Problem 1

对于连续线性时不变系统 $\dot{x}(t)=Ax(t)+Bu(t)$，ZOH离散化方法的状态空间表示为 $x(k+1)=Cx(k)+Du(k)$

其中 $C=e^{AT}$ $D=\int_0^T e^{A\tau}B\,d\tau$ $A=\begin{pmatrix}0 & w\\ -w & 0\end{pmatrix}$ $B=\begin{pmatrix}0\\ w\end{pmatrix}$

$$(SI-A)^{-1}=\begin{pmatrix}s & -w\\ w & s\end{pmatrix}^{-1}=\frac{1}{s^2+w^2}\begin{pmatrix}s & w\\ -w & s\end{pmatrix}=\begin{pmatrix}\frac{s}{s^2+w^2} & \frac{w}{s^2+w^2}\\ -\frac{w}{s^2+w^2} & \frac{s}{s^2+w^2}\end{pmatrix}$$

$$\cancel{e^{At}}\ \mathcal{L}^{-1}\left[(SI-A)^{-1}\right]=\begin{bmatrix}\cos(wt) & \sin(wt)\\ -\sin(wt) & \cos(wt)\end{bmatrix}$$

代入 $t=T$

$$C=e^{AT}=\begin{bmatrix}\cos(wT) & \sin(wT)\\ -\sin(wT) & \cos(wT)\end{bmatrix}$$

$$D=\int_0^T e^{A\tau}B\,d\tau=\int_0^T\begin{bmatrix}\cos(w\tau) & \sin(w\tau)\\ -\sin(w\tau) & \cos(w\tau)\end{bmatrix}\begin{bmatrix}0\\ w\end{bmatrix}d\tau$$

$$=\int_0^T\begin{bmatrix}w\sin(w\tau)\\ w\cos(w\tau)\end{bmatrix}d\tau=\begin{bmatrix}1-\cos(wT)\\ \sin(wT)\end{bmatrix}$$

$-\cos(w\tau)\Big|_0^T$

∴ 离散形式表示为

$$x(k+1)=\begin{bmatrix}\cos(wT) & \sin(wT)\\ -\sin(wT) & \cos(wT)\end{bmatrix}x(k)+\begin{bmatrix}1-\cos(wT)\\ \sin(wT)\end{bmatrix}u(k)$$

Problem 2

状态

$G(z)=\frac{z-0.5}{z-1}$ 单位反馈控制器 $C(z)=\frac{k}{z^n-1}\frac{z-1}{z-0.5}$

(a) 闭环传递函数 $T(z)=\frac{C(z)G(z)}{1+C(z)G(z)}=\frac{\frac{k}{z^n-1}}{1+\frac{k}{z^n-1}}=\frac{k}{k+z^n-1}$

(b) $G(z)=\frac{z-0.7}{z-1}$，且 $n=1$ 时，分析稳定性

开环传函

$C(z)G(z)=\frac{k}{z^n-1}\frac{z-0.7}{z-0.5}$

$$T(z)=\frac{k(z-0.7)}{(z^n-1)(z-0.5)+k(z-0.7)}\overset{n=1}{=}\frac{k(z-0.7)}{(z-1)(z-0.5)+k(z-0.7)}$$

$$=\frac{k(z-0.7)}{z^2+(k-1.5)z+(0.5-0.7k)}$$

对于 $z^2+a_1z+a_2=0$，Jury判据 $a_1=k-1.5\quad a_2=0.5-0.7k$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | $a_1$ | $a_2$ | $D(1)>0\quad D(-1)>0$ | $1+k-1.5+0.5-0.7k>0\quad k>0$ |
| 2 | $a_2$ | $a_1$ | 1 | $\lvert a_2\rvert<1$ | $1-k+1.5+0.5-0.7k>0\quad k<\frac{30}{17}$ |
| 3 | $1-a_2^2$ | $a_1-a_1a_2$ | | | $\lvert 0.5-0.7k\rvert<1$ |
| 4 | $a_1-a_1a_2$ | $1-a_2^2$ | | | $-\frac{5}{7}<k<\frac{15}{7}$ |
| 5 | $(1-a_2^2)^2-(a_1-a_1a_2)^2$ | | | $(1+a_2-a_1)(1+a_2+a_1)>0$ | |

