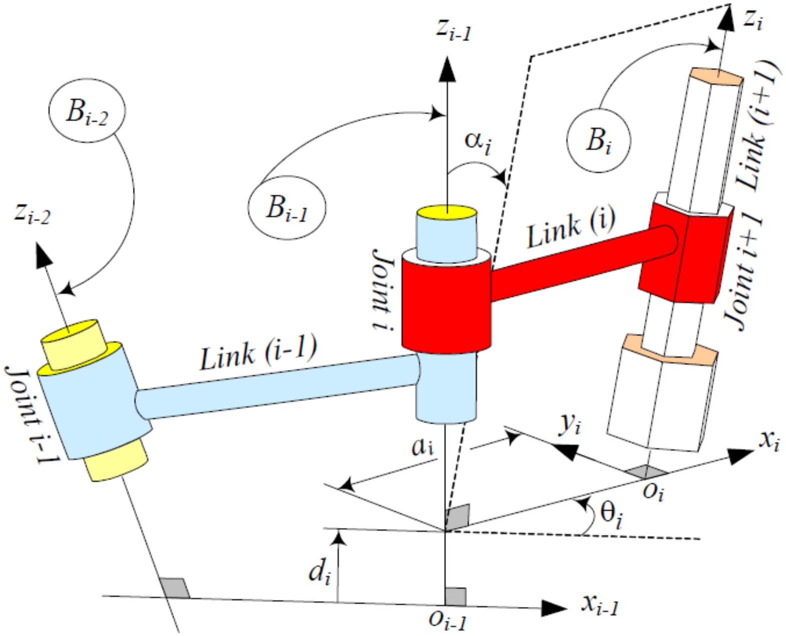
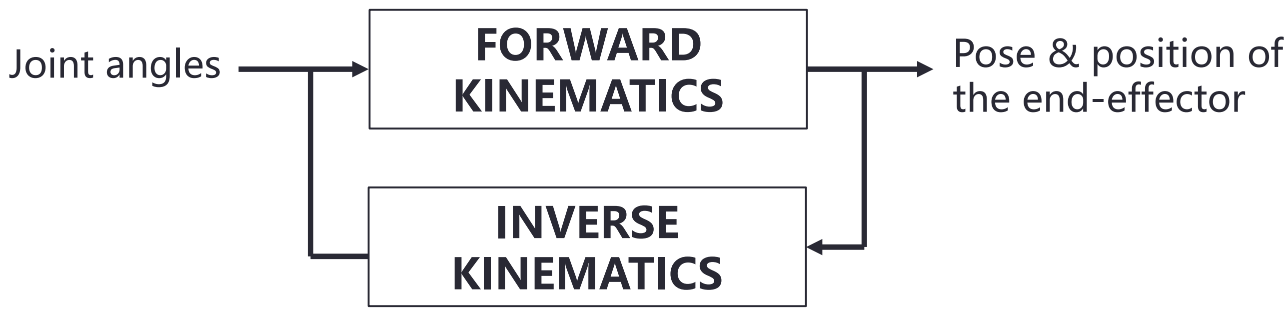
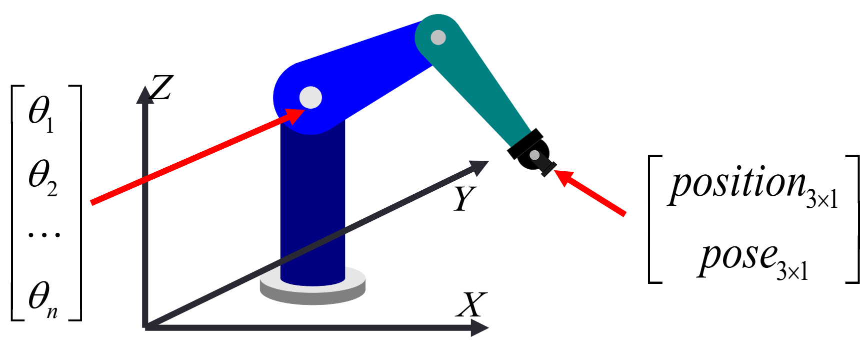
3. FORWARD KINEMATICS II



3.3 Forward Kinematics

✓ The forward (*or direct*) kinematics is the transformation of kinematic information from the robot joint variable space to the Cartesian coordinate space;





