1. 离散时间的闭环~~传递函数~~响应 $\frac{0.4z}{z^3-2.2z^2+1.9z-0.7} = \frac{Y(z)}{\cancel{R(z)}}$

$z^3-2.2z^2+1.9z-0.7=0$

$(z-1)(z^2-1.2z+0.7)=0$

$z_1=1 \quad z_2=\frac{6+\sqrt{34}i}{10} \quad z_3=\frac{6-\sqrt{34}i}{10}$

$U(z)=\frac{z}{z-1} \quad \cancel{Y(z)=\frac{0.4z^2}{(z-1)^2(z^2-1.2z+0.7)}}$

$$\frac{Y(z)}{z}=\frac{0.4}{(z-1)(z^2-1.2z+0.7)}=A\cdot\frac{1}{z-1}+B\frac{1}{z-\frac{6+\sqrt{34}i}{10}}+C\frac{1}{z-\frac{6-\sqrt{34}i}{10}}$$

$$=\frac{A(z^2-1.2z+0.7)+B(z-1)(z-\frac{6-\sqrt{34}i}{10})+C(z-1)(z-\frac{6+\sqrt{34}i}{10})}{(z-1)(z^2-1.2z+0.7)}$$

$\cancel{B=C}$

$$\begin{cases} A+B+C=0 \\ -1.2A-B-\frac{6-\sqrt{34}i}{10}B-C-\frac{6+\sqrt{34}i}{10}C=0 \\ 0.7A+\frac{6-\sqrt{34}i}{10}B+\frac{6+\sqrt{34}i}{10}C=0.4 \end{cases}$$

$\cancel{\begin{cases} A+2B=0 \\ 0.7A+1.2B=0.4 \end{cases}}$

$\cancel{-1.2A-3.2B=0}$

$$\begin{cases} A=0.8 \\ B=\frac{0.4}{-0.68-0.8j\sqrt{0.34}} \\ C=\frac{0.4}{-0.68+0.8j\sqrt{0.34}} \end{cases}$$

$y(KT)=A+B\left(\frac{6+\sqrt{34}i}{10}\right)^K+C\left(\frac{6-\sqrt{34}i}{10}\right)^K$

$y[k]=A+B(re^{j\theta})^k+C(re^{j\theta})^k$

$=A+|B|e^{j\phi}\cdot r^k e^{jk\theta}+|B|e^{-j\phi}\cdot r^k e^{-jk\theta}$

$=A+|B|r^k(e^{j(k\theta+\phi)}+e^{-j(k\theta+\phi)})$

$=A+2|B|r^k\cos(k\theta+\phi)$

$B=-0.4+0.2744i \quad C=-0.4-0.2744i$

$|B|=0.485 \quad \phi=145.55^\circ\doteq 2.5403$

$\frac{6+\sqrt{54}i}{10}=\overset{r}{0.83666}\angle\overset{\theta}{0.7711}=re^{\pm j\theta}$

$y[k]=[0.8+0.9701(0.83666)^k\cos(0.7711k+2.5403)]u[k]$

$y[0]=0 \quad y[1]=-0.143 \quad y[2]=0.2437 \quad y[3]=0.9311$

$y[4]=1.5271 \quad y[5]=1.7013 \quad y[6]=1.3676$

∴ 首次峰值出现在 $k=5$

发现不如递推

$Y(z)=2.2z^{-1}Y(z)-1.9z^{-2}Y(z)+0.7z^{-3}Y(z)+0.4z^{-2}$

$y(k)=2.2y(k-1)-1.9y(k-2)+0.7y(k-3)+0.4\delta(k-2)$

k	y
0	0
1	0
2	0.4
3	0.88
4	1.176
5	1.1952
6	1.01104
7	0.776608

∴ $k=5$ 时首次峰值

$$G(z)=\frac{K}{z(z-0.2)(z-0.4)}$$

$(1+w)^3-0.6(1+w)(1-w^2)+0.08\cancel{(1-w)}(1-w^2)$
$+K(1-w)^3=0$

特征方程 $z(z-0.2)(z-0.4)+K=0$

$z^3-0.6z^2+0.08z+K=0$

$z=\frac{1+w}{1-w}$ 代入~~化简~~得

$(1+3w+3w^2+w^3)-0.6(1+w-w^2-w^3)+0.08(1-w-w^2+w^3)+K(1-3w+3w^2-w^3)=0$

$(0.48+K)+(2.32-3K)w+(3.52+3K)w^2+(1.68-K)w^3=0$

令 $A_0=0.48+K$ $A_1=2.32-3K$ $A_2=3.52+3K$ $A_3=1.68-K$

w^3	A_3	A_1
w^2	A_2	A_0
w^1	$\frac{A_2A_1-A_3A_0}{A_2}$	0
w	A_0	0

$$\begin{cases} 0.48+K>0 \\ 2.32-3K>0 \\ 3.52+3K>0 \\ 1.68-K>0 \\ \frac{A_2A_1-A_3A_0}{A_2}>0 \end{cases}$$

