作业2

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Problem 1

> A discrete time system under step input was found to have a closed-loop response:
>
> $$\frac{0.4z}{z^3 - 2.2z^2 + 1.9z - 0.7}$$
>
> Task: Find the time $k = 1, 2, \ldots$ when the time response reaches its first peak by hand, and verify using MATLAB.

by hand

$$Y(z) = \frac{0.4z}{z^3 - 2.2z^2 + 1.9z - 0.7} \Longrightarrow \frac{Y(z)}{z} = \frac{0.4}{(z-1)(z^2 - 1.2z + 0.7)}$$

$$z = \frac{-(-1.2) \pm \sqrt{(-1.2)^2 - 4 \times 0.7}}{2} \approx 0.6 \pm j\sqrt{0.34}$$

得到三个极点 $p_1 = 1, p_2 = 0.6 + j\sqrt{0.34}, p_2 = 0.6 - j\sqrt{0.34}$
且

$$\frac{Y(z)}{z} = \frac{0.4}{(z-1)(z^2-1.2z+0.7)} = \mathbf{A}\cdot\frac{1}{z-1} + \mathbf{B}\cdot\frac{1}{z-(0.6+j\sqrt{0.34})} + \mathbf{C}\cdot\frac{1}{z-(0.6-j\sqrt{0.34})}$$

其中

$$\mathbf{A} = 0.8$$
$$\mathbf{B} = \frac{0.4}{-0.68-0.8j\sqrt{0.34}} \approx -0.4+j0.2744$$
$$\mathbf{C} \approx -0.4-j0.2744$$

根据Z变换对（$Y(z)$为因果信号） $\mathcal{Z}\{a^n u[n]\} = \frac{z}{z-a}$

Z反变换后，得到

$$y[k] = \left[0.8 + 0.9702\,(0.8367)^k \cos(0.7716k + 2.543)\right] u[k]$$

计算得到

$$y[0] \approx 0$$
$$y[1] \approx 0.0102$$
$$y[2] \approx 0.3829$$
$$y[3] \approx 0.8818$$
$$y[4] \approx 1.1780$$
$$y[5] \approx 1.1948$$
$$y[6] \approx 1.0096$$

可以看出首次峰值出现在 $k = 5$

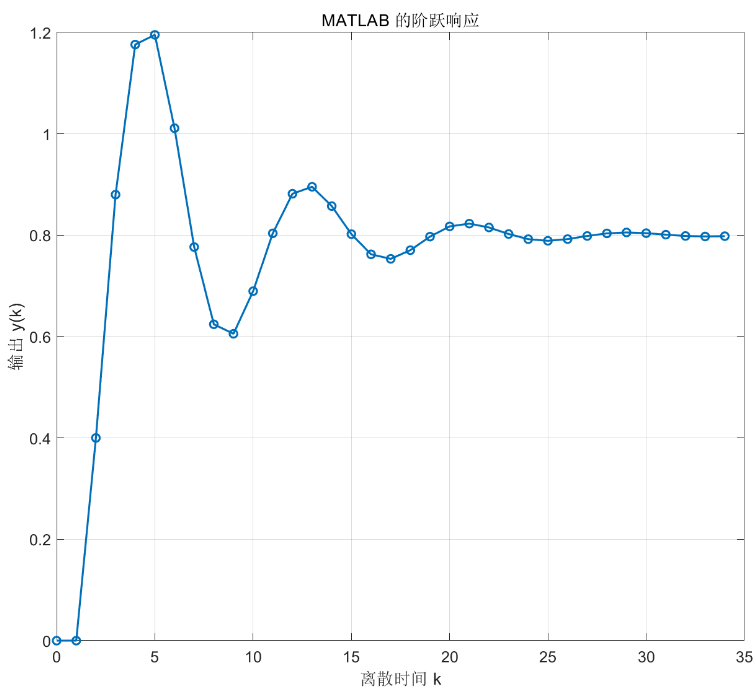
by MATLAB

```
num_T = [0.4, -0.4];
den_T = [1, -2.2, 1.9, -0.7];
Ts_sim = -1; % 离散 k 的未指定采样时间。
sys_T = tf(num_T, den_T, Ts_sim);
[y_matlab, k_matlab] = step(sys_T);
% 绘制响应
figure;
plot(k_matlab, y_matlab, 'o-');
xlabel('离散时间 k');
ylabel('输出 y(k)');
```

```
title('MATLAB 的阶跃响应');
grid on;

% 使用 findpeaks 识别首次峰值
[pks, locs] = findpeaks(y_matlab);
if ~isempty(pks)
    first_peak_value_matlab = pks(1);
    % locs 给出 y_matlab 中的索引。k_matlab(locs(1)) 给出实际的 k 值。
    first_peak_time_k_matlab = k_matlab(locs(1));
    fprintf('MATLAB: 首次峰值 %f 出现在 k = %d\n', first_peak_value_matlab, first_peak_time_k_matlab);
else
    fprintf('MATLAB: 未找到峰值。\n');
end
```

