

Problem 1

对于连续线性时不变系统 $\dot{x}(t) = Ax(t) + Bu(t)$, ZOH离散化方法的状态空间表示为 $x(k+1) = Cx(k) + Du(k)$

其中 $C = e^{AT}$ $D = \int_0^T e^{Az} B dz$ $A = \begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ \omega \end{pmatrix}$

$$(sI - A)^{-1} = \begin{pmatrix} s & -\omega \\ \omega & s \end{pmatrix}^{-1} = \frac{1}{s^2 + \omega^2} \begin{pmatrix} s & \omega \\ -\omega & s \end{pmatrix} = \begin{pmatrix} \frac{s}{s^2 + \omega^2} & \frac{\omega}{s^2 + \omega^2} \\ -\frac{\omega}{s^2 + \omega^2} & \frac{s}{s^2 + \omega^2} \end{pmatrix}$$

~~$$e^{At} = \mathcal{L}^{-1}[(sI - A)^{-1}] = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix}$$~~

代入 $t = T$

$$C = e^{AT} = \begin{bmatrix} \cos(\omega T) & \sin(\omega T) \\ -\sin(\omega T) & \cos(\omega T) \end{bmatrix}$$

$$D = \int_0^T e^{Az} B dz = \int_0^T \begin{bmatrix} \cos(\omega z) & \sin(\omega z) \\ -\sin(\omega z) & \cos(\omega z) \end{bmatrix} \begin{bmatrix} 0 \\ \omega \end{bmatrix} dz$$

$$= \int_0^T \begin{bmatrix} \omega \sin(\omega z) \\ \omega \cos(\omega z) \end{bmatrix} dz = \begin{bmatrix} 1 - \cos(\omega T) \\ \sin(\omega T) \end{bmatrix}$$

离散形式
表示为

$$x(k+1) = \begin{bmatrix} \cos(\omega T) & \sin(\omega T) \\ -\sin(\omega T) & \cos(\omega T) \end{bmatrix} x(k) + \begin{bmatrix} 1 - \cos(\omega T) \\ \sin(\omega T) \end{bmatrix} u(k)$$

Problem 2

状态 $G(z) = \frac{z-0.5}{z-1}$ 单位反馈控制器 $C(z) = \frac{K}{z^n-1} \frac{z-1}{z-0.5}$

(a) 闭环传递函数 $T(z) = \frac{C(z)G(z)}{1+C(z)G(z)} = \frac{\frac{K}{z^n-1} \frac{z-1}{z-0.5}}{1 + \frac{K}{z^n-1} \frac{z-1}{z-0.5}} = \frac{K}{K+z^n-1}$

(b) $G(z) = \frac{z-0.7}{z-1}$, 且 $n=1$ 时, 分析稳定性

开环传递函数 $C(z)G(z) = \frac{K}{z-1} \frac{z-0.7}{z-0.5}$

$$T(z) = \frac{K(z-0.7)}{(z-1)(z-0.5) + K(z-0.7)} \stackrel{n=1}{=} \frac{K(z-0.7)}{(z-1)(z-0.5) + K(z-0.7)}$$

$$= \frac{K(z-0.7)}{z^2 + (K-1.5)z + (0.5-0.7K)}$$

对于 $z^2 + a_1 z + a_2 = 0$, Jury 判据 $a_1 = K-1.5$ $a_2 = 0.5-0.7K$

1 $1 \quad a_1 \quad a_2$

$D(1) > 0 \quad D(-1) > 0$

$1 + K - 1.5 + 0.5 - 0.7K > 0 \quad K > 0$

2 $a_2 \quad a_1 \quad 1$

$|a_2| < 1$

$1 - K + 1.5 + 0.5 - 0.7K > 0 \quad K < \frac{30}{17}$

3 $1 - a_2^2 \quad \frac{a_1 - a_1 a_2}{a_2 - a_1}$

~~$a_2 > 0$~~

$|0.5 - 0.7K| < 1$

4 $a_1 - a_1 a_2 \quad 1 - a_2^2$

~~$a_2 > 0$~~

$-\frac{5}{7} < K < \frac{15}{7}$

5 $(1 - a_2^2)^2 - (a_1 - a_1 a_2)^2$

$(1 + a_2 - a_1)(1 + a_2 + a_1) > 0$

$\therefore 0 < K < \frac{30}{17}$

Problem 3

$$C(s) = \frac{as+1}{s+1}$$

(a) Tustin 变换 $s = \frac{2}{T} \frac{z-1}{z+1}$

$$C(z) = \frac{a \cdot \frac{2}{T} \frac{z-1}{z+1} + 1}{\frac{2}{T} \frac{z-1}{z+1} + 1} = \frac{(2a+T)z - 2a + T}{(2+T)z - 2 + T}$$

(b) 阶跃不变法

$$\frac{C(s)}{s} = \frac{as+1}{s(s+1)} = \frac{1}{s} + \frac{a-1}{s+1}$$

$$h(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s} + \frac{a-1}{s+1} \right\} = (1 + (a-1)e^{-t}) u(t)$$

对其采样 $h(kT) = 1 + (a-1)e^{-kT}$, $k=0, 1, 2, \dots$

z变换得 $H(z) = \frac{z}{z-1} + (a-1) \frac{z}{z-e^{-T}}$

则 $C(z) = \frac{z-1}{z} H(z) = 1 + (a-1) \frac{z-1}{z-e^{-T}}$