- ✓ Finding the end-effector position and orientation for a given set of joint variables is the main problem in forward kinematics;
- ✓ Solving the forward kinematics problem is a process to determining transformation matrices ${}^0T_i$ to describe the kinematic information of link (i) in the base link coordinate frame;
- ✓ By usg the Denavit-Hartenberg notations and frames, we have

$$T_n = {}^0T_1(\theta_1)\,{}^1T_2(\theta_2)\cdots{}^{i-1}T_i(\theta_i)\cdots{}^{n-1}T_n(\theta_n)$$

$$= \begin{bmatrix} R_n & \mathbf{d}_n \\ \mathbf{0} & 1 \end{bmatrix}$$

Orientation of frame B_n ← R_n

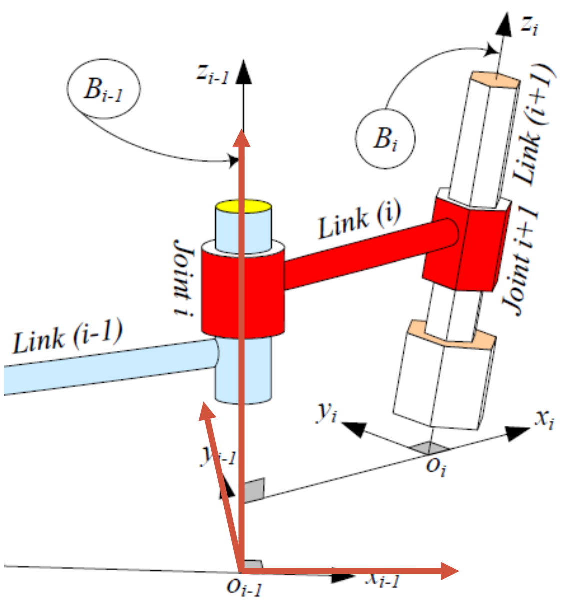
$\mathbf{d}_n$ → Position of o_n

Transformation of Two Frames

- ✓ The transformation matrix between two adjacent frames attached to link (*i*) and link (*i* + 1) is a fundamental block to the forward kinematics problem.

$$\boxed{{}^{i-1}T_i \ : \ B_{i\text{-}1} \longrightarrow B_i}$$

- ✓ Initially, frame B_i coincides with $B_{i\text{-}1}$. It becomes the present state after four steps of homogeneous transformations.



$B_{i\text{-}1}$ is a global frame and B_i is a local frame

1. $B_{i\text{-}1}$ translates d_i along $z_{i\text{-}1}$ $D_{z_{i-1}}(d_i)$
2. $B_{i\text{-}1}$ rotates about $z_{i\text{-}1}$ by θ_i $R_{z_{i-1}}(\theta_i)$

commutative

3. $B_{i\text{-}1}$ translates a_i along x_i $D_{x_i}(a_i)$
4. $B_{i\text{-}1}$ rotates about x_i by α_i $R_{x_i}(\alpha_i)$

commutative

Transformation Matrix

✓ By pre- or post-multiplications of four homogeneous transformation matrices, the overall transformation from frame $B_{i\text{-}1}$ to B_i can be obtained by

$$
{}^{i-1}T_i = D_{z_{i-1}}(d_i)R_{z_{i-1}}(\theta_i)D_{x_i}(a_i)R_{x_i}(\alpha_i)
= \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
D_{z_{i-1}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad
R_{z_{i-1}} = \begin{bmatrix} \mathrm{c}\theta & \text{-}\mathrm{s}\theta & 0 & 0 \\ \mathrm{s}\theta & \mathrm{c}\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
D_{x_i} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad
R_{x_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \mathrm{c}\alpha & \text{-}\mathrm{s}\alpha & 0 \\ 0 & \mathrm{s}\alpha & \mathrm{c}\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{i-1}T_i^{-1} = {}^{i}T_{i-1} = \begin{bmatrix} c\theta_i & s\theta_i & 0 & -a_i \\ -c\alpha_i s\theta_i & c\alpha_i c\theta_i & s\alpha_i & -d_i s\alpha_i \\ s\alpha_i s\theta_i & -s\alpha_i c\theta_i & c\alpha_i & -d_i c\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Solution to Forward Kinematics Problem

- ✓ The position of a point P in frame *n*

$$\begin{bmatrix} \mathbf{r}_P \\ 1 \end{bmatrix} = {}^0T_n \begin{bmatrix} {}^n\mathbf{r}_P \\ 1 \end{bmatrix} = \begin{bmatrix} R_n & \mathbf{d}_n \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} {}^n\mathbf{r}_P \\ 1 \end{bmatrix} = \begin{bmatrix} R_n\,{}^n\mathbf{r}_P + \mathbf{d}_n \\ 1 \end{bmatrix}$$