并得到输出:




```
MATLAB: 首次峰值 1.195200 出现在 k = 5
```

Problem 2

> Consider a unity feedback system with open-loop pulse transfer function $G(z)$:

$$G(z) = \frac{K}{z(z-0.2)(z-0.4)}$$

Where $K > 0$.
Task: Determine the values of K for which the closed-loop system is stable using:

1. Bilinear transform & Routh-Hurwitz criteria
2. Jury criteria
3. Root locus (manually or using MATLAB commands `rlocus` , `rlocfind` , `zgrid`)

闭环特征方程:

$$1 + G(z) = 0 \implies z^3 - 0.6z^2 + 0.08z + K = 0$$

1. Bilinear transform & Routh-Hurwitz criteria

令

$$z = \frac{1+w}{1-w}$$

则,

$$\left(\frac{1+w}{1-w}\right)^3 - 0.6\left(\frac{1+w}{1-w}\right)^2 + 0.08\left(\frac{1+w}{1-w}\right) + K = 0$$

化简后得到,

$$(1+3w+3w^2+w^3) - 0.6(1+w-w^2-w^3) + 0.08(1-w-w^2+w^3) + K(1-3w+3w^2-w^3) = 0$$

$$\implies (1.68+K) + (3.52+3K)w + (2.32-3K)w^2 + (0.48+K)w^3 = 0$$

令 $A_3 = 1.68+K$, $A_2 = 3.52+3K$, $A_1 = 2.32-3K$, $A_0 = 0.48+K$

根据劳斯判据，构造劳斯表如下:

幂次	系数1	系数2
w^3	$A_3 = 1.68+K$	$A_1 = 2.32-3K$
w^2	$A_2 = 3.52+3K$	$A_0 = 0.48+K$
w^1	$\frac{A_2A_1 - A_3A_0}{A_2}$	0

幂次	系数1	系数2
w^0	$A_0 = 0.48 + K$	0

系统稳定需满足：

1. $A_3 = 1.68 + K > 0 \implies K > -1.68$
2. $A_2 = 3.52 + 3K > 0 \implies K > -1.173$
3. $A_1 = 2.32 - 3K > 0 \implies K < 0.773$
4. $A_0 = 0.48 + K > 0 \implies K > -0.48$
5. $\frac{A_2 A_1 - A_3 A_0}{A_2} > 0$

将 $A_2 A_1 - A_3 A_0$ 展开并化简：

$$A_2 A_1 - A_3 A_0 = (3.52 + 3K)(2.32 - 3K) - (1.68 + K)(0.48 + K)$$

$\frac{A_2 A_1 - A_3 A_0}{A_2}$化简后得到：

$$\frac{7.36 - 5.76K - 10K^2}{3.52 + 3K} > 0$$

结合 $K > 0$，解得 $0 < K < 0.705$。

综上，闭环系统稳定的 K 取值范围为：

$$0 < K < 0.705$$

2. Jury criteria

z 域中的特征方程为：

$$P(z) = z^3 - 0.6z^2 + 0.08z + K = 0$$

对于 Jury 判据，按 $a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0 = 0$，有 $n = 3$，系数如下：

