1. 离散时间的闭环~~传递函数~~响应 $\frac{0.4z}{z^3-2.2z^2+1.9z-0.7}=\frac{Y(z)}{\cancel{R(z)}}$

$z^3-2.2z^2+1.9z-0.7=0$

$(z-1)(z^2-1.2z+0.7)=0$

$z_1=1 \quad z_2=\frac{6+\sqrt{34}i}{10} \quad z_3=\frac{6-\sqrt{34}i}{10}$

$U(z)=\frac{z}{z-1} \quad \cancel{Y(z)=\frac{0.4z^2}{(z-1)^2(z^2-1.2z+0.7)}}$

$$\frac{Y(z)}{z}=\frac{0.4}{(z-1)(z^2-1.2z+0.7)}=A\cdot\frac{1}{z-1}+B\frac{1}{z-\frac{6+\sqrt{34}i}{10}}+C\frac{1}{z-\frac{6-\sqrt{34}i}{10}}$$

$$=\frac{A(z^2-1.2z+0.7)+B(z-1)(z-\frac{6-\sqrt{34}i}{10})+C(z-1)(z-\frac{6+\sqrt{34}i}{10})}{(z-1)(z^2-1.2z+0.7)}$$

$\cancel{B=C}$

$$\begin{cases} A+B+C=0 \\ -1.2A-B-\frac{6-\sqrt{34}i}{10}B-C-\frac{6+\sqrt{34}i}{10}C=0 \\ 0.7A+\frac{6-\sqrt{34}i}{10}B+\frac{6+\sqrt{34}i}{10}C=0.4 \end{cases}$$

$$\cancel{\begin{cases} A+2B=0 \\ 0.7A+1.2B=0.4 \end{cases}} \quad \cancel{-1.2A-3.2B=0}$$

$$\begin{cases} A=0.8 \\ B=\frac{0.4}{-0.68-0.8j\sqrt{0.34}} \\ C=\frac{0.4}{-0.68+0.8j\sqrt{0.34}} \end{cases} \qquad y(KT)=A+B\left(\frac{6+\sqrt{34}i}{10}\right)^K+C\left(\frac{6-\sqrt{34}i}{10}\right)^K$$

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$y[k] = A + B(re^{j\theta})^k + C(re^{j\theta})^k$

$= A + |B|e^{j\phi}\cdot r^k e^{jk\theta} + |B|e^{-j\phi}\cdot r^k e^{-jk\theta}$

$= A + |B| r^k (e^{j(k\theta+\phi)} + e^{-j(k\theta+\phi)})$

$= A + 2|B| r^k \cos(k\theta + \phi)$

$B = -0.4 + 0.2744i \quad C = -0.4 - 0.2744i$

$|B| = 0.485 \quad \phi = 145.55° \doteq 2.5403$

$\frac{6+\sqrt{54}i}{10} = \overset{r}{0.83666} \angle \overset{\theta}{0.7711} = re^{\pm j\theta}$

$y[k] = [0.8 + 0.9701(0.83666)^k \cos(0.7711k + 2.5403)]u[k]$

$y[0]=0 \quad y[1]=-0.143 \quad y[2]=0.2437 \quad y[3]=0.9311$

$y[4]=1.5271 \quad y[5]=1.7013 \quad y[6]=1.3676$

∴ 首次峰值出现在 $k=5$

发现不如递推

$Y(z) = 2.2z^{-1}Y(z) - 1.9z^{-2}Y(z) + 0.7z^{-3}Y(z) + 0.4z^{-2}$

$y(k) = 2.2y(k-1) - 1.9y(k-2) + 0.7y(k-3) + 0.4\delta(k-2)$

k	y
0	0
1	0
2	0.4
3	0.88
4	1.176
5	1.1952
6	1.01104
7	0.776608

∴ $k=5$ 时首次峰值

3. $G(z) = \dfrac{K}{z(z-0.2)(z-0.4)}$ $(1+w)^3 - 0.6(1+w)(1-w^2) + 0.08\cancel{(1-w)}(1-w^2) + K(1-w)^3 = 0$

特征方程 $z(z-0.2)(z-0.4)+K=0$

$z^3 - 0.6z^2 + 0.08z + K = 0$

$z = \dfrac{1+w}{1-w}$ 代入~~化简~~得

$(1+3w+3w^2+w^3) - 0.6(1+w-w^2-w^3) + 0.08(1-w-w^2+w^3) + K(1-3w+3w^2-w^3) = 0$

$(0.48+K) + (2.32-3K)w + (3.52+3K)w^2 + (1.68-K)w^3 = 0$

令 $A_0 = 0.48+K$ $A_1 = 2.32-3K$ $A_2 = 3.52+3K$ $A_3 = 1.68-K$

w^3	A_3	A_1
w^2	A_2	A_0
w^1	$\dfrac{A_2A_1-A_3A_0}{A_2}$	0
w	A_0	0

$$\begin{cases} 0.48+K>0 & \cancel{K<0.48}\ K>-0.48 \\ 2.32-3K>0 & K<\dfrac{58}{75} \\ 3.52+3K>0 & K>-\dfrac{88}{75} \\ 1.68-K>0 & K<1.68 \\ \dfrac{A_2A_1-A_3A_0}{A_2}>0 & \end{cases}$$

$\Rightarrow -0.48<K<\dfrac{58}{75}$

0.7733

$A_2A_1 > A_3A_0$

$(3.52+3K)(2.32-3K) > (1.68-K)(0.48+K)$

$8.1664-3.6K-9K^2 > 0.8064+1.2K-K^2$

$8K^2+4.8K-7.36<0$

$K^2+0.6K-0.92<0$ $\dfrac{-3-\sqrt{101}}{10} < K < \dfrac{-3+\sqrt{101}}{10}$

-1.305 0.70498

又 $K>0$

$0<K<\dfrac{-3+\sqrt{101}}{10}$

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$\cancel{a_{n-1} - a_1 \frac{a_n}{a_0}} = \cancel{\frac{a_{n-1}a_0 - a_1 a_n}{a_0}}$ $\frac{a_1 a_0 - a_n a_{n-1}}{a_0}$

$a_1 - a_{n-1}\frac{a_n}{a_0}$

Jury criteria

$D(z) = z^3 - 0.6z^2 + 0.08z + K = 0$ $\quad \frac{1}{a_0}\begin{vmatrix} a_0 & a_{n-1} \\ a_n & a_1 \end{vmatrix}$

>0

1	1	-0.6	0.08	K
2	K	0.08	-0.6	1
3	$1-K^2$	$-0.08K-0.6$	$0.6K+0.08$	
4	$0.6K+0.08$	$-0.08K-0.6$	$1-K^2$	
5	$K^4-2.36K^2-0.096K+0.9936$	$0.08K^3+0.648K^2+0.2864K-0.552$		
6	$0.08K^3+0.648K^2+0.2864K-0.552$	$K^4-2.36K^2-0.096K+0.9936$		
7	$(K^4-2.36K^2-0.096K+0.9936)^2-(0.08K^3+0.648K^2+0.2864K-0.552)^2$			

解得 $0 < K < \frac{-3+\sqrt{101}}{10}$

$$\begin{cases} D(1) > 0 \\ D(-1) < 0 \\ 1-K^2 > 0 \\ K^4-2.36K^2-0.096K+0.9936 > 0 \\ 0.08K^3+0.648K^2+0.2864K-0.552 > 0 \\ (K^4-2.36K^2-0.096K+0.9936)^2-(0.08K^3+0.648K^2+0.2864K-0.552)^2 > 0 \end{cases}$$