~~$K<0.48$~~ $K>-0.48$

$K<\frac{58}{75}$

$K>-\frac{88}{75}$ $\Rightarrow -0.48<K<\frac{58}{75}$

0.7733

$K<1.68$

又 $K>0$

$A_2A_1>A_3A_0$

$0<K<\frac{-3+\sqrt{101}}{10}$

$(3.52+3K)(2.32-3K)>(1.68-K)(0.48+K)$

$8.1664-3.6K-9K^2>0.8064+1.2K-K^2$

$8K^2+4.8K-7.36<0$

$K^2+0.6K-0.92<0$ $\frac{-3-\sqrt{101}}{10}<K<\frac{-3+\sqrt{101}}{10}$

-1.305 0.70498

$\cancel{a_{n-1} - a_1 \frac{a_n}{a_0}} = \cancel{\frac{a_{n-1}a_0 - \ldots}{a_0}}\ \frac{a_1 a_0 - a_n a_{n-1}}{a_0}$

$a_1 - a_{n-1}\frac{a_n}{a_0}$

~~Jury criteria~~

$D(z) = z^3 - 0.6z^2 + 0.08z + K = 0$

$\frac{1}{a_0}\begin{vmatrix} a_0 & a_{n-1} \\ a_n & \cancel{a_{n-1}}\ a_1 \end{vmatrix}$