- $a_3 = 1$
- $a_2 = -0.6$
- $a_1 = 0.08$
- $a_0 = K$

Jury 判据的稳定性条件如下：

1. $|a_0| < a_n$
   $|K| < 1 \implies 0 < K < 1$

2. $P(1) > 0$
   $P(1) = 1 - 0.6 + 0.08 + K = 0.48 + K > 0 \implies K > -0.48$
   由于 $K > 0$，此条件总成立。

3. $(-1)^n P(-1) > 0$
   $P(-1) = -1 - 0.6 - 0.08 + K = -1.68 + K$
   $(-1)^3 P(-1) = -P(-1) > 0 \implies P(-1) < 0 \implies K < 1.68$

4. **Jury 表条件**
   构造 Jury 表：

行/系数	z^0 (a₀, b₀)	z^1 (a₁, b₁)	z^2 (a₂, b₂)	z^3 (a₃)
第 1 行	K	0.08	-0.6	1
第 2 行	1	-0.6	0.08	K
第 3 行	$b_0 = K^2 - 1$	$b_1 = 0.08K + 0.6$	$b_2 = -0.6K - 0.08$	/

需满足 $|b_0| > |b_2|$，即

$$|K^2 - 1| > |-0.6K - 0.08|$$

结合 $0 < K < 1$，有 $K^2 - 1 < 0$，即 $1 - K^2 > 0.6K + 0.08$，化简得

$$K^2 + 0.6K + 0.08 < 1 \implies K^2 + 0.6K - 0.92 < 0$$

解得 $-1.305 < K < 0.705$，结合 $K > 0$，最终有 $0 < K < 0.705$。

由 Jury 判据，闭环系统稳定的 K 取值范围为：

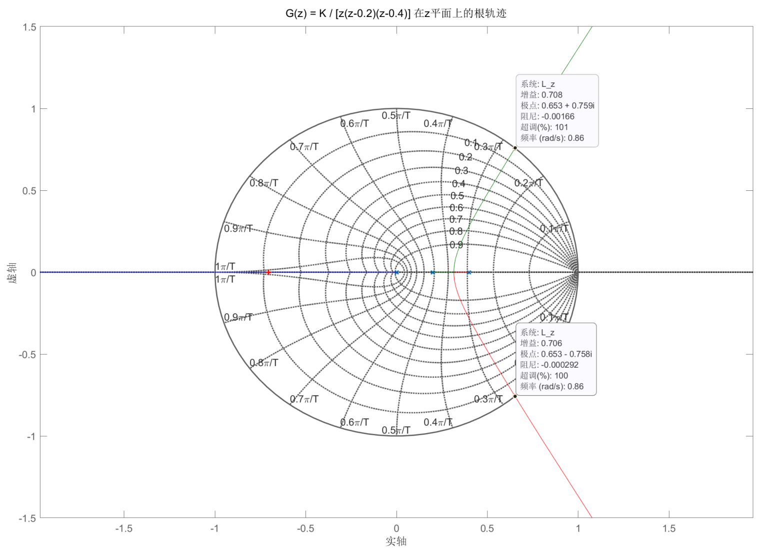
$$0 < K < 0.705$$

3. Root locus

```
L_z = tf([1], conv([1 0], conv([1 -0.2], [1 -0.4])), -1);
figure;
rlocus(L_z);
title('G(z) = K / [z(z-0.2)(z-0.4)] 在z平面上的根轨迹');
```

```
axis equal; % 确保单位圆是圆形的
zgrid;
[k_selected, poles_selected] = rlocfind(L_z)
```

并得到输出:




```
Select a point in the graphics window

selected_point =

   0.6504 + 0.7585i


k_selected =

    0.7040


poles_selected =

   0.6523 + 0.7574i
   0.6523 - 0.7574i
  -0.7046 + 0.0000i
```

得到K的取值范围为:

$$0 < K < 0.7040$$