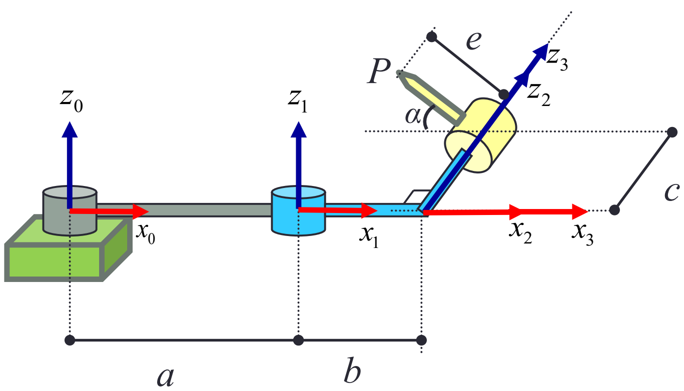
$$ {}^0T_n = {}^0T_1\,{}^1T_2 \ldots {}^{n-1}T_n $$

- ✓ The pose of the end-effector where B_n is attached

$$R_n = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix}$$

Ex 3-3-1

$\alpha=\pi/6$. Find the position of tip point P.



No.	a_i	α_i	d_i	θ_i
1	a	0	0	θ_1
2	b	-90°	0	θ_2
3	0	0	0	θ_3

$$^{i-1}T_i = D_{z_{i-1}}(d_i)R_{z_{i-1}}(\theta_i)D_{x_i}(a_i)R_{x_i}(\alpha_i)$$

$$= \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^1T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & b c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & b s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^2T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

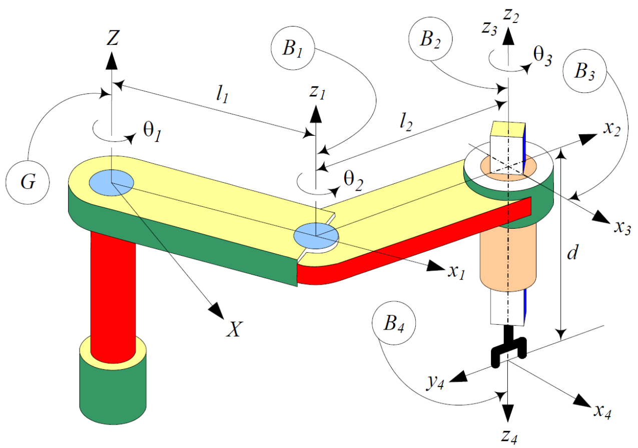
$$^0T_3 = {}^0T_1\,{}^1T_2\,{}^2T_3$$

$$= \begin{bmatrix} c(\theta1+\theta2)c(\theta3), & -c(\theta1+\theta2)s(\theta3), & -s(\theta1+\theta2), & bc(\theta1+\theta2)+ac(\theta1) \\ s(\theta1+\theta2)c(\theta3), & -s(\theta1+\theta2)s(\theta3), & c(\theta1+\theta2), & bs(\theta1+\theta2)+as(\theta1) \\ -s(\theta3), & -c(\theta3), & 0, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

$$P = {}^0T_3 \begin{bmatrix} ^3\mathbf{r}_P \\ 1 \end{bmatrix} = {}^0T_3 \begin{bmatrix} -e\cos\alpha \\ -e\sin\alpha \\ c \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} b*c(\theta1+\theta2)-c*s(\theta1+\theta2)+a*c(\theta1)-e*c(\theta1+\theta2)c(\alpha)c(\theta3) +e*c(\theta1+\theta2)s(\alpha)s(\theta3) \\ c*c(\theta1+\theta2)+b*s(\theta1+\theta2)+a*s(\theta1)-es(\theta1+\theta2)c(\alpha)c(\theta3) +e*s(\theta1+\theta2)s(\alpha)s(\theta3) \\ e*s(\alpha+\theta3) \\ 1 \end{bmatrix}$$