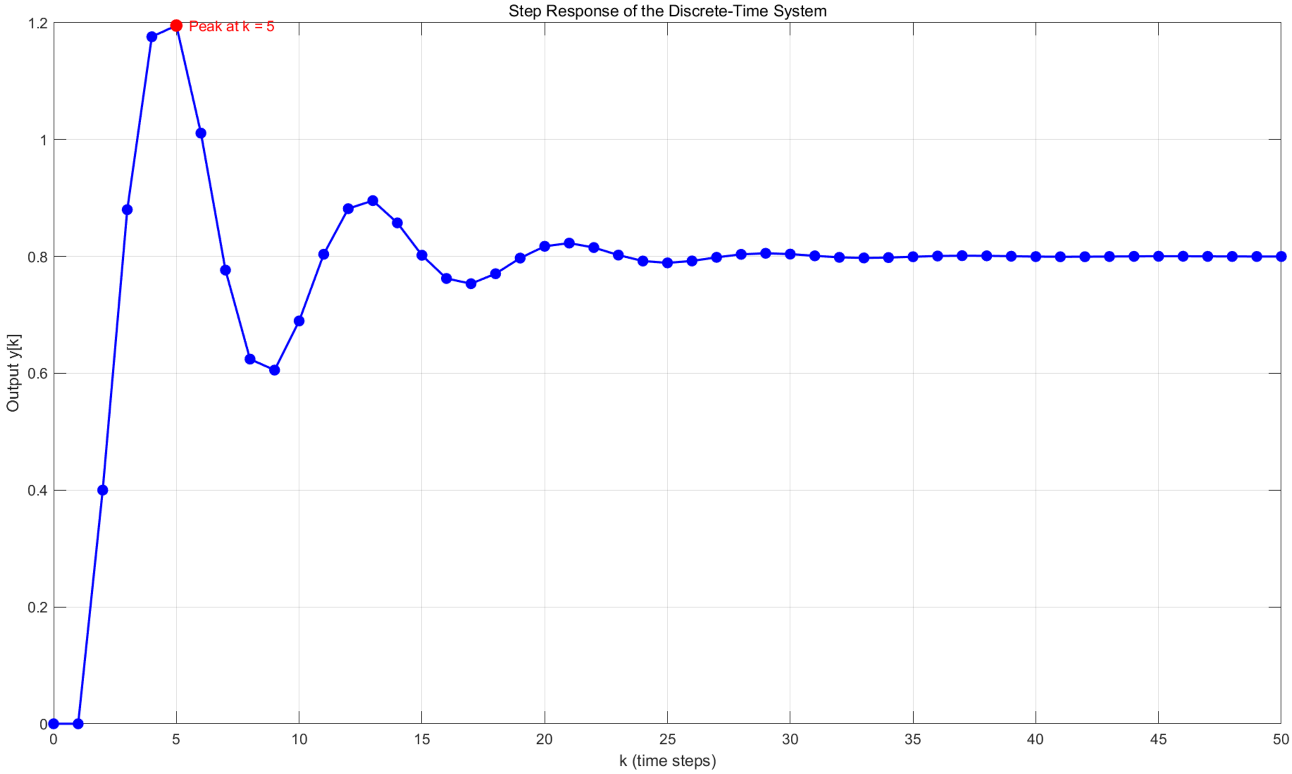
$a_0 > 0$

1	1	-0.6	0.08	K
2	K	0.08	-0.6	1
3	$1-K^2$	$-0.08K-0.6$	$0.6K+0.08$	
4	$0.6K+0.08$	$-0.08K-0.6$	$1-K^2$	
5	$K^4-2.36K^2-0.096K+0.9936$	$0.08K^3+0.648K^2+0.2864K-0.552$		
6	$0.08K^3+0.648K^2+0.2864K-0.552$	$K^4-2.36K^2-0.096K+0.9936$		
7	$(K^4-2.36K^2-0.096K+0.9936)^2-(0.08K^3+0.648K^2+0.2864K-0.552)^2$			