$\therefore 0<k<\frac{30}{17}$

Problem 3

$$C(s) = \frac{as+1}{s+1}$$

(a) Tustin变换 $s = \frac{2}{T}\frac{z-1}{z+1}$

$$C(z) = \frac{a \cdot \frac{2}{T}\frac{z-1}{z+1} + 1}{\frac{2}{T}\frac{z-1}{z+1} + 1} = \frac{(2a+T)z - 2a + T}{(2+T)z - 2 + T}$$

(b) 阶跃不变法

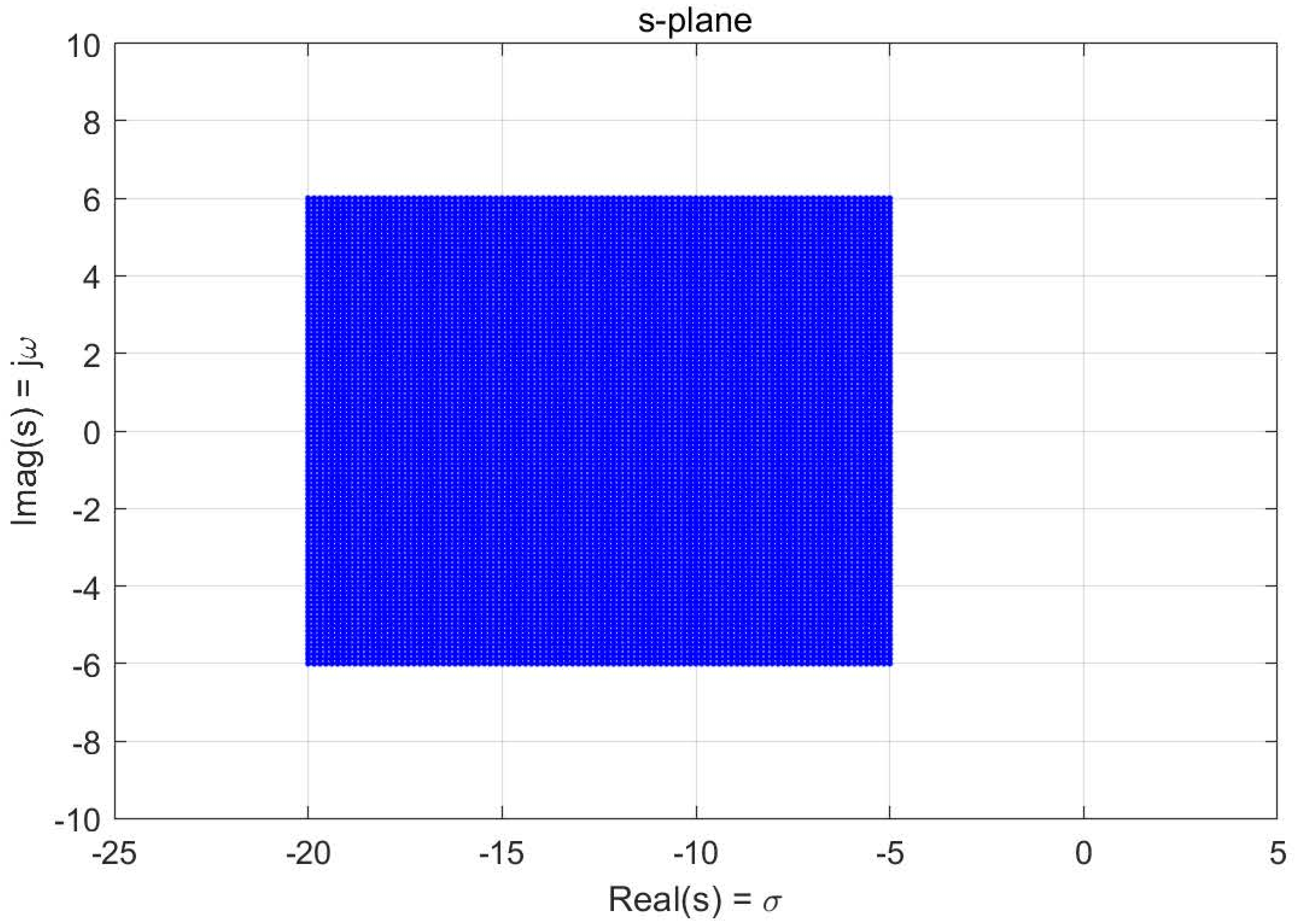
$$\frac{C(s)}{s} = \frac{as+1}{s(s+1)} = \frac{1}{s} + \frac{a-1}{s+1}$$

$$h(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{a-1}{s+1}\right\} = (1 + (a-1)e^{-t})\,u(t)$$