Forward Kinematics - SCARA Arm



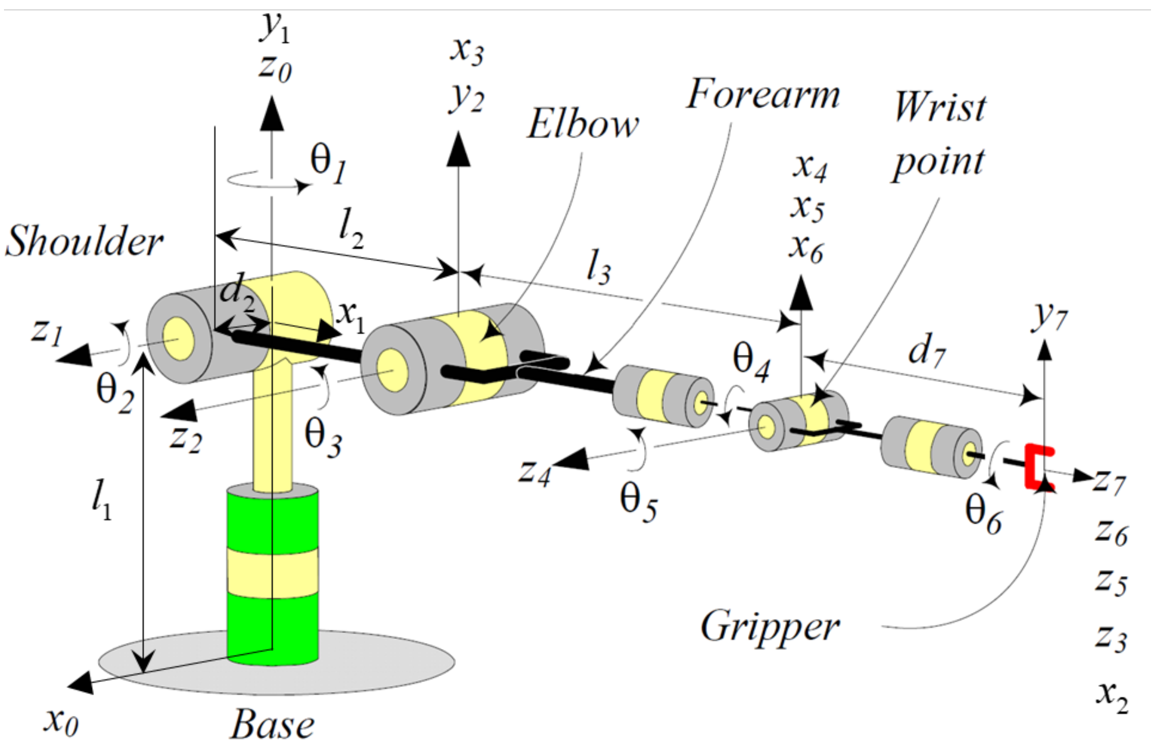
$$
{}^{0}T_{1} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & l_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & l_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^{1}T_{2} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_{3} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^{3}T_{4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

No.	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	θ_2
3	0	0	0	θ_3
4	0	-180°	$d(d_0)$	0

$$
{}^{0}T_{4} = {}^{0}T_{1}\,{}^{1}T_{2}\,{}^{2}T_{3}\,{}^{3}T_{4}
= \begin{bmatrix} c(\theta_1+\theta_2+\theta_3) & s(\theta_1+\theta_2+\theta_3) & 0 & l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) \\ s(\theta_1+\theta_2+\theta_3) & -c(\theta_1+\theta_2+\theta_3) & 0 & l_1 s\theta_1 + l_2 s(\theta_1+\theta_2) \\ 0 & 0 & -1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Forward Kinematics – 6R Manipulator



No.	a_i	α_i	d_i	θ_i
1	0	90º	l_1	θ_1(90º)
2	l_2	0	d_2	θ_2
3	0	90º	0	θ_3(90º)
4	0	-90º	l_3	θ_4
5	0	90º	0	θ_5
6	0	0	0	θ_6

$$
{}^{i-1}T_i = D_{z_{i-1}}(d_i)R_{z_{i-1}}(\theta_i)D_{x_i}(a_i)R_{x_i}(\alpha_i)
$$

$$
= \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Forward Kinematics – 6R Manipulator

$$ {}^0T_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2T_3 = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3T_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^4T_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^5T_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^6T_7 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0T_3 = {}^0T_1\,{}^1T_2\,{}^2T_3 = \begin{bmatrix} c_1c_{23} & s_1 & c_1s_{23} & l_2c_1c_2 + d_2s_1 \\ s_1c_{23} & -c_1 & s_1s_{23} & l_2s_1c_2 - d_2c_1 \\ s_{23} & 0 & -c_{23} & l_1 + l_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^3T_6 = {}^3T_4\,{}^4T_5\,{}^5T_6 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -s_4c_6 - c_4c_5s_6 & c_4s_5 & 0 \\ c_4s_6 + s_4c_5c_6 & c_4c_6 - s_4c_5s_6 & s_4s_5 & 0 \\ -s_5c_6 & s_5s_6 & c_5 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