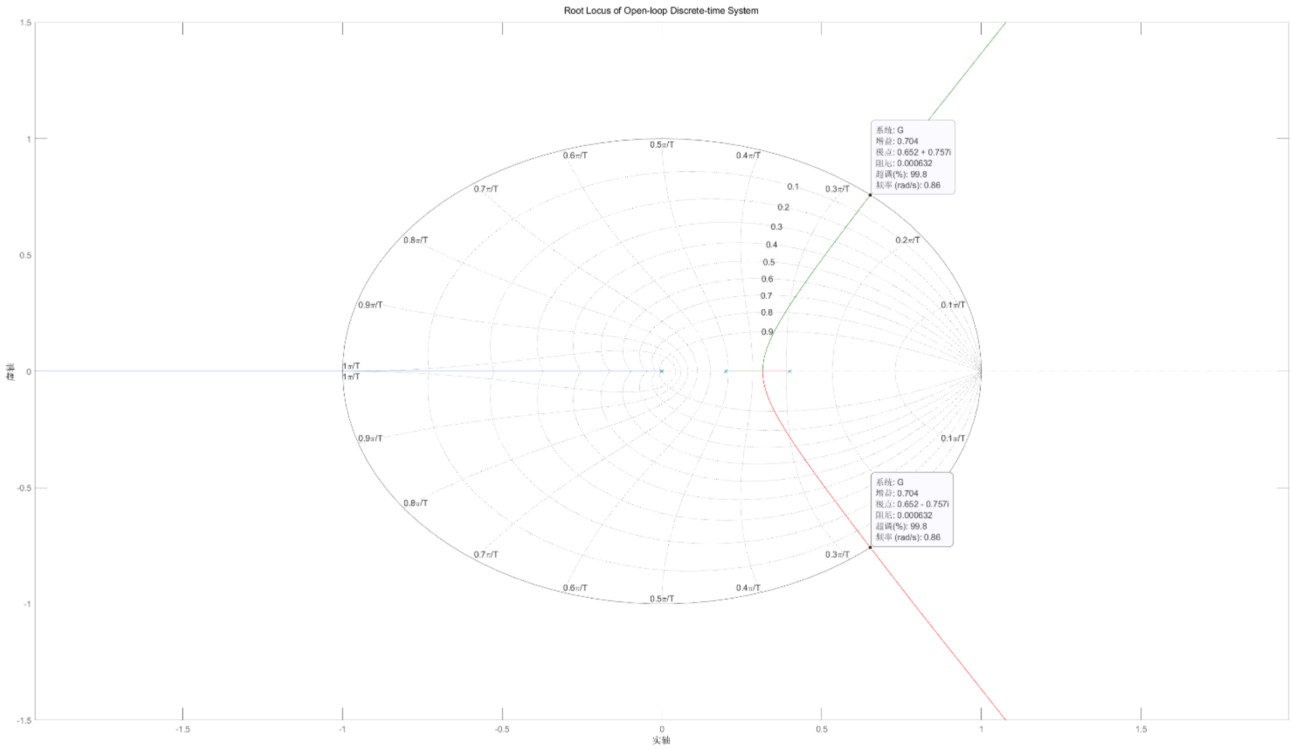
解得 $0 < K < \frac{-3+\sqrt{101}}{10}$

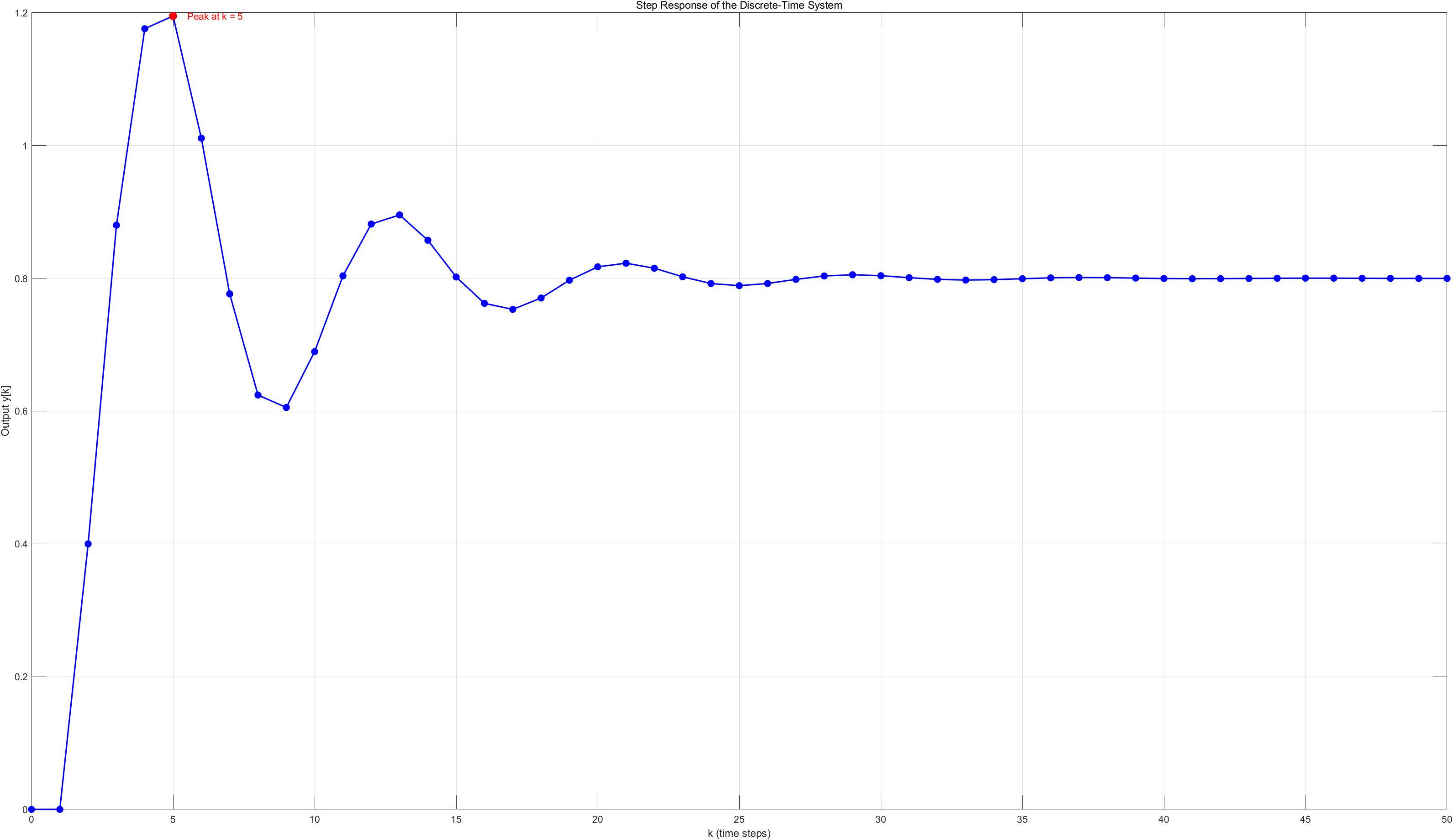
$$\begin{cases} D(1) > 0 \\ D(-1) < 0 \\ 1-K^2 > 0 \\ K^4-2.36K^2-0.096K+0.9936 > 0 \\ 0.08K^3+0.648K^2+0.2864K-0.552 > 0 \\ (K^4-2.36K^2-0.096K+0.9936)^2-(0.08K^3+0.648K^2+0.2864K-0.552)^2 > 0 \end{cases}$$

