[1] Show that discrete time ZOH equivalent state space representation of the continuous-time system

$$\dot{\mathbf{x}}(t)=\begin{bmatrix}0 & \omega\\ -\omega & 0\end{bmatrix}\mathbf{x}(t)+\begin{bmatrix}0\\ \omega\end{bmatrix}u(t)$$

is

$$\mathbf{x}(k+1)=\begin{bmatrix}\cos(\omega T) & \sin(\omega T)\\ -\sin(\omega T) & \cos(\omega T)\end{bmatrix}\mathbf{x}(k)+\begin{bmatrix}1-\cos(\omega T)\\ \sin(\omega T)\end{bmatrix}u(k)$$

[3] For a stable and minimum-phase system $G(z)=\frac{z-0.5}{z-1}$, a unity-feedback controller $C(z)=\frac{K}{z^n-1}\frac{z-1}{z-0.5}$ was suggested to deal with the tracking control of periodic reference input signals (period = n samples, n>1).

(a) Assuming that the stable pole/zero cancellation is exact. Derive the closed-loop transfer function (5%)

(b) It is known that under the ideal condition, the stability bound is 0<K<2, regardless of the value of n. However, when the plant inversion is not exact, this stability bound can change (usually reduce). Find out the range of K for a stable closed-loop when the actual plant is $G(z)=\frac{z-0.7}{z-1}$, for the case n=1.(10%)

[5] For a continuous-time lead controller $C(s)=\frac{as+1}{s+1}$, find its discrete-time equivalent $C(z)$ using

(a) The Tustin approximation (10%)

(b) The step invariant method (5%)

[6] For the regions in the s-plane described below, find out the corresponding region in the z-plane using MATLAB (assuming the sampling time T=0.1sec).

(a) The rectangular region, $s=\sigma+j\omega,\ \sigma\in[-5,-20],\ \omega\in[-6,6]$ (10%)

(b) The pizza-slice shaped regions (complex conjugate) governed by $\zeta\in[0.5,0.9],\ \omega_n\in[0,20]$ (10%).