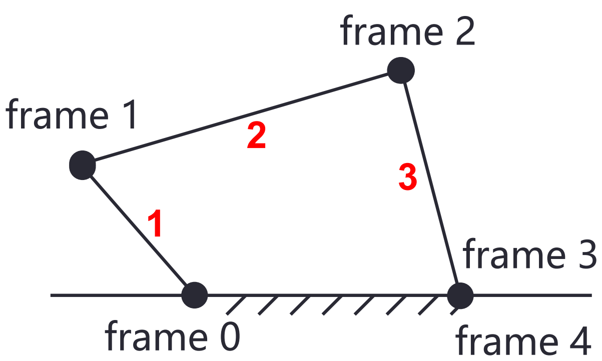
$$ {}^0T_6 = {}^0T_1\,{}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5\,{}^5T_6 = {}^0T_3\,{}^3T_6 \qquad {}^0T_7 = {}^0T_3\,{}^3T_6\,{}^6T_7 = {}^0T_6\,{}^6T_7 $$

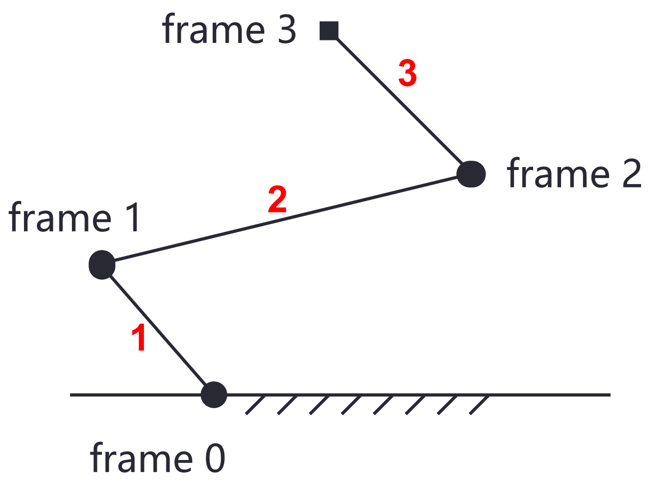
Code Session

Ch3_3.m

3.4 Non-standard DH Parameters

✓ The standard DH parameters will be inefficient if the mechanism is a closed chain where the base binary link connects both link 1 and link *n*, respectively. It will cause the ambiguity because the base link will have two different attached coordinate frames.



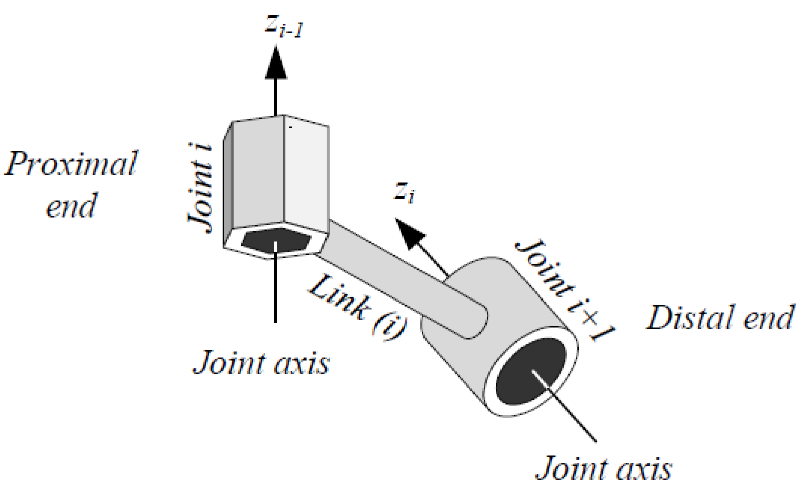


Closed chain
3 active links
4 joints

Open chain
3 active links
3 joints

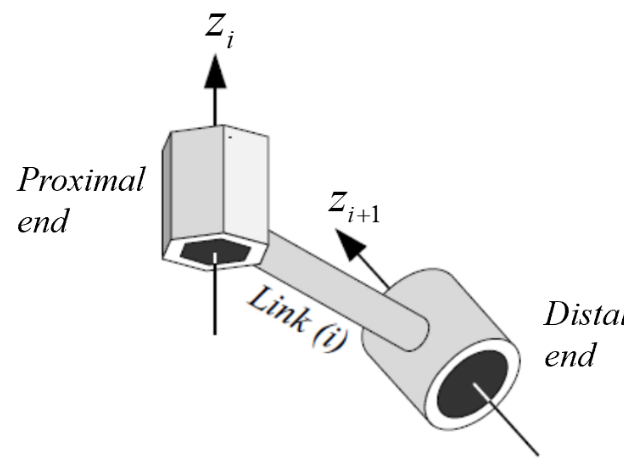
3.4 Non-standard DH Parameters

✓ In order to cope with the closed chain mechanism, standard DH parameters can be modified in the way that changing the location of the body attached coordinated frames.