Problem 1

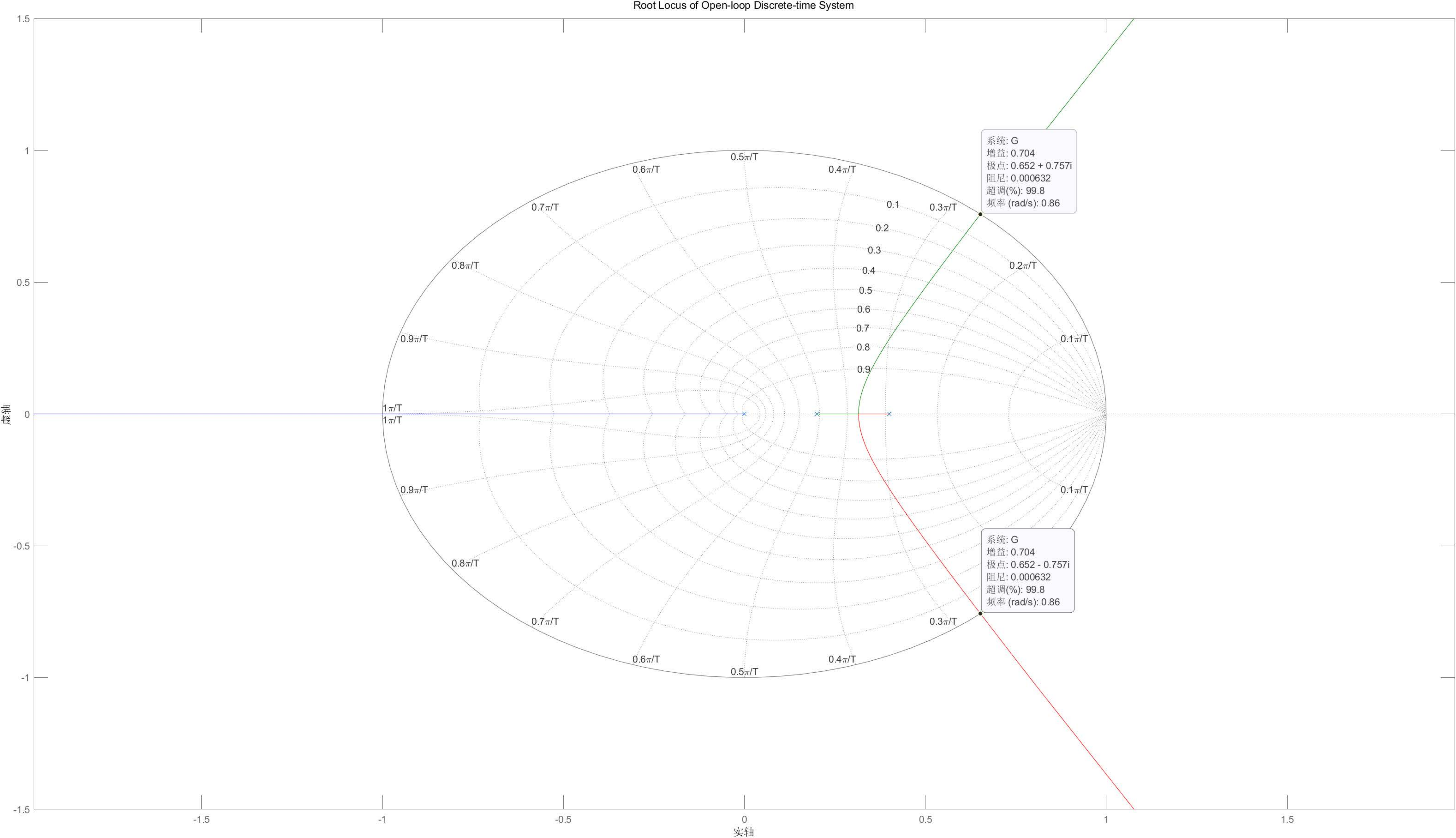


Problem 2(3)





Step Response of the Discrete-Time System
Peak at k = 5
Output y[k]
k (time steps)
0
0.2
0.4
0.6
0.8
1
1.2
0
5
10
15
20
25
30
35
40
45
50


Root Locus of Open-loop Discrete-time System
系统: G
增益: 0.704
极点: 0.652 + 0.757i
阻尼: 0.000632
超调(%): 99.8
频率 (rad/s): 0.86
系统: G
增益: 0.704
极点: 0.652 - 0.757i
阻尼: 0.000632
超调(%): 99.8
频率 (rad/s): 0.86
实轴
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