对其采样 $h(kT) = 1 + (a-1)e^{-kT}$, $k = 0, 1, 2, \cdots$

Z变换得 $H(z) = \frac{z}{z-1} + (a-1)\frac{z}{z-e^{-T}}$

则 $C(z) = \frac{z-1}{z} H(z) = 1 + (a-1)\frac{z-1}{z-e^{-T}}$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8
6
4
2
0
-2
-4
-6
-8
-10
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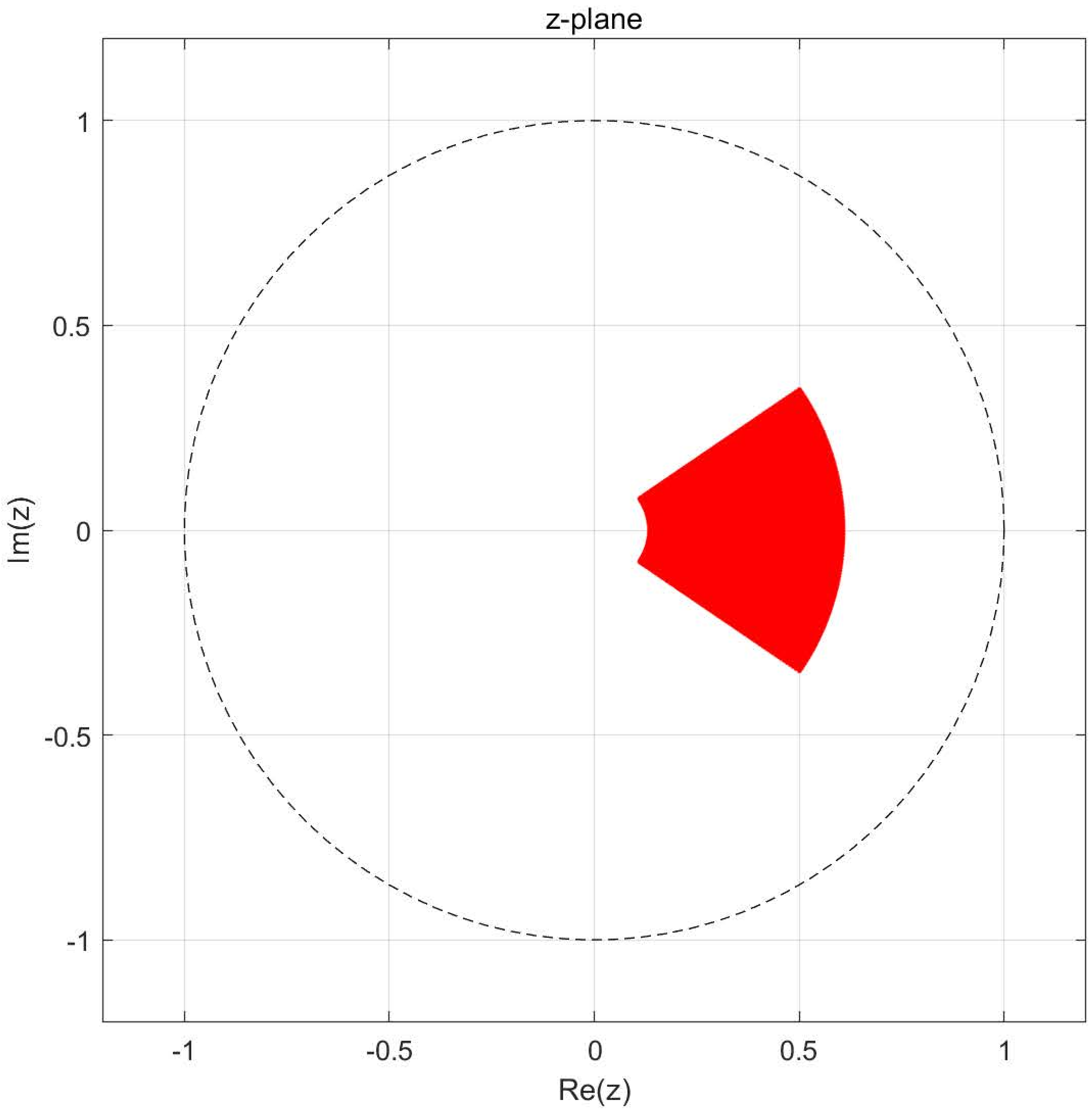