Standard

Local frame *i* is setup at the distal end of the binary link

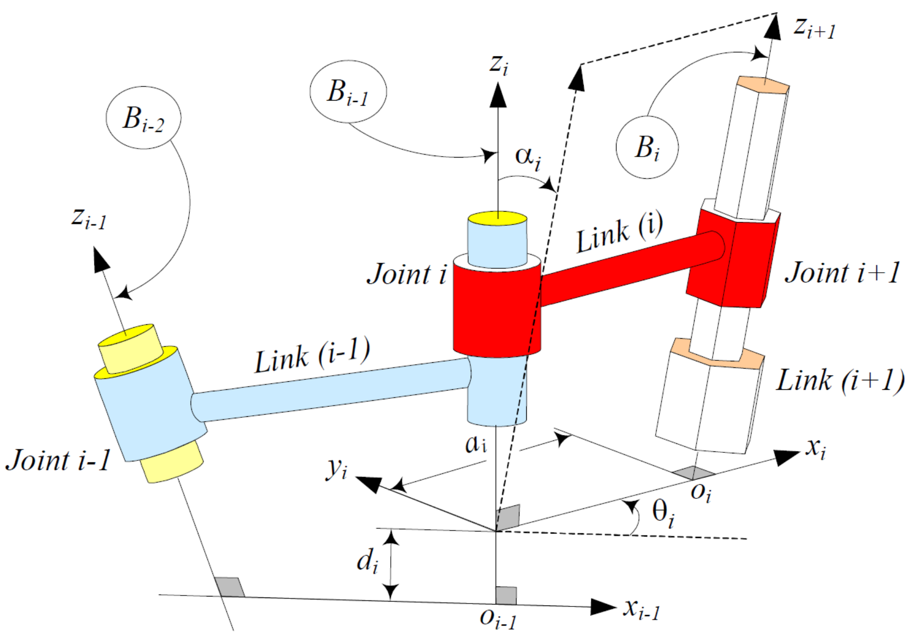


Non-standard

Local frame *i* is setup at the proximal end of the binary link

3.4 Non-standard DH Parameters

1. a_i is the distance between the z_i and z_{i+1} axes along the x_i-axis.
2. α_i is the angle from z_i to z_{i+1} axes about the x_i-axis.
3. d_i is the distance between the x_{i-1} and x_i axes along the z_i-axis.
4. θ_i is the angle from the x_{i-1} and x_i axes about the z_i-axis.



Link parameters α_i a_i

Joint parameters θ_i d_i

1. B_{i-1} rotates α_i about x_i
2. B_{i-1} translates a_i along x_i
3. B_{i-1} translates d_i along z_i
4. B_{i-1} rotates θ_i about z_i

Homogeneous Transformation Matrix

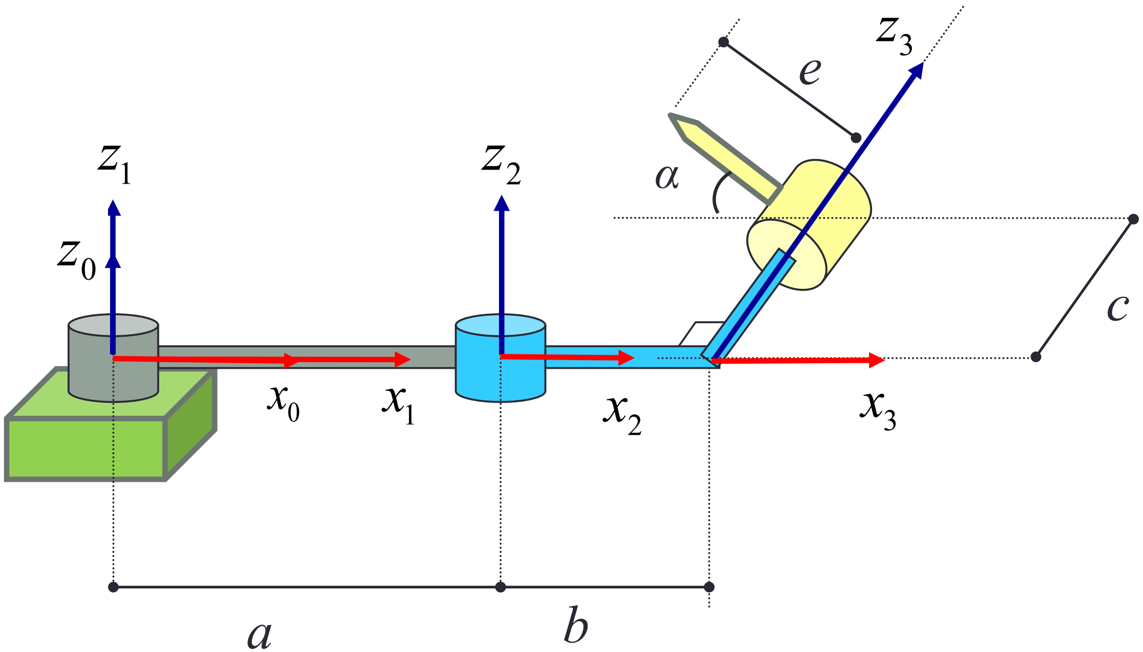
- ✓ By pre- or post-multiplications of four homogeneous transformation matrices, the overall transformation from frame $B_{i\text{-}1}$ to B_i can be obtained by