-20
-15
-10
-5
0
5
Real(s) = σ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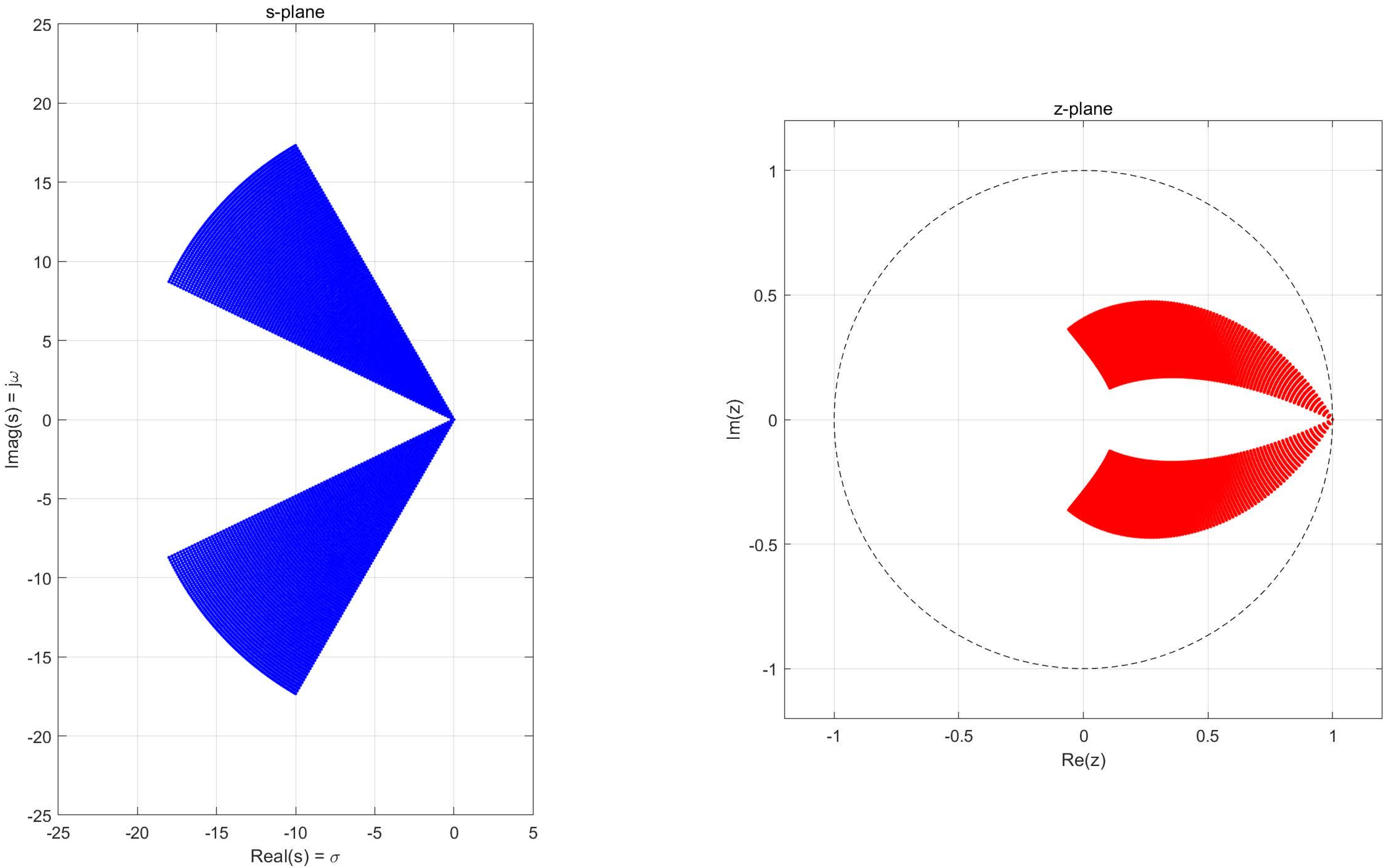
0.5
0
-0.5
-1
Im(z)
-1
-0.5
0
0.5
1
Re(z)


s-plane
Imag(s) = jω
Real(s) = σ
25
20
15
10
5
0
-5
-10
-15
-20
-25
-25
-20
-15
-10
-5
0
5
z-plane
Im(z)
Re(z)
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1