$$
\begin{aligned}
{}^{i-1}T_i &= R_{x_i}(\alpha_{i-1})D_{x_i}(a_{i-1})D_{z_i}(d_i)R_{z_i}(\theta_i) \\
&= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\alpha_{i-1}c\theta_i & -s\alpha_{i-1} & -d_i s\alpha_{i-1} \\ s\alpha_{i-1}s\theta_i & s\alpha_{i-1}c\theta_i & c\alpha_{i-1} & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

- ✓ An advantage of the non-standard DH method is that the rotation θ_i is around the z_i-axis and the joint number is the same as the coordinate number;
- ✓ A disadvantage is that the transformation matrix is a mix of $i-1$ and i indices.

Ex 3-4-1

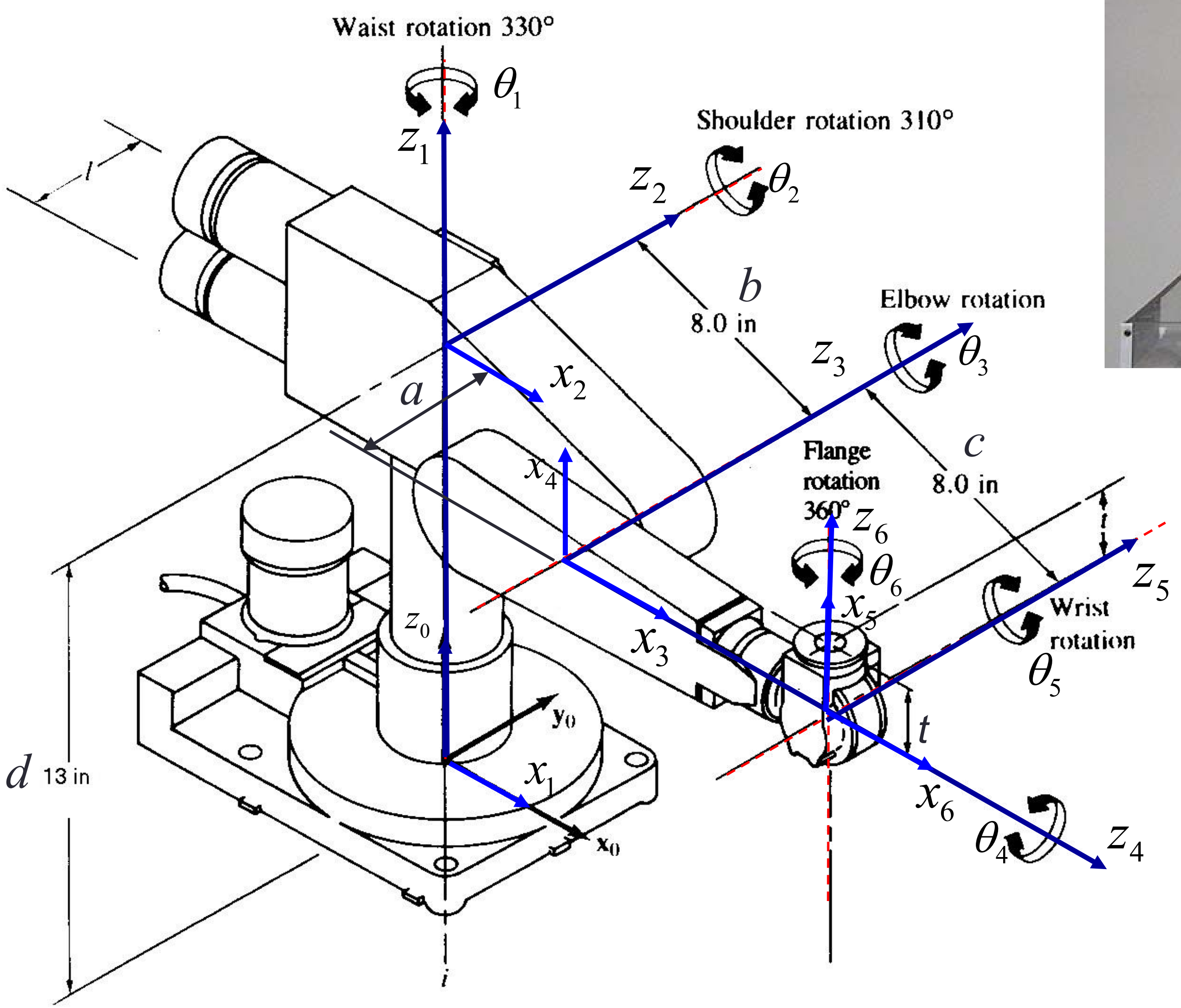
Fill in the table of non-standard DH parameters for the 3R robotic arm



Frame No.	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	a	0	0	θ_2
3	b	-90°	0	θ_3

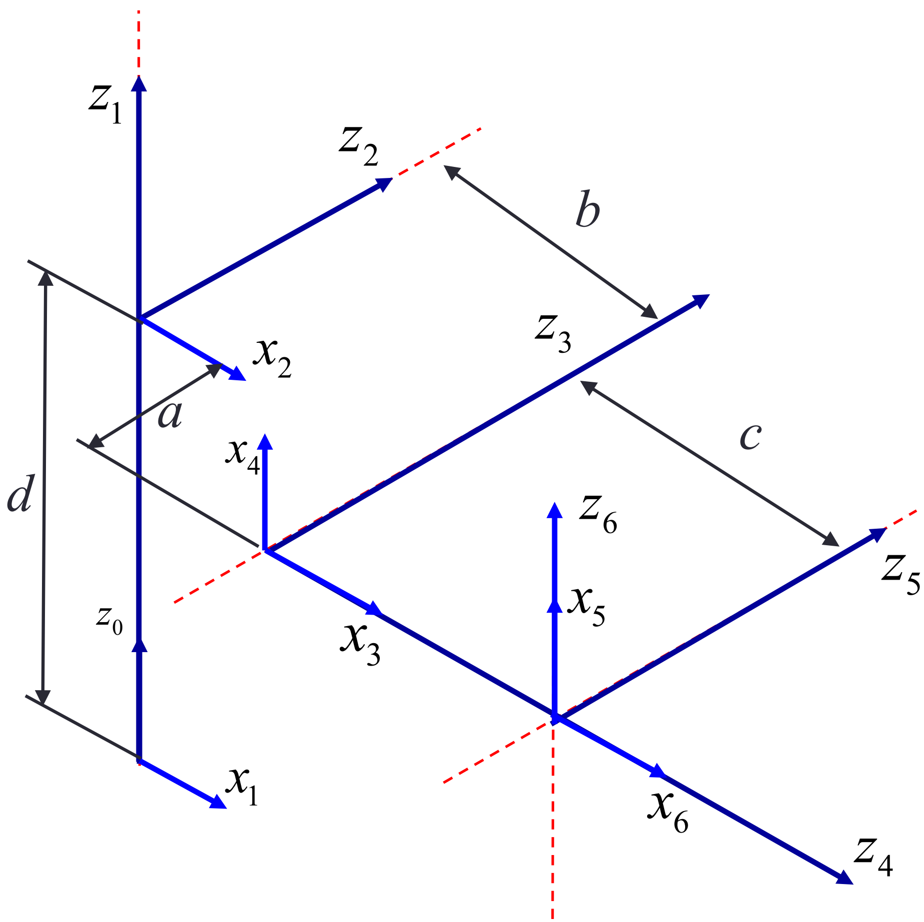
Ex 3-4-2

Fill in the table of non-standard DH parameters for PUMA200 robot



Ex 3-4-2

Fill in the table of DH parameters for the PUMA200 robot



No.	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	-90^o	d	θ_2
3	b	0	-a	θ_3
4	0	-90^o	0	$\theta 4(-90^o)$
5	0	90^o	c	θ_5
6	0	90^o	0	$\theta_6(90^o)$

Code Session

Ch3_4.m